

## REVIEWS

*Life and Other Contingencies*. Volume 1. By P. F. HOOKER, F.I.A., A.S.A., and L. H. LONGLEY-COOK, M.A., F.I.A., F.C.A.S., A.S.A.

[Pp. viii + 312. Cambridge. Published for the Institute of Actuaries and the Faculty of Actuaries at the University Press, 1953. 22s. 6d.]

THIS book, another in the ambitious series of text-books designed for students preparing for the actuarial examinations, differs from most in that it is the third in a distinguished line of text-books on this important section of the development of actuarial science. The authors have given generous acknowledgment of the aid they have received; but it is only fair to point out that the influence of these existing works is not without its disadvantages, and tribute should be paid to the authors' ability impartially to accept, reject or modify what had been accepted by many generations of students as the essential fundamental approach to actuarial techniques.

The authors have not been entirely free to develop their own line of approach, as they have been directed to treat the subject precisely in accordance with the requirements of the syllabus of the Institute examinations laid down in 1951. There have been modifications in the examination syllabus from time to time, and it is perhaps questionable whether it was wise to put the authors into such a strait-jacket. More is said about this later.

The book has been written on the principle of a progressive approach, where by progressive is meant the segregation of techniques dealing with a mortality table with one decrement only, from those associated with more than one decrement. Treatment of the latter is reserved for Volume 2 still to be published, and to this extent the division in Spurgeon's text-book is retained. The balance of the book is taken up with a consideration of valuation methods and of benefits depending on marriage, sickness, etc.

The reviewer has strong recollections of the tribulation he experienced in passing from the mathematical preparation in the earlier stages of the examinations to the conception of a life table. While at this distance he finds some difficulty in assessing the impact of Chapters 1-3 on the student, his impression is that the latter will find them of great assistance. The introduction of graphs is to be commended, as they give the reader a much more lively understanding of the subject than does a study of the actual figures in a life table. It is hoped, however, that the expression 'ordinary mortality table' will not cause any confusion; it seems possible that the word ordinary may be connected with 'ordinary branch' assurance, whereas it is intended to cover any table where the mortality does not depend on duration.

Early opportunity is taken in the examples at the end of these chapters to stress the desirability of a rough check on a numerical answer. A greater exercise of this in the examination room in the past would no doubt have increased the number of successes. It could have been added that a sense of proportion is also of great value—that d...d dot can play strange tricks!

The progressive development of the theory of single-life contingencies proceeds on orderly and lucid lines in the first fourteen chapters. The reviewer

was struck, however, by the, to him, almost casual introduction of commutation functions. The need for the student to understand the fundamental nature and possible uses of the functions is so great that a rather fuller treatment could with advantage be given—this would prove of particular value in later work in any problem involving the use of different mortality or interest assumptions in the same calculation. In the section dealing with conversion table relationships, the criticism may perhaps also be made that the limitations of the relationship are not fully explained—many students tend to use it automatically in circumstances showing that they do not really understand it. Perhaps this is a casualty of the progressive approach.

The authors have avoided interrupting their progressive development by the introduction of approximate methods until they were able to devote a whole chapter to them. This approach is undoubtedly to be preferred, and the student having mastered Chapters 1-8 is in a better position to appreciate the merits of the various methods and to consider them as a whole. Admirable though the printing of the book is, a little wider spacing in setting out the approximate methods would in this particular chapter make for greater clarity in reading.

The treatment of office premiums is so essentially a practical matter that any progressive treatment must leave many questions unanswered, and the authors are to be congratulated on having trodden a knife edge with considerable success. A marked defect in the previous text-book has been remedied without trespassing too far into considerations arising from a more advanced study of the subject.

Policy values are dealt with so clearly, graphs again being used with advantage, that one is a little puzzled why there is not in the opening paragraph a more detailed explanation of how in most cases a policy value arises. Alternatively, the point could have been made in earlier chapters that a level premium cannot normally be charged for a decreasing risk except by limiting the term of payment, and the reasons given.

There is a valuable chapter on extra risks—which received inadequate treatment in previous text-books—followed by suitably pruned and modified sections on the laws of mortality and stationary populations, etc.; both the expansion and the contraction in reading matter will be to the advantage of students.

It is when attention is turned to the remaining chapters, dealing with valuation, and marriage and sickness benefits, that some doubt arises whether the particular progressive approach adopted is necessarily the best. This is not a criticism of the authors' treatment of the subjects—within the confines of their directive they have done excellently—but it is questionable whether so wide an extension of life contingencies should be attempted at once. One of the difficulties the student faces in passing from the earlier mathematical section to the subject of life contingencies is the new conception necessary. After the initial shock of the change the new text-book will lead him along most efficiently and lucidly in the manipulation of the various functions. Suitable hints are thrown out that in passing from theory to practice practical considerations may introduce complications. He is then thrust suddenly into the very practical subject of valuation grouping and formulae—a big departure which he will probably not touch again in the examination for at least a year—followed by a rather artificial treatment of marriage benefits and then by sickness, which has special characteristics but can hardly be called a decrement, and which again would probably be left in abeyance for at least a year. In Volume 2 the student will have to revert to the earlier development, but applied to more than one life.

Would it not have been better to have dealt with joint-life probabilities, assurances, etc., in continuation of single-life work and so armed the student completely in the more simple but essentially basic sections of life contingencies? One has only to read the first part of Chapter 17, on marriage benefits, etc., to realize the difficulties with which the authors have been faced. It is indeed, not surprising that at the end of 17.6 they refer to a point of practical importance which does not affect the theory of the subject 'within the limits of the present treatment'.

In the preface the authors refer to the practice of marking certain items with an asterisk as not being, in their judgment, within the syllabus of the examinations. Where the item is of academic interest only and involving mathematical demonstrations of no particular value in later work, the practice has its merits, although, possibly, an appendix to the chapter in question would have served the purpose at least as well. In other cases the introduction of the item is, no doubt, intended as background, and there is always room for difference of opinion on this aspect of the matter. Some people will consider certain asterisks unnecessary, and others that a particular item should not be there at all—the reviewer would put the third paragraph of 10.4 in this category as being far outside the syllabus.

In conclusion, tribute should be paid to the printers, Robert MacLehose and Co. Ltd., who, unlike the Cambridge University Press, had no previous experience of actuarial work. They have maintained a uniformly high standard, although the abridged table at the end of Chapter 18, presumably reduced in size by a photographic process, will come as something of a shock to those students whose eyesight is less than excellent. F.C.T.K.

*Compound Interest and Annuities Certain.* By D. W. A. DONALD, O.B.E., T.D., F.F.A.

[Pp. viii + 300. Cambridge. Published for the Institute of Actuaries and the Faculty of Actuaries at the University Press, 1953. 20s.]

THE subject of compound interest lends itself more readily than many parts of the examination syllabus to instruction by means of a text-book because it calls primarily for understanding and application of basic principles, and to a small extent only for judgment and practical experience. For this same reason the ground to be covered is not subject to those frequent revisions which complicate the task of tutors in other parts, and it was, therefore, to be expected that almost precisely the same ground should be covered as in the familiar 'Todhunter'. Nevertheless, the author's treatment fully justifies the labour and expense of producing a new text-book.

Calculus has, in recent years, become increasingly recognized as a valuable tool for the actuary, and it is reasonable to suppose that students will benefit by more extensive use of this technique in clarifying the particular problems of compound interest. Conversely, the *expertise* obtained at this stage should be helpful when the same methods are used later to analyse more complicated problems. It can, therefore, be counted a great improvement that Mr Donald has used the calculus throughout the book to elucidate each point as it arises, instead of reserving one chapter for this aspect as has been done previously.

An equally valuable improvement in the treatment appears in the breaking down of the longer passages of exposition. This point is clearly illustrated in the

chapter on the analysis of the annuity, where Mr Donald presents a little more subject-matter in roughly the same amount of space as was used by Todhunter. By different arrangement, and by the use of headings in heavier type, he avoids both long paragraphs of closely reasoned argument and large tables of similar, but different, expressions. The resulting contrast with Todhunter may be compared with that between a meal in Soho and a meal of the same calorific value in an Army cookhouse.

An interesting departure from Todhunter appears in Chapter 7, where, taking the view that a most important application of compound interest is in valuing securities quoted on the Stock Exchange, the author describes briefly the main types of securities and certain technical terms used in connexion with them. Unfortunately, this chapter, though sound in principle, offers some scope for improvement in detail. No doubt the reference on p. 130 to  $3\frac{1}{2}\%$  War Loan being quoted 106 $\frac{1}{2}$  cum dividend on 25 April and 105 ex dividend on 26 April is deliberate, to avoid confusion, but in fact prices are not adjusted by the full gross dividend unless the general level of markets moves overnight. Similarly, on p. 136, it is regrettable that 4% Funding Loan 1960/90 should have been chosen as the example of the treatment of optional dates of redemption in view of the uncertainty as to the date by which the sinking fund will have extinguished this loan. Furthermore the reviewer would have wished to see some emphasis on the fact that as almost all interest income is paid half-yearly, yields are rarely quoted on a basis other than a nominal rate, convertible half-yearly.

A more troublesome point arises in connexion with securities valued between two interest dates and with bond tables. Mr Donald states, quite accurately, that if the value of a security immediately after a dividend payment is  $A$  its value  $1/m$  of a year later is  $A(1+i)^{1/m}$ , and one may use  $A(1+i/m)$  as an approximation to this value. In practice, however, there is an accounting view that if the security is bought on the later date the price is divided into accrued interest  $g/m$  and firm price  $A(1+i)^{1/m} - g/m$ . Indeed, in certain cases securities are bought and sold on the basis of a price plus accrued interest. This basis is widely used in the United States and bond tables of American origin which show values of  $A(1+i)^{1/m} - g/m$  for terms proceeding by monthly steps are commonly used. A fuller discussion of this practical application is desirable for an understanding of investment book-keeping.

Finally, the very comprehensive examples and exercises will be invaluable, and the writer finds it hard to believe that any student who has conscientiously worked through them all will trouble the examiners in this subject more than once.

H.C.C.

*Faster than Thought.* By B. V. BOWDEN, M.A., PH.D. (CANTAB).

[Pp. xix + 416. London: Sir Isaac Pitman and Sons Ltd., 1953. 35s.]

OF the post-war developments in scientific instruments, few have attracted so much attention in the popular press as the electronic digital computer. Various descriptive names have been devised and the title 'Electronic Brain' has persisted in spite of many efforts of the pioneers in this field to explain the scope of such machines.

In the past few years a number of books have been published on the subject of these electronic devices, but they have catered for the technician rather than

for the informed layman; there has been a great need for an authoritative text from which potential users could assess their own problems in the light of the facilities rapidly becoming available. Dr Bowden has now filled this need and has produced a book which should be widely read, particularly by all who are concerned with the processing of information and data on any appreciable scale.

The principles upon which such machines are based are straightforward enough; in fact they were clearly laid down a hundred years ago by Charles Babbage in the detailed design of his Analytical Engine (not to be confused with his Difference Engine which had a limited functional application). This fact is emphasized by Dr Bowden, who devotes, with considerable effect, about a quarter of his book to some very interesting historical material about Babbage and the Analytical Engine. This machine, however, was never completed, the mechanical standards required being too great for the resources available at that time, but the advent of electronic devices has made it possible to construct machines to fulfil all the functions which were the object of the mechanisms Babbage devoted his life to designing.

Initially, the modern machines have been used for computational problems arising from applied and theoretical science; but their remarkable speed of operation has given rise to investigations into their application in other fields of human activity. Popular examples are the playing of games such as chess and draughts or of the synthesis of music, all of which are valuable in showing the scope of the machines; more directly useful experiments have, however, been made in commercial work, of which the calculation of wage statements with P.A.Y.E. and other deductions is a first-class example. As yet the machines are still in their early stages of development; but already the work done with them has shown them to be practical propositions, with potential applications in any field where data are handled and are subject to operations which can be defined by simple rules. It may well be that more development is required before the machines can be utilized economically in insurance offices, but development is so rapid and the possible savings in staff so attractive that their impact cannot be ignored. Furthermore, even the study of office routine with the object of assaying the field of application will reveal many directions in which substantial savings can be obtained provided re-orientation of responsibilities is accepted.

Within this background Dr Bowden's book is a 'must'. It is described as a symposium and twenty-four contributors have provided material which has been welded into a readable whole. Inevitably some repetition occurs thereby but this has been carefully turned to advantage by the author. The book is divided into four parts of roughly equal length dealing with history and theory, descriptions of many of the machines now working, applications, and, finally, an appendix on the Analytical Engine. The section on history and theory includes descriptions of the circuit components of the machines, of their construction and performance, and of programming; included in this section is a thorough discussion of maintenance and similar problems. The section on applications is particularly valuable and brings under review most of the fields in which the machines have been used, including a very useful chapter (No. 22) on business and commerce drawn from Dr Bowden's personal experience in this field. This chapter also includes fairly detailed descriptions of the application of the machines to wages-statement calculations, a typical commercial application which well merits close study. Detailed descriptions are provided of the playing of chess, both as a full game and as an end game, of draughts, which the Manchester machine is reputed to play tolerably well, and of Nim.

The book is well produced and few misprints have been noted, minor ones in the chapter on the solution of logical problems and in the detailed description of a game of Nim being unlikely to mislead the reader. It makes good straightforward reading, and from time to time the hand of Dr Bowden is detected in interpolations which whet the appetite for more; the problem in the middle of p. 37 and the description of the reaction of a machine on p. 75 are typical of these delightful touches.

R. E. B.

*Sampling Methods for Censuses and Surveys.* By FRANK YATES, F.R.S.

[2nd Ed., pp. xvi + 401. Charles Griffin and Co. Ltd., 1953. 38s.]

THE first edition of this book was reviewed in *J.I.A.* 75. In the second edition two further chapters have been added and some errors in the original text corrected. The extra chapters contain a rather oddly-assorted range of topics. It has been possible to give a more or less apt title—'Further Notes on the Critical Analysis of Survey Data'—to one of these chapters, but the other has to be called 'Miscellaneous Developments' and contains subjects varying from punched card machines to the British sample census of 1951 *via* lattice sampling and road traffic censuses.

It is regrettable that it was not found possible to incorporate the fresh material at appropriate points in the original text. At the same time some of the evidences of haste in the preparation of the first edition might have been removed, with a gain in clarity and coherence of the book as a whole. The author gives reasons for the course he has pursued but one hopes that he may not find them so pressing if and when a third edition is under consideration.

The above adverse comments do not in the least detract from the general value of the book, as acknowledged in the review of the first edition. Nevertheless, an increase of more than 50% in price does not appear to be adequately compensated by the addition of eighty pages of heterogeneous matter (even when these include a useful extension to the original bibliography) and the use of better quality paper.

N. L. J.

*Cardano, the Gambling Scholar.* By OYSTEIN ORE.

[Pp. xiv + 249. Princeton University Press, 1953. \$4.00.]

TODHUNTER'S *History of Probability* opens with a brief reference to Cardan's *De Ludo Aleae*. Although this work was not published until 1663, 87 years after Cardan's death, it was written nearly a century before the famous Pascal-Fermat correspondence (1654) which is commonly supposed to represent the beginnings of the theory of probability. Todhunter remarked that Cardan's 'treatise is so badly printed as to be scarcely intelligible', that it is 'best described as a gambler's manual' and that 'the discussions relating to chances form but a small portion of the treatise'. Actually, Todhunter's comments on the mathematical parts are confined to certain arithmetical difficulties occurring in one brief chapter. He does not mention Cardan's positive achievements.

The subject of this review is a popular and very readable biography of Cardan but its importance lies in the fact that it gives in an appendix the first English translation of *De Ludo Aleae*. The author also devotes a chapter to a detailed discussion of the work, claiming for Cardan an important place as a pioneer in probability theory.

These early writings on probability, including those of Pascal and Fermat, are concerned exclusively with gambling odds and expectations—the famous ‘problem of points’ is a problem in expectations; references are made to chances, and in particular to equal chances, but measurement on the scale 0 to 1 does not seem to have been invented until much later. (The phrase on page 204 of *Cardano* ‘cases where the probability is not  $1/2$ ’ is probably a free translation for ‘cases where the chances are not equal’).

An appreciation of this preoccupation with gambling is particularly necessary for an understanding of Cardan’s treatise. In fact, it is suggested by the reviewer that the author of the present book has done less than justice to Cardan when he takes too literally the false reasonings and wrong answers that Cardan produces before reaching the truth. ‘It seems peculiar that Cardano still has not discovered that there are 91 throws...’ (p. 158), ‘for immediately afterwards he realised that there was an error...’ (p. 163)—there is a more plausible (and more generous) explanation for these parts of Cardan’s work. It is that Cardan may well have been discussing and discrediting certain gambling rules in current use before giving the correct reasoning and correct answers to his problems. The fact is that he does always in the end reach the right answers for the right reasons. Moreover, the incorrect solutions examined by Cardan seem all to favour the banker!

It is hardly credible that the simple problems involved in throwing two and three dice should have required the trial-and-error approach suggested by Oystein Ore, from such a man as Cardan who was not only a leading mathematician of his century but probably also the first to use complex numbers in the correct solution of really difficult mathematical problems.

That Cardan had the essence of probability in him is evident not only from the fact that he always ‘counted cases’—‘there is one general rule, namely, that we should consider the whole circuit, and the number of those casts which represents in how many ways the favourable result can occur, and compare that number to the remainder of the circuit, and according to that proportion should the mutual wagers be laid so that one may contend on equal terms’ (p. 202)—but because he also realizes that the conditions must be equal—‘The most fundamental principle of all in gambling is simply equal conditions, e.g. of opponents, of bystanders, of money, of situation, of the dice box, and of the die itself’ (p. 189). He even knew about ‘utility’! The ‘circuit’ is the total number of ‘ways’ or ‘the reference class’ and it operates as a different ‘radix’ for each problem; but there was evidently a curious confusion, even at that early stage, between probability and frequency, e.g. ‘one-half of the total number of faces always represents equality; thus the chances are equal that a given point will turn up in three throws, for the whole circuit is completed in six’. The pursuit of ‘equality’, i.e. even odds, is the source of much difficulty. How many throws of two dice may be allowed at evens for a double six? (This problem turns up again with Pascal-Fermat.) The whole circuit is 36, therefore 18 throws is equality! ‘...and so occurs once in 18 throws’ (p. 197). Now if a punter wagers 1 for a prize of 36 for each of 18 successive throws he can put down 18 at the beginning and lose the lot if he doesn’t get a double six. If he does get a double six he should receive 36 plus the return of the balance of his total wager or go on throwing for a second, third, etc., prize. Cardan fully appreciated this, despite his traffic with the false idea of ‘equality’ for ‘half the circuit’, because he refers (p. 196) to ‘a repeated succession’ and goes on: ‘So the whole set of circuits is not inaccurate, except in so far as there can be

repetition, even twice or three times in one of them. Accordingly, this knowledge is based on conjecture which yields only an approximation....'

Cardan does not state or use the product rule in general—nor apparently did Fermat and Pascal—but he had the germ of the idea because he uses it for doubles and trebles. 'Thus if it is necessary for someone to throw an ace twice' (in two throws of three dice) (p. 202) he says that in 'the opinion of some' the stakes should be doubled. He considers squaring the odds (cubing for a treble) and concludes, by reference to a double or treble of an even chance, that 'this reasoning seems to be false'; in fact, the rule favours the banker! He then discusses doubles of other than even chances and reaches the correct rule of squaring (and cubing) the number of favourable ways and the total number of ways, and then using as the odds the square of the former to the remainder of the square of the total—and he appeals to 'counting cases' for justification (pp. 203-5). He concludes that 'this reasoning is generally true'.

It has not, of course, been possible to check the deciphering of the original Latin or the translation, which is evidently fairly free, but the evidence produced strongly suggests that Cardan should certainly be remembered more for his 'probability' than for del Ferro's and Tartaglia's solutions of the cubic. W.P.

*Exercises in Theoretical Statistics.* By Professor M. G. KENDALL.

[Pp. vii + 179. London: Charles Griffin and Co. Ltd., 1954. 20s.]

This book is a collection of 400 problems in theoretical statistics, 100 on distribution theory, 100 on sampling, 75 on statistical relationships, 75 on estimation and inference, and 50 on time series. Many of the problems are taken from the statistical examinations of the universities and of the Royal Statistical Society. About one-third of the book consists of answers and hints on the solution of the problems, so that it will be very useful to students reading privately, including those taking the advanced statistics papers of the Institute's examinations.

R. H. D.

*Stochastic Processes.* By J. L. DOOB.

[Pp. vii + 654. New York: John Wiley; London: Chapman and Hall. 80s.]

IN many fields of scientific activity there is a common underlying drift towards the use of an ever more sophisticated mathematical background. The process is, in part, a natural result of increasing precision of ideas, and in so far as it satisfies the need for a correspondingly adequate technique of thought it is to be welcomed. The value of fresh mathematical concepts can be judged by results, in the same way as other new techniques are assessed. Their value should not be decried merely because of dislike or, even worse, lack of comprehension of the ideas involved. On the other hand, new methods should not be introduced simply because of some supposed prestige attaching to the use of fashionable words and phrases. Nor should the new methods be allowed to take command and impose their values on the science they should serve.

This somewhat lengthy preamble is necessary for a proper appreciation of the remarkable book on stochastic processes produced by Professor Doob. Many arguments and concepts used in this work will be unfamiliar to the majority of actuaries and statisticians. We may instance the definition of a random variable:

A (real) function  $x$ , defined on a space of points  $\omega$ , will be called a (real) random variable if there is a probability measure defined on  $\omega$  sets, and if for every real number



$\lambda$  the inequality  $x(\omega) \leq \lambda$  delimits an  $\omega$  set whose probability is defined, that is, a measurable  $\omega$  set.

If one is not conversant with the concepts 'space of points' and 'measure', such a definition will be a rather troublesome start to a study of the book. Nevertheless the author finds these, and allied, concepts absolutely essential to a satisfactory treatment of probability theory—and this is, in spite of its apparently specific title, a definitive book on probability theory. A stochastic process is defined, as 'any family of random variables  $\{x_t, t \in T\}$ ' and, as Doob remarks, the definition is 'embarrassingly inclusive in that there are not many problems in probability that cannot be formulated as problems in families of random variables'.

The value of the book lies, to a great extent, in the success with which it picks out the topics to be included and systematizes them in such a way as to give the subject an overall form which can be comprehended without undue strain. In the opinion of the reviewer, this aspect of the book is more important than the detailed results obtained, numerous though these are. The ways in which the general definition of a stochastic process is specialized are of sufficient interest to be described in outline. Processes can be classified according to

- (a) the structure of the set  $T$  (if  $T$  is an infinite sequence (e.g. 1, 2, 3, ...) we have a *discrete parameter* process, while if  $T$  is an interval (e.g.  $a < t < b$ ) we have a *continuous parameter* process),
- (b) relations between distributions of the  $x_t$ 's.

Under (b), the specializations studied by the author include the following:

- (i) *Markov processes*—the distribution of  $x_{t_n}$ , given  $x_{t_1}, x_{t_2}, \dots, x_{t_{n-1}}$ , where  $t_1 < t_2 < \dots < t_{n-1} < t_n$ , depends only on  $x_{t_{n-1}}$ ;
- (ii) *Martingales*—the expected value of  $x_{t_n}$  given  $x_{t_1}, x_{t_2}, \dots, x_{t_{n-1}}$ , where  $t_1 < t_2 < \dots < t_n$ , is equal to  $x_{t_{n-1}}$ ;
- (iii) *Stationary processes*—the joint distribution of  $x_{t_1}, x_{t_2}, \dots, x_{t_n}$  is unchanged if we replace  $t_i$  by  $t_i + h$ , where  $h$  is any number such that  $t_i + h$  is in  $T$  for all  $i$ ;
- (iv) *Independent, uncorrelated or orthogonal increments*—these are hypotheses about the joint distribution of increments such as

$$x_{t_n} - x_{t_{n-1}}, x_{t_{n-1}} - x_{t_{n-2}} \quad (t_n > t_{n-1} > t_{n-2}).$$

In addition to classification on the above lines, the author introduces a distinction between 'strict sense' and 'wide sense' properties of a process. The latter are properties possessed by a process for which the joint distribution of any  $x_{t_1}, \dots, x_{t_n}$  is multivariate *Normal* and has variances and covariances satisfying 'strict sense' conditions valid for general processes of the type considered.

Each specialized process is subjected to detailed study. Particular interest is evinced in 'large-sample behaviour' or more precisely theorems of central-limit type. In the final chapter linear least-square prediction of stationary Normal processes is discussed.

The book is intended to be a mathematical treatise. The student will find therein a thorough description of the various types of stochastic process, but little or no guidance on how to assign an observed series to one or other of the processes. This cannot be advanced as criticism, as the author makes it quite clear that he is concerned solely with the theoretical development of the subject. However, it is a factor in discouraging a prospective reader who is interested in

applications—and let it be clear that there are many applications. Such diverse fields as Brownian motion, sequential analysis, radioactivity, diffusion, and some aspects of risk theory all fall within the ambit of the theory developed in the book. Those results which are useful are, however, so well buried that only the specialist with time to devote to its study will be able to use the book with facility. For those with sufficient time the study of Professor Doob's work will prove richly rewarding, even if some will find his strictly mathematical outlook unhelpful and even perhaps annoying. It is a book the real worth of which can only be assessed after a considerable passage of time, but it will undoubtedly have a profound effect on the future direction of research in the field of stochastic processes.

N. L. J.