Risk & the Financial Crisis: Why we should look at the data rather than make excuses about ‘tsunami’ events.

9 March 2010

William T. Shaw

`william.shaw@kcl.ac.uk`

King’s College, The Strand, London WC2R 2LS

Overview I

There is a very short answer to the question posed by the seminar title.

Given an $\alpha\sigma$ movement, we can create a model to say this is a "one in N year" event for many choices of N by adjusting the probability distribution. You all know this.

There is a bias in those who have to spend the money on insurance to go thin-tailed so they can save money and also blame any subsequent failure of risk control on matters being ‘too extreme’.

Recent famous excuses

Hilary Benn, the Environment Secretary, on the Nov 2009 floods: whilst the area's flood defences had been built to withstand a "one in 100 year" flood, "what we dealt with last night was probably more like one in a thousand, so even the very best defences, if you have such quantities of rain in such a short space of time, can be over-topped".

David Viniar, CFO, Goldman Sachs: We were seeing things that were 25-standard deviation moves, several days in a row

Excuses ctd

We had it again in January with the snow and it not happening often enough here to justify buying the snow-ploughs that many other countries seem happy to buy. High cost of snow ploughs $\sim$ trader reluctance to have cash not working and sitting in capital reserve insurance account.

The environment-financial links are many - look up Paul Embrechts' wonderful Nomura lecture: "From Dutch Dykes to Value at Risk".

The financial mathematics community has not done enough to resist the thin-tailed bias, by continuing in some areas with a Gaussian philosophy, rather than paying attention to the statistics of the data.

Overview II

I wish to thank Ralph for his excellent survey.

In my talk I will try to drill down to some particular examples and issues raised by the management of this in the financial markets, including the historical mistakes and looking what we might do better for some key risk-drivers, e.g. index returns.

Overview III: Ralph's list

- Market consistent? Poor evidence that implied volatility can predict realized volatility. Biased, inefficient, different, **Strike-dependent!!**
- Make best possible use of the data, bearing in mind sparsity of tail
- Try to underpin with a model, but realize the model is neither necessary nor sufficient
- Experts best used for the itemization of non-distributional risks and consequence calculation - do we then insure?

Too much emphasis on Gaussian *marginals* and excuse it by treating the rest as "tsunami-class". Good academic FM/stats research ignored.

An incomplete time-line

- Observations by Mandelbrot and Fama 1960s,

An incomplete time-line

- Observations by Mandelbrot and Fama 1960s,
- Asset (log-)returns are not Gaussian,

An incomplete time-line

- Observations by Mandelbrot and Fama 1960s,
- Asset (log-)returns are not Gaussian,
- 1973: Black-Scholes Log-Normal model,

An incomplete time-line

- Observations by Mandelbrot and Fama 1960s,
- Asset (log-)returns are not Gaussian,
- 1973: Black-Scholes Log-Normal model,
- 2003: **Nature** Studies Gabaix, Gopikrishnan, Plerou, Stanley.

An incomplete time-line

- Observations by Mandelbrot and Fama 1960s,
- Asset (log-)returns are not Gaussian,
- 1973: Black-Scholes Log-Normal model,
- 2003: **Nature** Studies Gabaix, Gopikrishnan, Plerou, Stanley.
- Claimed universal cubic CDF tails.

An incomplete time-line

- Observations by Mandelbrot and Fama 1960s,
- Asset (log-)returns are not Gaussian,
- 1973: Black-Scholes Log-Normal model,
- 2003: **Nature** Studies Gabaix, Gopikrishnan, Plerou, Stanley.
- Claimed universal cubic CDF tails.
- Fergusson Platen, **AMF** 2006,

An incomplete time-line

- Observations by Mandelbrot and Fama 1960s,
- Asset (log-)returns are not Gaussian,
- 1973: Black-Scholes Log-Normal model,
- 2003: **Nature** Studies Gabaix, Gopikrishnan, Plerou, Stanley.
- Claimed universal cubic CDF tails.
- Fergusson Platen, **AMF** 2006,
- MLE amongst Generalized Hyperbolic

An incomplete time-line

- Observations by Mandelbrot and Fama 1960s,
- Asset (log-)returns are not Gaussian,
- 1973: Black-Scholes Log-Normal model,
- 2003: **Nature** Studies Gabaix, Gopikrishnan, Plerou, Stanley.
- Claimed universal cubic CDF tails.
- Fergusson Platen, **AMF** 2006,
- MLE amongst Generalized Hyperbolic
- Found Student T_4 for daily log index returns!

An incomplete time-line

- Observations by Mandelbrot and Fama 1960s,
- Asset (log-)returns are not Gaussian,
- 1973: Black-Scholes Log-Normal model,
- 2003: **Nature** Studies Gabaix, Gopikrishnan, Plerou, Stanley.
- Claimed universal cubic CDF tails.
- Fergusson Platen, **AMF** 2006,
- MLE amongst Generalized Hyperbolic
- Found Student T_4 for daily log index returns!
- Do not agree on detail, but kurtosis poor tool. Kurtosis theoretically ∞, unstable calibration tool.

Further evidence

- D. Taylor..., 2009, SA equities, VG and T
- Breymann, Luthi, Platen **EPJB** 2009, Multiple time-scales, T good for longer

Clear **options for marginals**. We must also think about

- **Choice of risk measure: VaR, CVaR, Ω,...**
- **Event frequency choice: which quantile?**
- **Dependency models**

There is poor understanding of the relative impact of these choices. But first look at the marginals choice and getting at the fat tails.

Fat Tails Levels 1 and 2

Level One is just getting them into routine risk management, without necessarily tying up all the issues it generates with option pricing. Based on the *data statistics*.

Level 2 is about having a comprehensive tie up, within each asset class, of observed data, basic maths, option pricing and risk management. This is applied maths view - you are modelling reality in self-consistent manner, though it is the human financial world - predictions are distributional.

There is no excuse for the past R&B failure to implement level one. Some have tried to kick it into the domain of UUs. There may be “tsunami” events, but there is no excuse for ignoring data.

L1: What do we have to do?

Fergusson & 2006 (AMF). Daily index log-returns MLE Student T_4 amongst hyperbolic distributions. Relation to BU/MIT study? FP work considered Generalized Hyperbolic Distribution and did MLE amongst that. Found T_4. CDF tail x^{-4}. See paper in EPJ-B also.

Student is one of many conditionally Gaussian distributions. Not that new! W. Gosset derived it in 1908 essentially using a variance that is inverse gamma, in his case arising as a sample-estimated variance.

What do we have to do 2

For Student T: I publicised the older methods in survey Shaw, J Comp Fin 2006. All known bar ratio of uniforms.

Ralph Bailey (1994!): how to modify Box-Muller/polar form to move from Gaussian to Student T. **ONE LINE OF CODE to change in a widely used banking algorithm.** Also $\nu < 0$.

Hill (1970) produced approximations for Student quantile(NAG). Shaw JCF 2006 produced some closed form quantiles (1,2,4) and series (3...) Quantile Mechanics gives us simulation for all $\nu \geq 1$.

Hyperbolic, Stable forms being pursued by many colleaues. But Student pragmatic and in data.

Yet, in June 2009

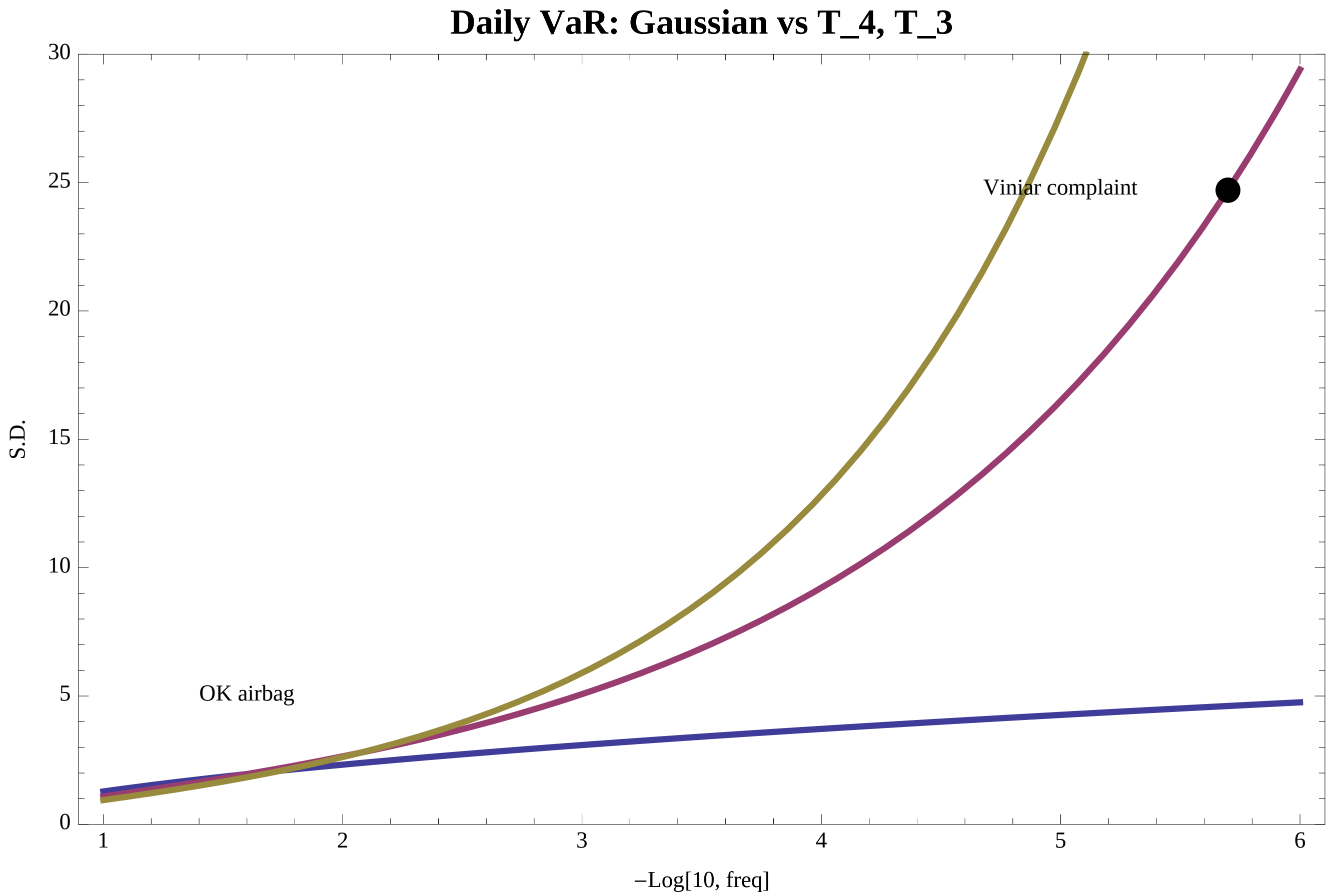
In the FT of 10th June 2009, Lord Turner, Chair of the FSA, is quoted as follows:

The problem, he said, was that banks' mathematical models assumed a "normal" or "Gaussian" distribution of events, represented by the bell curve, which dangerously underestimated the risk of something going seriously wrong.

While there is always the unpredictable tsunami event outside scope of historical data (the excuse), even the routine modelling from history was wrong. A 25 sigma event is over 10^{130} times more likely in T_4 than in Gaussian.

Running the numbers

Let's be univariate + VaR/quantile. Critical issue is distribution and frequency. If you pick Normal or T_4 the 2.5% quantile is 1.96! (working Gaussian airbag) Ignore mean shifts and variance explosion.



Practicalities

Those plots use inverse beta functions in Mathematica. Nice series for quantiles (EJAM 2008). Some closed-form quantiles in Shaw, JCF 2006 (see Wikipedia on quantile functions). For many apps Bailey's method will be more readily deployed: Bailey, R.W., 1994, Mathematics of Computation 62 (206), 779-781.

```
BaileyStudent[n_] :=
 Module[{W = 2, u, v, U, V},
  While[W > 1, (u = RandomReal[];
  v = RandomReal[];
    U = 2 u - 1; V = 2 v - 1;
    W = U^2 + V^2)];
  U*Sqrt[n (W^(-2/n) - 1)/W]]
```

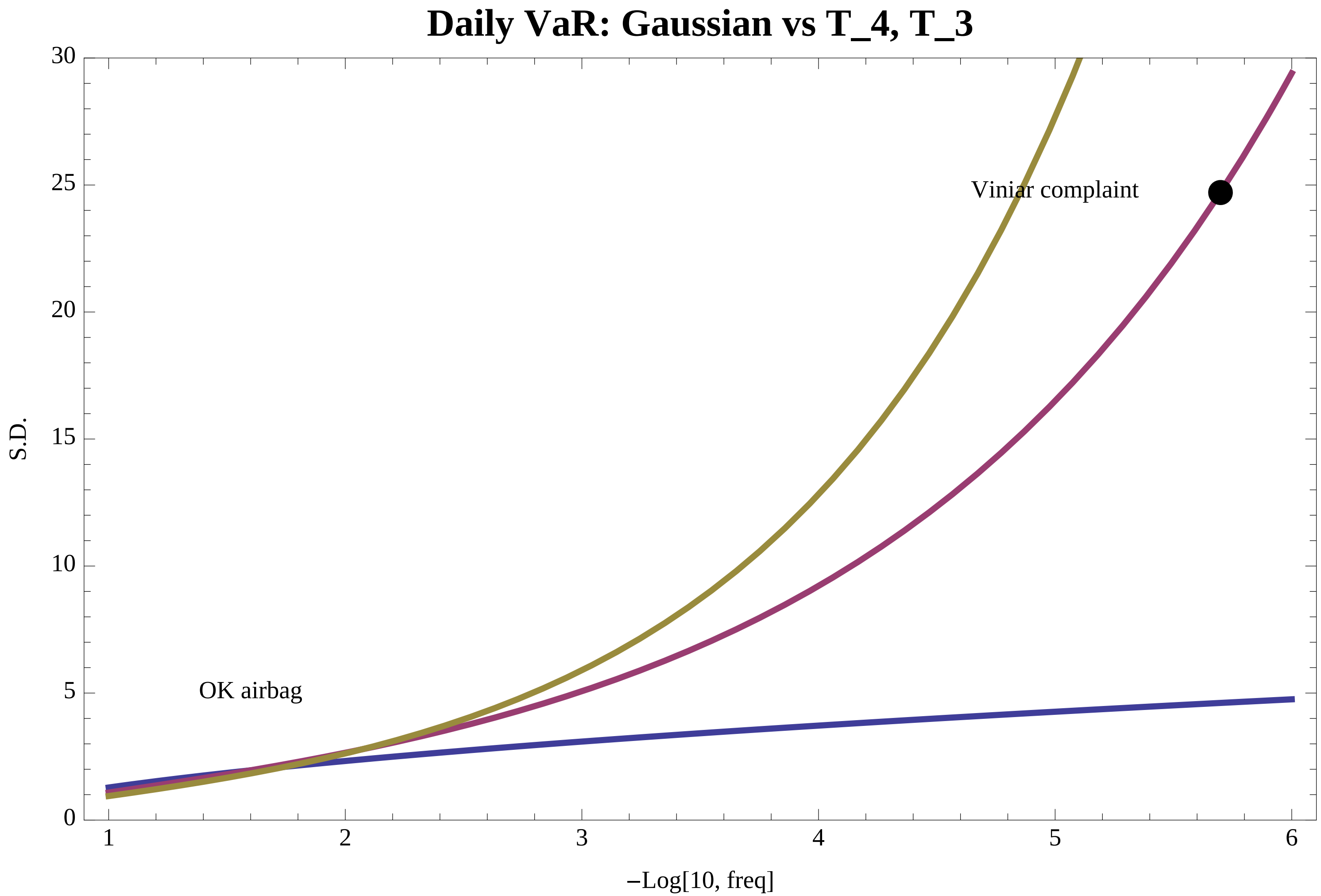
Other Risk Measures

So going fat-tailed with a distribution with strong roots in the data is easy. Will return to its theoretical underpinning. Now what about the risk measure?

Much fuss over use of VaR - can replace by CVaR - expected loss given you are in the tail. It amplifies risk numbers - tilts up the T curves, but is not as critical as swapping the marginal and reporting low frequency events. No argument from me about aggregative properties.

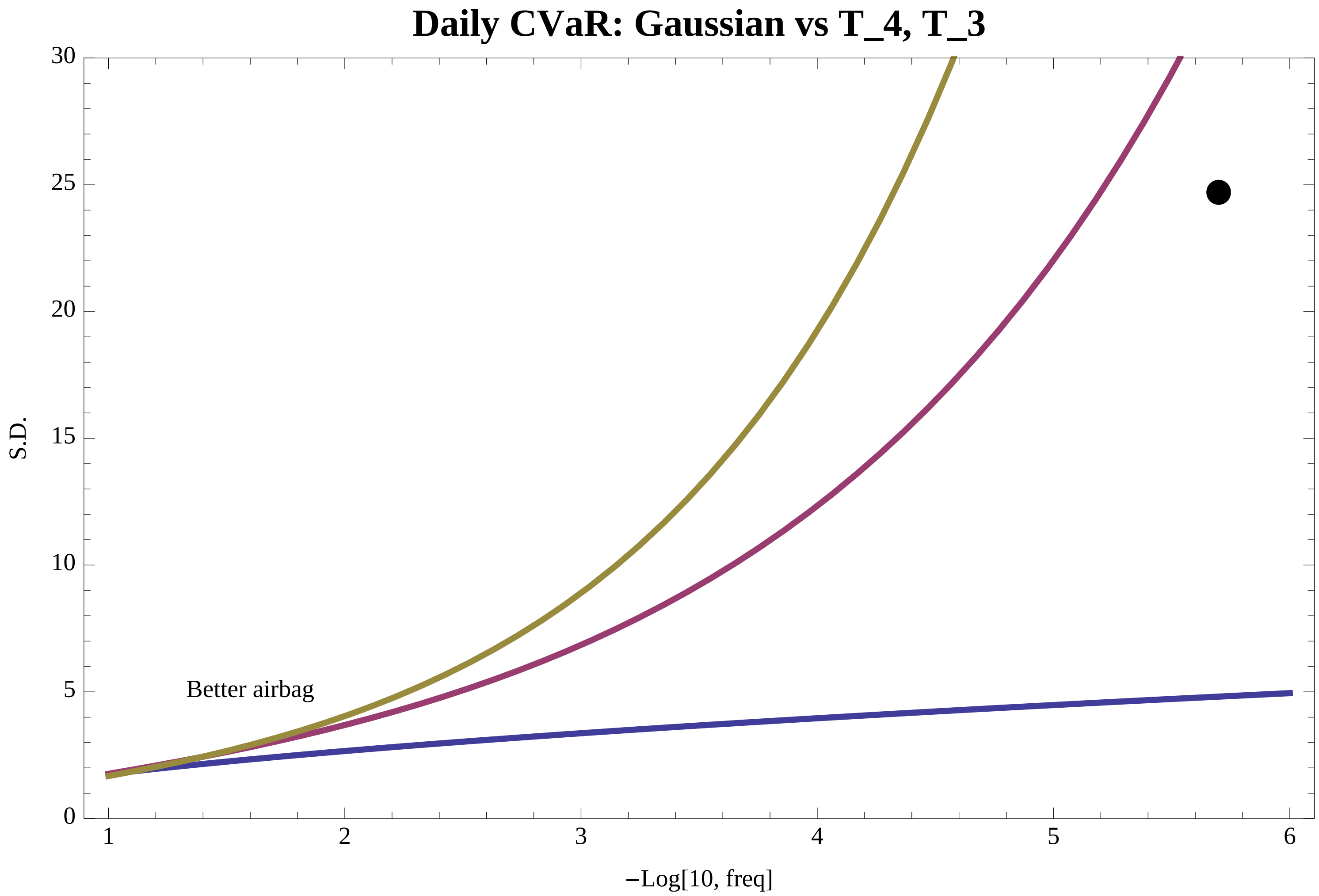
IT IS NOT A PANACEA!

VaR vs CVaR



Daily VaR: Gaussian vs T_4, T_3
30
25
20
15
10
5
0
S.D.
Viniar complaint
OK airbag
1
2
3
4
5
6
−Log[10, freq]

VaR vs CVaR



Daily CVaR: Gaussian vs T_4, T_3
S.D.
30
25
20
15
10
5
0
1
2
3
4
5
6
−Log[10, freq]
Better airbag

So what matters?

Replacing VaR by CVaR will produce more conservative numbers, but in the Gaussian world the numbers change by a tiny amount. What really matters is going fat on the marginals and reporting lower frequency risk numbers.

Note how little the Gaussian curved moved when we replaced VaR by CVaR.

Combination of T+CVaR gives better model resolution, even at 1% levels.

n.b. These t-quantiles are all based on unit variance models, not raw t.

Are there balls and urns?

So this is all based on data. Can we underpin with theoretical model and bolster the sparse tail data. The answer is yes - you can see my preprint: *A model of returns for the post-credit-crunch reality*, `http://arxiv.org/abs/0811.0182.`

to see the theory. Not all new with me but the rooting in technical trading and new dynamical PDFs are. The idea is to include a form of **price-feedback** in the governing SDEs, to model the presence of "technical trading"'. In general these are complex path-dependent rules, but we take simple non-Markov version.

The governing SDEs

Making assumptions (including linearizing a lot), we are lead (in the absence of jumps) to the following for intraday. Let $X_t = \log(S_t/S_0)$. Then, for some f, g with $f(0) = g(0) = 0$,

$$dX_t = (\mu_1 - f(X_t))dt + \sigma_1 dW_{1t} + g(X_t)dW_{2t}$$

With one set of simplifying assumptions, we are led to (technical trade arrival rate fixed, further linearization, Schofield)

$$dX_t = (\mu_1 - \mu_2 X_t)dt + \sigma_1 dW_{1t} + \sigma_2\sqrt{|X_t|}dW_{2t}$$

This is a hybrid ABM-CIR process. Investigation under way.

The ABM-GBM Hybrid

Linearization form of technical component:

$$dX_t = (\mu_1 - \mu_2 X_t)dt + \sigma_1 dW_{1t} + \sigma_2 X_t dW_{2t}$$

where the arithmetical terms arise from the fundamental (price insensitive) trades and the geometric terms arise from technical (price-sensitive) trades. W_{it} are standard BMs.

I first saw this in recent plasma physics literature (GS), but such hybrids go back some way in statistical physics lit.

The link to Pearson-Diffusions

If ρ is the correlation between the two Brownian motions, then we can write the SDE as

$$\begin{aligned} dX_t =& (\mu_1 - \mu_2 X_t)dt \\ &+ \sqrt{\sigma_1^2 + X_t^2\sigma_2^2 + 2\rho\sigma_1 X_t \sigma_2^2}\, dW_t \ . \end{aligned} \tag{1}$$

This is one of the class of “Pearson diffusions” considered by Nagahara, and Forman and Sørensen. This sub-family generates Pearson Type IV and Student equilibria. Dynamics under actiev study. Idea, with change of variables, goes back to Wong (1953). You can also build multivariate heterogenous. **But we have underpinning of power laws**.

Brief remark on copula choices

This talk has mainly been about marginals. However, the dependency structure matters too. There are many types of dependency:

- Company A has a real influence on company B
- Both A, B influenced by common external factor
- Spurious associations (same sector)

It is clear that a correlation number does not capture all the possibilities. This is not an excuse for throwing rocks at those who tried to come up with models to couple systems in a tractable manner.

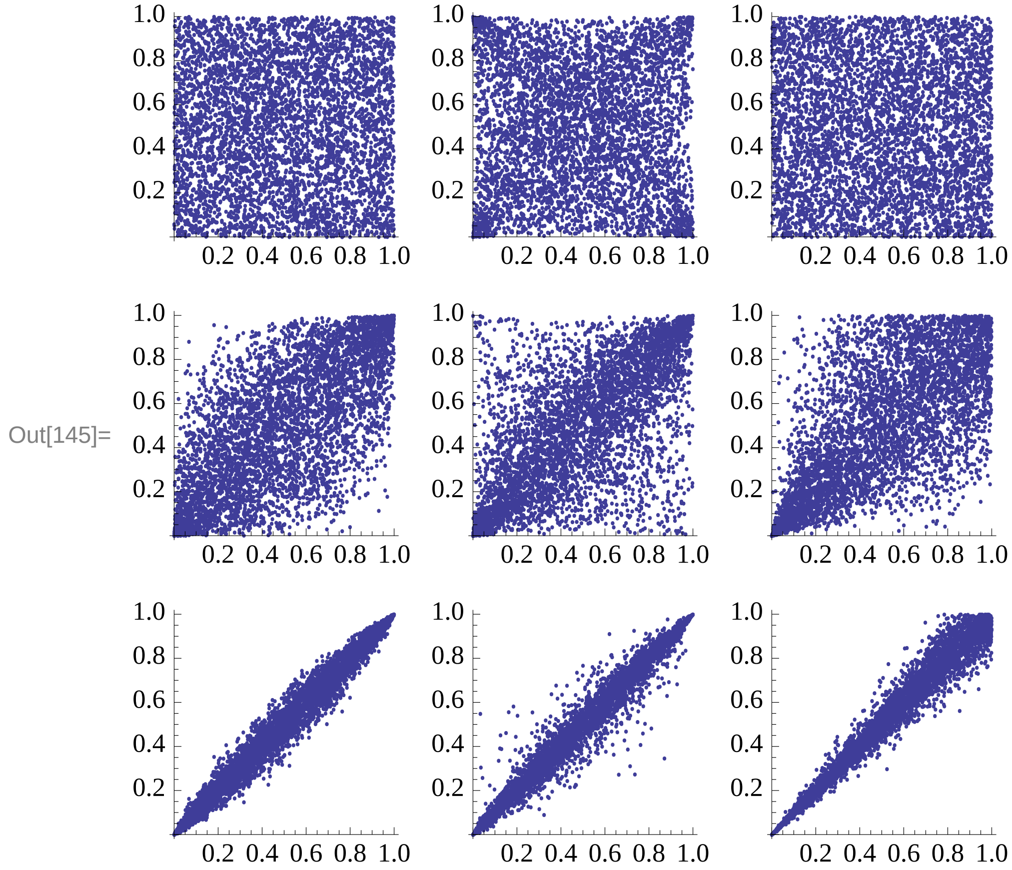
Who Killed Wall Street?

A low point in the discussion of the crisis has to be Felix Salmon's hype in Wired Magazine. Despite getting quotes from Wilmott and Taleb that, when read carefully, made it pretty clear that the real problem was a naive (constant, historical, poor) choice of correlation, Salmon tried to pin a lot of blame on the Gaussian copula.

Exercise for the viewer: Sample from Gauss, T (pick your own dof) Clayton copulas with Kendall's $\tau = 0, 0.5, 0.9$ and measure your favourite risk with function the same marginals. Deduce that the Gaussian copula is not the problem!

Copulae with Tau-spiking

Consider Gauss, T_2, Clayton (horizontal) with $\tau \sim 0, 0.5, 0.9$ (vertical).



Down tail starts to look the same.

Summary I

In this talk we have explored some statistics-based choices of marginal for index data. It is not Gaussian. It is easily modelled and the frequency-risk computations are easy. There is stats support and model underpin for Student T, but also other models. **There are serious options for stressing risk computation**.

With a decent marginal based on the history, high-σ movements are relatively frequent. There is no justification for burying the head in the sand and claiming it's a Gaussian and the odd tsunami. This is irresponsible and bad science. We should make best use of the data.

Summary II

Replacing VaR by CVaR helps resolve tail effects, is not a panacea and needs a sensible marginal.

Depedency is complex. You can do a lot by allowing for correlation-spiking.

We are looking at

- Statistics of data;
- Linking that to models;
- Network models (TDM), structural credit;
- Simulation methods;
- Real-world risk control : Optimal asset-allocation with general marginals, dependency and risk (objective) function.

Thank you!