

## REVIEWS

*Table of the Coefficients of Everett's Central-Difference Interpolation Formula.*

By A. J. THOMPSON, Ph.D.

[Pp. 32. No. V of *Tracts for Computers*, edited by E. S. PEARSON, D.Sc.  
2nd edition, Cambridge University Press, 1943. 5s.]

THIS new edition contains additions and rearrangement. The main table gives, for values of the argument from 0 to 1 by intervals of  $\cdot 001$ , the coefficients of Everett's formula up to the 8th order with their 2nd central differences, correct to 10 decimal places. Similar data for intervals of  $\cdot 01$ ,  $\cdot 1$ , and  $\cdot 2$  are given in other tables, which show in addition the 4th, 6th, etc., central differences of the coefficients and in the last two cases the coefficients themselves up to the 12th and 16th orders respectively, the figures being exact except in a few instances. A further table gives the coefficients up to the 12th order in the range  $-5$  to  $+6$  by intervals of  $\cdot 1$ , this being for use when interpolating near the beginning or end of a table.

An introduction gives some helpful advice on the use of the tables both for isolated calculations and for systematic interpolation (subdivision of intervals). The reader should bear in mind that the same symbol is used for the difference-operator when applied over a subinterval to the coefficients and when applied over a full interval to the function.

C. D. R.

*The New Population Statistics.* By R. R. KUCZYNSKI.[Pp. 32. No. I of *Occasional Papers*, National Institute of Economic and Social Research. Cambridge University Press, 1942. 1s. 6d.]

THOSE who have read Dr Kuczynski's previous writings on population statistics will be aware of his logical manner of exposition and his skill in detecting any weakness in argument. In this booklet he turns his attention to two official publications, *Current Trend of Population in Great Britain* and *The Registrar-General's Statistical Review of England and Wales for the year 1938, Tables, Part II, Civil* (see *J.I.A.* Vol. LXXI, p. 461). Though everyone may not agree with all his criticisms, it is often difficult to resist the persuasion of his reasoning.

In the *Current Trend of Population in Great Britain* the Registrars-General of England and Wales and of Scotland gave, it will be remembered, estimates of the future population of Great Britain on certain assumptions of mortality, fertility and migration. The reviewer in the *Journal* criticized the assumptions regarding the rates of mortality at the older ages; Dr Kuczynski is more chiefly concerned with the rates of fertility, which so far as population trend is concerned are probably a more important factor. The Registrars-General made two sets of forecasts, one based on a constant annual supply of 700,000 births (approximately the pre-war level), the other on estimated average fertility rates for the years 1934-37. Dr Kuczynski points out that the first assumption implies a large rise in fertility rates on account of the decrease which will take place in the future in the number of women of child-bearing age, and considers that even the second basis is optimistic. He sums up his analysis of the forecasts as follows:

'Even if fertility had remained constant since 1938 and should remain so for another generation, even if mortality had declined considerably in every year since 1938 and should decline considerably in each year for another generation, even if the total excess of immigration over emigration should be much greater in 1938-51 than in 1924-37, the population of Great Britain would begin to decline in the 1950's and the rate of decline would increase

rapidly. If, on the other hand, an annual supply of 700,000 births could be maintained, the population would ultimately not be smaller than in 1937 provided mortality were kept throughout on a somewhat lower level than in 1938; but to maintain a constant annual supply of 700,000 births would presuppose an enormous rise in fertility.'

As has already been mentioned in the *Journal*, the 1938 *Statistical Review* gives data for England and Wales which became available as a result of the coming into force of the Population (Statistics) Act. Students of the population statistics of this country have previously been somewhat hampered by lack of such data, particularly as regards fertility according to age of mother and to duration of marriage. Under the Act information must be given at birth registrations regarding age of mother, and (when parents are married to each other) date of marriage, number of previous children, etc.; and at death registrations regarding marital status, age of surviving spouse, and (for females who have been married) duration of marriage, whether any children, etc. These particulars are not entered in the registers but are used only for the preparation of statistics. Nearly a hundred pages of the *Statistical Review* are taken up by statistics derived from the new returns for the second half of the year 1938 as well as tables deduced therefrom with accompanying explanations. The following are extracts from two of these tables for full explanation of which reference should be made to the *Statistical Review* itself.

Estimated annual live births per 1000 women of  
the specified group and age, 1938

| Age   | Married women | Unmarried women | All women |
|-------|---------------|-----------------|-----------|
| 15-19 | 521           | 2.8             | 14.6      |
| 20-24 | 270           | 7.7             | 92.3      |
| 25-29 | 172           | 9.2             | 113.3     |
| 30-34 | 110           | 8.1             | 83.0      |
| 35-39 | 59            | 7.0             | 47.3      |
| 40-44 | 20            | 2.8             | 16.1      |
| 45-49 | 2             | 0.2             | 1.5       |

Estimated number of births (live and still) in the year per wife  
at the marriage duration indicated

| Age of wife in 1938 | 0-   | 1-   | 2-   | 3-   | 4-   | 5-   | 6-   | 7-   | 8-   | 9-10 |
|---------------------|------|------|------|------|------|------|------|------|------|------|
| 16-19               | .432 | .211 | .210 | .198 | —    | —    | —    | —    | —    | —    |
| 20-24               | .291 | .266 | .220 | .196 | .180 | .160 | .156 | .163 | .215 | —    |
| 25-29               | .230 | .264 | .209 | .186 | .162 | .147 | .137 | .123 | .115 | .105 |
| 30-34               | .228 | .252 | .200 | .173 | .153 | .132 | .122 | .107 | .100 | .087 |
| 35-39               | .163 | .161 | .148 | .123 | .116 | .103 | .098 | .085 | .077 | .068 |
| 40-44               | .066 | .065 | .050 | .049 | .043 | .038 | .038 | .035 | .031 | .028 |

Another interesting table relates to first maternities according to duration of marriage for the first 2 years of marriage; it is shown, for instance, that the proportion of marriages where a birth takes place within  $8\frac{1}{2}$  months is 49% for wives aged 18 last birthday at the date of birth, and 12% for wives aged 24 last birthday at the date of birth.

Dr Kuczynski discusses possible inaccuracies in the birth returns. In particular he criticizes the method of basing the annual returns on the births registered rather than on the births actually occurring, since there may be variations in the average lag between occurrence and registration. Though no doubt there is a weakness here in our birth returns for individual years, the total error over a series of years is not likely to be important; for the purpose of population forecasts it is probably more satisfactory to use the fertility experience of a period of years rather than that of a single year, and Dr Kuczynski himself considers that the use of mortality rates derived from the experience of certain years but omitting others is open to objection.

Some half-dozen pages of the *Statistical Review* are devoted to a discussion of the population reproduction rates of England and Wales and the possibilities of a rise in fertility rates. It is calculated that the gross and net reproduction rates in 1938 were .897 and .805 respectively, and the opinion is expressed that a 20 % increase in fertility rates would suffice to prevent a decline in population. It is stated that this might be achieved by (1) an increase of fertility within marriage, or (2) an increase in the marriage rate of spinsters at ages when births are likely to result, or (3) a lowering of the average age at marriage of those destined to marry. Under the second of these headings the Registrar-General contemplates the marriage between ages 20 and 24 of an additional number of women who under present conditions are destined not to marry until after age 40 or not to marry at all; Dr Kuczynski points out, however, that additional marriages on the required scale could not be maintained for many years as there would not be sufficient spinsters available. As regards the lowering of the average age at marriage, the Registrar-General examines the effect of marriages taking place 2 or more years earlier among women who at present marry at ages 23 to 30, so increasing the number of wives at the younger ages at which fertility rates are higher as shown in the second of the above tables. Dr Kuczynski here argues that the Registrar-General's calculations are fallacious, since it is not correct to assume that if women are induced to marry at an earlier age they will thereby necessarily experience the same fertility rates as women marrying at that age without such inducement; in this connexion he draws attention to the high proportion of women at the younger ages pregnant at the time of marriage as shown by the percentages quoted above.

C. D. R.

*Britain's Future Population.* By R. F. HARROD.

[Pp. 32. No. H. 4 of *Oxford Pamphlets on Home Affairs*.  
Oxford University Press, 1943. 6d.]

IN this booklet for the general reader the author points out the seriousness of the fall in the birth-rate. The booklet is divided into four sections: The Facts, Causes, The Consequences, and Remedies. It is explained how the size of the population ultimately depends on the net reproduction rate and that a decrease in our population is inevitable unless there is an increase in the size of the average family. The author is unable definitely to diagnose the causes of the present low fertility rates, but considers that financial inducements should prove a remedy. The family allowances under the Beveridge plan would not however be effective for members of the middle and higher income groups where encouragement is specially desirable. He therefore suggests augmenting them for persons whose income exceeds £250 a year by a compulsory 'insurance' scheme, the present income-tax allowances in respect of children being abolished. Under this scheme contributions would be payable by men over age 20 at the rate of 1s. in the £ of income with a maximum of £50 p.a.; and benefits would be granted in respect of children after the second according to a scale varying from £30 to £204 p.a. depending on income of father and age of child, half-benefits being granted in respect of the second child.

The pamphlet is lucidly written, but the proposed 'insurance' scheme might with advantage have been more fully explained. The statement that 'if the population came to live on average ten years longer, this... would cause the whole population to increase by one-tenth' does not seem quite correct.

C. D. R.

*Poisson's Exponential Binomial Limit.* By E. C. MOLINA.

[Pp. 92 + iii. New York: Van Nostrand. Lithog. 1942. \$2.75]

IT is believed that theoretical developments in actuarial theory will cause the Poisson law,

$$P(X = x | a) = e^{-a} \frac{a^x}{x!} \quad (x = 0, 1, 2, \dots),$$

to be of increasing interest to those concerned with mortality, sickness and accident statistics. Thus Molina's present collection of tables of the individual and cumulated terms of Poisson's distribution, although designed for use in connexion with quality control, may well prove useful to actuaries.

The standard tables of Poisson terms are Tables LI and LII of Pearson's *Tables for Statisticians etc.*, Part I; these provide six-decimal accuracy for  $a = .1$  (.1) 15.0 and five-decimal values for  $a = 1$  (1) 30. Molina's tables consist of seven-decimal values of the terms for  $a = .001$  (.001) .01 (.01) .3 (.1) .4 and six-decimal values for  $a = .5$  (.1) 15.0 (1) 100. Even with  $a$  as large as 100 there may be differences of a unit in the third decimal place from the corresponding areas of the appropriate normal curve.

It is well known that if individual function values are recorded on punched cards the standard six-counter Hollerith Rolling Total Tabulator can be plugged to print off the successive fifth differences at a rate of 1000 an hour. The reviewer used this method to check Molina's tables of the individual terms from  $a = 40$  to  $a = 100$ . In this range of values of  $a$  the fifth differences of the Poisson series only amount to a few units in the sixth place of decimals, so that any error of 2 or more in the last place retained would have been located at once, and even errors of a unit in the sixth place are very likely to have been detected. Some 20 possible cases of last figure misstatement were tested and, with one exception, were found to be the result of 'forcing' the last figure retained. The exception was the ordinate at  $x = 53$  with  $a = 62$ ; the printed value is .027516, the true value to ten decimals being .0275154996.

H. L. S.

*Birth, Poverty and Wealth.* By RICHARD M. TITMUSS.

[Pp. 118. Hamish Hamilton, 1943. 7s. 6d.]

HAS infantile mortality in England and Wales improved more rapidly in the higher or lower social classes? Mr Titmuss considers that between 1911 and 1930/32 the decline in infantile mortality has been steeper for Classes I and II than for Classes III, IV and V; that the gradient of inequality has tended to increase; and that the infants of the poor are to-day dying in relatively greater numbers than before the 1914 war.

The death rates in the following table (representing deaths under one year of age per 1000 legitimate live births, both sexes) are taken from the *Registrar-General's Decennial Supplements on Occupational Mortality*. Owing to differences

in the classification of occupations the corresponding figures for 1911 may not be strictly comparable.

| Social class of father                       | Death rate<br>1921 | Death rate<br>1930/32 | Decrease | Percentage<br>decrease |
|--|--------------------|-----------------------|----------|------------------------|
| All classes (including unoccupied)           | 79                 | 62                    | 17       | 22                     |
| Class I (professional, etc.)                 | 38                 | 33                    | 5        | 13                     |
| Class II (intermediate between<br>I and III) | 55                 | 45                    | 10       | 18                     |
| Class III (skilled workers)                  | 77                 | 58                    | 19       | 25                     |
| Class IV (intermediate between<br>III and V) | 89                 | 67                    | 22       | 25                     |
| Class V (unskilled workers)                  | 97                 | 77                    | 20       | 21                     |

In order to obtain comparable figures for 1911 and 1930/32, Mr Titmuss has selected certain clearly defined occupations within each class, and has obtained the death rates shown below.

| Social class of father | Death rate<br>1911 | Death rate<br>1930/32 | Decrease | Percentage<br>decrease |
|------------------------|--------------------|-----------------------|----------|------------------------|
| Class I                | 55                 | 32                    | 23       | 42                     |
| Class II               | 90                 | 47                    | 43       | 48                     |
| Class III              | 110                | 55                    | 55       | 50                     |
| Class IV               | 104                | 58                    | 46       | 44                     |
| Class V                | 160                | 84                    | 76       | 48                     |

In both the above tables it will be seen that the fall in the death rate, given in the fourth column, was least for Class I. The fall for Class V was three to four times that for Class I, and, generally speaking, the lower the class the greater the fall. Looking at the proportionate falls in the last column, between 1921 and 1930/32 these were greater for Classes III, IV and V than for Classes I and II. Over the longer period from 1911 to 1930/32, no marked difference in the proportionate falls is apparent between the higher and lower classes; the greatest proportionate fall was for Class III.

These results hardly confirm the author's statements mentioned above. It depends, of course, on what are meant by expressions such as 'steeper', 'gradient of inequality', and 'relatively'. But neither absolutely nor proportionately can it be said that improvement in infantile mortality was more rapid in the higher than in the lower classes. In interpreting the statistics the author attaches great importance to the ratios borne by the death rates for the different classes to the death rate for all classes combined; he overlooks the effect on these ratios of variations in the relative numbers of children in the different classes.

Broadly speaking it appears that over the period from 1911 to 1930/32 there was a general fall in the death rates of between 42 and 50 %, and it would seem that there was probably greater proportionate improvement in mortality in the higher classes during the first half of the period and in the lower classes during the second half. It is to be hoped that the latter trend has continued and will accelerate until the worst has become equal to the best. Admittedly the large disparities in infantile mortality rates shown in the above tables should be a matter of serious concern.

In the course of his study the author examines the death rates for the first year of life according to age at death in months, according to cause of death

(congenital or environmental), and according to district. It would be interesting to know whether the statistics on which the occupational death rates are based are in any way biased by errors in the returns; for instance, occupation of father recorded at birth of child differing from that recorded at death, or child recorded as illegitimate at birth being recorded as legitimate at death.

Since 1930/32 there has been further improvement in infantile mortality rates, though the war caused a setback. But we have a long way to go yet before we can compare favourably with the rates quoted by Mr Titmuss for certain other countries, e.g. Holland with a rate of 37 per 1000 for 1936/38.

C. D. R.

*The Advanced Theory of Statistics.* By M. G. KENDALL.

[Volume I. Pp. 457 + xii. C. Griffin and Co. Ltd., 1943. 42s.]

NUMEROUS books on the methods of statistics have been published in recent years. Besides works intended for the general reader there have been more specialized texts to meet the needs of biologists, agriculturalists, doctors, engineers, economists and actuaries, respectively. The intended readers being for the most part practical men inclined to stop at the mezzanine floor of the mathematical skyscraper, the development of the subject has been elementary and, for the most part, orthodox. In fact, the authors themselves often make no pretence of having studied the highly advanced mathematical treatments which characterize recent advances in theory.

Kendall, however, is a polished mathematician and has set out to provide the first complete text-book on mathematical statistics. His wide acquaintance with English and American (rather than Continental) literature is plain from the lists of references and valuable sets of problems at the end of each chapter. But his very strength is also a source of weakness, for the historical development of the subject as revealed by the successive volumes of *Biometrika* is apt to minimize the importance of the theory of probability as an essential mathematical and epistemological *point du départ*. That Kendall has been influenced by this historical background is evident from his relegation of Probability to Chapter VII, and his suggestion that this chapter can be read after Chapter I if desired is no solution, for the reasons which induced him to adopt his actual chapter numbering have implicitly affected his viewpoint throughout.

Although in the reviewer's opinion this is a misjudgment of the proper position of probability, Kendall has written a very fine text-book on mathematical statistics. The present work is the first of two volumes and forms the basis for the more modern developments of statistical testing, estimation, analysis of variance, etc., which are promised for Volume II. There are as yet no norms of the relative importance of the various portions of the subject, so the author cannot be blamed for showing that his interest lies with particular chapters. Of the sixteen in this book, the last, on rank correlation, is the longest and most original.

From the point of view of the actuary anxious to learn something of modern statistics two serious drawbacks present themselves. The mathematical knowledge required varies considerably from chapter to chapter: few actuaries will be able to manage the important Chapter IV on Characteristic Functions, the results of which are so essential to the understanding of some of the later and easier chapters. Furthermore, there is a good deal of 'dead wood' in the book: the reviewer wonders when 'tetrachoric  $r$ ' and 'biserial  $\eta$ ' were last used in a statistical analysis, and why a student has to face a whole chapter on 'Association

and Contingency' (with six of its seven references written in 1915 or earlier) before learning about Correlation. Kendall has nowhere indicated what method a practising statistician would use.

In a book of over 450 pages slips of pen and printer are bound to occur and this book is no exception; nevertheless, Kendall has produced a very satisfying text-book and work of reference, and his second volume will be awaited with keen anticipation.

H. L. S.

*Les Principes fondamentaux du Calcul des Probabilités et la Théorie de l'Assurance-Maladie.* By JULES DUBOURDIEU.

[Pp. 196 + xx. Paris: Gauthier-Villars, 1939.]

ANY student of probability theory who can understand, or gloss over, the intricacies of logistics, Jordan measure, and the geometry of a denumerable infinity of dimensions is likely to be enthusiastic about this brilliant monograph. Its expressed object is to resolve the difficulties felt by Galbrun in his text-book on the mathematics of sickness when, for example, by two different routes, which will be obvious to readers, he arrived at the following expressions for  $\kappa_{aa}(x, y)$ , the infinitesimal probability of recovery at age  $y$  from the last of one or more sicknesses occurring since healthy age  $x$ :

$$\kappa_{aa}(x, y) = \kappa_{aa}^{(1)}(x, y) + \int_x^y \kappa_{aa}^{(1)}(x, \xi) \kappa_{aa}(\xi, y) d\xi \quad (1)$$

$$= \sum_{n=1}^{\infty} \kappa_{aa}^{(n)}(x, y), \quad (2)$$

where  $\kappa_{aa}^{(n)}(x, y)$  is the infinitesimal probability of recovery at age  $y$  from the last of exactly  $n$  sicknesses suffered between  $x$  and  $y$ , and is defined by

$$\kappa_{aa}^{(n)}(x, y) = \int_x^y \kappa_{aa}^{(n-1)}(x, \xi) \kappa_{aa}^{(1)}(\xi, y) d\xi.$$

For (1) is an integral equation of the Volterra type and its solution is known to be (2) under certain conditions,\* but a ready-made solution in terms of probabilities must be regarded with suspicion. Dubourdieu is able to show that Galbrun has not based his theory on a minimum of hypotheses and that had he done so (2) and (1) would both have appeared as mathematical consequences of the same basic assumptions.

But it seems to the reviewer that the real importance of the book lies in the author's clear distinction between axioms necessary for the mathematical development and those intuitive 'definitions' which one is prone to make in the application of mathematics to a well-worn practical subject. The meaning of 'independence in probability' is particularly well explained.

This book can be warmly recommended to mathematically inclined readers.

\* See, for example, Whittaker and Watson, *A Course of Modern Analysis* (Cambridge, 1927), Chapter xi.

H. L. S.