

## **2004 Reinsurance Pricing GIRO Working Party**

### **Rating long-tail excess of loss reinsurance**

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#### **1. Introduction**

##### **1.1. Objectives**

We set three objectives for its report:

- Outline current techniques for rating reinsurance of long-tail risks and discuss their relative strengths and weaknesses.
- Identify issues specific to rating long-tail reinsurance business and discuss how well current techniques address the identified issues.
- Prepare case studies to draw attention to one or more of the issues identified in greater detail. Case studies are to be presented to GIRO in the form of a workshop.

Previous GIRO papers and other published work have covered the details of actuarial rating techniques. We do not attempt to duplicate the work in this report except in outline. Readers wanting to find out more about the techniques involved are referred to other sources, for example the Institute Sessional paper Sanders *et al* (1995) [1] and two previous GIRO working party papers Chandaria *et al* (1998) [2] & (1999) [3].

In this report, we are looking at rating from the viewpoint of the reinsurer, i.e. the party accepting a share of the long-tail risks written by the cedant. By long-tail, we mean that the original insured, or its interest, is exposed to claims that by their very nature are often not settled for several years from the time the original contract of insurance was issued. This may be a result of a delay in the manifestation of the loss itself or in the agreement of the final settlement amount.

We have considered rating only in the sense of technical pricing, i.e. producing a premium rate that is considered sufficient to achieve the desired profit margin. We do not consider market or strategic effects that may lead to a higher or lower actual premium rate. To quote Morton Lane (1996) [4]: “*There is no right price of insurance; there is simply the transacted market price which is high enough to bring forth sellers, and low enough to induce buyers.*” We do not use this quote to suggest that knowing a correct technical rate is of no significant importance. Instead, we

recognise that the rate is just one of a raft of factors underwriters use and are measured by.

## **1.2. Acknowledgements**

The authors would like to thank the people who assisted us in preparing this report, for giving their time and expertise freely and for proof-reading draft versions of this report.

Any errors or omissions that remain are solely the responsibility of the authors.

## **1.3. Disclaimers**

The views expressed in this report are not necessarily shared by our employers, the Institute and Faculty of Actuaries, GIRO or anyone who has assisted us in preparing this report.

Where we have had a variety of views, in writing the report we have tried to set out the issues in a fair and balanced way. Where we make recommendations or give views, they are not necessarily the opinions of all members of the working party.

## **1.4. Structure of the report**

We first lay out what we consider to be good principles of rating reinsurance for long-tail risks in Section 2. The remainder of the report splits naturally into two parts: techniques (or methods) of rating and then related issues.

In reporting on techniques, in Sections 3 to 5, we give a brief outline of a sample of the several methods in use and discuss each of the selected methods' strengths and weaknesses as measured against the ideals set out in Section 2.

Traditional rating methods are either experience-driven (relying on the historic claims experience of the cedant, allowing for changes in exposure over time) or exposure-driven (benchmarking against some standard allowing for class of business, region of exposure and layer being rated).

We discuss experience-based methods in Section 3, namely:

- Burning cost
- Individual claim projection
- Frequency / severity simulation.

Exposure-based rating (excluding retrocession) is discussed in Section 4 with retrocession rating in Section 5.

Section 6 lists the issues we identified as being important to long-tail reinsurance, their sources and their effects. In that Section, we also discuss how the various methods are able to deal with the issues. The issues discussed are as follows:

- Adjusting claims data for inflation.
- Historic exposure data.
- Open claims and unreported claims (IBNER and IBNR).
- Extreme event / common cause claims.
- New / latent claims.
- Capital requirements.
- Investment income and discounting.
- Reinsurance design – aggregate claims features.
- Allocated loss adjustment expenses.
- Non-insurance risks and loadings.
- Data availability and accuracy.
- Combining results from different methods.
- Consistency of approach between layers.
- Presentation of recommendations / feedback.

## **2. Ideals in rating reinsurance of long-tail business**

A reinsurer is aiming to create maximum profit on reinsurance contracts it issues, subject to an acceptable degree of risk that losses do not breach some threshold. Profit can be measured in many ways, such as return on capital employed or addition to the embedded value of the reinsurer, but essentially it is a measure of present value of the cashflows of the contract. There may be second order effects also, such as diversification benefit to the reinsurer.

Rating involves determining a target premium for the reinsurance contract such that the profit criteria are met allowing for all reasonably-foreseeable sources of risk. The actual premium a contract is written for will often be very different – higher or lower than the target rate – but this is the responsibility of the underwriter.

To determine the target premium, rating must therefore identify and quantify the risk of loss, and evaluate the effect of a loss under the contract to the reinsurer.

The rating techniques employed should allow for:

- Class, nature, location and currency of the risks being covered.
- Number, type and size of these risks to which the cedant is exposed.
- Likelihood of a loss under the contract.

- Severity of losses.
- Underlying factors that may influence the number and severity of losses.
- Terms and conditions of the contract.
- Costs, investment income and other cashflows arising out of the contract.
- Reinsurer's own outwards reinsurance arrangements.
- Risk appetite understood from the reinsurer's own management, interpreted as a target return on capital (or similar).
- Required contribution to overhead expenses and profit.
- Expertise and skill of the cedant's and reinsurer's claims management.
- Legal and supervisory aspects.
- Operational risks.
- Potential for model, parameter or data error.
- Availability of and quality of suitable data.

The above list is by no means exhaustive, but it serves to illustrate the great variety of risks to which a reinsurer is exposed and which ought to be addressed in the rating process.

Some items in the list, particularly the risk appetite, contribution to overheads and profit and wider operational risks, may have been included in the reinsurer's business plan, and perhaps in a DFA model. If so, they are likely to have been incorporated into a single profit criterion.

The profit criterion may allow for differing cashflow items under the contract, such as the reinsurer's own reinsurance costs, other indirect expenses and investment income. In this case, the profit criterion may be simply a gross loss ratio net of commission.

As losses and other factors are variable, the outcome is rarely known with confidence in the precision. Therefore a range of scenarios, each with its corresponding probability of occurring, has to be considered.

Ideally, any model should not produce significantly varying conclusions if there is a small change to the inputs, whether to historic claims data, the terms and conditions of the contract being rated, or to the assumptions used in the rating. Owing to the often sparse data (an issue explored in more depth in Section 6.11), this is an important but often overlooked requirement.

Finally, there are the very practical considerations that the model should be reasonably quick and easy to use and for the methodology and conclusions to be easy to explain to an underwriter.

The ideals of a rating model can be summarised in 5 ‘*R*’s thus:

- Risk*: The model produces a rate that allows for all reasonable sources of risk, considering both the severity and probability of the risk, allowing for the specific circumstances of the contract being rated. The model should also be approximately additive, e.g. the rate should approximately double given double the risk (subject to no limits on aggregation of risk).
- Return*: The rate determined, allowing for all reasonable risks, can be shown to meet the specified profit criterion.
- Range*: The model produces not only a single measure of the rate, but can be made also to produce a range or distribution of outcomes to illustrate the risks to the reinsurer from the cedant and the contract under consideration.
- Robustness*: The model should produce only small variations in output for small changes to inputs, and should produce a realistic rate in most circumstances. This is distinct from *Range*, which reflects an underlying distribution of outcomes. *Robustness* produces the same or very similar ranges under small changes to inputs: in this context it implies stability of *Range*.
- (Underw-)riter*: The model’s methodology, assumptions and conclusions should be easy to understand and to explain, should be transparent to the user and should be easy to change as necessary. It may be that the model itself resides with, and is used primarily by the underwriter.

### **3. Experience-based rating methods**

The descriptions of the experience-driven methods assume that the historic claims and exposure data have been adjusted to “as-if” for the contract being rated. The techniques and issues of data preparation have been explored in previous papers; see for example Chandaria *et al* (1998) [2]. Possible issues relating to data, inflation and exposure measures are explored in greater detail in Section 6.1 and 6.2.

#### **3.1. Burning cost**

This, the most traditional experience-based technique of rating XoL reinsurance, looks at the development of aggregate claims to the layer being rated.

##### **3.1.1. Outline of method**

Inflated claim values from the ground-up (FGU) are pushed through the XoL layers and a development triangle is formed of the claims to each layer aggregated by cohort year and development period. Because aggregate data behave more predictably than individual claims, assuming there are many historic claims into the layer, the ultimate losses to the layer can be projected using standard reserving techniques.

The estimated ultimate aggregate claims are compared against exposure data for the respective cohort years to identify any trends, and a selected layer loss per unit exposure is applied to the exposure for the new policy.

The analysis can be split into number of claims and average cost per claim to the layer, although this requires more historic data in order to be credible.

The burning cost rate can be “back-tested” by determining the rate ignoring the last diagonal in the development triangle. Any significant difference to the rate can be investigated.

### **3.1.2. Strengths and weaknesses**

As stated in the introduction to this paper, we are analysing the methods against the ‘ideals’ we set out in Section 2.

#### *Risk*

The burning cost method works well for simple, working layers, for which there should be a large volume of historic claims data. The burning cost method implicitly assumes the rate of aggregate claims per unit exposure in the future will mirror that of the past. As long as this assumption holds, the burning cost method, if applied correctly, should allow for all sources of reinsurance risk observed in the data.

For higher layers, the burning cost results are very dependant on fewer claims, by definition, so the effect of outlying claims will distort the rate considerably. With, typically, 10 years of claims, the largest observed claim that can be expected in the data is a 1-in-10-years event.

If an event with a return period of longer than 10 years is included in the observed large claims data, without adjustment, then it will receive an unduly high weight in the burning cost result. Similarly, the absence of exceptional claims will result in a rate below the long term average. See Section 6.4 for further discussion on extreme events and common cause claims.

Even for an intermediate layer, for which a few claims are likely to reach the layer, the burning cost method can give misleading results. If no inflated claim burns through the whole layer, the part of the layer above the largest inflated claim will not get any addition to the rate – there is, in effect, some “free” cover. Any explicit allowance for such free cover, eg through manual addition to the burning cost IBNR, is highly subjective.

For complex layers, i.e. those with aggregate loss sensitive features, the burning cost ultimate claims need to be adjusted before any aggregate loss feature is applied. The adjustment converts the ultimate claims as-if the future level of exposure had applied, i.e. allowing for changes in the volume of business written as well as the relative premium rate strength.

The effect of the aggregate loss feature can be calculated for each year of historic data, but there are unlikely to be enough years to have observed the

full range of results – for example years in which any minimum and maximum swing-rated premium adjustments are reached.

### Return

The method does not automatically test for profitability, although it is reasonably simple to add a profit test to the result for each historic cohort year.

### Range

Modern techniques on producing ranges of reserving estimates, such as bootstrapping, can be utilised to give a range on the average rate, if there is a large volume of data in the development triangle.

A bootstrapping exercise can be utilised further to determine an explicit contingency margin as a selected percentile of the distribution of results.

Bootstrapping can assist in rating more complex layers. As discussed above, the burning cost method on its own is not likely to rate properly swing-rated premiums, profit commissions, etc. But the range of results produced by bootstrapping is more likely to encompass the extreme results and to produce a reasonable rate given the terms and conditions.

However, bootstrapping cannot create data where none exist, so issues of the burning cost method relating to free cover and high layers remain.

### Robustness

The burning cost method is reasonably robust on simple, working layers: the existence or absence of individual outlying claims or a small change in inflation assumption is not likely to affect the derived rate materially. The method is also additive: doubling claims experience will, in most cases, double the rate.

On higher layers, the presence or absence of large claims has a significant effect, so care is required when using or allowing for such claims. Given that the highest layers are protecting reinsureds against loss events with longer return periods than the typical period of historic data, the burning cost cannot be used reliably on anything other than working layers.

### Underwriter

This method is simple to apply and simple to explain. It is entirely deterministic; the actual claims can be traced through the whole rating process, from inflating, through applying claims to layers and projecting for IBNR, to selection of ultimate. Its simplicity may also explain the method's near-universal use.

### **3.2. Individual claim projection**

Aggregate claims develop in a generally predictable manner, subject to there being a credible volume of homogeneous data. But individual claims do not. People may not respond to medical treatment as expected, courts can make unexpected awards: surprises occur constantly. That is the nature of and the reason for reinsurance – to protect against the downside risk of the unpredictable. It is also the difficulty in projecting individual claims rather than aggregate claims.

#### **3.2.1. Outline of methods – IBNER projection**

We present in the following paragraphs several methods of projecting ultimate values for individual claims. The first two are mentioned for completeness only and are not generally recommended practice (although in some circumstances they may be the only two options available).

In all cases, FGU claims are defined as those inflated claims whose values are, or have been at some point in their development, above a given threshold. An FGU claim should not be inflated beyond its appropriate policy limit (which should be “as-if” the period to be covered by the proposed reinsurance – see Section 6.11).

##### *IBNER Option 1: Ignore years not fully developed*

The option considers only mature cohort years of FGU claims data, with “mature” defined as a cohort of claims for which projection of the aggregate FGU claims no longer has any significant tail.

The assumption is that a mature year of claims should have few, if any, open claims, and the few that there are should not have significant movements from current values to settlement. “Pure IBNR” claims, relating to immature years, are determined separately (see below).

Ignoring recent experience has the clear disadvantage of ignoring the data that ought to carry more weight in the rating. Older years may be more mature, but they are also less relevant to the expected result of the proposed cover. The mix of business, the claims environment and the management of large claims may have changed significantly in the intervening period.

Whilst allowance for such changes can be made, it is inevitably subjective.

##### *IBNER Option 2: Ignore claims not yet fully settled*

Another option looks at the status of historic claims, and retains only those that have already settled (or “closed”). Currently open claims are ignored.

Considering only settled claims will include some of the latest claims, but ignoring open claims is likely still to distort analysis. Open claims are generally the largest and most complex, and there will be a greater proportion of open claims in the most recent cohort years. Just looking at the value of closed claims is likely to show a trend of decreasing average claim size when there is none in reality or to miss an upwards trend.



### IBNER Option 3: Develop open claims by a uniform factor

An IBNER development factor for a given cohort year is derived from the FGU claims data from prior cohort years and applied to all relevant claims in that cohort year, applying the IBNER factor only to known, open claims and separately to “create” pure IBNR claims. IBNER factors should be derived from only those claims in prior cohort years that were open at that stage of development.

Claims can be grouped in ranges by size and separate IBNER factors derived and applied depending on to which range the claim belongs.

This method will allow use of all known claims, and the overall ultimate claims will tally back to a projected aggregate claims figure.

However, applying a uniform IBNER ratio to open claims in a particular cohort year will mean that variation at the level of the individual claim is understated. Consider the case of three open claims on a particular cohort year, each with a current incurred value of £0.5m. In aggregate, these may be expected to increase by 10% to a combined ultimate of £1.65m (three times £0.55m). For a reinsurance layer of £0.5m xs £0.5m, the expected recovery is £0.15m. Note that the burning cost method might have missed these expected recoveries, as none had yet breached the excess point to the layer.

But not all claims develop in line with the average. In an extreme case, of the three open claims above, one may settle for costs only, one roughly in line with average development and one may “jump” in value at settlement – say £0.1m, £0.55m and £1.0m respectively. The combined total is still £1.65m, but the loss to the same layer is now £0.55m.

So, variation matters and what is required is a method to incorporate variation in claim development.

### IBNER Option 4: Develop open claims by a non-uniform factor

Capturing variation requires the introduction of a random component to future run-off of individual claims. At the same time, the development should have a basis in experience: no individual FGU claim development outside some kind of boundary conditions, average development in line with expected aggregate development, and a realistic distribution of claims.

In Appendix A.1, we describe a method of sampling historic claims to project current open claims. Alternatively, the development of historic claims can be modelled by fitting distributions to the development at each development stage and simulating developments for current open claims.

Random projections can throw up spuriously large claims not supported by the experience. This is most likely when claim values relating to the oldest cohort year in the historic data exhibit sudden jumps in the tail, or when a large claim was previously small in size. Banding claims by size (see Appendix A.1) and

artificially smoothing historical data can reduce or even eliminate spurious claim projections.

### **3.2.2. Outline of methods – IBNR claims**

Using experience-driven rating requires some allowance for claims not yet observed, i.e. new IBNR claims.

All the options described below rely on a preceding estimate of the number of IBNR FGU claims, for example with standard reserving techniques. Each option also assumes that IBNR claims are not materially different from known claims. In practice, this assumption is unlikely to matter materially to the rate. Like all assumptions, it needs to be tested for reasonableness. One approach is to perform a significance test on a linear regression of size of inflated claim against either date of loss or time to notification of loss.

#### *IBNR Option 1: Pro rate result*

The simplest option is to not create any new IBNR claims. Instead, the claims projected as above are pushed through the layers and the resultant layer losses for each year are increased pro rata for IBNR FGU claim numbers.

This method works well when there are a number of projected FGU claims to the layer. But it becomes unreliable when there are few claims, and fails altogether with no claims. The problems are exaggerated if the layers are indexed or have an inner aggregate deductible.

#### *IBNR Option 2: Sample projected FGU claims*

For each IBNR claim required, a projected FGU claim is selected at random.

#### *IBNR Option 3: Select from a fitted severity curve*

A severity distribution can be fitted to the projected FGU claims (see Section 3.3.1). For each IBNR claim, a value is selected from the curve at random.

### **3.2.3. Determining a rate**

Whatever methods are used for IBNER and new IBNR claims, the sum of the individually projected FGU claims is unlikely to agree with a projection of aggregate FGU claims (eg by standard reserving methods). We recommend at least that the comparison be made between aggregate projected FGU claims and projected aggregate FGU claims.

If the comparison shows an unreasonably large difference then this should be investigated. For example, the bands into which the claims are grouped by size may need adjusting, or one or more individual claims may have developments that are distorting projections (in which case claims data may need smoothing). The projected FGU claims can be adjusted (“normalised”) to agree with a projected aggregate FGU claims amount – see Appendix A.2 for details.

Finally, the projected FGU claims and IBNR claims are pushed through the layers. The layer losses are compared against exposure data for the respective cohort years to identify any discernable trends, and a selected layer loss per unit exposure is applied to the exposure for the new policy.

#### **3.2.4. Running repeated simulations**

With IBNER options 3 and 4, which both involve random development of FGU claims, the process can be repeated to achieve a range of results from which a rate can be determined. This is assumed to be the case in the discussion on the strengths and weaknesses below.

#### **3.2.5. Strengths and weaknesses**

##### Risk

The major weakness of projecting known claims, in common with the burning cost method, is that the method relies entirely on actual historical data. Genuinely extreme events are not likely to have been observed in the data, and if so are likely to be given unduly high weight.

Compared to the burning cost, there will be less “free cover” as some claims are likely to be projected higher than the current largest incurred value. But free cover remains an issue for higher layers.

As the method can be repeated as many times as desired, it can more accurately rate aggregate loss features of the reinsurance programme. For example, the effect of aggregate limits and paid reinstatement premiums can be rated, as the average of, say, 10,000 simulations rather than 10 years of claims data as under the burning cost method.

##### Return

The method can incorporate a specific profit test on each simulation of projected claims.

##### Range

The use of a random component means that the process can be rerun as many times as desired and a distribution of outcomes generated. This is a very powerful tool in deriving explicit contingency margins.

However, each simulation is likely to give claims amounts that are close to a burning cost rate, owing to the probable absence of extreme cases, so the method does not result in the full range of possible outcomes.

##### Robustness

In Appendix A.3, we discuss the number of simulations required to give an expected reliable result. Additional steps, such as ‘normalising’ projected claims, banding claim sizes and smoothing historic data also reduce artificial variation.

Small changes in assumptions, such as inflation, and changes to the data are unlikely to give large changes in results to working layers. As for the burning cost method though, the existence or absence of extreme claim values in the data does have a significant effect on higher layers.

#### Underwriter

In using a random element to the process, the method can no longer give precisely the same result given the same inputs of data and assumptions on a second or subsequent run. However, the number of simulations can be set to give a sufficiently low expected variation, as per Appendix A.3.

In addition, the random steps in the process of projecting individual claims means that there is no longer a full audit trail of how a rate was derived. The overall process has become a “black box”; it is more difficult to control and more difficult to explain to others.

### **3.3. Frequency-severity simulation**

This method is the most flexible and useful in terms of allowing for complex layer terms and conditions and in being able to model extreme events. However, careful judgment is required in selecting the input parameters.

#### **3.3.1. Outline of method – severity distribution**

Severity distributions can be parameterised using market data or some other external source. But in rating for different reinsureds, with different risk profiles and mix of business, it is better if the parameterisation can be based on that reinsured’s own claims experience.

Fitting a curve to claim severity makes sense only if the claims are reasonably mature, at or near their final settlement value. Pure IBNR claims are not needed in fitting severity distributions. Section 3.2.1 outlines various options for projecting IBNER on individual claims, although it should be noted that these methods assume the severity of IBNR claims are not materially different from that of known claims.

Care needs to be taken where there are original policy limits. FGU claim values are likely to be concentrated at common policy limit values. These policy limits will then have significant weight in the severity distribution fitting and may distort the parameterisations. Consideration should be given to excluding all but one data point at common policy limits.

The severity distribution should be fitted only to projected FGU claims that exceed a chosen threshold. FGU claims may have “peaked” already, i.e. they once reached a significant incurred value and then actually settled for much less. Equally, an FGU claim may have been projected randomly to an ultimate value much less than its current value. Small claims may distort frequency analysis of large claims and they need to be discarded. A lower threshold will result in a larger number of data points for the fitting, but may include claims that have no relevance to the reinsurance being rated.

If the chosen threshold is greater than the excess point for a given layer being rated then that layer cannot be rated by this method owing to the distribution not being a reliable fit to FGU claims below the threshold.

The individual claim projections can be repeated several times, with the severity distribution fitted to the combined set of projected claims. This is especially useful if the threshold has been set particularly high or if there is a particularly wide range of results from several projections; combining several sets of projected claims should increase the stability and reliability of the fitted severity distributions.

The fitting itself can be done several ways, and the following are the most common in use:

- Matching moments.
- Percentile matching.
- Linear regression (e.g. using GLM).
- Maximum likelihood.

More details of fitting techniques can be found in standard textbooks on the subject.

All of the fitting methods work on the projected claims, treating each as an equally likely data point (i.e. giving each claim equal weight in the fitting). The data, by definition, is sparser towards the tail of the distribution, so the fitting is weighted towards the bulk of the data. The severity distribution ideally should be fitted with data points in the tail being given more weight. One possible method is to define an empirical distribution (see Appendix A.4) so that the fit is weighted equally along the range of projected claims.

### **3.3.2. Outline of method – frequency distribution**

Frequency modelling is done by fitting a distribution to the number of claims per year. The exact form of this will depend on the severity distribution fitting method. If the severity distribution fitting is to all claims with an ultimate value above a threshold, then the frequency distribution should be fitted to the number of claims exceeding this threshold as well.

One method is to create a triangle of the number of inflated FGU claims that exceed the threshold and to project ultimate numbers using standard reserving techniques.

Before fitting frequency distributions, the projected number of claims for each historic cohort year should be adjusted to account for the change in exposure from that cohort year to the period being rated.

If there is a high proportion of years with zero claims, then separate models for zero claims and for the number of claims in non-zero years should be considered.

If the number of large claims is particularly high, then some approximation (eg using a Normal distribution) may be appropriate. As the Normal has an unbounded left tail to the distribution, any negative frequency needs to be interpreted as a zero number of claims instead.

### **3.3.3. Determining a rate**

There are several approaches to determining the rate given frequency and severity distributions.

Depending on the distributions used, it may be possible to calculate the expected loss to layers directly from the distribution functions.

An aggregate distribution for the layer losses can be derived, for example using Panjer's recursion method, for which a description can be found in Chandaria *et al* (1999) [3].

The simplest method though is a full Monte Carlo simulation. A random number of claims is generated using the fitted frequency distribution. For each claim, a claim size is selected at random from the severity distribution. The selected claims are pushed through the layers and the total layer loss determined.

The simulation can be repeated to give an average rate and also a range of outcomes; Appendix A.3 discusses the number of simulations likely to be sufficient.

### **3.3.4. Strengths and weaknesses**

#### Risk

A fitted curve, if it is an accurate model of the underlying large claim severity distribution, describes completely the behaviour of individual claims, including extrapolation beyond the largest observed claim. Hence, by definition, there is no free cover issue. Similarly, the rate allowing for aggregate loss features can be valued more accurately than the other methods.

#### Return

The method can incorporate a specific profit test on each simulation.

#### Range

As for the individual claims projection method, the use of a random component means that the process can be rerun as many times as desired and a distribution of outcomes generated.

#### Robustness

The fitted curves method is more robust to the presence or absence of very large claims to which the curves are fitted. However, outlier claims will affect

the parameterisation of the severity curves and may also affect which curve best fits the data.

The fit can be tested for robustness by repeating the fit on a new set of individually projected claims, and also by restricting the range of the empirical distribution to exclude the largest claims.

The method is sensitive to the choice of frequency and severity curves, especially in the tails of the distributions.

#### Underwriter

Monte Carlo simulation is simple to describe in principle, although details of the underlying technique can be left aside. Underwriters usually understand sample statistics, such as the number of claims to a layer and the return period for a particular loss size, and discussion of these with the underwriter can help refine the method.

It is impossible to distinguish mentally between two severity curves described as “*Pareto (2.1, 50,000)*” and “*Weibull (0.8, 55,000)*”. However, there are rules of thumb, such as Weibull is light-tailed, Pareto is heavy-tailed and Lognormal is in between.

### **4. Exposure-based rating (excluding retrocession)**

Exposure-based methods have a wide variety of uses in reinsurance rating, not just for estimating a technical rate. In many cases an experience-based method is not appropriate; for example the mix of business has changed considerably or a new class of business is being written.

Exposure-based methods can be used to provide an initial expected rate as an input to an experience-based method. The exposure-based rating techniques can be used also to rate higher layers, using increased limit factors on working layers for which experience-based techniques are reliable. Section 6.12 provides more discussion on using different methods together.

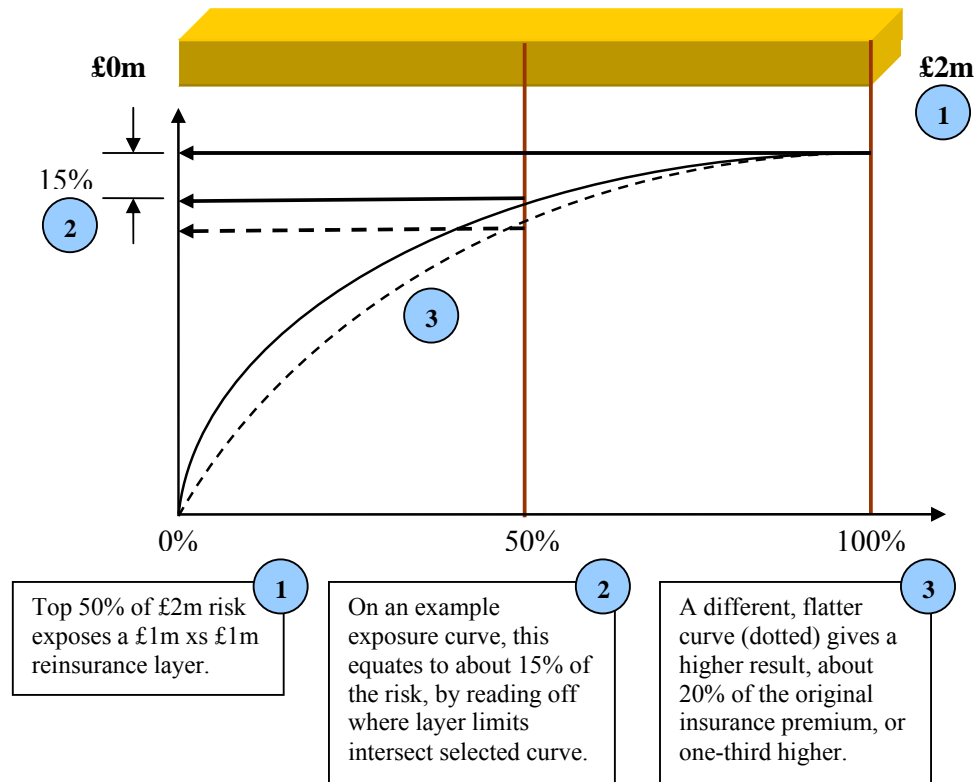
There exist a variety of different approaches, but most are concerned with allocating premium on original risks between reinsurer and reinsured using a standard curve for a given class and region of exposure and adjusting for the particular cedant’s premium rate strength. This is the common approach that we outline and discuss in this Section. There are many sources giving worked examples, including Sanders *et al* (1995) [1].

#### **4.1. Outline of method**

Each original contract that a cedant writes is analysed separately against an exposure scale, which has been selected as appropriate for rating the class of business and region of exposure. Selection of the right scale can be crucial – too flat a curve and the reinsurer’s rate may be uncompetitive, but too convex a curve may give too low a rate to be profitable for the reinsurer.

The original contract's exposure is divided in two parts: that which is retained by the cedant and that which is ceded to the reinsurer. The rate that is charged on that contract is the proportion of the exposure ceded (according to the scale) times the premium for the original contract. The following diagram illustrates:

*Diagram 1 – allocation of premium to reinsurance*



Several adjustments may be necessary. First, the cedant cannot know exactly what contracts will be written in the future, only those that have been written to date. The assumption is that the proportion of the cedant's original business that exposes the reinsurance will not change significantly. Alternatively, the percentage can be adjusted to reflect an expected change in risk profile, although any such adjustment is subjective.

Secondly, the cedant's original premiums may or may not be deemed to be sufficient, according to the reinsurer's profit criterion. An adjustment can be made to the reinsurance rate to reflect the profitability of the cedant. This may be calculated directly by performing a reserving exercise on the cedant's gross claims and premiums. The reinsurer may even give a discount if the cedant's own rates are more than sufficient compared to the reinsurer's profit criterion. For example, if the insurer is expecting an 82.5% loss ratio but the reinsurer requires only a 90% loss ratio on its own business, then the reinsurer can apply a factor of  $0.825/0.90 = 0.9167$ , i.e. an approximate 8.3% discount.



## 4.2. Strengths and weaknesses

### Risk

Exposure methods work in almost all situations, regardless of the existence or otherwise of relevant historic claims data. Historic data are useful in assessing profitability. But the only historic claims data required for this are triangulated aggregate claims, not development of all individual claims, which have exceeded some arbitrary threshold at some point in their development.

The scale can be derived from a much larger dataset than just the cedant's own claims data. Therefore the dataset is more likely to include large or unusual claims.

On the other hand, the scale is unlikely to be exactly specific to the contract being priced: a US medium industry scale is unlikely to be suited to rating public liability on Florida theme parks, to give an example.

There are a series of assumptions underpinning exposure-based rating, which may or may not be correct. In particular, the choice of scale can be crucial. Changing the scale can make a large difference to the result, and often scale selection is an arbitrary decision within the rating process.

In addition, the method assumes that the cedant has already allowed for all reasonable sources of risk, although potential shortfalls in the original premiums can be made up for in adjusting expected profitability or selecting a more penal exposure scale.

Exposure scales are usually specific first-loss or similar. Clash covers or layers involving aggregate claim features (such as paid reinstatements – see Section 6.8) can be difficult to rate on this basis.

### Return

Without any adjustment for the cedant's profitability, the expected return should match that of the cedant (assuming the correct apportionment of exposure to the reinsurance). Any adjustment is likely to be based on a target loss ratio and an estimated (or assumed) loss ratio for the cedant. Therefore any explicit profit criterion needs to be converted to a target loss ratio in order to be able to demonstrate a rate's profitability.

### Range

Exposure-based methods do not lend themselves to explicit modelling, so determining a range and distribution of outcomes is difficult.

### Robustness

The method is robust towards historic experience, as claims history is used only to assess profitability of the cedant's original business.

The method is much less robust as regards choice of exposure scale; different scales can have a significant effect on the rate, particularly on higher layers.

### Underwriter

Exposure-based rating is a frequently-used method, so the underlying principles and techniques are widely understood. Actuarial input can be restricted to revising the exposure curves based on recent claims data, with the model itself being used directly by the underwriter.

## **5. Retrocession rating**

### **5.1. Specific issues relating to retrocession rating**

This type of cover is variously called retrocession, excess-on-excess, XS-on-XS or LMX. Reinsuring a reinsurer's own long-tail risks presents additional problems for rating. Primarily, the difficulty over and above rating for "first-tier" reinsurance is that the experience of the cedant, now itself a reinsurer, is heavily influenced by its own line size and where in a programme of reinsurance it has accepted risks.

Consider the case of a £1m xs £1m retrocession cover for a reinsurer who has written the following four contracts:

A: £1.5m xs £0.5m (100% share)

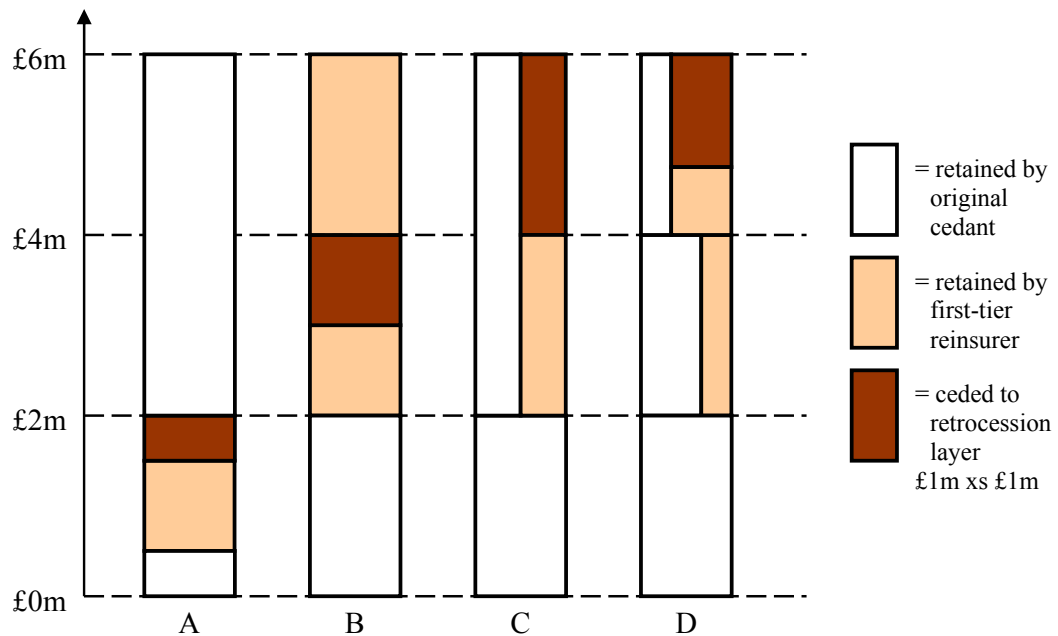
B: £4m xs £2m (100% share)

C: £4m xs £2m (50% share)

D: Section A £2m xs £2m (33% share) and Section B £2m xs £4m (67% share)

The different layers, and who carries the risk, are shown in the next diagram:

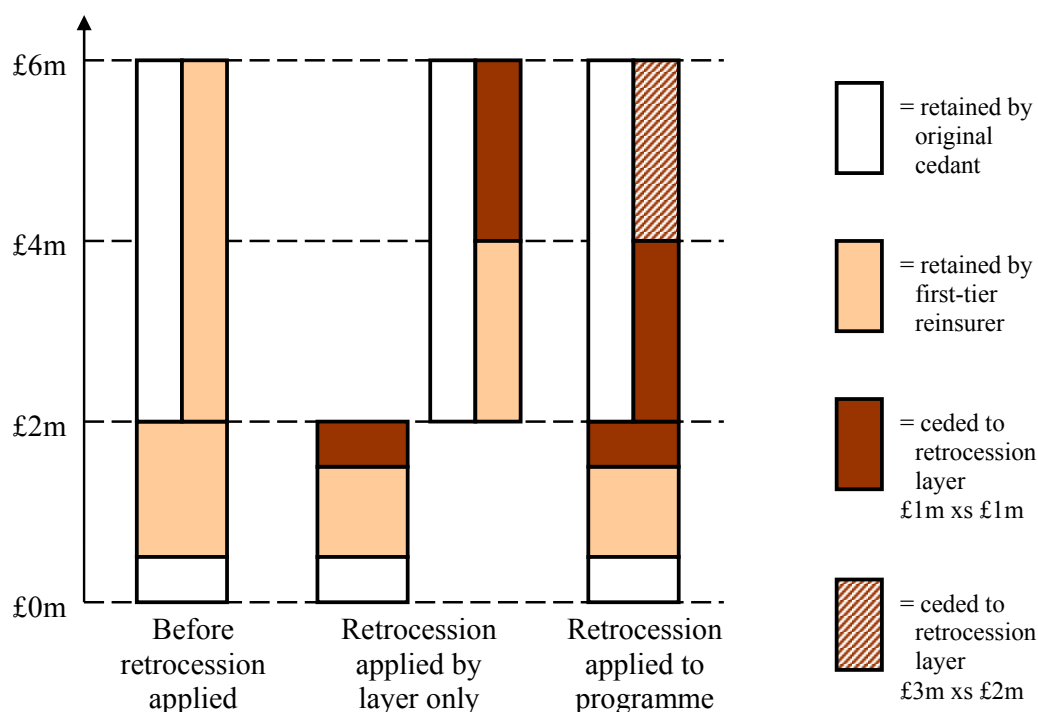
Diagram 2 – illustration of retrocession portfolio



It is quite possible that these four examples are from the same retrocession over time, with the first-tier reinsurer repositioning their portfolio further away from the original insurance risk.

Where the first-tier reinsurance is a programme of layers, perhaps with varying participation shares, the layers need to be considered simultaneously. In the examples above, contract A with one of contracts B, C or D could be part of a single programme. If, for the sake of example, the programme consists of contracts A and C, then retrocessional layers £1m xs £1m and £3m xs £2m are exposed as shown in the following diagram:

Diagram 3 – illustration of combining layers to retrocessional reinsurance



If the layers are rated for retrocession separately, then the rate for the £1m xs £1m and the £3m xs £2m retrocession covers are almost certainly going to be too light.

## 5.2. Outline of method

Rating retrocession reinsurance is an extension of the exposure-based method for rating first-tier reinsurance. The aim, as for first-tier reinsurance, is to determine what proportion of the cedant's premium to charge for the retrocessional cover. This requires going back to the original covers: "looking through" the first-tier reinsurance to the insurance risks.

There follow several options, depending on the viewpoint of the retrocession underwriter as to how well the cedant has managed to rate its own inwards reinsurance business:

### Retrocession Option 1: Share of first-tier reinsurance premium

A standard exposure scale is applied to the original covers to determine what proportion of the first-tier reinsurance premium to take as retrocession premium. In the example in Diagram 3 above, the cover for the £1m xs £1m retrocession may take 70% of the premium for the partial share of the £4m xs £2m first-tier reinsurance layer, with the remaining 30% of the premium allocated to the £3m xs £2m retrocession cover.

This option recognises that the first-tier reinsurer is close to the original insurer. Therefore, the first-tier underwriter will have more information with which to assess

the risk and is more able than the retrocession underwriter to rate the first-tier reinsurance accurately.

#### *Retrocession Option 2: Share of first-tier reinsurance premium with reallocation*

Different reinsurance underwriters will have available different scales that they use to exposure rate. Even if they all agree on the total exposure taken on by a reinsurer across a programme, they may differ on the allocation of that exposure to particular reinsurance layers. This is significant for retrocession rating.

In the above example, the first-tier reinsurer may be happy that the overall premium for the programme is adequate, but a retrocession reinsurer participating on the £3m xs £2m needs to be satisfied that the proportion of that premium allocated to the 50% share of the £4m xs £2m first-tier reinsurance is sufficient.

Therefore, retrocession rating can include the step of reallocating first-tier reinsurance premiums for a programme between layers using the retrocession reinsurer's own exposure scale. This recognises that the first-tier reinsurer is likely to be able to rate accurately a programme but may be less concerned about allocating premiums.

#### *Retrocession Option 3: Re-rating the first-tier reinsurance*

The final option is to re-rate exposing first-tier reinsurance layers from scratch. As discussed in Option 2, different reinsurers may use different exposure scales, which is likely to lead to a difference in opinion on the overall rate of a programme as well as the relative rates between different layers.

For example, the underwriter may have charged 50% of the original insurer's premium for the first-tier £1.5m xs £0.5m layer. Under his or her own rating assumptions, the retrocession underwriter may decide the cover should have cost 60% of the original insurer's premium. The retrocession underwriter should increase the rate that would otherwise be charged for the £1m xs £1m retrocession cover using share of premium methods by 20% to compensate for the under-rating, i.e. the same as the increase in the first-tier rate from 50% to 60%.

Re-rating may instead include a general allowance for the overall profitability of the first-tier reinsurance compared to the target profitability of the retrocessional reinsurance. If the estimated loss ratio of the first-tier reinsurance is 105%, and the target loss ratio for the retrocession reinsurer is 95%, the rate can be adjusted by a factor of  $1.05 / 0.95 = 1.1053$ , or an approximate 10.5% increase.

### **5.3. Strengths and weaknesses**

Apart from the additional steps outlined in the options above, the method is very similar to exposure-based rating of first-tier reinsurance. Therefore, the strengths and weaknesses from Section 4.2 apply in this situation also. There are some additional comments as follow.

## Risk

The process of “looking through” the first-tier reinsurance portfolio to the original risks means that this method of retrocession reinsurance rating is more likely to reflect actual risk than from just considering the first-tier reinsurance portfolio alone.

Because retrocessional reinsurance is further away from the original risk than first-tier reinsurance, the choice of scale is even more important. Retrocessional reinsurers are less likely to be familiar with the individual contracts written by the original insurer, so are more likely to miss salient features that can affect the choice of scale and hence the rate.

Other factors used in rating the first-tier reinsurance, such as changes in an insurer’s portfolio or level of profitability, are less likely to be available in a retrocession rating. This can be assessed by a review of a sample number of contracts written by the first-tier reinsurer even if it may not be possible to do so for all contracts in practice.

## Underwriter

The additional complication of analysing original risks, not just the reinsurance contracts, adds to the complexity of the underwriter’s task. The retrocession underwriter is effectively re-rating an entire year’s first-tier reinsurance portfolio in the time available when the retrocession proposal is submitted. In practice, re-rating is likely to be restricted to the largest exposures in the first-tier reinsurer’s portfolio, with a simple share of premium calculation on all other contracts.

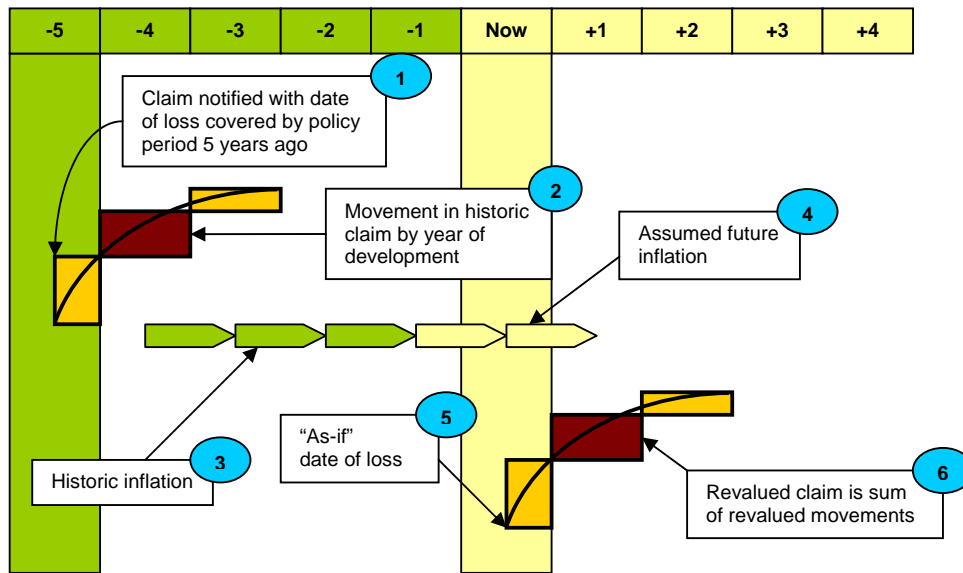
## **6. Issues in rating reinsurance for long-tail risks**

### **6.1. Adjusting claims data for inflation**

As discussed in Section 3, the experience-based methods require claims and exposure data that have been revalued “as-if” they are going to arise on the contract being rated. Inflation is a key part of that revaluation. It is a calendar year effect, rather than a year of account effect.

The claims should be converted to incremental movements over, usually, each year of development. All incremental movements should then be revalued to the corresponding future period. The revalued claim is the sum of the revalued incremental movements. Revaluation requires part historic inflation and part future inflation, as shown in the following diagram:

Diagram 4 – illustration of inflating incremental development



In practice, many reinsurers revalue all historic claims arising on a particular cohort year with the same inflation increase. In a stable inflation environment, the two methods will not produce significantly different outcomes. But if inflation rates have fluctuated, or there has been “shock” claims inflation over and above normal claim inflation levels, then the revalued claim amounts can be markedly different between the two methods.

The historic inflation rates can be built up from some suitable index of general economic inflation, e.g. wage inflation for employer’s liability business, plus an allowance for “social” inflation. Social inflation is the part of claims inflation over and above the selected general economic inflation. Future inflation assumptions should allow for future social inflation too.

## 6.2. Historic exposure data

Used for experience-based methods, ideally the exposure measure is a good predictor of claim frequency (claim severity for long-tail business shows almost no significant link to volume of business or line size). Exposure data is needed for each historic cohort year for which claims data has been provided.

Adjusting exposure data is a simpler exercise than for claims data above. The revaluation required to convert historic data to as-if for the period of reinsurance proposed depends on the type of data:

### 6.2.1. Exposure measure is a risk count

For example, the exposure measure for reinsuring a portfolio of private motor business is number of vehicle years insured. In this case, the measure does not need revaluation. The number of policies written by the insurer is also a measure that does not need revaluation.

The exception is when the nature of the risk has changed significantly (e.g. the motor portfolio now includes a much higher proportion of young drivers). Where the underlying risk has changed, the experience-based methods are not likely to be reliable unless the data can be amended to allow for the change in risk.

#### **6.2.2. Exposure measure comes from the original policyholders**

In some cases, the exposure measure will quantify the amount of business conducted by the original insureds. Examples are the total fee incomes on a book of professional indemnity business and wage roll for employer's liability business.

In these cases, there may be some revaluation required. Wage roll and fee income will each rise, at least approximately, in line with wage inflation for the same probability of a large loss.

#### **6.2.3. Exposure measure is subject premium**

Subject premium is the gross premium on risks written by the insurer for the business to be covered by the reinsurance. Premium rates fluctuate, so the revaluation must include allowance for the relative change in premium rates from each historic cohort year to the period of reinsurance being proposed.

It is important to be sure what the premium rate change information represents. In particular, premium rate changes may or may not include general rate changes to allow for inflation. In the former case, the rate change applies directly to the subject premium. In the latter case, the subject premium should be revalued first in line with an inflation index, such as the general inflation index used for claims, with the rate changes applied afterwards.

### **6.3. Open claims and unreported claims (IBNER and IBNR)**

IBNER and IBNR can present the greatest sources of uncertainty in the estimate of the ultimate cost of claims for long-tail risks. Loadings for IBNER and IBNR need to be projected reliably to be incorporated into the technical rate when using experience-based methods.

The burning cost method projects IBNER and IBNR together, but other experience-based methods need separate consideration for IBNER and IBNR.

#### **6.3.1. IBNER loadings**

Open claims are generally those claims that are more complex or disputed. These tend to involve larger sums of money than claims that are settled more quickly. They also tend to involve greater volatility in their development than that seen in average claims. Hence there may be a large proportion of historic claims that are still open.

Determining the IBNER loading for XoL reinsurance claims can be made more uncertain owing to the often sparse data presented to reinsurance



underwriters (see Section 6.11). Several options for projecting IBNER in claims were given in Section 3.2.1.

However, changes in the reserving philosophy for large claims by the reinsured need to be allowed for, in particular:

- the speed of setting large reserves,
- the size of initial reserves,
- level of prudence in large claim reserves,
- authority levels of claims staff, and
- turnover in claims staff.

It may be that claims data from earlier cohort years may need to be discarded, although it may be that just the initial development is not applicable and the last few periods of development can still be included in the analysis.

#### **6.3.2. IBNR loadings**

For reinsurers, there are two types of IBNR claims: those that relate to claims that have been notified to the reinsured but have not yet reached the XoL layers, and claims that have not yet been notified to the reinsured.

For the FGU claims that have notified to the reinsured, it is normal that the reinsurer imposes a reporting threshold, as low as 50% of the lowest excess point, and that all claims that have reached the reporting threshold are notified to the reinsurer as well. Allowing for the possibility that these FGU claims may then grow to reach the XoL layers is then an extension of the IBNER exercise in Section 3.2.1.

Options for IBNR loadings were discussed in Section 3.2.2.

#### **6.4. Extreme event / common cause claims**

Common cause claims can be defined as a series of claims that can be identified as arising from one particular source (not necessarily one event). For reinsurance purposes, each loss is a separate potential claim. We consider only those cases in which there are multiple reinsurance claims.

Extreme event claims are single large claims lying in the tail of a severity distribution; they can be either a single large risk loss or, depending on the terms of the reinsurance, the aggregation of many claims arising directly from a single event.

The difference between the two types of claim is that extreme event claims occur in the tail of the severity distribution whereas common cause claims occur in the tail of the frequency distribution. Both types of claim are expected to occur infrequently, but are expensive if they do. If such claims exist in the historic data, they are likely not to be representative of the underlying distribution and may distort experience-based methods.

One option is to exclude such claims from the historic data and rating as if the claims had not existed in the data, as described further below.

Another option is to retain the data but to reduce the impact of the claims on the rate. For example, in fitting a severity curve with an extreme event or a frequency curve if there are common cause claims, the weight of the data points involved is relatively lower than the “regular” claims.

More likely is that such claims are absent from the historic data and a separate allowance is required. Allowance for such claims can be implicit in the rating, for example, to model extreme event claims that impact only high layers an increasingly common approach is to use a fitted Generalised Pareto distribution (GPD). Further information on fitting a GPD and using Extreme Value Theory in XoL reinsurance rating is given in many sources, including A. McNeil (1997) [5].

With other methods, an additional allowance is necessary. For extreme event claims, a conditional frequency / severity model can be used. This can perhaps be a low probability Binomial distribution to randomly select whether such an event is included in the simulation. The severity of the claim, conditional on a claim occurring that exceeds the extreme value threshold, can be modelled with a GPD.

For example, to model a one in fifty year event, each simulation selects randomly from the Binomial distribution with a 2% probability of “success” and, if the Binomial sample is a “success”, a random value is selected from the top 2% of the distribution of the fitted GPD.

For common cause claims, a similar approach of modelling additional claims can be used. In this case, it is necessary to model the probability of any one simulation having such claims by a Binomial distribution and then model the claims themselves from a chosen frequency / severity model. Calibration of the frequency and severity distributions can come from internal or market data on suspect cases of common cause claims in the past. The severity distribution fitted to the FGU claims data (see Section 3.3.1) could also be used.

## **6.5. New / latent claims**

Long-tail risks generally involve liability from one party to another, including personal injury. Therefore, reinsurance of long-tail risks is exposed to latent claims, or claims from new, previously unforeseen sources.

Examples of where new claims can arise from, include: latent claims from new sources (workplace stress, say), a change in legislation, a change in interpretation of wordings following a court case, and changes in the nature of the underlying risk. New claims can also include changes in the severity of claims (such as after the Ogden tables in the UK).

Latent claims can be described as unexpected claims arising on known sources. Reserving techniques for valuing latent claims – see, for example, P. Archer-Lock *et al* (1999) [6] – can be used to ensure that experience-based methods are fully loaded for claims that may arise on past cohort years.

Section 6.4 described common cause claims. Latent claims and new claims are a form of common cause claims, so the same methods can be applied. The difficulty is in calibrating any model of latent or new claims owing to the “unknown” nature of such claims. Calibration is necessarily subjective.

Some grounding in reality is possible using specific industry-group data. For example, one possible source of latency is mobile phone use: an estimated exposure can be derived from the number of phones in use and the number of signal masts, coupled with the likelihood and severity of successful lawsuits against the manufacturers and network companies (which can be estimated by rough comparison with previous product liability claims). In practice, reinsurance contracts now routinely exclude claims from mobile phones and other transmission devices, but the principle remains for other cases.

Other approaches by reinsurers can be used. Strictly, these replace allowances for latent and new claims by seeking to exclude or at least reduce the possibility of such claims and therefore reduce the uncertainty in the rate. Examples are:

- Change wordings: this can easily exclude claims from known sources of latent claims, but wordings to exclude new claims are, by necessity, vague and open to adverse interpretation by a court of law.
- Change basis of reinsurance: moving from a risks-attaching during (RAD) or losses-occurring during (LOD) basis to claims-made during (CMD) can virtually eliminate claims arising long after the period of reinsurance has expired.

Latent claims from past periods of exposure are already excluded as they are covered under previous RAD and LOD reinsurance. This can be made explicit by the CMD cover being limited only to those cohort years since the RAD and LOD covers ceased.

- Compulsory commutation: the contract wording provides for the reinsurer to cede back remaining exposure on payment of a commutation premium covering known reserves and an allowance for IBNR claims. The basis for calculating the commutation premium can be determined in advance or left open (eg “by an independent actuary whose appointment is subject to mutual agreement”).

The effect on historic data of having the changes needs to be included in the rating analysis, whatever method is being used, as the effect may be to exclude more than just possible future latent claims.

A significant potential problem is that the cover on offer may no longer be marketable as it not regarded as useful by the reinsured, who may require a substantial discount to accept the proposed changes.

## **6.6. Capital requirements**

In Section 2, we discussed briefly how a profit criterion might be set for determining whether a rate is sufficient for a contract of reinsurance. In some cases, the rating

does not need to make special allowance of capital or investment income, or perhaps even general expenses, as these will have been incorporated into a business plan or DFA model, with a profit criterion established in terms of a target gross loss ratio net of commission.

Even in such cases, the level of capital and expected investment income are likely to have been tested at a line of business level, which may be considered to be a special case of a profit test in a rating model.

Traditionally, the capital has been set as a percentage of premiums, possibly adjusted for expected loss ratio, and a target set as a rate of return on capital. This is suitable in a deterministic rating world. But in a stochastic world, each simulation can be tested for its own capital requirements (to cover any shortfalls in accumulated cashflows).

A stochastic approach to capital required is described in detail in Mango (2003) [7]. The overall capital required is the capital “consumed” by the loss-making scenario, with progressive penalties for using capital resources, multiplied by the probability of that scenario. There is no *a priori* assumption about capital requirements and the process adjusts automatically for higher or lower premium rates.

An alternative, for when there is no stochastic model, is the proportional hazards (PH) transform approach, which uses a suitable uplift in the rate given the level of reinsurance and the risk aversion of the reinsurer. The PH transform method is described by, among others, Wang (1997) [8], and a brief outline is included as Appendix A.5.

## **6.7. Investment income and discounting**

Income and discounting are different things – the former is an expected cashflow from holding reserves, the latter is the process of determining the present value of a series of cashflows – and in long-tail reinsurance it is important to recognise the difference. Both are of particular importance to long-tail risks as a large element of the expected profit, or perhaps the entire profit, may be in the income on reserves and capital.

Investment income can be modelled explicitly within a stochastic model, given development patterns for premiums, commissions, expenses and claim payments. Paid claim development patterns can be generated directly from reserving techniques on aggregate paid claims to the layer, or by modelling payment patterns on FGU claims in a stochastic rating model and pushing the simulated FGU payments through the reinsurance programme.

An investment income model, such as a Wilkie-type random walk model, can allow for varying returns.

Discounting, in traditional actuarial literature, has used a risk-adjusted rate in excess of a suitable risk-free rate. There is an increasing body of actuarial literature taking an alternative view, that discounting ought to be at the risk-free rate alone (allowing for the yield curve). Mango (2003) [7] uses the phrase “conditional certainty”, meaning that each scenario is in effect a series of certain cashflows and ought to be discounted at a risk-free rate. The uncertainty in the premium rate comes from which

scenario of all possible scenarios the future will most closely resemble, and this is modelled by the range of outputs of a cashflow simulation model.

This argument mirrors the approach taken in determining capital requirements above and in the new international accounting standards. The theme is taken up and expanded in greater detail elsewhere, see for example Halliwell (2004) [9].

## **6.8. Reinsurance design – aggregate claim features**

Aggregate claim features allow insurers and reinsurers to adjust the level of risk that each carries in relation to the other, generally by reducing the likelihood and/or severity of a loss to the reinsurer. Examples are aggregate limits, inner aggregate deductibles, paid reinstatements, profit commissions and swing-rated premiums.

### Risk

Accurate rating for reinsurance design needs knowledge of the likely distribution of aggregate results to a particular layer and the correct understanding of how a particular feature operates.

The basic burning cost method gives partial information on the distribution of aggregate results, based on the actual experience over, typically, ten years of claims data. However, an inner aggregate deductible may reduce the rate based on historic claims to nil five years in ten, say, so that the actual rate is the average of only the five years with a non-zero rate. In practice, the basic burning cost method may be unreliable when rating for aggregate features.

Bootstrapping techniques can be used on the development triangle of losses to the layer before any aggregate claim feature has been applied. Doing so may enable more accurate rating of aggregate claim features owing to the large number of simulations that can be run. But bootstrapping and other techniques are applicable only if there is a large volume of data in the development triangle.

The individual claims projection method, if using the stochastic IBNER options in Section 3.2.1, does allow many more data points to be used. However, the aggregate results by year are likely to be limited to near the range of actual claims so the separate simulations are unlikely to give the full range of aggregate outcomes.

The frequency / severity simulation method, with sufficiently many simulations (see Robustness below), will provide the full range of aggregate outcomes, the associated distribution of the outcomes and a high degree of confidence in the average rate.

Exposure-based methods can allow for aggregate features but require much more computation than before. One approach is described in Mata *et al* (2002) [10].

### Return

As discussed in Section 4.2, exposure-based methods do not have an explicit profit test element to the method.

Of the experience-based methods, the requirement of being able to determine a rate given aggregate features in the reinsurance design means that the ability to profit test is in reality limited to the frequency / severity simulation method only. The technique is exactly the same as before, with a specific profit test on each simulation and the overall ability of a reinsurance programme to meet a target return given by the average return across all simulations.

### Range

As discussed before with exposure-based methods, they are not suited to determining a range of results. With experience-based methods, the only way to derive a range is from a simulation approach.

### Robustness

Aggregate reinsurance features increase volatility, making robustness more of an issue. Under the simulation method, robustness requires that the number of simulations is enough that the results from two different runs are very likely to be the same to within an acceptable tolerance. The approach discussed in Appendix A.3 to determine a sufficient number of simulations applies with aggregate features too.

### Underwriter

Many underwriters have rules of thumb that they apply in cases where aggregate features are significant. Rules of thumb generally are applied where there is no other way of quickly or easily finding a more accurate answer. This is an area that actuaries can add a lot of value. The methods themselves are no more complex when dealing with aggregate features. So the main issue is finding a way that explains the effect of reinsurance design, for example showing a graph of the rates at various levels of confidence “before and after” a particular feature is applied.

## **6.9. Allocated loss adjustment expenses (ALAE)**

ALAE refer to the reinsurer’s share of reasonable costs of defending a claim, such as legal fees. The reinsurer’s share is determined usually on one of two bases:

- (A) ALAE are added to the pure indemnity part of any claim and the combined value is the loss subject to the reinsurance, or
- (B) ALAE are recoverable in addition to the claim; it is split between reinsurer and reinsured *pro rata* to the share of the claim.

Reinsurers on high layers prefer basis B, as it is less likely that a claim reaches the higher layers.

Historic ALAE data also need revaluation for inflation, which may or may not be the same as claim inflation. If the development of ALAE data is available, then it can be revalued in the same way as claim development data (see Section 6.1), albeit using a different set of inflation assumptions. If not, ALAE data can be revalued either *pro rata* the claim to which it attaches, so that ALAE are implicitly revalued with claim inflation, or by applying revaluation factors on a cohort year basis.

Ideally, historic ALAE data forms a separate part of the submission to reinsurers, with development on a claim-by-claim basis. This enables the ALAE data to be revalued separately as above. It is acceptable for ALAE data to be included as part of the claim amount if ALAE form part of the subject loss (basis A), in which case revaluation is implicitly in line with claims inflation. Otherwise (i.e. basis B), ALAE data must be stripped out of the claims data.

On basis A, once the ALAE data have been revalued and combined with the revalued pure indemnity part of the claims data, the experience-based methods proceed as before. On basis B, an additional step of estimating ALAE loading to the layer loss cost is required, and the approach will vary by method.

On the burning cost method, if there is full ALAE development data, then a triangle of ALAE data applicable to the layer can be created and a suitable ALAE loading projected using standard reserving techniques. Otherwise, the loading will have to be an assumed fixed percentage of estimated claims to the layer.

For the individual claims projection method, for each projected claim the ALAE percentage loading can be selected at random from past claims. For the frequency / severity method, the simulated FGU claims can include a separate estimate of ALAE. The percentage load can be modelled randomly from a distribution fitted to the ALAE as a proportion of the FGU claims data, perhaps with different models fitted to small and large FGU claims. As for the burning cost method, an alternative approach is to assume an explicit ALAE loading.

With exposure-based methods, the reinsurer needs to be confident that the cedant has sufficient allowance for likely ALAE in its rates. Adjustment can be made as part of an overall profitability adjustment (see Section 4.1).

## **6.10. Non-insurance risks and loadings**

Rightly, given the sources of risk to a reinsurer, the great majority of this paper has its focus on estimating a pure risk rate, as would be the case for rating long-tail reinsurance contracts in practice. Doing so then leads onto capital required to support the marginal business and, perhaps, stochastic modelling for investment return too. But these are not the only risks. Section 2 lists many other risks to which a reinsurer is exposed.

The new FSA capital regime is the driver for more accurate estimation of other risk sources. Some of these are not reliant on marginal business: for example, operational risk is arguably independent of the volume of new business written.

But other risks are dependent on the marginal business: for example, bad debt risk increases with the reinsurer's use of its own retrocessional reinsurance cover and expense overrun is related in part to the volume of claims. Therefore, those risks that are related to the reinsurer's experience should be modelled explicitly and allowed for in the rate as an addition to the pure risk premium.

Some loadings, such as variable expenses (e.g. brokerage and the reinsurer's internal claim handling expenses) need to be allowed for as a proportion of premium or claims as appropriate.

Other loadings, such as a contribution to overhead expenses and an allowance for non-insurance risks that are not related to marginal business, need to be applied. The loading factor depends on business plans and on the estimated capital required to support the line of business. The loading factor itself is likely to be a fixed percentage of the gross premium.

## **6.11. Data availability and accuracy**

The main issues relating to data are its availability and accuracy, which must be acknowledged when considering the rate to be charged. Lack of credibility in either availability or accuracy may indicate further, hidden problems with the reinsured's business.

### **6.11.1. Data is not available**

Each method has minimum data that is required to perform the analysis. For example, experience-based methods require exposure data for at least those cohort years having historic claims data.

Lack of sufficient data reduces the credibility of the rating methods, increasing uncertainty surrounding projected rates, or meaning that some methods (such as any experience-based method) cannot be used reliably. This occurs for new lines of business for which there are insufficient data to project IBNER or IBNR or too few data points to fit frequency and severity curves. Similarly, the data exist in the reinsured's own systems but cannot be (or otherwise just isn't) extracted for the reinsurance submission.

Alternative techniques, such as exposure-based methods, or alternative data sources, such as IBNER factors from a similar portfolio or from industry-level data, can be used.

### **6.11.2. Data is not accurate**

Reinsurers do not receive full FGU data for a particular portfolio or risks that the reinsured itself writes. For example, only FGU claims that have ever exceeded some threshold will be submitted, rather than the full set of claims. This cuts back on spurious data but it does mean that the data is difficult to reconcile back to independent sources, such as the reinsured's Report and Accounts.

Some checks by the reinsurer of the partial FGU claims data are possible, if it has reinsured the portfolio previously. Claims data can be checked against actual claims submitted for payment. Data can be checked for reasonableness against submissions in previous years. Data can be benchmarked against other similar portfolios to assess reasonableness.

Other data can be checked: for example historic exposure data can be checked against FSA returns data.

Reinsurers generally have the right to inspect reinsured's policy records, for example in case of disputed claims. One of the aims of inspecting records is



to review the policy data against data submitted at renewal. If submission data is found to have varied significantly from policy records at the time of submission, then this by itself is likely to be grounds for cancelling the reinsurance on the grounds of lack of utmost good faith on the part of the reinsured.

It should be borne in mind that reinsurers are always going to receive data later than the reinsured, and that some delays are a feature of the business. Delays may be compounded where intermediaries are involved. Reinsurers though may be aware that a reinsured is exposed to a claim or type of claim (because of general market knowledge) and can allow for this in the rating.

### **6.11.3. Data adjustments**

Sections 6.1 and 6.2 discussed adjusting historic data to “as-if” by allowing for claims inflation and relative premium rate strength. More adjustments may be required if there are suspected to have been significant changes in the underlying risk. Examples are:

- Change in mix of business (eg a motor book taking on cover for high-performance vehicles).
- Change in policy terms and conditions (eg increasing policy deductibles) or in coverage.
- Changes in reserving philosophy (see Section 6.3.1).
- Latent claims (see Section 6.5).

Other distortions relating to the randomness of claims data have already been mentioned too, such as catastrophes and extreme event claims (see Section 6.4).

Data need to be adjusted for changes in risk (for example, applying new policy deductibles to revalued FGU claims) and for suspected distortions (for example, reducing the weight of extreme event claims in the analyses). Alternatively, the changes in risk or the distortions may render the historic data unusable, leaving just the exposure-based methods.

## **6.12. Combining results from different methods**

Having more than one independently derived result is, in itself, beneficial to the rating process, as it can either confirm or challenge a preferred rate. Where results from two or more methods differ, the reasons for the difference need to be explained. Once the differences can be accounted for, it should be possible to judge which method is more appropriate in the circumstances.

There are as well a few ways different methods can be combined:

### **6.12.1. Rating higher layers from a base layer**

For layers with lots of historic data, the experience-based methods will agree to within reasonable tolerances. For higher layers, the past data becomes sparser and the results from different methods are likely to diverge.

The rate for a working layer that has a high volume of past data can be used as a base for rating higher layers. In the case where an actual programme does not include a working layer, then a notional working layer can be invented, although the excess has to be not less than the inflated observation point of the data.

There are two main ways to rate higher layers from a base layer.

- Use the base layer to confirm that the frequency / severity simulation method gives a sensible result on the base layer so that, qualitatively at least, the results from the same method on higher layers are credible and reliable.
- Use ILF factors from an exposure scale to rate the higher layers relative to the base layer.

### **6.12.2. Creating an *a priori* rate for experience-based methods**

The experience-based methods can use an initial expected rate from exposure-based methods. In particular, the exposure-based rate can be used as a Bornhuetter-Ferguson initial expected rate for the burning cost method.

### **6.12.3. Blending results from exposure-based and experience-based methods**

Rather than discard the results from all methods except one, and take only the result from that one method, it is possible to take an average of the rates. The usual weight to give to the different rates is a Bayesian credibility factor. Unfortunately, the factors in most situations are not simple and must be estimated from numerical estimation methods.

The complications arise because there are two random variables: claim frequency and claim severity. Patrik & Mashitz (1990) [11] simplified the case by just considering claim frequency, using a Poisson distribution with a Gamma prior.

To include claim severity as well, so that the credibility weighting is based on the same variable as being estimated – aggregate claim amounts to a layer – is much more complex.

M. Cockroft (2004) [12] presents an extension of Patrik & Mashitz's work, using a Poisson distribution with a Gamma prior for numbers of claims and a two-parameter Pareto with a Gamma prior on the Pareto shape parameter for claim severity. The formulae are unwieldy and involve infinite series, but can be coded into a macro and converge reasonably quickly for practical use.

In many cases though, there has already been use of the results of one method in another. Examples, including cases mentioned above, are:

- The exposure curve has been calibrated from revalued historic claims, which may include claims data from the contract being rated.
- One way of rating higher layers is by applying ILF factors from a lower layer that has been rated using experience methods, so involving a combination of experience and exposure methods.
- The experience methods may use an initial estimate of the loss ratio from an exposure method.
- The exposure method may adjust the rate to allow for an estimate of the insurer's profitability based on a reserving exercise.

### **6.13. Consistency of approach between layers**

It is preferable for the rates for all the layers in a programme to be consistent relative to each other. If rates are not consistent, then the reinsurer is in danger of being selected against by the reinsured (or rather the broker). Layers for which the reinsurer is relatively high may be placed elsewhere in the reinsurance market, leaving only those layers that at best are just adequately priced. Overall, the effect is likely to reduce the expected profitability of the cover for the reinsurer.

Consistency may imply using the same method for each layer, and in this respect the burning cost needs to be considered as a separate method for each layer, owing to layers being rated one at a time.

The individual claim projection method can rate several layers simultaneously, but the rates for higher layers are unlikely to be reliable. Frequency / severity simulation and exposure-based methods are able to rate any combination of layers simultaneously. If any layer has complex, aggregate loss features, then only the frequency / severity method is able to accurately rate all layers simultaneously.

In discussing combining results from different layers, this paper mentioned further ways to rate different layers using a common method, such as using ILFs to project results from a base layer. Although the different layers are not rated using the same method (only the base layer is rated using the burning cost method, say), the rates between layers are consistent.

In addition, using credibility weighting, as described above, means that the rates for successive layers should be consistent: rather than jump from one method to another, credibility weighting smoothes the change in methodology.

### **6.14. Presentation of recommendations / feedback**

This paper has used underwriters' considerations as one of the 5 'R's against which the various methods were discussed. Actuaries who price reinsurance have frequent contact with underwriters. Reserving actuaries have regular communications and

meetings with underwriters too. But the actuary / underwriter interface is still too often a minefield, rather than a meeting of minds.

The many reasons, and possible solutions, were fleshed out in more depth by Wrenn *et al* (2003) [13]. Some of the suggested solutions for action by actuaries are:

- Spend time on the relationship: meet with the underwriter often, including time outside the office too.
- Understand the underwriter: what makes an underwriter tick?
- Understand the business: get to know the standard risks and rate lots of examples (even as practice).
- Use common language: get to know the jargon, use it, be familiar with it, and avoid actuarial jargon.
- Select the location: distance is a barrier, so be present nearby as much as possible.
- Explain the thought processes: explain assumptions and methodology, but in underwriters' terms, using real-life examples.

Presentation of recommendations to underwriters does not imply “dumbing down”. But using common language, examples, illustrations, and graphs may get the same point across more effectively and quickly than a page of numbers on which the most relevant information is less than prominent.

Underwriters are very close to the risks that they write, so will be familiar with specific issues and interested in changes or new issues that crop up. Actuaries can use an underwriter's knowledge, for example warnings about movements in large claims that may distort analysis.

The conclusion is that presentation of the results and getting an underwriter's buy-in is not just about making sure that the central message of, say, the technical rate is understood. Communicating concerns, such as the volatility of the rate to the inflation assumptions, is just as important and is easier if the relationship between actuary and underwriter is a positive one.

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## Appendices

### A.1. Sampling method for projecting individual FGU claims

#### A.1.1. Outline of method

The method samples development from prior open claims at a given point in development. If there are inflation-adjusted claims from  $n$  cohort years then, for a cohort year now  $l$  years old ( $l < n$ ), the development of open claims is

broken down into single future development periods:  $(l, l + 1)$ ,  $(l + 1, l + 2)$ , ...,  $(n - 1, n)$ .

For each open claim, at each future development period, an older claim is sampled at that development period (provided it was still open then) and its development in the subsequent period is used on the claim being projected. For example, if the sampled older claim increased in incurred value by 5%, then 5% is added to the claim being projected and the projection moves on iteratively to the next development period.

#### **A.1.2. Using claim status as a guide to projection**

As mentioned, the sampled claim needs to have been open at the start of the development step. This is obvious enough: open claims can develop whereas closed claims do not, assuming they are not reopened. The sampling can use the claim status in another way: if the sampled older claim settled (i.e. goes from open status to closed status) in the development period, then the claim being projected can also be “closed” at that point.

If claim status is not given in the data provided, it can be guessed from the development of the claims. There are various decision rules that can be applied, with the default status being “open”:

- If separate paid data is available, the claim can be said to be closed if the outstanding is nil. The period the claim closed is when paid first equalled the final incurred amount.
- If separate paid data is not available, the claim status can be taken as closed if the incurred value has not moved for the latest, say, two development periods. The period the claim settled is when the incurred first equalled the final incurred amount.

#### **A.1.3. Grouping claims by size**

The sampled claim should be of a similar size to the partially projected claim. This is because small claims and large claims develop in different ways:

- More senior, experienced claims handlers examine larger losses.
- Larger losses may relate to a different type of loss (e.g. a motor claim that includes bodily injury).

It is also worth noting that a small-value claim can multiply in size many times before it affects reinsurance layers; but apply the same large development factor to a claim already in the reinsurance layers and its size will become extreme.

Therefore it is advisable that claims are banded by size. The sampled claim should belong to the same band as the claim being projected at that point in the development. Typically three bands (low, medium and high) are sufficient to ensure few or no claims are projected to spuriously high values.

Setting the claims bands is an inexact science at best. The low/medium threshold can depend on where initial claims reserves are set by the reinsured or authority levels for claim staff. Typically, the low/medium thresholds are 5% of the lowest excess point of the programme or about £10,000 (or equivalent in the contract currency). The medium/high threshold typically can be set initially so that approximately 10% of claims fall in the high category.

Thereafter, trial and error can be used to determine the number and boundaries of the bands. If banding claims fails to stop extreme values being produced, then it may be necessary also to amend the historical data, especially in the tail of the oldest claims, so that the development is smoother

## **A.2. Normalisation of projected individual FGU claims**

The combined total of a set of individually projected FGU and IBNR claims is unlikely to be the same as that for projected aggregate FGU claims. For most situations, trial and error in selecting process parameters (such as claim size bands, see above) can give a combined total to within a tolerance of, say, 5% of a projected aggregate FGU amount. However the combined total can be and often is outside this range. An optional extra stage is to “normalise” the total projected FGU and IBNR claims. Normalisation here means adjusting the combined total of a set of individual claim ultimates to agree with a projected aggregate FGU amount.

If  $S_{ind}$  represents the sum of FGU and IBNR claims from the projected individual claim method, and  $S_{agg}$  represents the projected aggregate FGU claims, then normalising is multiplying the results from the individual claim projection by  $S_{agg} / S_{ind}$ . The factor can be calculated once for all years combined or separately for each cohort year.

However, care must be taken in how the adjustment is made. Adjusting the projected FGU and IBNR claims before pushing them into the reinsurance layers is likely to change the overall proportion of claims hitting the layers. Instead, the adjustment factor should apply to losses to the layer, i.e. to losses after the layers have been applied. The same effect can be made by giving all projected FGU and IBNR claims a weighting of  $S_{agg} / S_{ind}$  in subsequent stages of rating.

## **A.3. Determining a sufficient number of simulations**

There is a trade off between a high number of simulations to achieve some form of precision (the average result does not unreasonably change when the same number of simulations is rerun) and a low number of simulations for speed and efficiency. Clearly, more simulations are required for higher layers than for lower layers to achieve the same degree of precision.

The required number of simulations depends on the expected variation in the result. From the Central Limit Theorem, the distribution of the average result of the simulation will tend towards a normal distribution. The variance of the average varies according to a  $\sigma/\sqrt{N}$  rule where  $\sigma$  is the standard deviation of the average result and  $N$  is the number of simulations.

One way of deciding how many simulations is enough is to continue running simulations until the average result is unchanging within a predetermined tolerance. For example, the simulation process can be iterated indefinitely until the average result from  $N$  simulations is the same as the average result from first  $(N-100)$  simulations to within 0.1%. Clearly, some method of preventing the simulations carrying on indefinitely is required, for example imposing a very large upper bound to the number of simulations.

But the required number of simulations can be determined during the exercise, even with no prior knowledge of the likely variation in average result. This approach is suggested in Papush (1997) [14]. The mean and unbiased sample variance from running, say, 1,000 simulations can be used to approximate the mean and variance for the average result. Hence for a given tolerance, say 2% of the mean, and a required confidence level, say 95% (equivalent to 1.96 standard deviations in a standard normal distribution), the required number of simulations is calculated as follows:

$$N \geq \left( 1.96 \times \frac{\bar{\sigma}}{2\% \times \mu} \right)^2$$

If the estimate average result from the first 1,000 simulations is £4.5m and the estimate standard deviation is £2.5m, then the required number of simulations is 2,965, which can be rounded up to 3,000 simulations.

#### **A.4. Empirical distribution of claim sizes**

The empirical distribution can be derived several ways, but one of the simplest is to set a scale that covers all or most of the data set and calculate the cumulative probability of observing a sampled data point at regular intervals along the scale.

The procedure more explicitly is, given a set of  $N$  data points,  $X_1, \dots, X_N$ , all  $X_i > T$ , generate  $M$  points  $Y_1, \dots, Y_M$ , with  $M \leq N$ , along a scale as follows:

*Step 1:* Set  $Y_1 \geq \min_{i=1, \dots, N} (X_i)$ .

*Step 2:* Set  $Y_k = Y_{k-1} + \Delta$ ,  $k = 2, \dots, M$ , with  $\Delta$  a constant such that  $Y_M \leq \max_{i=1, \dots, N} (X_i)$ .

*Step 3:* Calculate the empirical probability  $P_k$  for each  $Y_k$  from the conditional sum  $\sum_{i=1, \dots, N} (1 | X_i \leq Y_k) / N$ .

The empirical distribution, the series of pairs  $(Y_k, P_k)$ , has data points spread evenly along the range of the distribution. So the fit of a given distribution to the empirical distribution will have greater bias towards the tail than the fit based on the original projected individual claims.



## A.5 Proportional hazards transform for risk margin loading

The proportional hazards (PH) transform method uses the assumed distribution of claim severity and a risk aversion factor. Wang (1997) [8] uses a factor in the range  $[0, 1]$ , with a lower factor representing increasing risk aversion. More recent work uses the reciprocal, with the range  $[1, \infty)$ .

If  $F_X(x)$  is the cumulative distribution function of the severity of FGU claim, then define  $S_X(x)$  as the survival function, or decumulative distribution function  $1 - F_X(x)$ . The PH transform of  $X$  is  $H_\rho(X)$  defined by:

$$H_\rho(X) = \int_0^\infty (S_X(t))^{1/\rho} dt, \text{ where } \rho \geq 1 \text{ is the risk aversion factor.}$$

Increasing  $\rho$  increases the value of  $H_\rho(X)$ . For  $\rho = 1$ ,  $H_\rho(X)$  is the mean of the distribution of  $X$ ,  $E(X)$ . As  $\rho \rightarrow \infty$ ,  $H_\rho(X) \rightarrow \max(X)$ , which may be unbounded. Other useful properties of the PH transform are:

- Scale and translation invariance:  $H_\rho(aX + b) = aH_\rho(X) + b$  for any  $a, b \geq 0$ .
- Sub-additivity:  $H_\rho(X + Y) \leq H_\rho(X) + H_\rho(Y)$ .
- Layer additivity: if risk  $X$  is divided into layers  $(l_1, l_2], \dots, (l_{n-1}, l_n]$ , then  $H_\rho(X) = \sum_i H_\rho(X_i)$ , where:

$$X_i = X_{(l_{i-1}, l_i]}, \text{ and } X_{(D, U]} \equiv \begin{cases} 0 & , X \leq D \\ X - D & , D < X \leq U \\ U - D & , X > U \end{cases}$$

$$\text{and } H_\rho(X_i) = \int_{l_{i-1}}^{l_i} (S_X(t))^{1/\rho} dt.$$

- Decreasing risk-adjusted premiums: for layers  $(a, a + h]$  and  $(b, b + h]$ ,  $a < b$ ,  $H_\rho(X_{(a, a+h]}) > H_\rho(X_{(b, b+h]})$ .
- Increasing relative load: for layers  $(a, a + h]$  and  $(b, b + h]$ ,  $a < b$ :

$$\frac{H_\rho(X_{(a, a+h]})}{E(X_{(a, a+h]})} < \frac{H_\rho(X_{(b, b+h]})}{E(X_{(b, b+h]})}$$

The last three attributes mean that a programme of layers has a constant total risk-adjusted rate regardless of the division into layers, that the risk-adjusted premium decreases as the excess point increases, and that the relative risk-adjustment increases with excess.

Risk-adjusted rates using the PH transform are easy to calculate for some claim severity distributions:

- Pareto distribution with parameters  $(\psi, \lambda)$  and risk aversion factor  $\rho$  is transformed to risk-adjusted distribution Pareto with parameters  $(\psi/\rho, \lambda)$ :

$$[S_X(t)]^{1/\rho} = \left[ \left( \frac{\lambda}{\lambda + t} \right)^\psi \right]^{1/\rho} = \left( \frac{\lambda}{\lambda + t} \right)^{\psi/\rho}$$

- Burr distribution with parameters  $(\psi, \lambda, \alpha)$  and risk aversion factor  $\rho$  is transformed to risk-adjusted distribution Pareto with parameters  $(\psi/\rho, \lambda, \alpha)$ :

$$[S_X(t)]^{1/\rho} = \left[ \left( \frac{\lambda}{\lambda + t^\alpha} \right)^\psi \right]^{1/\rho} = \left( \frac{\lambda}{\lambda + t^\alpha} \right)^{\psi/\rho}$$

- Weibull distribution with parameters  $(\alpha, \beta)$  and risk aversion factor  $\rho$  is transformed to risk-adjusted distribution Pareto with parameters  $(\alpha, \beta\rho^{1/\alpha})$ :

$$[S_X(t)]^{1/\rho} = \left( \exp \left[ - \left( \frac{t}{\beta} \right)^\alpha \right] \right)^{1/\rho} = \exp \left[ - \left( \frac{t}{\beta\rho^{1/\alpha}} \right)^\alpha \right]$$

Other distributions, including the Lognormal, need the PH transform to be calculated by numerical methods.

The PH transform method with a risk aversion factor in the range 1.5 to 1.8 matches real-life reinsurance rates across a programme of layers in very many cases.