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SCOTTISH BANKERS' MORTALITY AND MARRIAGE EXPERIENCE, 1950-1966

by

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1. INTRODUCTION

1.1. There have been three previous papers investigating the mortality and marriage experience of the widows' funds of Scottish Banks. In 1894 Messrs. Hewat and Chatham presented a paper to the Institute of Actuaries covering the experience of the funds from their inception, varying from 1808 to 1866, up to 1892-93. Mr. Alexander Fraser presented a paper to the Faculty in 1925 covering the period from 1903 to 1923 and in 1949 a further paper was produced by Messrs. Borthwick, Denholm, Wallace and Waugh for the years 1923 to 1943. As it had been suggested that an investigation of a more recent period was now due the present authors volunteered to undertake this task in view of their office's association with the previous papers.

1.2. This paper presents the results of our investigation into the mortality and marriage experience of six widows' funds of the Scottish Banks over the period 1950 to 1966. The changes that have taken place over this period have been examined and, where considered significant, have been allowed for when calculating the full range of monetary functions required for valuing these funds. Monetary functions to enable a valuation to be made at 5% interest by the reversionary method are published in the paper together with the results of valuing a model fund and the financial effects of the various changes. A valuation of this fund was carried out using the collective method and the result is shown for comparison.

2. GENERAL PRINCIPLES AND METHOD OF INVESTIGATION

2.1. The previous papers used the experience available from five banks whose head offices were in Edinburgh. In order to obtain as much data of a homogeneous nature as possible the Clydesdale Bank Ltd. in Glasgow was approached and agreed most readily to provide the required information concerning its own widows' fund and this

was incorporated with that obtained from the banks which had previously participated.

2.2. In the past the investigations extended from the anniversary date of each fund in the first year of the period under review to the corresponding date in the final year. As the funds had different anniversary dates the data obtained did not cover a uniform period. On this occasion, since a computer was to be used for tabulation, information was obtained from each bank covering the period from the anniversary date in 1947 to the corresponding date in 1967 but the period of investigation was to be limited to an exact number of calendar years. The years from 1943 to 1946 were excluded to avoid any distortion in the rates which might have arisen due to the war and its immediate aftermath. As we were unable to reconcile the data for the years 1947 to 1949 with the statistics produced in the annual reports on certain of the funds, we were forced to limit the investigation to the period from 1st January 1950 to 31st December 1966.

2.3. Instead of cards for each member, specially devised computer coding sheets were issued to the Clerks to the funds. The coding instructions are set out in Appendix I. It will be noted that information was sought about members' children born during the period of the investigation, as we had hoped to be able to produce issue rates, but the data proved to be so scanty that this idea was subsequently abandoned.

2.4. The information on the coding sheets was transferred to punched cards and thence to magnetic tape for subsequent analysis using an I.C.L. 1500 computer. The use of the computer allowed approximately one hundred validity and consistency checks to be imposed on the data. Although the investigation was to cover an exact number of calendar years a programme was written to produce in force figures for each fund at the anniversary dates within the period under review, together with numbers entering and leaving within each year. This information was compared with the totals recorded in the annual reports on the widows' funds of each bank and very satisfactory reconciliations were eventually obtained—the statistics recorded at Register House being of assistance in obtaining data on members when a bank's records were incomplete.

2.5. An exposed to risk formula was used which allowed the calculation of mortality and marriage rates for exact integral ages

attained during each calendar year of the period covered by the investigation, allowing for the exact duration of the exposure to the appropriate rate of decrement. This exact age and duration method is sometimes referred to as the life-year method. Computer programmes were written which enabled all the crude rates required for this paper to be produced from the basic coded information which had been supplied by the banks.

2.6. In outline the method adopted when calculating mortality rates was to calculate for each life included in the experience on 1st January of a given year the period of exposure up to the next birthday, when he would attain age x let us say, which would form part of the exposed to risk at age $x-1$ for that year. The exposure for the balance of the year would similarly contribute to the exposed to risk for age x. If the life left the experience during the year his period of exposure ceased at the exit date, except for deaths for whom the appropriate exposure up to the birthday following exit was calculated and allocated to the appropriate calendar year. New entrants during the year were exposed from date of entry.

2.7. When calculating mortality rates for a given range of calendar years the fragments of exposed to risk at each age for these years were added together and divided into the sum of the corresponding fragments for deaths, thus ensuring that the deaths were given the same period of exposure in both the numerator and the denominator. If the period to which the rates related included 1966, the last year for which data were available, the numerator should be adjusted to allow for those who died between 31st December 1966 and their birthday in 1967 and an approximation to this adjustment, based on deaths in 1966, was made. In theory a somewhat similar adjustment should have been made for those who died after their birthday in 1949 when the year 1950 was included, but since the effect of this would have been trivial it was ignored.

The procedure for calculating marriage rates followed the same lines, and in both cases the fragments of exposed to risk were classified so that rates could be segregated by bank or marital status, as desired.

3. EXTENT OF THE EXPERIENCE

3.1. Table 1 indicates the extent of the 1950-66 investigation compared with that for 1923-43.

Once again there has been a large increase in the exposed to risk although it must be remembered that an additional bank has been included on this occasion. The further substantial increase which has

taken place in the crude marriage rate of bachelors is reflected in the increased percentages of the total exposed to risk attributable to married men, widowers and wives, and the corresponding reduction in the percentage attributable to bachelors. There have been marked reductions in the crude rates of mortality for all classes, but for widows this reverses the trend which appeared in the 1923-43 investigation although the rate remains above that for 1903-23. Throughout the paper male members who have been divorced have been included in the same category as widowers.

TABLE 1

Summary of Exposed to Risk of Death, Deaths and Marriages

Status	1950-66			1923-43		
	Exposed to risk of death	Deaths	Marriages	Exposed to risk of death	Deaths	Marriages
Bachelors	33,539	167	2,083	37,790	262	1,886
Married men	96,233	1,270	—	54,149	962	—
Widowers (incl. divorcees)	3,626	244	169			
Wives	95,853	481	—	56,118	352	—
Widows	20,062	815	51	12,539	573	—
Total	249,313	2,977	2,303	160,596	2,149	1,886

4. GRADUATION

4.1 Since so much of the work was being done by computer we had hoped that the graduation, particularly of the mortality tables, could also be carried out by this means. A considerable amount of work was undertaken in an attempt to fit, by the least squares method, a single polynomial equation of up to the 15th degree to the mortality experience. Unfortunately the equations tested did not produce satisfactory results when normal actuarial criteria for adherence to data were applied, there invariably being a tendency for the sum of the accumulated deviations to become positive and remain positive after a few ages. We are aware that more success might have been achieved by fitting separate polynomials to parts of the experience and merging the resultant curves, or even by attempting to fit more complicated formulae, but these were not pursued.

4.2. The graphic method of graduation was used, therefore, throughout the paper but certain modifications to the normal procedure were of considerable assistance in producing the mortality rates. Since the mortality curve is approximately exponential in form over the age

range covered by this experience it was thought that a first attempt at graduation could be made by fitting a straight line to the crude rates for quinquennial ages plotted on graph paper with a logarithmic scale and then modifying it to improve the fit. Tests with previous graduations were encouraging and once a suitable curve had been drawn the rates for each age were read from it and handpolished to the requisite degree of smoothness. Handpolishing was necessary since the rates from the graph could not be read to more than three significant figures. This logarithmic method considerably reduced the work involved in fitting a satisfactory curve to the crude mortality rates and in Appendix III the graphs of some of the graduated rates have been plotted on a logarithmic scale to display the advantages of the method.

4.3. To facilitate the testing of the graduations, computer programmes were written in COBOL for use with a Burroughs B.3500 machine. In addition to bringing out first and second differences information was provided which allowed eight further tests to be applied. Details of these tests, which were applied to all the graduations, are shown in Appendix II together with the results.

5. MORTALITY EXPERIENCE

5.1. The graduated mortality rates q_x at quinquennial ages are shown in Tables 2 to 8 along with the rates from various other investigations for comparison. Graduated rates for individual ages are included in Appendix IV.

TABLE 2
All Males' Mortality Rates

Age	Bankers 1950-66	Bankers 1923-43	E.L.T. 12 males	C.M.I. Assces. 1950-66	A49-52 ult.	$a(55)$ ult. males
17	·00065	·00120	·00099	—	·00111	—
22	·00067	·00190	·00114	—	·00111	·00123
27	·00073	·00220	·00100	—	·00113	·00131
32	·00098	·00250	·00128	·00100	·00120	·00148
37	·00158	·00330	·00181	·00130	·00147	·00187
42	·00269	·00510	·00287	·00210	·00231	·00269
47	·00462	·00670	·00505	·00380	·00420	·00417
52	·00812	·00970	·00930	·00700	·00750	·00659
57	·01427	·01660	·01668	·01220	·01272	·01050
62	·02417	·02720	·02778	·02050	·02096	·01706
67	·03927	·04070	·04332	·03220	·03409	·02803
72	·06187	·06150	·06570	·05270	·05489	·04602
77	·09483	·09810	·09963	·08700	·08709	·07463
82	·14268	·15490	·14934	—	·13500	·11798
87	·21428	·23500	·21369	—	·20205	·17910
92	·32499	·42040	·28396	—	·28819	·25699

5.2. Table 2 shows the rates for all males and from this it can be seen that there has been a substantial improvement over the 1923-43 experience, especially at the younger ages, but that the rate of improvement diminishes with age until a point is reached between ages 70 and 73 where the rates come together only to diverge again. As might be expected, the Bankers' rates are generally lighter than those for the general population; the crossing-over of the rates between ages 86 and 87 can be discounted since the shortage of data at the high ages makes these rates unreliable. The figures for C.M.I. Assurances 1950-66 are the arithmetic mean of the rates for that period included in the report on the Continuous Mortality Investigation in *T.F.A.* Vol. 32, p. 79 and demonstrate that during this time Bankers' mortality was from 10% to 20% higher than that of assured lives.

5.3. An examination of the male experience by marital status confirmed the expectation that in general widowers' mortality is heavier than that of bachelors, which in turn is heavier than that of married men and therefore it was decided to produce separate tables of graduated rates for each of these categories. Since no distinction was made between married men and widowers in the previous investigation the rates for this combined category were also graduated for the 1950-66 data for comparison.

5.4. Table 3 shows a marked improvement in the mortality rates for bachelors at all ages amounting to a reduction of about 50% throughout. The rates are below those published for bachelors in E.L.T. 12 as one would expect but the rates approach each other in the 35-45 age range before diverging again.

TABLE 3
Bachelors' Mortality Rates

Age	Bankers 1950-66	Bankers 1923-43	E.L.T. 12	Age	Bankers 1950-66	Bankers 1923-43	E.L.T. 12
17	·00066	·00120	—	57	·01308	·01980	·02100
22	·00073	·00190	·00130	62	·02049	·03270	·03200
27	·00104	·00220	·00160	67	·03329	·05300	·04600
32	·00179	·00320	·00220	72	·05385	·08600	·06700
37	·00304	·00500	·00320	77	·08531	·14700	·09900
42	·00478	·00730	·00520	82	·13351	·25700	·14400
47	·00683	·01020	·00780	87	·20972	·41700	·20300
52	·00923	·01350	·01200	92	·33283	·62600	—

5.5. While the improvement for married men and widowers combined is even more marked at the younger ages, the rate of

TABLE 4

Married Men's and Widowers' Combined Mortality Rates

Age	Bankers 1950-66	Bankers 1923-43	Age	Bankers 1950-66	Bankers 1923-43	Age	Bankers 1950-66	Bankers 1923-43
17	·00041	·00100	47	·00445	·00640	72	·06302	·06020
22	·00043	·00110	52	·00805	·00940	77	·09573	·09600
27	·00049	·00160	57	·01440	·01640	82	·14323	·15200
32	·00076	·00210	62	·02475	·02690	87	·21448	·23300
37	·00127	·00300	67	·04036	·04000	92	·32464	·42000
42	·00242	·00480						

TABLE 5

Married Men's Mortality Rates

Age	Bankers 1950-66	E.L.T. 12	Age	Bankers 1950-66	E.L.T. 12	Age	Bankers 1950-66	E.L.T. 12
22	·00043	·00070	47	·00445	·00480	72	·06180	·06200
27	·00049	·00080	52	·00805	·00860	77	·09150	·09300
32	·00076	·00100	57	·01440	·01600	82	·13209	·13500
37	·00127	·00160	62	·02474	·02700	87	·18864	·19500
42	·00242	·00270	67	·04014	·04100			

TABLE 6

Widowers' Mortality Rates

Age	Bankers 1950-66	E.L.T. 12	Age	Bankers 1950-66	E.L.T. 12	Age	Bankers 1950-66	E.L.T. 12
32	·00303	·00250	52	·01328	·01400	72	·06765	·08100
37	·00438	·00340	57	·02000	·02400	77	·10120	·11700
42	·00626	·00480	62	·03001	·03900	82	·15130	·16600
47	·00899	·00860	67	·04506	·05600	87	·22636	·23600

improvement gradually falls until between ages 66 and 76 the 1950-66 rates are higher than those for 1923-43 by as much as 5% at some ages. At older ages there is an increasing rate of improvement. This is the same feature as was noticed in 5.2 when comparing all males' mortality, but it is now seen to apply to married men and widowers only.

5.6. Except at the very young ages the mortality rates of married men approach the general population statistics much more closely than did the bachelors' rates, but remain below them.

5.7. The rates for widowers at ages under 45 are quite hypothetical. Since there were no deaths at these ages, these rates were obtained by extending the exponential curve fitted to data at higher ages so that a complete mortality table would be available for the calculation of monetary functions. The financial effect of widowers' mortality rates is negligible at these ages and, since they are hypothetical, there is no point in comparing them with rates on any other table.

5.8. On examining the 1950-66 rates by marital status as set out in Table A of Appendix IV, we observed that, although bachelors' rates are generally higher than those of married men, between ages 55 and 81 this situation is reversed, the difference at some ages approaching 20%. Our first suspicion was that this might be a quirk of the graduation but on referring back to the crude data it became clear that only by deviating markedly from the observed experience over this not inconsiderable range of ages could we hope to avoid this crossing and recrossing of bachelors' and married men's rates. We were therefore forced to the conclusion that since a graduation on this basis would not satisfy our graduation tests the original rates should be allowed to stand, thus reviving a feature which has not been seen since the original paper by Hewat and Chatham and which is, as far as we are aware, unique among modern mortality experiences.

5.9. Tables 7 and 8 show similar data in respect of wives and widows. The graduated rates for widows are hypothetical at young ages as no deaths were recorded below age 42.

TABLE 7

Wives' Mortality Rates

Age	Bankers 1950-66	Bankers 1923-43	Bankers 1903-23	E.L.T. 12 Wives	C.M.I. Assces. 1950-66
17	·00034	·00120	—	—	—
22	·00038	·00170	·00300	·00040	—
27	·00049	·00210	·00350	·00050	—
32	·00073	·00220	·00330	·00080	·00100
37	·00116	·00240	·00240	·00120	·00130
42	·00181	·00270	·00290	·00200	·00210
47	·00281	·00390	·00570	·00320	·00380
52	·00432	·00710	·00810	·00500	·00700
57	·00675	·01100	·01050	·00770	·01220
62	·01057	·01530	·01640	·01300	·02050
67	·01692	·02180	·02900	·02100	·03220
72	·02840	·03080	·05300	·03600	·05270
77	·04924	·04900	·07600	·06200	·08700
82	·08504	·09350	·09600	·10100	—
87	·14429	·15850	·12300	·14800	—
92	·24248	·26100	·24500	—	—

TABLE 8
Widows' Mortality Rates

Age	Bankers 1950-66	Bankers 1923-43	Bankers 1903-23	E.L.T. 12 widows	$a(55)$ ult females
22	·00041	·00170	—	—	·00119
27	·00058	·00220	·00360	—	·00126
32	·00098	·00320	·00390	·00130	·00138
37	·00158	·00460	·00410	·00170	·00165
42	·00242	·00610	·00450	·00290	·00216
47	·00375	·00770	·00500	·00420	·00302
52	·00583	·00980	·00630	·00610	·00437
57	·00902	·01380	·01280	·00920	·00653
62	·01441	·02090	·01850	·01500	·01032
67	·02311	·03290	·02900	·02400	·01695
72	·03652	·05020	·05300	·04100	·02839
77	·05772	·07530	·07600	·07000	·04776
82	·09297	·11040	·09600	·11400	·07938
87	·15230	·16100	·12300	·18200	·12808
92	·24999	·26100	·24500	—	·19684

5.10. The mortality of both wives and widows has shown considerable improvement, particularly that of widows. The 1923-43 experience contained the unexpected feature that mortality of widows in almost every age group was heavier than the rates produced for the period 1903-23, and the improvement which has now taken place, especially in the middle age group, seems to have corrected this. The rates for 1903-23 are shown to illustrate this unusual trend. Widows' mortality remains heavier than wives' mortality at all ages.

5.11. The improvement in wives' mortality has similar characteristics to those shown by the combined mortality of married men and widowers, viz., a marked improvement at the younger ages gradually diminishing until the rates come together at age 75 before diverging again from age 77. While the trend in male mortality is mirrored in the general population statistics this is not true for wives as there have been considerable improvements at all ages according to the E.L.T. 12 figures.

5.12. The mortality rates experienced in the periods 1950-55 and 1962-66 were compared with the graduated rates for the whole period. While an improving trend was observed, particularly under age 70, this change from first to last period was not statistically significant and therefore only the aggregate rates have been produced.

5.13. The experience of each bank was also compared with the aggregate rates but there was no evidence to suggest that any one bank had a tendency to show mortality which was consistently heavier or lighter than that of the experience as a whole.

6. MARRIAGE EXPERIENCE

6.1. Marriage rates were produced for bachelors and widowers using the data for the whole period 1950-66 and the experiences for the periods 1950-54 and 1962-66 were compared with the overall rates. As the rates for bachelors had increased significantly between the first and last periods, separate graduated rates were produced for this class from the data for the latest period as current indications suggest no reversal of this trend. The data for widowers were not extensive enough to warrant this treatment. The results of the graduation tests are shown in Appendix II.

6.2. Table 9 summarises the exposed to risk and marriages of bachelors for the periods 1950-1966 and 1962-66 together with the corresponding figures for the 1923-43 experience.

TABLE 9
Bachelors' Marriages

Period	Exposed to risk	Marriages
1923-43	33,059	1,886
1936-43	7,980	601
1950-66	34,501	2,083
1962-66	13,307	810

6.3. Although there is little difference in the crude marriage rate for the two periods 1950-66 and 1962-66, both of which are significantly lower than the crude rate for 1936-43, it will be seen from Table 10 that significant changes have occurred at the younger ages.

6.4. In spite of the reluctance of many speakers in the discussion on the previous paper to accept the marriage rates of bachelors for the truncated period 1936-43 as the standard for the future, we have found that the trend towards marriage at younger ages, and towards higher marriage rates generally, has continued unabated. The changes which have taken place in the ages up to 30 are truly remarkable and the peak rate of ·187 which occurred at age 31 for the 1936-43 experience has risen by 14% to ·214 at age 27 for the 1950-66 period

and by 29% to ·242 at age 26 for the later 1962-66 data. The rates of marriage remain at least as high as those produced for the previous paper for most of the remaining ages and the effects are demonstrated in the later Table 16 showing the proportions married in quinquennial age groups. Marriage rates at individual ages are contained in Table C of Appendix IV.

TABLE 10
Bachelors' Marriage Rates

Age group	Bankers			England and Wales (general)		
	1962-66	1950-66	1936-43	1961-65	1951-65	1941-45
16-19	·00247	·00227	—	·01740	·01330	·00950
20-24	·07018	·05344	·00762	·15860	·14770	·10990
25-29	·23481	·20406	·11048	·18230	·18050	·14880
30-34	·17405	·16604	·17540	·08820	·09880	·11440
35-39	·10568	·09991	·09807	·04680	·05340	·07600
40-44	·05895	·05722	·05177	·02670	·03110	·04440
45-49	·03139	·03494	·03303	·01720	·01980	·02650
50-54	·01200	·02361	·02048	·01110	·01230	·01540

6.5. Table 11 gives the corresponding information for widowers, although the data from which the Bankers' rates have been produced are extremely limited.

TABLE 11
Widowers' Remarriage Rates

Age group	Bankers		England and Wales (general)	
	1950-66	1923-43	1951-65	1941-45
25-29	·3000	·1667	·3895	·2563
30-34	·2700	·1628	·3237	·2321
35-44	·1369	·1359	·1879	·1737
45-54	·0785	·0675	·1041	·0953
55 and over	·0311	·0211	·0206	·0169

At the important ages from 50 onwards significant increases in the rates for remarriage of widowers have occurred and the very high rates produced for bachelors at younger ages are reflected in the corresponding rates for widowers. Rates at individual ages appear in Table C of Appendix IV.

6.6. As a matter of interest the remarriage rates of widowers were investigated by duration only, all ages being combined for this purpose, and the results including a graduation of the unadjusted rates are given in Table 12.

TABLE 12

Remarriage Rates of Widowers by Duration

Curtate duration	Unadjusted remarriage rate	Graduated remarriage rate	Unadjusted 1923-43 rate
0	·031	·030	·035
1	·100	·100	·082
2	·074	·082	·093
3	·070	·064	·037
4	·056	·046	·023
5 and over	·018	·019	·024
5	·025	·034	—
6	·028	·028	—
7	·013	·024	—
8	·030	·022	—
9	·023	·021	—
10 and over	·009	·009	—

The figures suggest that the remarriage rate rises to a maximum around the second year of widowerhood compared to the third year previously, with the rate falling quite rapidly after the fourth year, the effects of duration from widowerhood remaining more apparent than in the earlier investigation.

6.7. Table 13 summarises the remarriage rates for widows according to the experience for the full period 1950-66 and compares them with the corresponding rates for widowers and general population rates. Rates at individual ages appear in Table C of Appendix IV.

TABLE 13

Widows' Remarriage Rates

Age group	Bankers 1950-66		England and Wales (general) 1951-65	
	Widows	Widowers	Widows	Widowers
25-29	·0718	·3000	·3343	·3895
30-34	·0432	·2700	·2114	·3237
35-44	·0183	·1369	·0846	·1879
45-54	·0055	·0785	·0292	·1041
55 and over	·0013	·0311	·0031	·0206

The rates confirm the expectation that widows have a lower chance of remarrying than widowers but perhaps the extent of the discrepancy is surprising. It is possible that the financial independence provided by the annuity and its loss on subsequent remarriage may be a contributing factor.

6.8. The remarriage rates of widows by duration since widowhood were also investigated and the results are shown in Table 14 together with the corresponding rates for widowers.

TABLE 14
Remarriage Rates of Widows by Duration

Curtate duration	Unadjusted remarriage rate	Graduated remarriage rate	Widowers' graduated 1950-66 rate
0	·002	·001	·030
1	·003	·002	·100
2	·005	·003	·082
3	·000	·004	·064
4	·007	·004	·046
5	·004	·004	·034
6	·004	·004	·028
7	·001	·004	·024
8	·001	·004	·022
9	·004	·003	·021
10 and over	·002	·002	·009

The duration since widowhood has obviously very little effect on the remarriage rates of widows.

6.9. The relative ages of husband and wife are important as these may have a considerable effect on the values of the benefits.

The ages last birthday of wives at husbands' birthday prior to marriage have been extracted for both bachelors and widowers and these are shown at quinquennial ages in Table 15 together with the single set of graduated ages which it was decided to adopt. The ages adopted in the two previous investigations are shown for comparison purposes. Table D of Appendix IV shows these values at individual ages and also gives the average age of the wife at each attained age of the husband which we adopted.

6.10. Proportions married of the male population at each age were required in order to carry out a valuation by the collective method and the proportions used were graduated from those applicable to the period 1962-66.

The proportions married in quinquennial age groups are shown in Table 16 below together with corresponding figures for the general male population in Scotland in 1964

TABLE 15
Age of Wife at Marriage of Husband

Age of husband	Bachelors actual 1962-66	Widowers actual 1950-66	Bachelors and widowers graduated 1950-66	Bachelors 1923-43	Bachelors 1903-23	General population Scotland 1964
17	20·0	—	18	—	—	17·5
22	22·0	—	23	22	24	20·9
27	24·1	26·5	26	26	26	23·3
32	26·1	30·0	28	29	29	26·8
37	—	—	32	32	32	31·1
42	43·7	35·0	35	34	34	35·8
47	25·5	38·7	40	36	36	40·2
52	43·7	46·5	45	39	39	45·2
57	—	51·0	50	43	43	48·5
62	50·0	49·6	53	46	46	53·2
67	—	63·0	54	48	48	56·4
72	—	—	54	—	51	61·6

TABLE 16
Proportions of Married Males (%)

Age group	Bankers graduated 1962-66	Scottish males 1964	Age group	Bankers graduated 1962-66	Scottish males 1964
15-19	0·2	1·6	45-49	92·0	86·7
20-24	6·0	32·5	50-54	93·3	84·7
25-29	57·6	70·6	55-59	90·6	83·1
30-34	83·6	82·2	60-64	88·1	79·4
35-39	89·6	84·5	65-69	86·0	74·5
40-44	91·4	84·6	70 and over	73·7	56·6

7. COMPARISONS OF MONETARY VALUES

7.1. To investigate the monetary effect of the changes in mortality and marriage rates since the 1923-43 investigation, the B.3500 computer was used to produce monetary functions, correct to three decimal places, at 5% p.a. interest on both experiences.

7.2. The lighter mortality of widows in the most recent investigation compared with the 1923-43 investigation is reflected in the higher

annuity values at all ages. The mortality of female annuitants in the $a(55)$ ult. experience was lighter than that of the new experience except at the youngest ages and all annuity values exceed those of the new experience. The effect of allowing for the cessation of a widow's annuity on remarriage is relatively small for those ages where widows are numerically significant.

TABLE 17
Single Life Annuity Values : Widows

Age	Widows 1950-66	Widows 1923-43	$\frac{(i)}{(ii)}\%$	$a(55)$ ult.	$\frac{(i)}{(iv)}\%$	Widows 1950-66 ceasing on remarriage	$\frac{(vi)}{(i)}\%$
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
30	17·312	16·375	105·7	17·538	98·7	12·131	70·1
40	15·894	14·887	106·8	16·328	97·3	14·197	89·3
50	13·909	12·940	107·5	14·540	95·7	13·489	97·0
60	11·300	10·323	109·5	12·055	93·7	11·163	98·8
70	8·241	7·397	111·4	8·932	92·3	8·212	99·6
80	5·109	4·706	108·6	5·674	90·0	5·109	100·0

TABLE 18
Single Life Annuity Values : Wives

Age	Wives 1950-66	Wives 1923-43	$\frac{(i)}{(ii)}\%$	$a(55)$ ult.	$\frac{(i)}{(iv)}\%$	Widows 1950-66	$\frac{(i)}{(vi)}\%$
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
30	17·617	17·126	102·9	17·538	100·5	17·312	101·8
40	16·326	15·825	103·2	16·328	100·0	15·894	102·7
50	14·475	13·843	104·6	14·540	99·6	13·909	104·1
60	11·952	11·456	104·3	12·055	99·1	11·300	105·8
70	8·763	8·559	102·4	8·932	98·1	8·241	106·3
80	5·335	5·060	105·4	5·674	94·0	5·109	104·4

7.3. The trend in wives' mortality referred to in paragraphs 5.10 and 5.11 results in higher annuity values at all ages. At all ages save the youngest the annuity values are, however, below those derived from the $a(55)$ ult. tables. Annuity values for wives exceed those for widows at all ages in the new experience, reflecting the lighter mortality of wives at all ages.

TABLE 19

Single Life Annuity Values : All Males

Age	All males 1950-66	All males 1923-43	$\frac{(i)}{(ii)}$%	A49/52 ult.	$\frac{(i)}{(iv)}$%
	(i)	(ii)	(iii)	(iv)	(v)
30	16·826	16·368	102·8	16·954	99·2
40	15·095	14·685	102·8	15·325	98·5
50	12·665	12·388	102·2	12·978	97·6
60	9·672	9·475	102·1	10·074	96·0
70	6·599	6·502	101·5	6·929	95·2
80	3·922	3·666	107·0	4·106	95·5

TABLE 20

Present Value of Annuity on Widowhood to Present Wife of a Married Man

Husband's age (x)	Wife's age (x)				
	Wives' and widows' mortality 1950-66	Wives' mortality 1950-66	$\frac{(ii)}{(i)}$%	Wives' and widows' mortality 1923-43	$\frac{(i)}{(iv)}$%
	(i)	(ii)	(iii)	(iv)	(v)
25	1·082	1·131	104·5	1·278	84·7
35	1·643	1·721	104·7	1·765	93·1
45	2·386	2·508	105·1	2·299	103·8
55	3·118	3·289	105·5	2·843	109·7
65	3·393	3·580	105·5	3·029	112·0
75	2·852	2·974	104·3	2·735	104·3
85	1·792	1·843	102·8	1·908	93·9
	Wife's age ($x+3$)				
25	0·964	1·008	104·6	1·163	82·9
35	1·454	1·525	104·9	1·582	91·9
45	2·091	2·200	105·2	2·016	103·7
55	2·681	2·829	105·5	2·452	109·3
65	2·813	2·959	105·2	2·530	111·2
75	2·219	2·303	103·8	2·127	104·3
85	1·275	1·306	102·4	1·370	93·1
	Wife's age ($x-3$)				
25	1·199	1·252	104·4	1·393	86·1
35	1·831	1·915	104·6	1·947	94·0
45	2·682	2·814	104·9	2·586	103·7
55	3·560	3·752	105·4	3·248	109·6
65	3·993	4·220	105·7	3·551	112·4
75	3·546	3·718	104·9	3·369	105·3
85	2·413	2·492	103·3	2·524	95·6

7.4. Annuity values for all males in the most recent experience are higher at all ages as one would expect by referring to 5.2. At most ages however the mortality rates for all males in the 1950-66 experience exceed those for the A49/52 ult. investigation, providing lower monetary values at all ages as shown in column (v) of Table 19.

7.5. The effect of using wives' mortality throughout marriage and widowhood is shown in column (iii) of Table 20. This varies with age but is roughly equivalent to an addition of 5%. The effect of changes in mortality since the 1923-43 investigation, however, varies considerably according to age of husband, but is substantially independent of age difference of husband and wife within the range shown above. One cannot generalise upon the monetary effect of the variation of this function, which is of great significance in the evaluation of the liabilities of widows' funds, though one may expect that in many such funds it will result in an increase in the liability for pensions to future widows of married men.

The annuity values have been calculated using the graduated mortality rates for married men based on the experience for the whole period 1950-66. The difference caused by using the combined mortality of married men and widowers, as was done in the previous investigation, is negligible.

7.6. The reserve required to provide for the future wife of a bachelor is less at all ages than that required using the 1923-43 experience. The considerable diminution in reserve required at the higher ages reflects the substantially lower marriage rates in the 1962-66 marriage experience.

TABLE 21

Present Value of Annuity on Widowhood to First Wife of a Bachelor

Age	1950-66*	1923-43	$\frac{(i)}{(ii)}$ %
	(i)	(ii)	(iii)
25	1·090	1·143	95·4
35	1·254	1·312	95·6
45	0·602	0·931	64·7
55	0·080	0·529	15·1

* The mortality basis used for males is: 1950-66 bachelors prior to marriage and 1950-66 married men thereafter.

The mortality basis used for females is: 1950-66 wives during marriage and 1950-66 widows thereafter.

The marriage basis is: 1962-66 bachelors.

7.7. A comparison of the reversionary annuity values for second wives has not been made as the financial effect of any differences is insignificant, but tables of these functions, based on the latest experience, appear in Appendix IV along with those for the other monetary functions mentioned above.

8. COMPARISONS OF VALUATIONS

8.1. To illustrate the financial effects of the various changes described in the paper, a model fund was constructed and valued in a number of different ways at 5% as set out below. The model fund exhibited at all ages proportions married similar to those observed in the 1962-66 experience of the data from the six funds studied.

TABLE 22

Model Fund Valuation Results

Basis of valuation	Liability for future and current annuities	Value of future contributions	Net liability
1. Scottish Bankers' mortality 1923-43 assuming widows' annuities continue after remarriage.	710,660	149,533	561,127
2. Scottish Bankers' mortality 1950-66 assuming widows' annuities continue after remarriage.	754,960	153,266	601,694
3. Scottish Bankers' mortality 1950-66 assuming widows' annuities cease on remarriage.	746,420	153,266	593,154
4. Scottish Bankers' mortality 1950-66 assuming widows' annuities continue after remarriage and using combined mortality for married men and widowers.	761,860	153,266	608,594
5. Scottish Bankers' mortality 1950-66 assuming widows' annuities continue after remarriage and using wives' mortality for both wives and widows.	794,140	153,266	640,874
6. Scottish Bankers' mortality 1950-66 using collective functions and assuming widows' annuities continue after remarriage.	724,532	153,266	571,266

In all cases the value of future contributions was calculated using all male mortality.

8.2. The main features of these results are firstly the need for larger reserves as the net result of the changes in the underlying marriage and mortality rates, and secondly the considerably lighter reserve produced using collective functions. The proportions married used in calculating the collective functions were those graduated from all males living during the years 1962-66 in the funds studied; these proportions reflect marriage rates over the preceding years and are considerably lower than those which would emerge in a stable state when the 1962-66 marriage rates prevailed. These latter rates underlie the reversionary method valuations 2 to 5, which explains the observed results.

9. ACKNOWLEDGMENTS

9.1. Our thanks are due to the Clerks to the various funds for their generous co-operation in supervising the completion of the coding sheets from the records of the funds which were kindly made available to us by the banks, and for their whole-hearted assistance in clearing up subsequent queries.

9.2. A great deal of the work involved in using the computer consisted of the writing and testing of programmes and we are indebted to colleagues in our own office who assisted with this work.

APPENDIX I
EXAMPLE OF CODING INSTRUCTIONS FOR EACH WIDOWS' FUND

Note: These instructions relate to a Widows' Fund of which the anniversary date is 1st June.

1. A line of coding is to be written for the following classes of member :—

(a) All members who had joined the fund prior to the commencement date, viz. 1/6/46, and were still alive on the commencement date and still members of the fund ;

(*b*) All members who had joined the fund prior to the commencement date and, though deceased on the commencement date, were survived by a widow claiming on the fund at the commencement date ;

(*c*) All members who joined the fund during the period under investigation, viz. 1/6/46 to 31/5/67.

2. Members of the messenger class should be excluded, but all other members should be included.

3. The following guidance is given on completing a line of coding :—

Columns	*Information*	*Remarks*
1-2		To be left blank.
3-7	Member's number	Required in all cases. Where the member's number consists of less than 5 digits, leading spaces can be left (i.e. the number 100 appears as space, space, 100, not as 00100).
8-15	Member's date of birth	This is required accurately for members of classes (*a*) and (*c*). For members of class (*b*) complete the date accurately if known, otherwise leave blank.
16-23	Member's date of entry	Remarks as under " Member's date of birth ".
24-31	Member's date of first marriage	Complete only if applicable, otherwise leave blank. Other remarks as under " Member's date of birth ".
32-39	Member's date of second marriage	Remarks as under " Member's date of first marriage ".
40-47	Member's date of exit	Required for all members of class (*b*). If unknown, a date one day prior to the beginning of the investigation (i.e. 31/5/46) should be coded. For members of classes (*a*) and (*c*) the date should be completed if the member left fund during the period of the investigation.
48	Mode of exit	Code in all cases where the member's date of exit is coded, as D if the exit was by death or W for withdrawal if the exit was for any other reason. Spaces if inapplicable.
49-56	First wife's date of birth	Required in *all* cases where the member has or has had a first wife. Spaces if inapplicable.
57-64	First wife's date of exit	Required in all cases where the first wife is no longer a potential claimant. Spaces if inapplicable.
65	First wife's mode of exit	As for column 48. Exits of widows by remarriage should be coded W.
66-82	.....	as for 49-65, for second wife.....
83-90	Date of birth of member's 1st child	Date of birth of member's first child *born within the period of investigation.* (Not necessarily the first child of the member.) Spaces if inapplicable.
91-98	Date of birth of member's 2nd child	do. second child
99-106	Date of birth of member's 3rd child	do. third child
107-114	Date of birth of member's 4th child	do. fourth child
115-122	Date of birth of member's 5th child	do. fifth child
	Notes	Include information relating to third and further marriages, third and further wives, sixth and further children and information called for in these instructions which you cannot supply. Under the heading " Remarks " write " See notes " and give details on a separate sheet, identifying the case by the member's number.

APPENDIX II

GRADUATION TESTS OF MARRIAGE AND MORTALITY RATES

The graduations were carried out graphically. The graduated rates produced were fed into the computer and the following information was printed out :—

Column 1—Individual ages over the range of the graduation.

Column 2—The graduated rates to five decimal places.

Column 3—First differences of the rates.

Column 4—Second differences of the rates.

The smoothness of the graduation was judged by inspecting the data in columns 3 and 4.

Column 5—The exposed to risk of the decrement being graduated.

Column 6—The actual recorded numbers dying or marrying at each age with an overall total recorded at the bottom.

Column 7—The expected numbers dying or marrying at each age when the graduated rates were multiplied by the exposed to risk; once again an overall total was recorded.

An examination of the overall totals for Columns 6 and 7 showed the difference between actual and expected deaths or marriages. This should approximate to zero if the graduation is to be accepted.

Column 8—The differences between actual and expected deaths or marriages at each individual age with a summation of these differences, ignoring sign, shown at the bottom.

The number of changes of sign in these differences should approximate to the number of non-changes of sign.

Column 9—The accumulated sum, at each age from the youngest, of the differences recorded in Column 8—the figure against the last age being the same as the overall difference between deaths or marriages shown by the totals of Columns 6 and 7. A total of this column, allowing for sign, was recorded.

The number of changes of sign in the accumulated deviations should approximate to the number of non-changes and the final total should approach zero.

Column 10—For each age the function $\sqrt{E_x q_x(1-q_x)}$ was calculated where E_x was the exposed to risk at that age and q_x was the graduated rate of decrement.

[*continued on p. 254*]

Graduation test	Mortality rates							Marriage rates			
	Bachelors	Married men	Widowers	Married men and widowers (combined)	All males	Wives	Widows	Bachelors		Widowers	Widows
								1950-66	1962-66		
Actual events	167	1,270	244	1,514	1,681	481	815	2,083	810	169	51
Total deviations (allowing for sign)	1	1	0	−3	−1	0	0	−2	0	−3	1
Total accumulated deviations (allowing for sign)	−77	10	129	326	271	−10	−76	119	−88	43	108
Sign changes in deviation	40	41	25	43	41	37	34	26	20	28	24
Non-changes in deviation	37	31	34	31	40	33	32	28	34	34	30
Sign changes in accumulated deviations	14	16	8	13	7	19	6	15	10	11	6
Non-changes in accumulated deviations	63	56	51	61	74	51	60	39	44	51	48

No. of deviations:											
> 2 × standard deviation	3	2	4	1	4	5	4	2	4	5	1
> 3 × standard deviation	0	0	2	0	0	1	0	2	3	3	0
No. of deviations:											
> probable error	30	30	18	27	35	33	28	21	24	25	26
< probable error	41	42	40	47	43	37	38	18	22	8	28
Total deviations (ignoring sign)	91	210	80	222	271	135	160	198	73	87	34
Total mean deviations	85	210	83	233	256	136	156	194	106	84	39
χ^2	9	14	12	12	14	8	14	8	5	9	1
No. of cells	14	11	8	12	14	12	10	11	7	12	3
$\cdot 05 < P < \cdot 95$?	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

The figure in Column 8, ignoring sign, should not be greater than twice this " standard deviation " figure.

Column 11—Two-thirds of the figure calculated for Column 10—the " probable error ".

About half the observed deviations brought out in Column 8 should be greater than the " probable error " and half should be smaller.

Column 12—Eighty per cent of the figure calculated for Column 10—the " mean deviation "—with a total of this column printed at the bottom.

The total produced here should be approximately equal to the total appearing in Column 8, ignoring sign.

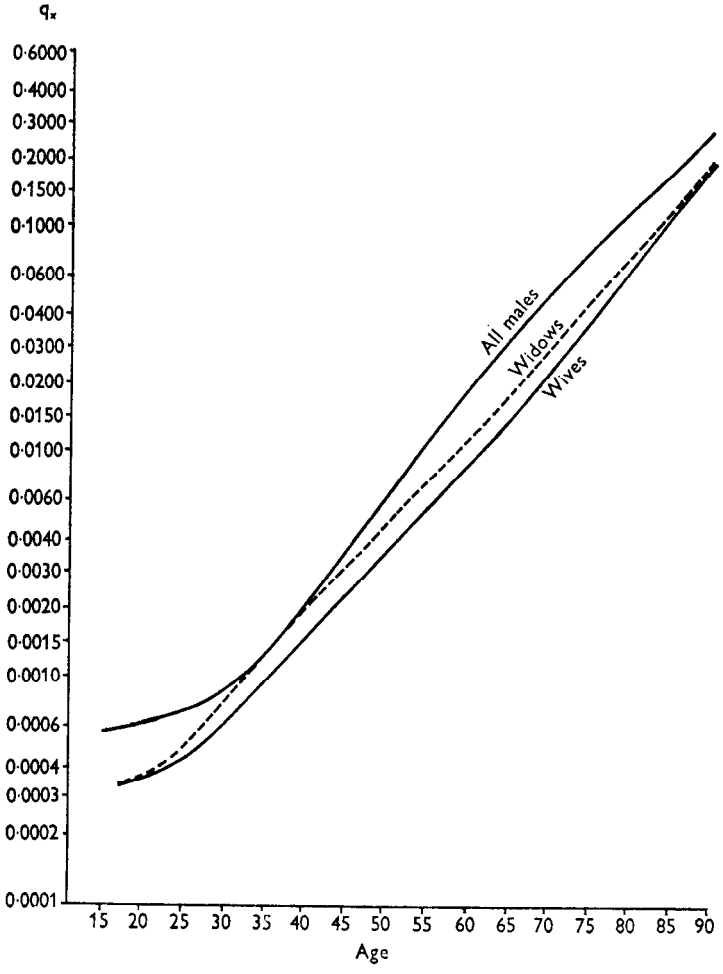
Column 13—For five-year age groups the function $\frac{(A - E_x q_x)^2}{E_x q_x}$ was calculated, where A was the actual number of deaths or marriages for that group, and a total for the figures for each group was obtained.

This enabled the chi-squared test to be carried out where the number of degrees of freedom is the number of groups for which a figure has been calculated less one. The probability, P, of obtaining a value of chi-squared greater than that actually observed is found by reference to a standard table.

The results of the above tests, applied to the mortality and marriage graduations which were adopted, are shown on pages 252-3.

APPENDIX III

GRADUATED MORTALITY RATES q_x FOR ALL MALES, WIVES AND WIDOWS



APPENDIX IV

TABLES OF FUNCTIONS

Note on construction of tables

The formulae and the marriage and mortality bases used in constructing the tables of monetary functions contained in this Appendix are set out below. It will be noticed that a new symbolic notation has been used to denote widows' annuities.

Table E. Single life annuities.

$$a_x = \frac{\sum_{t=1}^{\omega-x} v^t . l_{x+t}}{l_x}$$

based on mortality for all males, for wives and for widows (Table A and B).

Table F. Widow's annuity payable to present wife (y) of married man (x).

$$\mathrm{F}(1, \mathrm{M})_{x:y} = \frac{\sum_{t=0}^{\omega-x} v^{t+\frac{1}{2}} . d_{x+t} . l_{y+t+\frac{1}{2}} . \bar{a}_{y+t+\frac{1}{2}}}{l_{xy}}$$

where d_x and l_x are based on married men's mortality (Table A),

l_y is based on wives' mortality (Table B) and

$\bar{a}_y = \frac{1}{2} + a_y$ and is based on widows' mortality (Table B).

Table G. Widow's annuity payable to first wife of bachelor (x).

$$\mathrm{F}(1, \mathrm{B})_x = \frac{\sum_{t=0}^{\omega-x} v^{t+\frac{1}{2}} . (bm)_{x+t} . \mathrm{F}(1, \mathrm{M})_{x+t+\frac{1}{2}:y}}{(bl)_x}$$

where $(bl)_x$ is no. of bachelors attaining age x and $(bm)_x$ is no. of bachelors marrying between ages x and $x+1$, these two functions being derived from a double decrement table based on bachelors' mortality and marriage rates (Table A and C respectively),

$\mathrm{F}(1, \mathrm{M})_{x+t+\frac{1}{2}:y} = \frac{1}{2}[\mathrm{F}(1, \mathrm{M})_{x+t:y'} + \mathrm{F}(1, \mathrm{M})_{x+t+1:y''}]$

these values being obtained as in Table F,

y′ is average age of wife at marriage to a man aged $x+t$ and

y″ is average age of wife at marriage to a man aged $x+t+1$ (Table D).

Table H. Widow's annuity payable to next wife of widower (x).

$$\mathrm{F}(1,\ \mathrm{W})_x = \frac{\sum_{t=0}^{\omega-x} v^{t+\frac{1}{2}} . (wm)_{x+t} . \mathrm{F}(1,\ \mathrm{M})_{x+t+\frac{1}{2}:y}}{(wl)_x}$$

where $(wl)_x$ is no. of widowers attaining age x and $(wm)_x$ is no. of widowers marrying between ages x and $x+1$, these two functions being derived from a double decrement table based on widowers' mortality and remarriage rates (Tables A and C respectively). Other functions are as for Table G.

Table I. Widow's annuity payable to second wife of married man (x).

$$\mathrm{F}(2,\ \mathrm{M})_x = \frac{\sum_{t=0}^{\omega-x} v^{t+\frac{1}{2}} . (mw)_{x+t} . \mathrm{F}(1,\ \mathrm{W})_{x+t+\frac{1}{2}}}{(ml)_x}$$

where $(ml)_x$ is no. of married men attaining age x and $(mw)_x$ is no. of married men becoming widowers between ages x and $x+1$, these two functions being derived from a double decrement table based on married men's mortality, wives' mortality and average age of wife at husband's birthday (Tables A, B and D respectively) and

$\mathrm{F}(1,\ \mathrm{W})_x$ is obtained as in Table H.

Table J. Widow's annuity payable to second wife of bachelor (x).

$$\mathrm{F}(2,\ \mathrm{B})_x = \frac{\sum_{t=0}^{\omega-x} v^{t+\frac{1}{2}} . (bm)_{x+t} . \mathrm{F}(2,\ \mathrm{M})_{x+t+\frac{1}{2}}}{(bl)_x}$$

where $(bl)_x$ and $(bm)_x$ are as in Table G and

$\mathrm{F}(2,\ \mathrm{M})_x$ is obtained as in Table I.

TABLE A

q_x—*Males*

Age	Bachelors	Married men	Widowers	Married men and widowers combined	All males
15	·00065				·00065
16	·00065				·00065
17	·00066	·00041	·00105	·00041	·00065
18	·00066	·00041	·00112	·00041	·00065
19	·00067	·00042	·00120	·00042	·00066
20	·00068	·00042	·00128	·00042	·00066
21	·00070	·00042	·00137	·00042	·00067
22	·00073	·00043	·00147	·00043	·00067
23	·00077	·00043	·00158	·00043	·00068
24	·00082	·00044	·00170	·00044	·00069
25	·00088	·00045	·00183	·00045	·00070
26	·00095	·00047	·00197	·00047	·00071
27	·00104	·00049	·00212	·00049	·00073
28	·00115	·00053	·00228	·00053	·00076
29	·00128	·00058	·00245	·00058	·00080
30	·00143	·00064	·00263	·00064	·00085
31	·00160	·00070	·00282	·00070	·00091
32	·00179	·00076	·00303	·00076	·00098
33	·00200	·00083	·00326	·00083	·00106
34	·00223	·00091	·00351	·00091	·00116
35	·00248	·00101	·00378	·00101	·00128
36	·00275	·00113	·00407	·00113	·00142
37	·00304	·00127	·00438	·00127	·00158
38	·00335	·00144	·00471	·00144	·00176
39	·00368	·00164	·00506	·00164	·00196
40	·00403	·00187	·00543	·00187	·00218
41	·00440	·00213	·00583	·00213	·00242
42	·00478	·00242	·00626	·00242	·00269
43	·00517	·00274	·00672	·00274	·00299
44	·00557	·00310	·00722	·00310	·00333
45	·00598	·00350	·00776	·00350	·00371
46	·00640	·00395	·00835	·00395	·00414
47	·00683	·00445	·00899	·00445	·00462
48	·00727	·00501	·00969	·00501	·00516
49	·00772	·00564	·01046	·00564	·00577
50	·00819	·00635	·01131	·00635	·00646
51	·00869	·00715	·01225	·00715	·00724
52	·00923	·00805	·01328	·00805	·00812
53	·00982	·00906	·01441	·00906	·00911
54	·01048	·01019	·01564	·01019	·01021
55	·01123	·01145	·01698	·01145	·01143
56	·01209	·01285	·01843	·01285	·01278
57	·01308	·01440	·02000	·01440	·01427
58	·01422	·01611	·02170	·01611	·01591
59	·01552	·01799	·02354	·01799	·01771

TABLE A (*continued*)
q_x—*Males*

Age	Bachelors	Married men	Widowers	Married men and widowers combined	All males
60	·01699	·02005	·02553	·02005	·01968
61	·01864	·02230	·02768	·02230	·02183
62	·02049	·02474	·03001	·02475	·02417
63	·02255	·02738	·03254	·02741	·02671
64	·02484	·03023	·03529	·03029	·02947
65	·02738	·03330	·03828	·03340	·03247
66	·03019	·03660	·04153	·03675	·03573
67	·03329	·04014	·04506	·04036	·03927
68	·03670	·04393	·04889	·04425	·04311
69	·04044	·04798	·05304	·04844	·04727
70	·04453	·05230	·05753	·05295	·05177
71	·04899	·05690	·06239	·05780	·05663
72	·05385	·06180	·06765	·06302	·06187
73	·05914	·06702	·07334	·06864	·06752
74	·06489	·07258	·07950	·07469	·07361
75	·07114	·07850	·08617	·08120	·08017
76	·07793	·08480	·09339	·08820	·08723
77	·08531	·09150	·10121	·09573	·09483
78	·09334	·09863	·10968	·10384	·10301
79	·10209	·10622	·11886	·11258	·11182
80	·11164	·11430	·12881	·12201	·12132
81	·12208	·12291	·13960	·13220	·13158
82	·13351	·13209	·15130	·14323	·14268
83	·14604	·14189	·16399	·15519	·15471
84	·15979	·15237	·17775	·16818	·16777
85	·17489	·16360	·19267	·18231	·18197
86	·19148	·17566	·20884	·19770	·19743
87	·20972	·18864	·22636	·21448	·21428
88	·22979	·20264	·24534	·23279	·23266
89	·25189	·21778	·26590	·25278	·25273
90	·27625	·23420	·28817	·27462	·27467
91	·30313	·25207	·31232	·29850	·29868
92	·33283	·27159	·33858	·32464	·32499
93	·36570	·29300	·36734	·35329	·35386
94	·40215	·31659	·39910	·38474	·38559
95	·44266	·34271	·43446	·41933	·42053
96	·48779	·37178	·47412	·45746	·45909
97	·53819	·40430	·51888	·49960	·50175
98	·59461	·44087	·56964	·54630	·54907
99	·65791	·48221	·62740	·59820	·60170

TABLE B
q_x—*Females*

Age	Wives	Widows	Age	Wives	Widows
			60	·00883	·01191
			61	·00966	·01310
17	·00034	·00035	62	·01057	·01441
18	·00035	·00036	63	·01158	·01585
19	·00035	·00037	64	·01270	·01743
20	·00036	·00038	65	·01395	·01916
21	·00037	·00039	66	·01535	·02105
22	·00038	·00041	67	·01692	·02311
23	·00039	·00043	68	·01869	·02535
24	·00041	·00046	69	·02069	·02779
25	·00043	·00049	70	·02295	·03045
26	·00046	·00053	71	·02551	·03335
27	·00049	·00058	72	·02840	·03652
28	·00053	·00064	73	·03166	·03999
29	·00057	·00071	74	·03533	·04380
30	·00062	·00079	75	·03945	·04799
31	·00067	·00088	76	·04407	·05261
32	·00073	·00098	77	·04924	·05772
33	·00080	·00109	78	·05501	·06338
34	·00088	·00120	79	·06142	·06966
35	·00097	·00132	80	·06852	·07663
36	·00106	·00145	81	·07637	·08437
37	·00116	·00158	82	·08504	·09297
38	·00127	·00172	83	·09462	·10252
39	·00139	·00187	84	·10521	·11312
40	·00152	·00204	85	·11693	·12487
41	·00166	·00222	86	·12291	·13789
42	·00181	·00242	87	·14429	·15230
43	·00197	·00264	88	·16021	·16822
44	·00215	·00288	89	·17782	·18578
45	·00235	·00314	90	·19728	·20514
46	·00257	·00343	91	·21877	·22648
47	·00281	·00375	92	·24248	·24999
48	·00306	·00410	93	·26862	·27588
49	·00333	·00448	94	·29742	·30438
50	·00363	·00489	95	·32914	·33574
51	·00396	·00534	96	·36407	·37025
52	·00432	·00583	97	·40255	·40825
53	·00472	·00636	98	·44498	·45014
54	·00516	·00693	99	·49182	·49638
55	·00564	·00756			
56	·00617	·00825			
57	·00675	·00902			
58	·00738	·00988			
59	·00807	·01084			

TABLE C
Rates of Marriage

Age	Bachelors		Widowers 1950-66	Widows 1950-66
	1962-66	1950-66		
17	·001	·001		
18	·002	·002		
19	·004	·004		
20	·010	·008		
21	·024	·016		
22	·064	·046		
23	·144	·098		
24	·204	·148		·100
25	·240	·190	·300	·091
26	·242	·208	·300	·083
27	·234	·214	·300	·075
28	·224	·212	·300	·068
29	·212	·204	·300	·061
30	·198	·192	·300	·055
31	·184	·178	·290	·050
32	·170	·163	·277	·045
33	·156	·148	·261	·040
34	·142	·134	·243	·036
35	·128	·121	·224	·033
36	·115	·109	·205	·030
37	·103	·098	·187	·027
38	·092	·088	·170	·024
39	·082	·079	·154	·022
40	·073	·071	·139	·020
41	·065	·064	·126	·017
42	·058	·057	·115	·015
43	·052	·051	·106	·014
44	·046	·046	·099	·012
45	·041	·041	·093	·011
46	·036	·037	·088	·009
47	·031	·034	·084	·008
48	·026	·032	·081	·007
49	·022	·030	·079	·006
50	·018	·028	·078	·005
51	·015	·026	·077	·005
52	·012	·024	·007	·004
53	·010	·022	·076	·003
54	·008	·020	·076	·003

TABLE C (*continued*)
Rates of Marriage

Age	Bachelors		Widowers 1950-66	Widows 1950-66
	1962-66	1950-66		
55	·006	·018	·075	·003
56	·005	·016	·074	·002
57	·004	·014	·073	·002
58	·003	·012	·071	·002
59	·002	·011	·067	·002
60	·001	·010	·063	·002
61		·009	·058	·002
62		·008	·053	·002
63		·007	·048	·002
64		·006	·043	·002
65		·005	·039	·001
66		·004	·035	·001
67		·003	·031	·001
68		·002	·028	·001
69		·001	·025	·001
70			·023	·001
71			·021	·001
72			·019	·001
73			·017	·001
74			·015	·001
75			·013	
76			·012	
77			·011	
78			·010	
79			·009	
80			·008	
81			·007	
82			·006	
83			·005	
84			·004	
85			·003	
86			·002	
87			·001	

TABLE D
Average Age of Wife

Husband's age (x)	Age of wife at husband's birthday prior to marriage	Age of wife for husbands attaining age x	Husband's age (x)	Age of wife at husband's birthday prior to marriage	Age of wife for husbands attaining age x
			50	43	47
			51	44	48
17	18	18	52	45	49
18	19	19	53	46	50
19	20	20	54	47	51
20	21	21	55	48	52
21	22	22	56	49	53
22	23	23	57	50	54
23	23	23	58	51	55
24	24	24	59	51	56
25	24	24	60	52	57
26	25	25	61	52	58
27	26	26	62	53	59
28	26	27	63	53	60
29	27	28	64	53	61
30	27	28	65	53	61
31	28	29	66	54	62
32	28	30	67	54	63
33	29	31	68	54	64
34	29	32	69	54	65
35	30	33	70	54	66
36	31	34	71	54	66
37	32	35	72	54	67
38	32	35	73	54	68
39	33	36	74	55	69
40	34	37	75	55	69
41	35	38	76	55	70
42	35	39	77	55	70
43	36	40	78	55	71
44	37	41	79	55	71
45	38	42	80	55	72
46	39	43			
47	40	44			
48	41	45			
49	42	46			

TABLE E

Single Life Annuity Values

Mortality Rates 1950-66—Interest 5%

Age	All Males	Wives	Widows*	Age	All Males	Wives	Widows*
15	18·348			60	9·672	11·952	11·300
16	18·277			61	9·360	11·662	11·008
17	18·202	18·663	18·494	62	9·047	11·364	10·712
18	18·123	18·603	18·426	63	8·735	11·060	10·412
19	18·041	18·540	18·354	64	8·423	10·749	10·109
20	17·955	18·474	18·279	65	8·113	10·431	9·803
21	17·864	18·404	18·200	66	7·804	10·108	9·494
22	17·769	18·332	18·118	67	7·498	9·779	9·183
23	17·669	18·256	18·031	68	7·195	9·444	8·870
24	17·655	18·176	17·941	69	6·895	9·105	8·556
25	17·456	18·093	17·847	70	6·599	8·763	8·241
26	17·341	18·005	17·748	71	6·307	8·417	7·924
27	17·221	17·914	17·646	72	6·020	8·069	7·608
28	17·095	17·819	17·539	73	5·738	7·720	7·291
29	16·934	17·720	17·427	74	5·461	7·372	6·974
30	16·826	17·617	17·312	75	5·189	7·023	6·658
31	16·682	17·509	17·192	76	4·924	6·678	6·344
32	16·533	17·397	17·067	77	4·664	6·335	6·031
33	16·376	17·280	16·938	78	4·411	5·996	5·720
34	16·213	17·159	16·804	79	4·163	5·662	5·413
35	16·044	17·033	16·666	80	3·922	5·335	5·109
36	15·867	16·902	16·522	81	3·686	5·013	4·810
37	15·685	16·765	16·374	82	3·457	4·699	4·516
38	15·495	16·624	16·219	83	3·234	4·393	4·227
39	15·298	16·478	16·060	84	3·017	4·094	3·946
40	15·095	16·326	15·894	85	2·807	3·805	3·672
41	14·884	16·168	15·723	86	2·603	3·524	3·405
42	14·666	16·005	15·546	87	2·405	3·253	3·148
43	14·441	15·835	15·363	88	2·214	2·991	2·899
44	14·209	15·660	15·174	89	2·029	2·740	2·659
45	13·969	15·478	14·979	90	1·851	2·499	2·429
46	13·722	15·290	14·777	91	1·680	2·269	2·209
47	13·468	15·096	14·569	92	1·516	2·050	1·999
48	13·207	14·896	14·355	93	1·357	1·841	1·798
49	12·939	14·689	14·135	94	1·206	1·644	1·608
50	12·665	14·475	13·909	95	1·060	1·456	1·427
51	12·385	14·254	13·676	96	0·921	1·280	1·256
52	12·099	14·026	13·437	97	0·789	1·113	1·094
53	11·808	13·791	13·191	98	0·663	0·956	0·941
54	11·512	13·549	12·940	99	0·543	0·808	0·796
55	11·213	13·300	12·681	100	0·425	0·669	0·659
56	10·909	13·045	12·417	101	0·324	0·539	0·530
57	10·603	12·782	12·146				
58	10·294	12·512	11·870				
59	9·984	12·236	11·587				

* Widow's annuity continues on remarriage.

TABLE F

Value of Widow's Annuity payable to Present Wife (y) of Husband (x)

Basis: Husband's mortality—Married men
Wife's mortality —Wives, until death of husband; thereafter, widows
Interest —5%

Husband's age (x)	Wife's age (y)					
	$y = x$	$y = x-6$	$y = x-12$	$y = x-18$	$y = x-24$	$y = x-30$
23	0·995					
24	1·037					
25	1·082					
26	1·129	1·372				
27	1·177	1·432				
28	1·228	1·496				
29	1·281	1·562				
30	1·336	1·631				
31	1·393	1·702				
32	1·452	1·776	2·076			
33	1·513	1·853	2·168			
34	1·577	1·933	2·263			
35	1·643	2·015	2·362			
36	1·710	2·101	2·464			
37	1·780	2·189	2·570			
38	1·852	2·280	2·680	3·032		
39	1·925	2·374	2·792	3·162		
40	2·000	2·469	2·907	3·295		
41	2·076	2·567	3·025	3·432		
42	2·152	2·666	3·146	3·572		
43	2·230	2·767	3·269	3·716		
44	2·308	2·869	3·395	3·862	4·260	
45	2·386	2·973	3·523	4·012	4·429	
46	2·465	3·077	3·652	4·164	4·601	
47	2·543	3·183	3·783	4·319	4·776	
48	2·622	3·288	3·916	4·476	4·955	
49	2·699	3·394	4·049	4·634	5·136	
50	2·775	3·499	4·183	4·795	5·319	5·754
51	2·849	3·604	4·317	4·955	5·504	5·960
52	2·921	3·706	4·450	5·117	5·691	6·168
53	2·990	3·807	4·582	5·277	5·877	6·377
54	3·056	3·905	4·711	5·437	6·063	6·586
55	3·118	3·999	4·838	5·595	6·249	6·795
56	3·175	4·089	4·962	5·750	6·432	7·004
57	3·227	4·174	5·081	5·902	6·613	7·210
58	3·273	4·254	5·195	6·049	6·791	7·414
59	3·313	4·327	5·304	6·192	6·964	7·614

TABLE F (*continued*)

Husband's age (x)	Wife's age (y)					
	$y = x$	$y = x-6$	$y = x-12$	$y = x-18$	$y = x-24$	$y = x-30$
60	3·346	4·393	5·406	6·328	7·132	7·810
61	3·372	4·452	5·500	6·458	7·295	8·002
62	3·390	4·502	5·587	6·580	7·450	8·187
63	3·399	4·544	5·664	6·694	7 598	8·365
64	3·400	4·577	5·733	6·799	7·739	8·537
65	3·393	4·600	5·792	6·895	7·870	8·700
66	3·376	4·613	5·840	6·982	7·992	8·855
67	3·351	4·617	5·879	7·057	8·105	9·001
68	3·317	4·611	5·906	7·122	8·207	9·137
69	3·274	4·594	5·923	7·176	8·298	9 264
70	3 222	4·567	5·929	7·219	8·378	9·379
71	3·163	4·529	5·923	7·250	8·447	9·485
72	3·095	4·482	5·906	7·269	8·504	9·579
73	3·021	4·425	5·879	7·276	8·550	9·661
74	2·939	4·358	5·840	7·271	8·583	9·732
75	2·852	4·282	5·790	7·255	8·605	9·791
76	2·759	4·196	5·730	7·227	8·614	9·839
77	2·661	4·103	5·659	7·187	8·610	9·874
78	2·559	4·001	5·577	7·136	8·595	9·897
79	2·453	3·892	5·486	7·074	8·567	9·908
80	2·346	3·777	5·385	7·000	8·528	9·907
81	2·236	3·655	5·275	6·917	8·476	9·893
82	2·125	3·529	5·156	6·822	8·413	9·868
83	2·014	3·398	5·030	6·718	8·339	9·830
84	1·902	3·264	4·896	6·604	8·254	9·780
85	1·792	3·128	4·755	6·481	8·159	9·719
86	1·683	2·990	4·609	6·349	8·053	9·647
87	1·576	2·851	4·458	6·209	7·938	9·563
88	1·471	2·712	4·302	6·062	7·814	9·469
89	1·370	2·573	4·144	5·907	7·680	9·365
90	1·271	2·436	3·983	5·746	7·538	9·250
91	1·176	2·301	3·820	5·580	7·388	9·126
92	1·086	2·169	3·657	5·408	7·230	8·993
93	0·999	2·040	3·494	5·233	7·066	8·851
94	0·916	1·914	3·332	5·055	6·895	8·701
95	0·838	1·792	3·792	4·875	6·718	8·543
96	0·764	1·675	3·015	4·692	6·535	8·377
97	0·694	1·563	2·860	4·510	6·348	8·205
98	0·628	1·457	2·710	4·328	6·158	8·026
99	0·567	1·355	2·564	4·147	5·964	7·840
100	0·508	1·259	2·422	3·967	5·767	7·649
101	0·454	1·169	2·286	3·790	5·569	7·452
102	0·398	1·081	2·152	3·613	5·365	7·246
103	0·347	1·005	2·033	3·449	5·172	7·046
104	0·278	0·916	1·899	3·268	4·956	6·820

TABLE F (*continued*)

Value of Widow's Annuity payable to Present Wife (y) of Husband (x)

Husband's age (x)	Wife's age (y)			Husband's age (x)	Wife's age (y)		
	$y = x$	$y = x+6$	$y = x+12$		$y = x$	$y = x+6$	$y = x+12$
20	0·879	0·695	0·521	65	3·393	2·269	1·345
21	0·916	0·722	0·540	66	3·376	2·233	1·305
22	0·955	0·751	0·561	67	3·351	2·191	1·262
23	0·995	0·782	0·582	68	3·317	2·142	1·216
24	1·037	0·814	0·604	69	3·274	2·088	1·167
25	1·082	0·847	0·628	70	3·222	2·028	1·115
26	1·129	0·882	0·652	71	3·163	1·963	1·062
27	1·177	0·919	0·678	72	3·095	1·894	1·007
28	1·228	0·958	0·705	73	3·021	1·822	0·951
29	1·281	0·998	0·733	74	2·939	1·746	0·894
30	1·336	1·039	0·762	75	2·852	1·668	0·838
31	1·393	1·082	0·791	76	2·759	1·587	0·781
32	1·452	1·126	0·822	77	2·661	1·506	0·725
33	1·513	1·172	0·853	78	2·559	1·423	0·670
34	1·577	1·219	0·886	79	2·453	1·340	0·617
35	1·643	1·268	0·919	80	2·346	1·258	0·565
36	1·710	1·318	0·953	81	2·236	1·176	0·515
37	1·780	1·369	0·988	82	2·125	1·095	0·467
38	1·852	1·422	1·023	83	2·014	1·017	0·421
39	1·925	1·475	1·058	84	1·902	0·940	0·378
40	2·000	1·529	1·094	85	1·792	0·866	0·336
41	2·076	1·584	1·129	86	1·683	0·794	0·297
42	2·152	1·638	1·164	87	1·576	0·726	0·261
43	2·230	1·693	1·198	88	1·471	0·660	0·226
44	2·308	1·748	1·232	89	1·370	0·598	0·194
45	2·386	1·802	1·265	90	1·271	0·539	0·164
46	2·465	1·855	1·297	91	1·176	0·484	0·134
47	2·543	1·908	1·327	92	1·086	0·431	0·107
48	2·622	1·960	1·356	93	0·999	0·382	
49	2·699	2·010	1·384	94	0·916	0·335	
50	2·775	2·058	1·409	95	0·838	0·292	
51	2·849	2·104	1·432	96	0·764	0·249	
52	2·921	2·148	1·452	97	0·694	0·208	
53	2·990	2·188	1·469	98	0·628	0·168	
54	3·056	2·225	1·483	99	0·567		
55	3·118	2·258	1·493	100	0·508		
56	3·175	2·286	1·499	101	0·454		
57	3·227	2·310	1·501	102	0·398		
58	3·273	2·327	1·498	103	0·347		
59	3·313	2·339	1·490	104	0·278		
60	3·346	2·344	1·478				
61	3·372	2·343	1·461				
62	3·390	2·335	1·439				
63	3·399	2·320	1·412				
64	3·400	2·298	1·380				

TABLE G

Value of Widow's Annuity payable to First Wife of a Bachelor

Basis : Bachelors' Marriage Rates 1962-66
Bachelors' Mortality Rates until Marriage ;
thereafter factors from Table F
Interest 5%

Bachelor's age	Value	Bachelor's age	Value
15	0·666	40	0·977
16	0·700	41	0·908
17	0·735	42	0·834
18	0·773	43	0·757
19	0·812	44	0·679
20	0·853	45	0·602
21	0·896	46	0·526
22	0·942	47	0·452
23	0·990	48	0·384
24	1·041	49	0·322
25	1·090	50	0·266
26	1·137	51	0·217
27	1·184	52	0·174
28	1·227	53	0·138
29	1·264	54	0·107
30	1·291	55	0·080
31	1·309	56	0·059
32	1·315	57	0·041
33	1·308	58	0·026
34	1·288	59	0·013
35	1·254	60	0·005
36	1·213	61	0·000
37	1·166		
38	1·109		
39	1·045		

TABLE H

Value of Widow's Annuity to Next Wife of a Widower

Basis: Widowers' Remarriage Rates 1950-1966
Widowers' Mortality Rates until Marriage;
thereafter factors from Table F
Interest 5%

Widower's age	Value	Widower's age	Value
		45	2·048
		46	2·057
17	0·740	47	2·066
18	0·773	48	2·070
19	0·810	49	2·074
20	0·850	50	2·070
21	0·896	51	2·061
22	0·949	52	2·052
23	1·004	53	2·030
24	1·062	54	1·998
25	1·122	55	1·954
26	1·182	56	1·898
27	1·251	57	1·830
28	1·322	58	1·752
29	1·395	59	1·660
30	1·468	60	1·556
31	1·541	61	1·454
32	1·610	62	1·359
33	1·675	63	1·220
34	1·732	64	1·103
35	1·780	65	1·028
36	1·825	66	0·921
37	1·869	67	0·784
38	1·905	68	0·730
39	1·932	69	0·655
40	1·958	70	0·557
41	1·983	71	0·423
42	2·003	72	0·240
43	2·018	73	0·000
44	2·033		

TABLE I

Value of Widow's Annuity payable to Second Wife of a Married Man

Basis: Married Men's and Wives' Mortality up to Widowerhood; thereafter factors from Table H
Interest 5%

Married Man's age	Value	Married Man's age	Value
		45	0·092
		46	0·093
17	0·036	47	0·094
18	0·037	48	0·095
19	0·039	49	0·095
20	0·040	50	0·095
21	0·042	51	0 095
22	0 044	52	0·094
23	0·046	53	0·093
24	0·048	54	0·092
25	0·050	55	0·090
26	0·052	56	0·087
27	0·054	57	0·084
28	0·056	58	0·080
29	0·058	59	0·076
30	0·060	60	0·071
31	0·063	61	0·066
32	0·065	62	0·061
33	0·067	63	0·055
34	0·070	64	0·050
35	0·072	65	0·043
36	0·074	66	0·038
37	0·076	67	0·032
38	0·078	68	0·026
39	0·081	69	0·020
40	0·083	70	0·013
41	0·085	71	0·007
42	0·087	72	0·002
43	0·089	73	0·000
44	0·090		

TABLE J

Value of Widow's Annuity payable to Second Wife of a Bachelor

Basis : Bachelors' Marriage Rates 1962-66
Bachelors' Mortality up to Marriage ; thereafter factors from Table I
Interest 5%

Bachelor's age	Value	Bachelor's age	Value
15	0·029	40	0·030
16	0·031	41	0·027
17	0·032	42	0·024
18	0·034	43	0·021
19	0·035	44	0·019
20	0·037	45	0·016
21	0·039	46	0·014
22	0·041	47	0·012
23	0·043	48	0·010
24	0·045	49	0·008
25	0·046	50	0·006
26	0·047	51	0·005
27	0·048	52	0·004
28	0·048	53	0·003
29	0·049	54	0·002
30	0·049	55	0·002
31	0·048	56	0·001
32	0·047	57	0·001
33	0·046	58	0·000
34	0·044		
35	0·042		
36	0·040		
37	0·037		
38	0·035		
39	0·032		

SYNOPSIS

This paper presents the results of an investigation into the mortality and marriage experience of six widows' funds of the Scottish Banks over the period 1950 to 1966 and compares the results with similar investigations which have been made in the past. Tables of mortality rates have been produced for bachelors, married men, widowers, married men and widowers combined, all males, wives and widows, and marriage rates have been produced for bachelors, widowers and widows.

The examination of the male mortality experience shows a substantial improvement over the previous 1923-43 investigation and in general terms demonstrates that widowers' mortality may be expected to be heavier than that of bachelors which in turn is likely to be heavier than that of married men. Women's mortality also shows considerable improvement but widows' mortality is heavier than wives' mortality at all ages.

The trend towards higher marriage rates and towards marriage at younger ages which was so marked in the previous investigation has continued unabated and the marriage rates for bachelors for the period 1962-66, which have been used for calculating the appropriate monetary functions, demonstrate this trend when compared with the rates for the whole period and with the results of previous investigations.

Monetary functions to enable valuations to be made of widows' funds at 5% interest by the reversionary method are published in the paper together with the results of valuing a model fund by this method using various assumptions. The result of valuing the model fund using the collective method is shown for comparison.

DISCUSSION

Mr. W. M. Morrison, introducing the paper, said :—I do not suppose that many of you today will object, as some did 20 years ago, to our use of the heavy marriage rates of the latest period 1962-66. The increasing rate of marriage is now part of everyday experience.

It will be interesting to hear your comments on the anomalies : why is married men's and widowers' mortality, though considerably improved at the younger ages, actually higher than that for 1923-43 at certain of the older ages? Why is bachelors' mortality lighter than married men's at the older ages, as it was in 1903-23? Why are widowers' remarriage rates so much heavier than widows'?

In some ways the most significant result to emerge from our work is the comparative insignificance of the change in net liability which arises from the various changes in the underlying marriage and mortality rates since the 1923-43 experience—an increase, but only of 7%. It is also worth noting that the collective method of valuation understates the net liability by 5% simply because it uses proportions married derived from marriage rates which have been increasingly steadily.

The published tables of monetary functions have been calculated at 5% interest. Arrangements can be made to use the computer to produce tables, for anyone who may be interested, at other rates of interest, or indeed on other marriage and mortality bases.

All four of us are looking forward very much to hearing the discussion, to which Mr. Limb will reply.

Mr. J. A. Cairns :—It is now over twenty years since the last information on tonight's subject was presented and the papers involved now have what I would call a certain classical element about them. The new paper touches on many aspects which are relevant to basic actuarial techniques, covering the observation and collection of basic data and turning these into graduated mortality and marriage tables so that some of us, if not all of us, can use them in a practical manner when dealing with the many problems encountered when considering widows' pension funds. Any student who wonders just how much work goes into turning observed data into something practical should look through this paper. In particular, he will see from Appendix II just how many tests had to be carried out before an actuary is satisfied with the final results of a graduation. This work in itself must have been tremendous.

For this latest contribution we are greatly indebted to the authors and, indeed, we are still indebted to the authors of the previous paper, some of whom are here tonight and I think at this stage we must also mention the Clerks of the various Funds who must have done considerable work to make the basic data available in a suitable form.

The timing of this paper is also appropriate in view of the increasing interest which proposed legislation has created in this type of employee benefit. Certainly actuaries can expect to be confronted by a substantial increase in the number of problems to be tackled in connection with widows' pension funds in the immediate future. Apart from actuarial aspects, the paper also provides further information on certain aspects of life in Scotland over the last twenty years within a particular group. The paper not only shows how mortality has improved since the 1939 war within a major occupational group, but also it highlights the rapid trend to earlier marriage.

Turning now to the more detailed aspects, the history of male mortality

is possibly not as interesting as that for females. Many actuaries today will, however, be using an adjusted form of the A49-52 Table as a basis for making calculations and I was interested to note that the male mortality experienced by the bankers in 1950-66 was heavier than the A49-52. Reference back to the 1923-43 experience showed that the same feature applied at most ages in comparison with the A24-29. One explanation may be that Scottish mortality is heavier than English mortality and the mortality experience of assured lives almost certainly contains a high proportion of English rather than Scottish data. If population comparisons between Scottish and English mortality can be applied to bankers, then the mortality for English bankers during the period 1950-1966 might be about the A49-52 level. Even so, such an adjustment might not be sufficient to bring their mortality down to the level indicated for C.M.I. Assurances 1950-1966. Finally, on this particular topic, I tried to estimate the financial effect of using assured lives mortality for the male lives in the valuation and my estimate was that the liability for future annuities would probably have been underestimated by about 5%.

In view of the general improvement in mortality, it would have been surprising if the pre-war trend to heavier mortality for widows (shown in Table 8) had not been reversed. It is interesting to look back to the 1949 paper where the authors commented, " It may be that the scale of pension payable has been inadequate to cope with the increasing cost of living ". This reference to rising costs of living could well be appropriate today, although fortunately we find that a lot of employers are aware that some cash must be made available to improve the pensions payable to former employees and their widows. This quotation from the previous paper is, however, a reminder that mortality is affected not only by medicine but also by simple economics.

Again, turning to a table which many actuaries use in connection with valuing widows' pensions, namely, the $a(55)$ ultimate female table, I note from paragraph 5.9, Table 7, that wives' mortality is very close to the $a(55)$ table but that widows' mortality in Table 8 is considerably heavier. A valuation using this standard table would (if my very approximate calculations are correct) probably more than offset the artificial reduction in liability created if assured lives mortality were used for the men.

I would now like to turn to marriage rates. At the time of the 1949 paper, it was noted that the 1923-1943 marriage rates were considerably higher than the rates obtained by Mr. Fraser in his earlier investigation into the 1903-1923 experience. It was further noted that the rates were highest in the very last seven years round about the start of the war. Because of the war, doubts were then expressed about the admissibility of such a trend for the future, but I think we would all agree that tonight's paper shows that the trend has been maintained, particularly in the age range 20-29. The obvious question is whether this trend has stopped. As far as the Scottish bankers are concerned, my forecast is that the trend will continue. The second half of Table 10 shows that the general population of England and Wales has an even earlier average age of marriage. Although part of this difference may be caused by the requirement for a longer period of full-time education it does mean that there is still room for a further drop in the average age for bankers. This particular trend is partly caused by simple family economics and the higher state of affluence today : but a fashion, as we all know, is very difficult to stop once it has started. I wonder how many, if any, of the young married bankers included in the previous experience had working wives, for example. Looking at the 1970 Annual Report of the Registrar-General for Scotland, it can be seen that the average

age at which bachelors marry spinsters was fairly constant at age 24 in the whole of the period 1967-1970 and the same appears in England and Wales. Whether this indicates a turning point or not, remains to be seen.

How much effect these trends will have on the liabilities of a pension fund is difficult to judge because we are talking about ages at which the chances of dying in any one year are very small. What is probably just as important is how many men will never marry. Perhaps I am wrong, but I would have thought that if all members of an age group have a fixed number who will never marry and that group has high initial marriage rates, then it will be found that marriage rates will tend to be lower than for the previous generation as the group gets older. A reasonable case for this theory seems to be supported by the England and Wales figures given in Table 10 ; but the bankers' half of that table certainly cannot be used in favour of it. Possibly, it is because there is a much smaller proportion of unhealthy lives. It does, however, make me feel that the use of the 1962-66 marriage experience as a guide to the future may well result in a very small over-estimate of the liabilities of a fund.

Another consequence of earlier marriage is that the average age difference between husband and wife will, on average, be less in future than it was in the past. If the average age difference at marriage remains constant for bachelors marrying at any given age, then the average age difference for all men aged 65 in forty years' time, say, could be built up and this should, in my estimation, be less than it is for men now aged 65 because of the fact that the age difference tends to increase with an increase in the age at marriage. Table 15 further indicates that the age difference at date of older marriages seems to be getting considerably less although it is presumably graduated on comparatively few marriages. Although such trends may be small, it is worth noting that at age 65 the reversionary annuity value to a wife aged 62 is very nearly 4·0, whereas if she was one year older the value would be about 5% less. Compare this difference with the figures given in Table 22 for the value of the net liabilities where the value for future and current annuities by the collective method is about 10% less than the highest value using the reversionary method. I think it would be instructive if the authors could indicate how much of the liability in that table is for current annuities as opposed to future annuities because I have little doubt that many actuaries, including myself, prefer to use the collective method rather than the reversionary method, especially in times of very complicated benefits which is what we are likely to be faced with in the future. Both methods undoubtedly have their advantages and known disadvantages, both theoretical and practical, and one of the merits of this paper is that it gives a measure of the differences for at least one large body of data.

Turning now to the subject of re-marriage rates of widows in Tables 13 and 14, it is interesting that high pensions should be put forward as a reason for lower remarriage rates whereas, as I have just mentioned, low pensions were partly blamed for the heavier mortality during the period 1923-43. Presumably in the earlier period, remarriage was just not the done thing for, as the basic data given in Table 1 indicate, there were no remarriages whatsoever in the earlier period. Perhaps the authors of that paper decided that if there had been only one or two they had better not show them. Even now, the remarriage rates are very low and I think we must congratulate the authors on their ability to graduate the widows' remarriage rate from exactly 51 remarriages.

The comparison with England and Wales certainly emphasises the differences which can apply to different sections of society and I have no doubt that the widows' pension plays its part in keeping down remarriage

rates. I think, however, the real explanation is simply that we are dealing with a thrifty section of the community where personal saving, life assurance and buying a house are parts of the normal way of life. Table 22 shows that the ignoring of widows' remarriage has very little effect on the value of the liabilities in this case, but the difference would presumably have been several times greater had the authors used population remarriage rates. I think that this particular aspect of the paper is a warning to actuaries who may be working out rates of premium for widow's pension ceasing on remarriage. Population rates of remarriage may result in substantially smaller premiums than required for a thrifty section of the community. In Table 13, the population remarriage rate in the age range 35-44 is 0·0846, but it must be remembered that within that age range the remarriage rate in the early years of widowhood is probably several times 0·0846. This, therefore, would have a considerable effect on the annuity value for a new widow.

I have already mentioned the financial effects of some of the rates of decrement, etc., and will comment on these no further. If anything, like Mr. Morrison, I am surprised how small some of the differences are. The authors have rightly illustrated which bases and methods can give high or low estimates of the liabilities of a widows' pension fund and, once again, I would thank them for writing this paper, particularly at this point of time.

Mr. W. H. Lawson :—This is an excellent and concise paper which must be of great value to students of mortality and pension funds as well as those actuaries concerned with the practical results obtained.

The conciseness of the description of the method of finding the crude mortality rates gave me some difficulty. As I understand it, the experience is subdivided into calendar years and the life year rate interval is applied in each sub-experience. The exact exposure at age x last birthday is calculated for each life in each calendar year and the deaths at age x last birthday which occurred between birthdays in successive calendar years are counted as partly exposed in the first and partly in the second calendar year. The death itself is deemed to have occurred partly in the first calendar year and partly in the second, the fraction of the death in each year being the fraction of exposed to risk. Thus, deaths which occurred between birthdays in 1966 and 1967 will count for a fraction of the deaths in 1966 whether or not the death actually occurred in 1966. Thus, account must be and is taken of pre-birthday deaths in 1967. In the same way, deaths occurring after birthdays in 1949 should count in the exposed to risk and deaths for 1950. I am not sure why the adjustment for the year 1950 was ignored ; perhaps an approximation could have been made by counting one for all pre-birthday deaths in 1950 instead of the fraction of exposed to risk.

In Appendix II the χ^2 test is applied with a deduction of one from the number of cells in obtaining the degrees of freedom for the graphic method of graduation. The results tabulated are all favourable, but one hopes that the first graphic graduation was successful in all cases, because if the first is unsuccessful and the second graph is drawn, partly with reference to the first, further constraints are surely involved and a greater deduction should be made when deciding the degrees of freedom.

One of the most interesting and apparently statistically significant results of the investigation is set out in Tables 13 and 14—the remarriage rates of widows. The remarriage rates of bankers' widows are so low in relation to those of the general population that we must conclude that

either remarriage is inhibited by the loss of pension or the declaration of such remarriage is inhibited. We can deduce from Table 14 that of every 100 widowed male bankers 40 may be expected to remarry within eleven years, whilst from 100 bankers' widows 96 are still unmarried eleven years later. It would appear that the remarriage rates of widows are so greatly affected by whether or not there is a pension ceasing on remarriage that this factor is of paramount importance and completely overshadows the actuarial considerations of age and duration of widowhood. Remarriage appears an almost insignificant mode of decrement from the ranks of pensioned bankers' widows; to all intents and purposes the pension actually ceases on death; to recognise this by removing the remarriage condition would involve an additional real cost of much less than 5% of the cost in respect of widows' pensions.

Throughout history, widows have been famed and feared for their supposed man-hunting propensities. In an erudite contribution to the discussion of a paper presented by G. H. Maddex to the Institute in 1936, which dealt with the national remarriage experience of widows, the author appears to have concluded that widows could not be thought of as rendered harmless to the male population until fifteen years had elapsed or age 50 had been attained, but the statistics concerning Scottish bankers' widows appear to put the charms of men as opposed to those of money pretty firmly in their place.

Mr. D. W. C. Bell :—I would like to take up two or three points and to hope that my banker friends who are present will follow my remarks.

In fairly recent months I have been concerned with two of the widows' funds which are included in the experience we are now considering. Although a merger has taken place between the banks themselves, these are still two quite separate funds. Benefits are different and so are contributions. So also are the nature of the investments and the amounts of the invested funds, the smaller membership being associated with the larger fund. One fund has hitherto been valued using the reversionary method and the other the collective. For certain good reasons, it became necessary to consider if both funds could properly be valued on the same valuation basis, one which would, of course, fairly reflect the probable future experience of both. It was going to be helpful if both funds could be seen to have been judged by the same standard by being measured against the same valuation yardstick, one which applied to assets as well as to liabilities. After the inter-valuation experience of each had been investigated, it was concluded—and not without minor misgivings—that it would, in fact, be quite proper for a common valuation standard to be adopted for the valuations then due. It was therefore with considerable pleasure that I came upon paragraph 5.13. I take this paragraph to mean that the bachelors' mortality experience of each of the funds can be considered as being a random sample from the same universe of bachelors' experience and so also with the mortality of married men, of married women and of widows. In passing, I noticed that there was no paragraph similar to this which related to marriage experience. Table 1 shows that this apparent omission was not due to lack of information, as there were more than ten times as many marriages among bachelors as deaths.

But to return to mortality. It is interesting to compare the mortality brought out by the authors in Tables A and B with the mortality being adopted by the Government Actuary in connection with the proposed State Reserve Pension Scheme which will, of course, provide widows' pensions. In the White Paper (Cmnd. 5143) tables of q_x are set out for

quinquennial ages and the different years to be passed through. Taking the values of q_x which are expected to apply in the year 1977, these are only very slightly heavier than the rates in Table A for married men at ages 25 and 30 ; while for ages 35, 40, 45, 50 and 55 the Government Actuary's rates are lighter than any rates in Table A ; for ages 60 and 65 the Government Actuary's rates lie between those quoted for bachelors and those quoted for married men. For female lives the Government Actuary's rates at all ages quoted are lighter than those for wives and widows in Table B. If the banks' widows' funds should follow a pattern like this for the future, there seems a little more scope for improvement in female mortality than in male so that the financial effect will possibly be an increase in liabilities.

When considering Table A, it must be remembered that normal retirements may have taken place from age 60 onwards for males and that in at least one of the funds, bachelor and widower members without dependants have the right to withdraw upon retirement and, again, that in at least one of the funds bachelor and widower members have no such right. Thus, the data available at the higher ages is to some extent self-selected.

All funds will probably have been valued once and some twice since the experience closed in 1966. For what it is worth I would mention that the marriage rates in Table C, where the authors have highlighted the changes over the period 1950-1966, seem to have altered still further. The build-up has become more rapid with a peak at age 24 and a very gentle fall over the next ten years. These observations are based on a very limited amount of data with, of course, a hand-polished graduation. But a more rapid build-up of marriages would seem to conform to the present pattern of social pressures as observed more generally.

Perhaps we might now turn back to paragraph 1.2 of the paper. The authors refer, and I quote, to " calculating the full range of monetary functions required for valuing these funds ". This is not altogether so. These funds were originally intended to be financed almost entirely by members' contributions and, hence, it was most important to preserve equity as between members. This was intended to be achieved by including, in the table of contributions payable, a tax to be paid on marriage plus a further tax whose amount would depend on the age difference between bride and bridegroom. There are, of course, different ways of valuing these items and the method chosen would doubtless reflect the importance of these taxes to the finances of the fund. One way to value future taxes on first marriage is simply to adapt the formula in Appendix IV for Table G by striking out the F function and substituting the average amount of tax payable deduced from Table 15, which shows the average age of wife at marriage of husband and, of course, according to the actual scale of taxes used by that particular fund.

It is possibly a little ungrateful to have expected more from the authors, but monetary functions at two rates of interest, at 6% for example as well as 5%, would have been extremely valuable and I am glad to learn from Mr. Morrison that this can be done. Do I understand that this service will be provided free of charge? Nowadays, widows' funds such as these are advised by professional investors and the actuary is expected to justify his valuation rate of interest to the same extent that he has chosen to justify the rates of mortality and marriage which he is using. It seems to me that he is not entitled to use a low interest rate in order to put away surplus which he would prefer to see undistributed although he is, of course, entitled to recommend that a widows' fund should be run for the time being with an undistributed surplus, if he thinks this to be desirable and explains why ;

or, indeed, should be run with a deficiency, for some reason special to the fund. Underlying these points is the desirability for the valuation factors to be available at more than one rate of interest or, alternatively, for some quick means of estimating the financial effect of a change in the valuation rate of interest.

Incidentally, these widows' funds require payment of an entrance fee which increases with age at entry. Usually the fees are quite inadequate and, hence, steps must be taken to ensure that the banks, who themselves control the recruiting policies which nowadays can extend to financial specialists, to computer experts and, indeed, to North Sea oil men, do provide sufficient funds to alleviate any new entrant strains.

One final comment. I have seen the typescript of a new textbook for our students and I quote, " There is an alternative method of valuation of widows' benefits known as the reversionary method which has certain theoretical advantages over the collective method. The reversionary method is seldom used in practice ". If it does nothing else for our students, this paper will at least counterbalance that last assertion.

Mr. M. C. Polman :— I was introduced by the President as a Fellow of the Institute of Actuaries and I trust that my somewhat exaggerated London accent will not prevent you understanding what I have to say.

When I first read this paper, I was impressed by it as a competent professional job. I have reread it and I have seen no reason to change my opinion. Perhaps actuaries should not change their opinions too frequently. Nevertheless, it would seem that certain earlier speakers have found things to criticise in it. I don't propose to do just that, but I would rather give you certain ideas which have occurred to me as a result of reading the paper.

There are two aspects on which I should like to comment. The first is regarding the technical process and the second regarding the results. Now, as far as the technical part is concerned, my attention was first directed to graduation because, for my sins, for some eleven years I set examination questions on this particular subject. I was therefore more than impressed by what was done. I have never yet found a set of figures which could be satisfactorily graduated. I somehow feel that columns of figures have the same quality of " dumb insolence " as is ascribed to certain types of machinery. I would, however, suggest to the authors that there is possibly one other method which, had they had the time, they might have tried. Since I have also studied some of the American literature, I know that there are formulae known as Henderson-Whittaker formulae which do the graduation process for you somewhat mechanically. They might perhaps have provided the results slightly more easily than having to sort these things out by drawing curves. However, I only offer it as a suggestion to the authors in case they might like to try it in twenty-five years' time when they write the next paper.

In passing, I noted the marriage rates of bachelors, particularly for those under 30 for the period 1936-43. Again, working from general observation, I was surprised that they were so high. Having grown up in a period before the 1939 war when there was active discouragement among both bank employees and insurance employees to get married before the age of 30, I am surprised that there were any at all. Perhaps that is overstating human nature, however. Now, having commented on those particular items, I would like to mention something that was said to me last week by an actuarial colleague in London. He questioned whether this detailed technical process was necessary because a change of one point of 1% in the

rate of inflation could upset the figures very considerably. I would suppose this was the reason why the authors picked 5% to start with because if you work on a gross rate of interest of 8% and you take 3% for inflation, you get back to 5%. It may be as simple as that, but perhaps somebody would like to provide the answer later. However, I did fly to the authors' rescue in this particular case because I drew attention to the fact that if you make life office calculations for obscurer benefits, and if you don't go through the whole process and leave your signposts up to follow, you come back to the job in five years' time and you wonder why on earth you arrived at these particular results. If for no other reason, I feel that it is necessary to go through this kind of process. Here again I hope the authors will agree with me.

Having said that, I would like to come back to the results, because here I am perhaps changing my line slightly. I note that the authors do not approve of the collective method and that they have used the reversionary method. I think, perhaps, in this they have left out of account—or at any rate they may not have done, but they may not have stated it—the person who is most important in this calculation, which is the client whom you are advising. I would rather feel that the methods to be employed need to be justified to him. We have a situation at the present time in insured pension plans when funding rates, not always being recommended by actuaries, have taken on quite an unusual aspect. I think you are therefore advising the client that the funding rate on this particular scheme should be based on the most expensive possible method and this may well be the best advice to give to the clients concerned. I raise the point because sooner or later somebody will have to face up to it. In this respect, I would like perhaps, even by way of comic relief, to give an account of one of my own experiences in connection with this kind of thing.

For some years I had the duty of advising a small craft trade union about the management of its superannuation fund. There were no withdrawals, which may be different from the bankers these days, and retirement was voluntary : there was no compulsory age of retirement at all, with the result that you had the most amazing form of reverse selection that one has ever seen, because nobody retired between the ages of 50 and 80 unless he was too sick to work. So it was rather an unusual type of fund. As this fund accumulated over the years, naturally there was a demand from members for increased benefits and the various calculations were made on the best figures available and a short statement distributed to the members. And at the annual meeting, somebody got up at the back and enquired who the actuary of somewhat doubtful parentage was who had recommended that they needed to pay more money and, also, wasn't it a lot of adjectival nonsense to have an actuary at all. Now, equally naturally, this was passed back to me by the officials and my first reaction was one of some annoyance at having my professional capacity called into question. I then went away and thought about it and, in due course, briefed the officials about the nature of the calculations as intelligently as I could and they in due course explained it at the next meeting of their union. Now, I mention this, not because I think the Scottish bankers who have been advised here will use the same sort of language and description of the authors of this paper as was applied to me. I think they would be much too polite to say so, or even to use such language, but, nevertheless, it is an exercise in having to convince the client that not only do you think the figures are right but also that he should think the figures are right and between us I think, as a profession, we owe it as a duty to be able to explain these things to an intelligent layman who may ask questions.

Mr. A. D. Wilkie :—I am disappointed that the authors, having fitted polynomials up to the 15th degree to the mortality experience, did not persist—not with further polynomials but with some other type of function. The traditional actuarial function, $a+bc^x$, or something like that, might not have fitted too badly. I have not got their crude data available but I amused myself today with fitting bits and pieces of curves to their all male q_x and by doing a bit of approximate least squares activity, not producing a proper least squares answer, I fitted a function in the form of

$$a+be^{-cx}+b_2e^{-c_2(x-d)^2}-fx.$$

That is a usual Makeham type plus a bit of normal curve with a peak at 78 which accounts for the bulge at those ages. The fx is a curious little bit that seemed to make it fit better, though I don't like it at all ; it introduces funny results at the very young ages. Now, I can't tell whether my graduation of their graduation fits their data at all because I have not got the data, but I notice that the total number of deaths of the all males is about, adding them up, 1,681 and the square root of that actual number is exactly 41, which means that about one standard error is about $2\frac{1}{2}\%$ of the actual number. So that about $2\frac{1}{2}\%$ up or down on those final q_x's would presumably represent the data approximately equally well : I mean $2\frac{1}{2}\%$ off the values in each case, not plus or minus 0·025 to the q. Certainly, my curve fits their graduated curve from about 40 to 90 anyway, within $2\frac{1}{2}\%$. It is not very good at 15—20 and it is not very good at 95—99, but I suppose I could put another couple of terms in and get that fitting better.

The point about all this seems to me that, while fitting with a graphic method is a way of producing practical answers fairly quickly, it seems to me to be theoretically " funking it ". What you are, in effect, doing is fitting by eye a curve of unknown form by least squares but not bothering to do the algebra to find the right answers. Your graduation tests, which then fill most of Appendix II, in effect are asking whether the residuals are random? Actually, I think a serial correlation coefficient test might do that without seventeen lines of test results. Further, a difficulty about the tests here is that for the bachelors' columns there are only 167 deaths split into fourteen cells which means on average 12 deaths in each cell. I am not surprised that the χ^2 test fits ; almost any mortality table that they produced would have fitted that experience. The " all males " experience, of course, has very many more deaths and the test is much more valid. But if one fits, as I have done, a curve with seven parameters, I think one has to take off seven degrees of freedom, so for " all males ", instead of 14 cells giving thirteen degrees of freedom, I would have to use fourteen cells minus seven which is only seven degrees of freedom ; this gives a graduation much more chance of fitting. For a graphical graduation, I am sure that to reduce the number of degrees of freedom by only one is not right unless you draw a straight line, and that's that ; or you can draw a straight line with a curve at the young ages. But any graphically graduated curve must involve a lot more than one parameter.

As I say, my curve was produced fairly quickly : it is not a proper least squares fit and I shall give the actual values of the parameters in writing, but I should be interested in having a look at the actual data and see how well it fits. To conclude : curve fitting should be done by algebra and not solely by bits of paper and pencil.

Mr. J. G. Wallace :—I was honoured to be associated to a small extent with some of the work carried out by the late Alexander Fraser towards the end of his life and I met him on several occasions. He was an actuary of the

highest calibre and his actuarial calculations were a model of accuracy and precision. He would, I am certain, have greatly enjoyed reading the authors' paper tonight and would have much envied them the calculation advantages afforded them by the electronic computer for he carried out all his extensive calculations in his own handwriting with the aid of a table of logarithms.

It is interesting to note the comparison between bachelors' mortality and married men's mortality. In Fraser's investigation, bachelors' mortality was also heavier than married men's mortality and Fraser suggested as an explanation that " banking is a sheltered existence and it is possible that a certain number of delicate young men may go into banking simply because of this, thus tending to send up the rates for bachelors ". Judging from the delicate young men I have to deal with when arranging my overdraft, this may have been true in Fraser's day, but I am inclined to think it has somewhat changed at the present date. On the other hand, there is a somewhat surprising crossing of the bachelors' mortality rates and the married men's mortality rates at ages 55—81 where the bachelors evidence much lighter mortality. Could this feature be associated with the major increase in bachelors' marriage rates, to which I think Mr. Polman has referred, particularly in the age group 25—30? I was able to have access to the marriage experience of bachelors in an Irish Bank for each of the last eight quinquennia up to 1965. I took the authors' 1962-1966 marriage rates as my base and over these forty years the ratio of actual marriages to expected marriages, excluding the vital age group of 25-30, was nearly 70%. In the vital age group 25-30, the ratio was only 25%, which demonstrates the huge increase in marriage rates which have taken place in this particular group, and, clearly, what is happening is that these delicate young bachelors, to whom Mr. Fraser referred, are now getting married much earlier and they are being well looked after by their wives and kept going until middle age, and are now dying after that as married men and not as bachelors : and I think that that might explain the points to which I think Mr. Cairns referred. And all this is taking its toll of the wives, because if you look at their mortality, they gallantly start off at the young ages with a mortality rate much lower than their 1923-43 predecessors ; but the strain tells and, contrary to the general population mortality, their mortality is approaching the 1923-43 mortality about age 70. So perhaps Mr. Fraser's explanation is not entirely without foundation!

While I was calculating these marriage rate comparisons, I thought it might add to the interest of the paper if I compared for the quinquennium 1960-1965 the Irish bankers' married men's, widowers' and wives' mortality (there was insufficient data to compare the widows') with the mortality rates produced by the authors. For the married men and widowers taken together, the ratio of actual deaths to expected was 112%, and for the wives 104%. The numbers involved were relatively small but the results are sufficiently close to support the authors' conclusions.

I have also had access to the mortality experience of another very old Edinburgh widows' fund—the widows' fund of the Company of Merchants of the City of Edinburgh for which I have records going back to 1827. I have compared the experience during 1965-1971 for all males (there are practically no bachelors in this fund) with the authors' 1950-1966 all male rates, rated down one year ; against actual deaths of 98 I found an expected of 98. And for wives, I used the unadjusted 1950-1966 rates and, against actual deaths of 30, produced expected deaths of 30. The 1950-1966 tables produced for widows, however, expected deaths of 88 but there was actually

an extraordinarily light mortality as only 67 actual deaths occurred. In fact, I found that the closest approximation to the widows' mortality was the $a(55)$ ultimate table rated down one year to give 65 expected against the actual 67 deaths. This is a considerably lighter mortality than that experienced by the up-to-date bankers' rates and it would be interesting indeed to know if any other fund has found a similar experience. It may be, even (as some other speakers I think have mentioned), that it is connected with economic circumstances as the widows of merchants may not be to such an extent dependent on the widow's annuity as the perhaps less affluent widows of bankers.

Mr. Cairns referred to the age difference between husband and wife and in view of the records available to me from the Merchant Company's Fund, I thought it might be interesting to refer to the average age of the married members of the fund and the average age of the wives. In the first period of fifty years the average age of the husband was 41·5 ; in the second fifty years, the average age was also 41·5 ; and in the third period, from 1926 to the present time, it is 40. For wives, the three corresponding ages were 36, 37·6 and 37·1. These showed remarkably little change considering the long period of time, but the notable feature is the steady decrease in the age differences : 5·5 years in the first fifty years, 3·9 years in the second fifty and, latterly, 2·9 years.

There is one other comment I would like to make. It is in connection with a point which I think Mr. Polman raised where he suggested that the authors felt that the reversionary method was superior to the collective method. I must confess that I did not get this impression from reading their paper : I thought they were merely comparing the results. I understand that Mr. Polman has found it easier to explain to clients using a collective method rather than using a reversionary method. Mr. Bell indicated that he had experience of both methods and I think it would be very interesting to know from him if he too found that it was easier to explain to clients using the collective method as distinct from the reversionary.

Dr. J. J. McCutcheon :—Lest I seem critical of this evening's paper, let me begin by repeating what so many of our speakers have said already—how much I have enjoyed reading the paper and how instructional and how educational I have found it. Certainly a great deal of effort has gone into it.

The only point which I wish to discuss has already been raised by Mr. Wilkie. I am referring to the graduation of the mortality rates. I suggest that I might possibly take the authors of the paper to task for using the graphic method of graduation. However, I must admit to having myself used this method in connection with a banker's pension fund, so the criticism which I offer is not only against the authors but also against myself. Like Mr. Wilkie, I do feel that there is perhaps something rather unsatisfactory about the graphic method. It is an intuitive feeling that I have, for which I am unable to give very detailed reasons, but I feel that some form of curve fitting might have been very helpful, particularly where there are rates of mortality for three different classes—bachelors, widowers and widows. I would have hoped that one might possibly have been able to find a type of curve, or a family of curves, for which simply by varying the parameters one could have graduated the crude rates satisfactorily. I think that perhaps Mr. Wilkie has over-simplified the case. He referred to a curve with seven parameters. Just how one finds the values of these seven parameters is something I would be very interested to hear from Mr Wilkie! I have spent some time in recent weeks trying to do this for

some rates and I found it far from a trivial task. I think one has to seek a happy medium.

The only other possible method of graduation, if one does not like the idea of curve fitting, would be by some form of summation. In the United Kingdom, it seems that the classical formula of "summation n"—just the straight arithmetic average of n consecutive terms—is the form which is known to actuaries and actuarial students. Reference has been made to the Whittaker-Henderson type of formula used in North America. There is a more general type of summation technique, usually referred to as the "moving weighted average method", where the graduation coefficients of the rates are not all $\frac{1}{n}$. One can put certain criteria for smoothness on the coefficients and come up with uniquely defined graduation formulae. I think perhaps that with an experience of this nature something of this kind might have been better, unless Mr. Wilkie can convince me of his ability to find seven parameters with such ease. But really I am just adding my plea to Mr. Wilkie's that perhaps one could have found some other method than the graphic method.

Again, let me thank the authors for a most instructive paper.

Mr. T. W. Walker :—My actuarial friends will not expect me to attempt to be technical: I would be quite unable to do so even if I wanted but I feel I must destroy immediately the illusion of the delicate young men to whom Mr. Wallace referred. What I want to say is that these former delicate young men are probably now the most shrewd in the business. They are marrying earlier, certainly, and that is due to the attention to their welfare paid by our staff managers, some of whom are present tonight. And, furthermore, when they realise that they can get a house loan at $2\frac{1}{2}\%$, they are very quick actually to see that it is one of the best investments that can be had today. So I don't think we should be too surprised at the increase or improved rate of bachelor marriages in the banking profession.

There is one other thing that I would like to say (I am a bit alarmed at this because I am afraid that this is going to be on my conscience for some time). When I look at the remarriage rates of widows, I am beginning to wonder indeed if quite soon I am going to have a charge against me of *contra bonos mores* and whether I shall have to take out of my widows' funds this prohibition (if that is the right word) of widows remarrying and forfeiting the annuity which had been provided by their former husbands. The point has been taken, and I should like to say that it is something to which my attention has been drawn by my own bank's actuary.

These are all general observations I would like to make. Speaking on behalf of my banking colleagues tonight, I would like to say before I sit down what a pleasure you have given us to be here tonight in participating in this very stimulating exercise.

Mr. G. D. Gwilt :—This paper does two things, I think. One is the obvious one of producing new sets of mortality rates and marriage rates which together with the tables given will be great assistance to those engaged in giving advice in connection with bankers' widows' funds. The other is to provide theoreticians with a homogeneous experience on which can be tested theories of mortality and to which various mathematical formulae can be fitted. (Incidentally, I was slightly surprised that the

authors had tried to fit a fifteenth degree polynomial to q_x and not to $\log q_x$, which latter function they found was nearly a straight line over a wide range of ages.) It would, I think, be of great use to theoreticians if the crude rates and the exposed to risk at each age could be given, as indeed they were in the two previous papers, and presumably will again be in twenty-five years' time.

Finally, I would like to point out that in Table 13, which contains widows' remarriage rates, the rates shown for widows under the heading " Bankers 1950-1966 " are in fact almost exactly half, at all these ages, of those rates shown by the Government Actuary in his First Quinquennial Review of the National Insurance Act, 1946, which rates presumably refer to the years 1946-1951 just before the period considered here. These rates are so very much less than the England & Wales rates shown by the authors for comparison.

Mr. M. D. Thornton :—I would like to start by reassuring Mr. Walker. There is no evidence in the paper that the annuity ceasing on remarriage is the cause of the low remarriage rates of widows. The figures are equally consistent with the provision of an annuity to a widow reducing her need to remarry : thus, the cessation may well be irrelevant and any deduction from these figures to the contrary should be resisted. It could only be proved by seeing how much the remarriage rates went up if the annuity was continued after marriage. I am quite sure they would not go up to the level of the general population which up to now has not had widows' annuities at all.

On the graduation, I was very happy to see the use of the graphical method. I remember that, when we discussed the A 49-52 Table in this Hall, I made some gentle criticism of its graduation by curve fitting methods and said then that I thought that a graphical graduation would have given more scope for judgment. I was taken to task for that, not in the course of the discussion but on the way out of the Hall, by the late Mr. Haycocks of the Actuarial Tuition Service who asked me if I did not realise that graphical graduation was completely out of date and that at the time of the classical paper to which I had referred (Sprague's paper on Graphical Graduation in *J.I.A.* 26), the graduators, in order to get readings to even three significant figures from their graph, had to have squared paper so large that no table would hold it and they performed the operation on the floor. For the benefit of our visitors, I should say that Dr. Sprague was for close on thirty years the most distinguished actuary in Britain and you can see him on the pedestal just to the left of the four authors of the paper tonight. In those days, they did not use logarithmic graph paper which perhaps would have made the task possible on a table rather than the floor. Actually, graphical graduation by reference to a logarithmic scale is probably in substance nearly a hundred years old. There was a paper on *Graduation of a limited experience by reference to a standard table*, by, I think, G. F. Hardy ; and he ratioed his crude rates on to the rates of a standard table which, of course, would be basically logarithmic in character and found them very much easier to graduate thereby.

As far as the graduation by curve fitting is concerned, I can see the considerable advantages of fitting a Makeham curve since this enables joint life functions to be more easily calculated. I cannot see any corresponding advantages over graphical graduation from fitting other curves by algebraic methods. The method of getting to the curve is just the scaffolding : unless it has the permanent advantages of the Makeham curve or some other residual advantages, when the scaffolding is taken away

and the curve stands on its own, the type of scaffolding used is no longer important. The algebraic method rather than the graphical method is simply a matter of taste, a matter of fashion and a matter of people who are expert in a special skill naturally enjoying the exercise of that skill. It gives them very good results, but an experienced graphical graduator also gets good results, and when there are limited data, graphical graduation has much to commend it.

The other matter arising from the paper—and it is a delightful paper that brings up so many topics of interest—is, of course, mortality. I too was interested in the bachelors' experience being lighter than the married men's. A mortality experience shows only what happened in the past; it is no use to you for future valuations unless you have reason to expect the experience will be repeated, and you can only assume this if it seems reasonable that it should have happened in that way. We therefore have to account (and Mr. Wallace has already done so or had an attempt at it) for the bachelors' mortality being lighter than the married men's mortality over a great area of the table. Marriage is a selective force and, therefore, in the early years of adult life, married men show a lighter mortality than bachelors. In the later years when this selection has worn off, the true situation is revealed; namely, that bachelorhood is basically healthier than matrimony. For the benefit of our visitors, I should say that the fact that the mortality experience of bachelors looks heavier at certain ages is not significant. The mortality experience in a hospital is very much higher than the experience outside of a hospital. That does not mean that a hospital is an unhealthy place, it means that unhealthy people go to it. Much the same applies to being a bachelor. (Talking about explaining to laymen, I explained this to my wife and told her how greatly I had shortened my expectation of life by my marriage to her. I was not successful in convincing her.)

We had corroboration of this feature, I think, a couple of years ago when we were addressed by Dr. Comfort who, speaking on the postponement of old age and the prolongation of life, gave strong evidence to show that fasting was a great contribution to long life. Now, the Church, particularly the Catholic Church, whether Anglican or Roman, has for many years recommended fasting though recently it has weakened on this, but not having Dr. Comfort's experience, it had to recommend it for spiritual reasons. But these later figures and discoveries of Dr. Comfort showed that really in their stumbling way the Church was getting at a mathematical truth. In the same way, the Catholic Church has recommended celibacy as a spiritually higher state than marriage. That may be disputed. We are not experts on spiritual matters. We should have perhaps raised this at the last meeting when we had some ministers of the Church of Scotland present. But it seems clear that there is now material support for the Catholic viewpoint. I hope this will give pause to any of our younger members contemplating matrimony; for the older members, all I can say is that they should eat no dinner tonight.

The remaining point that I would like to refer to is the choice of reversionary or collective method. It has been said that Oxford is the home of lost causes. In actuarial circles, Edinburgh is the home of lost causes because it is practically the only place in the United Kingdom where the reversionary method of valuing widows' annuities continues to flourish. Long may it do so! It is a beautiful thing to see the successive formulas of the reversionary method. It takes somewhat longer than the collective method; it does not necessarily bring a more satisfactory result. In these days particularly, where interest rates are so compelling a force in the

valuation of any fund, the question of relative error and the degree of accuracy which you can achieve in a valuation, which you expect to correct three years later, or five years later, with a further valuation, is not such that the theoretical accuracy of the reversionary method justifies its use in preference to the shorter, easier collective method. But although actuaries are professionals, many are also, in the sense of loving our work, amateurs and find it a real joy to carry out a valuation of a widows' fund by the reversionary method. I look forward—I may not be here but I look forward—in twenty-five years to a further paper on the subject and I am very grateful to the authors for their paper tonight.

Mr. D. D. McKinnon, closing the discussion, said :—" I have ", Wellington once remarked, " been much exposed to authors." This remark appeared recently in a review of the latest biography of the Duke of Wellington. By now the Scottish bankers, mortal or immortal, married or unmarried, or indeed whatever state they happen to be in, could not be faulted for having a similar feeling. The present authors make no apology for this exposure and none is required for, as has been pointed out, the present paper is a continuation of the previous three papers. The President has referred to the number of authors and to Mr. Reid's extrapolation but the effect of restricting the number of present authors to four means that the total now comes to thirteen. This may or may not be significant but even so it is with some diffidence that I rise to close this discussion because I suspect that my remarks may, in fact, be the last—apart from those of the President and the authors—to be made on the very restricted experience of the marriage and mortality of Scottish bankers. This is not a reflection either on the authors or indeed on the Scottish bankers. Extrapolating, as some have done, to find the date of the next instalment, brings us exactly to the centenary of Hewat and Chatham's paper but, as mentioned by Mr. Cairns, 1975 and all that will mean that another instalment in the same form will not be possible. I do not share Mr. Gwilt's optimism or Mr. Thornton's enthusiasm. Changes there are bound to be, changes which by making widows' benefits universal will remove the distinctive features of the older widows' funds. The subject has not, at least until this evening, been one to attract contributions to the discussion in numerical keeping with the authorship. The contributions this evening, including my own, bring the total to 30. The lack of numbers in previous discussions did not affect the quality and I think the number of contributions this evening, which amount to 11 out of the 30, have not affected the quality either. The papers, the previous discussions and the one this evening will be interesting and instructive to students of the subject displaying, as they do, the differing approaches to the techniques of an analysis of a marriage and mortality experience, which all the papers have stimulated. They do not set out the rationales of either the existence of or the benefits within the funds. Very occasionally did a speaker digress and perhaps I may be forgiven if I mention one such digression.

In the discussion of the paper by Hewat and Chatham, R. P. Hardy expressed the indebtedness of the members of the Institute " for bringing before them the almost forgotten subject of pension and superannuation funds ". At the same time, he criticised the existence of these funds on the grounds that " man was learning slowly but gradually that all fruitful association must be with his brethren generally and that all artificial limitations of interest or sympathy were opposed to the common good and defeated the ends of the promoters ". We have now reached the stage when

the State is about to compel employers to look after the widows of our brethren in general; this may be long overdue, but in Scotland we can look back with some satisfaction that as far back as 1808 the first bankers' widows' fund was established and we were reminded only at our last meeting of the work done even earlier in connection with the widows' fund of the Church of Scotland. It is interesting that Scots put their ministers first and their bankers second.

Tribute has been paid (particularly by Mr. Cairns) to the tremendous volume of work carried out by the authors. Mr. Wallace commented on the labours of Mr. Fraser, who had not had the advantage of a computer but had carried out his calculations by logarithms.

I hope that the authors will not think me churlish if I mention that the uses of the functions produced are limited in several ways. The most obvious limitation is that mentioned by Mr. Bell, namely, the single interest rate and, in view of this, I was surprised to see the publication of full tables from which approximation will need to be made in any event in future. However, I was pleased to hear from Mr. Morrison that arrangements can be made, quite readily, to provide the functions at different rates of interest, and I, like Mr. Bell, would like to be quoted the appropriate fee for the service.

Further, looking at all four papers, it is surprising that details are not given in any, apart from Hewat's, and there only brief details, of the rules of the funds. It may be that the rules do not affect the relative mortality experience between one group and another or between one investigation and another. Mr. Bell mentioned the differing rules between one fund and another, although the authors state that this has had no effect on the mortality experience. He mentioned, however, that there was no statement about the marriage experience. Until the second world war, this probably did not matter, but the experience since then must have been affected considerably by the rules of the fund. I refer to the effect of withdrawals. No information is given whether members can withdraw and, if so, at what stage they may do so. There was a suggestion, I think in Hewat's paper, that members could continue to subscribe to the funds after they left the bank. There has been a large turnover of staff in banks and insurance offices since the second war. In fact, one gathers, if only after a visit to one's local bank, that the career pattern has changed. The process appears to have accelerated since the end of the period of the investigation. There is now an emphasis on job evaluation both in banking and insurance. It may be that the authors do not feel free to comment on this, but it does seem to me that the withdrawals could well have a selective effect on the marriage experience. I assume that young unmarried bankers, for example, are more free to move than those who are possibly betrothed and so on, and this could raise the marriage rates of those who stay.

This question of marriage rates leads on to that of collective valuation and it would be surprising if, in a discussion such as this one, we did not have the rival claims of reversionary and collective methods brought out before us once again. I realise that the authors' collective valuation was done simply for comparative purposes, it having been done on the previous occasion. Surely, however, an actuary to a fund would not use the collective method with unadjusted proportions married after such a period of altering marriage rates and where, as mentioned by Mr. Cairns, the relative ages of husband and wife would not be appropriate either. As Mr. Wallace has pointed out, in another fund that he was dealing with there were diminishing differences between the ages. Undoubtedly, the

result of the collective valuation produced by the authors does provide an illustration of the effect of changing marriage rates, but I hope that Mr. Cairns does not suggest that the discrepancy indicates any disadvantage of the collective method. With respect to Mr. Bell and to Mr Thornton, it would be unfortunate if the paper resuscitated the old arguments.

One of the main contributors to the discussion in Hewat's paper was G. F. Hardy, the G. F. Hardy referred to by Mr. Thornton. He would jump with delight, as would many of our older members here tonight, at the rehabilitation of the graphic method of graduation, only to be knocked on the head again by Mr. Wilkie whose remarks were not intended for intelligent laymen; there was no possibility of misinterpretation of what he meant by even the unintelligent professionals. Mr. Lawson and Mr. Wilkie in fact doubted the reliability of the tests and Mr. Polman, together with Dr. McCutcheon, has suggested that other methods could have been used and possibly would have been more appropriate. However, it is good to know that we have not yet sold all the special skills of Mr. Thornton to the computer.

Like Mr. Thornton, I am intrigued by the bachelors' marriage experience at older ages. Mr. Bell has mentioned about the rights in one fund to withdraw as a bachelor at retirement. Do the unhealthy or unmarriageable bachelors withdraw on retirement? If they soldier on, the results seem surprising indeed and I am afraid I find Mr. Thornton's remarks more amusing than convincing. It strikes me that the married men have been cheated indeed twice over, or thrice over, because not only is married life shorter but also one is taxed on entering it and it feels longer at the end of the day.

I have one criticism to make of the terminology in the paper, namely the use of the expression the " life-year " method. An attempt is made in the recent book by Benjamin and Haycocks to remove the stumbling block in the appreciation of exposed to risk formulae by students by inserting the words " rate interval " to make it quite clear that the only distinction is between the " census method " and the " rate interval " method and the life year, calendar year, policy year methods are merely the same method with a different rate interval. Mr. Lawson has been confused by a difficulty about the exposures. This difficulty was referred to in all three previous discussions and various answers were given there.

One of the advantages of a paper such as the one we have had this evening is that it forces us to examine trends over a very long period, including trends in actuarial thought. In the *Journal* and *Transactions* which contain the previous papers, there is much emphasis on mortality and on graduation. Today, we sit back, or at least I thought so until this evening, and wait for the C.M.I. publications and worry about inflation. Ironically, when the modernists remove investment, we will all be back to mortality. The paper reminds us that times change, and even bankers and we change too. The paper demonstrates that the cobbler when he sticks to his last is quite efficient and, indeed, one of the longest actuarial discussions we have had in this Hall for some time has taken place this evening. We shall keep the paper and the discussion handy against the time when it may be " All change " again.

May I thank the authors for giving us an opportunity to discuss the subject.

The President :—We have had an excellent discussion. In the first place, we are very glad to have contributions from two of our visitors, Mr. Polman and Mr. Walker: in the second place, it has been one of the most instructive

and indeed entertaining actuarial discussions I have heard for a long time and as regards the latter description I must confess that I did not quite anticipate that on first reading the paper.

On more than one occasion this evening comment has been made on the tremendous amount of work that has been devoted by our four authors to producing their paper, and it is a model of its type. In it, the results of their extensive investigations are beautifully displayed—deceptively simply and briefly—and these attributes may be taken as marks of their success. So often one can get confused by a mass of data but in this paper the data and the results are so very clearly laid out. We are most grateful to the authors for all this. May I now ask Mr. Limb to reply to the discussion.

Mr. A. P. Limb :—Mr. President, Lady and Gentlemen : I have always wondered why T. B. Sprague looked faintly disapproving : I now realise that he must have known for some time that we were not going to perform these operations on our stomachs on the floor.

I would like to begin by thanking both the Clerks to the various funds involved for the patient co-operation extending over quite a long period and also the speakers in the debate this evening which has afforded all of us a great deal of satisfaction and pleasure. I propose to make brief comments on some of the observations that have been made this evening in replying to the debate.

Mr. Cairns referred to the question of whether or not the trend towards earlier marriage is approaching a limit. Quite clearly, just as widows' mortality depends on economic factors and on social factors, so these factors have similarly influenced the trend towards earlier marriage. One can speculate at length and in the delightful absence of evidence which can only be provided in years to come on the likely trend which will develop in this and other areas. There must obviously be a limit to earlier marriage ages. My own guess would be that we shall see a continuation of the current trend for some time to come and I understood Mr. Watson Bell to say that he had already observed a continuation of this trend in the last six years. We were asked also to give figures as to how much of the liability in the various comparative valuations appearing at the end of the paper could be attributable to current and how much to future wives. I don't have these figures with me and I would prefer not to rely on my memory, but this information will be provided in the published reply.

Mr. Lawson spoke of the exposed to risk formulae which we had used. It is quite true that no correction was made regarding the beginning of the period under investigation analagous to the correction that was made at the end of the period and, while the matter is of course of small practical significance in an investigation extending over some sixteen years, it can be shown that the significance of that correction at the beginning of the period is very much less than it would be at the end of the period. I would not like to embark on a description of why this is so verbally but we may consider perhaps whether we should include that in our published remarks.

Mr. Bell pointed out, quite rightly, that we had not made any comment in the paper on the homogeneity or heterogeneity of marriage rates and various other functions. We did, in fact, compare these functions for the various funds involved and there was no conclusive evidence of heterogeneity in any of them. Mr. Bell also pointed out that we had not supplied all the functions required for valuing marriage funds. I must apologise : this is quite true. I would imagine that the revamping of the rules of these funds will gradually tend to removal of the imposition of a tax on marriage, the feeling being that marriage is enough of a difficulty without having to

cope with a tax as well. Perhaps I might reply to the question posed by both Mr. Bell and Mr. McKinnon. As Mr. Morrison mentioned, the ability to produce monetary functions at different rates of interest all with different mortality and marriage tables does exist. This will clearly require the use of clerical and computer time and it would be our present intention that some contribution towards defraying the cost of that time might be appropriate, but that would have to be considered when we were asked to produce rates.

I was very interested indeed by Mr. Polman's remarks on the subject of graduation. He said to my considerable relief that he had never yet found a set of figures which could be, shall we say, conclusively graduated. Well, we did not either. He referred to Henderson/Whittaker formulae and one might also speak of various other tests which exist for determining the significance of departures from accepted limits when testing graduations. I think myself that this is a fascinating theoretical field, but beyond a certain point one has to say, " Well, for the purpose for which it is intended, is this graduation sensible, convincing and will it do the job ? " If so, then we will accept it providing there is nothing obviously wrong with it. If not, we will have to try again. I propose to return to the subject of graduation when replying to Mr. Wilkie and Dr. McCutcheon. Mr. Polman drew from the paper the apparent inference that the authors did not approve of the collective method. If we gave that impression, that was unintentional. My own views between collective and reversionary methods are frankly ones of impartiality. I would have thought that both are perfectly adequate tools and if you use the right rates and use them in the right way it is perhaps unnecessary to be too partisan in defending either of them. I certainly would not agree that the collective method is easier to explain to a client than the reversionary method. Frankly I think both are pretty difficult to explain. I hope I am never put on the mat and asked to explain either in front of a Trustees meeting—at least not without adequate warning.

I would like to group together Mr. Wilkie's remarks and Dr. McCutcheon's remarks on the subject of graduation. There was, I think—I understand anyway—a debate running possibly for centuries amongst theologians on the subject of how many angels could dance on the tip of a pin and I must confess that I feel rather this way when discussing whether one should attempt to graduate by curve fitting or whether one should attempt graphical methods. As I have already indicated, my own feeling is very pragmatic on the subject. It seems to me that the expectation that any curve will do for more than a short period of time and then only by chance is not one that is logically justifiable. I admit that I had cause to review this feeling somewhat on reading Mr. Redington's paper some time ago on the ageing process and the use of exponential functions in cell ageing and I thought for a time, " Well, possibly, this does justify some sort of exponential curve in mortality graduations ". Life is subject to so many hazards of various kinds, both at different times and with the evolution of medicine and pressures of life and so on, that it just seems to me inherently unlikely that any curve or family of curves will really do. They may do for a time : they may work very well for a period. Whether the work involved in fitting to a particular family of curves is justifiable is, however, in my view very much another matter. We could, of course, have tried various osculating functions but these seemed inappropriate in the context of Scottish bankers.

Mr. Wallace attempted some explanation of the reasons which might lie behind some of the phenomena which we observed. The whole subject, as

we have seen this evening in a very entertaining debate, is a fertile one for speculations of this kind and perhaps I might end my remarks by making a further speculation on why it is that Scottish bankers' widows' remarriage rates are light. Before doing that, however, I might suggest that Mr. McKinnon has perhaps graduated the authors wrongly. I think that we make up eleven, not thirteen. Yes, I was going to suggest that possibly if you have been married to a Scottish banker and you are unfortunately widowed after life with so agreeable a companion as a Scottish banker, life with another male just simply does not appeal.

The President :—May I ask you again, this time formally, to join with me in according a hearty vote of thanks to our four authors—Mr. Limb and Mr. Morrison, from whom we have heard this evening and their joint authors Mr. Bews and Mr. Murray. We thank them all for this excellent paper and for the discussion that it has produced.

The authors subsequently wrote as follows :

Mr. Cairns in opening the discussion asked what proportion of the liabilities shown in Table 22 was attributable to current annuities. The answer is approximately 40%.

Mr. Lawson expressed doubt over the question of why an adjustment analogous to that used at the end of the period in calculating crude rates of decrement was ignored at the start of the period. Suppose that for two consecutive calendar years, year 1 and year 2, we consider the progress of a group of lives P_x attaining age x at time t in year 1 and exposed to the risk of death, where θ_1 die in year 1 and θ_2 die in year 2. Let E_x represent the Exposed to Risk between ages x and $x+1$ we seek to build up, and d_x the deaths appropriate thereto.

Then we have the following table :

Item	*Contribution in year 1*		*Contribution in year 2*	
	E_x	d_x	E_x	d_x
θ_1 deaths	$\theta_1(1-t)$	$\theta_1(1-t)$	$\theta_1 t$	$\theta_1 t$
θ_2 deaths	$\theta_2(1-t)$	$\theta_2(1-t)$	$\theta_2 t$	$\theta_2 t$
$P_x-\theta_1-\theta_2$ survivors	$(P_x-\theta_1-\theta_2)(1-t)$	nil	$(P_x-\theta_1-\theta_2)t$	nil
Total	$P_x(1-t)$	$(\theta_1+\theta_2)(1-t)$	$P_x t$	$(\theta_1+\theta_2)t$

Now suppose at the end of the period under review (i.e. at the end of year 1) θ_2 is unknown. Ignoring it would lead to a crude rate of mortality of $\theta_1(1-t)/P_x(1-t)$ at age x in year 1. This requires correction by adding $\theta_2(1-t)$ to the numerator, and since $\theta_2(1-t) \doteqdot \theta_1 t$, the correction is readily made, and significant.

When the beginning of year 2 is the start of the period under review, however, and θ_1 is unknown, the crude rate of mortality in year 2 at age x, ignoring θ_1, emerges as $t\theta_2/t(P_x-\theta_1)$.

Now $\dfrac{t\theta_2}{t(P_x-\theta_1)} = \dfrac{t(\theta_1+\theta_2)-t\theta_1}{tP_x-t\theta_1} \doteqdot \dfrac{t(\theta_1+\theta_2)}{tP_x} = \dfrac{\theta_1+\theta_2}{P_x}$ so the correction is less important.

Mr. Lawson also queried the number of degrees of freedom used in applying the χ^2 test to the graduated rates. There are several schools of thought as to how many degrees of freedom should be allowed when testing the results of a graphic graduation and we are aware that the one we followed has as many dissenters as it has disciples. However, the values of χ^2 produced give satisfactory results for all the graduations whether 0, 1 or

2 degrees of freedom are allowed for, which should be sufficient to satisfy most investigators.

Mr. A. D. Wilkie subsequently wrote :

The authors have very kindly provided me with the data for their male mortality experience, and I have been able to do some experiments on fitting curves to the data. The formula I quoted in the discussion was :

$$q_x = a + b_1 e^{-c_1 x} + b_2 e^{-c_2 (x-d)^2} - fx$$

and with values of the parameters as follows, it fits the data adequately well :

$$\begin{array}{ll} a = 1{\cdot}88856 \times 10^{-3} & c_2 = 0{\cdot}54075 \times 10^{-2} \\ b_1 = 1{\cdot}4648 \times 10^{-4} & d = 78{\cdot}0 \\ c_1 = 0{\cdot}083904 & f = 1{\cdot}0 \times 10^{-4} \\ b_2 = 6{\cdot}97254 \times 10^{-3} & \end{array} \qquad (1)$$

By altering the parameters to those below one can obtain a maximum likelihood best fit, which fits the data slightly better :

$$\begin{array}{ll} a = 1{\cdot}8956441 \times 10^{-3} & c_2 = 0{\cdot}53997807 \times 10^{-2} \\ b_1 = 1{\cdot}0868367 \times 10^{-4} & d = 69{\cdot}609323 \\ c_1 = 0{\cdot}087337 & f = 0{\cdot}91152756 \times 10^{-4} \\ b_2 = 6{\cdot}4036961 \times 10^{-3} & \end{array} \qquad (2)$$

However, this formula is quite unnecessarily complicated for this relatively small quantity of data. The following formulae fit just about as well ;

$$q_x = a + be^{cx} - fx$$

$$\text{with} \quad \begin{array}{ll} a = 2{\cdot}7858 \times 10^{-3} & c = 0{\cdot}078468 \\ b = 2{\cdot}4221 \times 10^{-4} & f = 1{\cdot}6006 \times 10^{-4}, \end{array} \qquad (3)$$

$$\text{and} \quad q_x = e^{a+bx+cx^2+dx^3}$$

$$\text{with} \quad \begin{array}{ll} a = -8{\cdot}65398 & c = 0{\cdot}166104 \times 10^{-2} \\ b = 0{\cdot}0165294 & d = 0{\cdot}105304 \times 10^{-4} \end{array} \qquad (4)$$

The values of q_x produced by each of these formulae are surprisingly different. Nevertheless, overall they each fit the data adequately. Values of q_x are shown in the table below.

By maximum likelihood best fit I mean one which maximises the log likelihood function, or

$$\Sigma\{A_x . \log q_x + (ER_x - A_x) . \log (1 - q_x)\}$$

where A_x is the actual number of deaths at age x and ER_x is the number of years exposed to risk at that age (see e.g., E. S. Keeping " Introduction to Statistical Inference ", Ch. 6). To minimise χ^2 is an approximation to this (see N. L. Johnson's comments on H. A. R. Barnett's paper in *J.I.A.* 77, p. 63).

By a satisfactory fit I mean one where the χ^2 value is within a one-tailed 5% probability level, and where the number of runs of the same sign is also within a one-tailed 5% probability level using the Wald-Wolfowitz runs test (see e.g., H. M. Blalock, *Social Statistics*, Ch. 14).

It is of interest also to see whether the subdivisions of the male data—bachelors, married men and widowers—can be treated as coming from the same " all male " population. I applied the authors' graduated " all males " rates to each group separately and a χ^2 test shows that married men and bachelors are not significantly different from all males (at a probability level of well within 5%), but that widowers showed significantly higher mortality (at a probability level of less than 1%). I therefore

conclude that the discussion on why bachelors had better or worse mortality at various ages than married men, although of great interest, was based on an almost wholly spurious hypothesis.

Age	Authors' all Males	Formula (1)	Formula (2)	Formula (3)	Formula (4)
17	·00065	·00080	·00083	·00098	·00035
22	·00067	·00062	·00063	·00063	·00050
27	·00073	·00060	·00058	·00051	·00074
32	·00098	·00084	·00076	·00048	·00115
37	·00158	·00145	·00130	·00065	·00183
42	·00269	·00266	·00243	·00128	·00300
47	·00462	·00478	·00461	·00260	·00499
52	·00812	·00836	·00855	·00880	·00837
57	·01427	·01432	·01520	·01488	·01405
62	·02417	·02404	·02535	·02428	·02343
67	·03927	·03928	·03976	·03857	·03849
72	·06187	·06198	·06004	·06012	·06182
77	·09483	·09476	·09017	·09240	·09629
82	·14268	·14252	·14964	·14058	·14430
87	·21428	·21437	·21202	·21229	·20642
92	·32499	·32472	·32945	·31884	·27966
97	·50175	·49459	·51240	·47697	·35603

The Authors replied to Mr. Wilkie as follows:

'Admittedly the result of the χ^2 test does not indicate that bachelors' mortality rates are significantly different from those for all males, but the results of the other graduation tests suggest that the data do not fit the graduated rates for all males sufficiently well for us to conclude that bachelors' and married men's data should be amalgamated for graduation.'