

REVIEWS

Catalogue de la Bibliothèque de la Compagnie d'Assurances sur la Vie 'Utrecht'

[Eighth edition, in two volumes. Vol. I, pp. 736, Vol. II, pp. 570. Utrecht, 1949. Privately printed.]

THE compilers of this catalogue have paid the Institute a graceful compliment by beginning their preface with the words 'No library is of much value without an effective catalogue', quoted from the preface to the first catalogue of the Institute library. Although these words are still true, times have changed since they were first written and many librarians, at least in Great Britain, now regard the issue of a printed catalogue as being impracticable. On glancing through the book under review, one cannot fail to be impressed both by the magnitude and scope of the library to which it relates and by the patience and industry which must have gone into the preparation of such a catalogue. Anyone who thinks that post-war conditions made the production of the catalogue difficult, however, is apparently in error, for we learn from the preface that 'les difficultés de l'époque' have ceased to exist. Perhaps this statement ought not to surprise those of us who visited Holland for the Thirteenth International Actuarial Congress and who discovered then that the Dutch people have a way of making difficulties disappear.

When the previous catalogue was issued in 1916, the library contained over 11,000 books. We are not told how many there are at present (and it is difficult to form an estimate from the catalogue because some books are mentioned in it more than once), but it is stated in the preface that the number has been continually increasing. The library is therefore a good deal larger than that of the Institute; it also covers a wider field. The first volume of the catalogue is devoted to insurance in general and to life assurance; the second volume includes other branches of insurance (fire, sickness, accident, etc.) and allied subjects such as finance, legislation, taxation and political economy.

Although, as is only to be expected, entries in the Dutch, French and German languages occupy a large proportion of the catalogue, English titles occur quite frequently; it is interesting to see that there is an excellent representation of the work of early writers on actuarial science in Great Britain, including Babbage, Bailly, Griffith Davies, de Moivre, Dodson, Morgan and Price. There is also a set of *J.I.A.* complete from Vol. 1.

Two early works in languages other than English are deserving of mention. One is Tetens's two-volume work *Einleitung zur Berechnung der Leibrenten und Anwartschaften* published in Leipzig in 1785 and 1786, which contained the first commutation columns in modern form. The other—even more noteworthy—is the *Waerdije van Lijfrenten naer proportie van Losrenten* of Johan de Witt, 1671. Dutchmen are rightly proud of this, the earliest known treatise on life annuities. British actuaries have long been familiar with it as a result of the researches of Hendriks, who gave an English translation in *J.I.A.* 11, 232. The book has always been rare and when Hendriks tried to obtain a copy he was unsuccessful; he therefore made use of an extract from the 'Resolutiën' of the States of Holland and West Friesland but this differs slightly from the published treatise. The Utrecht library is to be congratulated on the possession of an original copy; the Institute is not so fortunate.

The library contains a considerable number of books (many of them written during the first half of the nineteenth century) which deal with the early history of friendly societies in Great Britain, a subject in which our own library is unfortunately very poorly equipped. Another striking feature is the number of deeds of settlement of early English and Scottish assurance societies. There is also a good selection of books which have been published in the United States of America and which deal with such subjects as life assurance and mortality tables; only a small proportion of these are in the Institute library. A glance through the 'Biography' section of the catalogue confirms the impression, already formed from the Institute library, that biographies which are of direct interest to actuaries are rare.

Any member of the Institute who turns eagerly to the two subsections of the catalogue dealing with mathematical statistics in order to see whether the library contains modern works by such writers as R. A. Fisher and M. G. Kendall will be disappointed. The only recent book on mathematical statistics, written in English, which he will find there (apart from *Actuarial Statistics*, Vol. I by Tetley, and Vol. II by Anderson and Dow, which he will note with pleasure) is the 1944 edition of Caradog Jones's *First Course*; the only edition of Yule's *Introduction* which the library possesses is the second, published in 1912. David's *Probability theory for statistical methods*, however, is listed in the 'Probability' subsection; it is one of the very few modern books included there.

This is a fascinating catalogue and a few minutes spent in turning its pages will cause the mouth of any actuarial bibliophile to water. It may be of interest to refer here to some of the entries which caught the reviewer's eye; none of these books is in the Institute library. *A full account of the rise, progress and advantages of Dr Assheton's proposal* (London, 1711) promises information about a little-known project—a scheme for reversionary annuities operated by the Mercers' Company. A book by E. Laurence, published in London in 1730, has the provoking title *A dissertation on estates upon lives and years, whether in lay or church-hands. With an exact calculation of their real worth, by proper tables, and the reasons for their different valuations*. It would be interesting to know what were the 'most rational probabilities' stated to have been used for constructing the tables of values of annuities on lives, in the second edition of *The banker's sure guide: or monied man's assistant* by S. Thomas (London, 1768). Th. Paine (presumably the well-known Tom Paine), in a book published in London in 1797, describes a plan for creating in every nation a fund to pay £15 to every person on attaining age 21 and £10 per annum to everyone over age 50. *The International Insurance Encyclopedia* (New York, 1910) is stated to be 'a descriptive record of the history, theory and practice of all branches of insurance throughout the world and from the earliest times to the present day' and to be 'embellished with more than twelve hundred illustrations'. Only Vol. I is listed; possibly the work, like Walford's *Insurance Cyclopaedia*, was never finished.

So much for the library itself. The manner in which the books have been classified calls for some comment. The catalogue is divided into twenty-one sections, denoted by letters of the alphabet, and most of these are divided into two or more subsections to which numbers are allotted. There are thus seventy different subsections, in each of which the books are arranged in alphabetical order of authors.

It may seem ungracious to find fault with a book which has been presented to us by some who were our hosts at the recent Congress and who generously offer, in their preface, to lend books 'à tous ceux qui desirent se documenter

sur la science de l'assurance', but the conscientious reviewer must harden his heart and draw attention to weaknesses where he thinks that he sees them.

Although some broad grouping according to subject-matter was obviously desirable, or even necessary, it does seem to have been a mistake to attempt as many as seventy subdivisions in an 'author' catalogue. There are many books which cannot be confined within such narrow limits. The compilers have tried to surmount this difficulty by including some of the books in two or more subsections of the catalogue and by providing an index, but in the former respect they do not appear to have been entirely successful. Numerous examples to illustrate this could be given, but it will be sufficient to consider three books—Henry's *Calculus and Probability for Actuarial Students*, Harry Freeman's *Elementary Treatise on Actuarial Mathematics* and the same author's *Examples in Finite Differences, Calculus and Probability*. We should have expected to find all three books in section E (Mathématique actuarielle), subsection 1 (Notions générales, manuels actuariels, congrès d'actuaire), and we should not have been surprised if the compilers, with their customary thoroughness, had included them also in section R (Mathématiques), subsection 1 (Généralités sur les mathématiques, calcul infinitésimal, géométrie, nomographie) and subsection 2 (Calcul des probabilités). Actually, Henry's book is in section E (Mathématique actuarielle), subsection 2 (Statistique mathématique, construction et ajustement des tables de mortalité), Freeman's *Treatise* is in E1 and the *Examples* are in E2 and R1.

Incidentally, it seems to be a pity that, whereas the theory of probability is in section R (Mathématiques), books on mathematical statistics are included in one or both of sections E (Mathématique actuarielle) and S (Economie politique).

The alphabetical arrangement of the books within the subsections leaves much to be desired. When a book is written by an author whose name is known, all is well, but when no author is mentioned the compilers have followed the practice of extracting a word from the title of the book and treating this word as if it were the author's name; this produces some incongruous effects. Here again numerous illustrations could be found but a few will suffice. The booklet, *Interest and Life Contingencies*, issued in 1919 by the Students' Society, is treated as if the author were named 'Contingencies' and it therefore appears among books written by authors whose surnames begin with C—where we should never have looked for it. The H^m experience, entitled *Mortality Experience of life assurance companies, collected by the Institute of Actuaries*, is attributed to the author 'Experience, Mortality' but the author of *Mortality Experience of Government life annuitants, 1900-1920*, is taken as 'Mortality'. The subsection devoted to actuarial tables consists of sixteen pages, three of which are occupied by books which the compilers were content to regard as having been written by the author 'Tables'. No—not quite content, for they have tried to be helpful by including the *Short Collection* under 'Collection' as well as 'Tables'. For this kind of book it would have been much better to take as the author the Society or other Institution conducting the investigation or publishing or sponsoring the work.

Despite these little imperfections, the catalogue forms a useful and imposing record of a splendid collection of books and is a challenge to the Institute to do as well for its own library. Should any member of the Institute be visiting Utrecht, he would do well to ask for the privilege—which would no doubt be readily granted—of viewing the library itself.

Interpolation. By J. F. STEFFENSEN, Sc.D.

[2nd ed. Pp. ix + 248. New York: Chelsea Publishing Company,
1950. \$3.50.]

THE first edition of this well-known book, published in 1927, was reviewed in *J.I.A.* LVIII, 325. Though the present is described as the second edition, it seems in fact to be merely a reprint. However, the many friends of Prof. Dr Steffensen in this country will be pleased that the book is once more available and that a new generation may profit from his excellent work.

Calculus of Finite Differences. By CHARLES JORDAN, D.ès Sc., Dr.Phil.

[2nd. ed. Pp. xxi + 652. New York: Chelsea Publishing Company,
1950. \$5.50.]

THE first edition of this book was published in Budapest in 1939. The present volume, though described as a second edition, contains no new material and is identical apart from a few trifling amendments.

The treatment is that of a mathematical text-book and follows much the same plan of development as a standard work on the infinitesimal calculus. Thus the author defines his operators, investigates their properties and proceeds to examine the n th differences of functions such as $\cos mx$ and $\Gamma(x)$, long before he deals with numerical work or, indeed, gives any hint of the type of practical problem that the finite calculus is designed to solve; the reader can study half the book without learning that interpolation in prepared tables is one such problem. These observations are not intended as derogatory criticism. On the contrary, there is no doubt that the customary emphasis on the numerical side has caused many students of this subject to regard it as a mere means to a practical end, and has blinded them to the fact that it can be built up rigorously as a branch of pure mathematics, rich in elegant results of satisfying generality. At the same time, this is not a book to be recommended to students coming fresh to the subject; it is a treatise rather than an introduction and will therefore serve best those who are already familiar with the everyday uses of finite differences, whose aim it is to deepen, widen and consolidate their knowledge. This may seem curiously at variance with the author's avowal that the book 'has been written especially for practical use, with the object of shortening and facilitating the labours of the computer'. In truth, however, there is no contradiction; Prof. Jordan caters for the computer who insists on understanding 'why' as well as 'how', and who is prepared to embark upon a serious course of theoretical instruction before being considered competent to apply himself to practical problems.

The book is encyclopaedic in content, and besides being a rigorous text-book it constitutes a first-class work of reference. In this latter capacity, however, its usefulness would have been still further enhanced if the author had kept more rigidly to standard notation. He states in his preface that unfamiliar notations have been avoided as far as possible, but he nevertheless uses, for example, a Gothic capital letter for the divided difference operator—a serious drawback to the computer in his manuscript work—and the symbol $(x)_n$ instead of the widely used $x^{(n)}$ for the factorial $x(x-1)\dots(x-n+1)$. Some of the departures from modern standard notation are doubtless due to the fact that no changes have been made in this edition to bring it up-to-date in this respect. Not all,

however, can be so explained; for instance, the author writes \sum_a^{b+1} for the sum usually written \sum_a^b ; the reason for this will be obvious to anyone acquainted

with finite difference methods of summation, but the slight simplification of formulae so effected surely does not compensate for the inconvenience occasioned to the reader. However, it must be mentioned in the author's favour that the book is liberally supplied with bibliographical footnotes which set out clearly the notation of other authors; also, the question of notation cannot be said to be fundamental in this subject in the sense that it is in certain other branches of mathematics such as the tensor calculus. In the course of this volume many numerical tables appear scattered throughout the text; for ease of reference these might with advantage have been gathered together at the end of the book.

In the first two chapters Prof. Jordan deals with the fundamental operators, generating functions and the expansion of functions by various methods, and makes a study of a number of particular functions important in finite differences. The basic operators introduced, apart from E and Δ , are the mean $M \equiv \frac{1}{2}(1 + E)$, the central difference $\delta \equiv E^{\frac{1}{2}} - E^{-\frac{1}{2}}$, and the central mean $\mu \equiv \frac{1}{2}(E^{\frac{1}{2}} + E^{-\frac{1}{2}})$, where Sheppard's notation is followed in preference to Thiele's symbol \square adopted by Steffensen in his *Interpolation*. The mean M as an operator is employed throughout the book instead of the central mean—with a gain in simplicity at the expense of symmetry. The receding difference operator ∇ , of which Steffensen makes much use, is defined, described as having no advantage whatever, and promptly dismissed. We may remark in passing that the author uses the sign of equality in stating relations between operators, as, incidentally, does Steffensen also. In a text-book, however, the sign of equivalence is surely to be preferred, if only because it serves to remind the novice that, like the handling of ammunition, the treatment of operators as if they were algebraic quantities is safe only so long as he remembers that it is dangerous.

The third chapter covers inverse difference operations and summation. Chapter 4, of some 90 pages, is concerned with the properties and applications of Stirling's numbers of the first and second kinds, and treats related topics such as Stirling's polynomials, Thiele's seminvariants and the operators $\Theta \equiv x\Delta$ and $\Psi \equiv x\Delta$. The author lays great stress on Stirling's numbers and employs them in his developments and results in much of the book. The next two chapters deal with the polynomials of Bernoulli, Euler and Boole, with applications to the summation of the powers of the natural numbers and of their reciprocals. In chapter 7 we meet the practical problem of interpolation and the author here derives and explains all the usual formulae, together with a method of his own which does not require the calculation of differences. The next chapter is entitled 'Approximation and Graduation', the first word being used as a synonym of Curve Fitting; the properties and applications of orthogonal polynomials are discussed and there is an interesting section on approximation by trigonometric functions. Chapter 9 deals with numerical solution of equations and quadrature, closing with a short discussion of the numerical solution of differential equations. The theory and application of the calculus of finite differences to problems involving two and three variables are dealt with in the following chapter, of only 13 pages, and are covered somewhat summarily. There is incidentally a curious set of correlated errors on the last page of this chapter where the algebraic coefficients of eight functions printed in vertical alignment have been allotted to them in the reverse of the correct order. The brevity of this chapter is more

than atoned for by the last two chapters of the book, which contain an excellent treatment of difference equations both complete and partial, including those involving as many as four variables. This is a particularly welcome inclusion, for works on difference equations are as rare as those on differential equations, for instance, are abundant: Steffensen in his *Interpolation* devotes only four pages to the subject which, as he points out, is properly beyond the scope indicated by his title. In several places in the work under review the author introduces problems in probability as illustrations; in particular he discusses the problem of Parours (better known to British readers as Random Walk) in as many as three dimensions, leading of course to difference equations in four variables, but we must warn the student of this subject that the discussion is on a very simple level and the classic complications of absorbing and reflecting barriers are not even mentioned.

The printing of this book cannot be praised. Small Greek letters are frequently indistinct and subscripts and indices even indecipherable in one or two places. Horizontal lines of division appear here and there to have been reproduced from a crude freehand stroke; and where a word of the earlier edition has been removed to improve the English there is now a meaningless blank space in the middle of a line. We would advise anyone intending to obtain the book to endeavour to secure a first edition whose printing is definitely superior.

The book contains an Introduction contributed by Prof. Henry C. Carver, of the University of Michigan, who predicts that it is destined to remain the classic treatment of its subject for many years to come. He regards it as the fourth of the main milestones in the development of the calculus of finite differences, the preceding three being in his opinion the works of Boole, Nörlund and Steffensen. British actuaries will probably be familiar at least with the first and third of these, and will think also of Whittaker and Robinson's *Calculus of Observations*, which though published as long ago as 1924 (the same year as Nörlund's book) remains one of the computer's practical handbooks. It is not easy to compare Jordan's work with 'Steffensen' or with 'Whittaker and Robinson'; it supplements both without replacing either. Steffensen is concerned primarily with interpolation and deals exhaustively with this aspect of the subject in particular, while Whittaker and Robinson's book is a treatise on numerical mathematics—not on finite differences—and includes such topics as correlation and Fourier analysis. Jordan, on the other hand, has written a treatise which abides faithfully by its own titular description and accordingly devotes itself to the calculus of finite differences, the whole calculus and nothing but the calculus.

M. T. L. B.

Australian Life Tables, 1946-48. By WALTER C. BALMFORD, F.I.A.

[Pp. 19. Published for the Rt Hon. the Treasurer by R. Wilson,
Commonwealth Statistician, 1950. No price.]

NATIONAL Life Tables for the Commonwealth of Australia, based on the census of 30 June 1947, and the deaths during the years 1946-48, have been constructed by the Commonwealth Government Actuary, Mr W. C. Balmford, F.I.A. The method of construction follows the pattern widely adopted for national life tables, viz. (a) the use of King's system of quinquennial pivotal values of population and deaths for the main body of the tables, the ratios for individual ages being inserted by osculatory interpolation; and (b) the completion of the tables at the

infantile ages by direct computation from registered births and deaths, and at the end of life by a Gompertz formula; with (c) special treatment to ensure a smooth junction between the rates at the two ends of the tables and the rates of the central sections from ages 16 to 81.

Though it is conventional to adopt three years as the period over which death-rates should be aggregated for a national life table—any longer period would perhaps have been precluded in the present instance by the influence of war conditions on the years 1945 and earlier—some figures might have been quoted to show whether the average of the three years gives a reasonable measure of the trend of the most recent mortality experience of Australia. The extent to which the mortality experience of a triennium may depart from the 'true' values may be illustrated from the course of the 'comparative mortality index' for all persons in England and Wales between 1921 and 1948. The three-year averages of these standardized civilian mortality rates from 1920-22 to 1947-49 (except for 1938-40 to 1941-43, which are affected by war conditions) show differences from the underlying trend, obtained by a simple smoothing process, of up to $3\frac{1}{2}\%$, and exceed 1% in nine of the twenty-four groups of three years.

Mr Balmford investigated, for infantile mortality rates, the errors occasioned by using the figures for births according to date of registration instead of actual date of birth and also by assuming an even distribution of deaths over the quarter year of age. The resulting conclusion that the error for the first year of life was an overstatement of only $\cdot00015$, or about $\frac{1}{2}\%$, seems to be hardly a sufficient reason why the correction, when ascertained, should not have been made, even though the correction was itself subject to minor assumptions that rendered it not necessarily precisely accurate.

The main characteristics of the life tables, in comparison with those for 1932-34, are summarized in the report as follows:

- (a) the mortality rates for the first year of life show a substantial reduction;
- (b) the vitality of both males and females up to age 40 shows a marked improvement;
- (c) a less pronounced improvement in vitality is shown for both sexes between ages 40 and 80, the male rates of mortality between 60 and 80 being, in general, in excess of those recorded in the 1932-34 tables;
- (d) had the methods of graduation been similar, the new mortality rates over age 80 for males would probably have been somewhat higher than, and those for females approximately equivalent to, the rates fourteen years earlier;
- (e) on the whole, the vitality of the female population shows greater improvement since 1932-34 than that of males.

It is interesting to compare the Australian experience with the experience of this country in the same three recent years. It is found that Australian mortality in the first year of life is nearly 30% lighter than it is in Britain for both boys and girls. During childhood the ratios to British mortality for the two sexes follow a very different course, those for boys rising till Australian mortality slightly exceeds ours between ages 10 and 20, and those for girls falling to little more than half ours between ages 15 and 20. After age 20 the ratios for males show a sudden fall to less than 90% of our rates from 25 to 35, but then rise again to rough parity with rates in Britain from age 40 onwards. The relative rise from age 20 is more rapid for females, equality with British rates being reached soon after age 35. Thereafter, Australian rates for females are, in general, rather more than 5% above our own.

The expectation of life at birth on the basis of the new Australian Life Tables is 66·1 for boys and 70·6 for girls, each being about a year more than the expectation of life for British babies. After age 10, however, the expectations of life for Australian males are almost the same as the British ones, and those for females start with a similar close agreement in childhood, but change to a defect of from a quarter to a half of a year for ages 25 and over.

It would be interesting to learn if there is any simple reason to account for the fact that, whereas in Great Britain the mortality of the two sexes between ages 15 and 25 is almost equal, in Australia the young women experience only about 60 % of the mortality of the young men. Comparison with the experience for Australia fourteen years earlier shows that the improvement has been much more rapid amongst females than males. This is, however, a matter for the Commonwealth Statistician rather than the Commonwealth Actuary, and we are indebted to Mr Balmford for a useful contribution to our knowledge of mortality trends among the races of the world.

W. S. H.