

REVIEWS

The Registrar General's Statistical Review of England and Wales.

[1940-1945 Text, Vol. I, Medical. H.M.S.O. 6s. 6d.]

[1940-1945 Text, Vol. II, Civil. H.M.S.O. 5s. od.]

[1946-1947 Text, Vol. I, Medical. H.M.S.O. 6s. 6d.]

THE war of 1939-45 interrupted the regular publication of the Registrar General's *Statistical Review of England and Wales*. This used to appear annually in three parts: two volumes of tables presenting medical and civil statistics respectively and a third volume known as the *Text*, which provided a commentary on both volumes of statistics. In making up the war-time arrears it was decided to condense the *Text* for the six years 1940-45 into two volumes, one of which should deal with medical statistics and the other with civil statistics. A similar plan has been adopted for the two-year period 1946-47, although the *Civil* volume for this second period has not yet been published. Ultimately, however, it is intended to revert to the pre-war practice of issuing a single *Text* volume every year.

The two *Medical* volumes 1940-45 and 1946-47 together give a useful conspectus of what has been happening to English death-rates during a period of war, of far-reaching social changes and of great progress in medical science. They enable us to follow the course of mortality during the period for each cause of death and within each geographical region. Information such as is given in the following extract from Table XXII of the 1940-45 volume and Table XXIII of the 1946-47 volume is full of significance for the student of mortality, be he actuary or medical officer.

Deaths from certain selected causes at ages under 1 year
per 100,000 related live births

Cause of death	Greater London			North Region		
	1936-39	1947	Decrease	1936-39	1947	Decrease
Bronchitis and pneumonia	1,001	603	398	1,308	991	317
Diarrhoea and enteritis	847	350	497	521	781	-260
Congenital malformations	524	414	110	676	555	121
Premature birth	1,222	680	542	1,798	1,152	646

Regional differences, such as are illustrated in the above table, are important to any actuary who is engaged on projecting mortality rates into the future, since it is not unreasonable to suppose that areas which now exhibit higher mortality (other than may arise from specific occupational risks) will eventually catch up with other parts of the country where current experience is more favourable.

Unfortunately, it is not possible to include in the *Statistical Review* any analysis of mortality by social class. In the past it has been amply shown by the Registrar General's Decennial Supplements that social class differentials in

mortality are substantial and it would be valuable to know how much of the observed regional differences is due to an uneven distribution of the social classes over the various regions of the country. Is the experience of, say, Social Class III in the North Region significantly different from that of Social Class III in Greater London? Unless questions of this kind can be answered, regional comparisons have only a limited value.

The main theme of these volumes is reduction in mortality. There are, however, two causes of death which have now been increasing over many years. These are cancer of the lung and diseases of the coronary arteries. Of the former cause it is remarked (1946-47 volume, p. 165):

Comparing the rates in 1947 with the mean annual rates in 1921-30, men between 45 and 75 show a tenfold increase.

As regards diseases of the coronary arteries there is a possibility that some part of the recorded increase is due to changing fashions of nomenclature in entering death certificates; but, even when all due allowance has been made for this possibility, the increase remains remarkable. Its relative incidence is greatest among men aged 45-75, i.e. the same age-group as has been so heavily affected by cancer of the lung. It is not without significance that this was also the age-group at which in a recent note on the Continuous Mortality Investigation into Assured Lives (*J.I.A.* LXXVII, 103) a marked hump was observed in the ratio of the death-rates experienced in 1947-48 to the rates of the A 1924-29 table.

In the 1946-47 volume (p. 204) attention is paid to the difference between the two sexes in the recent trend of total death-rates under 70 years of age from coronary, cardiovascular and chronic myocardial disease and senility. The proportionate excess of male over female mortality in this group has steadily increased for many years and it is stated:

Changes in diagnosis and death certification clearly could not account for these remarkable contrasts, i.e. between the male and female trends, and it must be concluded that the recent war had cumulative effects on the circulatory systems of men without any such effects appearing in the case of women.

It may be questioned whether the observed feature can be so readily regarded as an effect of the war. But, however that may be, the sustained level of heart diseases, and particularly of coronary disease, as a cause of death among male lives is a fact of considerable importance in the study of mortality.

Like its *Medical* counterpart the *Civil* volume for 1940-45 is also concerned with events of some magnitude in British demographic history. The war period saw some sharp fluctuations in the annual numbers of births and at the same time the marriage experience underwent considerable changes—the chief of which was an appreciable reduction in the average age at marriage. Although the period covered nominally ends in 1945, many figures are quoted for subsequent years up to 1948. Hence the high level of births in the post-war period comes to some extent into the general survey—particularly when ‘long-term trends’ are under discussion.

In recent years marriage-rates have shown substantial increases. Table XIII of the 1940-45 *Text (Civil)*, which gives women’s marriage-rates in age-groups for 1911, 1932, 1938 and thence every year consecutively to 1947, illustrates this development with telling effect. The most spectacular increases are, as would be expected, at the youngest ages. Yet it is noteworthy that substantial changes have also occurred in the older groups. Thus in the 45-50 age-group the marriage rate in 1947 was 50% above the corresponding rate in 1938.

For many years after the first world war there was a shortage of men of marriageable age and women's marriage rates fell accordingly. It was inevitable that the subsequent levelling up of the sex ratio should result in increases in these rates. However, by 1947 the rates for all age-groups had not merely reverted to what might have been considered a normal level but had risen substantially above the level of 1911. In fact, at the present time a higher proportion of people are marrying than at any time during the present century and, moreover, they are marrying younger than their forebears.

The increase in the marriage-rate at the younger ages is particularly important in its effect upon fertility. As the 1940-45 *Text (Civil)* justly remarks (p. 36):

All the available statistical evidence goes to show that, other things being equal, the earlier the marriage age, the greater is the average family, and there can be little hesitation in expecting the future families of the women marrying to-day to be significantly larger than they would have been if the said women were to have delayed their marriages to the ages which were more or less customary during the long period prior to 1932 when the fall in the birth-rate was so pronounced.

This seems a more reasonable expression of opinion than the alternative view advanced by some demographers that the size of the family is largely independent of age at marriage.

Among many other informative tables bearing upon recent marriage experience, there is space here to mention only Table XVI. This shows proportions married among women of various age-groups for 1911, 1932, 1938 and every year thereafter to 1945. A comparison of 1945 with 1911 yields striking results. In the age-group 20-25 the proportion married in 1911 was .242; in 1945 it was .428. In the 30-35 group the corresponding proportions were .711 and .790 respectively. It is noteworthy that in the group 45-50, the proportion married in 1945 was only .765, i.e. less than the proportion at 30-35. These older women are the survivors of a generation a certain number of whom lost their potential husbands in the first world war. As subsequent generations move into this age-group the proportion married will steadily increase and the anomaly whereby the proportion married assumes a maximum value at a relatively low age should eventually disappear.

The sections of the 1940-45 *Text (Civil)* which deal with fertility contain much valuable material for the demographer. They include an analysis of recent changes in the E.R.R. (effective reproduction rate) with a view to discerning the genuine trend underlying the disturbances and fluctuations caused by the war. From 1933 to 1947 the trend of the E.R.R. was upwards and the total increase during this period of .458 has been analysed according to the following contributory causes (Table XXXI):

Increase in legitimate fertility	·200
Increase in illegitimate fertility	·040
Increase in proportion married	·192
Decrease in mortality	·026
	·458

Until 1946, legitimate fertility was, on the whole, declining. The years 1946 and 1947 experienced abnormally high fertility which cannot be expected to continue—indeed 1948 and 1949 were both well below the 1947 level. The increased proportion married, however, is for all practical purposes a permanent feature and its contribution to increased fertility is of the utmost importance.

The general picture of the current demographic situation which emerges from these instructive pages may be summed up as follows:

(i) Age-rates of legitimate fertility (i.e. legitimate children per 1,000 married women) decreased slowly but steadily up to 1945. The decline may have been arrested since then, although as yet it is impossible to form a definite opinion on this question.

(ii) There have been spectacular increases at all ages in the proportions of married women to all women. Consequently there have been relatively more women exposed to risk of legitimate maternity with resultant increases in the rates of total fertility (i.e. ratios of all births to all women at each attained age).

(iii) The advent of a younger mean age at marriage has lengthened the period during which women are exposed to risk of maternity. Consequently, the average size of family per married woman is likely to grow larger even though, as indicated in (i), legitimate fertility rates are in all probability not increasing.

These conclusions on the family pattern of contemporary Britain illustrate the value of the contribution which demography can make to the study of society.

Controversy regarding the value or validity of the E.R.R. is long-standing and it is continued in the volume now under review. Much argument would be avoided if it could be generally understood that the E.R.R. is an index of secular trend in fertility and that it is not an efficient measure of the extent to which a given generation is replacing itself. For the latter purpose it is necessary to observe the reproductive performance of a given generation of women through time—a procedure which will in due course become possible with the admirable series of fertility statistics which the Registrar General has been able to publish since 1938. An alternative approach is, of course, that adopted by the Royal Commission on Population, which investigated the average size of family among a sample population of women.

Both types of index—the generation and the secular—are indispensable to the demographer and they serve different purposes. The secular index enables him to keep his finger on the pulse of current trends; but only the generation index can answer the ultimate question whether the population is replacing itself.

It is perhaps a little unfortunate that in presenting a modification of the E.R.R., the 1940-45 *Text (Civil)* refers to it as a type of 'replacement rate'. The new concept, which is designated the 'capacity replacement ratio', is introduced with an exposition which suggests greater novelty than actually appears justified. The new index may be developed as follows:

Let P_x = number of women aged x in a given year,

f_x = fertility rate at age x in the same year.

Then the E.R.R. = $\frac{1}{l_0} \sum f_x \cdot l_x$, where the summation extends over the reproductive ages and l_x is the usual life table function on an appropriate mortality basis.

By multiplying both numerator and denominator by $\{\sum f_x \cdot P_x\}$ the E.R.R. may be written

$$\{\sum f_x \cdot P_x\} \left\{ \frac{1}{l_0} \sum f_x \cdot l_x \right\} / \{\sum f_x \cdot P_x\}.$$

If a set of arbitrary weights, k_x , be substituted for f_x in both the denominator and in the second factor of the numerator of this expression, we obtain

$$\{\sum f_x \cdot P_x\} \left\{ \frac{1}{l_0} \sum k_x \cdot l_x \right\} / \{\sum k_x \cdot P_x\}.$$

This is the 'capacity replacement ratio'.

The weights, k_x , are chosen to resemble f_x fairly closely with the condition that $\sum k_x = 1$. The advantage of the capacity replacement ratio over the E.R.R. is that the values of k_x can be held constant from year to year. The first factor in the numerator, $\sum f_x \cdot P_x$, is the number of births recorded in the year, so that the capacity replacement ratio can be written

$$\text{Births in year multiplied by } \left\{ \frac{1}{l_0} \sum k_x \cdot l_x \right\} / \{\sum k_x \cdot P_x\}.$$

As will be seen, the difference between the E.R.R. and the capacity replacement ratio is not particularly striking, and it may be felt that the elaborate biological *mystique* with which the latter index is expounded is somewhat out of place. At the same time it is only fair to add that the close similarity between the two indices is recognized in the concluding paragraphs of Appendix III to the 1940-45 *Text (Civil)* where it is stated that the object of the enquiries pursued has been 'to seek a clearer understanding of the E.R.R. rather than to change its substance'.

It is not possible in the scope of the present review to discuss all the topics covered by the 1940-45 *Text (Civil)*. Many tables have been included, based upon the new fertility statistics, which form valuable supplements to the data published in the annual numbers of the *Statistical Review*. Chief among these are the tables which exhibit maternity rates by age, by duration of marriage and by age and duration combined. The computation of an exposed-to-risk at each duration of marriage presented some difficult problems and their solution has provided material for a most absorbing report which has been included as Appendix II. This should be read by every actuary who is interested in exposed-to-risk techniques.

R. D. C

A Survey of Public Attitudes towards and Ownership of Insurance in Norway, 1949.

[An abbreviated edition translated, edited and published (presumably privately) by Institute of Life Insurance, New York.]

THE original survey was made and the Norwegian edition prepared by the Statistical Bureau of the Norwegian Life Insurance Companies. This translation has been abbreviated and edited by the Institute of Life Insurance, New York, by whom it has been published. It was hoped that the translation would be of interest to executives in the U.S.A. because of the close parallel between public relations problems in Norway and the U.S.A.

The object of the survey was to ascertain the extent of the knowledge of the people in Norway of life insurance, their attitude towards life insurance, their opinion of the companies and their agents, and whether they are satisfied with the companies and the distribution of life insurance in different social groups and in different parts of the country. The results of the survey would undoubtedly be considered by the Royal Commission appointed in 1947 which is soon to report on its proposals for the future of life insurance in Norway.

Through two market-research institutes, Fakta and Gallup, the same series of questions was asked of 3000 self-supporting persons aged 21 and upwards. One half of the group of persons was questioned by Fakta and one half by Gallup. The two institutions were engaged to make independent surveys as a check on each other and the results correspond closely notwithstanding that Fakta used a probability-sampling method and Gallup a quota-sampling method.

Seventeen main questions with one or two subsidiary questions were asked in all and the questions may best be analysed into three categories.

(1) Those which asked for statements of fact, e.g. What life insurance policies have you? How much do you pay each year in premiums?

(2) Those which sought to test the knowledge of the person questioned, e.g. Are life insurance companies under public control?

(3) Those which asked for the opinion of the person questioned, e.g. Are you satisfied with the work of the companies?

No information is given to show how the statements made in reply to the first category of questions were checked. It is hard to believe that the persons questioned kept the details of their life insurance policies in their heads and equally hard to believe that they were asked to produce their policies so that the questioner could extract details. Yet so accurate is the information obtained both by Fakta and Gallup of the amount of insurance in force that the percentages which the thirteen native companies have of the total in the group questioned agree very closely with the percentage which the thirteen companies have of the total insurance in force in the country. Answers to questions relating to visits of agents in 1948 show that 58 % of the policies issued were sold on the first visit of the agent. It must be borne in mind however that the majority of persons questioned came from the lower income groups (two-thirds of the policies which they held were under £125), that a large proportion lived in rural districts, and that many would be subjected to salesmanship on the visits ostensibly for collection of premiums as well as on those where the sale of a new policy was the main object. Only 35 % of persons questioned had been asked to buy insurance in 1948 and only 9 % had actually bought, which would seem to indicate that there is scope for further canvassing. Although the information brought out by some questions in this category could no doubt have been obtained from other sources, such as government returns and agents' records, the summaries of the replies are valuable to the Statistical Bureau as a check on the sample and to the companies in reorganizing field work.

Questions in the second category covered such items as the knowledge of the persons questioned of the allowable deductions for life insurance premiums in assessing income tax. The replies would be a guide to the type of information which the companies should disseminate to the public either through agents or through co-operative advertising.

Replies to questions asking for opinions may be of doubtful reliability because so many factors have a bearing: the exact wording of the question, the personal views of the questioner, the intonation and inflection of the voice when the question is asked, the surroundings in which the questioning takes place and the topics of political and general discussions at the time. A classic example occurred during the campaign for the last Presidential election in the U.S.A. when a question in the form 'Do you expect Dewey or Truman to be elected President?' brought an overwhelming majority in favour of Dewey. Whether or not the widely publicized result affected the voting at the election will probably

never be known but it is a historical fact that Truman was elected. With this in mind it is difficult to know what scientific value to attach to the replies to questions of the type 'Do you want more information about life insurance?' 'Have you anything to say against the way the agents are working?'.

Taking the questions as a whole the answers are what might have been expected, but none the less useful because of that. It is understood that the Norwegian companies have made considerable use of the summaries of the replies. The value of the survey to the English-speaking world lies in the example which it affords and the lessons which it teaches to any organization which may contemplate a similar survey as a counter to a threat of nationalization.

C. F. W.

Demography. By PETER R. COX

[Pp. xii + 326. Published for the Institute of Actuaries and the Faculty of Actuaries at the University Press, Cambridge, 1950. 20s.]

THE important use of population statistics made by actuaries in mortality and sickness investigations and in the special problems of social insurance, and the increasing contribution of the actuary to the development of techniques of analysis of population statistics, have both compelled the Institute of Actuaries to give particular regard to demography in its examination syllabus. This book has been written to meet the needs of students training for the examinations. Since the book is strictly related to examination needs the author was doubtless handicapped by certain inhibitions imposed by the syllabus. For the syllabus places emphasis on descriptive statistics while, for example, such aspects as mortality investigations and applications of demography in the field of social insurance are transferred to other parts of the examination syllabus. The author sets out, despite this handicap, to provide a connected account of the principles and methods employed in the study of population for actuarial students and for all who are concerned with population statistics. Mr Cox emphasizes that

the quality which is perhaps the most needed for success in demography is a proper understanding of the difficulties involved in the interpretation of the statistical data and the dangers inherent in their use to provide estimates applicable to future years and this objective is apparent throughout the work.

It is a brave man who would attempt a definition of demography in these days when old names are often given new meanings for the sake of novelty. The author speaks of 'the study of human populations by statistical methods'. This is deliberately general and to be more particular one would have to list the chapter headings of the book. Exception might be taken to one or two minor points. For example, the statement that 'for some purposes qualitative as well as quantitative factors are included in the subject' is confusing since unless the factor were measurable, i.e., were quantitative, it could hardly be studied. Again the author says 'various phases in the growth and decline of civilizations can be shown to be associated with demographic developments' and pursues this theme as if to suggest that the population changes were the cause of the industrial and economic disturbances. The historical introduction could be improved. The reference to Halley on p. 8 is misleading—his table was not based upon the enumerated population but on the registers of births and deaths. There is no mention of Gregory King or Richard Price, or of Milne, who was an actuary. These names are more important than that of Malthus, who was

hardly a demographer. John Finlaison, the first President of the Institute, deserves mention for his part in the inauguration of registration in 1837.

The author begins with raw materials. The discussion of census procedure and the practical difficulties involved, bias and inaccuracy in the data, is comprehensive in length and sufficiently deep for actuarial students. Mr Cox leads on from a history of the registration of births and deaths to an outline of the publications of the General Register Office. The Registrar-General caters for a wide field of varied interests and a tabulation which is essential for one observer may be unavoidably buried among a number of tables which do not interest him. In the past finding one's way among the returns of the General Register Office has been a matter of practical experience and, though students will find this chapter valuable, it is still true that the only way to get a real appreciation of the data provided by the Registrar-General is to use them and become thoroughly acquainted with them. Since the publication of this book the General Register Office has issued a guide to census reports, so the labours of the author may be lightened in subsequent editions.

Information about migration is scanty, but the author makes a systematic reference to possible sources with useful illustrations. Migration is at times violently affected by political and economic influences; it may have important effects on the age and sex distribution of the population and it is a pity that students often have only the vaguest ideas of its measurement.

Statistics of sickness are still insufficient, but are rapidly becoming more and more comprehensive; and a second edition of this book will have to contain fuller treatment of the General Register Office's exploitation of hospital records and more extensive reference to direct surveys of general practitioner records as distinct from their indirect use in the analysis of National Insurance statistics. There have also been important developments in the central registration of cancer and of mental disease. Local authorities have maintained a register of tuberculosis since 1912. The author rightly emphasizes the limitations of mortality rates as indices of sickness and one would, therefore, expect the measurement of disability to be treated as exhaustively as the measurement of mortality.

The author stresses both the advantages and the practical difficulties of sampling; but not enough is made of the fact that these difficulties may be less than the difficulties of a total survey, and illustrations of the reliability of samples, of which there are many, are lacking. Of advantages Mr Cox mentions the saving in time and cost (but does not mention that a small staff of skilled enumerators replaces a vast army of unskilled workers), and goes on to say 'Against these gains a wider margin of statistical error must in fairness be set'. It could have been pointed out that if the technique is properly applied the margin of error is measurable and can be reduced by increasing the sample (and by other means) until it is within acceptable limits. The author also states 'the extent to which any conclusions... (from local surveys)... can be regarded as generally applicable over wider areas is, however, uncertain'. This appears to overlook the very techniques for taking local variation into account which Mr Cox himself describes in subsequent paragraphs of the same chapter, e.g. stratification and in particular the sampling-efficiency schemes perfected in the Census Bureau of the United States Government and in the Bureau of Statistics of the Canadian Government. Of the practical, as distinct from theoretical, difficulties, too much is made of those which can be avoided by good management but not enough of those which cannot be so avoided, viz. the dependence

upon voluntary co-operation or the possible extra cost of a pilot survey (where the prior knowledge for stratification is not available).

The analysis of mortality data is an exercise in clear thinking. A death-rate is a ratio of two numbers. The choice of numerator, i.e. the number of deaths from, say, bronchitis (in a specified year, sex and age-group) appears simple, but the assignment of deaths to a particular cause is a process conditioned by diagnostic fashion and other sources of variation in the precision of certification and by arbitrary procedures which are superimposed, after the certificate has been produced, by the registration authority. To construct a meaningful time-series it is necessary to be a medical historian. The choice of the denominator too is difficult. Deaths 'at all ages' may in fact be (e.g. as in cancer) deaths within a certain age-range. There is a danger that having taken pains in the construction of the ratio the operator will elevate it to a meaning which its arbitrary nature does not sustain. The author's treatment of the factors involved in the comparison of mortality indices is thorough (and, in his own word, 'connected') and the various indices, e.g. the comparative mortality index, are carefully analysed by expressing them in mathematical formulae and reducing them to their simplest terms. The formulae are, for this purpose, essential, but the student would benefit also from a little plain arithmetic.

Treatment of the difference between the actuarial and the demographic approach to mortality statistics in terms which will be understandable to students must always be a difficult task. The differences between the demographer and the actuary are less in attitude than in method, and are to be sought in the way in which their differing responsibilities determine their choices of method.

In his reference to the difficulty of disentangling the influences on mortality of various characteristics Mr Cox might have mentioned the use of statistical techniques such as multiple regression analysis and the important paper by Buckartzsch (*Population Studies*, 1, 229 [1947]).

In the chapter on fertility the different types of 'reproduction rate' in current use will become less mystifying to the student under the author's skilful guidance. Mr Cox was perhaps anxious lest by indulging in criticism he might confuse the student, but a more critical approach would be instructive, particularly in drawing more attention to practical difficulties of interpretation. Most of the author's criticism is contained in a single paragraph on 'reproductivity', emphasizing the pre-occupation with generation replacement and the difficulties of stretching data of single years to encompass this concept. The author does also point out that a reproduction rate is not a forecast but an index of current fertility 'just as the life-table death-rate is an index of current mortality'. The manner in which each and every index falls short as an index ought, however, to be stressed. Gaps in information are underlined in an historical account of the various fertility inquiries which have been made from time to time, an important assembly of information.

A short account of the growth of the population is neatly illustrated by diagrams. The introductory passages of this chapter can be criticized; e.g. there is barely enough evidence to substantiate the stability of the population in the seventeenth century. There is an excellent review of changes in mortality from the principal causes. Some figures of world population are given but the author does not effectively extend the scope of the book to world dimensions. Such an extension might have advantages if the data were available; but it would not add to the value of the book to actuarial students—for whom it is intended—

who read illustrative material mainly in order to improve their absorption of the fundamental principles of demography, which hold no more strongly in Timbuktu than in London.

Population projection as a method of illustrating the relative influence of various forces is given adequate space and the principal projections hitherto made are critically compared in relation to bases and results. The value of projections to the politician may be controversial, but to the student they are an essential technique of demonstration.

The mathematics of population growth are sufficiently explored and avenues of future research are indicated. As Mr Cox says of the mathematics:

contact with reality may be lost because attention becomes concentrated on convenient assumptions by means of which the analysis can be completed.

Nevertheless, the student will find this chapter stimulating.

The Royal Commission on Population did not publish its *Report* until this book was in a late stage of preparation, but Mr Cox has introduced an addendum covering the new data made available and the principal questions discussed by the Commission.

The student will find at the end of each chapter a bibliography. Great care has obviously been taken to keep the list as short as possible without omitting essential sources. There is so much written in the demographic field that the student would be bewildered without some assistance in discrimination. There are three excellent appendices giving a summary of the information given in the 1931 Census tabulations, a list of 1931 Census volumes and decennial supplements for England and Wales, and a similar list for Scotland.

This is a first text-book on a subject not only new to the Institute syllabus but also comparatively new to the field of formal teaching. The author's task was unenviable and it has been much easier to criticize his work after its accomplishment than it would have been to do it from scratch (especially considering the circumstances in which it was written). The Institute is indebted to him; and future generations of students will find in this book, which will improve from edition to edition, invaluable help through their examination ordeals.

B. B.

Life Insurance Mathematics. By ROBERT E. LARSON and ERWIN A. GAUMNITZ

[Pp. vii + 184. New York: John Wiley and Sons, Inc.; London: Chapman and Hall, Ltd. 30s.]

BRITISH actuaries are so used to Spurgeon's official text-book on Life Contingencies that they are hardly aware of the large number of other text-books which exist on the subject. The book under review has been prepared by two members of the School of Commerce in the University of Wisconsin; Mr Larson is a lecturer in commerce and a Fellow of the Society of Actuaries, Dr Gaumnitz is a professor of commerce.

Commercial subjects, such as insurance, are nearly always included in the curriculum of universities in the United States of America and some universities have professors of insurance. It is not surprising, therefore, that text-books on Life Contingencies have been published in the U.S.A., particularly when it is remembered that portions of Spurgeon's book do not reflect American practice.

Life Insurance Mathematics is elementary in its scope, and the calculus is not

employed. There is no reference to the force of mortality, premiums payable continuously or assurances payable immediately on death. Problems involving more than one life or more than one decrement are outside the scope of the book.

To the British actuary the main interest in the book lies in the chapter headed *Modified Reserves*. This chapter explains the 'Commissioners Reserve Valuation Method' and other modified reserve methods of valuation. The supervision of insurance in the United States is on a state level and each state has its own insurance laws. The administration of these laws is in the hands of an official who, in most states, is called the Commissioner of Insurance. The insurance industry is subject to much greater control in the U.S.A. than it is in Great Britain. The section of the New York Law (where the term Superintendent is used for Commissioner) on the valuation of life insurance policies begins 'The superintendent shall annually value, or cause to be valued, the reserve liabilities of all outstanding life insurance policies . . .', and it goes on to specify a minimum valuation basis. Minimum bases, which make an allowance for initial expenses, have in the past been set by various states, and in 1942 the National Association of Insurance Commissioners agreed to a minimum valuation standard, known as the Commissioners Reserve Valuation Method, which has since been enacted as part of the insurance laws of most states. For whole-life assurances, the net valuation premium (P') under the Commissioners basis is the 'full preliminary term' net premium, i.e. the premium on the assumption that the reserve at the end of the first year is zero. Hence we have:

$$P'_x a_x + P'_{x:1} = P_x \ddot{a}_x.$$

For limited-premium life policies and endowment assurances, when P' is greater than the corresponding premium for a whole-life assurance with premiums limited to 20 years the full preliminary term net premium is not used. The authors set out the Commissioners method in some detail and compare it with some of the more important earlier methods. L. H. L.-C.

Economics: An Introductory Analysis. By Professor PAUL A. SAMUELSON.

[Second Edition. Pp. xxii + 762. McGraw-Hill Publishing Co. Ltd., 1951. 36s.]

THIS book is an introduction both to economic theory and to applied economics. The author explains concisely and illustrates thoroughly the principles of modern economic analysis embodied in such theories as those of production, distribution, exchange, imperfect competition, interest and employment, and he applies these principles to the discussion of current economic questions such as the control of monopoly, collective bargaining, cheap money policies, inflation, subsidies, high taxation and currency depreciation. He uses the national income approach, proceeding directly from a description of the institutional framework and the concept of national income to the Keynesian theory of employment and the manner in which changes in saving and investment affect the national income level. Up-to-date statistics are given of national income, employment, income distribution, consumption expenditure and Government finance and, although the sources are mostly American, they retain a considerable interest for the British reader.

The book is divided into five main parts, each self-contained and providing sufficient material for a book in itself. The first part outlines the fundamental economic laws which apply to all forms of society, the production-possibility

curve, the law of increasing cost, the economies of scale and the law of diminishing returns. It contains several descriptive and factual chapters on income distribution, business organization, the elements of accountancy, public finance, trade unionism and family expenditure. There is also a valuable chapter explaining in detail the estimation and analysis of national income. The second part covers the theory of employment and the causes of fluctuations in national income. There are chapters on price changes, the American banking system, monetary theory and policy, the trade cycle and the fiscal problems associated with a full employment policy and the growth of national debt. The third part deals with the composition and pricing of the national output and contains chapters on the laws of supply and demand, the effect of speculation, the theory of consumers' choice, the nature of indifference curves and the equilibrium position of the firm under perfect and imperfect competition. The fourth part is concerned with the distribution of the national income among the different factors of production and embraces the theory of production and the special problems of rent, wages, interest and profit. The fifth part covers international trade, including the balance of payments, exchange rates, the theory of comparative advantage and tariff protection. Finally, a brief sixth part compares capitalism with some of the alternative economic systems.

This book is a recognized text-book in this country for students in the first year of their reading for a final degree in Economics. It can be strongly recommended to anyone wishing to obtain a sound understanding of the elements of present-day economic analysis, but it is probably too long and too comprehensive for the actuarial student, as such. He would undoubtedly profit, however, from a study of Chapter 11 on national income, Part Two on the determination of national income and its fluctuations, and Chapter 29 on interest and capital.

C. J. B.

Methods of Operations Research. By PHILIP M. MORSE, Ph.D. and GEORGE E. KIMBALL, Ph.D.

[1st ed. (revised). Pp. vii + 158. New York: John Wiley and Sons.
London: Chapman and Hall, Ltd. 1951. 32s.]

THE first edition of this book, published just after the end of the Second World War, was written largely for the benefit of officers of the armed services. Originally, parts of the book were contributed by various Scientific Officers serving in the Operations Research Group, U.S. Navy, and described in detail some of their more interesting statistical problems. The authors have collected all this material, rewriting some of it in the light of later knowledge. They have expanded certain chapters to make them more generally intelligible and have included chapters dealing with organization and general procedures.

The authors consider that the methods of operations research have important peace-time applications, and the revised edition, therefore, also discusses the scope of operations research in solving non-military problems. The book describes the methods and tools of the subject, for the instruction of the worker in this newly developed science.

Operations research differs from ordinary statistical analysis in that the aim is not simply to record or to explain past operations but to predict future operations and to modify them to produce new or better results. Although the

most important single mathematical tool of operations research is probability and statistical theory, all scientific techniques are used.

The first chapter opens with the following definition:

Operations research is a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control.

The operations research worker must not himself be in an executive position. He should, however, have direct and personal contact with the officer who makes executive decisions; thus he can find out what problems have to be examined and provide the executive with a scientific basis for his actions. Interesting examples given in this chapter cover a wide range of problems. There is the apparently trivial question of the long line-up of soldiers causing much delay in the washing-up of their mess-kits at a field mess station. More important problems include questions on the optimum size of military convoys, the magnitude of the weekly sales from city retail stores, the effect of road width on the flow of motor traffic, and the results of an advertising campaign. For all these varied matters, practical recommendations, based on the use of operations research principles, proved very valuable. Unfortunately, many of the non-military problems, dealing for example with the statistics of a specific mercantile trade or with the success of an advertising and mail order campaign, are of a highly confidential nature and, like numerous military problems which have to be kept a close secret, are never likely to be published.

Chapter 2 sets out in detail, for the instruction of the operations research worker, those parts of the theory of probability which are of greatest use for his practical duties. This section deals with simple laws, such as the binomial and Poisson distributions, together with problems of sampling and significance tests.

Practical war-time problems which can be solved by means of comparatively simple probability theories are dealt with in Chapter 3. For this purpose, the authors explain that most of the operational statistics available are only approximate; generally, these statistics can be used to one significant figure only, any greater accuracy merely serving to add unessential detail. In some strategic problems, the results obtained from operations can be compared with expected theoretical values. If the practical result is less than one-third of the theoretical figure, the authors suggest that the poor performance is significant of some operational faults or errors and that a distinct improvement is possible. Frequently, successful applications of operations research resulted in improvements by factors of 3 to 10 or more. We were at first rather surprised that the authors did not apply the usual significance tests to their data instead of relying upon the 'one-third rule' mentioned above. It is probable, however, that as they were dealing with approximate figures derived from intelligence reports etc. the simpler treatment was adequate. Of special interest in this chapter are the examples dealing with important aspects of naval, air and submarine warfare in both the Pacific and the Atlantic.

Chapter 4 is mainly theoretical. Strategic problems frequently involve a comparison between the losses of opposing forces; such comparisons require a similar technique to that used in studying chemical reactions or the biological increase or decrease of opposing species. For this purpose the authors give extracts from *Aircraft in War: the Dawn of the Fourth Arm* written by F. W. Lanchester in 1916. Lanchester dealt with the mathematical laws relating the losses of opposing forces under differing conditions of warfare. By means of differential equations derived from Lanchester's Laws, solutions may be

obtained to vital problems, such as the measurement of the attrition of the U-Boat forces operating in the Battle of the Atlantic.

Tactical analysis is the subject of attention in Chapter 5. By studying the statistics of a large number of Japanese 'Kami Kaze' or 'suicide bomber' attacks on the Pacific Fleet, research officers were able to suggest the best form of evasive action by naval vessels. Subsequent experience showed that 29% of attacks were successful when the new defensive tactics were employed, as compared with 47% of successes against ships using less suitable types of evasive action. Further examples in this chapter involve the 'theory of search' or the calculation of the probability of locating, damaging or colliding with a target placed at random within a known area.

Chapter 6 demonstrates the application of probability theory to gunnery and bombardment problems. In this way it is possible to solve important armament questions, such as the optimum spread of a salvo of torpedoes or the best way of dropping a series of bombs or a pattern of depth charges. For some such problems the authors have developed an interesting practical method, involving the use of Tippett's *Random Sampling Numbers*.

Frequently it is necessary to supplement operational data with material obtained from operational experiments under controlled conditions with a specially designated task force. In chapter 7, the authors describe the various practical considerations which arise when planning these experiments and interpreting the results.

Finally, the authors discuss in Chapter 8 the important subject of the organization and procedural problems which are involved in the selection, training and administration of a group of operations research workers.

The reviewer would have liked more examples of the peace-time applications of operations research. In this direction, the authors were, no doubt, handicapped by the fact that many of the non-military problems upon which they were consulted are still on the secret list. Certain questions, however, such as expediting traffic flow or improving telephone services, might have been dealt with more fully. *Methods of Operations Research* is a readable and interesting text-book. It will prove useful for the training and guidance of the operations research workers who will undoubtedly be required in connexion with the rearmament campaign. At the same time, it will be of particular interest to those actuaries who serve industrial concerns or nationalized corporations, some of whom have been engaged on work of an analogous nature for many years.

J. P.

Peter Moen's Diary

[Translated from the Norwegian by Kate Austin-Lund. Pp. 146.
Faber & Faber Limited, 1951. 10s. 6d.]

A DIARY is, perhaps, the last kind of book that is likely to be reviewed in the *Journal* because it deals with intimate, personal experience which is far removed from the subjects of a learned Society. Yet this diary earns its place as the record of the experience of an actuary caught up in the war, a Norwegian imprisoned by the Germans for his part in the illegal press. It opens a door into a world of which we have only indirect knowledge—one which, probably, can be fully shared only by those of our members who themselves were prisoners of war.

Peter Moen, who was born in 1901, became *beregner* in the Idun Life Insurance Company of Oslo—a position corresponding to an actuarial clerk; he

had studied mathematics at the University and had joined the Norwegian Actuarial Society, though he did not fully qualify as an actuary.

During the war he began by editing an illegal newspaper for circulation among the staff of his office; but later he became attached to *London News*—one of the most widely read of the illegal newspapers—and ultimately was promoted to be chief of all the illegal papers in the country. After his arrest in February 1944 he was imprisoned in the ordinary prison cells of Oslo, in solitary confinement at first, and afterwards in a small cell with two others.

With a tack from the black-out curtains he patiently pricked out the diary letter by letter on toilet paper; five pieces, each about six by seven and a half inches, he rolled in a sixth unwritten piece, numbered each roll and dropped it through a small ventilation grill at the foot of the wall under his table. Though he lost his life when the prison ship *Westphalen* struck a mine off the Swedish coast, one to whom he had confided the secret of the diary was among the five who were saved from the shipwreck; after the war his confidant told the police who broke up the floor of the cell and recovered the diary.

Why did he write it?

It is my diary which comforts and strengthens me... One summer day when Norway is once again a free country Bella and I will walk in the woods and sing: 'What is the country where you live called? Happiness!'

Under torture he had betrayed his friends and the fear of torture burdened him; he felt he was not worthy of his job in the illegal press and the tension quickened a moral crisis within him. The diary records his struggle for a faith to bear him through the evil days, a faith which seemed to conflict with his intellect, a faith which seemed within his grasp when in solitary confinement but which receded later when he shared his cell with others. His impressions are set down stark in these pages, for there was no possibility of revision.

The pettiness of spiteful guards he despised, but regarded the temptation to return tit for tat as defeat.

But the victory lies here: that we remain true to ourselves.

Everything that could pass the time was forbidden. The prisoner was forbidden to:

Smoke—read—lie on the bed—look out of the window—sit or lie on the floor—sit on the table or bed—write anything or play any kind of game—receive or send any letters or parcels.

Much of his time was spent on mathematical problems, some of which work was found with the diary. Thus on his seventy-sixth day he says:

I work solidly and industriously with my mathematical problems—ten and twelve hours a day. I do not evade any toil to clear up any obscurities. I use costly methods—because for me there is no time limit. I check everything with one or another converse proof or by another method. Yesterday and today I worked out a numerical integration with twenty terms and checked this with a mathematical progression in which I calculated a hundred terms—all with five decimals. It gave the same result to three decimals.

The previous day the guard had found him writing and had taken the paper.

Little of the quality of the diary can be conveyed by this review; the book is strong meat and scarcely suitable to be left where children could read it. But the reviewer hopes that many readers of the *Journal* will buy this diary for themselves and share the experience of one who would wish no memorial, for he felt himself to be no hero but an ordinary citizen caught in the toils of war.

M. E. O.

Les Opérations Financières et les Opérations Viagères. By H. MAURICE.

[Pp. 394. Les Editions Comptables, Commerciales et Financières,
Rue de Florence, 57, Bruxelles, 1951. No price.]

THIS work is No. 39 of the series 'Bibliothèque Générale des Sciences Économiques', and the section dealing with life contingencies is based on an earlier work *Les Opérations Viagères* by L. Maingie and the present author (reviewed in *J.I.A.* LXIV, 57), which has been out of print since 1938. The chapters on compound interest are new and, although the general treatment of life contingencies in the earlier work has been followed, the text has been considerably altered and chapters have been added dealing mainly with group insurance, reserves and life office accounts.

The book has been written for students of the 'Instituts Supérieurs de Commerce', as well as for actuarial assistants and others requiring an introduction to the subjects. Part I (Compound Interest, pp. 9-95) covers rates of interest and discount, successive payments, debts repayable by annuities, redeemable securities and capital redemption insurances. Part II (Life Contingencies, pp. 99-362) deals with the elements of probability including its application to mortality, mortality tables, single life annuities and assurances, annuities and assurances on two or more lives, survivorship annuities and contingent assurances, annual and fractional premiums, office premiums, group insurance, debts repayable by life annuities, life interests and reversions, policy values, office valuations, life assurance accounts and their analysis, and State control of insurance. A series of questions for students (without answers) is given at the end of the book. Mortality and monetary tables are not included as they are the subject of a separate volume, No. 34, in the same series (*Tables pour les Opérations Financières et Viagères* by H. Maurice).

The demonstrations are in general clear and well illustrated with numerical examples. The wide scope of the book has forced the author to condense the treatment of certain sections to keep the work within a manageable size and it is possible that the student may have some difficulty in grasping all the basic principles without further assistance; this is especially true of the chapter on probability and mortality. A knowledge of the methods of finite differences and the calculus is not assumed and this has inevitably imposed some limitation on the approach to the subject, particularly in the case of payments m times a year, sums payable at the moment of death, survivorship annuities and contingent assurances. It seems strange that no reference is made to the expectation of life nor to premium conversion tables, although the basic formula for the latter is given. While these and a number of other minor points may be open to criticism, none are sufficiently important to detract from the value of the book as a whole.

The notation used follows that of the earlier work, viz.

$$C_x = v^{x+1} d_x \quad \text{and} \quad C_x = v^{x+\frac{1}{2}} d_x;$$

$$N_x = D_{x+1} + D_{x+2} + \dots \quad \text{and} \quad N_x = D_x + D_{x+1} + \dots;$$

with corresponding use of thick and thin type for M , R and S . The use of thick type for two different purposes in the assurance and annuity commutation columns must, it is felt, lead to some confusion in the mind of the student. English readers should be careful to note the differences from the notation used in the new Institute of Actuaries text-book. It may be noticed that in dealing with joint-life functions the factor $D_{xy} = D_x \times l_y$ is suggested instead of the more usual $D_{xy} = l_x \times l_y \times v^{\frac{x+y}{2}}$.

The wide ground covered, the frequent reference to practical points and the notes on Belgian Law governing financial, insurance and pension transactions should make the work a valuable text-book for those wishing to acquire sufficient knowledge to deal intelligently with practical actuarial problems, without going into the theory as far as the professional actuary. It should also form a very useful work of reference for those engaged in this type of work.

B. R.

Register of research in the social sciences in progress and in plan.

[Published annually by the Cambridge University Press for the National Institute of Economic and Social Research, No. 8, 1950-51, August 1951. Pp. 174. 15s.]

THE purpose of the *Register*, which is published annually, is to provide a continuous but not cumulative record of the research which is being carried out or has been planned in a number of subjects collectively described as the Social Sciences. Those of most interest to actuaries, namely Demography, Economics, Social Medicine and Social Surveys take up nearly half of the present number in which Economics is the largest single subject. The research described is that undertaken by Universities, Research Institutions and professional bodies but does not include research being undertaken by private persons as such. It has been found impracticable to collect directly information about research in Government Departments except in some special cases such as the research units of the Medical Research Council. A notable omission therefore is the work done by the General Register Office.

The information given for any piece of research includes the names of the persons in charge of it, the date of its beginning and the approximate date of its completion together with particulars of interim results and final results already published or expected to be published. It follows therefore that the *Register* should be of considerable assistance to research workers in providing information about work being carried out in their own subjects.

The book also includes a directory of research institutions giving information in considerable detail of their objects and organization. It is well indexed so that it is easy to discover the work being done in a particular subject or being sponsored by a particular research institution or in the charge of a particular person. Although not cumulative, it is arranged so that reference back to earlier numbers, where necessary, can be conveniently made.

The *Register* is edited by Mrs Feodora Stone, Secretary of the Institute, and the Editorial Advisory Committee includes Professor D. V. Glass, Vice-Chairman of the Population Investigation Committee. Mrs Stone points out in her preface that the *Register* is not, and cannot be, a complete record of all work undertaken in the field covered. There is certainly a case for the inclusion of the Institute of Actuaries and Faculty of Actuaries in the directory of research institutions and, in the main body of the *Register*, for details of the progress of work such as that of the Joint Mortality Committee and perhaps of other research being carried out by members of the Institute or Faculty. There can be no doubt, however, that the claim that the *Register* is 'a contribution to communication in a field where communication is difficult' and that 'it provides information which for the most part cannot be found elsewhere' is justified.

R. G. B.