

## REVIEWS

*The Investment of the Funds of Social Insurance Institutions.* (Studies and Reports—Social Insurance—No. 16.)

[Pp. v+196. 5s. International Labour Office (League of Nations), Geneva, 1939.]

THIS is an account of an interesting effort in international collaboration. The International Labour Office of the League of Nations (referred to in this book and in this review as 'the Office') made plans during several years before 1939 for the preparation of a series of studies of various aspects of Social Insurance. One such study was planned to deal with the Investment of Social Insurance Funds.

The Office prepared the way by making a comparative study of the regulations in various countries governing the investment of these funds. Experts in social insurance in various countries were then consulted in three stages as follows:

- (1) At meetings spread over three days in Geneva in December 1937, experts from various countries with particular competence in the investment of social insurance funds drew up a questionnaire intended to elicit opinions on the essential aspects of the subject.
- (2) On the basis of this questionnaire the members of an International Correspondence Committee on Social Insurance, which had been set up by the Governing Body of the Office, were consulted in writing. The Office analysed the replies, and prepared a series of recommendations based on them.
- (3) The analysis and recommendations were then placed before the investment experts at a series of meetings spread over five days in Geneva in December 1938, and taking the recommendations of the Office as a basis the experts drew up a series of conclusions intended to bring out the main problems involved in the investment of social insurance funds and possible ways of solving these problems.

At the meeting in December 1937, experts were present representing Austria, Belgium, Czechoslovakia, France, Great Britain, Hungary, Luxembourg, the Netherlands, Poland, Roumania, Sweden and the United States of America; while at the meeting in December 1938 experts were present representing Argentina, Belgium, Brazil, Bulgaria, France, Great Britain, Greece, Hungary, Jugoslavia, the Netherlands, Poland, Portugal, Sweden, Switzerland and the United States of America. There were also present at the meetings representatives of the Office, the League of Nations, and the Bank for International Settlements, Basle. The British representative at both series of meetings

was Mr E. Hackforth, C.B., Controller of the Insurance Department, Ministry of Health.

The aspects of the subject which were considered were: basic principles of regulations as to investments of social insurance funds, general conditions to be satisfied by investments (safety, yield, liquidity, social and economic utility), classes of investments to be regarded as permissible, possible guarantee by public authority against depreciation in the value of assets or of their purchasing power, and the composition of bodies responsible for selecting investments.

For each point considered there are set out:

- (a) notes on the practice of different countries,
- (b) the questions which were included in the questionnaire,
- (c) a brief analysis of the replies from the correspondents,
- (d) the conclusions recommended by the Office,
- (e) a summary of the discussion by the experts, and
- (f) the experts' conclusions.

It may be noted that the experts brought the touch of practical experience which was necessary to control the provisional recommendations of the Office. For example, it was reported that in Argentina and Brazil the regulations permitted social insurance funds to be invested in loans on personal security to insured persons, and the Office recommended that such loans should be permitted, but the experts decided to delete all reference to them in their recommendations as to permissible securities. As representatives of Argentina and Brazil were present, a report of the discussion on this point might have been interesting, but none is given.

A fuller illustration of the method of working may be found in the account of the consideration given to the question of investment in variable-yield securities, particularly ordinary shares (unfortunately described on p. 118 as 'investment in invariable-yield securities'). The Office was evidently much impressed with the possibility that currency might depreciate to such an extent that the purchasing power of the benefits payable to insured persons might become insufficient to realize the objects of the scheme, and suggested for consideration the question whether a means of meeting this difficulty might be found in investment in ordinary shares.

In answer to objections that losses might be incurred by investment in ordinary shares, the Office brought forward the well-known arguments of Professor Irving Fisher and Mr E. L. Smith of New York, to the effect that in certain periods investments in ordinary shares had proved not only more profitable but also more stable both in income and in capital value than investments in fixed-yield securities, but the experts were more cautious than the Office, and their approval of investment in ordinary shares was very limited.

Their eventual recommendation on this question was:

'Ordinary shares or stock of commercial or industrial companies should only be entertained as possible investments for insurance funds if the interest or dividend and redemption of capital are sufficiently guaranteed. Only the shares or stock of companies enjoying long-dated concessions or monopolies would appear to fulfil these conditions.'

The general meaning of this recommendation is fairly clear, but the wording seems to give an indication of the difficulties which beset persons, even 'experts', speaking different languages and not very familiar with the practice of countries other than their own, when they try to make joint recommendations on complex problems.

The International Labour Office is to be congratulated on having successfully organized what is described as 'the first study on an international plane of the investment policy of social insurance institutions,' and the efforts of all those who took part were unquestionably a valuable contribution towards international understanding.

H. B.

*Actuarial Technique and Financial Organization of Social Insurance (Compulsory Pension Insurance).* By LUCIEN FÉRAUD. (Studies and Reports—Social Insurance—No. 17.)

[Pp. vi+568. 15s. International Labour Office (League of Nations), Geneva, 1940.]

THIS book offers to serve as an introductory guide to the financial organization, including the underlying statistics and actuarial technique, of certain schemes of social insurance in six countries, viz. Belgium, Czechoslovakia, France, Germany, Great Britain, and Italy, up to the end of 1937.

The social insurance schemes which are dealt with primarily are those in which the benefit is a pension. Other social insurance benefits, such as sickness insurance, are only mentioned incidentally.

The book was planned several years ago by M. Adrien Tixier, at that time Chief of the Social Insurance Section of the International Labour Office, and the work was carried out by M. Lucien Féraud, the Actuary of the Social Insurance Section. The book was originally published in French; the English translation was made by Mr W. E. P. Loraine, F.I.A., of the British Government Actuary's Department.

The information given has been compiled from the official publications of the different countries, and is presented without comment. In bringing together a great deal of up-to-date official information on certain aspects of social insurance in different countries the compiler has done much useful work. An introduction is followed by six separate

studies, one for each of the countries whose schemes are described, and the six studies are drawn up, as far as possible, on a uniform plan dealing in succession with:

- (1) The statistics on which the scheme is based, and the actuarial methods by which the statistics are brought into practical use.
- (2) The rates of contribution and benefit, and State grants.
- (3) The nature of the financial equilibrium intended to be maintained.
- (4) The organization of actuarial and financial control, including the recording of experience.

An account of British disablement benefit is included, but in this section the compiler seems to have been cramped by the plan of the book, which laid down as its primary concern benefits in the form of pensions. It would have been wiser to plan the book so as to deal with social insurance as a whole (for in this country, and to some extent elsewhere, all parts of social insurance are linked together), and to subdivide it according to the contingency covered, e.g. invalidity, rather than according to the form of benefit.

The outstanding schemes described are those of Great Britain and Germany. While the descriptions here given may serve as a useful introduction, the reader who wishes to make a serious comparison between the schemes of the two countries will feel the need of a more critical account, with fuller information as to historical origins and as to the ideas which lie behind the divergent developments of practice.

The German scheme of invalidity pensions appears to have been developed from workmen's compensation. It is in cases of severe accidental injury that the idea of permanent incapacity first arises. In some such cases, after several months, the physical condition becomes stabilized, but with the permanent loss of some important physical organ, which makes it impossible for the injured man to resume his work. For such cases a pension is an appropriate benefit. In the German invalidity scheme this idea was extended to long-continued incapacity arising out of disease. It was assumed that, in cases where treatment over several months failed to bring about recovery, disease, like injury, led to a condition from which, once entered upon, recovery was comparatively rare. A pension was regarded as the appropriate benefit, and the actuarial work in connexion with such a scheme was naturally based on a probability of becoming permanently incapacitated.

The British scheme of disablement benefit, on the other hand, was developed out of the sickness benefits of Friendly Societies. In accordance with the practice of those Societies, even long-continued disablement was regarded as a condition needing constant review. In some cases of severe disease there may be a considerable degree of slow, gradual recovery, while in others there may be successive stages of improvement

and relapse. Even if a man's physical condition becomes stabilized with some permanent loss of function, he may learn to adapt himself to his new condition and eventually regain ability to earn a livelihood, perhaps in a different line of work, and possibly subject to a risk of relapse. With this conception of incapacity, it is natural that the form of benefit granted should be, not a pension, but temporary payments subject to frequent review, that the administration should be closely linked with that of sickness benefit for shorter illnesses, and that the actuarial work involved should be based on an expected average number of weeks' disablement in a year rather than on a probability of becoming permanently incapacitated.

In Germany, as the invalidity scheme developed, it became recognized that there were many recoveries from long-continued invalidity. Periodical reviews of cases of incapacity were instituted, although these reviews were still far from being so frequent as in Great Britain. In the actuarial work it became necessary to investigate the probability of recovery, though for most purposes it was sufficient to deal with the probability of an invalidity pension ceasing in any year from any cause, the rates of cessation being tabulated in select form, according to the age and the number of years elapsed since entering upon a pension. But in the German scheme the emphasis has still been on the need for a permanent benefit in the form of a pension, with recovery regarded as exceptional, whereas in Great Britain greater stress has always been laid on the possibility of recovery. These different fundamental conceptions have influenced the two schemes in a number of ways.

Again, the development of the British scheme has been strongly influenced by the desire to maintain as large a measure as possible of independence for the Friendly Societies. No mere compilation from Blue Books will bring this out adequately, but no student from another country, or even from our own, will understand the organization of British sickness and disablement benefit until this is explained to him.

A prominent place is given in this book to actuarial notation, as used in those countries on the Continent of Europe whose schemes are described. A large number of symbols are well set out and clearly defined, and good use is made of them in the text. The array of German symbols is particularly extensive—much more so than in the Schaertlin prize essays.

There is not the slightest reference to British sickness and disablement notation. This may have arisen from a difference in official practice. In the Continental countries official reports on social insurance often make considerable use of actuarial symbols, whereas in this country the aim has always been to make the official reports intelligible without symbols, except in an occasional appendix. In a book of this kind, however, in which actuarial technique is stated to be one of the main subjects, the compiler would have done better to push his inquiries a

little further, so as to make it clear, to any reader who is unaware of the fact, that there is a British notation and actuarial technique for disablement benefit which, even if it is less complicated than the German, has sterling merits of its own.

One wishes, too, that half the space devoted to notation and to minor details had been used to give a fuller account of the statistical experience on which the schemes are based. The most remarkable features of those statistics which are given are the discrepancies between the experience relating to the grant of invalidity pensions in different countries. The most recent statistics which are given for Germany, France and Italy (in all of which the test of invalidity is stated to be inability, arising from disease or injury, to earn one-third of the earnings of a normal worker in the same occupational group) are as follows:

*Probability of Becoming Invalid*

Number becoming invalid in a year per 1000 insured, according to the experience of the respective State insurance schemes.

GERMANY

Age group	1925-30		
	Men	Women	All Persons
20-25*	0.99	1.22	0.50
25-30	3.15	4.07	3.21
30-35	5.37	6.50	5.47
35-40	5.81	7.83	6.10
40-45	7.38	13.40	8.67
45-50	9.97	18.24	11.89
50-55	17.57	36.24	21.68
55-60	34.72	66.36	42.68

\* There is evidently some error in the figures given for this age group.

FRANCE

Age group	1933-6
Under 20	1.18
21-25	2.02
26-30	1.87
31-35	2.00
36-40	2.12
41-45	2.26
46-50	2.68
51-55	3.51
56-60	5.01

ITALY

Age group	1929-32	1932 only
25-29	0.98	0.86
30-34	1.35	1.49
35-39	1.70	2.35
40-44	2.24	3.47
45-49	3.00	5.17
50-54	4.78	8.29
55-59	8.33	15.16

These figures are given by the compiler in different parts of the book without comment. Among all the matters with which he deals they seem to cry out for further investigation.

H. B.

*Reports of the Unemployment Insurance Statutory Committee on the Financial Condition of the Unemployment Fund:—Eighth General and Fifth Agricultural Reports, as at 31 December 1940.*

[Pp. 14. 3d. H.M. Stationery Office, 1941.]

In the General Account of the Unemployment Fund, the income in 1940 was about £67,000,000 and the expenditure about £33,000,000, leaving a surplus for the year of about £34,000,000. The balance brought forward at the end of 1939 was about £57,000,000, of which £37,000,000 was, on the recommendation of the Statutory Committee, applied to reduce the debt of the fund, so that the balance at the end of 1940 was about £54,000,000.

The Committee again take the view that no part of this balance can be regarded as a disposable surplus. They repeat their opinion that large reserves should be built up in the Unemployment Fund to provide for unemployment after the war.

It was urged by the representatives of the Trades Union Congress in their representations to the Committee that it was wrong to take so pessimistic a view, and that it should be assumed on the contrary that steps would be taken by the Government which would prevent a repetition of the high rate of unemployment which arose after the last war.

The Committee say: 'As individuals we have full sympathy with the hopes thus expressed. We believe that much could and should be done in the aftermath of war to maintain employment rather than to pay for unemployment. As the Unemployment Insurance Statutory Committee we dare not base our financial administration of the Fund on such hopes.'

In view of the complete uncertainty as to when the end of the war will come, and how soon thereafter and how rapidly unemployment will increase, the cautious attitude of the Committee should be welcomed.

They recommend that part of the balance in hand should be applied to extinguish the remainder of the debt of the Fund, amounting to about £38,000,000, subject to the right of re-borrowing given in the Unemployment Insurance Act, 1938.

On this occasion the Committee do not feel justified in treating as a disposable surplus even the saving on income account which will result from the repayment of debt. On the assumption that the war will continue at least throughout 1941, they estimate that at the end of that year the Fund will have a credit balance of about £55,000,000, and they regard this as 'by no means an excessive provision for the loss which we may anticipate in the aftermath of war'.

In the Agricultural Account, the income in 1940 was about £1,200,000 and the expenditure about £800,000, leaving a surplus for the year of about £400,000. The balance brought forward at the end of 1939 was

about £3,400,000, so that the balance at the end of 1940 was about £3,800,000. The Committee recommend that no part of this sum should be regarded as a disposable surplus.

The reports throw little light on the difficult subject of the variations in the proportion of unemployment which ranks for insurance benefit, merely stating that in 1940 the average proportion of the total recorded unemployment which ranked for insurance benefit was about 55% in the General Account and a little more in the Agricultural Account.

H. B.

*Theory of Probability.* By HAROLD JEFFREYS.

[Pp. 380 + vi. 21s. Clarendon Press, Oxford, 1939.]

IF a text-book is to interest the specialist it must either develop familiar methods along novel lines, or startle him from complacency by its contention that his accepted maxims are logically inadequate. For more than twenty years Dr Jeffreys, now Reader in Geophysics at Cambridge, has consistently applied to the interpretation of numerical data a theory of probability which no other statistician has employed in published work. He has now collected together the results of his researches in a book which is stimulating and provocative, but emphatically for the specialist.

It would be of interest to trace historically how, in England, America and the Scandinavian countries, probability has come to be regarded as a section of the larger subject of statistics, whilst in other continental countries the statistical treatment of observations is classed among the applications of probability theory. The reasons for this state of affairs would appear to be connected with the progress of the biometric and agricultural schools in England, with the advance of actuarial science in Scandinavia, and with the discoveries of physicists and pure mathematicians on the Continent. So it is somewhat anomalous that the first text-book written with the intention of dealing mathematically with the problems of statistical estimation and the testing of statistical hypotheses (problems which have received adequate attention only from workers in England and the United States) should be called *Theory of Probability, tout court*.

There are eight chapters in the book. Didactically it would have been improved by the rearrangement and omission of parts of the last three, and by a fuller statement of some of the mathematical proofs. On the other hand, the habitual discussion of degenerate cases is very valuable to the student. In the penultimate chapter Jeffreys outspokenly criticizes three other definitions of probability, citing Neyman, von Mises,



and R. A. Fisher, respectively, as their modern exponents; here, and in other places, he shows himself unfamiliar with Continental work.

The fundamentals of probability theory are derived from an axiomatic basis, and then in the second and fourth chapters there are mathematical discussions of many conceptions familiar in statistics: for instance, frequency laws (including an interesting new classification of the Pearson laws), maximum likelihood, Sheppard's corrections, rank correlation. The approach is always original. Less satisfactory is the mathematically advanced treatment of characteristic functions which are only introduced for the purpose of finding the probability law of the resultant of a number of independent random variables; the  $\Omega$  notation for the characteristic function, however, seems worthy of general adoption. The third, fifth, and sixth chapters deal with the theory of statistical estimation and tests of statistical hypotheses; all the other chapters are subsidiary in importance to these. A few numerical examples are scattered through the text, but mathematics predominate.

Given any relevant specified data  $h$ , the probability  $P(A | h)$  is defined to be the degree of impersonal belief in the truth of the proposition  $A$ . One is reminded of De Morgan's definition of probability as the degree of belief one *ought* to have in a proposition; nor does the resemblance end here, for Jeffreys's 'Convention' that probabilities should obey the law of addition is paralleled by De Morgan's 'Postulate' to the same effect.\* The reviewer, however, doubts whether it is possible to have a degree of belief in a proposition  $A$  unless the data  $h$  are of a special and perhaps unusual type. They must, that is, permit of some estimate of the truth frequency of the proposition  $A$  in a series of occasions when either  $A$  or  $\sim A$  is true; it is suggested that this notion, however expressed, is inseparable from the concept of probability whether or not defined as degree of belief. To put it another way: Jeffreys expresses the hesitancy experienced when one is confronted with a disjunctive proposition, as a degree of belief, even when there are no data available except the mere knowledge that a disjunction is involved (for instance, before experiment with a coin known to be biased, the probability of heads is  $\frac{1}{2}$  because 'neither a head nor a tail is more likely than the other at the first throw'†). To the reviewer, however, a belief is something he is prepared to act upon; if he were faced with a disjunction of ten terms of which nine were favourable to a certain course of action, he would not necessarily choose this course unless previous experience had demonstrated that, in fact, the tenth term of the disjunction was seldom true. A belief, and

\* See De Morgan, A., *Formal Logic* (London, 1847).

† Cp. Stumpf's remarks concerning the first throw with a new die: 'Ich zweifle gar nicht, dass eine der 6 Seiten durch beständig wirkende Ursachen begünstigt ist. Ich weiss nur nicht, welche, und habe nicht der geringsten Anhaltspunkt für eine von ihnen. Daher  $\frac{1}{6}$ .' *Sitzungsberichte d. königl. bayr. Akad. d. Wissenschaften (philos.-philol. Classe)*, 1892.

thus a degree of belief, can only be founded on a proportion of previous successes; no belief would ever be held if there were no consistency of statistical ratios in the universe. This is, surely, the answer to Jeffreys's statement that to make relative frequency the basis of a theory is to say 'that we know *a priori* something about observations or the structure of the world'.

In its less sophisticated form the De Morgan-Jeffreys probability definition is based on what has been called the Principle of Indifference.\* When this was applied to the interpretation of Bayes's Rule of inverse probability glaring anomalies appeared, and both Principle and Rule were abandoned. Since then mathematicians have rehabilitated Bayes's Rule as a legitimate theorem but have admitted that it is only of schematic value. However, Jeffreys is able to show that closer attention to the logical implications of the Principle frees it from the charge of absurdity, and permits the use of Bayes's Rule on every occasion when inferences are to be drawn from observations. In particular, it is of interest to mention that whereas he accepts the old 'uniform distribution of ignorance' if the parameter  $s$  may vary from  $-\infty$  to  $\infty$ , viz.

$$P(s < s < s + ds | h) \propto ds \quad (-\infty < s < \infty),$$

he introduces and defends the statement

$$P(t < t < t + dt | h) \propto \frac{dt}{t} = d \log t \quad (0 < t < \infty).$$

This assumption is very ingenious, and leads unerringly to results similar in form to those previously obtained by statisticians who have not used Bayes's Rule because of their suspicions of prior probabilities. Problems in statistical estimation become, for Jeffreys, straightforward applications of Bayes's Rule to obtain posterior probability laws of specified parameters; the concept of 'confidence interval' does not, of course, arise.

In testing a statistical hypothesis  $H$  against an alternative hypothesis  $\sim H$  involving one additional parameter (for example,  $H$  could be the assumption of a Gompertz law, and  $\sim H$  the assumption of a Makeham law), it is reasonable to compare the posterior probability of  $H$  given the observations with the posterior probability of  $\sim H$  given the observations. In calculating these probabilities the corresponding prior probabilities are taken to be equal, for, according to Jeffreys, 'we have no information initially as to whether the new parameter is needed or not'. The criterion actually used is  $K$ , the ratio of the two posterior probabilities mentioned, and the hypothesis  $H$  is considered supported if  $K$  is large, and disproved if  $K$  is small (decisively if  $K < 10^{-2}$ );  $K = 1$  is the critical value, and values of  $K$  near this boundary lead to doubtful decisions. Chapter v details the derivations of explicit expressions for  $K$

\* Cp. Keynes, J. M., *A Treatise on Probability* (London, 1921), Ch. iv.

appropriate to a series of hypotheses  $H$  likely to be required in practice. In an Appendix tables are provided which are of service in the numerical computation of  $K$  in standard cases.

The methodological simplicity of Jeffreys's treatment of statistical hypotheses and estimation in comparison with the Neyman-Pearson technique familiar to readers of *Biometrika*, is due entirely to the use by Jeffreys of prior probabilities. In calculating these it would not be contended, for example, that the Gompertz assumption is correct as often as the Makeham in the long run, but that this is immaterial when a choice is to be made between these two hypotheses which at first sight appear to be as likely one as the other. The orthodox statistician, on the other hand, cannot ascribe a value to a prior probability, and must therefore construct a theory which makes no use of Bayes's theorem. However, it should be mentioned that there are occasions when arguments involving prior probabilities are very illuminating; an example is Jeffreys's discussion in Chapter VII of the Neyman-Pearson minimization of their second kind of error.

Value of criterion	Number of times such values were observed		Theoretical value
	In test of kurtosis	In test of asymmetry	
·000 to ·004	—	—	1·3
·004 to ·016	—	—	1·3
·016 to ·064	1	2	2·6
·064 to ·148	1	2	2·6
·148 to ·455	—	7	5·2
·455 to 1·074	8	5	5·2
1·074 to 1·642	4	3	2·6
1·642 to 2·706	8	5	2·6
2·706 to 3·841	2	—	1·3
3·841 to $\infty$	2	2	1·3
	26	26	26·0

General reasoning is a useful adjunct to, but a poor substitute for, mathematical symbolism. In pursuance of his view that it 'is doubtful whether the normal law ever holds'\* Jeffreys, in *Phil. Trans. A*, Vol. CCXXXVII (1938), p. 231, derived some tests specifically to discover whether Type II or Type VII variations from the normal are exhibited

\* This agrees with Karl Pearson's dictum, 'I have never found a normal curve fit anything if there are enough observations' (*Nature*, Vol. CXXXVI (1935), p. 296). But it is important to add that the divergence from strict normality is usually small enough to be neglected; 'I have never known difficulty to arise in biological work from imperfect normality of the variation....' (R. A. Fisher, *Nature*, Vol. CXXIV (1929), p. 266).

by a set of observations. In brief, one method, not reproduced in the text-book, was to compare the observations, grouped in a way which would emphasize the leptokurtosis of the Type VII deviation from normality, with the corresponding expectations by the normal curve, and calculate a criterion which, it was plausibly argued, would be distributed as  $\chi^2$  with one degree of freedom if the normal law held. At Professor Pearson's suggestion the reviewer tested this experimentally; 26 samples each of 400 random normal deviates were obtained from the 26 Plates of Mahalanobis's Tables [*Sankhya*, Vol. 1 (1934), p. 289], and the appropriate values of the criterion were calculated.\* These 26 values are reproduced on p. 350 in comparison with the theoretical values which Jeffreys would assign, namely, the  $\chi^2$  distribution with  $f=1$ . A similar procedure was adopted to verify Jeffreys's test of the asymmetry of a set of observations; the resulting values of the criterion should, according to Jeffreys, also obey the  $\chi^2$  distribution with  $f=1$ . The table on p. 350 shows that 'general reasoning' has almost certainly led to error in the test of kurtosis, but that the test of asymmetry cannot be disproved with so few samples. This example is cautionary; Jeffreys uses many arguments from general reasoning in his book.

A keen critic of the epistemological basis of Jeffreys's theory, the reviewer yet thinks that this book must take its place as the fifth important English text-book on probability methods: De Moivre, De Morgan, Venn, Keynes, and—Jeffreys.

H. L. S.

\* Miss I. D. Reeves, B.Sc., kindly helped with some of these computations.