**Risk free rates and the calculation of realistic balance sheets**

David Dullaway and Andrew Smith

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This note sets out our thoughts on the appropriate choice of risk free rates to use when calculating realistic liabilities for realistic balance sheet ("RBS") reporting purposes. It is intended to set out the issues and stimulate discussion rather than provide a definitive answer.

## Definition of realistic liabilities

The appropriate risk free rate assumption depends at least in part on the definition of realistic liabilities adopted. Two possible choices present themselves: a "fair value" and a "hedging cost". The definition for regulatory purposes is a policy question rather than a technical one, and we look forward to the FSA's decision.

A fair value approach means we are trying to answer the questions "what value do investors put on the liabilities of an insurance company?" or "if the liabilities were traded, what price would they trade at?" We ignore questions about whether it would be theoretically possible to actually hedge the liabilities.

The alternative is to adopt a "hedge cost" definition of the realistic liabilities. This asks the question "how much would it cost to hedge the liabilities of an insurance company?"

The fair value approach is probably the right one for market-consistent reporting to shareholders (and associated uses such as pricing and internal management reporting), as it concentrates purely upon value. It is less clear whether it appropriate for prudential regulation, in that a company holding assets just equal to the fair value of liabilities, i.e. just solvent, would not necessarily have sufficient monies to allow it to move to a matched position if the need arose.

Unlike the fair value approach the hedge cost approach explicitly ensures that a company having assets equal to its realistic liabilities would be able to move to a matched position if the need arose. A hedging approach would usually give a higher liability than the fair value approach because it recognises the need to pay margins to third parties to put hedges in place. On the other hand, if the hedging approach is required to hedge only market (and not credit risk), then the hedging approach may give a lower answer in cases (such as fixed liability flows) where the effect of credit risk on discount rates exceeds the transaction costs of implementing the hedge.

The bank paradigm (for traded instruments) is closest to a hedging approach; inter-bank instruments are valued at a market price which reflects inter-bank credit risk, but in most cases a mid-market valuation is used.

## Implications of a "fair value" approach

In practical terms, a fair value would imply that we value liabilities in line with mid-market prices and that, where an appropriate hedging asset does not exist, we simply try to establish what it would cost if it were to exist. To do this requires a

"true" risk free rate (which we need to derive). We may find that observable instruments all deviate from risk-free for one reason or another, and in that case the “true” risk free rate may be a theoretical construction which does not correspond to any single investable instrument.

This is the area of the risk-free rate discussion that has received the most attention to date, as there are a number of candidates for a true risk free rate, and it is unclear which, if any, of them should be adopted. We discuss this at length later in this note.

Moving on from risk free rates to option prices, it is a point for discussion as to whether "profit margins" would need to be stripped from option prices; the key question being whether any such margins exist in the mid-market price or whether they are only present in the bid-offer spread. The treatment of any credit risk embedded in option also requires discussion, and we treat this below. If the FSA wishes to adopt the fair value approach then we will need to consider such questions further.

**Implications of a “hedge cost” approach**

Hedge cost calculations are in principle simpler. The implications of the hedge cost approach are that we use actual asset prices, even if they are not a perfect hedge, with additional capital in the ICA to cover basis risk. Bid or offer prices would be appropriate rather than mid, and where a dynamic trading strategy is required (e.g. where appropriate assets do not exist) then transaction costs should be allowed for.

Under a hedge cost approach it is possible that more than one “risk free” rate will be used, depending upon the approach that would be taken to hedge a particular liability. For example, a liability that would need to be hedged with swaptions would effectively be valued using the rate implicit in such an instrument, while a liability that an office hedges using gilts might be valued using a gilt rate.

The use of multiple rates, perhaps with different rates adopted by different companies, is not inconsistent, as the hedge cost approach asks whether a company has sufficient capital to hedge its liabilities, given available instruments, not whether it has selected the best hedge or what the cost would be if other instruments were available.

However, the hedge cost approach does encounter difficulties if no suitable hedge exists. For example, options on real estate do not exist. In effect the bid price is zero and the ask price is infinite. If would be unrealistic to set prudential requirements based on such a high ask price.

**Which approach should we use – the current state of play?**

The FSA have not issued any guidance on the appropriate approach to use. We have discussed this briefly with them and they appear to appreciate that there are merits to both approaches. Currently internal discussions are continuing.

Because the implications of the hedge approach are relatively clear – we do not need to derive a "true" risk free rate – it seems appropriate to concentrate on the implications of the fair value approach for the rest of this note.

**The need for a "true" risk free rate**

Under the fair value approach we must value all liabilities consistent with the way similar assets are valued in the market. However, if these assets exhibit credit risk or suffer from other market imperfections then they are not considered sufficiently similar to the liabilities we want to value. Hence the asset prices must be adjusted.

One significant adjustment is to find a set of fixed interest assets to use for valuing all fixed cash flows, i.e. define a "true" risk free rate. Another potential set of adjustments is to the value of other traded instruments (e.g. options) to allow for this new risk free rate.

**What is the "true" risk free rate?**

There are a number of possible candidates for a true risk free rate. These include:

- Gilts – based on the most liquid gilts, typically on the run benchmark issues
- Less liquid gilts
- The most highly credit-worthy corporate and super-national debt,
- Swap rates,
- LIBOR (and LIBID), and
- General collateral Repo rates.

LIBOR and repo rates are typically short term rates but the other three categories are longer term. Currently swap rates are 20-30bp above the most liquid gilts, but 10 bp of this spread is attributable to the bid/mid spread (as swaps are derivatives on LIBOR and not LIBMID). High quality super-nationals (e.g. World Bank, EIB) are 15-20bp above the most liquid gilts [figures are roughly right – data is available to harden them up]. It is difficult to compare repo rates to gilt yields, because the repo rates are only liquid at the very short end of the yield curve (2 weeks or less). However, from a credit perspective, GC repo rates should be close to risk free, as the default risk on a repo can occur in either direction.

The deep puzzle is to explain why the spread between swaps/super-nationals and the most liquid gilts appears to be greater than that which we would expect for credit risk alone. There are various academic studies around this, but in brief the essential point is that the spread between LIBOR and repo rates is smaller than the spread between swaps and the most liquid gilts. If the most liquid gilts represented a pure risk free rate, then these two spreads should be very similar.

The academic consensus, if there is a consensus, appears to be that the most liquid gilts attract a convenience yield of around 10bp. This is because, especially in times of financial distress, the most liquid gilts are in special demand for their benchmark properties. They are also the most likely to be use in short positions which have to be covered with physical delivery. The spread of the most liquid to

least liquid gilts can also be explained in terms of repo-specialness. The most liquid gilts have an embedded option to offer to short sellers in the repo market and special (ie discounted) repo rates. Several studies have suggested this is factored into gilt prices, and largely accounts for the liquidity discount on the yield.

This suggests that the true risk free rate is somewhere between

- the least liquid gilts and
- swaps adjusted for LIBOR/LIBMID spread and LIBMID/Repo credit spread

This is a fairly narrow range, lying from the least liquid gilt yields at most 5 bp above that level in current market conditions. It is difficult to say where in that range the "true" risk free rate lies.

It is worth noting that the spread between swaps and gilts is currently low historically. Part of this may be random variation but it is also possible that as the swap market has developed, the credit risk in swaps has genuinely reduced. This is consistent with the pattern of the spread between swaps and super-nationals, which has narrowed over time, while the spread between super-nationals and gilts has remained more constant.

**Does the true risk free rate affect the price of other instruments?**

Adopting a different risk-free rate will clearly have implications for the value of fixed liabilities. It will not affect the value of purely asset share linked liabilities. The remaining question is whether it affects the value of embedded options.

The argument for this is that the risk free rate is a key input to the calculation of option values; changing it will therefore change the value of such options.

The argument against is that when calibrating to option prices we pick a set of assumptions – risk free rates and volatilities – that reproduce market prices. Thus changing the risk free rate assumption does not change the value, simply the implied volatilities that we back out of the prices. So a model based on a "true" risk free rate can reproduce either option prices or option implied volatilities, but not both. If it logically more consistent to seek to reproduce implied volatilities, not least because this would still allow put-call parity to be preserved across a range of prices.

The approach values would change if we believed that traded options are themselves credit risky. Exchange traded options are almost certainly sufficiently credit-worthy that any such impact would be minor, as exchanges act as credit enhancers, and furthermore the credit risk effect is small anyway given the short term of typical exchange traded options. The case is less clear for over the counter options, and will depend upon whether they are governed by credit enhancement agreements. Data suggests that they are at least partially collateralised and netted, but not fully so. Furthermore, if the collateral takes the form of margin deposits with another bank, the credit risk of that bank may enter the equations. This suggests that some adjustment to option prices might be appropriate, but further work is required to establish the size of any adjustment.