

REVIEWS

The Elements of Probability Theory and some of its applications. By Professor HARALD CRAMÉR.

[Pp. 281. New York: John Wiley and Sons, 1954. 56s.]

TEXT-BOOKS on probability can take many forms, ranging from those concerned solely with coins and dice to those where measure theory is the main basis. The present work under review, written by an Honorary Fellow and Corresponding member of the Institute, falls somewhere in between the two extremes and is a carefully balanced mixture of probability theory with the statistical applications kept constantly in view and referred to at the appropriate places even if the argument necessary is not further developed.

The sixteen chapters of the book are divided into three parts headed (1) Foundations, (2) Random variables and probability distributions, (3) Applications. The first part on foundations contains four chapters dealing first of all with the history of probability from about 1650 up to Von Mises of the present century. This is followed by a chapter defining probability where the difficulties associated with the classical definition and its utilization of 'equally likely' events are discussed and the definition discarded in favour of one where we consider the frequency of success in a series of random experiments. Using this definition of probability the elementary theorems of probability such as the addition and multiplication theorem are proved in Chapter 3. In illustrating them it is rather unfortunate that the classical definition is tacitly dragged in to enable the author to use examples concerning playing cards and the like. Chapter 4 then takes the basic theorems a stage further by considering such matters as whether the samples are drawn from a finite or infinite population, the multinomial probability distribution (called polynomial distribution here) and the basic ideas of the matching problem.

The following six chapters forming the second part of the book have as their theme the notion of a random variable which necessitates the consideration in some detail of the basic probability distributions. The binomial and the Poisson distributions are discussed in Chapter 6, and Chapter 7 turns to continuous distributions with a discussion of the normal distribution. Chapter 8 deals with those three distributions that are so useful in statistical work, the χ^2 , t and F distributions. Chapters 9 and 10 deal with the concepts of distributions involving two or more variates, and a good description of regression, including partial and multiple correlation, is included.

The remaining six chapters form the third part of the book. Here the transition from probability to statistics is made, and it opens with a short chapter describing the methods by which a probability is used in the making of statistical judgments and predictions. Chapter 12 describes tabulation and grouping—the system, in Table 12.1.1, of grouping incomes does not seem quite foolproof here—graphs, histograms and the calculation of moments. Chapter 13 deals with sampling distributions such as those of the sample standard deviation and mean. The following chapter discusses statistical inference. The properties of estimates that are desirable and undesirable are described in detail and illustrated with examples. The maximum-likelihood method of obtaining estimates is then dealt with in some detail. This is followed by a good description of confidence

intervals and the use of approximate distributions to obtain such intervals. The chapter ends with a brief introduction to the concept of the power of a test. Chapter 15 deals with the χ^2 test for the goodness of fit of a hypothesis. The restriction, p. 218, that all the cell frequencies should be at least 10 even if pooling is necessary to attain this aim, is probably somewhat stringent and a lower bound is usually considered satisfactory. The application of the χ^2 test to several different types of example is considered in some detail. The final chapter contains a mixture of topics. There are sections on the theory of errors and the 'normal' equations, on some more complicated regression problems, on the principle of splitting up the total sum of squares in an analysis of variance, stratified sampling and quality control. Throughout the book there are numerous worked examples and questions for the reader to do for which answers are given. Short tables of the normal distribution and its derivatives (useful for the Edgeworth type of expansions) are given together with tables of the χ^2 , t and F distributions.

A text-book of this kind poses many problems to the writer. Although probability theory is the basis of modern statistical techniques it is not easy within the compass of one volume of some 280 pages to get across both the basic probability theory and the application to statistical methods. As a result something usually suffers, and in this case it is the statistical part where a lot of topics have, of necessity, to be treated in somewhat cursory manner. Nevertheless, this is a valuable book from the point of view of an actuarial student for two reasons. In the first place it is written at just about the correct mathematical level for the student to be able to follow and yet be stimulated by it. Secondly, the book makes a conscious and successful attempt to bridge a difficult gap and link together probability theory and the practical side of statistical analysis. It is always troublesome for the student to find that the probability studied in the Institute's Part IA examination appears to bear little or no relation to the probability that seems to be used in the Part IB examinations, and there grows up an artificial barrier between the cards and dice of the probability problem and the significance level of the statistical test. In Cramér's book the reader is led gently through the various stages of statistical thought without the conscious realization that this problem even exists. However, it is pertinent to add that the statistical theory is not developed here fully enough for the IB or IIB examinee who must have recourse as well to the official reading.

A few omissions of a minor nature struck the reviewer. For instance, in the historical review in Chapter 1 the dates of the various persons mentioned might be given. In Chapter 6 no mention is made of the use of a continuity correction when approximating to the binomial frequencies with a normal distribution. In Chapter 14 the standard errors of a number of sample statistics are used, few of which have actually been obtained, and a little help as to how to obtain the more complicated of them would be useful. In the chapter on the χ^2 test no mention is made of how Karl Pearson originally derived it as an approximation to the multinomial probability. The index to the book is a little too condensed and not entirely consistent in its method of positioning items. Finally, in a few places there is a certain unevenness in the printing, mainly of figures.

These points in no way detract from the value of the book as an elementary text. It is a worthy introduction to the same author's *Mathematical Methods of Statistics*, a book that is pre-eminent at a more advanced level. The style is lucid and the whole work is liberally sprinkled with examples to drive home the points

made. It is somewhat of a pity that the relatively high price of the book will put off many readers, especially if they have to buy the book for themselves.

P. G. M.

Collective Risk Theory. By Professor HARALD CRAMÉR.

[Pp. 92. Reprinted from The Jubilee Volume of Försäkringsaktiebolaget Skandia.]

THIS short book is in the nature of a review of the work that has been done by various authors over the past quarter of a century. Prof. Cramér himself has contributed much and stimulated many others, and a perusal of the very useful bibliography in the book bears witness to this fact. As a great deal of the work has been published in Scandinavian books and journals it is very pleasant to have this edited English edition.

To explain the theory the author first gives a concise summary of stochastic processes. The level of discussion is somewhere between that in Feller's *Introduction to Probability Theory and its Applications* and Doob's *Stochastic Processes*. From here the author tackles the problem proper. An insurance company, viewed in a simplified fashion, has two types of movement, namely, the claims of the policyholders and the net risk premiums paid in to the company. By assuming the latter to be paid in continuously and the former to take discrete finite amounts it is possible to regard the whole process as a stochastic one and investigate the properties of such a system. Amongst these properties we can find the probability distribution of the total amount of claims paid by time t , and thence the net gain to the insurance company at time t . Also it is of interest to see the way the net gain fluctuates in the period up to time t . This leads to a consideration of the probability of ruin, in that the insurance company may have at its disposal a fixed sum to cover any losses due to random fluctuations, but if the accumulated loss up to any point exceeds this amount the fund is ruined. It is possible to obtain the probability of this event happening under various assumptions, and the ruin problem is generally stated as being the probability of ruin at some time given the net risk premiums, the safety loadings and the distribution of claims. Cramér considers the cases of ultimate ruin, ruin within a time T , and both cases where the only points of time that matter are a series of finite points and not the whole continuum of time, e.g. the end of each financial year. The last chapter deals with a few generalizations in some of the basic assumptions which have had, of necessity, to be kept very simple, but they point the way for future work on the subject. For example, the introduction of compound interest into the expressions would result in more general equations.

P. G. M.

Introduction to Demography. By MORTIMER SPIEGELMAN.

[Pp. xxi + 309. The Society of Actuaries, Chicago, 1955. \$6.00.]

THE present reviewer in writing an account of the recent new edition of Wolfenden's *Population Statistics* expressed the opinion that there was 'a difference in the teaching of population statistics encouraged by the profession on the two sides of the Atlantic'. Hard on the heels of that statement has come stern correction in the shape of an official text-book of the Society of Actuaries with a preface which begins:

Demography, or the study of population, is one of the fundamental subjects of professional interest to the actuary. Its methods and concepts spring from the same

sources as those employed by the actuary in the insurance business. Many of its techniques provide valuable tools for the actuary not directly engaged in demographic studies, who will often find them applicable to insurance company experience, to pensions problems, and in other connections.

Clearly the opinion was wrong and it is therefore withdrawn.

This book is a model of arrangement. As the author himself says:

Most chapters begin with a statement of purpose, which is followed first by an account of basic concepts, definitions, and methods, and then by a description of characteristics and trends. The result of this effort to systematize the subject matter is evident in the detailed table of contents. In order not to interrupt the continuity of the text, the number of footnotes has been kept to a minimum. For the same reason, the lengthy list of references has been relegated to the end of the volume. The references are so keyed to the text that there should be no difficulty for those who wish to locate source material or to pursue the subject further.

Furthermore, the arrangement of the chapters themselves is such that there is a logical progression from one to another. Basic materials (census data and vital registration systems) come first followed by a discussion of the validity of this material; then mortality measurement and projection; morbidity measurement and trends; marriage statistics and family composition; and fertility measurement and projection. Finally, certain special and important characteristics, e.g. areal distribution, labour statistics and population estimation and projection are given thorough treatment.

The proper treatment of statistical methods applicable to demography involves considerable descriptive material, and any manual which attempts compression runs the risk of giving inadequate rationalization of currently accepted methods. It is a tribute to Spiegelman's lucidity that he avoids this risk throughout most of the book, though one would have wished to see more critical treatment of such topics as occupational mortality, population concepts (particularly that of usual residence), and mortality and fertility indices. Greater emphasis might also be given to tests of reproducibility in the classification of verbal data.

Students in this country will be interested to read of the United States Post-Enumeration Survey as a means of assessing errors in the census. The inevitability of census errors does not prevent their assessment; and if this assessment is designed also to reveal the true sources of the errors much can be done to minimize them. In dealing with age errors the author does not stress the value of graduation as a means of adjusting for these errors, especially where the graduation is applied not to crude figures but to the ratio of the figures to the relevant births in order to retain true irregularities due to birth fluctuations; but this is understandable in the North American context with heavy immigration.

In dealing with measures of mortality, the author's desire for conciseness has perhaps led him to give insufficient explanation of the underlying reasoning which has led to the development of foetal and perinatal mortality measures. The use of the term 'death ratio' for proportionate cause mortality will be new to British students (it does not appear in the English volume of the United Nations Multilingual Demographic Dictionary). Factors operating to produce recent mortality reductions are summarized in one very short paragraph. This seems a pity, since the author is himself better equipped than most to explain the relevant medical advances, knowledge of which is essential to those demographers who may wish to use this information in forecasting mortality changes. Less attention is given to the difference in the rate of mortality improvement for

the two sexes than seems appropriate to current experience, and to the needs of projection.

The chapter on the construction of life tables is clear but rather formalistic. It is fair either to assume that the data are in census form and that the separation factors are either provided or may be estimated, but for the purpose of teaching demography the British practice of leaving the treatment of populations at risk to a separate text-book on mortality and investigations is preferred. It would then only be necessary to indicate the procedure for developing the life table given the central death-rates.

The chapter on the projection of mortality is excellent both in relation to the method of presentation and its full reference to recent developments. A later edition will presumably include reference to recent work in Sweden.* Coale's work is reported without comment, although it would appear necessary to warn students of the dangers of using a single pattern of projection for application to all countries. There is no real substitute for adequate analysis.

As might be expected from a knowledge of the author's background the treatment of morbidity statistics is also comprehensive and well set out.

The need to understand fully the interrelationship between fertility and sociological factors has led demographers to concentrate their attention much more than hitherto upon family formation, composition and dissolution, and this need has been recognized in the present text-book. (The author might perhaps have referred to the detailed tabulations on family composition which were published as part of the One Per Cent Sample Tables of the Census in Great Britain 1951.) In discussing fertility a balanced view is taken of the various statistical measures used, though rather more mathematical analysis is introduced than would seem to be necessary for actuarial students.

The separation of areal distribution into a distinct chapter of the book is highly commendable, since such a distribution is just as much a separate characteristic of population as age or occupation. There is a full and interesting treatment of migration studies. The author also provides an excellent chapter on labour statistics, a subject of growing importance as the relationship between economics and population changes becomes more clearly recognized. The desirability of achieving maximum productivity in an era of full employment has given impetus to the study of labour turnover, and the topic is adequately dealt with in the book.

The subject of population estimation and projection is developed in an essentially practical manner in a final chapter. The author wisely stresses that methods must be related to the needs of the problem, and proceeds to give a full range of possible methods.

A full list of references is provided at the end of the text-book. Some of these are a great deal more advanced in their treatment of the subject than the text-book context demands. The student would be helped if there were some method of marking these references to distinguish the essential reading from those papers which represent more recondite treatment.

This is a text-book of very high quality which will be of great value to students; but it is also so readable that it will itself do much to stimulate an interest in the study of population changes in a wider circle of readers.

B.B.

* Prawitz, H., Swedish Actuarial Society, 50th Anniversary Commemoration Volume, 1954.

Versicherungsmathematik I. By WALTER SAXER.

[Pp. ix + 249. Berlin-Göttingen-Heidelberg: Springer-Verlag, 1955. DMks 39.60.]

ON THE occasion of the reopening of Staple Inn Hall the President of the Association des Actuaire suisses, Prof. Marchand, presented to the Institute a copy of the latest actuarial text-book to emanate from Central Europe. This volume, the 79th in a series called *The Foundations of the Mathematical Sciences*, caters for the needs of both students and practical men, and with this in mind the author has avoided, except in one chapter, the use of the calculus in the derivation of his formulae. He plans, however, to publish a second, advanced, volume in the future, and no doubt this volume will make use of more sophisticated mathematical and statistical tools.

When considering this book it is necessary to bear in mind the difference in background of both authors and readers on the continent as compared with this country. Most continental text-books are written by university professors and the present work is no exception; Dr Saxer is a professor at the Swiss Federal Institute of Technology at Zurich. Also, since continental students of actuarial science pursue their studies before having gained any experience in a life office or similar institution, they tend to view problems from a more theoretical angle than do their British counterparts. In consequence the book contains practically no worked numerical examples and devotes far less space to such practical problems as premiums payable more frequently than yearly than do comparable British texts.

The first chapter, on the theory of compound interest, consists of six pages only, but the ground covered should be sufficient for the understanding of the rest of the book. The concept of a life table is introduced in the next chapter, and methods of finding ungraduated rates of mortality are explained. The treatment of the life, policy and calendar-year methods is descriptive rather than mathematical, and the reviewer feels that students may experience some difficulty in distinguishing between the various meanings of the age x . In view of the author's aim to avoid the use of the calculus no mention is made of the force of mortality at this stage; multiple decrement formulae are derived by the use of algebraic methods only, and the treatment is of necessity less rigorous than that in *Bailey and Haycocks*. While the subject of graduation is to be dealt with in the second volume, the author points out the use of Makeham's law in this connexion, particularly as applied to recent Swiss population life tables. It is interesting to note, in the discussion on select tables, that the period of selection in recent German annuitants' tables has been reduced to one year.

Chapter III introduces commutation functions and their use in calculating single-life annuity and assurance values and net premiums. The various formulae are very clearly set out and are easily available for reference, but no attempt is made to go beyond their algebraic derivation. For instance, on p. 43 the single premium for a family income benefit payable annually in advance is shown to be $\ddot{a}_{\overline{m}|} - \ddot{a}_{x:\overline{m}|}$. It is felt that an explanation of this expression by general reasoning would have materially assisted a student in appreciating its significance. The second part of the chapter deals with the calculation of office premiums and with the application of select tables. As is usual on the continent, a distinction is made between bases of the first and second order. In the former, assumptions of interest, mortality and expenses are sufficiently cautious to make a profit to

the insurer reasonably certain; in the latter an attempt is made to forecast as nearly as possible the experience during the future course of the policy.

The fourth chapter deals with functions involving more than one life. The more important annuity and assurance values for two lives are derived with the aid of commutation functions and the application of Makeham's law is explained. The whole chapter takes up 11 pages only, and the British student will miss the thorough treatment of contingent assurances and reversionary annuities which depends on the use of approximate integration.

Chapter v contains a very useful treatment of pension schemes. Considerable attention is paid to the calculation of lump sums and pensions payable on disablement and to the effect on annuity values of variations in disablement rates. All benefits are assumed to be independent of salary, and the commutation functions are therefore free from the complications introduced by the use of salary scales. Formulae for widows' pensions are set out both by the reversionary and the collective method, and a clear distinction is made between pensions arising from death in active service and after disablement or retirement. The question of select remarriage rates is ignored, presumably because the publications of recent Swiss pension fund experiences (reviewed in *J.I.A.* 77, 317) are based on aggregate rates only. Figures for the mean number and mean age of children under the age of 18 (or 20) are to be found in the same publications, and it is probably for this reason that the author computes the value of orphans' pensions arising on a member's death by multiplying the mean number of children by a temporary annuity commencing at the mean age. While this multiplication of averages may be defended on practical grounds the reviewer feels that a more satisfactory result is obtained by calculating, for all members dying at age x , an average annuity value depending on the age distribution of all eligible children. In equations (5.9.1) and (5.9.3) children's mortality is taken into consideration, and it would appear that the annuity functions should read $\ddot{a}_{x:\overline{s-g}|}$ and $\ddot{a}_{xz:\overline{s-g}|}$ respectively.

The following chapter deals with the calculation of valuation reserves. After explaining the building-up of unmodified net premium reserves by splitting net premiums into risk and investment premiums, the author gives a useful list of reserve formulae for the main types of assurance; as before, the complication introduced by premiums payable more frequently than yearly is ignored. The calculation of reserves as at the end of a financial year is next dealt with and consideration of expenses leads to the treatment of Zillmerized reserves. The second half of the chapter contains an excellent summary of valuation methods under the following headings: (i) the method using valuation constants, impartially described as 'Karup-Altenburger's'; (ii) methods based on the use of an average age for each valuation group, i.e. Lidstone's Z in the case of prospective valuation and Jecklin's t in the case of retrospective valuation; (iii) methods based on interpolation with particular reference to Jecklin's F -method which fits a rectangular hyperbola to the reserve curve; (iv) the collective reserve method which is particularly applicable in the case of frequent changes of valuation bases provided that records are kept on punched cards. Using the mortality basis only, an estimate is made of the premiums expected to be received and the sums assured expected to be paid out in every future year and these amounts are valued by the use of interest functions only.

The subject of conversions and surrender values takes up the last two pages of this chapter and the practical man may well find the treatment somewhat inadequate. The strength of this book lies, however, in its treatment of actuarial

theory, and this is well to the fore in the next chapter in which the theory of the effect of changes in interest and mortality on changes in various actuarial functions is developed. Starting with two arbitrary series of real numbers and applying certain operators, the author deduces some basic equations of a general nature from which he obtains, by substitution, results of considerable practical importance. He proves Schärff's theorem of invariance and shows changes in policy values in a perfectly general type of assurance where the insurer receives π_s from each of the l_s survivors at the beginning of the year $s+1$ and pays out $U_{s+1}^{(i)}$ at the end of the year in respect of the cause of exit i . These general results are used to show changes in policy values resulting from changes in the underlying rates of interest and mortality. The effect of a variation in the rate of interest on various assurance and annuity values (including widows' and disability pensions) is also investigated.

Chapter VIII forms a short excursion into the continuous method. The concepts of force of interest and mortality are explained and the calculus is used to prove formulae for continuous annuity and assurance values. The relationship, for $\bar{d}_x$, between changes in the age x and in the underlying forces of interest and mortality is investigated both for tables obeying Makeham's law and under more general assumptions, and the conditions for transformation from one system of δ , μ to another are proved.

In Chapter IX the author leads us back into the realm of actuarial practice. After a discussion of the items which make up the asset and liability side of a balance sheet (for assurance companies and for pension funds) the various sources of surplus are described qualitatively rather than quantitatively. In the opinion of the reviewer, the reader wishing to try his hand at an analysis of surplus would have been considerably assisted by a statement of the basic formulae for interest, loading and mortality surplus. The second part of the chapter deals with the distribution of surplus, mainly by reference to the contribution system. Formulae for the various sources of surplus that make up the components of a dividend are clearly set out; they are calculated on the basis of second order mentioned above. The difference in viewpoint between the continental and British actuary is emphasized by the fact that only five lines are devoted to the reversionary bonus system. It is also interesting to note that the author suggests the use of a double decrement table for a bonus reserve valuation, surrender values being included among the prospective payments.

Chapter X is extremely useful for the understanding of the future development of pension funds and social insurance schemes and for the calculation of emerging costs. Various assumptions are made as to the initial age distribution of a population and as to new entrants, and it is shown how the population stipulated by one set of assumptions converges eventually towards another type. The treatment of this subject is well up-to-date and deserves careful study. The ideas put forward are used in the final chapter in which methods of financing social insurance are discussed. The following methods are considered: (i) the Individual method: the contribution of every individual member is dependent on his entry age, so that the value, at entry, of his contributions equals the value of his prospective benefits; (ii) the Pay-as-you-go method: all benefits payable in any year are paid for by the contributions due in that year; (iii) the Terminal Funding method (applicable only when all benefits are taken in the form of pension): the contributions in any year are equal to the purchase price of all pensions vesting in that year; (iv) the Generation Average method: entrants in any one year (termed a 'generation') pay a contribution independent of age which

is sufficient to purchase the benefits in respect of that generation; (v) the General Average method: the rate of contribution is such that the value of the contributions in respect of all present and future members is equivalent to the value of their benefits.

Methods (ii)–(v) are applied to the types of population considered in the previous chapter, and the effect of the various population assumptions, and the funding methods, on the cost of social insurance schemes is brought out very clearly.

While the discussion in the main body of the book is based on the orthodox mortality table, a short excursion into the theory of stochastic models is made in the Appendix. It is, of course, impossible to treat this subject exhaustively without assuming a fair knowledge of statistics, but the author succeeds in whetting the student's appetite for further study. He introduces the concept of an extended mortality table—first proposed by Lukacs and Vajda in their papers submitted to the Twelfth International Congress of Actuaries—and proves Sverdrup's theorem that the probability of the insurer incurring a loss tends to zero as the number of policies tends to infinity provided the premiums include a contingency loading.

The notation used follows the International Actuarial Notation. On p. 38, however, a temporary assurance deferred m years and terminating at age s is denoted by ${}_m|s-xA_x$ instead of either ${}_m|s-x-mA_x$ or ${}_m|A_x:s-x-m$. In accordance with continental custom the symbol r is used for $i+i$, but the simultaneous use of the same symbol for annuity per annum (as on p. 5) ought to be avoided.

A few minor errors have been noted among which the following may be mentioned: On p. 28 q_{x+t} should be defined as the probability of dying in the $(x+t+1)$ th, not the $(x+t)$ th year of life. On p. 86 it is stated that for duration of selection h , $l_{[x]+h+1} = l_{x+h+1}$. While this is strictly correct it is nevertheless felt that the term $+1$ should be omitted on both sides. In the retrospective formula for ${}_tV_x$ on p. 116 N_{x+n} should be replaced by N_{x+t} and the formula for δ on p. 157 should read $i - \frac{i^2}{2} + \frac{i^3}{3} - \dots$. Some misprints have crept into the reference to Seal's paper in the footnote on p. 17.

All these are, however, minor blemishes in a book which can be heartily recommended to all actuaries interested in an up-to-date treatment of life contingencies from the Central European angle.

H. F. F.

Mortality and Other Investigations, Volume I. By H. W. HAYCOCKS, B.Sc. (ECON.), F.I.A. and W. PERKS, F.I.A.

[Pp. ix + 164. Cambridge: Published for the Institute of Actuaries and the Faculty of Actuaries at the University Press, 1955. 20s.]

THIS book covers a subject so fundamental to actuarial teaching that it must be regarded as occupying a key position in the whole range of text-books which have been designed to meet the needs of the Examination Syllabus. In the very nature of things therefore this must be regarded as a supreme test for the authors. However, any single assessment of their degree of success must be approached with diffidence, for it is impossible to be free of prejudice in favour of a particular mode of approach, and in such an essentially practical subject it is impossible to claim that any one approach is uniquely correct. Judgment must be pragmatic, and it will be some time before examination results provide a full measure of the influence of this text-book. One general note of congratulation may be made at

the outset; the book deals in 164 pages with the toughest segment of the Actuarial Syllabus—for no student conquers this field without very hard thinking—formerly covered by a wide range of papers in the *Journal*, and it does this without sacrificing lucidity to brevity. No stronger tribute could be paid to its succinctness.

Any text-book on this subject must begin with the fundamental notion of a mortality rate and the correspondence between numerator and denominator. This immediately implies a time interval. The authors have chosen to begin with a single year, partly in order to be able to sweep aside seasonal and secular variation but mainly for simplicity. New entrants and leavers are also for the sake of simplicity considered at a later stage. Thus the first impact of the notion of a rate is not confused by these extraneous elements. This notion is then portrayed in relation first to a calendar year with the ratio of deaths to 'starters' as the direct analogy of q , and the ratio of deaths to 'mean population' as the analogy to m ; and then in relation to the life year, and the policy year in turn. The essential similarities and differences in the derivation of rates are emphasized as a means of assisting the later understanding of the statistical requirements in office practice; and age and duration are finally introduced as fundamentally 'time' variables, the adequate treatment of which forms the object of the whole exercise.

The principles of continuous exposed-to-risk formulae then follow as a natural development. The life year is a simple interval requiring only reference to birthdays; the calendar year brings the need for approximation to age (and duration); and the policy year also requires age approximation. These problems of definition and approximation imply methods of classifying lives and deaths by age and duration which are discussed and the derived rates are expressed symbolically. Consideration of the 'mean population' denominator for the central rate leads easily to the census method. The whole picture is then illuminated by graphical representation (as an aid to clear thinking the importance of this device cannot be too strongly stressed).

Extension to the combination of more than one year of experience is next seen as a matter of statistical organization. Finally, the complication of new entrants and leavers is disposed of. Though this part of the exposition is rigorous and the practical considerations of office tabulation are emphasized, it might have added to the completion of the treatment if there had been a few arithmetical examples, especially those involving the actual assignment of lives, classified according to specified time references, to the elements of the exposed-to-risk.

From mortality the authors pass to sickness with its durational characteristic and difficulties of definition; to partial rates; and finally to rates for other types of decrement.

The student has been continually reminded that practical considerations are of paramount importance. This is further emphasized in the next section dealing with graduation. Nevertheless, a balanced view is taken. While reminding the student that other factors (e.g. interest) in actuarial calculations render inappropriate the application of too refined techniques, the proper understanding of these techniques and their underlying principles is stressed as a prior condition for the development of sound judgment in choice of method for particular situations. Only the graphic method is specifically described, but there is a chapter on pivotal values, osculatory interpolation, and abridged life tables, which virtually extends the treatment to formulae methods.

A final chapter deals with the practical methods adopted for the various members of the whole series of English Life Tables, including the adjustment of

census populations, especially at the youngest ages, and the completion of the tables at advanced ages.

This condensation of teaching is highly welcome and volume II will be awaited with eagerness. The book will have a wider appeal than is represented by its main intention. Statisticians outside the actuarial profession have often complained that though actuaries make light work of estimating populations at risk in a variety of circumstances, they have never, hitherto, made it look easy. They are referred, with confidence, to Haycocks and Perks.

B. B.

Government Finance and Fiscal Policy in Post-War Britain. By A. R. ILLERSIC, B.COM., M.SC. (ECON.).

[Pp. 278. London: Staples Press Ltd., 1955. 25s.]

The London Capital Market. By NORMAN MACRAE.

[Pp. 285. London: Staples Press Ltd., 1955. 25s.]

THESE books have in common an examination of the monetary policies pursued in this country since 1945.

The first half of Mr Illersic's book is concerned with the effects of taxation upon the national economy and especially with the long-term effects upon industry of high direct taxation. By way of introduction he outlines the modern objectives of monetary and fiscal policy, explains in simple language the Keynesian theory of employment requiring the maintenance of an adequate level of investment and consumer spending, and draws attention to some of the deficiencies in our national income statistics. The author is not primarily concerned with the theory of taxation nor with schemes of taxation reform. He points out that taxation nowadays is not merely a means of raising revenue but is also a method of persuading or coercing the community (or a particular section of it) to vary the distribution of its expenditure or to influence its propensity to earn and to save. He therefore devotes most of this part of the book to a discussion, very fully supported by numerous references, of the economic consequences of many of the fiscal problems which have arisen during or since the last war. Among the problems discussed are the disincentive effects of high taxation, the inadequacy of industrial depreciation allowances, the reduced capacity of individuals to save, the redistribution of the national income, and the advantages and disadvantages of a capital gains tax.

The first half of Mr Macrae's book is devoted to a description of the London capital and money markets in the post-war period. He enumerates the sources of savings and of investments and loans, and discusses the workings of the various financial institutions in the City. This part covers ground which must be familiar to many actuaries, but the author brings the subject up-to-date and puts the various parts into perspective by attempting estimates of the quantities involved in everything he describes, even when some of his estimates are admittedly no better than rough guesses. He reaches the conclusions that this country is not saving and investing enough to keep its place among the wealthy countries of the world, and that capital formation has been inadequate and misdirected since the war because taxation has warped the distribution of savings and the incentive for business growth.

The second parts of both books deal with monetary policy. Mr Macrae brings

the post-war changes in budgetary and monetary policy into an account of the present-day function of the Bank of England and the banking system. Mr Ilersic, after a brief description of modern credit control technique, gives a detailed and most interesting analysis of the changes in monetary policy between 1939 and 1954. He explains the marked revival in the attention paid to monetary policies in most developed countries since 1950 by the failure of fiscal policy to restrain the post-war inflationary pressure.

Both authors present the arguments for and against the use of the so-called orthodox monetary policy as a means of establishing equilibrium. Mr Macrae concludes that budgetary policy is probably a more potent weapon than monetary policy when the object is to maintain demand; that monetary policy can be changed at any time of the year, whereas budgetary policy can generally only be brought into effect once a year; that reliance on the budgetary rather than on the monetary weapon in times of inflation will not necessarily bring about more investment and less consumption; that monetary policy may prevent excessive stock-piling of raw materials; and that high interest rates induce foreigners to leave funds in London. Mr Ilersic, in a much more closely reasoned discussion, quotes several authorities holding somewhat diverse views on this subject. He considers it logical that monetary policy rather than controls should be used to regulate a free economy which depends on the efficient working of the price mechanism in order to maximize the national product. He also considers that deliberate restriction of the volume of bank credit will exercise a far greater deflationary effect in conditions of full employment than any practicable increase in the level of interest rates. The effectiveness of interest-rate policy is based upon its psychological effects, and if these cannot be assessed then the value of bank-rate policy as a means of correcting instability in the economy is open to question. The bank-rate increases in 1951 and 1952, while contributing to the subsequent easing of the inflationary pressure, were materially assisted by a favourable change in the terms of trade and by a lower level of economic activity in other parts of the world.

Of the two books Mr Ilersic's is written in a more scholarly way and covers the ground more thoroughly, with abundant quotations from other writers. Mr Macrae writes in a journalistic style which makes his subject very readable but his treatment is more superficial.

C. J. B.

World Population and Resources. A Report by PEP.

[Pp. xxxvii + 339. PEP (Political and Economic Planning), 1955. 30s.]

'MANKIND has always had, and probably will always have, a population problem.... There may be at least two distinct... problems in adjoining regions... at the same time. The character, symptoms and trends of these problems are often obscure, and the possible remedies difficult and contentious. There is accordingly much temptation to regard them as academic or insoluble, yet they are much too important to be neglected.' This thesis, which might well serve as a motto for demographers of all kinds, gives an excellent justification for the survey that has just been completed by a distinguished (but almost entirely anonymous) group of natural and social scientists—a systematic survey in which the world's resources have been reviewed in relation to the expected growth of human population over the next few decades.

The statement quoted above is not a preconceived notion; it is a deduction from the facts, and is not encountered by the reader until thorough examination of the available evidence has shown the firmness of the basis on which it rests. This examination is effected early in the Report, which falls into two sections. In the first of these, Parts I and II set forth the evidence, first for the world as a whole and then for a wide selection of representative countries; in the second section, Part III is concerned with a description and discussion of the population policies of certain Governments, and finally Part IV outlines the programme of research and action that the Group consider necessary for the formulation of policies to improve man's circumstances, particularly in the underdeveloped areas.

An unusual feature of the Report is that each of its four Parts has been made very largely self-contained, with its own introduction and summary of conclusions. This rounding-off of the sections is of material help to the reader of a longish work, even if it makes for some repetition. It is one of a number of devices which have been successfully used to ensure clarity, a virtue which is enhanced by a well-written text and an effective use of tables and charts. It is slightly marred by the confusion created by having chapters, tables and figures (but not pages) numbered the same in each part, so that, for instance, four Chapters 3 and three Tables 2 are to be found within the covers of the book; and there is no index of tables and diagrams. The reader quickly adjusts himself, however, to these conditions.

In recent years, a number of studies of population and resources have been made, and a mass of information and varied opinion on nearly every aspect of each of the two fields has been published. Relatively little, therefore, remained to be added to the common stock of knowledge, but the Group found an important assignment in seeking to stimulate a more responsible and practical discussion than could be expected from many contributions to these subjects. In particular, they have concentrated attention on the inequalities and pressing problems of the present and coming decades instead of arguing them away by reference to a remote future period when every conceivable ultimate development favourable to man has occurred; such a time may well come, but not before the passage of many generations. An attempt has been made by the Group to put the whole matter in a new perspective, more closely related to the problems of the underdeveloped areas, which are the main centre of interest in the Report. In these areas, the problems of population and resources are inextricably linked. The weakness of many modern writings lies in their treatment of the two independently rather than as related functions; from this failing, rendered almost inevitable by the absence of a suitable background of completed research and collection of information, the present Report is not entirely free. Nevertheless, one of the early chapters has been given over to a consideration of the question of interrelationships, and the Group's recommendations for research are also largely concerned with improving the state of knowledge about them.

The second of the four Parts of the Report consists of a series of short sketches of nineteen countries and islands, classified in groups according to degree of economic development and density of population. The descriptions given of these varied areas are simple and concise. They do not go deeply into detail, and occasionally over-simplify, but they give the essentials and serve both as a valuable supplement to the main text and, for those who wish to pursue matters further, as a brief introduction to the demography and economy of the chosen regions.

The title of Myrdal's book, *Nation and Family*, has been chosen for Part III, in which are contrasted the national policies of to-day that are most strikingly concerned with population size or welfare. It is shown that the Governments of two countries—France and Russia—have, each in its own fashion, put into effect measures to prevent restriction of births; in Japan and India, however, steps have been taken, in differing ways, to encourage restriction. Sweden, by further contrast, has shown a greater concern for the care of the family than for the number of children in it. A similar tendency is observed by the Group in the United Kingdom and is described in a short history of the main demographic and related sociological developments of the last hundred and fifty years in this country, including references to the Royal Commission on Population and its aftermath. In times when Governments are increasingly being called upon to formulate population policies, such a survey of the desirable and undesirable features in recent legislation and action is a necessary adjunct to demographic studies.

By the beginning of Part IV the authors are ready to weave together many threads. They have observed how great an effect on the family State intervention can have if it is applied as vigorously as it was in Russia in the middle 1930's. They have also seen how fertility has sometimes been transformed by voluntary action in the face of contrary belief and custom. But while the great changes that are needed are possible in a variety of ethical backgrounds, they can hardly take place while most of the world is suffering from a gigantic unawareness of the problems of population. It is against this unawareness that the Report is mainly directed, and in seeking to promote understanding and informed discussion the Group have performed an important service. It will be inquired, as must be asked about any contribution to the literature of the subject, to what extent personal idiosyncrasy has prevented the fulfilment of the aims of the Report. The authors themselves do not claim to be entirely free from bias, just as they freely admit the fallibility of the predictions on which some of their arguments rest, but they suggest that their own diversity of backgrounds and interests should have minimized any distortions. There are, indeed, signs within the Report that conflicts of opinions were not always avoided; but these have been smoothed over without the need for minority reservations. Many of the final recommendations for research and action are unexceptionable. They make it evident that one of the major fields of investigation in the near future will be concerned with the interplay of population and economic resources. It is of particular interest to actuaries that, at least until a class of specialists in this field is developed, the joint action of workers in many techniques will be needed, and that there is thus fresh scope for the activities of our own profession.

P. R. C.

A First Approach to Economics. By E. VICTOR MORGAN.

[Pp. 456. London: Sir Isaac Pitman and Sons Ltd, 1955. 20s.]

THIS book is based on the author's lectures given to first-year University undergraduates. He states in the Preface 'The plan of the work is based on the belief that the more abstract ideas of economic theory are more easily grasped if they can be presented against a background of facts and events with which the student is familiar'. To attain this objective the book is divided into three parts: Part I, The Development of the Industrial Economy (about 100 pages); Part II,

The Economic Organization of Modern Britain (about 120 pages); and Part III, The Theory of Economic Organization (about 230 pages).

Part I is largely historical and informs the student how the most important economic institutions have become what they are to-day. The treatment is necessarily brief but sufficient for those requiring only an introduction to economics. Part II deals with the present-day structure of economic institutions. Some teachers may think that too much space is given to finance; for example, 41 of the 120 pages deal with Money and the Banking System, whilst another 24 pages deal with National Income and Expenditure. However, some students, for example, those studying for actuarial or accounting examinations, will find this weighting of the material an advantage. Part III covers what is normally termed Economic Theory or the Principles of Economics. It deals with such topics as consumers' choice, supply and demand, perfect and imperfect competition, the earnings of labour, land and capital, the price level, the maintenance of the level of employment and international trade. The treatment is simple and lucid and much use is made of diagrams.

The layout of the book is ideal for the Institute student who would like some reading matter additional to Whyte, Vol. 1. The objectives of Parts I and II are similar to those of Whyte, Vol. 1, namely, to provide an introduction to economics which is partly historical and which emphasizes the financial aspect. Part III should be read by such a student, but he need give serious study only to the chapters on Money, the Trade Cycle and International Trade.

H. W. H.

SONDERVERÖFFENTLICHUNG DER DEUTSCHEN GESELLSCHAFT
FÜR VERSICHERUNGSMATHEMATIK

No. 1. *Lebensversicherungstechnisches Wörterbuch* (1954). Edited by W. SACHS.

THIS dictionary of life assurance terms is an expansion of the original version of Gardner and Sachs in Vol. 2 of the *Blätter* (see *J.I.A.* 62, 185). It now provides the verbal equivalents in five languages: English, French, German, Italian and Spanish, each of these languages being used in turn as the basis for the translations.

No. 2. *Ableitung von Sterbetafeln für die Rentenversicherung und sonstige Versicherungen mit Erlebensfallcharakter* (1955). By F. RUEFF.

IT is explained in Chapter I that the sparseness of the relevant data makes it difficult to derive a mortality table for German annuitants that takes account of expected future improvements in mortality. The investigation that follows thus proceeds in three steps:

(1) the mortality rates of the German population are first studied as a function of their calendar year of observation;

(2) these rates are then reconstructed on a 'generation-table' basis and are projected into the future;

(3) a time-independent relationship is sought between population and annuity data.

'The objectives of the investigation were (1) a dynamic system of mortality tables by means of which expected future mortality changes could be synthesized, and (2) a simple technique for the application of this system in practice.'

In Chapter II, after a review of some previous attempts to forecast mortality, the author describes the method he used to obtain the (male or female) rate of mortality at age x during the year N , $^{(N)}q_x$ say. His formula is

$$^{(N)}q_x = {}^{(t)}q_x \cdot 10^{-(N-t)F_x},$$

where
$$F_x = \sum_{j=1}^5 (t_5 - t_j) \log ({}^{(t)}q_x / {}^{(t_5)}q_x) / \sum_{j=1}^5 (t_5 - t_j)^2,$$

and $t_1=1886$, $t_2=1896$, $t_3=1906$, $t_4=1925\frac{1}{2}$, $t_5=1933\frac{1}{2}$, $t_6=1950\frac{1}{2}$. The data used were German population figures and x proceeded in multiples of 5 from age 25.

It was thus possible to compute annuity values based on mortality of the year N , $^{(N)}\ddot{a}_x$, and also those based on mortality suffered by individuals born in the year τ , $^{(\tau+x)}\ddot{a}_x$.

The next step, described in Chapter III, was to seek a relation of the form

$$^{(N)}q_x \equiv {}^{(\tau+x)}q_x = {}^{(t)}q_{x-\Delta\tau} \quad (\Delta\tau \text{ independent of } x).$$

Experiments showed that a reasonable approximation could be obtained by assuming, for both males and females,

$$\Delta\tau = \begin{cases} 0 & (\tau \leq 1855), \\ 10^{(\tau-1855)/100} - 1 & (\tau > 1855). \end{cases}$$

For example, for year of birth 1905 ($=\tau$) the following 3% values of $^{(\tau+x)}\ddot{a}_x$ were obtained ($\Delta\tau = 2.162$):

x	Males		Females	
	${}^{(t)}\ddot{a}_{x-\Delta\tau}$	True value	${}^{(t)}\ddot{a}_{x-\Delta\tau}$	True value
50	17.592	17.513	18.664	18.629
60	13.640	13.642	14.513	14.514
70	9.516	9.518	9.997	9.987
80	5.826	5.782	6.092	6.060
90	3.346	3.201	3.564	3.431

Chapter IV explains how earlier German annuity tables (centred on the years 1877, 1900 $\frac{1}{2}$ and 1928 $\frac{1}{2}$, respectively) were combined with the 1948-51 observations of the Allgemeinen Rentenanstalt to produce values of δx in the relation

$$^{(N)}q'_x = {}^{(N)}q_{x-\delta x},$$

where the dash indicates (aggregate) annuitant mortality. Although δx decreased steadily as N increased, it was found possible to write, for both sexes,

$$^{(\tau+x)}q'_x = {}^{(\tau+x-2)}q_{x-2}.$$

A final chapter (Chapter V) discusses the practical use of the single table of $\Delta\tau$ in conjunction with published tables of $\ddot{a}_x$ based on the German population table of 1949-51 (or of 1924-26).

Readers need no reminding that there is considerable variation to-day between the mortality of different countries as a whole and of their annuitants. For

example, the following table compares decennial values of 10^3q_x according to six different mortality bases:

x	German population ADSt 1949-51	England and Wales 1948	British annuitants, 1947 (ultimate)	U.S. White, 1948	U.S. non- refund annuitants, 1941-46 (ultimate)	Dutch population, GBM 1947-49
Males						
50	7	9	6	11	8	6
60	17	23	17	15	19	15
70	42	51	45	39	40	39
80	117	121	112	105	93	105
90	280	—	244	265	220	265
Females						
50	5	6	5	6	4	
60	12	13	11	13	9	
70	36	34	27	37	25	
80	110	92	74	91	69	
90	257	—	187	210	188	

Nevertheless, working on the population data of six German censuses, Dr Rueff has succeeded in producing in essentially practical form a set of forecast annuity rates that bears great similarity to the corresponding values of the American and British investigations. Consider, for example, the following (generation) values of $\bar{a}_x$ at 3% appropriate for annuity purchasers in 1965:

Age in 1965	Rueff (aggregate)	$\bar{a}(55)$. ult. (pro- jected 10 years)	$\bar{a}$ -49 with pro- jection (ultimate)
Males			
50	18.66	18.24	18.49
60	14.45	14.08	14.40
70	10.07	9.78	10.20
80	6.09	6.10	6.40
90	3.45	3.62	
Females			
50	19.72	20.00	20.52
60	15.39	16.04	16.37
70	10.60	11.64	11.74
80	6.37	7.47	7.33
90	3.65	4.37	