# THE INSTITUTE OF ACTUARIES 

# REPRODUCTIVITY IN GREAT BRITAIN: A NEW STANDARD OF ASSESSMENT 

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## INTRODUCTION

Since the formation of the Demographic Study Group by the Students' Society at the end of 1949, much attention has been devoted by a small but keenly interested body of members of the Institute to the analysis of fertility and reproductivity. One of the aims of their researches has been to arrive at an independent method of assessing the degree of sufficiency of the numbers of births in this country for the replacement of the population. The present paper, founded as it is upon the ideas expressed at the Group meetings, is in essence a joint production with members of the Group, and their share in it is gratefully acknowledged.
2. The nature of the problems that have attracted the attention of the Group may be illustrated by recalling briefly some of the fundamental points in the investigations of the Royal Commission on Population; the activities of the Commission did much to advance the study of population replacement, and in particular to point out the main difficulties inherent in the subject with which it is not possible adequately to cope by means of the conventional approaches based on the net reproduction rate and similar measures. As a consequence two things have become apparent:
(a) in recent years there have been large variations from year to year in the numbers of births and marriages, as a result of which indices of fertility have fluctuated considerably in size; although such fluctuations render the true current situation uncertain, they may be of relatively little consequence in the long run-for instance, when the history of a generation is summed up;
(b) at any time there are in existence many sets of marriages of different durations and therefore at different stages in the process of having children; it is of doubtful validity to attempt to measure reproductivity by condensing the experiences of all these sets of marriages into a single index figure.

The Royal Commission in their Report( r ) did, however, make an effort to find one precise indication of current reproductivity in Great Britain; on certain assumptions the conclusion was reached that 'the deficiency in the present size of the family below that required in the long run for replacement may be roughly computed as being of the order of six per cent'.
3. The object of the present paper is to trace statistically the various elements contributing to population replacement. First, the uses and methods of construction of self-reproducing models are discussed; then some statistical models, built up in the manner described, are used as standards of reference in a new approach to the examination in detail of population replacement in Great Britain in recent years. Differences between what has actually occurred and what would be expected to happen according to the standards may be
traced approximately, as profits and losses are traced at an actuarial valuation of an insurance fund. In its application to the two important features mentioned in $\S 2$, the method permits
(a) the total numbers of children born so far in families not yet completed to be contrasted with the numbers of children in the models at corresponding marriage durations, thus enabling a generation analysis to be made instead of a mere study of secular variations, and
(b) the experience of different sets of marriages to be separately compared with the standards, thus avoiding the summation of all the data into one toocomprehensive index.

The idea of using statistical models in demography is not, of course, new. Recently, for example, Brown (2) constructed a self-reproducing population model as an aid to the understanding of the long-term housing requirements in the New Towns; it showed the relative numbers of families of various sizes. The existence of this earlier work has been of encouragement and assistance in the preparation of the present paper, to which it has certain basic similarities.

## ELEMENTS OF SELF-REPRODUCING POPULATION MODELS

4. The various elements in the construction of standards for the assessment of fertility and reproductivity will now be briefly described, even though they form part of the life-cycle with which everyone is familiar. First, one may consider the average number of children born to a married couple who are not separated by death or divorce before the wife reaches an age (taken here as 45) beyond which the chance of her having a child is very slight. In a selfregenerating population in which most people marry this cannot be far removed from two. Next there is the proportion of children who themselves marry in due course and so found the succeeding generation. As the proportions of the sexes that marry are not equal, it is necessary to distinguish between males and femalcs, and the relative numbers of boys and girls born will be the starting point. At this stage it is convenient to introduce a simple adjustment for illegitimate births.

The proportion of children who eventually marry may be regarded as the product of two factors for each sex: the proportion who do not die unmarried before an assumed upper age-limit for marriages (taken for convenience as 65 ) and the proportion of these 'survivors' who marry before that age. Some adjustment for migration should perhaps be added to the allowance for the chance of death before age 65. It must be remembered that the sexes marry in equal numbers and in approximate accordance with a conventional pattern of husbands' and wives' ages. Finally, account should be taken of the effects of widowhoods and divorces in reducing fertility below that of couples who are not so separated while still fertile; this reduction will be diminished to some degree on account of remarriages.
5. In order to relate the models as closely as possible to the main features of the current demographic situation, including the immediate past and perhaps also the next few years to come, the latest experience should be used in the bases. In certain instances, however, this may be prevented by the absence of suitable data. For this reason, and in order to examine the effects of any possible inconsistencies between the assumptions about marriage and
fertility on which the models are founded, the changes brought about by variations in basis should be tested, thus providing alternative models.

To be of practical utility, any models must not only be constructed with the aid of such relevant data as are available but also be arranged in a form suitable for ready comparison with regularly published marriage and fertility statistics. The three principal characteristics by which fertility data in this country are classified when a legitimate child is born are the mother's age, the duration of the marriage and the number of her previous children. To pay attention principally to number of children in the family, although perhaps desirable in theory, is unsatisfactory in practice owing to the lack of a sufficiently accurate exposed to risk against which to measure the number of birth registrations every year, classified solely by the number of children the mother has already had. Analysis of fertility mainly by age of mother provides an important method; it may perhaps be argued that, in some respects at least, it is the most fruitful means of approach to the subject of reproductivity and is therefore worthy of further research. Owing mainly to the form of the fertility data which are the most plentiful at present, however, the remaining characteristic, duration of marriage, has been adopted as the principal item of reference in constructing the models. For the more extended analyses, married women have been subclassified according to their age at marriage and number of children.

Fertility data going beyond the scope of those regularly published by the Registrar General are necessary for the work in hand; statistics are required, for instance, of the numbers of children born to mothers of various ages at marriage during each year of married life. The Family Census of 1946(3), conducted under the auspices of the Royal Commission on Population, is the latest inquiry to provide such material, and it is fortunate that the Director of the Family Census has permitted some access to the data in advance of their publication; the necessary researches have thus been enabled to proceed without delay. The use made of these data in this paper is entirely unofficial. It is understood that an official report on the Family Census is to be published before long.
6. As indicated in $\S 4$, an important item in the construction of a statistical model of the kind under consideration is the total number of children born on the average to families of completed fertility. If it is a selfreproducing model this will be the reciprocal of the number of marriages per birth, i.e. of the proportion marrying, subject to minor adjustments. There will thus be a fixed relationship between the average size of completed family per married couple and the proportion of children that eventually marry. This relationship offers alternative approaches to the construction of a model. Either a certain ultimate level for the proportion marrying may be assumed and then the average number of children per couple deduced so as to ensure exact replacement, or the average number of children per couple may be decided upon first and the corresponding proportion marrying ascertained afterwards. Owing to the nature of the available data it is somewhat more convenient to adopt the first of these courses rather than the second.
7. Table 1 shows, according to recent experience in this country, the average numbers of children in the completed families of couples whose marriages were not broken by death or divorce before the wife attained age 45 , classified according to the age of the wife at marriage. The bracketed figures
refer to marriages where the age-limit had not quite been reached; they have been included because they add to the information without rendering it materially inaccurate. The Family Census data from which the figures have been derived have not been adjusted in any way, for instance on account of the underestimation of childlessness that is known to have been a feature of the enumeration; such errors are unlikely to have had an important influence upon the comparison of the relative fertilities of the different marriage ages. Only first marriages of women are included in the table.

Table x . Average numbers of children born to married women, according to mother' age at marriage and year of marriage, Great Britain. (Data applicable to wives married for the first time, who were not separated from their husbands by deatl or divorce before reaching the age of 45)

| Age of wife at marriage | Period during which marriage took place |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $1900-09$ |  | 1910-14 |  | 1915-19 |  | 1920-24 |  | 1925-29 |  |
|  | Number of children | Index | $\begin{aligned} & \text { Number } \\ & \text { of } \\ & \text { children } \end{aligned}$ | Index | $\begin{aligned} & \text { Number } \\ & \text { of } \\ & \text { children } \end{aligned}$ | Index | Number of children | Index | Number of children | Index |
| Under 18 | 5.16 | 180 | 4.63 | 185 | (4.08) | ( 183 ) | - | - | - | - |
| 18-20 | $5 \cdot 20$ | 181 | $4 \cdot 57$ | 183 | 3.95 | 177 | - | - | - | - |
| 20-22 ${ }^{\frac{1}{2}}$ | $4 \cdot 27$ | 149 | $3 \cdot 79$ | 152 | $3 \cdot 29$ | 148 | (2.98) | (148) | - | - |
| 22 $\frac{1}{2}-25$ | 3.41 | 119 | $3 \cdot 00$ | 120 | $2 \cdot 70$ | 121 | 2.41 | 119 | - | - |
| 25-27 ${ }^{\text {d }}$ | $2 \cdot 87$ | 100 | $2 \cdot 50$ | 100 | $2 \cdot 23$ | 100 | 2.02 | 100 | ( $1 \cdot 74$ ) | (100) |
| $27 \frac{1}{2}-30$ | 2.49 | 87 | $2 \cdot 21$ | 88 | I•99 | 89 | I.78 | 88 | 1.54 | 89 |
| 30-35 | $2 \cdot 06$ | 72 | 1.83 | 73 | I. 68 | 75 | I 50 | 74 | 1.27 | 73 |
| 35-40 | 1.27 | 44 | $1 \cdot 07$ | 43 | 1:81 | 45 | . 88 | 44 | 75 | 43 |
| 40-45 | -69 | 24 | $\cdot 46$ | 18 | '40 | 18 | 40 | 20 | 3 I | 18 |

The table shows, not unexpectedly, a marked association between age at marriage and family size; the younger the age at marriage, the longer the period of exposure to risk of fertility, and the greater the number of children. The extent of the association appears to have changed little from the marriages of 1900 to those of 1929 . The indices, expressed for convenience as percentages of family size at ages $25-27 \frac{1}{2}$ at marriage, altered hardly at all from 1900 to 1929 in spite of the fall of about $40 \%$ in the average number of children per mother during that period. A preliminary inspection of the sample results of the 1951 Census (4) reveals that in all probability the index numbers have not materially changed for marriages during the nineteen-thirties. These figures may well have been influenced to some extent by the exercise of 'selection' in limiting attention to mothers who have survived to the Census date. Table $A_{3}$ in the Appendix shows, however, that the age-distribution of women's marriages changed but little during the period 1900-29 and Table A i demonstrates the relative unimportance of changes in the proportion of women ultimately marrying, according to the experience of those years; such factors as these may not, therefore, have influenced to any important degree the indices of numbers of children per mother according to the mother's age at marriage that are given in the Table. (An explanation of how the index numbers might be affected by such changes is given in $\S A_{9}$ of the Appendix.)

On the whole, the association expressed in these index numbers seems to be a feature of which cognizance should be taken in the construction of the
models in contemplation. The series of indices for 1915-19 has, in fact, been adopted as onc alternative for use in the succeeding calculations.
8. The association between the age-distribution of mothers and the indices of their average numbers of children is only one of a series of interrelationships among the various factors that enter into the computation of model populations of the kind under consideration. Because of these interrelationships the bases of the calculations need to be selected with care and to be discussed in appropriate order in some detail. As the main text should not be encumbered with such matters of procedure, the examination of the following bases has been relegated to the Appendix: sex-ratio at birth, mortality, migration, proportion marrying, age-distribution of marriages, and termination of marriage by death and divorce. For the reasons there given, the population models discussed helow are founded upon the following assumptions:
(a) that 106 boys are born for every 100 girls;
(b) that mortality is in accordance with the experience of Great Britain during the period $1948-50$ and will be higher for single men than for all men and higher for single women than for all women;
(c) that on balance inward and outward migratory movements cancel each other;
(d) that $90 \%$ of men and $95 \%$ of women marry, in two alternative agedistributions;
(e) that $95 \%$ of births are legitimate and $5 \%$ illegitimate;
$(f)$ that marriages are dissolved by death and divorce to the same general extent as in Great Britain during recent years.

## CONSTRUCTION OF THE MODELS

9. The first steps in the construction of the models are illustrated in Table 2. The first column gives the number of first marriages of women in a marriage and mortality table commencing with 1000 female births, on the assumption that $95 \%$ of those who do not die before attaining age 65 marry and that their age-distribution at marriage is the same as that of first marriages in Great Britain during the years 1915-19. On this basis, the number per thousand female births of first marriages of women at ages under 45 is 873 . The addition for remarriages-obtained in the manner described in §A6 of the Appendix-may be put at 60 . For the model to be self-reproducing the wives of these marriages must bear 1957 live legitimate children, that is, the $95 \%$ of legitimate children among the 1000 girls and 1060 boys who commence the nuptiality table. The number of children per marriage is thus $2 \cdot 10$ on the average.

The national fertility statistics with which the model is to be compared when it has been completed have to be expressed in the form of the total numbers of births at each marriage duration divided by the corresponding number of original marriages; the fertility rates so obtained are adversely affected by any separations of couples that may be caused by death or divorce. In this computation, first marriages and remarriages cannot be distinguished from one another. To construct the model, however, it is appropriate first to use the data for non-separated couples and then to adjust for the effects of separations. Column (2) of Table 2 shows the adjusting factors referred to in §AIr of the Appendix, and column (3) shows the number of unbroken marriages equivalent to the 873 first marriages mentioned above.

Losses of children of remarriages due to separations of remarried couples may be ignored, and in passing from column (3) to column (4) all the 60 remarriages are added. The total of effective unbroken marriages under age 45 is thus 889 , and so the average completed family size of an unseparated couple is 1957/889, or $2 \cdot 20$. If marriages at ages 45 and over are included, the average family size becomes 1957/913, or 2.14.

Table 2. Population Model A-Some basic calculations

| Age of woman at marriage | Number of women's first marriages if $95 \%$ marry (1915-19 age-distri(I) | Proportionatc loss of children through breakage of marriage <br> (2) | Equivalent number of unbroken marriages $\underset{(\mathrm{I}-(2))}{=(\mathrm{I})}$ (1-(2)) <br> (3) | Col. (3) adjusted to include remarriages <br> (4) | Indices of relative family size (1915-19) (from Table r) <br> (5) | Equivalent number of marriages at 25-271 (4) $\times(5)$ <br> (6) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 16-20 | 67 | -07 | 62 | 62 | I.80 |  |
| 20-22 $\frac{1}{2}$ | 198 | -06 | 186 | 188 | 1.48 | 278 |
| 22 2-25 | 219 | -05 | 208 | 211 | 1.21 | 255 |
| 25-272 | 160 | -05 | 152 | 161 | 1.00 | 161 |
| 27-1-30 | 90 | -04 | 86 | 99 | $\cdot 89$ | 88 |
| 30-35 | 87 | -03 | 84 | 102 | 75 | 76 |
| 35-40 | 36 | $\bigcirc 2$ | 35 | 45 | '45 | 20 |
| 40-45 | 16 | -OI | 16 | 21 | - 18 | 4 |
| Total | 873 |  | 829 | 889 |  | 994 |

10. To obtain the average number of children per marriage at a given age of woman that corresponds to $2 \cdot 14$, it is necessary to use the figures in column (5) of Table 2. The weighted average index is 994 divided by $9 r_{3}$, or $\mathrm{r} \cdot 09$, and thus the average number of children per marriage required for a self-reproducing population is 2.14 divided by 1.09 and multiplied by the index figure in column (5); it ranges from 3.53 for those marrying at ages under 20 to $\cdot 35$ for those marrying at ages $40-45$. The indices of family size are assumed to apply to remarriages as well as to first marriages.
in. The smaller the average family size of a group of married couples the lower is likely to be the proportion of children of the higher birth-orders that it contains. The Family Census data shown in Table A4* in the Appendix to this paper bear this out. Table A 5 * shows that the proportionate distribution of first births by duration of marriage varied, over the period shown, relatively little by age of mother at marriage or by time of marriage, and the same is broadly true of second births, third births and so on. Naturally, however, the higher the ordinal number of child the later it arrives.

For the purpose of constructing the model populations it seems desirable to express the average numbers of children per mother at different marriage ages in terms of distributions by parity (i.e. by number of children) with reference to Table $\mathrm{A}_{4}$ and thus determine by means of Table $\mathrm{A}_{5}$ the proportions of the average family that have arrived at specified marriage durations. Table 3 shows the average completed family sizes required from unbroken

[^0]marriages for exact replacement in Model A-these follow directly from Table 2-and the corresponding proportions of couples with no children, one child, two children and so forth. In the lower part of the table the calculation of the number of children per couple at any given marriage duration is illustrated by reference to the position at the end of the fourth year of marriage.
12. To refer in more detail to Table 3, the percentages of married women with various numbers of children must be found to conform with the average family sizes shown in the top line of the table, and with the distribution data given in Table A4. The distributions can be assigned without a large margin of error; that in the 'total' column was obtained by using as weights the proportions of marriages in the various age-groups, derived from column (4) of Table 2 and shown in the third line of Table 3.

In order to be able to apply factors, similar to those shown in Table A5, so as to determine the proportions of the completed families that are born after specified marriage durations, it is necessary to sum the birth-order distributions and thus calculate the proportions of married women with at least one child, at least two children and so on. The results of such summations are set out in the next segment of Table 3. Finally, the relative numbers of children of each birth-order at the end of the fourth year of marriage, obtained by applying percentages of the kind illustrated in Table A 5 , are shown; figures may of course be similarly obtained for any other marriage duration, and the results of doing so in respect of Model A are summarized in the upper part of Table 4. The corresponding results for two alternative models (termed Models B and C respectively) appear in the lower part of Table 4. The differences of assumption on which these other models have been based are described in the Appendix, § A9, and may be referred to, briefly, as:

Model B: the same as Model A in all respects except that the age-distribution of marriages is much younger.

Model C: the same as Model B except that the indices in column (5) of Table 2 have been replaced by a new series that varies less from age to age.

The three alternatives give closely similar results in regard to the numbers of children of each birth-order at the various durations.

The figures in Table 4 are all founded on the same assumptions as to the proportion of women who marry, and therefore they do not differ in regard to the average number of children per married woman whose period of fertile life is over. Evidently such changes as are likely to occur do not materially alter the weighted means. This is of great advantage in subsequent work, for whatever the actual age-distribution of women's marriages in a given year, a model can be found to match it and yet give the same result as Model A for all ages at marriage in combination in regard to the proportion of children born at the end of any period of years after marriage.
13. The fertility data for England and Wales collected under the 1938 Act, which related mainly to maternities, were analysed by marriage duration and birth-order in the paper of March 1951 (5). In the present connexion it is necessary to bring in the Scottish figures and also to adjust for the relatively small number of cases in which no statement of marriage duration or number of previous children was made when the birth was registered. Over the period 1938-50 about $\cdot 5 \%$ of maternity registrations were unaccompanied by this
Table 3. Population Model A-average size and corresponding distribution according to number of children of (a) com-

| Age at marriage | 16-20 | 20-22 ${ }^{\frac{1}{2}}$ | 22-25 | $25^{-27 \frac{1}{2}}$ | 27-30 | 30-35 | 35-40 | 40-44 | Total (including 45 and over) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Average number of children per family (from Table 2) | 3.53 | $2 \cdot 91$ | $2 \cdot 38$ | $1 \cdot 96$ | ${ }^{1} 75$ | 1*47 | -88 | $\cdot 35$ | 2.14 |
| Proportion of marriages in age-group (from Table 2) | .o68 | - 206 | -231 | '177 | $\cdot 108$ | -112 | -049 | $\cdot 023$ | 1.000 |
| Percentage of married women with undermentioned number of children in completed family |  |  |  |  |  |  |  |  |  |
| Number of children |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & \circ \\ & \mathrm{I} \end{aligned}$ | ${ }_{12}^{4}$ | 6 18 | 11 22 | 16 28 | 21 29 | 30 28 | 55 22 | 80 12 | 20 |
|  | 21 | 26 | 28 | 26 | 24 | 21 | 11 | 4 | 24 |
| 3 | 19 | 18 | 17 | 16 | 15 | 13 | 7 | 2 | 15 |
| 4 | 14 | 12 | 10 | 7 | 6 | 4 | 3 | $\underline{1}$ | 8 |
| 6 and over | 12 18 | -9 | 6 | 4 3 | 2 3 | 2 | I | $\underline{1}$ | 5 |
| Total | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| Percentage of married women with at least the undermentioned number of children in completed family |  |  |  |  |  |  |  |  |  |
| Number of children |  |  |  |  |  |  |  |  |  |
| 1 | 96 | 94 | 89 | 84 | 79 | 70 | 45 | 20 | 80 |
| 2 | 84 | 76 | 67 | 56 | 50 | 42 | 23 | 8 | 58 |
| 3 | 63 44 | 50 32 | 39 22 | 30 14 | 26 11 | 21 8 | 12 5 | 4 | 34 19 |
| 5 | 30 | 20 | 12 | 7 | 5 | 4 | 2 | I | 11 |
|  | 18 | 11 | 6 | 3 | 3 | 2 | 1 | - |  |
| Total (including higher orders) | 353 | 291 | 238 | 196 | 175 | 147 | 88 | 35 | 214 |
| Percentages similar to those immediately above but at the end of the fourth year of marriage |  |  |  |  |  |  |  |  |  |
| At least I child |  | 83 | 74 | 70 | 64 | 62 | 42 | 19 | 68 |
| At least 2 children | 47 | 38 | 32 | 26 | 25 | 23 | 12 | 4 | 29 |
| At least 3 children At least 4 children |  | 7 | 4 | 4 | 4 | 5 | 5 | 2 | 5 |
| At least 4 children Average family size | $\stackrel{1}{1} 48$ | 1 1.29 | $\stackrel{\mathrm{I}}{\mathrm{I}} \mathrm{I}$ | $\stackrel{\mathrm{I}}{1}$ | I <br> 0.94 | 1 0.91 | I 0.60 | 1 0.26 | 1 1.03 |

information；the apportionment of the non－stated cases each year rateably among the various birth－orders and marriage durations gives rise to no difficulty．

In England and Wales in 1938 about $3.8 \%$ of maternities were still－births， but $1.2 \%$ arose from multiple pregnancies；thus the deduction necessary in order to pass from maternities to live－births was about $2 \frac{1}{2} \%$ ．This had fallen by 1950 to $1 \%$ ．The corresponding figures for Scotland have been slightly higher．The actual data have been suitably adjusted to a live－births basis．

Table 4．Number of children per hundred non－separated couples according to Models A，B，and C，by order of birth and duration of marriage

| Order of birth | Marriage duration in completed years |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 4 | 7 | 10 |
|  | Model $A$ |  |  |  |  |
| 1st child | 28 | 52 | 68 | 76 | 77 |
| 2nd child | 2 | 6 | 29 | 47 | 53 |
| ${ }^{3}$ 3th child child |  | 二 | 5 | 19 | 27 12 |
| 5 th child |  | － | － | 1 | ＋ 5 |
| 6 th child | － | － | － | － | I |
| Total | 30 | 58 | 103 | 149 | 175 |
|  | Model B |  |  |  |  |
| rst child | 33 | 56 | 71 | 77 | 79 |
| 2nd child | $\stackrel{2}{-}$ | 7 | 24 | 47 | 51 |
| $4{ }^{\text {3 }}$ th ch child | 二 | － | ${ }_{1}$ | 17 6 | ${ }_{11}^{26}$ |
| 5 th child | － | － | － | I | 6 |
| 6th child | － | － | － | － | 1 |
| Total | 35 | 63 | Lo | 148 | 174 |
|  | Model C |  |  |  |  |
| 1 Ist child |  |  | 72 | 79 | 8 I |
| 2nd child | 2 | 7 | 29 | 49 | 53 |
| $3^{\text {rd }}$ child | － | 二 | 5 | 18 | 27 |
| ${ }_{\text {th }}{ }^{\text {th }}$ chinild | － | 二 | － | I | 12 5 |
| 6 th child | － | － | － | － | 2 |
| Total | 36 | 64 | 107 | ${ }^{153}$ | 180 |

A series of factors giving numerical expression to the effects of deaths and divorces of married couples on the numbers of children born have been obtained in a way consistent with the adjustments mentioned in §Air of the Appendix．The factors and the adjusted numbers of children per family in Model A are shown in Table 5.

## COMPARISON OF RECENT FERTILITY WITH EXPECTATIONS ACCORDING TO THE SELF－REPRODUCING MODELS

14．In Table 6 the actual numbers of children per couple according to the experience of the marriages in Great Britain in recent years have been expressed as percentages of the numbers of children per couple required for
population replacement according to Model A. As has been indicated, the alternative Models B and C are similar to A and their use would give similar results. This comparison is not quite in its final form, howcver, for there are one or two respects in which the bases of these models do not fit closely to actual demographic history in recent years. Over the period in question illegitimate births have averaged $6 \%$ and not $5 \%$ of all live births. The proportion of women ultimately marrying has, so far as can be ascertained, been rather less than $95 \%$, perhaps by $2 \%$ on average.

Table 5. Adjustment of Model A for the effects of separations of married couples by death and divorce

| $\begin{aligned} & \text { Pcriod } \\ & \text { since } \\ & \text { marriage } \end{aligned}$ | Number of children born allowing for the cffects of separation expressed as proportion of number of children of unseparated couples | Adjusted number of children per hundred couples, born up to end of period shown in first column, by order of birth |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Ist | 2nd | 3rd | 4th | 5th | 6th and over | Total |
| First year | -998 | 28 | 2 | - | - | - | - | 30 |
| Second year | -990 | 52 | 6 | - | - | - | - | 58 |
| Third and fourth years | -966 | 67 | 28 | 5 | 1 | - | - | 101 |
| Fifth to seventh years | -92I | 74 | 45 | 18 | 6 | I | - | 144 |
| Eighth to tenth years | -873 | 75 | 50 | 25 | II | 4 | 1 | 166 |

For the period up to 1945 the models may be regarded as reasonably representative in regard to survivorship of marriage partnerships. Since 1945, however, the resumption of emigration-including the movement abroad of women who had married United States and Canadian troops during the war-and an increased rate of divorce have caused the production of children in this country to fall below the standard expressed in the models. At a rough guess these recent features, taken together, have brought about an additional loss of perhaps $4 \%$ of the children of the couples married since 1938. This helps to explain some of the shortfall in the numbers of children below the requirements for replacement purposes.

On balance, it is thought that the percentages shown in Table 6 for the latest marriage durations should properly be increased on average by about $3 \%$; the corrected figures appear in Table 7.
15. For a variety of reasons, but mainly owing to the difficulty of ascertaining the true ultimate proportion marrying corresponding to the marriage experience of any single year, comparison of the percentages quoted in Tables 6 and 7 between the different years of marriage should be made, if at all, with caution. Nevertheless, when the effects of temporary separations of newly married couples owing to the husband's service abroad and of marriages to allied troops are remembered, it is not entirely surprising that the percentages for war-time marriages appear to be lower than those for peace-time marriages. The effects of separations are seen in the lower percentages for high birth-orders and in the trend of the figures with marriage duration in Table 6.

Table 6．Number of live－born children per marriage，Great Britain，according to date of marriage and marriage duration，expressed as an approximate percentage of the numbers required for exact reproduction according to Model A

| $\begin{gathered} \text { Period } \\ \text { of } \\ \text { marriage } \end{gathered}$ | Duration of marriage in completed years | Birth－order of child |  |  |  | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | First | Second | Third | Fourth and over |  |
| 1937－38 | $\begin{array}{r} 4 \\ 7 \\ 10 \end{array}$ | $\begin{aligned} & 95 \\ & 88 \\ & 87 \\ & 94 \\ & 99 \end{aligned}$ | $\begin{aligned} & \overline{77} \\ & 67 \\ & 79 \\ & 94 \end{aligned}$ | 7 62 62 76 | $\overline{53}$ 62 | 91 87 8 r 84 84 91 |
| 1938－39 | $\begin{array}{r} 2 \\ 4 \\ 7 \\ 10 \end{array}$ | $\begin{array}{r} 88 \\ 84 \\ 89 \\ 95 \\ 95 \\ 100 \end{array}$ | $\begin{aligned} & 66 \\ & 64 \\ & 78 \\ & 95 \end{aligned}$ | 50 58 75 | - <br> 49 <br> 62 | $\begin{aligned} & 84 \\ & 83 \\ & 80 \\ & 83 \\ & 83 \\ & 92 \end{aligned}$ |
| 1939－40 | $\begin{array}{r} 2 \\ 4 \\ 7 \\ 70 \end{array}$ | 72 73 86 95 100 | $\begin{aligned} & 43 \\ & 54 \\ & 57 \\ & 93 \end{aligned}$ | 7 36 51 68 | 二 <br> 3 S <br> 48 | $\begin{aligned} & 68 \\ & 70 \\ & 73 \\ & 81 \\ & 88 \end{aligned}$ |
| 1940－4I | $\begin{array}{r} 2 \\ 4 \\ 7 \\ 80 \end{array}$ | 66 73 83 94 98 | $\begin{aligned} & - \\ & 38 \\ & 53 \\ & 77 \\ & 88 \end{aligned}$ | 二 <br> 36 <br> 51 <br> 63 | 二 <br> 38 <br> 48 | $\begin{aligned} & 63 \\ & 69 \\ & 72 \\ & 8 \mathrm{z} \\ & 85 \\ & 85 \end{aligned}$ |
| 1941－42 | $4$ | 66 74 83 96 | $\begin{aligned} & \overline{38} \\ & 54 \\ & 80 \end{aligned}$ | 二 34 53 | $\bar{\square}$ | 62 71 72 83 83 |
| 1942－43 | $\begin{aligned} & 2 \\ & 4 \\ & 7 \end{aligned}$ | 72 81 89 89 96 | $\overline{45}$ <br> 59 <br> 59 | -72 60 | 44 | 69 77 77 85 |
| 1943－44 | $\begin{aligned} & 1 \\ & 2 \\ & 4 \end{aligned}$ | 80 83 93 94 94 | 7 <br> 48 <br> 69 <br> 83 | 50 62 | 二 | 76 80 83 85 |
| 1944－45 | $\begin{aligned} & \text { I } \\ & 2 \end{aligned}$ | 73 85 93 | 48 <br> 72 | 56 | － | $\begin{aligned} & 69 \\ & 8 \mathrm{x} \\ & 85 \end{aligned}$ |
| 1945－46 |  | 79 95 95 | 57 80 | － | 二 | 75 90 89 |
| 1946－47 | $\begin{aligned} & 1 \\ & 2 \\ & 4 \end{aligned}$ | 102 99 97 | 77 84 84 | $\overline{78}$ | － | $\begin{aligned} & 97 \\ & 97 \\ & 93 \end{aligned}$ |
| 1947－48 | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | 100 96 | 82 | 二 | 二 | 95 95 |
| 1948－49 | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | $\begin{aligned} & 92 \\ & 91 \end{aligned}$ | $\overline{79}$ | －－ |  | $\begin{array}{r} 88 \\ 87 \\ \hline \end{array}$ |

It should be emphasized that the data in Tables 6 and 7 show only the performance of certain recent sets of marriages so far in bringing children into the world; they can tell nothing of the additions to their families that these couples may have in the future. In this connexion, it is pertinent to remember that, although the durational distributions in Table A 5 which have been used as a basis for the models remained roughly the same for marriages up to 1929, they may since have altered, and children may now be slower in arriving. A preliminary study of the sample results of the 195r Census, however, while not fully conclusive, does not suggest that there has been any marked change in distribution so far. Nevertheless, only the future can tell for certain whether the post-war marriages, with their young age-distribution, will take longer to have all their children than the couples married in previous years, and thus whether their families will build up to a level higher than that indicated in Table 7.

Table 7. Percentage ratios of the numbers of live-born children actually produced by the couples married in various years to the numbers needed for exact replacement

| Year of marriage | Number of years in latest marriage duration examined | Percentage of number of children to number needed for replacement |
| :---: | :---: | :---: |
| 1937-38 | 10 | 94 |
| 1938-39 | 10 | 95 |
| 1939-40 | 10 | 8 Pr |
| 1940-41 | 10 | 88 |
| $1941-42$ $1942-43$ | 7 | 85 88 |
| 1943-44 | 7 | 88 |
| 1944-45 | 4 | 88 |
| 1945-46 | 4 | 92 |
| 1946-47 | 4 | ${ }_{98}^{96}$ |
| $1947-48$ $1948-49$ | 2 | 90 |

16. Subject to the point made at the end of the preceding paragraph, the results obtained in this paper are broadly in accord with the conclusion of the Royal Commission on Population that, taking into account the mortality rates of 1942-44 and not allowing for any future improvement in longevity, the rate of replacement resulting from the present size of the family is about $90 \%$ (the shortfall of $6 \%$ mentioned in $\S 2$ was assessed on the basis of projected mortality). This accord may perhaps be held to have followed from the general similarity of approach between the present investigation and that of the Royal Commission, in both of which some importance has been attached to duration of marriage as a factor in fertility and reproduction.

To make an alternative approach to the problem, it may be observed that the female net reproduction rate in recent years has remained fairly close to unity; the men's rate is undoubtedly higher at the present time. It would be informative, if enough data were available, to compare the numbers of children born to different generations of women up to specified ages, disregarding the proportion married. At the moment, however, little can be added to the material exhibited in Table 8 of the paper of March 1951, and a complete investigation by age is thus impracticable. It must be remembered that
difficulties also attend the approach to fertility and reproduction by means of analyses by attained age of parent, for instance the disparity between the results for men and for women and the distorting effects of changes in the numbers and age-distribution of marriages. It seems unlikely that a series of alternative models along these lines would give such consistent results as the three described in this paper have done. Probably the 'age' and 'duration' approaches ought both to be made in order to obtain the nearest approximation to the truth about reproductivity during times when the age-distribution at marriage does not remain constant. A reduction in the average age at marriage causes the fertility at a given attained age to relate to marriages which have been exposed to risk for longer periods, and as regards duration of marriage it increases the length of time for which a couple are exposed to risk of having a child.

Models A, B and C are, of course, all based upon the same proportion of women marrying. This proportion is so high that there is little latitude for its upward movement. Even if all women were assumed to marry, the numbers of children per mother required for replacement purposes would be reduced only by $5 \%$, and the percentages in Table 7 would thus be increased by no more than $5 \%$ also.
17. Reference has been made in two places above to the one per cent sample statistics derived from the 1951 Census of Great Britain. These were not published until after most of this paper had been written, and there has been insufficient time for a full investigation into their implications. It has, however, been observed that the numbers of children per marriage at durations of up to ten years recorded at the Census are apparently somewhat higher than those for unbroken marriages that would be consistent with the 1938 Act data. Without fuller investigation it cannot be said for certain which of the two sources is the more reliable; but the discrepancy is worthy of note when the results of Table 7 are under consideration.
18. As has already been remarked, this paper is in essence a product of the Demographic Study Group formed by the Students' Society. Thanks are expressed in particular to those who have recently taken an active part in its affairs, for contributing basic ideas and making helpful comments from time to time.

## APPENDIX

NOTES ON THE MORTALITY, MARRIAGE AND OTHER BASES USED IN CONSTRUCTING THE POPULATION MODELS

## The sex-ratio at birth

Ar. Among the live births registered in Great Britain during the last four decades the proportion of boys to girls has shown a tendency to increase slowly, and has risen from I .039 in 1910-14 to $\mathbf{1} .049$ in 1930-34 and to $\mathrm{r} \cdot 06 \mathrm{I}$ in 1945-49. During the later part of this period the age-distribution of women at marriage became younger, and part of the rise in masculinity may have been associated with the resulting reduction in the average age of mothers. The practical importance of this association, however, is not great enough to warrant elaborate treatment, as explained in § A8 below, and for present purposes it is adequate to assume a ratio of $\mathrm{I} \cdot 06$ boys per girl.

## Mortality

A2. When considering the loss of life at the younger ages that is likely to occur in future, some allowance may be made for improvement in vitality. In making a model for the study of current fertility experience, however, such projection is not required. Indeed, for a representation of the effect on population replacement of losses due to mortality in the years since 1938, it would not be wholly inappropriate to go back to, say, 1942-44 and to use the experience of those years as a basis. Bearing in mind the existence of special mortality due to the war, however, there is much to be said for adopting for the models the most recent mortality rates available and for treating any excess deaths, military or civilian, in recent years as departures from the standard, requiring to be traced along with other variations. The rates of 1948-50 have, in fact, been adopted for the calculations in relation to the mortality of men and women irrespective of marital status, and suitable adjustments have been made in order to arrive at the corresponding rates for unmarried men and women.

## Migration

A3. It is convenient, and not wholly inappropriate in relation to models for Great Britain, to assume that on balance inward and outward migratory movements cancel each other. The departures from this assumption during recent years may be accounted for, and their effects on fertility roughly measured, as part of the comparison between actual recent happenings and expected events according to the models.

## Marriage

A4. The proportions of men and women aged 60-64 in various years who were then married, or had been married at some time in the past, are set out in Table Ar, which extends over the first half of the twentieth century.

These figures give a broad indication of the national marriage experience, but only of that which occurred in the main many years before the date shown. The smallness of the proportions for women relative to those for men implies the long-continued existence of a surplus in the number of women available for marriage nompared with men. This surplus grew in size at first owing to
excess male emigration at the end of the nineteenth century; later it was prevented from falling again by the losses of men during the 1914-18 War.

The increases in the marriage rates of both sexes and the swing towards an excess in number of men over women at the younger ages that have occurred more recently find no expression in the table. On the strength of these tendencies it would be expected that tables expressed in a form sometimes referred to as 'net nuptiality', showing the numbers living unmarried, dying unmarried and marrying for the first time at each age in accordance with current marriage and mortality rates, would present a picture very different from that in Table Ai.

Table A i. Proportions of men and women aged $60-64$ who had married (including those who were still married at that age and also those whose marriages had been terminated by death or divorce) in Great Britain, 1901-5I

| Year | Men | Women |
| :---: | :---: | :---: |
| 1901 | .90 | .88 |
| 1911 | .90 | .87 |
| 1921 | .90 | .84 |
| 193 I | .90 | .84 |
| 1939 | .90 | .84 |
| 1951 | .92 | .84 |

Table A 2. Abridged marriage and mortality tables (net nuptiality tables) for men and women, based on the mortality rates of $1948-50$ and the first-marriage rates of 1948-49, Great Britain

| $\underset{x}{\text { Age }}$ | Men |  |  | Women |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Number alive and never married at age $x$ | Number dying unmarried between $x$ and next age below | Number marrying for the first time between $\boldsymbol{x}$ and next age below | Number alive and never married at age $x$ | Number dying unmarried between $x$ and next age below | Number marrying for the first time between $x$ and next age below |
| $\bigcirc$ | 1060 | 53 | $\overline{30}$ | 1000 | 40 | $\overline{7}$ |
| 15 | 1007 | 6 | 30 | 960 | 5 | 176 |
| 20 | 971 | 6 | 46 r | 779 | 4 | 516 |
| 25 | 504 | 4 | 287 | 259 | 2 | 143 |
| 30 | 213 | 2 | 102 | 114 | 1 | 37 |
| 35 | 109 | 2 | 36 | 76 | I | 14 |
| 40 | 71 | 2 | 14 | 61 | 1 | 6 |
| 45 65 | 55 27 | 14 | 14 | $\begin{aligned} & 54 \\ & 40 \end{aligned}$ | 9 | 5 |
| Total |  | 89 | 944 |  | 63 | 897 |

A 5. Table A. 2 contains one such 'net nuptiality' statement for men and one for women, at specimen ages, and fully bears out the expectations referred to at the end of the preceding paragraph. It is based on the mortality rates for single persons that have been mentioned above, the first-marriage rates in Great Britain during the years 1948-49, and a masculinity ratio at birth of 1.06. The marriage experience of the year r950 was not available when the table was constructed, but its inclusion would make no important difference.

The two main features of the table are the small proportion of each sex remaining unmarried at age 65 , and the disagreement between the numbers of first marriages of men and of women. Of the two thousand or so children attaining age 16 , only 27 men and 40 women are left unmarried, according to the table, at age 65 ; the corresponding numbers of married persons at that age are 944 men and 897 women, the numbers marrying, reduced by the numbers of married deaths between the time of marrying and the attainment of age 65 . It may be inferred, since any such deductions for married deaths would be relatively small, that the proportions married at age 65 according to the bases of Table A2 would be as much as $97 \%$ for men and $95 \%$ for women, whereas Table A i shows only $92 \%$ (men) and $84 \%$ (women) for the year 195r. The returns of the Registrar General for England and Wales show for any period that the numbers of marriages of men and women, while obviously involving an equal total of either sex, do not include as many remarriages of women as of men, except immediately after major wars, when young widows likely to remarry are unusually numerous. It follows that the numbers of first marriages of women normally exceed those of first marriages of men, a position contrary to that shown by Table A 2.

A6. An equality between men and women in the number of all marriages is essential for a realistic population model. Further, there should be a relationship between the ages of the sexes at marriage that is not out of accord with normal experience. If, however, the numbers of first marriages of men and women in Table A2 of the present paper are compared (after adjustment for remarriages) in the light of the customary relationship between husbands' and wives' ages at marriage in Great Britain, which has not greatly varied in recent years, it will be found that they are not wholly consistent. Some relevant details are given below:

| Age-group of <br> wife at marriage | Marriages of women <br> (numbers in Table A2 <br> augmented by remarriages) | Marriages of wives <br> (numbers dorivcd <br> from the men's data in <br> Table A2 augmented <br> by remarriages) |
| :---: | :---: | :---: |
| 16-24 <br> 25 and over | 698 | 690 |
| Total | 269 | 360 |

The additions for remarriages in these calculations were based on post-war experience and were evaluated by means of the ratios at the various ages of remarriages to first marriages; they amounted to 106 for men and 70 for women. The numbers of men's marriages according to age of wife were obtained by multiplying the numbers of men's marriages in each age-group of men by the proportions of wives of various ages according to the experience of the year 1949 in Great Britain, and then summing all the numbers of wives in the various age-groups of wife.

The disagreement in the above statement is almost entirely confined to ages 25 and over; below age 25 the results are quite satisfactory. Now the tendencies towards a new balance of the sexes that were mentioned at the end of § $A_{4}$ have so far been confined to the younger ages. If future developments proceed along the same lines, and are not affected by any factors operating to reduce the relatively high flow of men into the marriage market-
emigration, for instance, which has been resumed on a moderate scale re-cently-then further changes in the relative sizes of the marriage rates of men and women may be expected to occur; the age-distributions of the sexes at marriage may also alter. In these circumstances the disagreement between the numbers and age-distribution of marriages of men and women will be gradually resolved.

A 7. While on the one hand the data of Table AI are too out of date to be of much significance in modern conditions as a guide to marriage experience at the principal ages for bearing children, on the other hand Table A2 needs considerable modification in order to eliminate inconsistencies between the sexes. These inconsistencies are deep-rooted, and the necessary modifications cannot be effected simply by changing from the basis of the years 1948-49 to that of any other recent group of years, whether before or after the War of 1939-45. The extent of the difficulty is shown by the fact that the excess of 47 in the numbers of first marriages of men in Table A2 over those of women could not be accounted for even if every surviving woman were assumed to marry, for only 40 remain unmarried in Table A2 at age 65.

There are more boys than girls at age 15 in Table A2, the excess being about $5 \%$. In order that the number of first marriages of women may equal, or somewhat exceed, the numbers of first marriages of men in the model, the proportion of women eventually marrying must be greater than the corresponding proportion for men by about $5 \%$ also. Suppose, for instance, that it were decided to choose 920 instead of 897 as the total number of first marriages of women, and 910 instead of 944 as the corresponding figure for men. The number of those who died unmarried would be not far removed from the 63 (women) and 89 (men) shown in Table A2, and those remaining unmarried would number approximately 17 women and 61 men, or $2 \%$ and $6 \%$ respectively of the 960 women and 1007 men attaining age 15 -a difference between the sexes of nearly $5 \%$. The features of this corrected version of Table A2 may then be summarized in three sections for each sex, namely: (a) those who marry, (b) those who die unmarried before attaining age 65 , and (c) those who remain alive and unmarried at age 65 . The proportion of (c) to the sum of (a) and (b) would amount to about $2 \%$ for women and $6 \%$ for men.

A8. Although it would be too much to suggest that the male sex has the power of determining all marriages, it may reasonably be supposed that in general men have more say in the matter than do women. The men's proportion marrying may be expected to be more stable, and the women's proportion marrying to be more variable, as the relative numbers of the sexes available for marriage alter. This general rule is subject to the qualification that the proportion marricd among women cannot approach too close to unity: there are always some women (as there are men) who are unlikely to marry or who are certain not to marry because of permanent illness or injury or for other reasons. The relative numbers of each sex who are thus not available for marriage are unknown, and indeed they probably vary from time to time according to circumstances. They are doubtless far fewer than the unmarried residues of women shown in Table Ai but they could conceivably be as large as, or even larger than, the $2 \%$ mentioned at the end of $\S A_{7}$; $2 \%$ may therefore appear too low a figure to use for the purpose of the models. If it is assumed that $10 \%$ of men do not marry-as is shown
by Table $A_{I}$ to have been the case for a long time-the corresponding proportion of women would be not $2 \%$ but $5 \%$, on the basis described in the last sentence of § A 7 . This figure of $5 \%$ seems a not unreasonable amount and has accordingly been adopted throughout the model calculations.

All the fertility statistics used in constructing the models relate necessarily to women. It is not important, therefore, to know any more precisely than has been indicated above what is the proportion of men marrying, and thus no further inquiry is necessary into either the sex-ratio at birth (§ $\mathrm{AII}_{\text {I }}$ ) or the relative numbers of men's and women's remarriages. The approach to the subject by means of the data for men would give broadly consistent results.

A9. In Table A3, percentage distributions of women's marriages (including remarriages) by age are set out. The first three columns refer to the averages of groups of years during 1900-1930, the period to which the family-size data in Table I related; the fourth column gives the experience of 194I-45, and the last shows, for comparative purposes, the distribution derived from Table A2, adjusted for remarriages.

Table A3. Percentage distribution of women's marriages (including remarriages) according to age. Great Britain, 1906 onwards (approximate)

| Age of woman <br> at marriage | Period of marriage |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $1906-10$ | $1916-20$ | $1926-30$ | $1941-45$ | 'Ulimate' <br> (derived from <br> Table A2) |
| Under 20 | 7 | 8 | 8 | 16 | 18 |
| $20-24$ | 47 | 45 | 45 | 48 | 54 |
| $25-29$ | 28 | 26 | 27 | 17 | 17 |
| $30-34$ | 9 | 9 | 9 | 7 | 6 |
| $35-39$ | 4 | 5 | 5 | 4 | 3 |
| $40-44$ | 2 | 3 | 2 | 3 | 1 |
| 45 and over | 3 | 4 | 4 | 5 | 1 |
| Total | 100 | 100 | 100 | 100 | 100 |

During the first thirty years of the present century the distribution preserved a nearly constant pattern, but the figures for the early 1940's differ sharply from those preceding them in the table. Women married at younger ages; the 'ultimate' prospect is of a further development in the same direction.

It has been argued that as marriages tend to occur earlier in life there may be a change in the relationship between the number of legitimate children born to a woman and her age at marriage. As the index figures in Table I for the years 1915-19 have been adopted for use in the calculations, the only really consistent course, if this argument is valid, would be to use the age-distribution of women at marriage of $1915-19$ as the basis for the models. This has, indeed, been adopted as one alternative. As such a basis is so far out of accord with modern conditions, however, and in order to see the effect on the models of a substantial change of basis, the 'ultimate' distribution shown in the last column of Table $\mathrm{A}_{3}$ has been utilized as another alternative, also in conjunction with the family-size indices from Table $\mathbf{1}$.

As the marriages become younger the rate of change in the index
numbers of family size with advancing age at marriage may become slower: for instance, those who marry at 22, instead of at 26 as might perhaps have been the case formerly, may have a family no larger than that associated previously with marriage at age 26 . The effects of a series of such transfers to younger ages might be to reduce the extent of the differences between the index numbers shown in Table I . In order to see whether this theory has any significant influence upon the models now under construction, a new series of family-size index numbers has been used as an alternative to those shown in Table r , the excess of the average family size for ages under 20 at marriage over that for ages $25-27 \frac{1}{2}$ at marriage being reduced from $80 \%$ to $50 \%$.

## Illegitimate children

A ro. It is a simpler matter to make provision in the model for illegitimate births. Before 1939 the proportion of births classed as illegitimate was for some years less than $5 \%$. Although during the war this ratio rose to a peak of nearly $10 \%$ it has since fallen again to $5 \%$, and this lower 'normal' level seems to be the right quantity to import into the models; any war-time temporary excesses will emerge as features in the tracing of the differences between actual recent experience and the standards.

## Dissolution of marriage

Ain. When married couples are separated by death or divorce before the wife has reached the age of 45 , their average number of children will be lower than that of unseparated couples of otherwise similar characteristics; the difference will be referred to as a 'loss' of children due to separation. The causes of such 'losses' may be analysed as follows:
(a) in the event of the death of either partner, any reduced fertility occasioned by illness preceding death;
(b) in the event of a divorce, any reduction in fertility due to incompati-bility-often a cause of divorce-or to separation preceding divorce;
(c) after the occurrence of a widowhood or divorce, the loss of the opportunity to have legitimate children.

The fertility of any remarriage may do something to redress the loss. Separations of married couples that are not formalized by divorce, including temporary separations such as those brought about by war, are hardly capable of direct measurement and the extent of any losses can only be inferred generally.

Statistics of these causes of loss of children are rather scanty and space does not permit a discussion in detail of how the available information can be used. It must suffice to say that when all these causes of loss of children are added together, there is a range of possible variation in the extent of their total effect. While it might rise to as much as $8 \%$, it might be materially smaller. For the purpose of the models, a loss of no more than $5 \%$ has been assumed, a figure that finds some confirmation in recent experience. By a study of such facts as are available a series of percentages corresponding to the various ages at marriage has been obtained as an aid to the next stage of the calculations.

## Family data

A 12. Tables $\mathrm{A}_{4}$ and $\mathrm{A}_{5}$ give certain data derived from the Family Census. Their nature is discussed in the main text.

Table A4. Percentage distributions of married women according to the number of children they have borne: unadjusted data from the Family Census of 1945, Great Britain. Unbroken first marriages at selected age-groups

| Period of marriage | Number of children | Age of woman at marriage |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 20-22 $\frac{1}{2}$ |  | 271 $\frac{1}{2}-30$ | 32 $\frac{1}{2}-35$ | 40-45 |
| 1910-14 | - | 5 | 9 | 17 | 28 | 74 |
|  | 1 | 12 | 18 | 23 | 27 | 17 |
|  | 2 | 19 | 23 | 24 | 21 | 4 |
|  | 3 | 18 \| | 18 | 16 | 13 | 2 |
|  | 4 | 14 | 11 | 9 | 6 | 1 |
|  | 5 | 10 | 8 | 6 | 3 | 1 |
|  | 6 and over | 22 | 13 | 5 | 2 | 1 |
|  | Total | 100 | 100 | 100 | 100 | 100 |
| Average family size |  | 3.79 | 3.00 | 2.21 | 1.6i | 0.46 |
| 1915-19 | $\bigcirc$ |  | 10 | 18 | 31 | 78 |
|  | I | 16 | 20 | 26 | 27 | 14 |
|  | 2 | 22 | 25 | 26 | 21 | 4 |
|  | 3 | 18 | 18 | 14 | 12 | I |
|  | 4 | 13 | 11 | 8 | 5 | I |
|  | 5 | 8 | 7 | 4 | 2 | I |
|  | 6 and over | 16 | 9 | 4 | 2 | 1 |
|  | Total | 100 | 100 | 100 | 100 | 100 |
| Average family size |  | $3 \cdot 29$ | $2 \cdot 70$ | 1.99 | 1.47 | 0.40 |
| 1920-24 | $\bigcirc$ | 6 | 11 | 19 | 33 | 77 |
|  | 1 | 19 | 24 | 30 | 29 | 14 |
|  | 2 | 25 | 27 | 27 | 21 | 5 |
|  | 3 | 19 | 17 | 12 | 9 | 2 |
|  | 4 | 12 | 9 | 6 | 5 | 1 |
|  | 5 | 7 | 5 | 3 | 2 | $\bigcirc$ |
|  | 6 and over | 12 | 7 | 3 | 1 | 1 |
|  | Total | 100 | 100 | 100 | 100 | 100 |
|  | Average family size | $2 \cdot 98$ | 2.41 | 1.78 | $1 \cdot 34$ | $0 \cdot 40$ |
| 1925-29 | $\bigcirc$ | 7 | 12 | 24 | 38 | 82 |
|  | I | 23 | 28 | 31 | 32 | 12 |
|  | 2 | 28 | 30 | 26 | 18 | 3 |
|  | 3 | 19 | 15 | 11 | 7 | I |
|  | 4 | 10 | 8 | 4 | 3 | 1 |
|  |  | 6 | 4 | 2 | 1 | - |
|  | 6 and over | 7 | 3 | 2 | 1 | 1 |
|  | Total | 100 | 100 | 100 | 100 | 100 |
| Average family size |  | $2 \cdot 52$ | $2 \cdot 07$ | 1.54 | $1 \cdot 12$ | $0 \cdot 31$ |

Note. With the exception of the figures in italics, this table relates to completed, or virtually completed, families; the figures in italics relate to ncarly completed families.

Table A 5. Percentage proportions of the numbers of children that had been born to women, by the end of various specified periods of time after first marriage, to the corresponding numbers in their completed families, according to birth-order of child and to selected dates of marriage and selected ages of mothers at marriage. Unadjusted Family Census data, Great Britain: unbroken marriages

| Mother's age at marriage | Years in which marriage took place | Duration of marriage in completed years |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | I | 2 | 4 | 7 | 10 | 15 | End of fertile period |
| 20-22 $\frac{1}{2}$ | $\begin{aligned} & 1910-14 \\ & 1925-29 \end{aligned}$ | First children |  |  |  |  |  |  |
|  |  | 44 | 78 | 9188 | 96 | 9898 | 100 | 100 |
|  |  |  | 73 |  | 96 |  | 100 | 100 |
| 27-1-30 | 1910-14 | 32 | 65 | 87 | 95 | 99 | 100 | 100 |
|  | 1925-29 | 26 | 57 | 8 I9089 |  | 98 | 100 | 100 |
| $32 \frac{1}{2}-35$ | $\begin{aligned} & \text { 1910-14 } \\ & 1925-29 \end{aligned}$ | 3329 | $\begin{aligned} & 67 \\ & 66 \\ & 65 \end{aligned}$ |  | 97 | 100 | 100 | 100100 |
|  |  |  |  |  | 97 | 100 | 100 |  |
|  |  | Second children |  |  |  |  |  |  |
| 20-22 $\frac{1}{2}$ | 1910-14 | 2 | 13 | 55 | 82 | 93 | 99 | 100 |
|  | 1925-29 | 2 | 12 | 49 | 78 | 92 | 100 | 100 |
| 271-30 | 1910-14 | 2 | 8 | 47 | 80 | 94 | 100 | 100 |
|  | 1925-29 | 2 | 9 | 42 | $\begin{aligned} & 76 \\ & 88 \end{aligned}$ | 9398 | 100 | 100 |
| $32 \frac{1}{2}-35$ | $\begin{aligned} & 1910-14 \\ & 1925-29 \end{aligned}$ | 3 | 1114 | 5852 | $\begin{aligned} & 88 \\ & 86 \end{aligned}$ |  |  | 100 |
|  |  |  |  |  |  | $\begin{aligned} & 98 \\ & 98 \end{aligned}$ | 100 |  |
|  |  |  |  |  | ird | dren |  |  |
| 20-22, $\frac{1}{2}$ | $1910-14$$1925-29$ | - | 1 | 14 | 5454 | 8078 | 95100 | 100 100 |
|  |  |  |  |  |  |  |  | 100 |
| 271-30 | 1910-14 | - | 1 | 12 | 54 | 84 | 99 | 100 |
|  | 1925-29 | 1 | 2 | 15 | 56 | 82 | 10099 | 100100 |
| 322-35 | $\begin{aligned} & 1910-14 \\ & 1925-29 \end{aligned}$ | 1 | 35 | 1925 | 7069 | 9195 |  |  |
|  |  |  |  |  |  |  |  | 100 |
|  |  |  |  | Fourth children |  |  |  |  |
| 20-22 $\frac{1}{2}$ | 1910-14 | 一 | - | 2 | 24 | 59 | 89100 | 100100 |
|  | 1925-29 | - | - | 2 | 30 | 65 |  |  |
| $27 \frac{1}{2}-30$ | 1910-14 |  | - | 3 | 26 | 67 | 97 | 100100 |
|  | 1925-29 | 一 | 1 |  | 35 | 72 | 100 |  |
| 322-35 | $\begin{array}{r} 1910-14 \\ 1925-29 \\ \hline \end{array}$ | I | 23 | 812 | 4044 | 8189 | $\begin{array}{r}97 \\ 100 \\ \hline\end{array}$ | $\begin{aligned} & 100 \\ & 100 \end{aligned}$ |
|  |  |  |  |  |  |  |  |  |

Note. With the exception of the figures in italics, this table relates to completed, or virtually completed, families; the figures in italics relate to nearly completed families.

## REFERENCES

(1) Report of the Royal Commission on Population (1949). London: H.M.S.O.
(2) Brown, S. P. (195I). Analysis of a hypothetical stationary population by family units-a note on some experimental calculations. Population Studies, rv, 380.
(3) Papers of the Royal Commission on Population. Reports and Selected Papers of the Statistics Committee (1950) (p. 87). London: H.M.S.O.
(4) 195 I Census of Great Britain. One per cent Sample Tables. Part II (1952). London: H.M.S.O.
(5) Cox, P. R. (1951). Studies in the recent marriage and fertility data of England and Wales. Population Studies, v, 132.

## ABSTRACT OF THE DISCUSSION

Mr O.D. Cumming, in opening the discussion, said that the paper dealt with a new way of measuring whether births in Great Britain had in recent years been sufficient for the replacement of the population. It was perhaps as well to add that it had nothing to do with forecasting, or projection, or with the future performance of existing married couples.

It might be asked why a new method of measuring the extent to which the population was replacing itself was wanted. In an ideal population where mortality and fertility rates had continued virtually unchanged for many years, the conventional single-figure indices, such as the net reproduction rate, might have considerable value, but those indices were clearly defective in a country such as Great Britain, where in little more than a generation there had been great wars, vast social upheavals and far-reaching changes in marriage and family-building habits.

Two essential additional features of any new method of measurement of reproductivity were (i) that it should reflect the fundamental characteristics of the population rather than the fluctuating, temporary aspects of a single year or other short period, and (ii) that it should in some way reconcile the two widely different answers given by most other methods according to whether they were applied to the female element or to the male element of the population.

The author had concentrated throughout on the long-term aspect and had scrupulously ignored any ephemeral matters. His approach had necessarily been by way of the female population, in view of the form in which the returns were collected under the Population (Statistics) Act, 1938, but he had been at great pains to reconcile his results with such male data as were available. His method went further than mere quantitative measurement, as he claimed that it made it possible to test statistically the various items contributing to population replacement. The author had provided what might perhaps be called a new instrument, something essentially practical in its nature, and his work should be judged by the results which it achieved in the hard world of fact.

It might be desirable, therefore, to examine the assumptions underlying the author's models. Those assumptions were set out in $\S 8$ of the paper. Assumption (a) was that 106 boys were born for every 100 girls; in other words, that every 1000 girls born must between them give birth to 2060 children if the population were to be self-reproducing. Forty years earlier, for every 1000 girls born there had been ro39 boys, compared with the current level of upwards of 1060. If everything else were equal, the higher the masculinity the greater would be the number of births required from roo0 new-born girls; but, of course, it would not be a case of ceteris paribus, because higher masculinity would mean that more men entered the marriage market, with the result that the proportion of women ultimately married would rise or the average age of women at marriage would fall, or both. That in turn would lead to an increase in the total births, particularly at the younger ages of mothers, where it was believed that the children exhibited the highest degree of masculinity.

Assumption (b) was 'that mortality is in accordance with the experience of Great Britain during the period 1948-50 and will be higher for single men than for all men and higher for single women than for all women'. Bearing in mind that they were measuring events in the recent past and not in the future, and remembering also that the experience of the war years was probably exceptional, that seemed to be eminently reasonable.

Assumption (c) was 'that on balance inward and outward migratory movements cancel each other', and assumption (d) was 'that $90 \%$ of men and $95 \%$ of women marry, in two alternative age-distributions'. 'The assumption that $90 \%$ of men who survived to age 65 and $95 \%$ of women who survived to that age would have married was a fundamental assumption on which different demographers would have different ideas, and it was probably a case of quot homines, tot sententiae. It was a fact that over the last half-century or so the proportion of men surviving to the end of the fertile period-say age 65 -who had never married had consistently been close to $10 \%$. Taking that as a starting point, and leaving out of account complications such as remarriages, which did not greatly affect the main argument, the author reasoned in the following way. At the start of the fertile period--say age 15 -there were about 100 boys to every 95 girls. If $10 \%$ of the boys never married then, ignoring mortality (which at the ages concerned did not differ greatly for the two sexes), 90 boys would ultimately marry. That meant that 90 girls would ultimately marry out of the 95 at age 15, so that the proportion of women who ultimately married would be in the vicinity of $95 \%$. In other words, if the experience of the past few years continued for, say, another 50 years, it would be found at the end of that time that of the women then aged 65 only $5 \%$ were spinsters. At the time of speaking that proportion was $16 \%$, but there had previously been a big excess of females of marriageable age, due mainly to war and migration.

It should be emphasized that the author was not concerned with the future, but was trying to express the marriages of the past few years in terms of the proportion of women who, if conditions remained unchanged, would ultimately marry.

The remaining assumptions, (e), 'that $95 \%$ of births are legitimate and $5 \%$ illegitimate', and ( $f$ ), 'that marriages are dissolved by death and divorce to the same general extent as in Great Britain during recent years', called for no particular comment.

The methods of construction of the models were clearly set out in the paper. It was necessary to have not only completed family size but also the size at various marriage durations, for which the author had had access to the Family Census data. He had been in some doubt as to the correct age-distribution of women at marriage owing to changes of fashion in recent years, and he had therefore employed two alternatives, Models A and B. Likewise, to forestall any suggestion that the extent of the association between age of bride and ultimate family size might after all not be so constant as was indicated by Table i, he had used two alternative series of index numbers of relative family size, Models B and C. Fortunately, the results of the three models, A, B and C, as to the speed at which families were built up after marriage were not widely dissimilar.

In the final section of the paper the author compared the actual fertility with that expected according to Model A; the results were given in Table 6. It would be seen that, except at the very beginning and at the end of the table, where peace-time years were concerned, the percentages in the last column increased with duration of marriage. That was, no doubt, mainly due to the effect of temporary war-time separation of husbands and wives, coupled with the tendency to defer family building in times of uncertainty.

The author then attempted to trace approximately the 'profits' and 'losses' due to the various deviations from the standard bases. For example, the illegitimate births had averaged $6 \%$ instead of $5 \%$, and he estimated that the
true average ultimate proportion of women marrying had been $93 \%$ and not $95 \%$. Moreover, since 1945 emigration and an increased divorce rate had, at a rough guess, the author said, caused a loss of perhaps $4 \%$ of the children of couples married since 1938. The corrected percentages to allow for those differences from the expected were shown in Table 7.

It was important to appreciate clearly just what was being done. In Table 6 the actual number of children per marriage was expressed as a percentage of the number required for exact replacement according to the model. In Table 7, on the other hand, there were adjusted percentages in which the numerators represented not the actual number of children born but the number of children per marriage who would, in the author's opinion, have been born had there been no disturbance due to migration and no increase in the illegitimacy or divorce rates, and had the proportion of women ultimately marrying been $95 \%$ instead of $93 \%$. Possibly Table 7 might with advantage have had some cautionary wording included in the heading to show that it dealt with a hypothetical situation in which some of the 'profits' and 'losses' had been eliminated from the 'actual' before comparing it with the 'expected'.

The author concluded that the rate of replacement resulting from the current size of family was about $90 \%$, using up-to-date mortality. It might perhaps have been felt that that statement would have carried rather more meaning if a normal period had been under consideration, instead of one which included five or six years of war. The choice of method had been circumscribed by the form of the data collected under the Population (Statistics) Act. If the material had been available, it would have been interesting to look at the average family size analysed by age of the groom or by the disparity in age between spouses.

MrR.D. Clarke observed that the paper brought out clearly the importance of rates of marriage in all studies of reproductivity and the Appendix was a welcome contribution to the literature on this subject. One difficulty which the author had had to face when constructing the models had been the fact that the marriage experience of the past was not much use for predicting the future, because of the tremendous change which had taken place in the relative numbers of men and women and the uncertain effect which that would have. It was impossible to do anything but fall back on hypotheses. The situation was thus rather different from that usually met in actuarial work, where it was customary to use the past as a guide to the future.

He was particularly interested in several features of the paper because recently he had been carrying out some research on the Registrar General's data from a different angle, not according to year of marriage, but according to the year of the mother's birth. The interesting feature had emerged that the total fertility, as measured by relating all children to all women, showed a steady increase, whereas married fertility, as measured by relating legitimate children to married women, showed a continual decrease. The explanation, of course, was simple : nuptiality had been increasing, i.e. the proportions married at the younger ages had been steadily going up, and that had the effect of increasing total fertility despite a reduction in married fertility. The great question-mark in the subject seemed to him to be whether the one factor was going to offset the other, i.e. whether the increase in nuptiality would be sufficient to offset the decrease in fertility within marriage. What mattered was the family sizes, at the various ages at marriage, which would eventually accrue from marriages which had taken place in recent years. That, of course, was completely unknown; it was one of those things about which it was only possible to speculate.

He would like to make one or two speculations. It was interesting to find in the preliminary report on the Family Census by the Statistics Committee of the Royal Commission that, at a time when the families of manual workers were still declining, the families of the non-manual classes had apparently reached a minimum and become stabilized. He thought that it was a matter of common experience that in the professional classes the three-child family was more popular than had been the case before the war. It would be interesting to see whether there was a similar time-lag in the recovery of fertility and whether this new fashion for the three-child family would spread in time through the community.

Another speculation which he would offer tended in the opposite direction. The increased employment of married women might, he thought, react adversely on reproductivity. In connexion with these speculations, he would like to underline what was said in § 15 of the paper:
It should be emphasized that the data in Tables 6 and 7 show only the performance of certain recent sets of marriages so far in bringing children into the world; they can tell nothing of the additions to their families that these couples may have in the future.

The author had also emphasized the extent to which Tables 6 and 7 depended on the durational distributions in Table A5, and that was important. The speaker had spent a good deal of time in analysing the models in the paper, and Table $\mathrm{A}_{5}$ was fundamental; if Table $\mathrm{A}_{5}$ were altered, there would be consequential changes in the model which would be reflected in Tables 6 and 7 , particularly at the shorter durations. A self-reproducing model with wider spacing of children would lead to increased percentages in Table 7. He thought that that had to be borne in mind, because the current tendency for married women to remain in employment for some years after marriage might have some effect on the spacing of the family. For those reasons he was a little nervous about the percentages in the final column of Table 7 for the short durations; but the author was clearly equally nervous, and no criticism was implied by that remark.

Mr H. A. R. Barnett said that he would like to put forward one idea which was more or less what he had said in an earlier discussion and was in no way a criticism of the paper. Certain approaches to the subject were not possible because of lack of data, but the author said that his object was to trace statistically the various elements contributing to population replacement. That, he submitted, was a little misleading, because the elements of the problem could be compiled in a different way.

Reading the paper and listening to the two previous speakers might give the idea that everything was all right, and that all that had to be done was to get married and to decide to have $n$ children, but in fact there were two opposite factors involved which probably did not cancel out. In the first place there were some marriages which produced less than their desired $n$ children. He had been told by a member of the medical profession that approximately $10 \%$ of the married couples of child-bearing ages in this country were childless through no desire of their own. That statement had been made two or three years earlier, and it referred to a medical factor which might possibly decrease in intensity; it depended partly on medical science and partly on the extent to which those involuntarily childless couples consulted the medical profession. On the other hand, there were the people who ended up with more than $n$ children. That might also be a decreasing factor, and it was made up of two elements : those who
had more than $n$ children because of the unsuccessful application or lack of application of birth control, and those who were just expecting their $n$th child when they had a multiple birth, with the result that they ended up with $n+\mathrm{r}$ (or more) children.

If it were correct that $10 \%$ of couples were involuntarily childless, he would think-though it was a wild speculation-that the couples who had fewer children than they desired exceeded those who had more. If the medical factor could be partly eliminated, there might be a closer approach to the family size necessary for replacement. The actual average of rather less than two might indicate that the average desired family was about two, while the actual produced was on the average slightly less.

Mr V. P. A. Derrick suggested that the value of any method of measurement must be judged by its success in measuring actual events. If it did not succeed in measuring satisfactorily the events of the period it covered, that seemed to be a defect of the method itself.

The fertility to which the author's measures were applied covered a wide range of marriage cohorts spread over a period, and an average for the whole of that period might be regarded as relating to a point of time which must be somewhere near the centre of the period, say about the year 1945. The outstanding feature of the paper, he thought, was the indication that fertility, which was known to have been running at a level $20 \%$ below replacement standard for many years before the war, suddenly improved to such an extent that in seven or eight years the deficiency had been reduced to $10 \%$. There was no reason to suppose that the rise came to an end about 1945, and it was not unreasonable to assume that it had continued, so that it was possible that current fertility was approaching replacement level. He thought that that was supported by Table 7 of the paper, which seemed to show that there was a tendency for the later cohorts to be doing better than the earlier ones.

It was desirable not only to try to get a correct average over a wide range but also to bring out how the forces might be moving over that range. The author had done something in that direction which the Royal Commission did not do, and that was an advantage.

He thought that the author's method tended, in various ways, to overstate the deficiency of the period. The author had neglected to take any account of the fall in mortality, which had been continuous for many years, so that each generation of births, to be of the same productive capacity when it reached its reproductive period, need not be so large as its forerunner. A self-reproducing table, however, could not bring out that effect, because it was based on a continuous cycle in which a given number of births were traced to their reproductive ages and were then required to produce new births of the same number as those from which they were descended. $\Lambda$ form of measurement which did not take account of falling mortality suffered from a defect which might make a serious difference to conclusions drawn from it. The Royal Commission had made two alternative assumptions, of a static mortality and of a decreasing mortality, and the effect had been to reduce the deficiency of $10 \%$ to $6 \%$. That was something which mattered. If it was not possible to get a measurement right within that margin of error, the method was open to challenge. Moreover, allowance for the fall in mortality would steepen the improvement over the period, because the later cohorts in marriage would benefit more from the fall in mortality than would the earlier ones, so that there was a double effect.

His second point of criticism was in regard to the author's use of constant durational 'achievement proportions'. The last marriage cohort for which those proportions could be ascertained was that of 1925 or thereabouts. That was some time in the past. So far as he understood the paper-and he confessed that he found it difficult reading-the author seemed to claim that such duration achievement proportions-i.e. the proportions of the total ultimate family achieved at the end of successive marriage durations-were roughly constant irrespective of the actual size of family and that the constancy was supported by statistics for cohorts prior to 1925, and might therefore reasonably be assumed to be still applicable.

That seemed to him to be a startling and challenging assumption to make, because a small variation in the duration proportions would make a great deal of difference in the measurement of the ultimate deficiency. He had not seen the figures in the Royal Commission's Family Census, but some extracts were given in Table A5, in which the author compared the experiences of two cohorts, a 1910-14 cohort and a 1925-29 cohort. The speaker's reading of that comparison was that a definite change was taking place over the period. He did not think that the durational achievements were constant. The big falls were in the earlier marriage durations and the big increases were in the later marriage durations. There seemed to be a tendency for the durational incidence to be steepening in favour of the later cohorts.

That related to 25 years earlier, and he thought that there was reason to believe that there might have been further change in the same direction; but it seemed to him speculative to apply durational constants obtained from that period to the extraordinary fertility experience which had been characteristic of the war and post-war years. There were, he thought, reasons for expecting post-war conditions to have the effect of birth production over wider and later marriage durations than formerly. All classes of young mothers attended clinics, pre-natal and post-natal, and their whole reaction from contact with knowledgeable people at the clinics would tend, he was sure, to make them want to rationalize their families, so to speak, by spacing out their children more than in the past. That was a tendency which he would expect to flow from that situation, and it would have the effect of pushing the durational achievements into the later durations.
A similar tendency would almost certainly ensue from the practical difficulties of the time. Young people could not get houses. Some of them had had children in spite of that, but he was sure that the effect was some degree of postponement in normal family-building habits. A like result would follow from the tendency of women to remain at work after marriage.

Much of what he had said was purely personal opinion, but he thought that it was of such importance that the author should have given more attention to it, possibly by providing alternative measurements based upon conjectured changes. It would be a long time before it was known what changes were actually taking place, but that was no reason why they should not be foreseen, and it would be helpful rather than otherwise if the author would give some alternative measurements which would at any rate make it possible to see what difference to the ultimate deficiency measurement would be made by a small change in achievement proportions, and therefore the degree of error which might be expected in figures brought out by a mechanical self-reproducing model involving a single measure which seemed to be rather doubtfully based.

Mr E. Grebenik (a visitor) remarked that when he received a copy of the paper he had been reminded somewhat wistfully of a time about 12 years earlier when he read an article in Punch. The writer had seen a copy of an issue of the Fournal of the Royal Statistical Society containing an article on population. The writer, in effect, said 'I am a plain man, and if I want to know whether the population is increasing or decreasing in a particular year I count the number of children who are born and the number of people who die. If the former exceeds the latter the population is increasing; if it does not, it is decreasing. The statistician, however, is not content with that; he talks about potentials, reproductivity indices, net reproduction rates and a thing called "Kuczynski".' That was before the days of Vincent, Pollard, Karmel, Hajnal-and, it might be added, Cox-and he shuddered to think what the writer would have said of the author's paper.

In considering the paper, he wondered whether demographers were not 'barking up the wrong tree' to some extent in talking about reproductivity and the reproductive potential of a population. He gathered that they wished to answer the simple question which the writer in Punch had asked himself, namely, whether the population was increasing or decreasing. They had abstractions called reproduction rates or indices of one kind or another which all purported to give an answer to that question in the long run, but all those indices involved, if not explicitly then at least implicitly, some sort of idea about what was going to happen in the future.

If he had been the writer of that Punch article, and had known a little more about statistics, he might have asked himself 'what do I mean by a population replacing itself?' Presumably he would have said that a particular cohort would replace itself if the girls born in a particular year each had on the average either one girl, or, if that were preferred, 2.06 children, by the time they reached the end of their reproductive period. That was a concept which he could understand, and it lent itself to calculation, though with a certain amount of difficulty, from the statistics. Unfortunately, by the time the results were available they were usually i5-20 years out of date, because the reproductive period ended at the age of 45 , but the bulk of child-bearing was done between the ages of 20 and 30 , so that if it was desired to know anything about current fertility it was necessary to some extent to extrapolate.

In the case of the net reproduction rate, the extrapolation implicitly assumed that age-specific fertility rates or mortality rates would remain constant. The model which the author put forward, on the other hand, assumed a certain constancy of rates which were specific by duration of marriage, or alternatively of the total size of family which the married couple would ultimately have. It might be argucd-and he was not prepared to disagree - that those rates were a good deal more stable and changed much more slowly than did annual rates of age-specific fertility, but some assumptions had to be made about the future.

In the paper the author tested the achievement at various durations of marriage against his model (in practice he would probably use current rates for the model), and concluded that births were inadequate for replacement. In practice reproduction was virtually complete at a marriage duration of 15 years. The Family Census data showed that about $80 \%$ of all births in marriage took place within the first 10 years, and there, probably, a fair degree of constancy might be assumed for the immediate future ; it was not likely that that proportion would change radically or quickly. On the other hand, the position was very different when one looked at the marriages of short duration.

Reproduction in modern times was very much a matter of planning, and the model which he had in his own mind when he talked about reproduction was based on the view that people when they got married had some sort of idea of what constituted a reasonable family size. Most people had in mind a family size which they regarded as reasonable or desirable, but they varied greatly in the rate of family building. If times were bad, perhaps the birth of the first child would be postponed; the second birth might follow the husband's promotion, and so on. The point was, however, that if they married sufficiently youngand the average age of marriage was at that time fairly low-they might have a fair chance of achicving their goal, but to take a durational pattern of anything less than 10 years and extrapolate on that basis seemed to be a little risky, because in the Family Census there seemed to be an indication that there was an inverse relationship between the fertility of the first five years of marriage and the fertility of the second five years; an excess or deficiency in the fertility of the first five years tended to be compensated in the next five years.

He was prepared to put up with reproductive indices which were based on long durations of marriage-because people seemed to want them, he could not imagine why-but felt that in analysing current fertility it was better not to talk about reproductivity at all, but simply to try to trace to what extent there had been changes in the size of the families of people at different durations of marriage. Once those statistics were obtained-and they could be calculated from the data tabulated by the General Register Office-for a fair period of time, it would be possible to trace temporary fluctuations in fertility, but those temporary fluctuations in fertility ought not, he suggested, to be translated immediately into fluctuations in replacement indices. He would plead for a generation rate of replacement, for regarding reproductivity merely as a historical phenomenon, and for analysing current fertility without bringing reproductivity into it at all.

He did not mean his remarks to belittle the author's work, for which he was very grateful. All he wanted to suggest was that, for the purposes of measuring fertility changes, the concept of reproductivity might be irrelevant and in some cases misleading.

Mr C. J. Thomas (a visitor) congratulated the author on providing in the paper a very clear account of a new method of assessing reproductivity-a method which appeared to have great potentialities. The virtue of the method lay in the light which it threw on the fertility pattern of those marriages of the recent past which had not yet reached the end of the fertile period. By distinguishing separate ratios relating to births of the various orders it revealed in detail the merits and demerits, from the replacement standpoint, of the marriages of the various years. The method would, he believed, provide a useful tool for the investigation of the effects of social and economic changes on the reproductive situation. For example, the effects of the trade cycle on childbearing might vary with birth-order, and might be conveniently investigated by means of it.

The value of a method of that kind lay in the ease and reliability with which it could be used for appraising the recent position, and it was in that connexion that it could be suggested that further investigation was necessary. The paper was an exploratory one, and the author had not yet fully investigated the best ways of making the comparison between the model and the actual experience. Members would all look forward to his further investigations in that direction.

The author had been cautious in the way in which he had made the comparison, and, while recognizing that some of the published statistics were inadequate for making any refined comparisons, it remained to be demonstrated, in the speaker's view, that the reliance placed on the inertia of the weighted averages was fully justified. The use of Models B and C instead of Model A would not affect the relative positions of the corresponding ratios for the various groups of marriages within the 12-year period, but the model chosen would exert a small influence on the short-fall of births for complete replacement. The inertia of the weighted averages, to which the author had drawn attention, was therefore both an advantage and a disadvantage. On the credit side it facilitated a comparison between the model and actual experience, even when the conditions were somewhat dissimilar, but it did so at the expense of precision, and means of improving the responsiveness of the measure to fertility changes only could be the next subject of study.

The author had effectively shown that a self-reproducing population could provide a useful standard of reference, but conditions had to be reasonably stable in all respects save those of the fertility of marriage before precise comparisons could be made. To the extent that the fertility patterns, nuptiality, mortality and divorce conditions differed from the model, comparison became more hazardous. The 12 -year period which had been considered had not been demographically static. The war years had witnessed increased numbers of marriages, particularly at the younger ages, and the changes were persisting. Those changes had largely been brought about by bringing forward marriages from the future.

It might be useful to consider a change slightly different from that from Model A to Model B or C used by the author. The self-reproducing population of the model might undergo a change which took the form of a borrowing of marriages from the future in such a way that
(i) the proportion married at the upper fertile age-limit remained unchanged,
(ii) the marriages were borrowed from one or more of the younger age-groups and transferred to still younger age-groups,
(iii) the marriages brought forward followed the fertility pattern they would have had if they had not been hastened.

Such a change would occur, for example, if those who were marked out for marriage when aged $20-25$ decided to marry when passing through the agegroup ${ }^{15}-20$. There would then be an additional number of marriages over a five-year period followed, at the end of that time, by a complete cessation of marriages in the $20-25$ age-group. Such a population would still be a selfreproducing one, and the ratios, at specified marriage durations, of the number of first births, second births and so on, when referred to the original model, should, in the case of an ideal measure, assume a value of unity. However, the method of comparison adopted, which involved the calculation of the number of children of the various orders and related them to all the marriages which had occurred in a given year, would give, in the transition period, values for the average number of births of the various orders which were greater than those in the model. That was the result of the over-weighting of the younger marriages, and the effect would persist throughout the reproductive period of the group of marriages concerned.

Those changes might appear unrealistic when compared with actual changes, but the changes of the last few years might be considered to be somewhat
similar with others superimposed. It would be interesting to find out how much of the variation shown by the ratios in Tables 6 and 7 was due to that effect, and whether the ratios should not, in fact, have been slightly lower than those given in the tables.

One way of meeting the difficulty might be to consider the marriages of a given year and to obtain birth-order fertility rates specific by age at marriage and duration of marriage, and to combine them using the model proportions of marriages in age-groups as weights. That would be a somewhat complicated procedure, as it would be necessary to adjust, age-group by age-group, for the breakage of marriage by divorce and death of husband, and the necessary statistics did not exist. An easier and possibly equally effective course would be to devise a marriage-factor index which could be used to assess any bias in the figures. A possible index for the purpose would be the ratio of the weighted proportions married in age-groups in the comparison year and in the model using the model fertility functions as weights. That index could then be used to assess any bias in the figures, and he thought it would be found that the figures from 1939 onwards would be slightly smaller than those which had been given.

There was the further point which the author made, that the durational distributions incorporated in the model, which remained sensibly constant up to 1929, did not appear to be markedly different in the $1 \%$ 195r Census Tables. In those circumstances, had the author been carrying out his calculations after the appearance of those tables he might conceivably have used a more recent marriage-age distribution than the 1915-19 one. He could have used an average age-distribution for the 12 years under consideration, and, as that would be rather nearer to the age-distribution of the particular year in the intervening period, it might then be superfluous to use the suggested marriage-factor index. If by appropriate adjustment 'model' effects could be largely eliminated from the ratios the value of the method would be further demonstrated.

His point was that when the conditions were not static the comparison ratios were biased slightly upwards or slightly downwards, and other means of obtaining more refined comparisons under changing conditions should be investigated. Whenever the conditions appeared to be reasonably settled, then those conditions should be the ones incorporated in the model. A previous speaker had suspected an overstatement of the deficiency. He himself suspected, as he had tried to explain, that the marriage-age distribution incorporated in the model might be responsible for a slight understatement. The author had therefore obviously provided a topic for discussion, and his further stimulating contributions on the topic would be appreciated.

Mr R. E. Beard, in closing the discussion, said that when approaching problems in a scientific manner it was usual to put down a definition of what it was desired to measure. One speaker that evening had already referred to that point and had questioned whether 'reproductivity' was an appropriate word to introduce into the paper under discussion, and it was important to look at the meaning of that word before discussing the paper. It was a concept which had been introduced into demographic studies to indicate whether or not a group of births would replace themselves before the end of their reproductive period. It was essentially a generation concept, and only in a very special sense would actual population movements bear a direct relationship to an index of reproductivity defined in that way.

Within that concept, the factor which mattered was the number of births during the child-bearing period, and questions of marriage and legitimacy were immaterial to the concept, as could easily be seen by comparing, for example, two populations in which the numbers and distributions of births by age of parent were identical, but in one of which the marriages occurred at earlier ages. A reproduction index based on marriage fertility would show different values in the two populations, but the true positions with regard to reproductivity would be identical.

Against that concept of reproductivity it was of interest to look at the various methods which had been developed to measure it. The first crude measurement was, of course, the natural excess of births over deaths in a population, but it was obvious that that net increase conveyed very little information about the concept sought and was largely dependent on the age structure of the population. In historical development, the next index had been the 'replacement index' based on the calculation of the stationary population supported by the recorded annual births related to the actual population. There again the index largely depended on the actual age structure of the population, and conveyed little of the true measure of reproductivity.

The net reproduction rate had next been developed with the object of eliminating the effect of age structure. It was essentially a calculation of the total births expected from the survivors of a group of new-born infants, using the fertility and mortality rates at successive ages experienced in a given calendar year. Male, female and joint reproduction rates had been devised, and also an 'effective reproduction rate' which allowed for projected mortality. The criticism which could be directed against the use of that type of index flowed directly from the concept of reproductivity itself. The measure being sought was one of generations, and the use of factors calculated at successive ages in a particular calendar year ignored entirely the substantial fluctuations which occurred from year to year in all the factors concerned, and which might well give rise to results far removed from reality. For example, in a generation calculation there was inevitably a close relationship between the fertility of women aged, say, 19 in 1940 and of their survivors at age 24 five years later, since the factors affecting the fertility, such as the probability of marriage, were interdependent. The fertility rates of women of the two ages in a given calendar year, say 1940, would not, however, bear such a relationship, particularly in the light of external factors existing at that time, which had a profound effect on the probability of marriage and on fertility.

Instead of recognizing that the life-table technique which had been imported into that type of calculation was inappropriate in the circumstances, on account of the nature of the variations involved, and that no single index was appropriate for describing the particular problem, demographers had directed their attention to modifying the net reproduction rate, basing their arguments on the theory that fertility rates dependent on age had to be corrected to allow for the incidence of marriage. That led to further difficulties, however, as it had soon been recognized that the unstable nature of marriage rates made it necessary to use some set of hypothetical tables which made the resulting index largely a matter of judgment, and of extreme difficulty in interpretation. That line of thought finally led to the development of indices based on average family size, but the fundamental weakness still remained.

The author had developed a further refinement in which models were built up on certain assumptions regarding the various factors entering into population.
structure. It was probably correct to say, on the basis of completed fertility, that a comparison of actual data with his models could give rise to results having a certain usefulness. The real problem, however, was whether his models were appropriate criteria for measuring incomplete fertility, and in considering Table 6 it was pertinent to consider the figures against the background of the total number of marriages and total births. In 1939 and 1940 the number of marriages had been high, and Table 6 showed that the fertility in the early years of those groups had been low. However, in 1942 the total number of births rose substantially. In fact, on the concept of reproductivity mentioned earlier there had been a substantial increase, whereas the figures of Table 6 suggested the opposite effect. In the same way, Table 7 could be criticized as giving a biased view of the replacement problem. It was, in fact, only one aspect of the subject and one which, over the period concerned, almost certainly gave a pessimistic result.

In view of that criticism, it was important to appreciate that the paper provided only one aspect of the question of reproductivity, albeit a useful and thorough analysis for which the author deserved praise. It was, however, far from a final answer to the problem, and there would seem to be strong grounds for picking up the subject again at the point of the net reproduction rate and, ignoring the life-table technique, making a study of the generation structure of fertility by attained age of parent.

The President (Mr W. F. Gardner), in proposing a vote of thanks to the author, said that a paper which fell within the province of demography was always welcome at the Institute. The future numbers, and more particularly the future age-distribution, of the population were of the utmost importance to the actuarial profession. Not the least among their preoccupations was the relative numbers of young and old persons with which the economic balance both of social insurance and of private pension funds was so much interlocked.

He felt that all research into the rate of replacement of the population was of great usefulness, and the Institute was indebted to the Demographic Study Group of the Students' Society, and more especially, of course, to Mr Cox, for laying before them the results of a thorough exploration of one line of approach to the problem. The paper had involved much intensive computation, and the construction of the self-reproducing models could have been no light task. If the use of such models in demography was in itself not new, the work nevertheless represented an original piece of research, throwing light on the population situation of the country.

As had been said by many speakers, the subject was not one on which finality was possible; but by patient and unprejudiced pursuit of the facts they might slowly increase their understanding of what was a most complex problem. It seemed to him that in the quest three things were above all necessary: hard work, integrity of purpose, and sufficient humility to be satisfied with limited objectives. The author had amply fulfilled those requirements.

Mr P. R. Cox, in reply, remarked that it was fortunate that Mr Stephen Potter was not in the audience that evening; otherwise he might have been accused of 'groupmanship'-the art of getting work done by others and taking the credit for it oneself.

He would like to say how the Demographic Study Group had come to look at the subject from the point of view which had been adopted. Two years
earlier, in another paper (reference (5)) produced by the Group, an attempt had been made to analyse fertility as simply as possible, avoiding all the complexities. That paper should be considered in conjunction with the one under discussion. It might be asked why they had departed from the simple approach and taken up the complicated question of reproductivity. They had done so with their eyes open, and after many discussions on what could be hoped for from that approach. Simple measures could tell how fertility was varying from one year to another, but could not, so far as was known at the moment, tell much about how a population was progressing, whether it was above or below the line. That was one reason why they came to try measures of reproductivity. Another reason was that although the subject had been tackled from time to time the problem did not appear to have been solved. They did not cxpect necessarily to solve it, but there seemed to be a challenge implied by something which had not been completed and which invited another attack upon it. That was why they had tried the 'model' approach. The object of using models and making a tracing had been to treat the subject in a new way and to track down if possible the elements contributing to population replacement. Criticism had been made of the heading to Table 7, and he agreed to some extent with the opener on that point. The aim was to improve on Table 6, where the models-derived as they were from current experience-did not exactly match the experience of the past 10 years, and to try to eliminate all elements not directly germane to fertility, such as the migration of women who married Canadian and American soldiers and bore their children elsewhere. Table 7, therefore, was the last item in the tracing, showing the difference between the actual fertility and the more relevant self-reproducing model.

He referred to the criticisms by Mr Derrick and others in connexion with Table A 5 , saying that the question was whether the Table showed the durational build-up of families to have remained constant. A related question was what had happened to the marriages contracted since 1925-29. One of the difficulties in fertility analysis was to find anything which had been approximately constant for a time. The original and much fuller version of Table $\mathrm{A}_{5}$ contained the experience of all the cohorts between 1910-14 and 1925-29, and it seemed that over that period there had been approximate stability in the build-up rates. He agreed that the figures for 1925-29 might indicate a slower build-up than the figures for 1910-14, but the difference was not large. When the 1951 Census I $\%$ sample tables had been published, he had been anxious to see whether it was possible to draw from the fertility tabulations any definite conclusion as to whether or not the rate of building up families had altered. It was not easy to get a completely clear answer from the Census, but he had formed the imprcssion that there had been no slowing down.

With regard to the question of the future and the past, it had been said that, if there had been a slowing down, Table A 5 was an unrealistic basis for the models. There was, however, another way of looking at the matter. Table 7 gave the number of children born so far to couples in comparison with certain standards. It did not say anything about the future-the aim had been to avoid extrapolating. If it were true that couples were building up their families more slowly, that would show up in due time, but for the present such a development was in the category of those future elements the consideration of which had been deliberately avoided.

Hc was a little surprised at the suggestion that they had not looked ahead enough and that falling mortality should have been brought into account. It
had been the aim of the paper to be as factual as possible and to avoid any guesswork. It appeared from their remarks that certain speakers still regarded the analysis in the paper as incorporating an element of the future, but the aim had simply been to produce certain model calculations which could prove nothing, but which might guide opinion in a certain direction.

Mr Cox has subsequently written as follows:
Mr Grebenik is, of course, quite right in saying that the extent of reproductivity can be measured with certainty only in relation to the more distant past; for marriages of less than 10 years' duration there must be uncertainty because their families are not complete. Nevertheless, the analysis in Table 7 provides a convenient comparison for recent short-term variations in fertility.

Mr Beard's criticisms are destructive in nature, and although he offers an alternative approach, namely, the generation aspect of fertility by attained age of parent, the data available today do not permit extensive inquiries to be made in this manner. It is hoped that the possibilities will improve with the passage of time, and that a valuable additional method will thus be provided along the lines he suggests.

Mr Clarke, Mr Barnett and Mr Derrick have each provided plausible reasons for a slower rate of child-bearing in the early years of marriage in present circumstances. It must be remembered, however, that fecundity declines with advancing age and that if opportunities are lost early in life it may not be possible to catch up later on. 'Slowing up' does not necessarily mean that fertility has not declined.

Mr Beard seems to suggest that the rise in births in this country in 1942 is not reflected in Table 6, but if the appropriate years of marriage and durations of marriage are examined, the effect of the rise can be seen in increased percentage rates.
When Mr Derrick says that fertility is known to have run for many years at a level of $20 \%$ helow replacement standard and suddenly to have improved to a deficiency of only $10 \%$, he is surely comparing the results of two different techniques of measurement. It is not known what would be found if the methods in the paper or those of the Royal Commission could be extended to the years before 1.938 , but it scems unlikely that they would reveal any very sudden transformations.

Mr Thomas makes some ingenious suggestions for improving the models, which might well be profitably pursued. It seems doubtful, however, whether much change would follow from using a current age-distribution of marriages, because this would be intermediate between the 'old' one of 1915-19 in Model A and the 'future' one in Model B.

Mr Barnett refers to social influences affecting family size. He may be right in his speculation that the couples who have fewer children than they desire outnumber the couples who have more children than they desire. It must be remembered, however, that we have had full employment for the last 10 years. If this could not be maintained at any time in the future the numbers of children desired by married couples might rapidly be reduced.


[^0]:    * These tables have not been adjusted in order to correct the understatement of childlessness which was a feature of the Family Census; for the purposes of the present paper, such a correction does not seem important.

