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REVIEWS

Life Contingencies. By Professor C. WALLACE JORDAN, F.S.A.

[Society of Actuaries' Text Book. Pp. xi+331. Published by the Society of Actuaries, Chicago, Illinois, 1952. $8.00.]

MEMBERS of the Institute may well take pride in the fact that, in the past, the text-book on Life Contingencies which has been recommended for reading by the Society of Actuaries, and by its parents the Actuarial Society of America and the American Institute of Actuaries, has been the official text-book of the Institute. The study of the theory of Life Contingencies is not an end in itself, but only a means to an end, and accordingly any text-book that is to serve its purpose properly must deal with the practical as well as the theoretical aspects of the subject. The practice of life assurance in the United States and Canada differs in many respects from that in Great Britain and it is therefore a great tribute to the work of George King and E. F. Spurgeon that the *Institute of Actuaries' Text Book, Part II, Life Contingencies* by King, published in 1887, and subsequently *Life Contingencies* by Spurgeon, first published for the Institute in 1922, should for so long have served the needs of students in the continent of North America.

Spurgeon's book is now out of date and the scope of the new official text-book of the Institute and Faculty, *Life and Other Contingencies*, which, at the time when this review is being written, is awaiting publication, has been mainly determined by the requirements of the present syllabus of the Institute. The Society of Actuaries is therefore fortunate in having secured the services of so able an authority as Mr C. Wallace Jordan, Jr., Associate Professor of Mathematics at Williams College and a Fellow of the Society, to write a new text-book which gives special attention to the benefits commonly offered by the life assurance companies of the United States and Canada.

The author explains in his preface that he has been mindful of the needs of those many actuarial students who must acquire their knowledge of Life Contingencies without the assistance of an instructor. Accordingly, he has given full and painstaking explanations which should make it easy for the student who is approaching the subject for the first time to follow the demonstrations of the various formulae. He has also appended copious examples to all the chapters.

The book is divided into three parts. Part I, consisting of seven chapters, deals with the mortality table and with net premiums, office premiums and reserves for single-life assurances and annuities. The American flavour is specially evident in Chapter 6, which explains some of the more important modified reserve systems and non-forfeiture provisions. The development of the subject in this part of the book is neat and follows a logical order, although laws of mortality and annuities payable m times a year are introduced at an earlier stage than in Spurgeon's book. Those portions of Chapter 1 in which the author investigates the effect of assuming a simple algebraic formula for l_x are a little tedious and one wonders what benefit the student is expected to derive from the various examples, at the end of this chapter, which require an arithmetical evaluation of functions based on such formulae. In the chapter on annuities it is a pity that the author has followed the convention that the term 'immediate' means 'payable in arrear', because this has led him to the awkward expression 'deferred immediate life annuity'. Part II contains six chapters, covering joint-life annuities and assurances, contingent assurances and reversionary annuities. Although these matters are disposed of in about half as many pages as Spurgeon required, space has been found for some of the more complicated contingent assurances and reversionary annuities which rarely, if ever, occur in practice. In Part III a chapter on stationary populations is placed, somewhat illogically, with two chapters on multiple-decrement theory and combined tables. For multiple-decrement theory the author uses his own notation; the chapter on combined tables contains a useful explanation of the notation used in America for disability insurance.

An appendix to the book contains *The Commissioners* 1941 *Standard Ordinary* (*CSO*) *Mortality Table*, with monetary functions at $2\frac{1}{2}\%$, and the *Life Table for U.S. White Males*, 1939–41. When the latter table was added, the fact that a slightly different version of it had already been included in Chapter 1 seems to have been overlooked. Also appended to the book are some finite-difference formulae, calculus theorems, derivatives of actuarial functions, answers to the examples, a short bibliography and an index to the notation.

When George King wrote the *Institute of Actuaries' Text Book, Part II*, he could have had no inkling of the developments that were to take place in the theory of probability. He was therefore quite content to write, on the first page of his book:

> Could we find 100,000 children all born at the same moment, and could we follow them throughout life, and enter in a column the numbers who remain at the end of each successive year until all have passed away, we should form the column living, headed with the symbol l_x; where l_x represents the number who attain the precise age x.

This interpretation of the mortality table, so simple and satisfactory to the unsophisticated student of 1887, has an air of unreality to-day. As Mr Charles A. Spoerl has so forcibly put it in his paper, *Life Insurance and the Theory of Probability* (*Proc. Cent. Assembly Inst. Actuaries*, II, 289),

> To one who is steeped in the intricacies of the modern developments, just the thought of George King's bland assemblage of l_x's and d_x's enshrined at the base of the whole theory of life insurance must be well nigh intolerable. It suggests everything that is inadequate, outmoded and oversimplified.

This does not mean that the mortality table itself is outmoded. It means, among other things, (or so it seems to the reviewer) that a mortality table with a radix of 100,000 at age 0 may be regarded as representing, not the actual numbers of lives surviving to successive ages out of 100,000 births, but the numbers who are *expected* to survive to successive ages out of 100,000 lives aged 0 who are taken at random from an indefinitely large number of lives whose mortality is represented by the table. Similarly, when mortality table functions are applied to an individual life or a group of lives, the life or lives must be regarded as having been taken at random from an indefinitely large number of lives who are assumed to experience, as a whole, the mortality on which the table is based.

With these thoughts in mind, the reviewer turned eagerly to the opening pages of Prof. Jordan's book in order to see whether he shared these views, or whether he had some other ideas as to a modern presentation of the mortality table which would enable the student to take a firm grasp of the fundamental principles underlying the application of the table to the solution of practical problems. It was extremely disappointing therefore to find the author explaining a mortality table with a radix of 100,000 by the statement that

> the figures in the l_x column indicate the number of survivors at each age x and the d_x figures indicate the number of deaths in the year of age x to $x+1$.

Worse still, having introduced the reader to the function ${}_xp_o$, which he calls the 'survival function' and which he denotes by the symbol $s(x)$, he says (although he has defined the function as a probability):

> Let us use the survival function $s(x)$... to answer the following question; How many lives, out of a group of 100,000 births, will survive to age 1? The answer is clearly 100,000 $s(1)$.

No doubt Prof. Jordan had some good reasons for presenting these 'inadequate, outmoded and oversimplified' ideas to the student, but it is surprising that his reasons are not explained.

This failure to get to grips with fundamental principles is, not surprisingly, evident again in Chapter 15 on multiple-decrement theory, where the author omits to point out that he is assuming in several places that the decrements are non-selective, an assumption which is hardly ever fulfilled in practice. It is, however, mildly refreshing to see the multiple-decrement table referred to as a mathematical model.

In a work of this nature it is only to be expected that the author should nod occasionally. Perhaps therefore it is a little ungracious to point out that on p. 18 he has fallen into the common error of suggesting that a group of lives can be subject to the influence of a force of mortality. On p. 29 he says that, in a select table with a 3-year select period, l_{25} represents the survivors at age 25 of the $l_{[20]}$ lives insured at age 20 and of the $l_{[22]}$ lives insured at age 22; this was the misconception that lay at the root of Sprague's famous theory of damaged ives. There is a curious lapse on pp. 118 and 119 where the rather tricky problem of equality of policy values by two different tables is investigated. In his anxiety to avoid Spurgeon's error of first proving that a condition is necessary and then assuming that it is also sufficient, the author blindly follows Mr Thomas N. E. Greville (*T.S.A.* III, 533) in thinking that the most convenient way to complete the proof is by induction; he apparently fails to realize that when the problem is limited to a range of ages one has only to reverse the steps in the proof by which the necessary condition has been established in order to show that the condition is sufficient. He also misleads the student in the sentence beginning at the bottom of p. 118, where he omits to point out that the condition is not sufficient unless k is defined as he defines it in line 3 of p. 119.

The index shows signs of hasty preparation; a few may be mentioned here. It is not clear why the page references for 'Gross premiums' and 'Premiums, gross' are given as 128 ff. and 128 f. respectively. No entry is included for Temporary annuities (or Annuities, temporary), Deferred annuities (or Annuities, deferred), Net premiums or Premiums payable m times a year. The page reference for 'Reserves, contingent insurances' is incorrect.

Notwithstanding these various shortcomings, the book, on the whole, is excellent. It is obvious that a great deal of loving labour has gone into the writing of it and it is well printed. The author has the gift of being able to explain things clearly and he has spared no pains to make full use of this gift.

Students of the Society will, no doubt, find the book of great value in their preparation for the examinations and they will have good cause to be grateful to Prof. Jordan for the care and thoroughness with which he has developed a clear and logical explanation of the subject. Students of the Institute might well profit by reading the book, but they should be advised not to take Chapters 1 and 15 too seriously. P. F. H.

Logarithmetica Britannica, being a standard table of logarithms to twenty decimal places. By A. J. THOMPSON, Ph.D. (Lond.)

[Issued in 9 parts by the Department of Statistics, University College, London. Part II, the ninth and last part to be issued, Cambridge University Press, 1952. 45*s*. All other Parts, 21*s*.]

THIRTY years after he took up the task, Dr Thompson's major work has been completed by the publication of the ninth and last part of *Logarithmetica Britannica*. Each of the nine parts contains the logarithms to twenty decimal places of 10,000 numbers, eight parts being issued at fairly regular intervals from 1924 to 1937. The ninth and last part had been completed and would have been ready for publication in 1940, but for the war. With the completion of the full design it can be seen as a worthy memorial of the tercentenary of the publication of Briggs's *Arithmetica Logarithmica* in 1624.

The older readers of the *Journal* may remember an article on *The Tercentenary of Common Logarithms* (*J.I.A.* LVI, 72) which welcomed the publication of the first part to be issued, Part IX, containing the logarithms of the numbers 90,000 to 100,000. The article included some information about Henry Briggs. One of the features of the tables has been the series of prefaces to each part which have recorded all kinds of material of historical and general interest relative to Briggs's logarithms.

What is the need for a table of logarithms to so many decimal places? Most persons, such as actuaries, whose work may entail some computing will never need a large number of significant figures. Calculating machines will serve their purposes, probably with more speed in calculation than logarithmic tables with 4, 5, 6 or 7 decimal places. The author argues with cogency that the day of such tables is past.

There are, however, occasions when calculations have to be made with more significant figures than can be conveniently handled on a calculating machine. For any computer in that predicament the new tables will be a boon. Prof. Pearson states that in statistical and computing laboratories the original Briggs and original Vega are in greater demand than any more contracted logarithmic tables. The following are instances within actuarial experience when extended logarithmic tables may be required:

(*a*) The calculation of high powers, e.g. isolated or check values in compound interest tables,

(*b*) Computations based on recurrence relationships—because significant figures may be lost at each stage of the computation,

(*c*) Computations based on differences or derivatives,

(*d*) Any calculations involving operations in sequence, and

(*e*) Standard tables for the preparation of other tables.

Readers may be interested to know that *Logarithmetica Britannica* was used for the calculations of $\log_{10} K_{100}$ and $\log_{10} A_{100}$ in the expansions of e^{e^t} and

e^{-x^2}, the problem being the summation of an infinite series with terms of alternating sign (*J.I.A.* LXXVI, 155).

The second and fourth central differences, separately computed, are tabulated beside the logarithms of all 5-figure numbers. The lay-out is clear and is easy to read. The Introduction surveys the methods of interpolation for 'reading between the lines' of the tables, namely interpolation by the method of factors, by Everett's formula and other methods of differences, and by Lagrange's method. The author gives two tables to facilitate the method of factors: the first is of $\log(1+N/10^7)$, $\log(1+N/10^{10})$ and $\log(1+N/10^{13})$: the second is of antilogarithms for logs up to ·0000450. In the series of 'Tracts for Computers' there is a *Table of coefficients of Everett's central-difference interpolation formula* which was also computed by Dr Thompson.

The Introduction describes fully and clearly the methods used in the calculation of the tables. The author constructed an integrating and differencing machine which consisted of four simple machines of the rotary type, so arranged in steps that a number on the product register of one machine could be transferred mechanically to the setting levers of the machine below it and that a number on the setting levers of one machine could be similarly transferred to the product register of the one above it.

The *Logarithmetica Britannica* is a standard work of outstanding importance, a magnificent conception magnificently carried out. The author has enhanced the value of his work as a memorial to Henry Briggs by the historical material which he has published comprising:

Part I. The will of Henry Briggs.

Part II. Translation of a memoir on the life and work of Henry Briggs written in Latin by Thomas Smith, D.D., and published in 1707. List of errors in Briggs's *Arithmetica Logarithmica* of 1624.

Part III. Four letters to Sam Ward, Master of Sidney College, and one to Thomas Lydiat in which Briggs says 'I am still at my logarithmes and can nether finishe them to my minde nor lett them alone'.

Part IV. Title page of the work in which Henry Briggs's Treatise on the North-West passage to the South Sea was published.

Part V. Title page of Briggs's *Arithmetica Logarithmica*, 1624.

Part VI. Letter to John Pell at Trinity College, Cambridge.

Part VII. Title page and five other pages of John Napier's *Canonis Descriptio* from the later edition published in 1619, after his death. The reproductions were chosen to illustrate the respective contributions of Briggs and Napier to the subject.

Part IX. Title page and two other pages of Briggs's *Logarithmorum Chilias Prima*, 1617.

It is to be hoped that the author will be able to round off his labours by publishing the life of Henry Briggs for which he has been collecting the material.

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