REVIEWS

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THE object of this new journal is to present to practising statisticians original articles illustrating the applications of modern statistical methods. To fulfil this object it must cover a very wide field, and the first number contains articles on industrial applications, the estimation of social and economic status, and the use of factor analysis, as well as other features such as *Questions and Answers* (one has a pleasant sense of being taken behind journalistic scenes on learning from the Editor that for this number both questions and answers have been supplied by members of the Editorial Committee), and reports of the Industrial Applications Section and the Study Section of the Royal Statistical Society. Two-thirds of the space taken up by original articles is allotted to commerce and industry but, although these are always likely to be strongly represented, it need not be assumed that this bias will always occur. It is clear that it is intended to extend the range considerably beyond what might be inferred from the initial small sample now before us.

Of the articles concerning industry that of most general interest is undoubtedly Dr Bernard Dudding's article, *The Introduction of Statistical Methods to Industry*. Dr Dudding is a pioneer in this country in the application of statistical method to the investigation and control of factory processes. In this article he describes how he and his colleagues at the General Electric Co. Ltd. developed this application to its culmination in the use of the control chart technique. An interesting feature of the article is a section concerned with the practical points that arise in introducing this technique into a factory. The author stresses the important point (often ignored in text-books) that 'successful application of a statistical technique to improve efficiency of production also demands a good knowledge of the sciences and technologies concerned'.

Actuaries may feel that the article nearest to their own interests is that by Mr Leslie T. Wilkins of the Social Survey entitled *Estimating the Social Class of Towns*. Yet the value of a publication such as this is in danger of being lost if it is merely searched for reference to matters coming within the reader's own day-to-day experience. Its great advantage is that it affords an opportunity of seeing what is going on in entirely different spheres and of doing this without laborious study of detailed technical papers scattered through a large number of specialized journals. The first number shows promise that this purpose will be well served.

One feature that must not be left unmentioned is that great pains have been taken to produce a journal that is attractive to the eye and pleasant to handle; the attractiveness, due to the printer's art, may for some of its readers be enhanced by the sparing use of mathematical symbols. The Editor and his Committee, and all concerned with the foundation and production of this journal, have reason to be congratulated on their achievement. R. G. B.

Probit Analysis. By D. J. FINNEY, M.A., Sc.D.

[2nd ed. Pp. xiv+318. Cambridge University Press, 1952. 35*s*.]

THIS book was first published in 1947 and evidence of its success is the appearance of a second edition. Dr Finney gives a description of the methods of probit analysis with clear instructions and examples of the numerical calculations, together with tables of the necessary functions. Formulae are given, usually without proof, together with sufficient of the theory behind them to enable the methods to be applied with understanding. A good knowledge of statistical theory would be required of the reader who wished to derive the methods and formulae given.

Probit analysis was originally developed for the analysis of biological and toxicological experiments where the response to some stimulus under study was 'quantal' or 'all-or-none' (e.g. death or survival, germinate or not germinate). These types of experiment still provide the majority of applications of probit analysis, although its use is spreading to other fields. A typical situation where probit analysis is applicable is as follows. 300 insects of the same species are divided into six groups of about 50 and each group exposed to different measured concentrations of a poison; in each group the number dead after a given time is recorded. The object of the experiment is to determine the relation between dose and mortality and hence the dose which kills a certain proportion (e.g. 50%, or 90%) of the insects.

For the concentrations of poison which are of principal interest it is found that some insects survive exposure and some do not; it is therefore argued that each individual insect has a certain 'tolerance' for the poison—it will survive exposure to any smaller dose and succumb to any larger dose. A batch of insects will therefore have a certain tolerance distribution, and it has been found by experiment that this distribution is often approximately normal when the dose of poison is measured on a logarithmic scale. Thus a dose of λ_0 will kill all insects with a tolerance less than λ_0 and the proportion killed, P, will be

$$P=\frac{1}{\sigma\sqrt{(2\pi)}}\int_{-\infty}^{x_0}\exp\left[-\frac{1}{2\sigma^2}(x-\mu)^2\right]dx, \qquad (1)$$

where $x_0=\log\lambda_0$ and μ and σ^2 are the mean and variance of x. The basis of probit analysis is to replace the observed proportions killed, P, at each concentration of poison by the corresponding normal equivalent deviate, $(x-\mu)/\sigma$, found by entering a table of the normal curve with P. The probit, Y, is this normal equivalent deviate with 5 added to avoid negative signs, so that

$$Y=5+(x-\mu)/\sigma. \qquad (2)$$

Chapter 3 describes the above process and gives a graphical method of determining the straight line (2) from the pairs of values of Y and x. The log(dose) which gives 50% (or any other required percentage) kill is then read off and its standard error derived from the slope of the line.

When more complex experiments than that described above are being analysed the graphical method must be replaced by an analytical technique. Chapter 4 describes the fitting of the probit regression line by the method of maximum likelihood. The following chapters describe the comparison of two poisons, adjustment for the natural mortality which often occurs in addition to that produced by the poison, factorial experiments, and the action of mixtures of poisons. The reviewer wonders whether the classification of the combined

effects of two poisons (chapter 8) could be made the basis of a study of multiple causes of death in human populations.

The book and the examples deal almost entirely with the application of probit analysis to biological assays, but the author (p. xiv) does not consider the usefulness of probit analysis limited to this type of data. However, in his exposition he appears to assume, almost without comment, the reality of the tolerance distribution. On this assumption a theoretical basis can be claimed for probit analysis and, although Dr Finney does not explicitly make this claim, he does give the impression that he considers probit analysis to have some advantage over other methods for this reason.

Sometimes the existence of a tolerance distribution can be demonstrated (e.g. when the stimulus can be applied repeatedly to the same individual), but more often this cannot be done even for bio-assays and the only evidence may be the existence of a variability of response. Dr Berkson (1951, *Biometrics*, VII, 327) discusses this question and quotes the case of fruit flies exposed to X-rays. The generally accepted theory is that the number showing mutation is proportional to the intensity of radiation and 'is not effectively related to anything that can be intelligibly referred to as tolerance of the fly'.

In any particular application it is of course hardly a statistical question as to whether or not the tolerance distribution is a reality, but if it does not exist any claim to a theoretical basis for probit analysis is destroyed. This must not be taken as a criticism of probit analysis, which is one workable method—quite possibly the best for certain types of data—of transforming a sigmoid response curve into a straight line, so as to simplify statistical analysis and enable valid estimates to be made from the figures. It is interesting to note that on p. 185 the application of the probit method in this sense is described for data showing quantitative responses. If the tolerance distribution is not real, then other formulae which Dr Finney mentions may give as good results and his criticism that they 'have less theoretical recommendation than the probit (transformation)' (p. 47) loses its force. In this connexion it may be pointed out that Berkson (1944, *J. Amer. Statist. Ass.* XXXIX, 357) describes the use of a logistic function which, in each application he quotes, appears to give a very slightly better fit than the probit method. It is only fair to the author to add that the preface to the second edition seems to indicate some change in his views on this matter.

There does seem a danger in statistical analysis that a convenient hypothesis (e.g. a tolerance distribution) which can explain the observations may come to be looked upon as true on little evidence but the observations themselves. A similar situation was disclosed recently (Arbous & Kerrich, 1951, *Biometrics*, VII, 340) in accident statistics—the theory that individuals have different degrees of proneness to accident was found to be based on little evidence except that a negative binomial fitted the frequency distribution of number of accidents per person, in spite of the fact that a negative binomial distribution can arise in other ways than from the existence of accident proneness.

Those who know the first edition of this book will be interested in the changes that have been made in the second edition. These consist principally of rewriting four paragraphs and Appendix II (Mathematical Basis of the Probit Method), and the addition of a new chapter on recent developments. Among other things the new chapter considers alternatives to the maximum-likelihood method of estimation used throughout the book. It is admitted that, for samples which are not 'large', maximum-likelihood estimation may not be the most efficient and some other method, e.g. minimum-χ^2, might prove better. The question of

the best allocation of test subjects to poison doses when estimating a percentage point is dealt with and an interesting method related to sequential sampling methods described. The chapter closes with a section on 'The meaning of standard errors' which gives a justifiable warning against 'the uncritical acceptance of standard errors...as though they indicated the variation that might be encountered in *any* repetition of an experiment.'

Dr Finney deserves the thanks of all interested in probit analysis for bringing his useful book up to date. R.H.D.

Mathematical Models of Demographic Analysis. By L. YNTEMA.

[Pp. viii+78. Leyden: J. J. Groen en Zoon, N.V., 1952.]

THIS monograph, submitted as a thesis for a doctorate of science at Amsterdam University, expounds and criticizes the Lotka system of population mathematics, applying to either one sex or the other but not both; it appraises and develops the 'joint' analysis of Pollard (*J.I.A.* LXXIV, 288), in which one sex is considered as being followed by the other in the next generation; and finally it investigates a new idea—the treatment of both sexes together simultaneously in various combinations. The author has paid a compliment to English-speaking demographers by writing directly in their language instead of in his own native tongue. The result, while not perfect, is sufficiently comprehensible to reveal, as does the choice of an appropriate quotation from Aldous Huxley in the Introduction, a witty appreciation of the significance and the limitations of his subject.

The fundamental, or as he calls them the 'unisexual', equations of Lotka are set out rather fully, even the complex solutions being considered. Attention is drawn to most of the defects of this form of analysis and to the various ways in which attempts may be made to overcome them. The author draws a basic distinction between what he terms the 'prospective' and the 'retrospective' approaches; the phraseology here is not entirely clear, but the difference in question appears to be that between indices of current fertility on the one hand, and measures of reproduction on the other. Finding that none of the methods examined is adequate in connexion with both these aspects at the same time, he elevates this uncertainty, by a metaphor that is perhaps somewhat overstrained, into a demographic 'Heisenberg principle'.

An ingenious attempt is made to eliminate the inconsistency in Pollard's system, in which the sex-ratio at birth is not constant, by extending the analysis from two to n generations, the aim being to produce the correct balance of the sexes in the long run. In order to deal with other difficulties of population mathematics, the author turns to what he calls 'bisexual' treatment, the functions for the separate sexes being combined by means of the weighted arithmetical, geometrical or harmonic averages, and is able to show that the population will tend towards the Malthusian type as in unisexual analysis. At the end, however, it is confessed that a completely satisfactory solution to the problem of measuring reproduction has not been found and that so far as indices of fertility are concerned there is really nothing better than the mean of the male and female functions. Although unproductive of any very remarkable results, these researches are nevertheless of great interest and constitute a distinctive contribution to population mathematics. P.R.C.

L'estimation des papiers-valeurs selon la méthode mathématique. By PHILIPPE CHUARD, D. ÈS SC.

[Pp. 179. Lausanne: Editions H.E.C., 1949. Swiss Fcs. 15.]

A NUMBER of actuaries here and abroad have been credited with the authorship of the aphorism that a life assurance company valuing its stock exchange securities at their market price is placing the one value on them that they cannot possibly have; for their market values only represent their sale prices provided they are not offered for sale. Nevertheless, it is clear from the discussion of Henry Moir's paper on the so-called 'American method' of amortizing the bonds on a life office's books (1915, *T.F.A.* VII, 171) that, at the time of the first World War, few British companies employed anything but market values in their asset valuations.

In 1909 the State of New York passed a law requiring all insurance companies doing business in that state to value their well-secured bonds at or below a value based on the purchase price, the dividend rate, and the maturity value, and determined in such manner that the yield on any intervening book value is equal to that calculated from the purchase price. Certain companies in this country and abroad had used this method before, but 1909 was the first time it was specifically mentioned in a statute. Today the 'American method' is used pretty widely, though it was apparently still necessary to justify it to German actuaries as recently as the Eleventh International Congress (1937).

However, a basically different though formally similar method of valuation of the various types of asset of pension funds and life assurance companies has been suggested in recent years by writers in different actuarial journals. The fundamental idea is to value some or all the assets at a rate of interest related, if not equal, to that employed in the actuarial valuation of the liabilities. Lieuwen (1942, *Het Verzekerings Archief*, XXIII, 205) proposed to apply the valuation rate of interest to the assets of a life company or pension fund, whether these assets were in the form of irredeemable or redeemable securities, real estate or mortgages. The methods he suggested are detailed in his paper. Puckridge (1948, *J.I.A.* LXXIV, 1) made a similar proposal for pension funds only. Alexandre (1949, *Bull. Inst. Actuar. Portugueses*, IV, 15), while permitting a wide exercise of prudent judgment, was also in favour of a valuation of all kinds of pension fund assets at the valuation rate of interest. Even more recently, Scrimgeour (1951, *T.F.A.* XX, 145) proposed the same method for valuing the assets of widows' funds.

In fact, this method—which may, with some justice, be called the 'Swiss method'—was apparently first used in valuing the redeemable securities and mortgage holdings of certain Swiss pension funds (1938, Riethmann, *Mitt. Verein. schweiz. Versich.-Mathr.* XXXV, 1). By a federal decree dated 31 December 1939, Swiss life insurance companies were required to place values on their redeemable securities not in excess of their 'mathematical values' calculated at a rate of interest equal to that of the actuarial valuation plus $\frac{1}{4}\%$. If the value thus calculated exceeded the purchase price of a newly acquired security the difference was to be held in a special fund; in the contrary case the fund was diminished by the resulting 'profit'. Wyss has explained (1949, *Bull. trim. inst. actuair. franç.* LX, 323) how this method could be extended to mortgages and real estate.

The advantage claimed for the Swiss method is that an adjustment in the

valuation interest rate caused by well-founded expectations of a future increase or decrease in market yields is compensated by a similar change in the interest rate used in valuing the assets. An over-cautious actuarial valuation would, to some extent, be balanced by an over-valuation of the assets of the life office or pension fund.

An interesting and important problem is: 'how does this compensatory influence work out in practice?' If the valuation rate of interest is lowered by $\frac{1}{2}\%$, is the change in the value of the assets likely to be larger or smaller than that of the liabilities? It is this question that Dr Chuard has set out to answer in his excellent monograph.

The work is divided into three parts. In the first there is a short review of methods of asset valuation—market price, 'American method', 'Swiss method', cost price, linear 'write-up' to maturity value—and a history of the legal requirements for life assurance companies in Switzerland. Here, as throughout the book, there is little or no reference to work published on the general subject outside that country.

The second part provides a good summary of the mathematical theory of amortization of various types of loans: perpetuities, loans with a fixed maturity date, loans involving repayment of capital and interest by a level annuity, those with uniform repayments of capital at equal intervals, etc. In the third, and last, part, which extends to more than half the book, the author commences his study of the compensatory effects of the 'Swiss method' on the actuarial and balance-sheet valuations.

By changing the actuarial valuation rate of interest from i to $i+m$ while, at the same time, the interest rate used in valuing the assets is altered from j to $j+m$, the net effect on the balance sheet is the introduction of a profit Δ_θ where, in an obvious notation,

$$\begin{aligned}\Delta_\theta &= \{\mathscr{A}(j+m)-\mathscr{A}(j)\}-\{\mathscr{L}(i+m)-\mathscr{L}(i)\}\\ &= \{\mathscr{A}(j+m)-\theta\mathscr{L}(i+m)\}+(1-\theta)\{\mathscr{L}(i)-\mathscr{L}(i+m)\},\end{aligned}$$

where $\theta=\mathscr{A}(j)/\mathscr{L}(i)$. On dividing throughout by $\mathscr{L}(i)$

$$\begin{aligned}\frac{\Delta_\theta}{\mathscr{L}(i)}=\delta_\theta &= \theta\left\{\frac{\mathscr{A}(j+m)}{\mathscr{A}(j)}-\frac{\mathscr{L}(i+m)}{\mathscr{L}(i)}\right\}+(1-\theta)\left\{1-\frac{\mathscr{L}(i+m)}{\mathscr{L}(i)}\right\}\\ &= \theta\delta+(1-\theta)\,\delta^*, \quad \text{say.}\end{aligned}$$

Since δ^* may be obtained from δ by replacing $\mathscr{A}(j+m)/\mathscr{A}(j)$ by unity, it is only necessary to study the behaviour of δ for various combinations of i, j and m. However, this study is extraordinarily complex owing to the number of parameters involved both in $\mathscr{A}(j)$—the number and types of securities in the portfolio, the various periods to maturity, the dividend rates—and in $\mathscr{L}(i)$—the number and kinds of policies, the ages at entry, the original and unexpired terms, the mortality table utilized. The author proceeds by means of numerical illustrations in which he varies a single parameter at a time, the remaining parameters having fixed numerical values. The examples become progressively more complicated as the four chapters of the third part are worked through.

The results obtained are illustrated by reference to one of the examples provided by Dr Chuard. The securities representing the asset portfolio and the policies assumed to be in force are supposed to have been obtained by an insurance company in increasing numbers with the passage of years. These

increases in purchases of securities or issue of policies are assumed to have occurred in arithmetical progression. The assets consist of $3\frac{1}{2}\%$ debentures redeemable at par 20 years after the insurance company purchased them. The ratio of the common difference of the A.P. governing the purchases in successive years to its first term is $12\frac{1}{2}$. On the other hand, 99% of the policies in force consist of 25-year endowment assurances for a unit sum assured, all effected at age 30. The remaining policies are immediate unit annuities purchased at age 65. The common difference ratios of the two series of policy issues are 17 and 16, respectively. The valuation rate of interest is $3\frac{1}{2}\%$ and the mortality table used in the actuarial valuation is the Swiss male population table of 1921–30 (SM 1921–30). The following table of δ_θ reproduces some of the results obtained.

Values of δ_θ

θ	m			
	$-\cdot01$	$-\cdot005$	$\cdot005$	$\cdot01$
	%	%	%	%
0	−14·8	−7·1	6·5	12·4
1	− 4·9	−2·3	2·0	3·7

In the degenerate case when $\theta=0$, δ_θ becomes equal to δ^* and thus represents the increase in surplus (expressed as a percentage of the reserves) caused by an increase in the valuation rate of interest from ·035 to $\cdot035+m$. The table thus shows, for $\theta=0$, the proportionate gains resulting from changes in the valuation rate of interest from $3\frac{1}{2}\%$ to $2\frac{1}{2}\%$, 3%, 4% and $4\frac{1}{2}\%$, respectively, on the assumption that the assets have been valued independently of the liabilities, e.g. by the American method. On the other hand, $\theta=1$ represents the case where the assets exactly equal the liabilities when both are valued at $3\frac{1}{2}\%$ and simultaneous and equivalent changes are made in the interest rate at which assets and liabilities are valued. A comparison of the two lines of values of δ_θ in the above table shows the considerable damping effect caused by a valuation of assets by the Swiss method.

We cannot do better than end our review of Dr Chuard's book with his own concluding words:

...the use of the 'Swiss method' results in a compensation which is very satisfactory. From this point of view, the resulting advantages for insurance companies are thus very real.

H.L.S.

Bibliography of Basic Texts and Monographs on Statistical Methods. By WILLIAM R. BUCKLAND.

[Pp. 84. International Statistical Institute, The Hague, 1951. 5*s*.]

THE scope of this publication is sufficiently described in the following extracts from its Introduction:

The bibliography consists of about 100 basic texts and monographs on statistical methods and their application, in the English language. The items have been selected mainly from the last decade but include some earlier works of outstanding importance.

The general form of each entry is the routine publication details followed by a list of chapter headings and extracts from reviews taken from the four main statistical journals published in English which include book reviews regularly. These journals are (1) *Journal of the Royal Statistical Society*, (2) *Journal of the American Statistical Society*, (3) *Biometrika*, (4) *Sankhya*.

The works are classified under the two main headings of General Theory and Applications. There is further broad sub-classification which is helpful in directing the reader to those books in which he is likely to be particularly interested.

One cannot quarrel with the choice of books for inclusion in this bibliography. All the better-known works published up to 1950 have found a place in it. For this very reason, a large part of the bibliography contains little that will be fresh to anyone with an active interest in statistical literature. Bibliographies of publications in other languages are promised, and these should prove of more value to those wishing to extend their acquaintance with different viewpoints on statistical methodology.

The selection of reviews from only four journals is obviously a great convenience for the compiler, but it can, in certain cases, be a handicap in forming a balanced judgment of the merits of a book. This criticism is particularly relevant in the case of books on applications of statistical methods in specific fields. Reviews of these books in the appropriate specialist publications would be of considerable interest. For example, reviews of Hill's *Principles of Medical Statistics* written for medical readers, or of Cox's *Demography* for actuarial readers, would seem to be of at least as much importance in assessing the value of these books as are the reviews actually given.

Although the method of selection of reviews is open to the above criticism, little exception can be taken to the editing of the chosen reviews. The passages reproduced are (in the cases checked) fairly representative of the tenor of the reviews from which they are taken. Together with the other information provided they will, in most cases, convey a reasonably clear notion of the nature of the books to which they refer. It is to be expected that this collection of facts and opinions about statistical publications will prove useful to many people, saving them time and trouble in finding their way about the subject. N.L.J.

Some Accounting Terms and Concepts. A Report of a Joint Exploratory Committee appointed by The Institute of Chartered Accountants in England and Wales and by The National Institute of Economic and Social Research.

[Pp. 46. Cambridge University Press, 1951. 3*s*.]

IN 1945 a Joint Exploratory Committee was set up by the Institute of Chartered Accountants in England and Wales and the National Institute of Economic and Social Research, under the chairmanship of Sir Henry Clay, with the object of achieving a closer understanding between accountants and economists. The present report is an examination of the 'major accounting concepts and of the more important terms commonly used by both accountants and economists'.

The report first attempts to formulate a basic accounting structure consisting of three accounts—profit and loss account, balance sheet, and capital reconciliation statement—and the various items of which they are comprised are set out and briefly commented upon. Differences of opinion as between accountants

and economists are, however, deferred to the next section which deals with the main accounting concepts, and contrasts, where they differ, the economists' view and accounting practice. The main items on which it proved impossible to reconcile the two were the valuation of stocks and the valuation of fixed assets, both of which, as the report points out, turn on the conception of 'maintaining capital intact', and the effect thereon of the instability of money as a store of value. The report makes it clear that the failure to reconcile the views of the two groups arose primarily from the difference in their objectives, the accountants being concerned with the financial aspect of business transactions and the economists with the real goods represented by those transactions.

The report is perhaps disappointing in that it does not attempt to carry the controversy between 'historical cost' and 'replacement cost' as the basis for the valuation of capital assets beyond a brief statement of the opposing points of view. Nevertheless, it is valuable to have the main accounting terms clearly set out and described. The report also includes, as an appendix, three recommendations issued by the Council of the Institute of Chartered Accountants in England and Wales on the depreciation of fixed assets, the valuation of stock-in-trade, and rising price levels in relation to accounts. F.W.B.

The Economics of National Insurance. By ALAN T. PEACOCK.

[Pp. 126. William Hodge and Co. Ltd., 1952. 8*s.* 6*d.*]

THE author, after briefly describing the financial aspects of the national insurance system and comparing private and national insurance, discusses the system in relation to fiscal and monetary policy. He assumes throughout the book that, since the liabilities of the system can be greatly influenced by Government action and a large part of the income is derived from the Exchequer, the National Insurance Scheme is a social service dependent on Government policy and is not really autonomous or in any way similar to private insurance, so that the accumulation of reserves is for purposes other than the finance of the scheme alone. He concludes that a reduction in the current surplus on the Insurance Funds might involve an increase in taxation if saving and investment are to be kept in balance—that a reduction in contributions during periods of cyclical unemployment would have little advantage over other forms of deficit spending, when most of the working population is covered by direct taxation—that the system has little redistribution effect on income *per capita*—and that the reserves have in the past been applied, in furtherance of a monetary policy restricting a rise in interest rates, in supporting the gilt-edged market under Treasury direction without any consequential inflationary credit expansion. The author also propounds the Liberal Party scheme for the reform of income tax and social security payments under which everyone would receive an untaxed 'social dividend' in place of the present income tax and insurance allowances, the scheme being financed by a proportional income tax assessable on all incomes without the payment of any contributions.

The book, which contains much useful information and many references, really consists of a collection of essays on some of the broader fiscal aspects of National Insurance, and does not aim at encompassing the more ambitious field suggested by the title. C.J.B.