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REVIEWS

Reports of the Unemployment Insurance Statutory Committee on the Financial Condition of the Unemployment Fund.

[Seventh General and Fourth Agricultural Reports, as at 31 Dec. 1939. Pp. 27. H.M. Stationery Office. 1940. Price 6*d.*]

THE Report on the General Account as at 31 December 1939 is dominated by the uncertainty as to what will be the extent of unemployment after the end of the war.

In 1939 the revenue in the General Account exceeded the expenditure by over £16,000,000, but the Committee do not regard this as a disposable surplus.

Hitherto the Committee have adopted the principle of adjusting the finance of the Unemployment Fund with a view to equating income and expenditure over the period of a trade cycle, for which they have assumed an average length of 8 years (see Review of previous Reports, *J.I.A.* Vol. LXX, p. 249). They indicate that in their opinion a period of 8 years from the present time will cover the war and a substantial depression thereafter.

The burden of unemployment at the beginning of the present war was materially greater than at the beginning of the war of 1914–18, but the initial rise in unemployment due to the outbreak of war and the subsequent decline in the period covered by the Report were smaller. The Committee consider that there is every reason for expecting that, as the war proceeds and men are called to the fighting forces and to the munition trades, the rate of unemployment will fall to a level which it would not reach in time of peace.

But in the opinion of the Committee it is "as certain as any prophecy can be" that the end of the present war will be followed, as the end of the war of 1914–18 was, by severe unemployment, whatever action it may be possible to take to keep it within bounds; and as the expected boom in employment during the war will enable almost all employable people to acquire claims to unemployment insurance benefit the cost of their unemployment after the war will for a time fall almost entirely on the Unemployment Insurance Scheme and not on the Unemployment Assistance Board.

The Unemployment Fund (which, in spite of its name, is concerned only with unemployment insurance benefit and not with unemployment assistance allowances) is therefore likely to have to meet heavy claims within a short period after the end of the present war, but any close forecast of its financial working, even in the years immediately ahead, is impossible.

At the end of 1939 the Fund had in hand, on its General Account, £57,000,000, and also (if it should be necessary in the near future) the call on the greater part of £23,000,000 (allocated to repayment of debt on the recommendation of the Committee in their two previous Reports). The debt at the end of 1939 was just over £77,000,000. On this occasion the Committee recommend that £37,000,000 out of the £57,000,000 in hand should be allocated to repayment of debt, subject to the rights as to re-borrowing which were given, on the Committee's recommendation, in the Unemployment Insurance Act, 1938.

By this repayment the net immediate gain in the income and expenditure account will be about £1,100,000 a year, and the Committee recommend that this should be used to provide an additional benefit of 1*s*. a week in respect of each of the first two dependent children, with the proviso that the rates of benefit, as they will now be established, must be regarded as temporary and subject to revision if the Fund should prove unable to maintain them.

The Agricultural Account had in 1939 a substantial excess of income over expenditure, and in the opinion of the Committee the rate of unemployment in agriculture is likely to be less affected by the war than the rate in the general scheme. The Committee are in favour of utilising the surplus eventually to make the insurance scheme cover a larger proportion of the unemployment in agriculture than it does at present, but they do not consider this practicable during the war. They recommend the same increases in the rate of benefit for dependent children under the agricultural scheme as under the general scheme, and in the agricultural scheme where there is a maximum limit of total benefit they also propose that this limit should be raised from 33*s*. a week to 35*s*. a week.

It seems unfortunate that, since the outbreak of war, the Gazette published monthly by the Ministry of Labour no longer shows separately the number of unemployed persons entitled to unemployment insurance benefit. As a war-time economy, that number and the number entitled to unemployment assistance allowances are shown combined in one total, so that the proportion of unemployed entitled to insurance benefit from time to time can no longer be obtained from the published figures. The Statutory Committee in their Report give "estimates" of the proportion, but do not say on what basis the estimates have been calculated.

In years to come, when it will once more be necessary to make forecasts of the future income and expenditure under the Unemployment Insurance Scheme, the probable proportion of the unemployed who will be entitled to insurance benefit will be as important a factor as the estimated future rate of unemployment itself. It is true that, in the best of times, it is difficult to forecast changes in the proportion (e.g. the great increase in the proportion in 1938), but the only basis of forecasting must be a study of the variations actually experienced. It is to be hoped that no information which is likely to be of value in the future practical working of the Unemployment Insurance Scheme will be allowed to slip away into oblivion.

H. B.

Erster Nachtrag und Zweiter Nachtrag zu den Rechnungsgrundlagen für Pensionsversicherung. (First and Second Supplements to the Valuation Tables for Pension Insurance.) By WALTER MEEWES and WALTER MEISSNER.

[Published in 1938 and 1939 respectively by Neumanns Zeitschrift für Versicherungswesen, Berlin.]

THE original tables were published in 1936 and reviewed in *J.I.A.* Vol. LXVIII, p. 433. The supplements give the same range of commutation columns and other functions at $3\frac{1}{2}\%$, 4% and $4\frac{1}{2}\%$ as in the original tables, but after modification of certain of the bases employed. The supplements have been made possible by the publication of further experiences of the German Imperial Insurance Institute for Employees.

The first supplement substitutes the mortality amongst invalids according to the German Imperial Insurance Institute (Males) 1934–36 for the National German Life Tables (Males) 1924–26. The authors' earlier statement that the choice of the 1924–26 basis would be considered as too careful has been justified, for the rates of mortality derived from the new experience are much in excess of the old, being about fifty times as great at age 20 and nearly twice as great at age 60. The resulting annuity at $3\frac{1}{2}\%$ to an invalid at age 20 is about one-quarter of the value on the earlier basis, the difference decreasing to nil at age 65, from which age the National German Life Tables 1933 are used as before. The authors express the opinion that, in suitable cases, the future mortality will lie between the limits of that employed in the original tables and that emerging in the first supplement.

The second supplement employs revised rates of exit on account of "incapacity for work" and revised rates of mortality amongst actives. The new rates of exit are markedly lower than the old for ages 25 to 64. The new rates of mortality among actives are not markedly different from the old after age 30. The combined effect on an annuity at $3\frac{1}{2}\%$ to an active so long as he remains such is a slight increase in the value never exceeding six months' purchase.

The tables in the second supplement embody the modifications made in both supplements. The net effect is to decrease the value of an annuity to a man now active upon retirement at 65 or on earlier incapacity for work, and to increase the value of an annuity to the widow of a man now active.

W. F. G.

Tables for converting Rectangular to Polar-Co-ordinates. By J. C. P. MILLER, Ph.D.

[Pp. 16. Scientific Computing Service Ltd., 1939. Price 2*s.*]

THE tables give $\sqrt{1+k^2}$ (to 4 places of decimals), $\tan^{-1} k$ and $\cot^{-1} k$ in degrees (to 2 places of decimals), and $\tan^{-1} k$ in radians (to 4 places of decimals) for values of k from 0 to 1 proceeding by differences of $\cdot 001$. An introduction explains how to use the tables. With the usual notation, if x and y are both positive, then $r = x\sqrt{1+y^2/x^2}$, $\theta = \tan^{-1} y/x$ or $r = y\sqrt{1+x^2/y^2}$, $\theta = \cot^{-1} x/y$, according as $x \gtrless y$. In other cases the tables are used similarly but $\tan^{-1} k$ or $\cot^{-1} k$ requires to be increased by a multiple of 90° according to the signs of x and y. C. D. R.

The Probability Integral. By W. F. SHEPPARD.

[Pp. 34+xi. Vol. VII of British Association Mathematical Tables. Cambridge University Press, 1939. Price 8*s.* 6*d.*]

THE computation of the tables in this book was commenced by Dr W. F. Sheppard, and since his death in 1936 they have been completed and checked under the auspices of the British Association.

The probability integral is $\dfrac{1}{\sqrt{2\pi}}\displaystyle\int_x^\infty e^{-\frac{1}{2}t^2}dt$, i.e. the area bounded by the normal curve of error, the x-axis and an ordinate; and the tables enable values of this integral to be found. The first two tables give the ratio of the integral to the bounding ordinate, and the remaining four tables give the negative natural or common logarithm of the integral. In all the tables the argument runs from 0 to 10 by intervals of 1, $\cdot 1$ or $\cdot 01$, reduced derivatives or second central differences being given where required for interpolation. The values of the function are tabulated in two of the tables to as many as 24 places of decimals.

The Introduction by J. O. Irwin explains the method of construction of the tables, gives some illustrations of their use, and describes other tables published previously. C. D. R.

Statistical Mathematics. By A. C. AITKEN.

[Pp. 153. Oliver and Boyd, 1939. Price 4*s.* 6*d.*]

THE theory of probability—the basis of modern statistical methods—has of recent years made such advances, philosophically and mathematically, that the English actuary will soon have to decide how these developments affect his interpretation of mortality statistics and the theory of life contingencies which he has based upon it. Is he, for instance, to regard the intricate, and often fundamentally difficult, problems of Part I of the examinations (balls in a bag, target practice, dice throwing and the like) as a mere training in precise thought, or can he find some analogy between these problems of imagination and the combinations of idealized relative frequencies with which he has to deal in office practice? And he should resolve, further, whether to follow the statistician into the elaborations of his theory. Is it reasonable, for example, to devote attention to the calculation and interpretation of measures of correlation? Evidence that the subject is at present of no importance to the actuary is found in the rare articles on correlation in actuarial publications; the authors are invariably hard put to find an example illustrating the theory in its application to actuarial statistics.*

Whilst Dr Aitken's book cannot answer these questions for him, it can assist the actuary by giving him some idea of the trends in mathematical statistics. He can learn that the interpretation of statistics depends essentially upon the theorems of probability; that the choice of a measure to typify an observed distribution depends more upon the characteristics of this distribution than upon any criteria determined *a priori* from a list of properties which it is considered desirable that such a typical measure should possess—in particular that the mean of a set of observations may be no more reliable than any one observation; that zero correlation does not necessarily imply independence in the probability sense; and numerous other details. But although the mathematics required are well within the limits of the syllabus of the Preliminary examination, the book postulates a maturity of thought which makes it unsuitable for the beginner. Each sentence has been carefully weighed and appraised and Dr Aitken has succeeded in providing the maximum of information in the minimum of space; like the mathematical text-book, it needs perseverance or an accompanying series of lectures for its full appreciation.

As far as we know this is the first book written in English in which a systematic development of the theorems of mathematical statistics has been made by means of moment generating functions. Given a random variable which consists of a combination of other, simpler, random variables, it is often very easy to derive the corresponding generating function and then, by means of an integral equation, obtain the distribution function of the given complex variable. However, some of the standard random variables give rise to integrals which are extremely difficult to negotiate (e.g. the sum of the squares of k normal variates

* For example, F. Esscher, "On a method of determining correlation from the ranks of the variates", *Skand. Aktuar. Tidskrift*, Vol. VII, p. 201, or P. Riebesell, "Die Bedeutung des Korrelations-Koeffizienten für Theorie und Praxis der Versicherung", *Blätter für Versicherungs-Mathematik*, Vol. I, Part 3.

leads to a very awkward integration) and when the research student attempts to apply the theory to novel variables the resulting integral equation is often heartbreaking.

The author adopts what Neyman has called the modernized classical theory of probability, but mentions in a summary and rather unsatisfactory way the attempt of von Mises to base probability theory on relative frequencies alone. His attempt to bridge the gap between abstractions and the concrete data of observation is not very satisfying.

The book is entirely mathematical in character but occasionally Dr Aitken works through a numerical example. He fits a Type A and a Type B curve, he calculates a coefficient of correlation and a correlation ratio, illustrates the method of fitting a cubic by means of orthogonal polynomials, gives a short example of harmonic analysis, and briefly indicates the procedure to be adopted in a straightforward analysis of variance. So it appears disproportionate when the author shows a numerical application, not only of the usual method of calculating moments, but also of the summation method, (i) with the origin at one end of the distribution, (ii) with the origin at the centre of the distribution, and (iii) using central instead of the ordinary factorial moments. Of the summation methods, practical computing preference would be given to that utilizing a central origin but *not* in the way Aitken advocates, as this requires larger numbers and six more additions than the corresponding method demonstrated by Elderton on pp. 21–4 of his *Frequency Curves and Correlation*.

A few misprints were observed and we have some minor points of criticism. The apparently simple derivation of the normal curve as a limit to the point binomial by means of moment generating functions is deceptive, for it requires the solution of a difficult integral equation for its completion. The meaning of "fiducial interval" is poorly explained, and the definition of $erf(z)$ given on p. 62 is not that usually adopted [see e.g. H. Jeffreys, *Theory of Probability* (1939)].

To the serious student who is prepared to do more than scratch at the surface of a difficult subject, the book is a scholarly, and, in its way, complete, introduction to that "advanced" statistical technique which is so fascinating to its adepts and so irritating to its detractors.

H. L. S.

Number-Divisor Tables. By J. W. L. GLAISHER.

[Pp. 100+x. Vol. VIII of British Association Mathematical Tables. Cambridge University Press, 1940. Price 15*s*.]

THESE tables, commenced by J. W. L. Glaisher and completed after his death, will be of value in connexion with the theory of numbers and allied subjects. The first table gives, for every integer n up to 10,000:

(*a*) its decomposition into prime factors; (*b*) the number $\phi(n)$ of numbers not exceeding n and prime to it; (*c*) the number $\nu(n)$, and (*d*) the sum $\sigma(n)$, of the divisors of n. Further tables set out the values of n corresponding to $\phi(n)$, $\nu(n)$ and $\sigma(n)$ respectively as the argument.

C. D. R.

The Stock Market, Credit and Capital Formation. By FRITZ MACHLUP.

[Pp. 416 + xii. London: William Hodge, 1940. Price 21*s*.]

THE book is a very much revised edition of a book published in German in 1931. The translation into English has been very ably done by Dr Vera C. Smith.

It is not a factual description of the Stock and New Issue Markets, but contains a great deal of theoretical speculation. The majority of English books on this subject are of the former type and Professor Machlup's book is therefore welcome. It is an attempt to link up the Stock Market with the theory of capital formation, the theory of money, and hence with crucial problems of trade-cycle theory. The book reads rather like a treatise on money which gives undue weight to the problem of the Stock Exchange. This latter fact is rather irritating to the English reader, but it must be remembered that Professor Machlup is concerned mainly with American conditions and the first edition was written shortly after the famous Wall Street boom of 1929.

One very important difference between Wall Street and the London Stock Exchange is that in the American system all transactions are settled every day whereas in London, apart from the gilt-edged market, transactions were (until the outbreak of war) settled fortnightly and exceptionally, three-weekly. Speculators in New York who have bought in the hope of profiting from rising prices have consequently to borrow continuously, whilst those in London have to overcome the settlement days only. The result is that in New York the most important opening for short-term funds has been in loans to the Stock Market. In the appendices to the book Professor Machlup gives statistics of brokers' borrowings in New York City. The following figures give some idea of the wide fluctuations in these borrowings. In January 1927 the amount was 3139 million dollars; in September 1929, 8549 million dollars; and in July 1932 only 242 million dollars. These figures coupled with the boom and the disastrous slump inevitably led many commentators to criticize this system whereby loans for speculative purposes could be obtained so easily, and further, it was stated, mainly by representatives of agricultural interests, that the Stock Exchange was depriving industry of legitimate credit.

Professor Machlup is at great pains to refute these suggestions. He shows that there is no definite relation between the volume of loans to stockbrokers and funds 'absorbed' by the Stock Exchange, and he points out that whenever money is spent by a speculator in the purchase of a security, somebody else, selling the security, receives money and sooner or later the money finds its way into new issues. The 'sooner or later' forms the real problem, and although Professor Machlup does consider the question of time-lags, he dismisses them, contrary to the verdict of other economists, as being of little importance.

The author's greatest fear seems to be that increases in bank credit may bring about such changes in the structure of production that a crisis will become inevitable. In order to deal with the problem of bank credit it is necessary that short-term funds should be carefully distinguished as to their sources and nature. In a private enterprise economy this is impossible, but the author does not suggest any remedy; he is not concerned with what economic policy ought to be.

The book has little to say about the effect of a Stock Exchange boom on the propensity to consume, nor does it consider the influence of fluctuations in security prices on the even flow of money capital into industry. These questions are of great importance in connexion with the stability of the economic system, and they have received much attention from English economists under the leadership of Mr Keynes.

It is not easy to combine descriptive and theoretical economics into a well-balanced whole, particularly if the author is afraid to discuss economic policy at length lest he should be accused of descending from the heights of pure theory to the world of political controversy. However, Professor Machlup has written an able book within the limits which he has set himself, but it has been written for the theoretical economist rather than for the ordinary investor.

H. W. H.

Valeur Pratique et Philosophie des Probabilités. By ÉMILE BOREL.

[Pp. 182+ix. Paris: Gauthier-Villars. 1939.]

NUMEROUS treatises have been written during this and the preceding century discussing the meaning of probability and its susceptibility to measurement. As might be expected, the wider the preliminary definitions the more difficult it is to base a mathematical theory on it and to induce therefrom "rules of conduct". The actuary, from the very nature of his training and the reason therefor, prefers a theory which is based on observable frequencies or can be readily applied to such frequencies, and regards other theories of probability as outside his purview. Borel, however, adopts the view that measures of probability may be obtained other than by consideration of relative frequencies, although he does not subscribe to Keynes's view that some probabilities are inherently incapable of measurement. For these reasons the portions of the present book dealing with the possible interpretations of the probability to be ascribed to an isolated case can have only a passing interest for the actuary. Few of us, I think, would agree with the author when he writes that there is no doubt that "*des médecins habiles pourraient gagner des sommes importantes au détriment de compagnies d'assurances en constituant à leur profit des rentes viagères sur la tête de personnes qu'ils auraient pu examiner avec assez de soin pour être certains que leur probabilité de vie est notablement supérieure à la moyenne des personnes du même âge*".

Borel's brochure forms the eighteenth and last part of the extensive treatise on the calculus of probability and its applications which has been in preparation under his editorship for nearly 15 years. Progress in the subject has been so rapid that already four separate monographs have been issued as a supplement and a fifth is in preparation. Anyone familiar with the author's classic contributions to the theory of functions will not anticipate any difficulty in understanding the contents of this book but he will, perhaps, be prepared for the discursive and somewhat prolix style.

An interesting argument is developed to justify the statement that an individual should constantly neglect probabilities of an order less than 10^{-6}, that probabilities of an order less than 10^{-15} should be considered as negligible by the ensemble of humans who people the earth, and that a probability inferior to 10^{-50} represents absolute impossibility. An example of the latter is afforded by Borel's now famous fantasy of the army of monkeys who, by tapping on the keys of numerous typewriters, will ultimately reproduce all the works in the British Museum. Russel Maloney has cleverly parodied this by telling of a professor of mathematics who, in the interest of science, shoots six experimental chimpanzees which, in five months, had typed Oliver Twist, the works of Vilfredo Pareto in Italian, some of the prose of John Donne, some Anatole France, Conan Doyle, Galen, the collected plays of Somerset Maugham and numerous other works.

A chapter of the book is devoted to the discussion of various well-known paradoxes, for example, Laplace's calculation of the probability that the sun will rise on the morrow, the St Petersburg problem, Condorcet's application of probability theory to judicial decisions, etc., and another is given over to the superstitions of gamblers. A further chapter discusses some applications and some limitations of set-point theory in the calculus of probabilities. Only the latter will be new to readers of Bertrand, Poincaré, Czuber, etc. H. L. S.

The Variate Difference Method. By GERHARD TINTNER.

[Pp. 175+xiii. Principia Press, Inc., Bloomington, Indiana, 1940.]

IT would not be unfair to say that when comparison is made with recent developments of statistical technique in agriculture, biology, and psychology, the mathematical treatment of economic observations is still in its infancy. As Neyman pointed out in his Lectures at the Graduate School of the U.S. Department of Agriculture, nearly all the methods applied by economists have been empirical in the sense that they seek to substantiate hypotheses made *a posteriori* from the actual results of economic machinery rather than to formulate hypotheses relating to the machinery itself. Among these empirical procedures must be numbered the variate difference method which does not attempt to interpret economic processes but merely provides a means of distinguishing between the "permanent" and "non-permanent" causes affecting an economic series ordered in time.

The statistical theory underlying the variate difference method is simple enough. Observations of some economic phenomenon have been made at equidistant intervals of time. It is assumed that the variations of this series may be analysed into two additive portions, a part due to "permanent causes in economic life" and a part due to a random component whose distribution is independent of time. Advantage is taken of the fact that almost any mathematical function, except a periodic function of very short period, yields negligible differences of high degree, and it is argued that after a time series has been differenced a few times the resulting series of differences are approximately the same as if the original random component alone had been differenced. The variate difference method proper consists in the construction of statistical tests to decide at which stage the functional component has been removed. A smoothing formula may then be chosen to reduce the variability of the random component, but not to affect the functional item; a suitably constructed summation formula would serve here, but the author prefers formulae fitted to the observations by means of least squares (W. F. Sheppard, *J.I.A.* Vols. XLVIII and XLIX). It is well known that such a process does not remove the random element entirely, but produces a series that probably approximates closely to the true underlying values. The economist may then attempt to interpret the resulting smoothed series and, perhaps, project it into the future.

Symbolically, if y_t be the tth observation of the series, $\pi_n(t)$ represents a polynomial of the nth degree or some other mathematical function whose nth differences are nearly constant, and Z denotes a random variable of zero mean, normally distributed independently of time, then by hypothesis

$$y_t = \pi_n(t) + z,$$

where z is a sampling value of the random variable Z. The actuary will notice that although superficially analogous to the "set-up" underlying a summation graduation of a series of observed mortality rates yet a difference arises, since he would write

$$q_x = \pi_3(x) + z_x,$$

where z_x is distributed approximately normally about zero with a variance, $p_x q_x / E_x$, which is essentially dependent on x.

Now, writing z_t for the tth sample value of Z,

$$\begin{aligned}\Delta^{n+r} y_t &= \Delta^{n+r} z_t \quad (r = 1, 2, 3, \ldots) \\ &= (E-1)^{n+r} z_t \\ &= z_{t+n+r} - \binom{n+r}{1} z_{t+n+r-1} + \ldots + (-1)^{n+r} \binom{n+r}{n+r} z_t,\end{aligned}$$

and hence, if the true variance of Z is σ^2, the variance of the $(n+r)$th series of differences is

$$\left[1^2 + \binom{n+r}{1}^2 + \ldots + \binom{n+r}{n+r}^2\right] \sigma^2 = \binom{2n+2r}{n+r} \sigma^2.$$

If, therefore, each member of the jth $(j > n)$ series of observed differences is divided by $\sqrt{\binom{2j}{j}}$, the resulting series should possess a variance which is a random sample value of the true variance σ^2. Unfortunately the successive series of observed differences are not independent of one another so that it is unjustifiable to test s_j^2, the estimate of σ^2 derived from the jth series of differences, against s_{j+1}^2 for significant inequality by means of Fisher's z-test. There seem to be two alternative means of escape from this difficulty. The first is due to Oskar Anderson, who calculated the variance of the distribution of $s_j^2 - s_{j+1}^2$ on the hypothesis that both members are estimates of a true variance σ^2; if the actually observed difference $s_j^2 - s_{j+1}^2$ exceeds two or three times its estimated standard deviation, the hypothesis of the equality of the variances of which s_j^2 and s_{j+1}^2 are the respective estimates, is considered disproved. The obvious criticism that the difference $s_j^2 - s_{j+1}^2$ divided by its estimated standard deviation is not distributed normally, invalidates the use of the factors 2 or 3 and must be counted heavily against this alternative.

Recently the author himself developed a useful alternative method. Suppose it is desired to test the observed series $\Delta^3 y_t / \sqrt{\binom{6}{3}}$ and $\Delta^4 y_t / \sqrt{\binom{8}{4}}$ for evidence of the previous elimination of the function $\pi_n(t)$. The first term of the former series is derived from y_1, y_2, y_3, and y_4, whilst the fifth term of the latter series is derived from y_5, y_6, y_7, y_8, and

y_9; hence by forming an estimate $s_3'^2$ by using the first, tenth, nineteenth,... terms of the series $\Delta^3 y_t \Big/ \sqrt{\binom{6}{3}}$, and an estimate $s_4'^2$ by using the fifth, fourteenth, twenty-third,... terms of the series $\Delta^4 y_t \Big/ \sqrt{\binom{8}{4}}$, two completely independent estimates of σ^2 are obtained and, provided that the random residuals are normally distributed, may be tested for significant difference by the usual z-test. Tintner acknowledges that his suggestion involves the serious curtailment of the effective number of degrees of freedom available to test two of the observed series of differences for significant departure from the hypothesis of a uniform value of σ^2, but is not concerned at the multiplicity of possible tests arising, not only with respect to the freedom of choice of the first term to employ (for instance in the above example the second, eleventh,... terms of Δ^3 could be compared with the sixth, fifteenth,... terms of Δ^4) but also in the number of comparisons to make. Judging from his Table 31, successive pairs of series of differences up to the tenth order should be compared before the author would be satisfied that second differences are unaffected by the mathematical function of the time elapsed. Since the successive statistical tests are highly correlated with one another, such a procedure appears of doubtful validity. As a further alternative a test may be proposed which to some extent obviates this latter difficulty. If it is suspected that second and higher differences arise only from the random element, the first three observations may be utilized to form a second difference, the next four observations to form a third difference, the next five to form a fourth difference, and so on until about one-fifth of the original series has been exhausted. The procedure is then repeated and results in several series of differences each possessing about five terms. These series are then reduced in the usual way by division by the appropriate factor $\sqrt{\binom{2j}{j}}$ and the several estimates of σ^2 formed. The whole set of k (say) variances is then tested simultaneously for homogeneity (see Bishop and Nair, *J. Roy. Statist. Soc. Suppl.* Vol. VI, p. 89).

Now the calculations involved in any of the foregoing methods are considerable and it is of importance to decide whether, in fact, they are necessary or even desirable. It has already been mentioned that many mathematical functions subject to wave-like variations, and thus resembling the general trends of many economic series, possess differences which decrease in numerical value as the order increases. The effect of dividing these differences by the appropriate reduction factor $\sqrt{\binom{2j}{j}}$ is to reduce them still further so that they rapidly become negligible in comparison with the random element. The reviewer is of the opinion that it would be difficult to construct successive overlapping polynomials that displayed a general similarity with some actually observed economic series but which would not possess negligible differences of the fourth degree after these had been divided by $\sqrt{\binom{8}{4}}=\sqrt{70}$. As an example a fifth degree polynomial, $\pi_5(t)$, was fitted to six equidistant points of the series chosen by Tintner to illustrate the variate difference method, namely the annual American wheat-flour prices from 1890 to 1937, after those six prices had been reduced by the six corresponding values from 48 successive normal variates of zero mean and standard deviation ·65 (a similar value to that resulting from Tintner's investigations on the series concerned). The resulting series y_t was of very similar appearance to the original series of wheat-flour prices and, after division by the appropriate values of $\sqrt{\binom{2j}{j}}$, the following variances resulted:

	Order of Difference				
	1	2	3	4	5
Variance of $\pi_5(t)$	·462	·017	·000	·000	·000
Variance of z_t	·318	·307	·299	·291	·280
Co-variance	−·097	−·007	·001	·000	·000
Variance of y_t	·683	·317	·300	·291	·280

It is clear that even second differences of $\pi_5(t)$ can have no influence on any statistical tests devised to distinguish between the variances of the given series y_t. The implication is that the variate difference method is not sensitive enough to serve the purpose for which it was devised.

The practical procedure of employing the variate difference method is fully and lucidly explained by the author and the mathematical basis is clearly set out in a number of Appendices. But the following points appear to justify criticism: (i) the word "nonapplicable" used on page 17, (ii) the use of first difference interpolation in Table 10, (iii) the over-detailed analysis of a simple series set forth in Table 31, (iv) the anomaly of calculating the sum of the eighth powers of the tenth differences to *twenty-seven* significant figures and applying to this figure the value of $\left[\binom{20}{10}\times(48-10)\right]^{-1}$ to *three*, (v) the nomination of W. F. Sheppard as the originator of smoothing formulae after Wolfenden's pleas in favour of De Forest (*T.A.S.A.* Vol. XXVI, p. 81).

H. L. S.

The birthplace and genesis of life assurance. By J. G. ANDERSON, M.A.

[Pp. 104 and 7 plates. London: Frederick Muller, Ltd., 2nd edition, 1940. Price 5*s*. Original edition reviewed, *J.I.A.* LXIX, p. 100.]

IT had been intended to expand this book into *The birthplace, genesis and pioneers of life assurance*, but this ambition was frustrated by the present war.

The second edition gives, however, notes on the five fellows of the Royal Society—Halley, De Moivre, Simpson, Dodson and Montaine—whom the author regards as the pioneers of life assurance up to the year 1762, after which date their work was continued by Price, Morgan and others. It gives also a complete list of the "Old Equitable's" Actuaries and Actuarial advisers from James Dodson, senior, in 1756 to the present time.

The book has been greatly improved by the addition of seven plates, including a hitherto unpublished likeness of William Morgan who is described as "possibly the greatest personality ever connected with life assurance". M. E. O.