

GISG 1990

RITC Presentation

### Stochastic Method

Full details of this method are due to be published in the December 1990 JIA in a paper entitled "A Stochastic Method for Claims Reserving in General Insurance" by Tom Wright.

The method is based on a general underlying model of the claims process. Risk theory provides a basis for both the random and systematic components of the run off. The method is free from dependence on any particular form of error structure (for example Log Normal). The model is fitted by Fishers scoring method (this is the method used in the GLIM package) which has the advantage that it is fitted directly to the original data without transformation (for example taking logs). Thus negative incremental data do not present a problem. The variation in development pattern across years of origin is estimated from the data and modelled by use of the Kalman filter.

The method is able to project the run off beyond the observed range of the data and, where available, may be fitted to data whose development period is less than annual (for example quarterly). The model is also able to model the non uniform build up of exposure during early development periods.

Standard Errors of reserves are calculated, both for individual years of origin and for the overall reserve (or RITC). The model only requires data that is readily available in a RITC exercise, namely paid and (if available) incurred claims triangulations. Use of exposure information, for example premiums, numbers of claims, numbers of risks etc. can also be used. This often increases the reliability of the estimated reserves, particularly for the more recent underwriting years.

The following pages contain tables and graphs showing the result of modelling the example paid and incurred data.

Marine Liability - Sterling £000's

Year	Premium to Year Three	Paid to 31.12.89	Incurred to 31.12.89	Outstanding at 31.12.89	Estimated Ultimate	Estimated ULR
1975	134	159	159	-	159	118.5%
1976	169	251	251	-	252	149.1%
1977	188	276	323	47	305	162.5%
1978	145	188	208	20	201	138.6%
1979	177	180	180	=	183	103.2%
1980	215	138	148	10	148	68.9%
Subtotal	1,028	1,192	1,269	77	1,248	121.4%
1981	247	217	250	33	244	98.6%
1982	226	316	388	72	377	167.0%
1983	252	203	241	38	257	102.0%
1984	346	117	205	89	196	56.6%
1985	357	191	326	135	279	78.3%
1986	518	93	147	54	293	56.5%
1987	612	49	90	42	238	38.9%
Total	3,586	2,378	2,916	540	3,132	87.3%

Year	Estimated IBNR	Estimated RITC	Prudent Ultimate	Prudent IBNR	Prudent RITC	Prudent ULR
1975	0	0	161	3	3	120.2%
1976	1	1	256	5	5	151.4%
1977	(18)	29	312	(11)	36	166.1%
1978	(7)	13	209	2	22	144.4%
1979	2	2	192	11	11	108.3%
1980	(0)	10	162	14	24	75.4%
Subtotal	(22)	55	1,292	24	101	125.7%
1981	(6)	27	266	17	50	107.8%
1982	(11)	61	415	27	99	183.8%
1983	16	55	300	59	97	118.7%
1984	(9)	79	243	38	127	70.3%
1985	(46)	89	326	1	135	91.4%
1986	146	200	387	241	295	74.7%
1987	147	189	330	239	281	53.8%
Total	215	755	3,559	646	1,185	99.2%

## **Analysis of Paid Data**

BACON & WOODROW STOCHASTIC MODEL 13:21 20-09-1990

**Marine Liability - sterling £000's (Paid Claims)**

**INCREMENTAL DATA FROM FILE**

Year of Origin	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1975	4	25	47	24	11	8	40	-23	34	-13	0	0	0	0	0
1976	7	16	27	21	117	6	19	-8	2	24	15	3	1	0	
1977	8	11	44	28	29	55	4	72	-25	24	1	15	10		
1978	3	16	17	19	6	6	73	7	14	23	-4	6			
1979	32	9	19	23	38	31	15	-5	15	0	2				
1980	5	26	18	32	22	4	0	23	5	2					
1981	52	24	39	25	25	18	18	16	0						
1982	17	32	46	40	78	27	64	13							
1983	83	19	33	25	15	14	14								
1984	20	22	19	18	25	13									
1985	27	62	46	45	12										
1986	6	45	22	20											
1987	8	11	30												
1988	6	16													
1989	4														

BACON & WOODROW STOCHASTIC MODEL 13:21 20-09-1990

**Marine Liability - sterling £000's (Paid Claims)**

**FORCE OF CLAIMS INFLATION - FORCE OF INTEREST**

1990	0.085	1999	0.085	2008	0.085
1991	0.085	2000	0.085	2009	0.085
1992	0.085	2001	0.085	2010	0.085
1993	0.085	2002	0.085	Onwards	
1994	0.085	2003	0.085		
1995	0.085	2004	0.085		
1996	0.085	2005	0.085		
1997	0.085	2006	0.085		

**RESULTS IN CURRENT PRICES (To The End Of 24 Development Periods)**

	Reserve	St.Error	Paid-So-Far	Ultimate
1975	2.73	11.29	158.58	161.32
1976	3.81	13.43	251.25	255.06
1977	7.05	18.69	276.37	283.42
1978	8.03	19.82	187.83	195.86
1979	7.05	18.29	180.36	187.41
1980	13.16	25.43	138.22	151.38
1981	27.74	38.04	216.59	244.33
1982	61.27	59.06	315.97	377.24
1983	67.57	60.89	202.73	270.30
1984	76.21	63.95	116.61	192.83
1985	69.65	60.12	190.90	260.55
1986	228.22	117.87	92.75	320.97
1987	203.81	109.73	48.79	252.60
1988	0.00	0.00	21.61	21.61
1989	0.00	0.00	4.43	4.43
Totals	776.31	297.00	2,403.02	3,179.32

BACON & WOODROW STOCHASTIC MODEL 13:21 20-09-1990

**Marine Liability - Sterling £000's (Paid Claims)**

RESULTS BY PERIOD OF PAYMENT IN CURRENT PRICES

Period Of Payment	Reserve	Standard Error
1990	190	30
1991	153	28
1992	119	26
1993	90	24
1994	67	22
1995	48	19
1996	35	16
1997	25	14
1998	17	11
1999	12	9
2000	8	7
2001	5	5
2002	3	4
2003	2	3
2004	1	2
2005	1	1
2006	1	1
2007	0	1
2008	0	0
2009	0	0
2010	0	0
Grand Tot	776	297

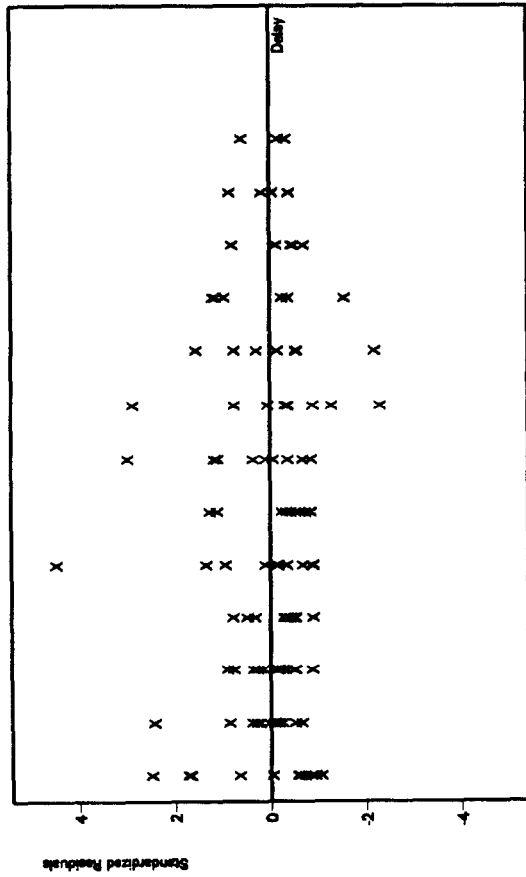
**BACON & WOODROW STOCHASTIC MODEL      13:21    20-09-1990**  
**Marine Liability - Sterling £000's      (Paid Claims)**  
**PROJECTED INCREMENTAL DATA (CURRENT PRICES)**

Year Of Origin	Development Period										19	20	21							
	1	2	3	4	5	6	7	8	9	10				11	12	13	14	15	16	17
1975 Observed: Estimate: St.Error:	4	25	47	24	11	8	40	-23	34	-13	0	0	0	N/A	N/A	-	-	-	-	-
	20	20	32	33	29	24	19	15	11	8	6	4	3	2	1	1	1	0	0	0
	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	6	5	4	4	3
1976 Observed: Estimate: St.Error:	7	16	27	21	117	6	19	-8	2	24	15	3	1	N/A	-	-	-	-	-	-
	18	18	29	30	26	22	18	14	10	8	6	4	3	2	1	1	1	0	0	0
	-	-	-	-	-	-	-	-	-	-	-	-	-	-	7	6	5	4	4	3
1977 Observed: Estimate: St.Error:	8	11	44	28	29	55	4	72	-25	24	1	15	10	-	-	-	-	-	-	-
	22	22	36	37	33	27	22	17	13	9	7	5	3	2	2	1	1	0	0	0
	-	-	-	-	-	-	-	-	-	-	-	-	-	9	8	7	6	5	4	3
1978 Observed: Estimate: St.Error:	3	16	17	19	6	6	73	7	14	23	-4	6	-	-	-	-	-	-	-	-
	17	17	28	28	25	21	17	13	10	7	5	4	3	2	1	1	1	0	0	0
	-	-	-	-	-	-	-	-	-	-	-	-	10	9	7	6	5	5	4	3
1979 Observed: Estimate: St.Error:	32	9	19	23	38	31	15	-5	15	0	2	-	-	-	-	-	-	-	-	-
	10	10	17	17	15	13	10	8	6	4	3	2	2	1	1	1	0	0	0	0
	-	-	-	-	-	-	-	-	-	-	-	9	8	7	6	5	4	4	3	3
1980 Observed: Estimate: St.Error:	5	26	18	32	22	4	0	23	5	2	-	-	-	-	-	-	-	-	-	-
	13	13	22	22	19	16	13	10	8	6	4	3	2	1	1	1	0	0	0	0
	-	-	-	-	-	-	-	-	-	-	12	11	9	8	7	6	5	4	4	3
1981 Observed: Estimate: St.Error:	52	24	39	25	25	18	18	16	0	-	-	-	-	-	-	-	-	-	-	-
	20	20	32	33	29	24	19	15	11	8	6	4	3	2	1	1	1	0	0	0
	-	-	-	-	-	-	-	-	-	18	16	14	12	10	9	8	7	6	5	4
1982 Observed: Estimate: St.Error:	17	32	46	40	78	27	64	13	-	-	-	-	-	-	-	-	-	-	-	-
	31	31	51	52	46	38	30	23	18	13	9	7	5	3	2	2	1	1	1	0
	-	-	-	-	-	-	-	-	26	23	21	18	16	14	12	10	9	7	6	5
1983 Observed: Estimate: St.Error:	83	19	33	25	15	14	14	-	-	-	-	-	-	-	-	-	-	-	-	-
	25	25	41	41	36	30	24	19	14	10	8	5	4	3	2	1	1	1	0	0
	-	-	-	-	-	-	-	26	24	21	19	17	15	13	11	9	8	7	6	5
1984 Observed: Estimate: St.Error:	20	22	19	18	25	13	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	21	21	34	34	30	25	20	15	12	9	6	4	3	2	1	1	1	0	0	0
	-	-	-	-	-	-	27	25	23	20	18	16	14	12	10	9	8	6	5	4
1985 Observed: Estimate: St.Error:	27	62	46	45	12	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	14	14	23	24	21	17	14	11	8	6	4	3	2	1	1	1	0	0	0	0
	-	-	-	-	-	25	24	22	20	18	16	14	12	10	9	8	7	6	5	4
1986 Observed: Estimate: St.Error:	6	45	22	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	36	36	59	59	53	44	35	27	20	15	11	8	5	4	3	2	1	1	0	0
	-	-	-	-	45	42	39	36	33	29	26	23	20	17	15	13	11	9	8	6
1987 Observed:	8	11	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

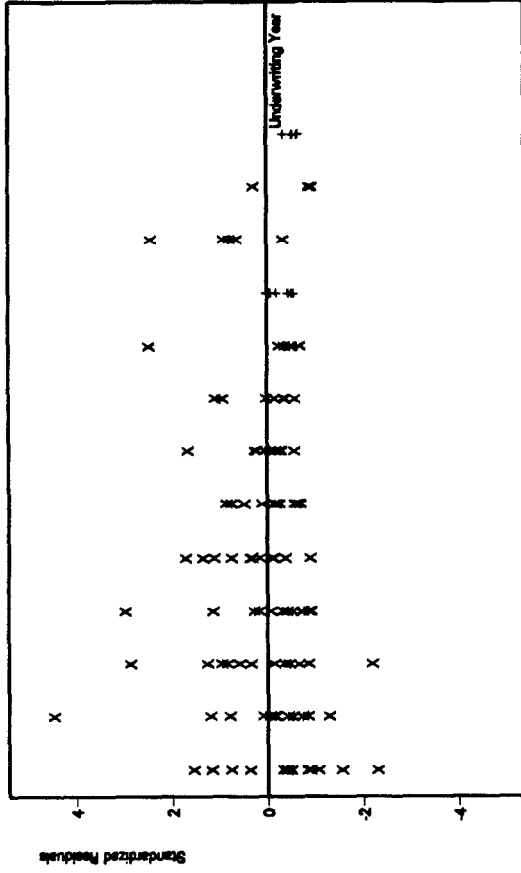
Year Of Origin	Development Period			Reserve
	22	23	24	
1975 Observed:	-	-	-	-
Estimate:	0	0	0	3
St.Error:	2	2	1	11
1976 Observed:	-	-	-	-
Estimate:	0	0	0	4
St.Error:	2	2	1	13
1977 Observed:	-	-	-	-
Estimate:	0	0	0	7
St.Error:	2	2	2	19
1978 Observed:	-	-	-	-
Estimate:	0	0	0	8
St.Error:	2	2	1	20
1979 Observed:	-	-	-	-
Estimate:	0	0	0	7
St.Error:	2	1	1	18
1980 Observed:	-	-	-	-
Estimate:	0	0	0	13
St.Error:	2	2	1	25
1981 Observed:	-	-	-	-
Estimate:	0	0	0	28
St.Error:	3	2	2	38
1982 Observed:	-	-	-	-
Estimate:	0	0	0	61
St.Error:	3	3	2	59
1983 Observed:	-	-	-	-
Estimate:	0	0	0	68
St.Error:	3	3	2	61
1984 Observed:	-	-	-	-
Estimate:	0	0	0	76
St.Error:	3	2	2	64
1985 Observed:	-	-	-	-
Estimate:	0	0	0	70
St.Error:	3	2	2	60
1986 Observed:	-	-	-	-
Estimate:	0	0	0	228
St.Error:	4	4	3	118
1987 Observed:	-	-	-	-
Estimate:	0	0	0	204
St.Error:	4	3	3	110
1988 Observed:	-	-	-	-
Estimate:	0	0	0	0



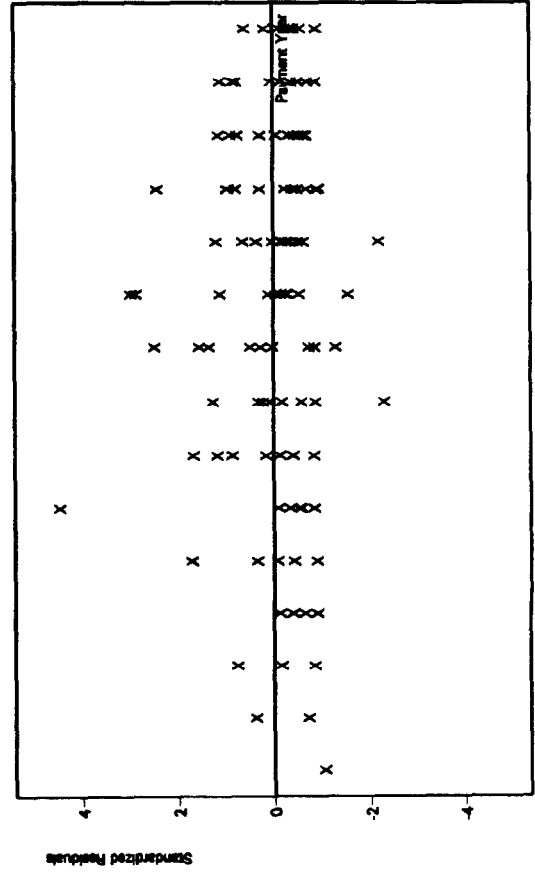
Plot Of Residuals Against Delay



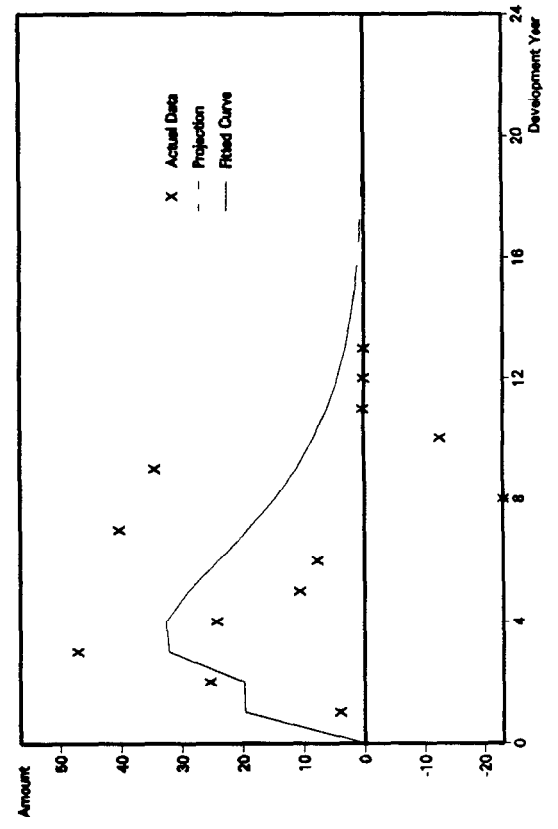
Plot Of Residuals Against Underwriting Year



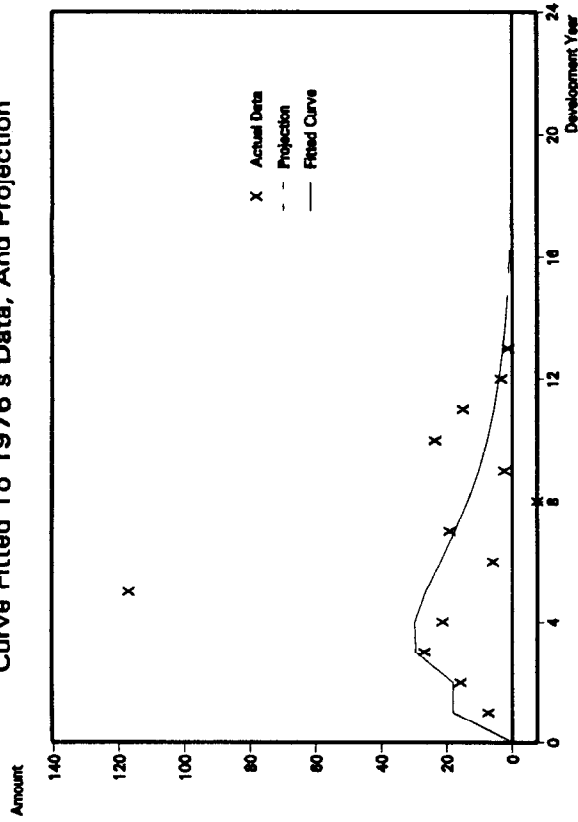
Plot Of Residuals Against Payment Year



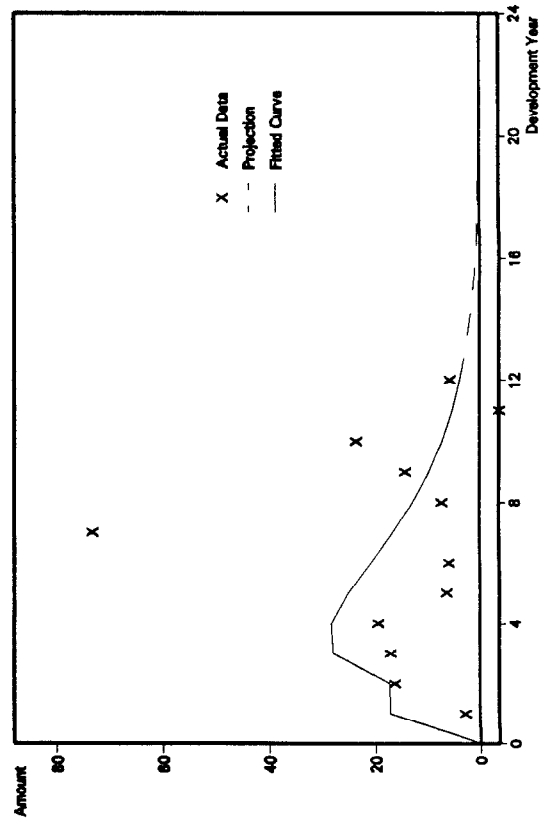
Marine Liability - Sterling 000's  
Curve Fitted To 1975's Data, And Projection



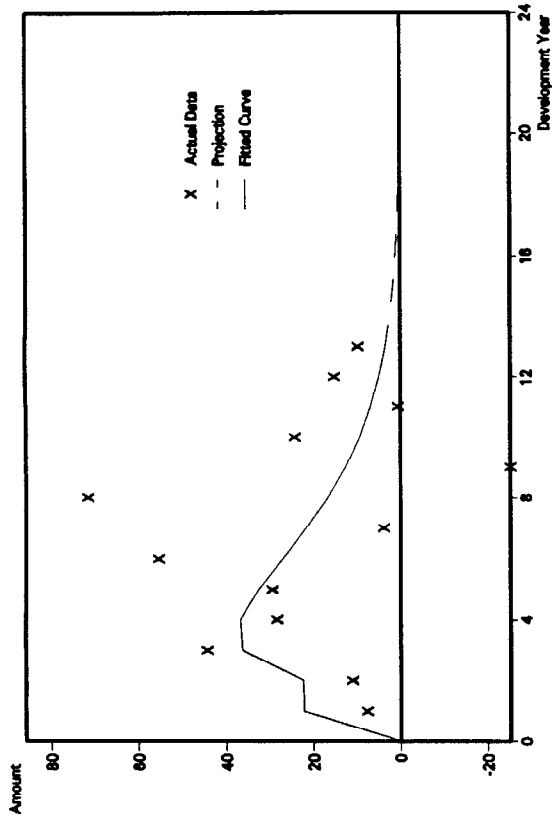
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Curve Fitted To 1976's Data, And Projection



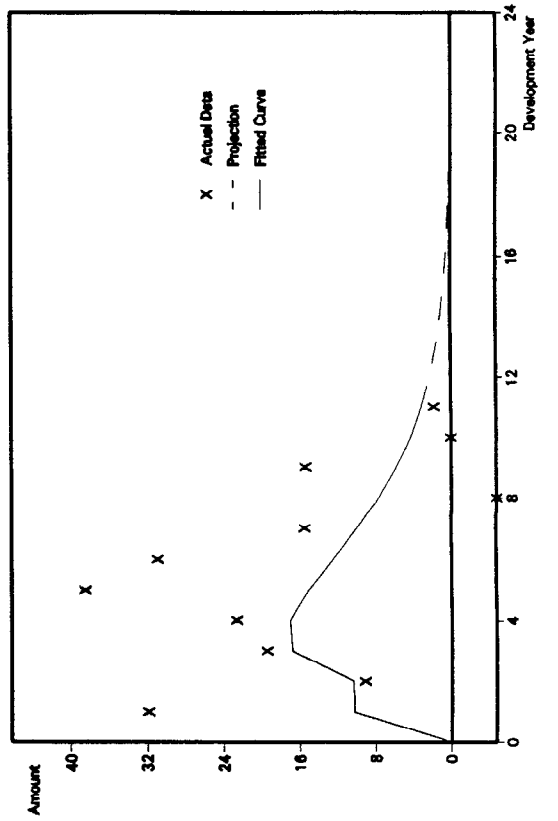
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Curve Fitted To 1978's Data, And Projection



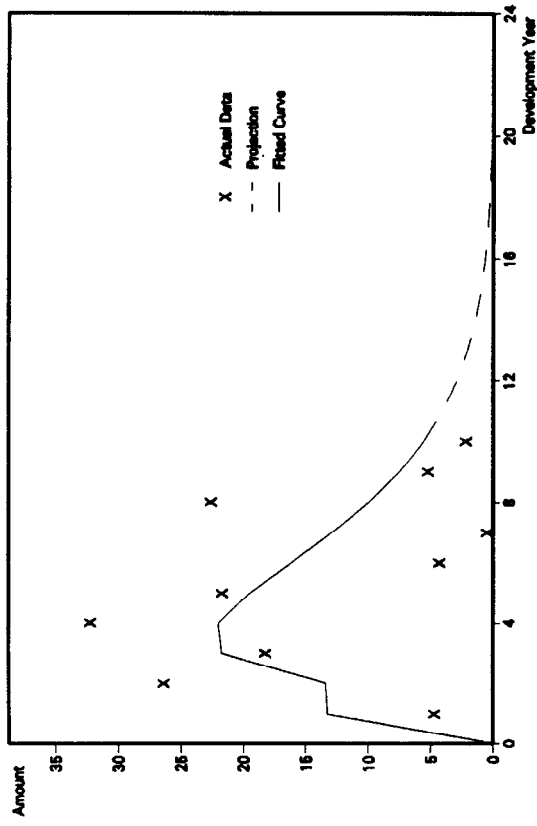
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Curve Fitted To 1977's Data, And Projection



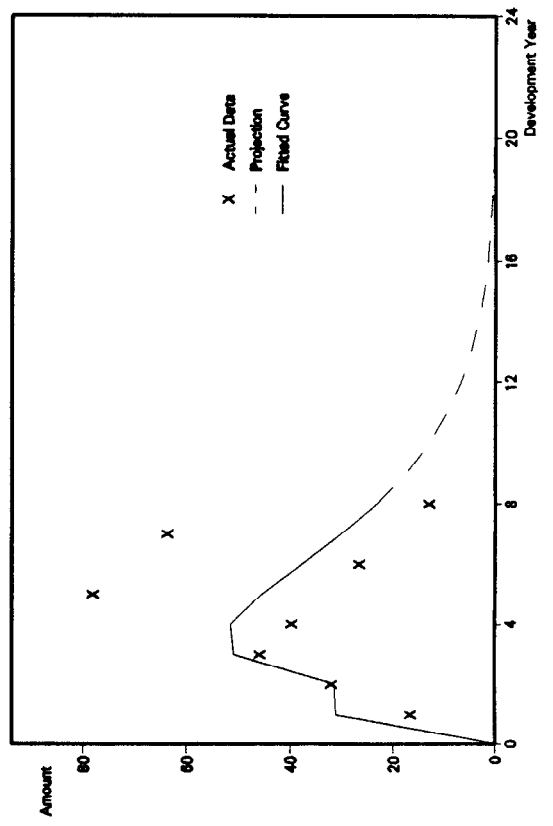
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Curve Fitted To 1979's Data, And Projection



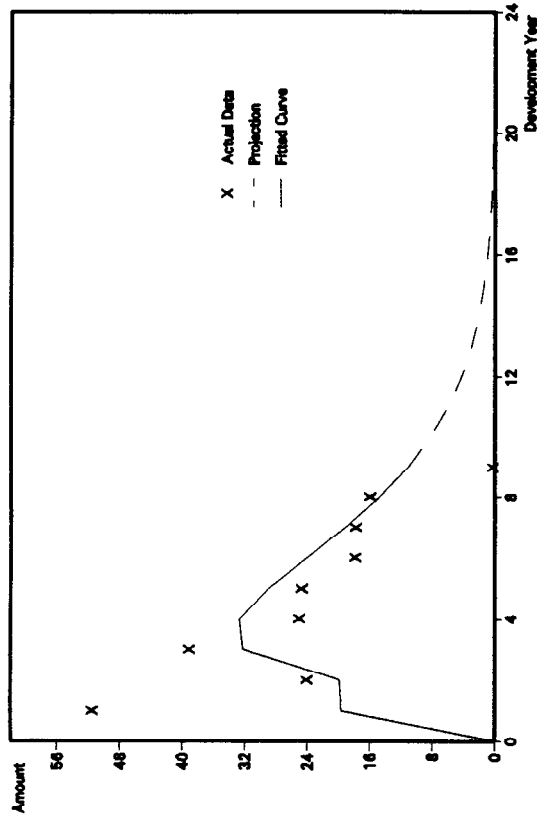
Marine Liability - Sterling 000's  
Curve Fitted To 1980's Data, And Projection



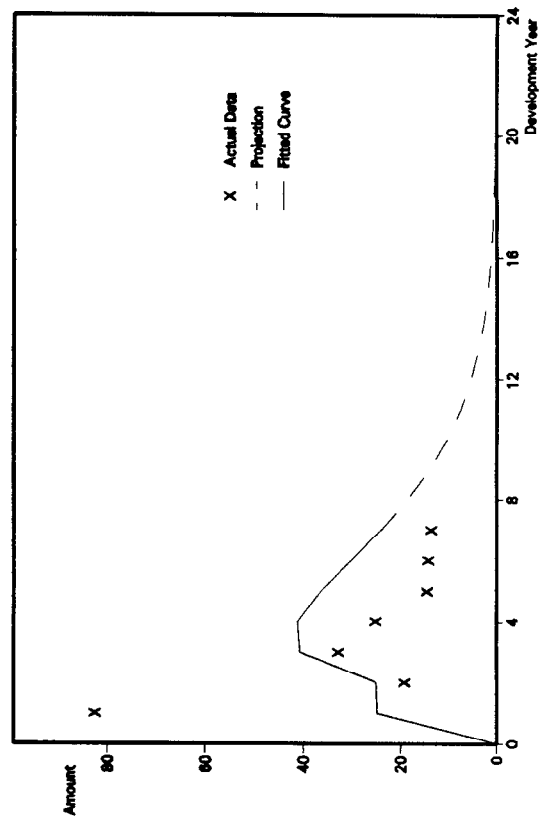
Marine Liability - Sterling 000's  
Curve Fitted To 1982's Data, And Projection



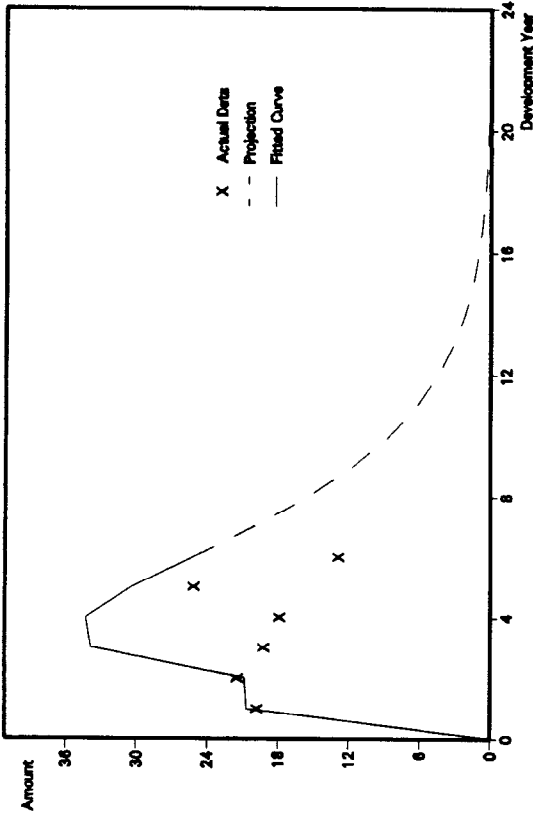
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Curve Fitted To 1981's Data, And Projection



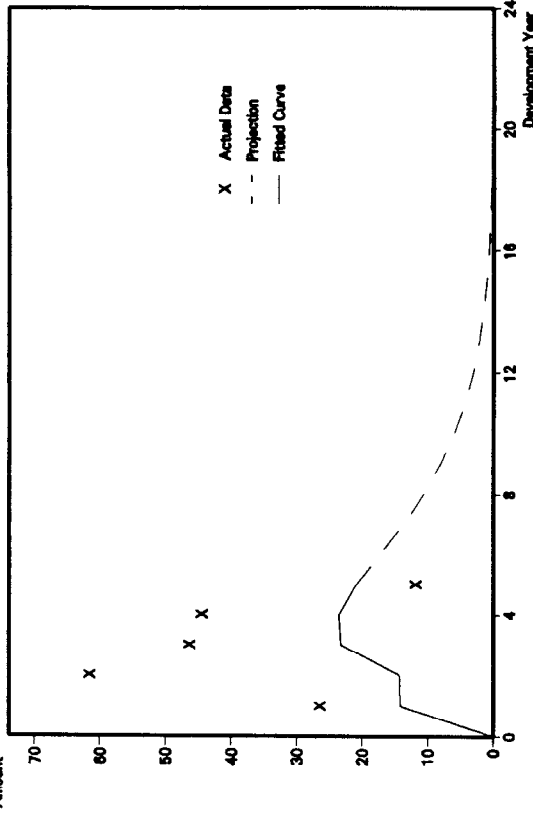
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Curve Fitted To 1983's Data, And Projection



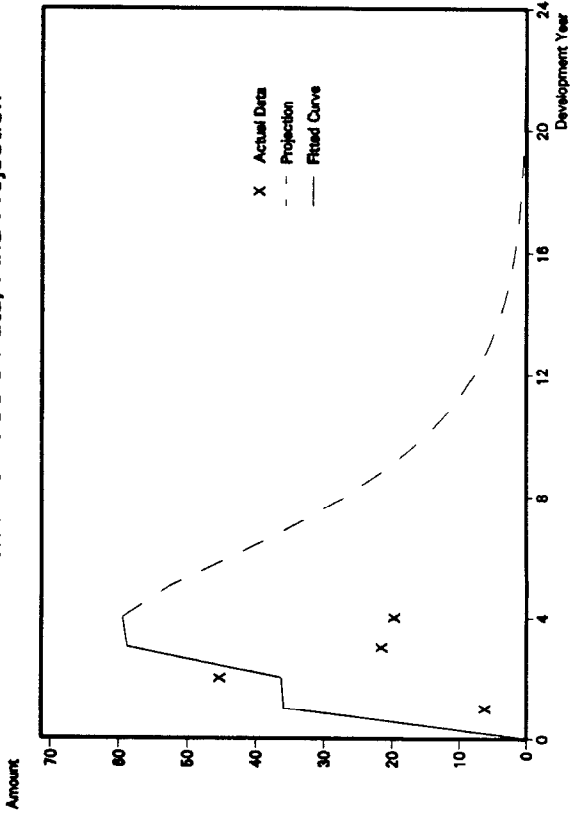
Marine Liability - Sterling 000's  
Curve Fitted To 1984's Data, And Projection



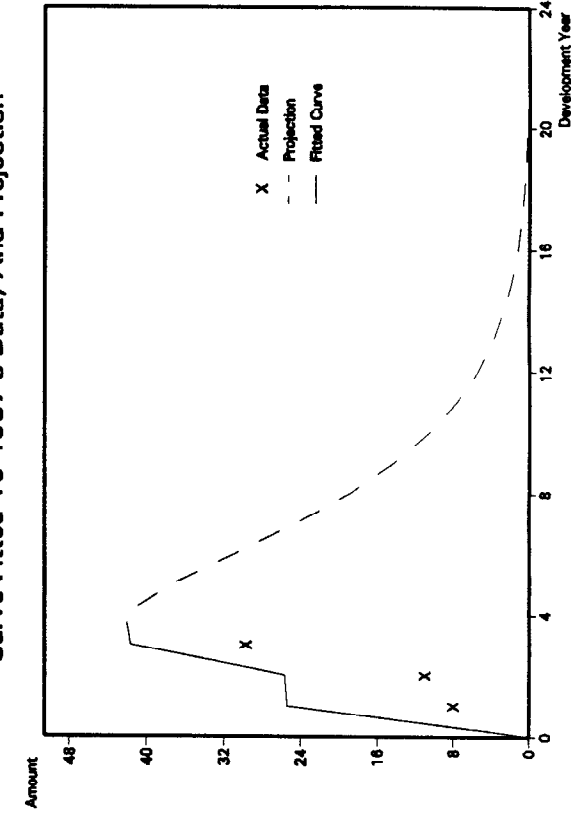
Marine Liability - Sterling 000's  
Curve Fitted To 1985's Data, And Projection



Marine Liability - Sterling 000's  
Curve Fitted To 1986's Data, And Projection



Marine Liability - Sterling 000's  
Curve Fitted To 1987's Data, And Projection



## **Analysis of Incurred**

BACON & WOODROW STOCHASTIC MODEL 14:36 20-09-1990

**Marine Liability - Sterling £000's (Incurred Claims)**

**INCREMENTAL DATA FROM FILE**

Year of Origin	1	2	Development Period				7	8	9	10	11	12	13	14	15
			3	4	5	6									
1975			107	64	10	-42	101	-22	4	-33	0	-31	0	0	0
1976			66	51	109	36	19	-49	-4	41	-10	3	-11	0	
1977			109	13	147	-8	-10	73	-15	2	-1	1	13		
1978			214	28	-28	58	90	-102	-42	-9	-7	6			
1979			221	-40	2	5	-9	5	-6	0	2				
1980			89	71	34	-37	-31	26	5	-10					
1981			214	-26	26	23	36	11	-35						
1982			145	108	94	-26	52	14							
1983			262	23	7	-39	-11								
1984			156	50	37	-38									
1985			283	38	5										
1986			159	-13											
1987			90												
1988															
1989															

BACON & WOODROW STOCHASTIC MODEL 14:36 20-09-1990

**Marine Liability - Sterling £000's (Incurred Claims)**

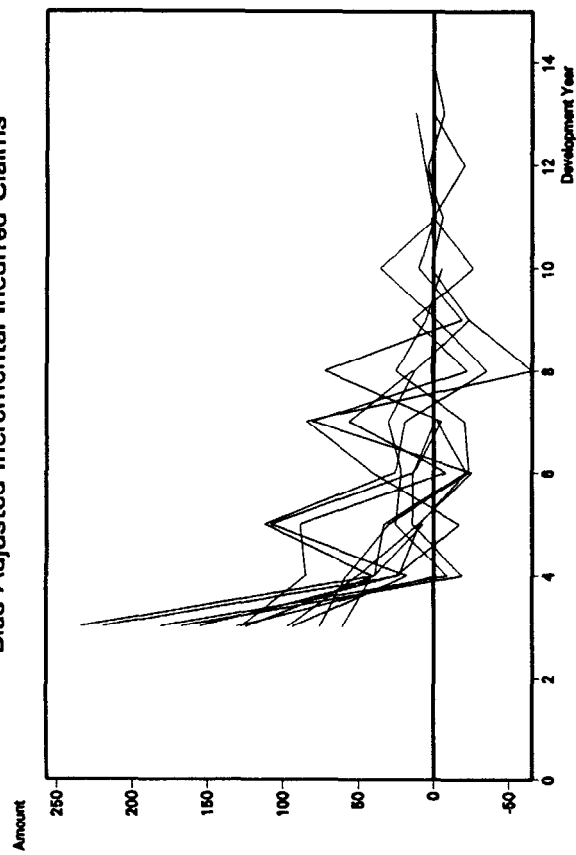
**FORCE OF CLAIMS INFLATION - FORCE OF INTEREST**

1990	0.082	1999	0.082
1991	0.082	2000	0.082
1992	0.082	2001	0.082
1993	0.082	2002	0.082
1994	0.082	Onwards	
1995	0.082		
1996	0.082		
1997	0.082		

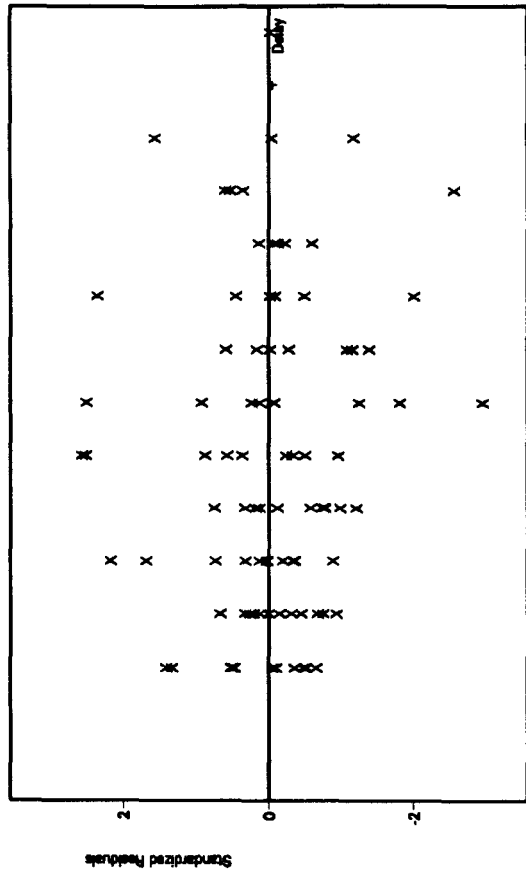
**RESULTS IN CURRENT PRICES (To The End Of 16 Development Periods)**

	Reserve	St. Error	Paid-So-Far	Ultimate
1975	0.03	2.41	158.58	158.62
1976	0.10	4.14	251.25	251.35
1977	31.67	7.22	276.37	308.04
1978	13.77	9.37	187.83	201.60
1979	0.57	10.39	180.36	180.93
1980	8.04	16.70	138.22	146.27
1981	26.06	28.37	216.59	242.66
1982	59.51	49.35	315.97	375.48
1983	42.11	58.68	202.73	244.84
1984	82.83	70.21	116.61	199.45
1985	117.37	74.87	190.90	308.27
1986	148.94	157.73	92.75	241.69
1987	155.04	165.95	48.79	203.83
1988	0.00	99,999.99	21.61	21.61
1989	0.00	99,999.99	4.43	4.43
Totals	686.04	N/A	2,403.02	3,089.06

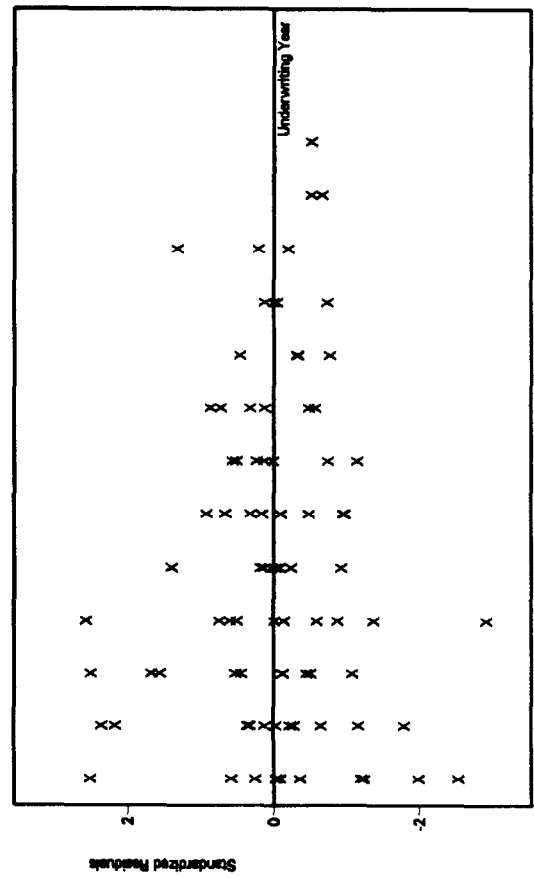
Marine Liability - Sterling 000's  
Bias Adjusted Incremental Incurred Claims



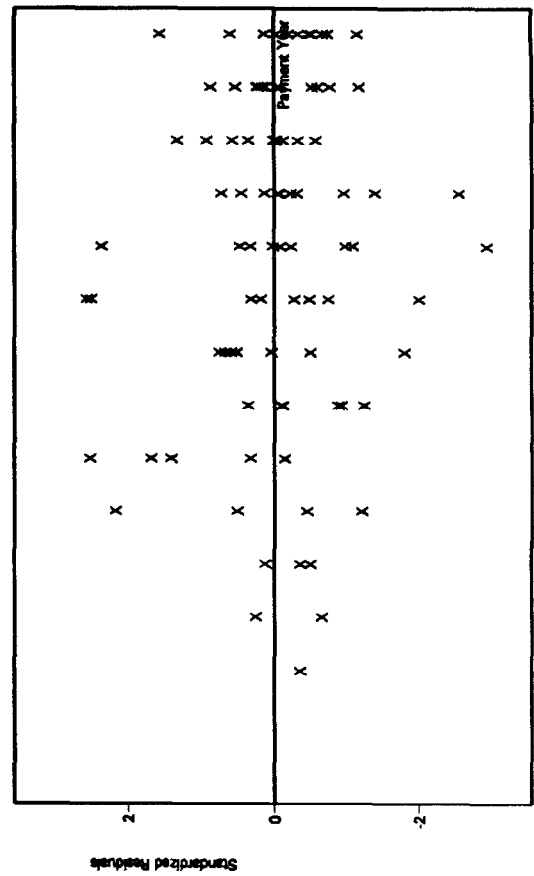
Plot Of Residuals Against Delay



Plot Of Residuals Against Underwriting Year

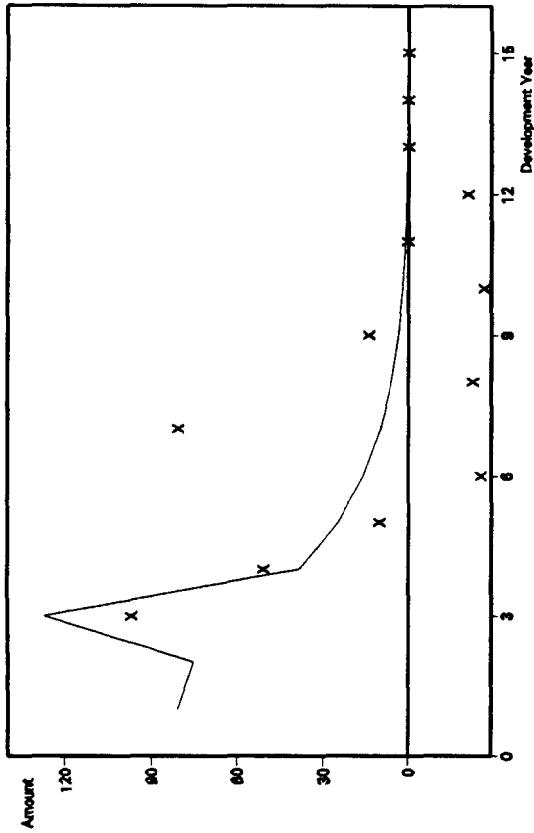


Plot Of Residuals Against Payment Year

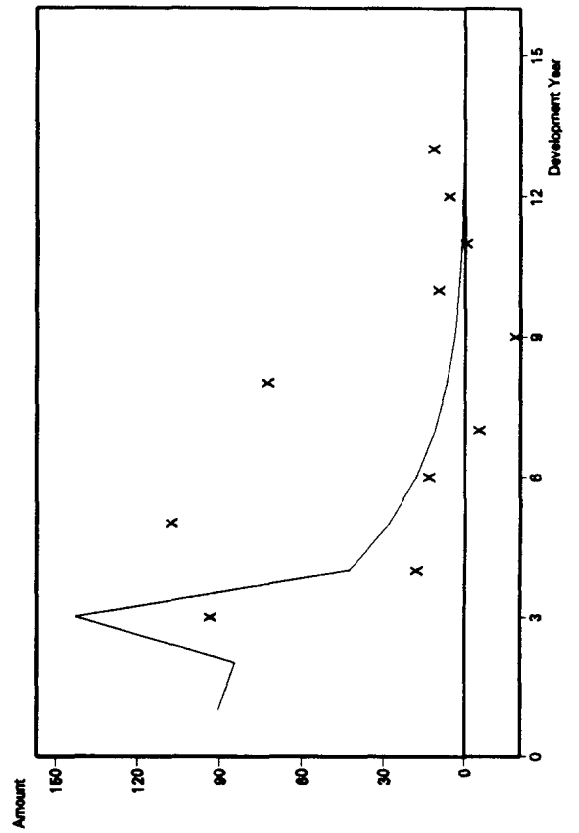




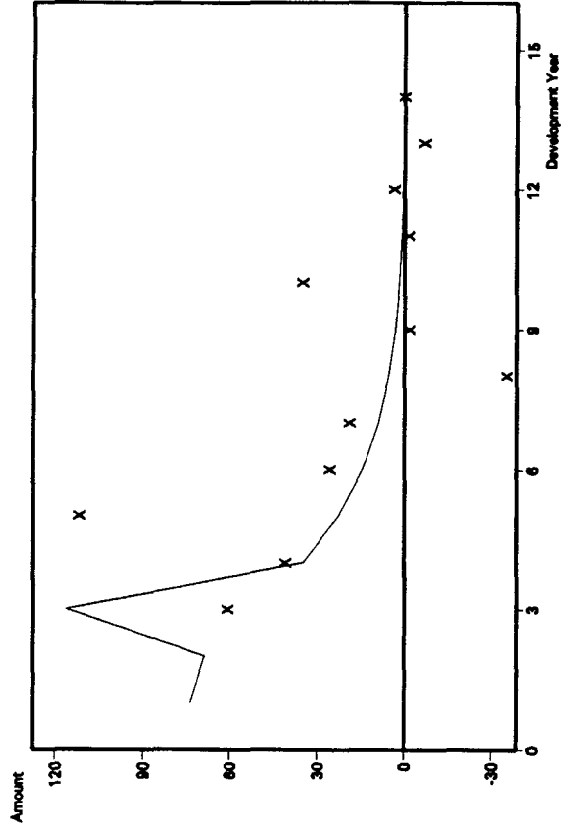
Marine Liability - Sterling 000's  
Curve Fitted To 1975's Data



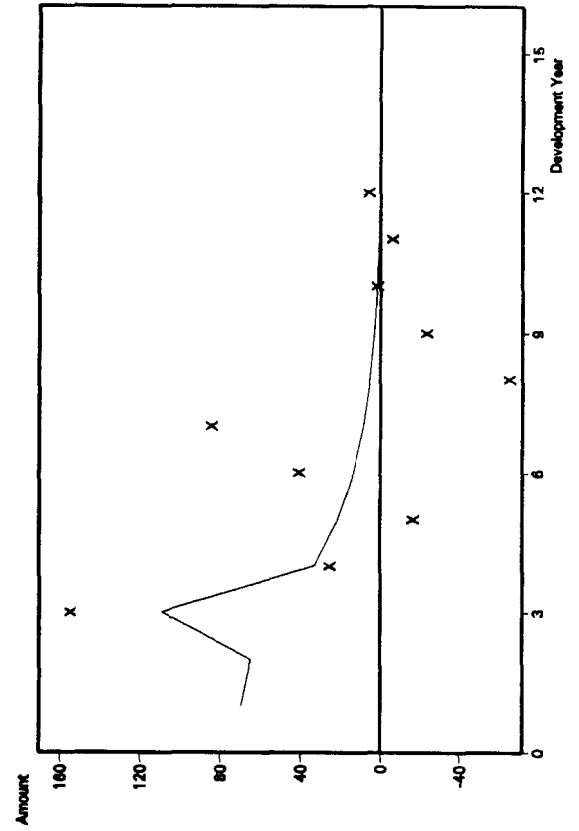
Marine Liability - Sterling 000's  
Curve Fitted To 1977's Data



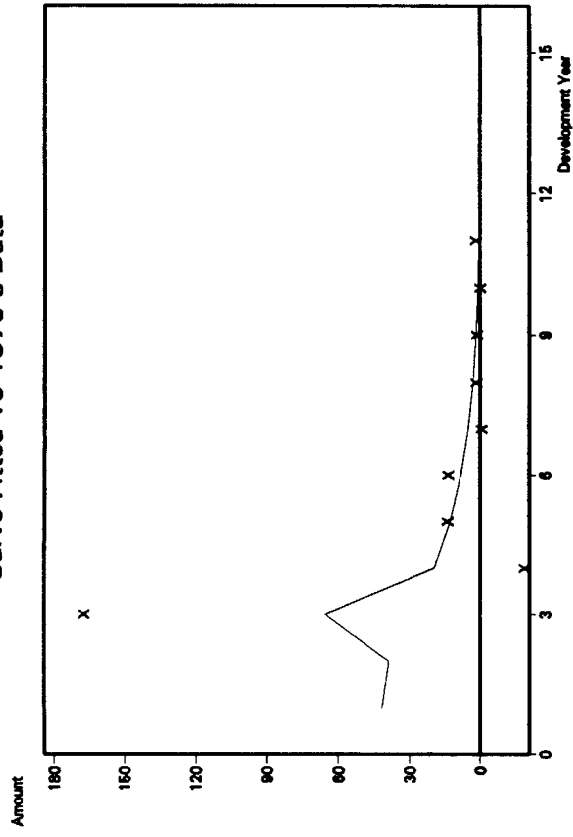
Marine Liability - Sterling 000's  
Curve Fitted To 1976's Data



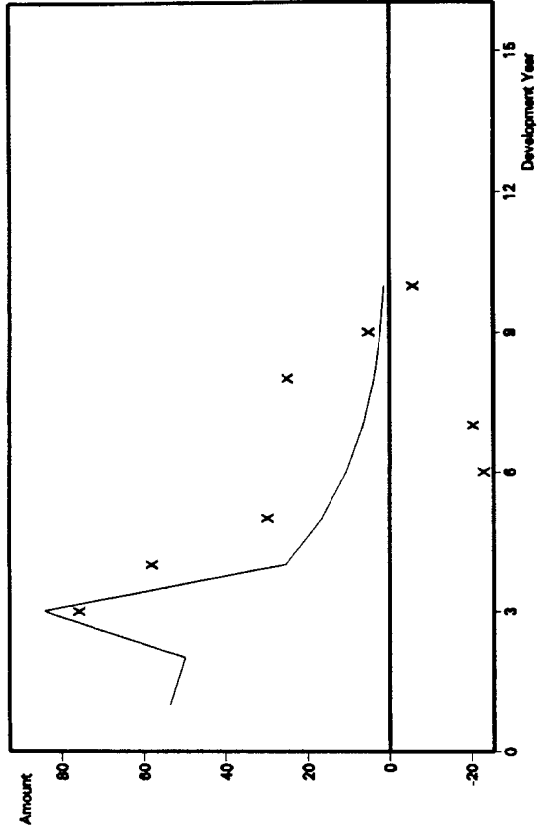
Marine Liability - Sterling 000's  
Curve Fitted To 1978's Data



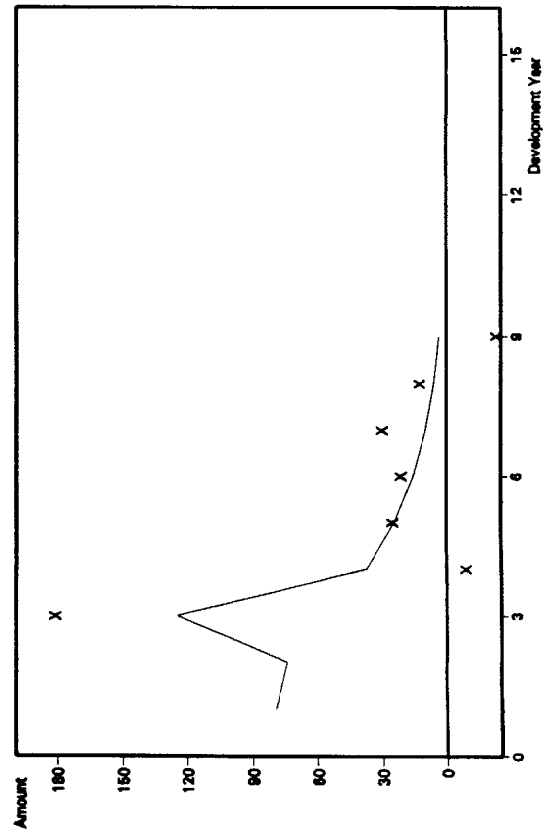
Marine Liability - Sterling 000's  
Curve Fitted To 1979's Data



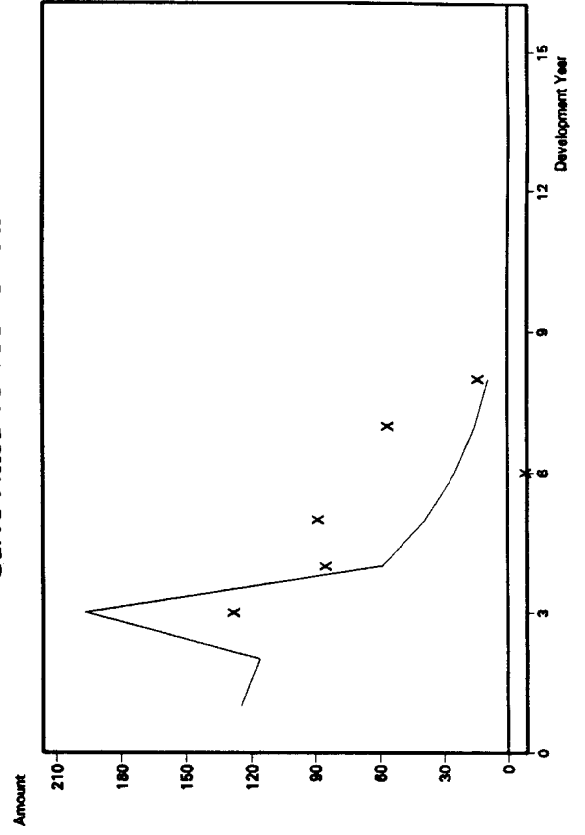
Marine Liability - Sterling 000's  
Curve Fitted To 1980's Data



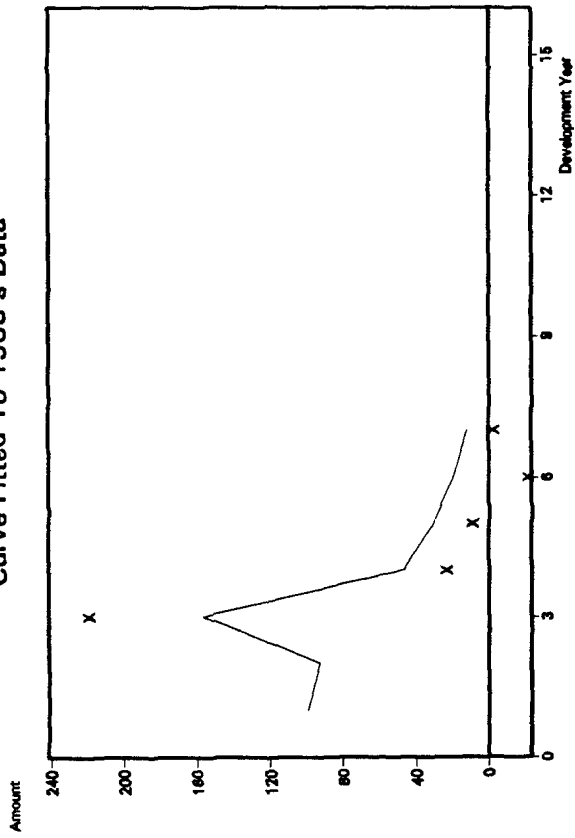
Marine Liability - Sterling 000's  
Curve Fitted To 1981's Data



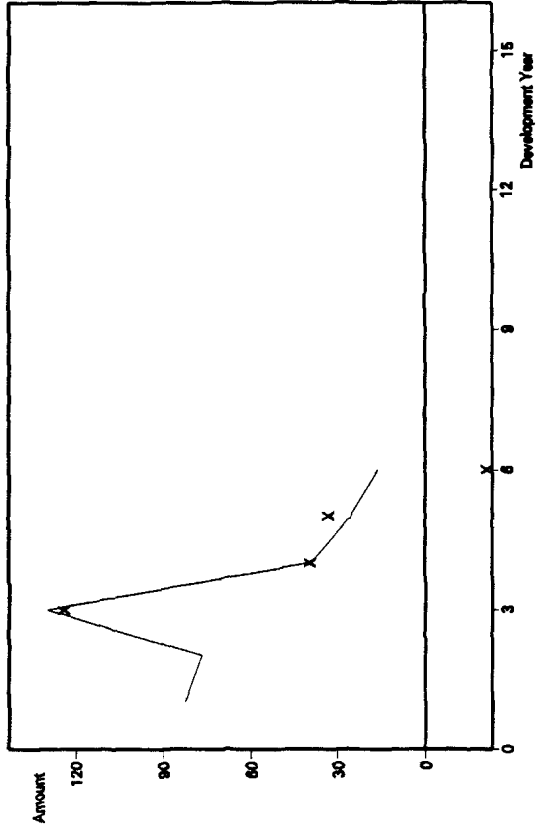
Marine Liability - Sterling 000's  
Curve Fitted To 1982's Data



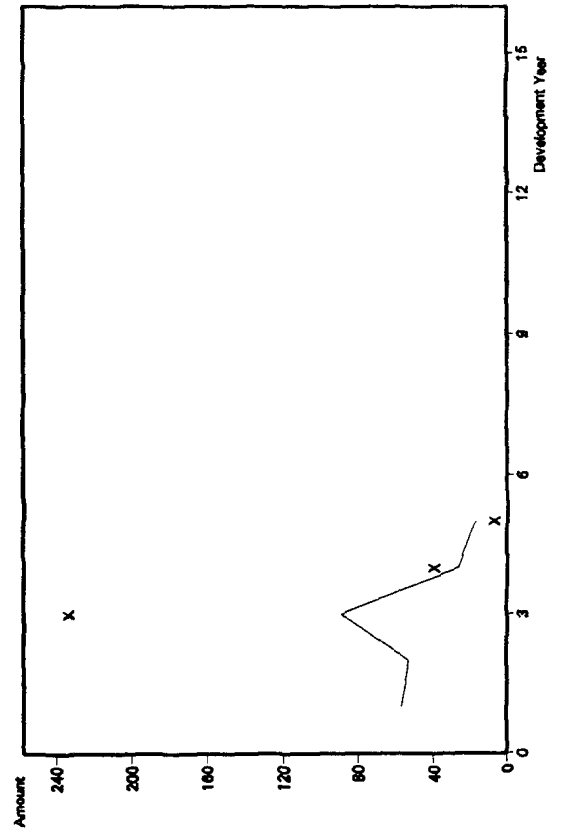
Marine Liability - Sterling 000's  
Curve Fitted To 1983's Data



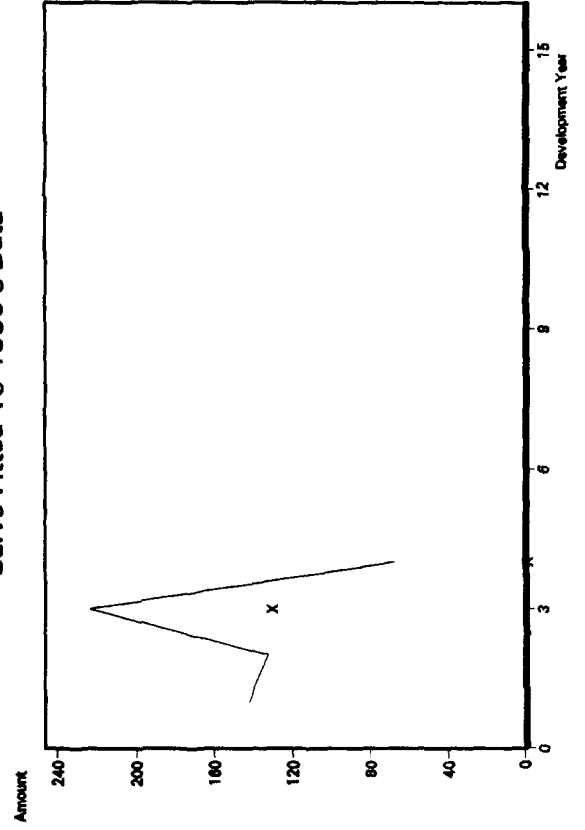
Marine Liability - Sterling 000's  
Curve Fitted To 1984's Data



Marine Liability - Sterling 000's  
Curve Fitted To 1985's Data



Marine Liability - Sterling 000's  
Curve Fitted To 1986's Data



# **Paid and Incurred Combined**

13:27 20-09-1990

BACON & WOODROW STOCHASTIC MODEL

**Marine Liability - Sterling £000's**

**COMBINED RESULTS**

Incurred Results Adjusted By Factor 1.00

Year	----- Paid -----		-- Incurred (Adj) --		----- Combined -----		
	Ultimate	St.Error	Ultimate	St.Error	Ultimate	St.Error	Paid-So-Far Reserve
1975	161.32	11.29	158.92	2.42	159.03	2.36	158.58 0.44
1976	255.06	13.43	251.83	4.15	252.11	3.96	251.25 0.86
1977	283.42	18.69	308.63	7.24	305.34	6.75	276.37 28.97
1978	195.86	19.82	201.99	9.39	200.87	8.49	187.83 13.03
1979	187.41	18.29	181.28	10.41	182.78	9.05	180.36 2.42
1980	151.38	25.43	146.55	16.73	148.01	13.98	138.22 9.78
1981	244.33	38.04	243.12	28.42	243.56	22.77	216.59 26.96
1982	377.24	59.06	376.21	49.44	376.63	37.91	315.97 60.66
1983	270.30	60.89	245.31	58.80	257.37	42.30	202.73 54.64
1984	192.83	63.95	199.83	70.35	196.00	47.32	116.61 79.38
1985	260.55	60.12	308.86	75.01	279.45	46.91	190.90 88.54
1986	320.97	117.87	242.16	158.03	292.79	94.49	92.75 200.05
1987	252.60	109.73	204.22	166.26	237.92	91.58	48.79 189.13
1988	21.61	0.00	21.65	100,192.24	21.61	0.00	21.61 0.00
1989	4.43	0.00	4.44	100,192.24	4.43	0.00	4.43 0.00
Total	3,179.32		3,095.00		3,157.89		2,403.02 754.88