The Actuarial Profession
making financial sense of the future

Risk Measures: Beyond Coherence?

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Agenda

- Risk measures: What are they good for?
- Risk measurement and risk aversion
- Coherent (and other) risk measures
- Beyond coherence:
  - Liquidity and aggregation
  - Background risk
  - A regulatory perspective
- Discussion (hopefully)

Risk Measures

- Risk measures are functions
  - You put in a (loss) distribution
  - You get out a number
- They can be used for
  - Comparing risks
  - Pricing
  - Capital allocation (as in portfolio management)
  - Capital allocation (as in regulatory requirements)

Some risk measures

- Expected loss: $(1+\lambda)\cdot E[X]$
- Standard deviation: $\sigma(X)$
- Percentile: $\Pr(X \leq VaR_a(X)) = a$
- Tail Conditional Expectation: $E[X|X>VaR_a(X)]$
- Lloyd's RBC: $E[\max\{X-(NP+RBC),0\}] = NP\cdot ELC$
- …

Risk aversion

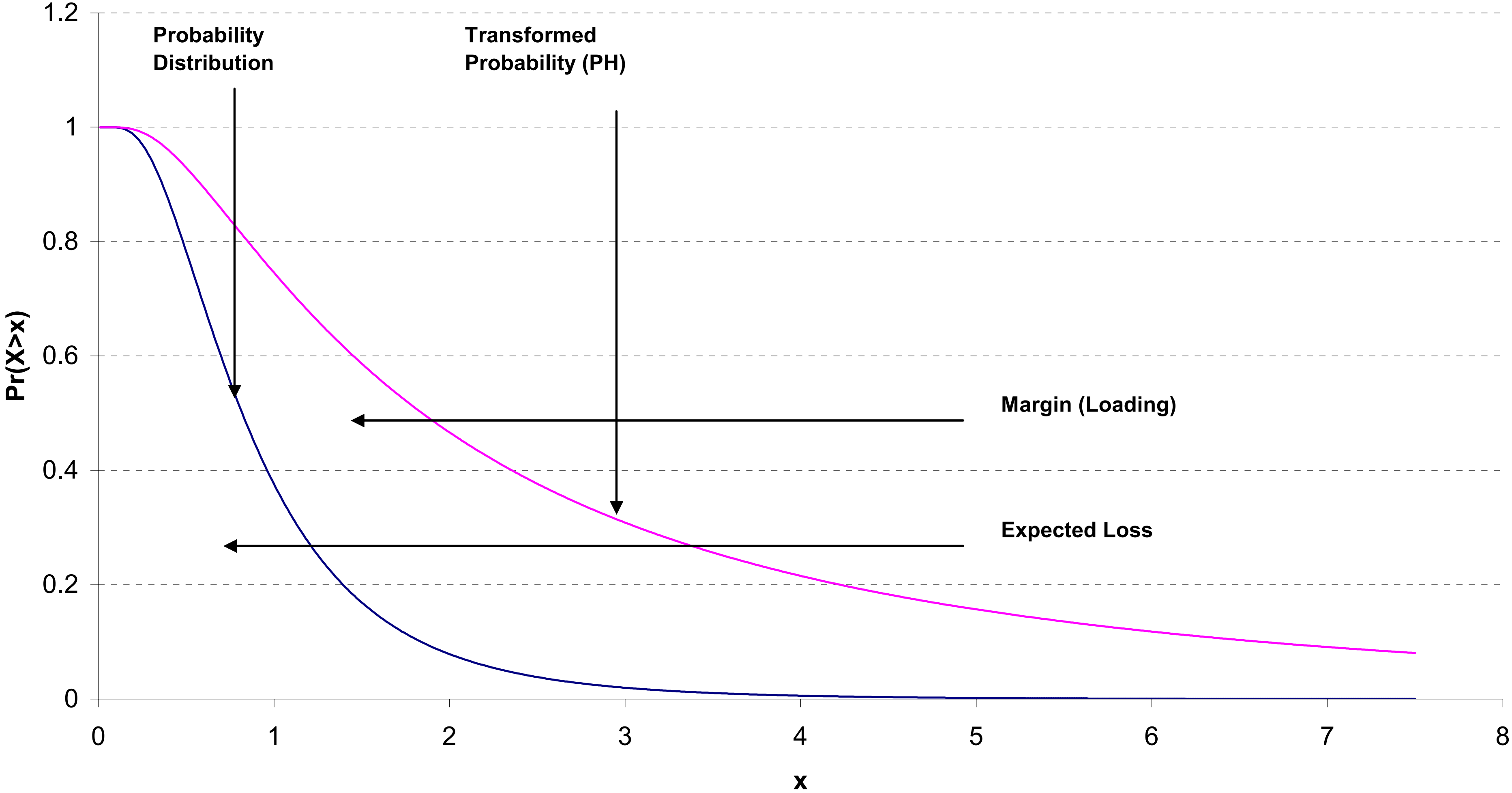
- A risk measure adds a margin to expected loss.
- Hence it forms a representation of risk aversion:
  - How risk averse are we? How much is the margin?
  - In which way are we risk averse? How do we calculate the margin?
- Ways in which to model risk aversion
  - Exaggerate the probabilities of adverse scenarios
  - Exaggerate the consequences of adverse scenarios

Distorting probabilities (1)

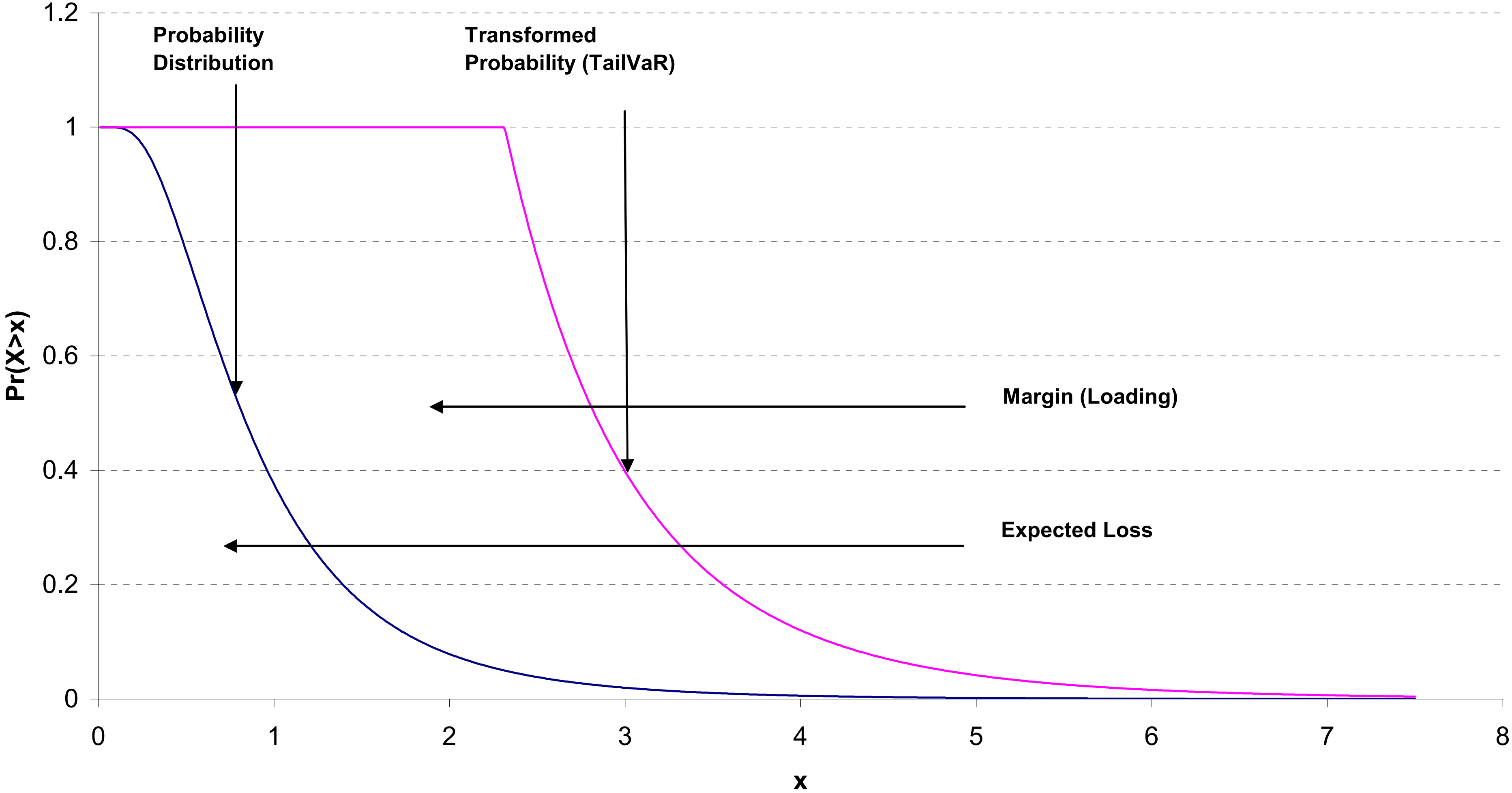
- If working with a probability distribution, the method consists of "blowing up" its tail.
- For a tail function $S(x) = Pr(X>x)$, apply the non-linear transform $S^*(x) = g(Pr(X>x))$.
- The risk measure equals the expected loss under the transformed probability distribution:

$$\rho(X) = \int_0^\infty S^*(x)dx \qquad \text{(recall that } \mu = \int_0^\infty S(x)dx \text{)}$$

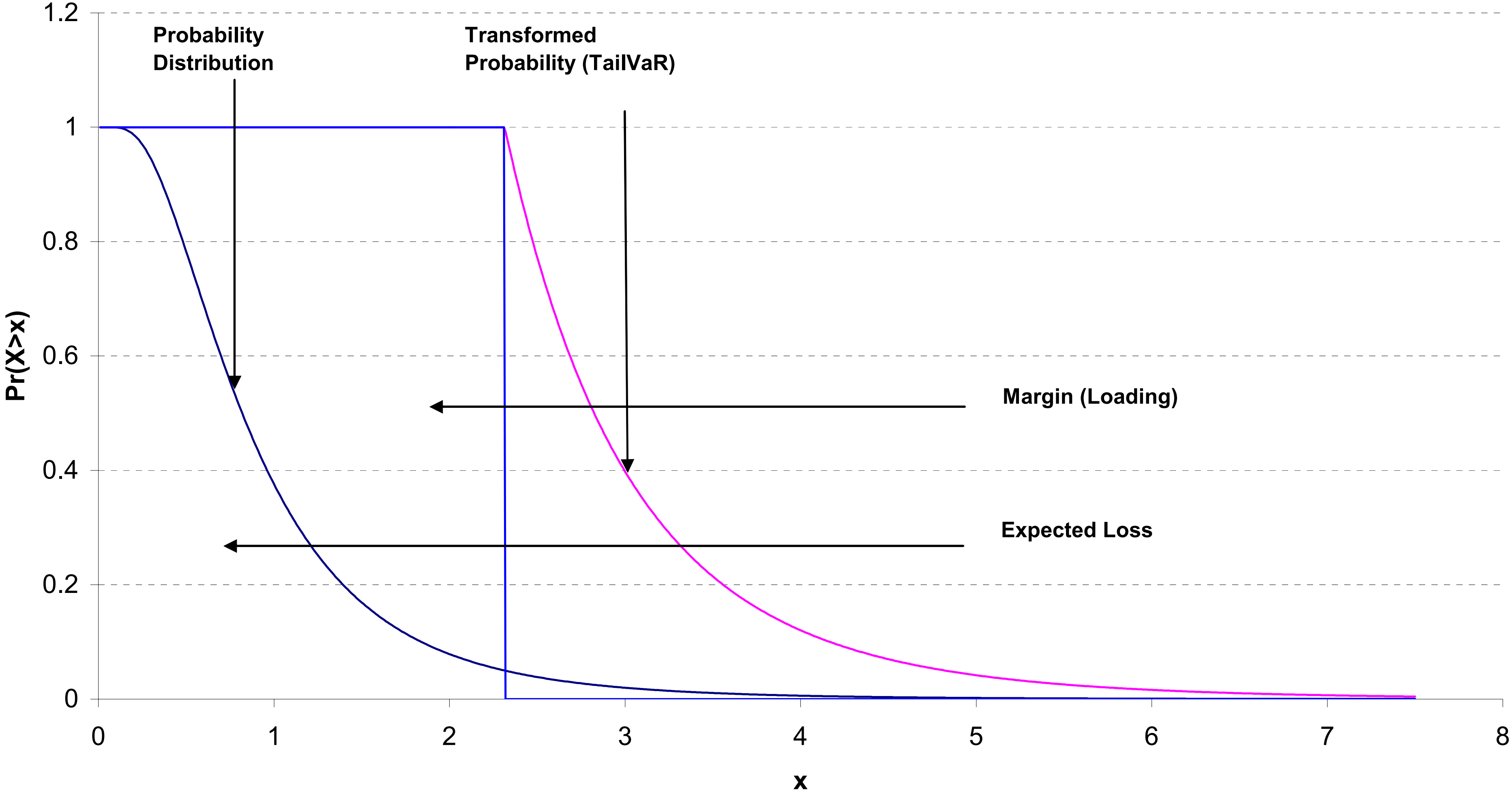
Transformed Probability Distributions



Transformed Probability Distributions



Transformed Probability Distributions



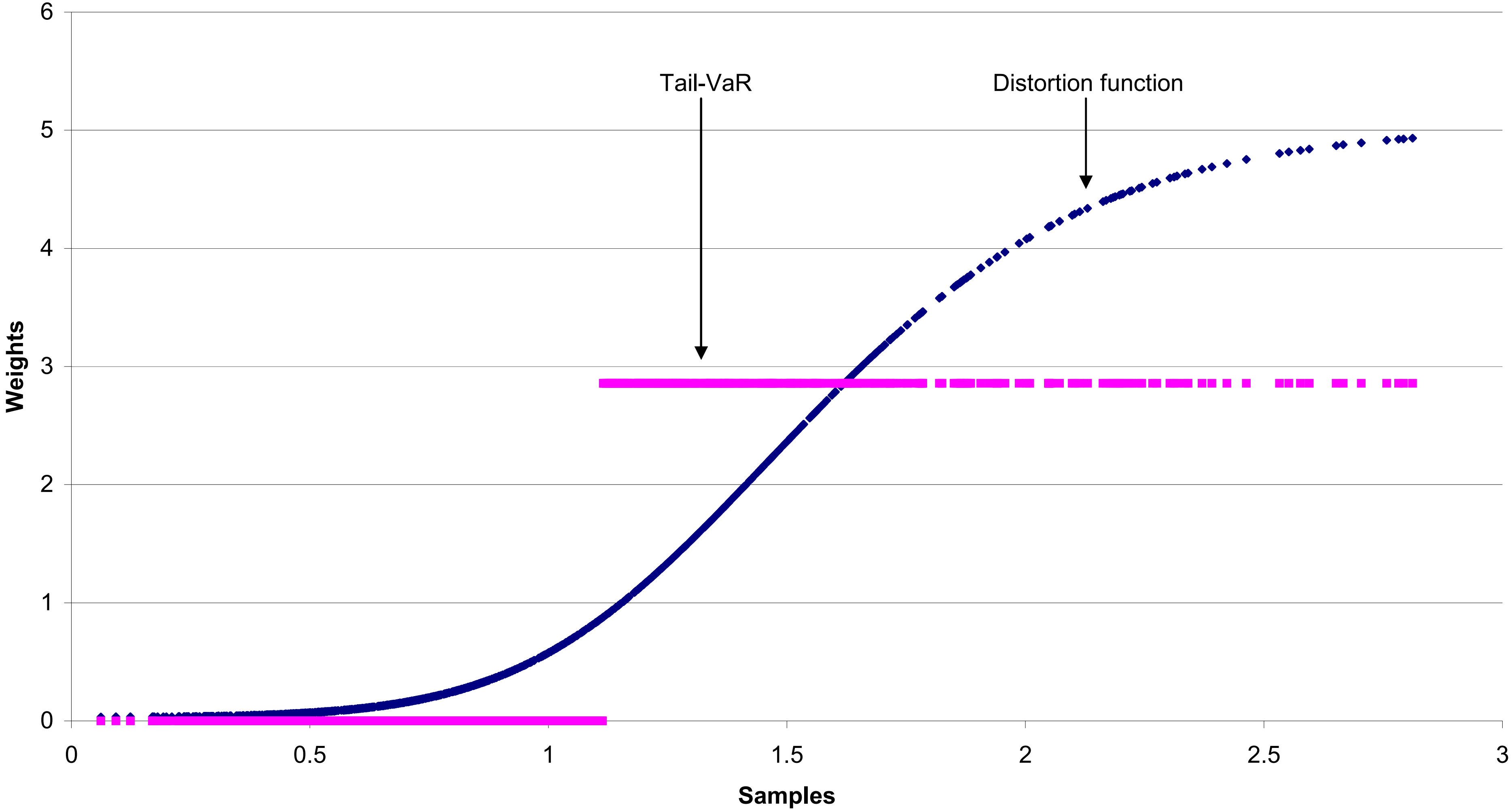
Distorting probabilities (2)

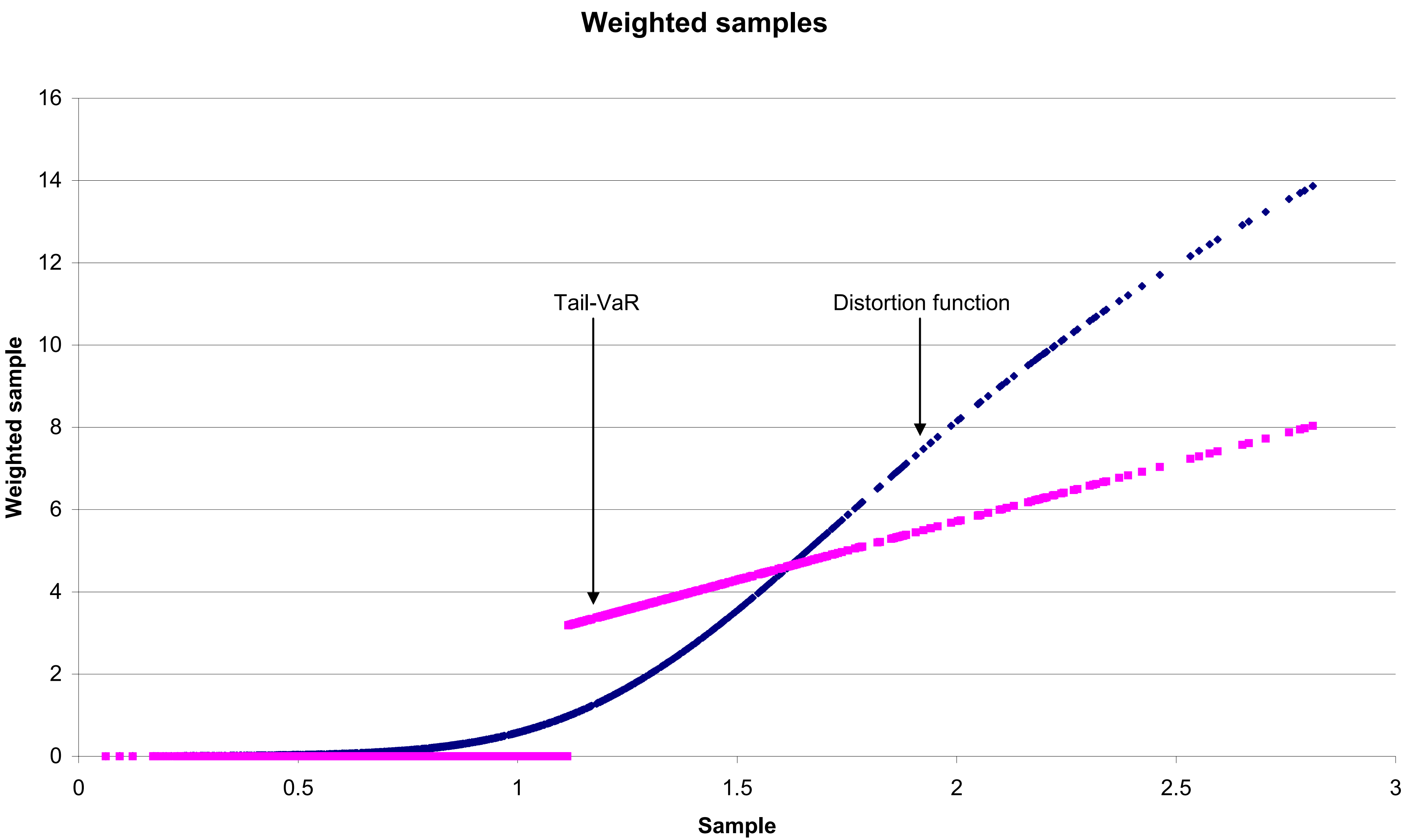
- If working with samples (e.g. from a DFA model), re-weight them according to (an increasing function of) their ranks.
- After re-weighting, take the average.
- If the CDF is $F(x) = \Pr(X \le x)$, this corresponds to:

  $\rho(X)=E[X \cdot h(F(X))]$

  for appropriate increasing function h

Weighting of samples in the distortion approach

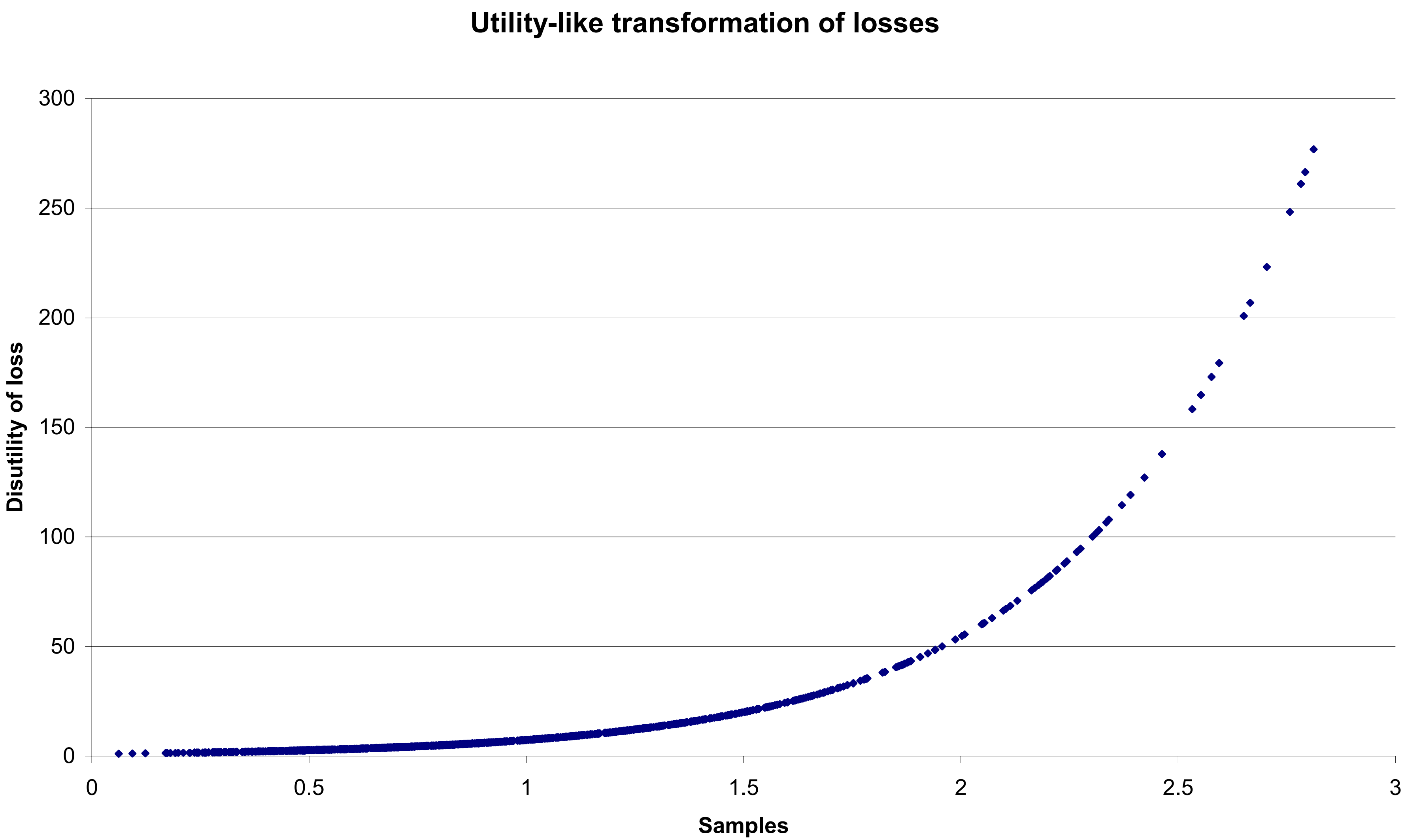



Weighted samples
Weighted sample
Sample
Tail-VaR
Distortion function
16
14
12
10
8
6
4
2
0
0
0.5
1
1.5
2
2.5
3

Distorting losses

- An alternative way of modelling risk aversion relies on the transformation of potential losses.
- Method: transform each sample x_i by $v(x_i)$.
- v is a convex and increasing “disutility” function.
- Take the average of the transformed samples.
- Apply inverse v^{-1} to recover original scale.

$$\rho(X) = v^{-1}(E[v(X)]$$


Utility-like transformation of losses
300
250
200
150
100
50
0
Disutility of loss
0
0.5
1
1.5
2
2.5
3
Samples

Properties of risk measures

- Different risk measures are characterised by alternative sets of properties
- Different sets of properties correspond to alternative notions of risk aversion
- Let’s have a look…

Coherence (Artzner et al., 1999)

- $\rho(X+Y) \leq \rho(X) + \rho(Y)$,
  meaning that pooling risks is always beneficial
- $\rho(a \cdot X) = a \cdot \rho(X)$, $a \geq 0$
  meaning that the scale of loss does not matter
- Can be constructed by distorting probabilities:
  $\rho(X) = E[X \cdot h(F(X))]$
- TailVaR is coherent, VaR isn't

Additivity (Gerber, 1974)

- $\rho(X+Y) = \rho(X) + \rho(Y)$ for independent (X,Y)
- $\rho(X+Y) \leq \rho(X) + \rho(Y)$ for negative correlation
- $\rho(X+Y) \geq \rho(X) + \rho(Y)$ for positive correlation
- Can be constructed by distorting the losses:

$$v(x) = \exp(\beta X) \Rightarrow \rho(x) = \frac{1}{\beta} \ln E[\exp(\beta X)]$$

- Such risk measures are quite sensitive to scale.

Convexity (Föllmer and Schied, 2002)

- $\rho(\lambda X+(1-\lambda)Y) \leq \lambda\, \rho(X) + (1-\lambda)\, \rho(Y)$
- "Must diversify in order to pool, not pool in order to diversify"
- The two classes described previously are special cases
- Can construct using a combination of distortion and utility approaches (Tsanakas and Desli, 2003 *BAJ*)

Departures from coherence

- Coherence nowadays forms the most widely accepted set of properties for risk measures.
- However, there are situations where the properties of coherent risk measures may not be appropriate.
- Typically this happens if the scale of potential losses is an issue.
- Three such situations are now described.

Liquidity risk

- In the case that a highly adverse scenario takes place, additional capital will have to be raised.
- Trying to raise £1m and £100m are two very different things.
- Coherent risk measures are scale invariant:

  $\rho(aX) \leq a \cdot \rho(X)$

  and do not address this issue.

Aggregation risk

- Some people would also argue that you should never accumulate highly correlated risks.
- This is intricately linked with liquidity – aggregating many highly correlated positions is like investing in one large risk.
- Coherent risk measures are subadditive:

  $\rho(X+Y) \leq \rho(X) + \rho(Y)$

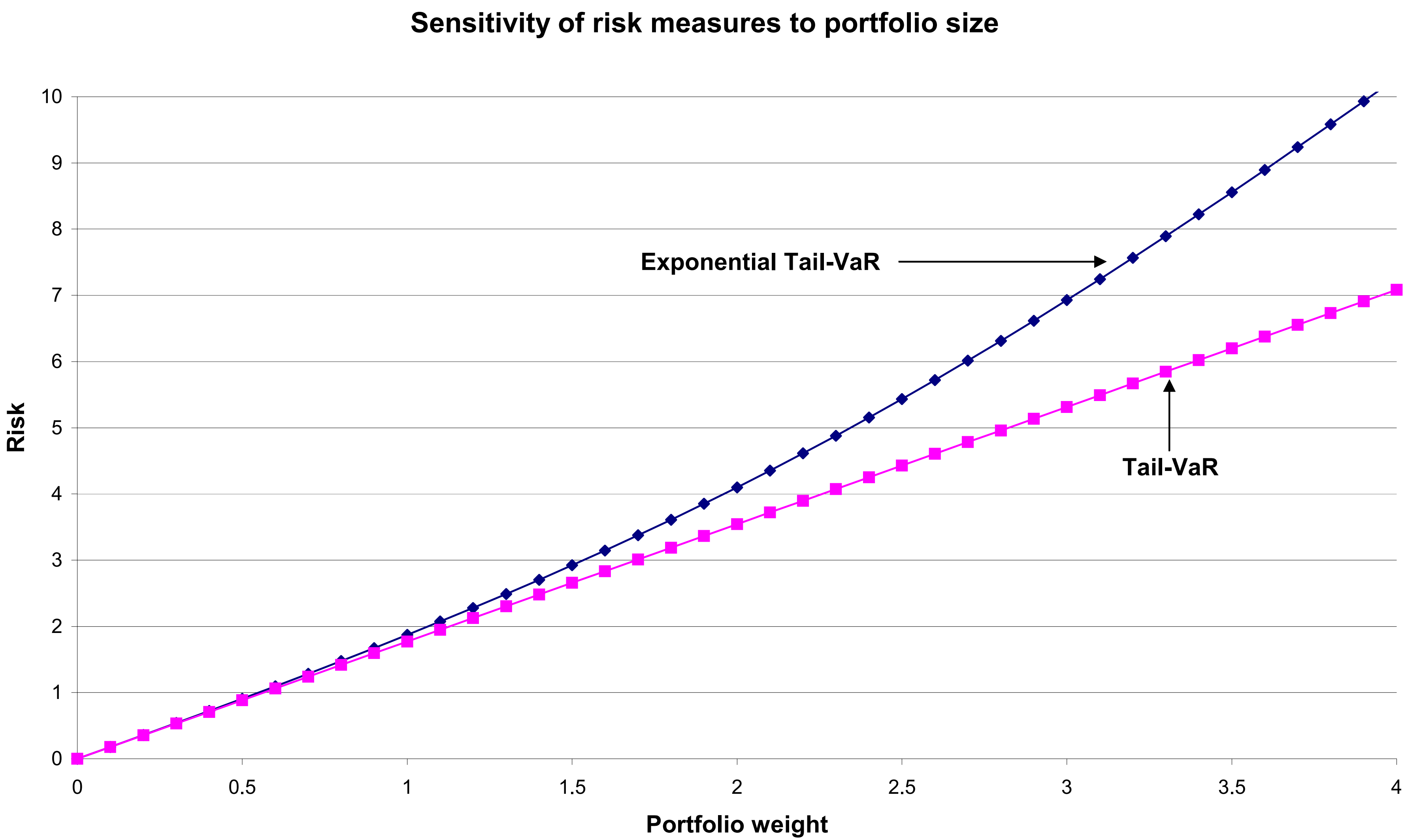
  and again do not take account of this issue.

Convex risk measures to the rescue

- If liquidity and aggregation are concerns, we could use a convex risk measure instead, e.g., “exponential TailVaR”:

$$\rho(X) = \frac{1}{\beta} \ln E\left[e^{\beta X} \mid X > Q_X(a)\right]$$

- For small losses behaves approximately like a coherent risk measure.
- For larger losses it becomes more and more sensitive to liquidity and aggregation.


Sensitivity of risk measures to portfolio size
Risk
Portfolio weight
Exponential Tail-VaR
Tail-VaR
0
1
2
3
4
5
6
7
8
9
10
0
0.5
1
1.5
2
2.5
3
3.5
4

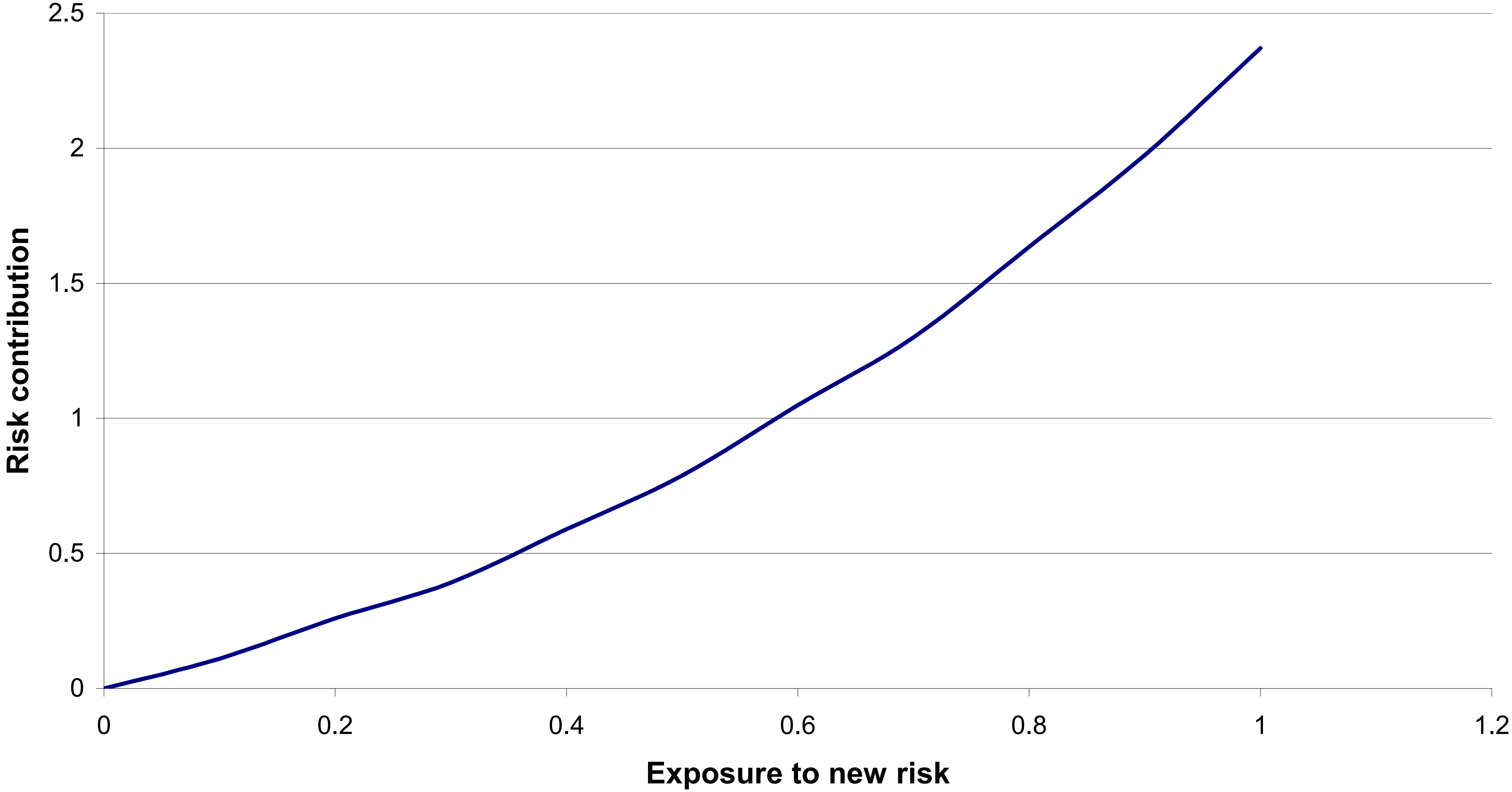
Background risk

- You hold a risk Y that you cannot get rid of.
- You add some new (say independent) exposure X to that.
- Small amounts of the new risk X diversify your initial exposure.
- The more of X you take on though, the more X dominates your portfolio, hence the less diversification benefit it contributes.

Background risk (cont'd)

- Capital allocation techniques can determine the benefit from taking on exposure in new risk X.
- Consider the portfolio $Y + \lambda \cdot X$
- Calculate the aggregate risk, using e.g. TailVaR
- Determine the contribution of the new exposure $\lambda \cdot X$ to the aggregate risk.
- Plot that against exposure λ.

Risk contribution in the presence of background risk



Regulation

- Coherent risk measures encourage the pooling of portfolios.
- It is desirable that such pooling does not increase the shortfall risk.
- Consider two loss portfolios X, Y.
- A regulator suggests a coherent risk measure ρ for determining respective capital $\rho(X)$, $\rho(Y)$.

Regulation (Cont'd)

- The loss from each portfolio, in excess of capital, is borne by "society".
- These losses are respectively:

  $\max\{X - \rho(X),0\}, \quad \max\{Y - \rho(Y),0\}$
- Suppose now that the holders of risks X and Y decide to merge them.
- New risk to "society" is:

  $\max\{X+Y- \rho(X+Y),0\}$

Regulation (Cont'd)

- It was shown by Dhaene et al. (2004), that, if ρ is subadditive, the shortfall risk to "society" after the merger can be higher than before.
- Hence pooling can be good for insurers, but bad for policyholders, due to increased shortfall risk!
- A bit controversial but there is something in it.

Conclusion

- There are different ways of constructing risk measures, depending on how our risk aversion is manifested.
- Coherent risk measures are the leading paradigm, but sometimes do not adequately capture risk.
- They can be enriched by introducing some sensitivity to the scale of potential shortfall.