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ON THE RISK ADJUSTED DISCOUNT RATE FOR DETERMINING LIFE OFFICE APPRAISAL VALUES

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1. INTRODUCTION

1.1 A number of papers have been written in recent years that discuss the valuation of the business of a life office, examples of these are Burrows and Whitehead (1987) and Bunch (1986). These papers recognize the need to use a risk adjusted discount rate in the calculations. Burrows and Whitehead devote a section of their paper to the determination of such a rate. In their abstract they state that " . . . an entire paper could be devoted to this subject". This latter point is supported by the existence already of a number of papers in the finance literature on the very topic of determining risk adjusted discount rates e.g. Robichek and Myers (1966), Fama (1977).

1.2 This paper outlines some results of the theory of determining risk adjusted discount rates and the valuation of risky cash flows as developed in the finance literature, and discusses the application of this theory to the determination of life office appraisal values. Some comments are made on Section 3 of the Burrows and Whitehead paper.

2. OUTLINE OF THEORY

2.1 The theoretical approach to the valuation of uncertain cash flows that has been developed in the finance literature is the Capital Asset Pricing Model (CAPM). This model was originally derived by Sharpe (1964) and Lintner (1965) and coverage of the model and its use can be found in Copeland and Weston (1983). The model provides a basis for pricing uncertainty in cash flows and specifies the type of uncertainty that is expected to be priced, in equilibrium, under the assumptions of the model. It is most often expressed in rate of return form but can easily be rearranged in terms of cash flows.

2.2 The model has some important implications for valuing cash flows. These are that the riskiness that is priced into the value of a cash flow is determined by its covariance with the returns on all assets in the economy (the beta of the cash flow), not by its total variability, and there is a linear relationship between the risk adjusted discount rate used to value the cash flow and its beta.

2.3 To put these results in symbols we can firstly consider the one period case. Using the CAPM the value of an uncertain cash flow of X received at the end of the period can be expressed as;

$$V=\frac{E(X)}{[1+E(R_x)]}$$

where its present value is V and the risk adjusted discount rate is $E(R_x)$, given by,

$$E(R_x) = R_f + [E(R_M) - R_f] \times \beta$$

and R_f is the risk free rate, $E(R_M)$ is the expected rate of return on all assets in the economy and β is equal to

$$\frac{\text{covariance }(X, R_M)}{V \times \text{variance }(R_M)} = \frac{\text{covariance }(R_x, R_M)}{\text{variance }(R_M)}$$

The risk free asset is an asset whose return is not affected by economic conditions so that its beta is zero.

2.4 In words this result says that we discount the expected value of the cash flow at a risk adjusted discount rate which is determined as the risk free rate plus a margin for risk. The margin for risk is related to the covariance of the cash flow with the returns on all assets in the economy. In fact, the adjustment is the product of the difference between the expected return on all assets of the economy and the risk free rate, often referred to as the market risk premium, and a measure of covariability with market returns, the beta of the cash flow.

2.5 It should be noted that in the one period case the relationship between rate of return and cash flow is

$$1 + R_x = X/V$$

where the cash flow X is a random variable. This result allows the CAPM rate of return result to be expressed in cash flow form as above.

2.6 Some of the assumptions of the model are restrictive when it comes to practical application of the results, amongst these is the assumption that we live in a one period world. However, the model can be extended to a multi-period setting.

2.7 The paper by Fama (1977) examines the use of the CAPM in the valuation of multi-period cash flows. His results were:

(a) the current market value of any future net cash flow is the current expected value of the flow discounted at risk adjusted discount rates for each of the periods until the flow is realized;
(b) the discount rates for the periods preceding the realization of a cash flow need not be the same in each period and the rates for any given period can differ across cash flows;
(c) the risk adjustments in the discount rates arise from uncertainties about reassessments of the expected value of a cash flow and the relationship between these reassessments and the corresponding reassessments of the expected cash flows of all assets in the economy.

2.8 Fama's results come from applying the one period CAPM result recursively to get the current value, at time 0, of a single cash flow in t periods time. In symbols:

$$V_o = E_o\,[X_t]\left(\frac{1}{1+E(R_l)}\right)\cdots\left(\frac{1}{1+E(R_t)}\right)$$

where $E_k\,[X_t]$ is the expected value of the cash flow at time k which is reassessed over time and is therefore not necessarily constant over time, and $E(R_k)$ is the risk adjusted discount rate for time period k given by:

$$E(R_k) = R_{fk} + [E(R_{Mk}) - R_{fk}] \times \beta_k$$

and

$$\beta_k = \frac{\text{covariance}\,(R_k\,,\,R_{Mk})}{\text{variance}\,(R_{Mk})}$$

It should be noted that the risk free rates and the beta values can vary in each period, and for different cash flows, but that the application of the CAPM to the multi-period case assumes that they are non-stochastic through time.

2.9 Another assumption of the CAPM that can be restrictive in practical applications is the allowance for taxation. The initial development of the CAPM ignored taxation. However, because the CAPM is based on the actual returns realized by investors it is effectively based on after tax returns when we allow for the existence of taxes in the model. The taxes to be allowed for are both company and personal taxes as investors will receive their returns net of both these taxes. Bowers and Ball (1987) discuss the adjustments to the CAPM to allow for taxes.

2.10 Under the Classical tax system equity returns are subject to taxation as profit at the company tax rate and as dividends in the hands of investors at the personal tax rate. Under an imputation system, as exists in the United Kingdom and as has recently been introduced in Australia, this double taxation of dividends is removed. Hence the adjustment to the CAPM for taxes will depend on the tax regime in the country where the asset is taxed.

2.11 If we assume that the company tax rate is T_c, the personal tax rate for interest income is T_d and the personal tax rate for equity returns is T_e, then Bowers and Ball give the risk adjusted interest rate as:

$$E[R_x] = R_f\,(1-T_d)/(1-T_e) + [E(R_M) - R_f\,(1-T_d)/(1-T_e)] \times \beta$$

where the returns R_x, R_M and R_f are the actual returns observed in the market. Hence R_x and R_M are net of company tax. The tax rates used are the effective rates applying to the investor and are adjusted to allow for the effect of financial intermediaries, such as life offices and superannuation funds, on these rates. These tax rates are those that apply to the marginal investor who is assumed to set the risk adjusted rate required. The beta value is defined as previously since the effect of tax is assumed to affect cash flows and total market returns in an identical way.

2.12 Under an imputation system the assumption could be made that T_e is zero since this is the aim of such a system. The risk adjusted interest rate would then simplify further.

2.13 In order to calculate values of cash flows that represent a combination of different types of cash flows, each type having a different covariance with the returns on all assets, it is necessary to use a different risk adjusted discount rate for each type of cash flow. Once values for each type have been determined they can be combined into one value by adding them together. If a single risk adjusted discount rate is to be applied to the sum of the cash flows then it is necessary to choose an appropriately weighted risk adjusted discount rate that allows for the different betas of the different types of cash flows. If the combination includes positive cash flows, such as revenues, and negative cash flows such as expenses then the covariance of the negative cash flows could well offset the covariance of the positive cash flows resulting in a reduction of riskiness of the total cash flows.

3. APPLICATION TO LIFE OFFICE APPRAISAL VALUES

3.1 The application of the above theory to life office appraisal values leads to the following points:

(i) expected values of future cash flows are taken into account and there is no explicit allowance for the variance of these cash flows;
(ii) cash flows with a different covariance with the returns on all assets in the economy should be discounted at different rates or, alternatively, a weighted risk adjusted discount rate should be used for the total cash flow that allows for the different covariance of the component cash flows;
(iii) allowance for tax needs to be made and the adjustment will depend on the tax regime that applies to the asset.

3.2 From point (i) we can note that standard actuarial techniques that use expected values in determining survivorship, death, lapse and surrender rates are used to determine the cash flow to be discounted at the risk adjusted discount rate. We do not need to allow for the total variability of the cash flows.

3.3 This result may seem counter intuitive. The explanation is that the owners of the cash flows of a life office should not be given any value to offset risks that they can otherwise avoid or reduce. The variability of cash flows that arise from small volumes of business can be reduced by increasing the number of policies or by reinsurance. The law of large numbers operates to reduce the variability of the cash flows of a life office. This reduction could take place within the life office, by it writing larger volumes of business, or outside the office by owners of the cash flows taking interests in a number of life offices or through the use of reinsurance.

3.3 This would suggest that, if the theory holds true, we would not expect the value of a small office, expressed as a per policy figure, to differ from that of a large office if they were identical in every other respect i.e. write much the same sort of business and follow much the same management practices.

3.4 Point (ii) highlights the need to value the different types of cash flows separately and to sum their individual values to obtain a total appraisal value. The different types of cash flows, ignoring tax, are premium income, expense outgoings, death claim payments, surrender payments and changes in reserves. If we consider the cash flows at a particular point of time and let:

NCF be the net cash flow,
P be premium income,
E be expense outgoings,
D be death claim payments,
S be surrender payments, and
R be the change in reserves,

we have
$$NCF = P - E - D - S - R$$
and
$$V = \frac{E(NCF)}{1+E(R_x)}$$

where $E(R_x)$ is the risk adjusted discount rate for the net cash flow given by
$$E(R_x) = R_f + [E(R_M) - R_f] \times \beta$$
with
$$\beta = \frac{\text{cov}(NCF, R_M)}{V \times \text{var}(R_M)}$$

3.5 We also have that
$$V = V_p - V_e - V_d - V_s - V_r$$
where
$$V_p = \frac{E(P)}{1 + E(R_p)}$$
with
$$E(R_p) = R_f + [E(R_M) - R_f] \times \beta_p$$
and
$$\beta_p = \frac{\text{cov}(P, R_M)}{V_p \times \text{var}(R_M)}$$
with similar definitions for V_e, V_d, V_s, V_r.

3.6 From the definition of β we have that

$$\beta = \frac{\text{cov}(NCF, R_M)}{V \times \text{var}(R_M)}$$
$$= \frac{\text{cov}(P, R_M) - \text{cov}(E, R_M) - \text{cov}(D, R_M) - \text{cov}(S, R_M) - \text{cov}(R, R_M)}{V \times \text{var}\,(R_M)}$$
$$= \frac{V_p \times \beta_p - V_e \times \beta_e - V_d \times \beta_d - V_s \times \beta_s - V_r \times \beta_r}{V}$$

Hence β for the net cash flow is a weighted average of the β's of the component cash flows using the ratio of the component present values to the net cash flow present value as weights. It can be demonstrated that, if the weighted average β is used to obtain the risk adjusted discount rate for the net cash flow, then the present value of the net cash flow is equal to the sum of the component cash flow present values, with each of these present values determined using the appropriate risk adjusted rate for that cash flow. This is left as an exercise for the interested reader.

3.7 The covariance of the cash flows with returns on all assets needs to be determined. Estimates can be made with past data from the particular life office under consideration using simple linear regression techniques. The returns available on the stock market index could be used as a proxy for the returns on all assets in the economy. This is the most often used proxy for empirical work with the CAPM.

3.8 It might be expected that most of the cash flows have very low correlation with the market return on all assets. Premium receipts and death claim payments are unlikely to be significantly affected by market conditions since their values will depend primarily on mortality which will be reasonably independent of market returns. Tax payments should also be reasonably independent of market returns on all assets. Surrender payments are the most likely to be correlated to market returns. High returns on assets may encourage investors in life insurance policies to surrender and invest elsewhere if they hold traditional policies. Unit linked policies are unlikely to show the same correlation.

3.9 One of the features of life office cash flows that needs to be allowed for is the inter-dependence of the cash flows across time. Higher levels of surrender payments than expected will result in lower than expected premium receipts and death claims in future periods. Uncertainty in the expected values of cash flows will diminish over time as the actual experience is revealed. As Fama pointed out this uncertainty is allowed for in the risk adjustments in the discount rates. In order to do this using the theory presented in this paper a model is needed to represent the process by which this uncertainty changes over the period until the cash flow is realized.

3.10 The adjustments for tax, as mentioned in point (iii), will depend on the tax regime that applies to the cash flows. The cash flows to be valued need to be after tax cash flows. After tax risk adjusted rates need not be the same for appraisal values of United States of America life offices and, say, U.K. life offices because of differences in tax rates and because of differences in the taxation of interest and equity returns. Similarly, a change in taxation arrangements at a particular point in time, such as a change to an imputation system from a Classical tax system, will alter the risk adjusted discount rates.

3.11 The risk adjusted rate should vary across time periods. Provided risk free rates for different maturity dates are similar and the reassessment of present values of the cash flows is likely to be stable over time then a constant rate across time periods should be adequate in practice.

3.12 Consideration needs to be given to the purpose of the determination of the appraisal value. As discussed in this paper the value in the hands of the owners of the cash flows, the shareholders in a proprietary office and the policyholders in a mutual office, will incorporate an allowance for personal taxes.

Personal taxes will generally not be taken into consideration in determining values of such cash flows on an aggregate basis. A justification for this could be that the owner of the cash flow is faced with similar personal tax effects on other alternate cash flows and so the relative values are not affected by personal tax. In practice, the calculation of appraisal values using the theory in this paper will be based on total cash flows and will allow only for corporate taxes. However, it should be pointed out that there may be other components of value that arise from the personal tax treatment of the owners of the cash flows.

4. COMMENTS ON BURROWS AND WHITEHEAD

4.1 There are a number of points raised in Section 3 of Burrows and Whitehead that are worthy of mention. In discussing rates of return they refer to a U.S. study to illustrate the risk premiums for equities. Because of the impact of tax, and other factors unique to a particular economy, it is important to use data on returns relevant to the cash flows to be valued. The returns on U.S. equities may not be relevant for valuing a U.K. life office. In fact the risk premium for U.K. equities, according to Bowers and Ball, has averaged 12·1% over the years 1973–82. This was a period when the imputation system was in operation. In the period 1963–72 the U.K. market risk premium averaged 9·1%.

4.2 Burrows and Whitehead also refer to the volatility of returns to quantify the riskiness of equities. In order to value uncertain cash flows the theory outlined in this paper suggests that we need to consider the riskiness of cash flows in terms of risks that can not be eliminated or reduced. Allowance should only be made for these risks in the appraisal value and higher returns will not be given for risks that can be appropriately eliminated.

4.3 They consider the following risks which may have an impact upon the earnings of a life insurance company; exogenous economic forces, performance of management and risk inherent in insurance products. Some of these risks can be eliminated or reduced by the owners of the cash flows of the life office. For example, reference is made to the extent of mismatching of assets and liabilities. This is a risk that could be reduced and should not be allowed for in the appraisal value. If some allowance is made, and we assume that the office is a proprietary office, then the company could be bought at the lower value required because of the added risk, a matching policy introduced and the company then sold at a higher value reflecting the reduced risk. A guaranteed profit with no risk!

4.4 One of the exogenous economic forces mentioned is inflation. Allowance for this will be made in the risk free interest rate used, which will incorporate a premium for expected inflation, and not necessarily in the risk adjustment. Inflation will affect the returns on all investments and will not make one investment riskier than another on its own. Inflation may affect the market risk premium required for equity investments as a whole, and hence the risk adjustment, but it will not affect the covariance of the cash flows with returns on all assets.

4.5 Mention was made earlier in this paper of the need to carry out some empirical work to determine the riskiness, in terms of covariances of cash flows, of a life office so as to calculate a risk adjusted discount rate. Burrows and Whitehead mention that the most commonly used risk discount rate over the last decade has been 15%. This figure is also mentioned in Bunch. This rate can be used to infer something about the riskiness of life office returns. The current yield on a U.K. 10 year government security is about 9%. This can be used for the risk free rate since life office cash flows will be long term and the rate will incorporate an allowance for inflationary expectations. The market risk premium is assumed to be 12% and if we allow for company tax but ignore personal tax the β implied is given by:

$$0{\cdot}15 = 0{\cdot}09 + [0{\cdot}21 - 0{\cdot}09] \times \beta$$

or

$$\beta = 0{\cdot}5.$$

4.6 An approximation of the risk adjusted discount rate for an Australian office could then be made by assuming that the riskiness of an Australian life office is the same as for a U.K. office. The risk free rate on 10 year government bonds in Australia is approximately 13% and Bowers and Ball gave the market risk premium as 8%. Assuming that under the imputation system introduced on 1 July 1987 the market risk premium does not change, the risk adjusted rate is:

$$0{\cdot}13 + [0{\cdot}21 - 0{\cdot}13] \times 0{\cdot}5$$

or 17%. Of course the introduction of the imputation system could result in changes to the market risk premium which will affect the risk adjusted rate.

5. CONCLUSIONS

5.1 This paper has outlined the theory developed in the finance literature that can be applied to the determination of risk adjusted discount rates for life office appraisal values. The application of this theory leads to some results that should be allowed for in the life office case. These are:

(i) expected values of cash flows are used and no specific allowance for total variability of cash flows is made;

(ii) risk adjusted discount rates are determined as a risk free rate plus a margin for risk based on the market risk premium and the covariance of cash flows with returns on all assets;

(iii) the risk adjusted rate will be different for different periods, especially if the risk free rate varies with maturity;

(iv) the riskiness of life office cash flows, as given by their covariance with returns on all assets, will be a weighted average of the riskiness of the component cash flows, with negative cash flows offsetting positive cash flows;

(v) the tax regime of the country in which the cash flows occur will affect the rate and, hence risk adjusted rates will be country specific.

5.2 This paper will hopefully allow actuaries to adopt a scientific approach to the calculation of appraisal values, at least in the determining of risk adjusted discount rates.

REFERENCES

BOWERS, J. & BALL, R. (1987) The Cost of Equity Capital Under the New Imputation Tax System. Draft. Australian Graduate School of Management.

BUNCH, T. S. (1986) The Valuation of With-Profits Business and the Estate. Paper presented to the Staple Inn Actuarial Society on 2nd December 1986.

BURROWS, R. P. & WHITEHEAD, G. H. (1987) The Determination of Life Office Appraisal Values. *J.I.A.*, **114,** 411.

COPELAND, T. E. & WESTON, F. J. (1983) *Financial Theory and Corporate Policy*, (2nd Edition). Addison-Wesley.

FAMA, E. F. (1977) Risk-Adjusted Discount Rates and Capital Budgeting Under Uncertainty. *Journal of Financial Economics*. **5,** 3–24.

LINTNER, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *Review of Economics and Statistics*. Vol XLVII, No 1, 13–37.

ROBICHEK, A. A. & MYERS, S. C. (1966) Conceptual Problems in the Use of Risk Adjusted Discount Rates. *Journal of Finance*. 727–730.

SHARPE, W. F. (1964) Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk, *Journal of Finance*. Vol XIX, No 3, 425–42.