SETTING INVESTMENT STRATEGY FOR THE LONG TERM

A Closer Look at Defined Contribution Investment Strategy

Really Long Term Investment Products Working Party

Robert Howie (chairperson)
Helen Davies

Presented to the Finance and Investment Conference
28 June 2002

Contents

Keywords:

asset allocation, asset liability modelling, defined contribution, investment risk, investment strategy, labour income, lifestyling, mean reversion, reinvestment rates, time diversification

Abstract:

This paper considers how investment time horizon affects investment strategy. We consider whether equity risk decays over time i.e. whether the notion of time diversification has validity. Noting that time diversification is mathematically equivalent to mean-reversion, we find that there is limited evidence that the effect exists, based on our own analysis and a review of other studies. If it does exist, the effect is both weak (the autoregression is small) and slow (only noticeable over the longer term) and consequently has little impact on investment strategy even for long time horizons. We examine defined contribution "lifestyling" strategies in this context and find that there is there is some support for a strategy of switching equity into bonds as members approach retirement. However, any equity allocation must be premised on the assumption that members have some risk tolerance, as we show that equity is risky relative to the "hedge portfolio" (the portfolio which most closely matches the target pension and comprises index-linked bonds) even for long time horizons. Given this assumption, maintaining some allocation to equity right up until retirement may be preferable than switching 100% in bonds.

1. Introduction

1.1 Really long term investment products can include capital projects and the like, but we have chosen to limit ourselves to pensions and life insurance. In this context, the longest term investment need we can think of is a defined contribution pension plan for someone just starting work. Because of pension legislation in most countries, the fund is tied up for 40 years or more, and on retirement must be used to purchase a pension.

1.2 This paper uses the example of this product to investigate the effect time horizon has on investment strategy. We consider time diversification, mean-reversion and the effect of reinvestment rates on how markets behave over long time horizons, and how this differs from their behaviour in the short term.

1.3 We examine "lifestyle" strategies in this context, including the use of asset-liability modelling. We also consider different asset allocation models, and whether they support lifestyle strategies.

2.   How does term affect investment risk?

*"In the long run we are all dead" - John Maynard Keynes*

2.1   The starting point for establishing how investment time horizon affects investment strategy is to establish how investment risk varies, if at all, over different time periods. By investment risk, we mean risk relative to a least-risk asset. A least-risk asset (or portfolio of assets) will, in most circumstances, consist of bonds and/or cash – bonds (either fixed or real of appropriate term) being the asset which most closely matches defined liabilities, and cash being the asset without capital risk (or at least with minimum capital risk). This means that for investment risk, read equity risk.

2.2   Time diversification is the notion that investment risk decays over time. This concept has fairly widespread acceptance in actuarial thinking and practice. For example, Yakoubov, Teeger & Duval (1999) incorporate equity risk decay in the stochastic investment model they describe which results in modelled "high short term volatility (which derives from market sentiment) and significantly lower long term volatility (which arises from economic fluctuations)" (p. 37). If time diversification did exist, one explanation would be that equity markets mean-reverted.

2.3   In this section we discuss the concept and evidence of time diversification, as well as mean-reversion in equity markets. We also discuss reinvestment risk, and the effect this risk when time horizons are long

Time Diversification

2.4   Exley, Mehta, Smith and van Bezooyen (1998) describe time diversification as the notion that "risk decreases over time as a result of 'diversification of returns in different periods'" (p. 2). Before we examine this concept, it is essential that we define what we mean by risk. Risk is typically measured by variance/standard deviation, but many other measures exist such as the 'Safety-First' measure first described by Roy (1952) which is the probability of obtaining a return less than some benchmark.

2.5   Over the period 31/01/1976 to 30/04/2002, UK long gilts (as measured by the FTSE A Over 15 Years Gilts Index) produced an average (arithmetic) annual return of 13.3% with a standard deviation of 11.3% compared to an average (arithmetic) return of 17.5% with a standard deviation of 17.1% for UK equities (as measured by the FTSE All-Share Index). UK equity and bond returns had a correlation of 0.40 over the period. The average (arithmetic) difference between UK equity and bond returns was 3.7% p.a. with an annualised standard deviation of 16.1%.

2.6   If we make the simplifying assumptions that the log differences between equity and bonds returns are independent from one period to the next, and are normally distributed with a mean and variance estimated from the period 31/01/1976 - 30/04/2002, we can plot the expectation and confidence intervals for the cumulative outperformance of equity over bonds as follows:

Cumulative outperformance of equities over bonds
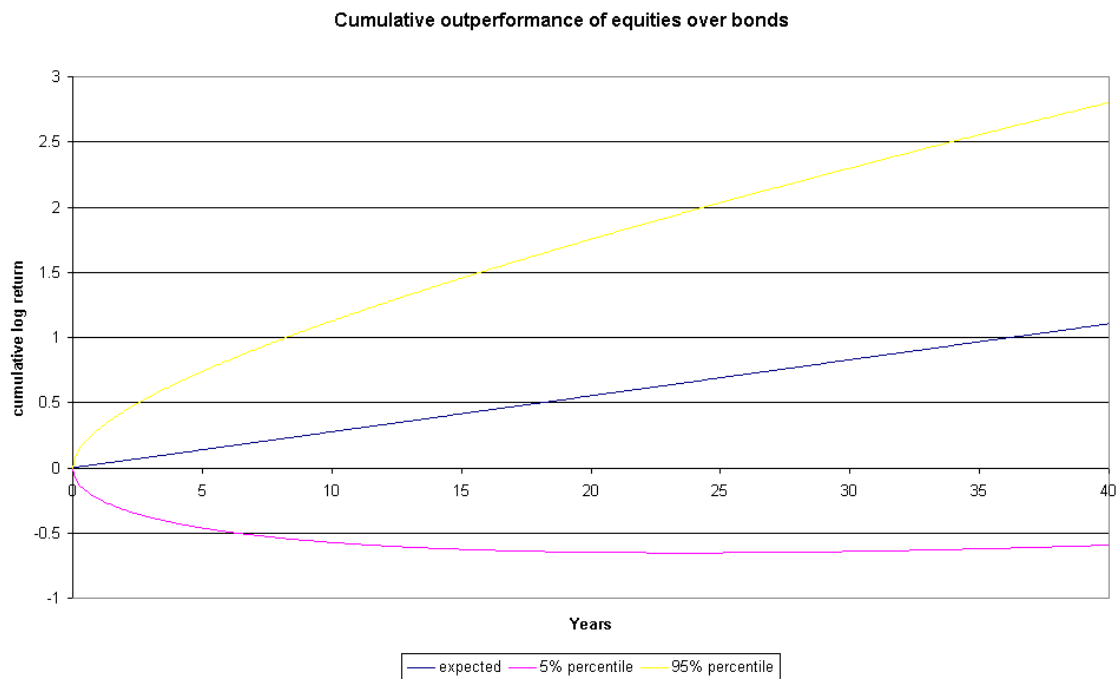
2.7 Under this simple model, the 5% percentile stops decreasing after around 24 years. This result illustrates the concept that returns compound over time to the power t, whereas risk (when it is measured as standard deviation) compounds at the square root of t, if returns are independent from one period to the next.

2.8 If we define risk as the probability of equity returns being lower than bond returns, and apply the same simple model, the risk profile is as follows:

Probabilty of cumulative equity returns lower than bond returns
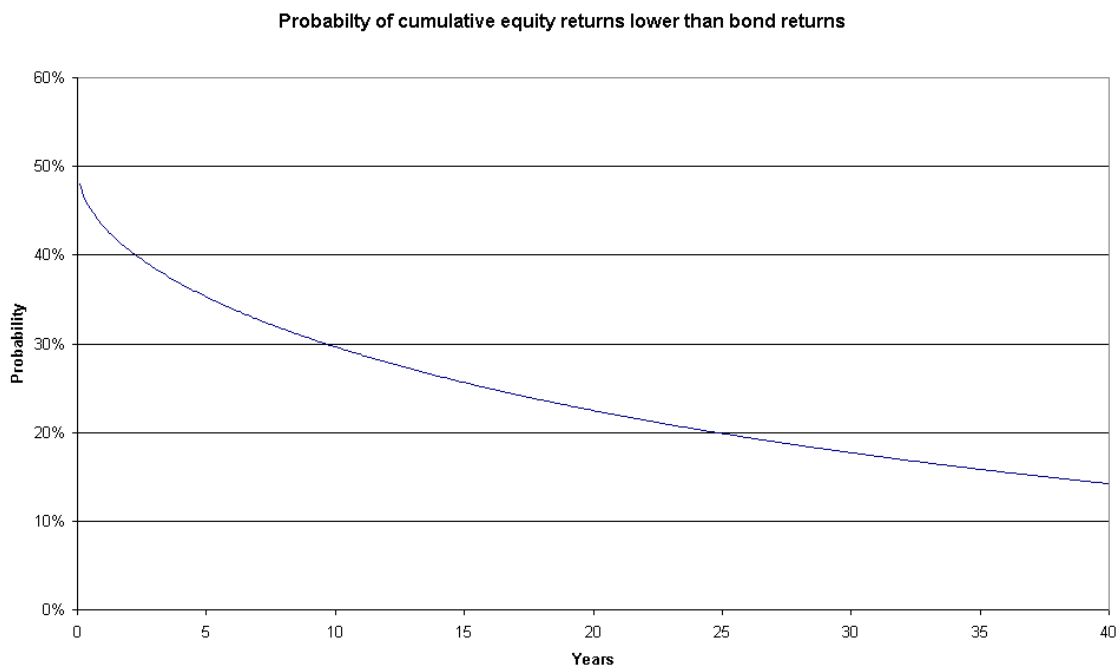
2.9　　Turning to index-linked gilts, over the period 31/01/1986 to 30/04/2002 (the longest period available), UK equities (as measured by the FTSE All-Share Index) outperformed UK long index-linked gilts (as measured by the FTSE A Over 5 Years Index-Linked Gilts Index) by an average of 4.8% p.a. The correlation between the asset-classes was 0.32 and the annualised standard deviation of the difference between equity and index-linked gilt returns was 15.8%. Making the same assumptions about the log differences as above, we can plot the same analysis above for equities versus index-linked gilts.

Probability of cumulative equity returns lower than index-linked bond returns
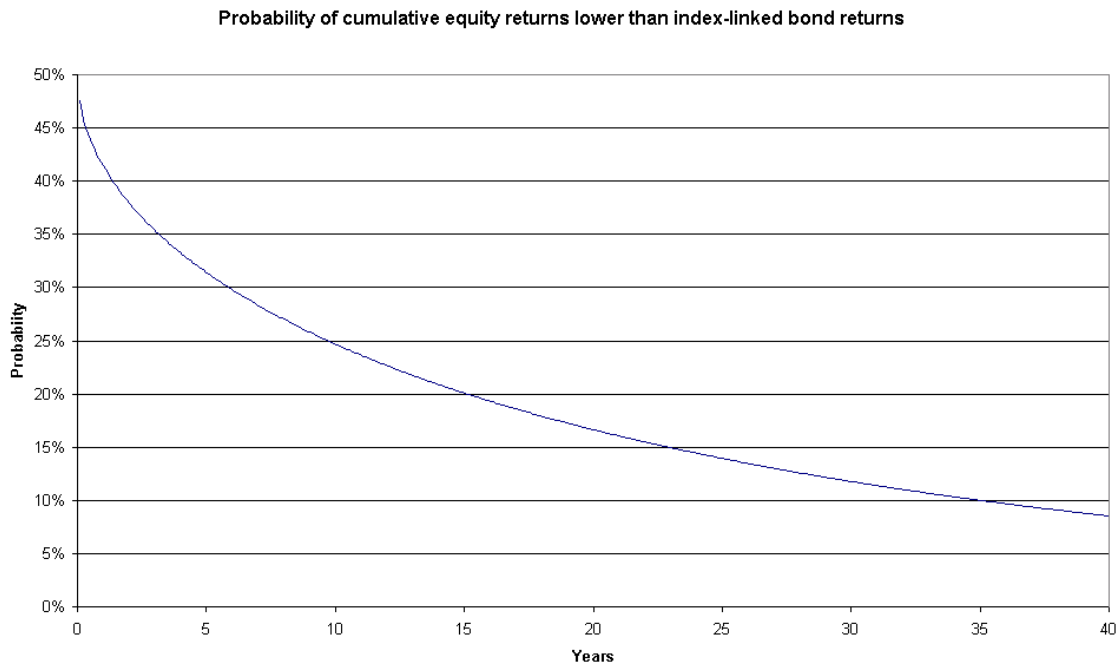
2.10　This shows the same pattern as for equity versus fixed income bonds, although the probability falls to a lower (although still significant) level over the longer term. This can be explained mainly by the extra inflation risk premium that fixed income bonds carry over index-linked bonds.

2.11　Clearly, when risk is measured as the probability of cumulative underperformance, it does decay over time. However, neither this nor the fact that, over very long periods, mean dominates standard deviation (because it compounds at t as opposed to root t) means that time diversification exists. For time diversification to exist, risk needs to compound at a less than root t.

2.12　Howie (1997) shows that there is no evidence that this occurs, based on 36 years of South African data. This analysis compared the annualised standard deviation of returns over different non-overlapping time periods ranging from 1 month to 3 years. The graph below plots some of the same analysis for UK data for the 36 year period ended 20/04/2002.

Annualised standard deviation of UK equity log returns
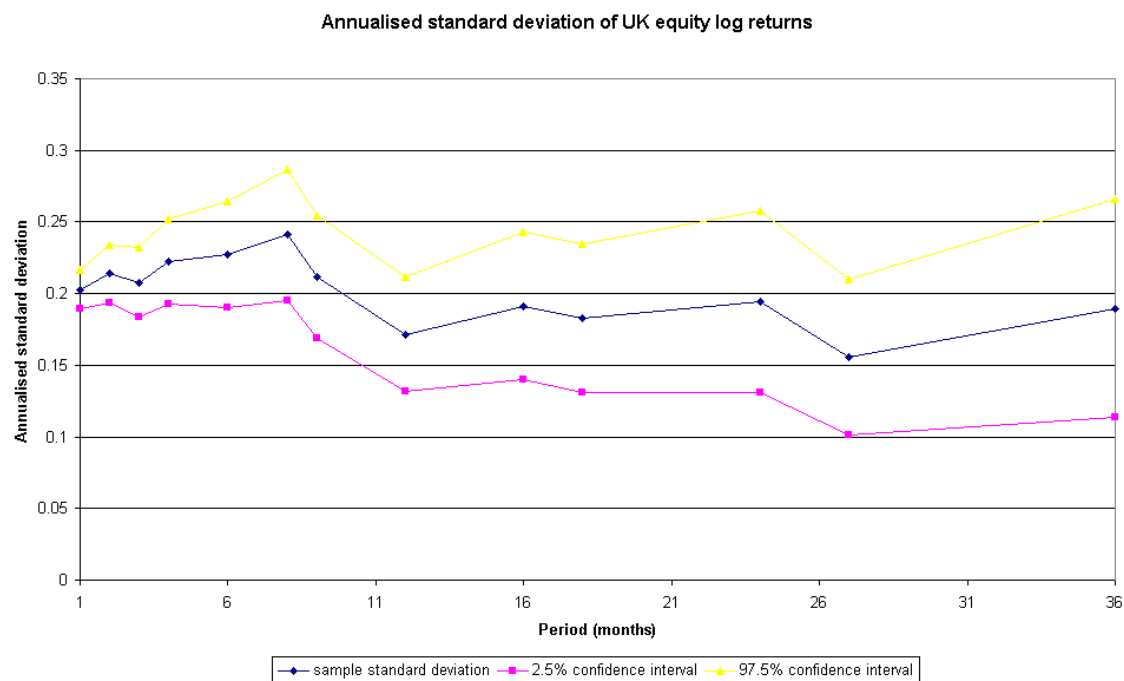
2.13 In contrast to the analysis of Howie (1997), the UK data suggests there might be evidence that equity risk decays over time, although this is not conclusive. This is supports the suggestion by Siegel (1994) that the volatility of real equity returns is substantially lower in the long run than that for bonds or cash, but in the short term is higher than the volatility of bonds and cash.

2.14 It is widely accepted in the academic literature that equity volatility changes over time - Campbell & Viceira (2002) quote numerous studies on page 147. The plot below of the (centered 3 year average) annualised volatility of monthly returns clearly confirms, at least visually, these findings.

Volalitlity of UK equities
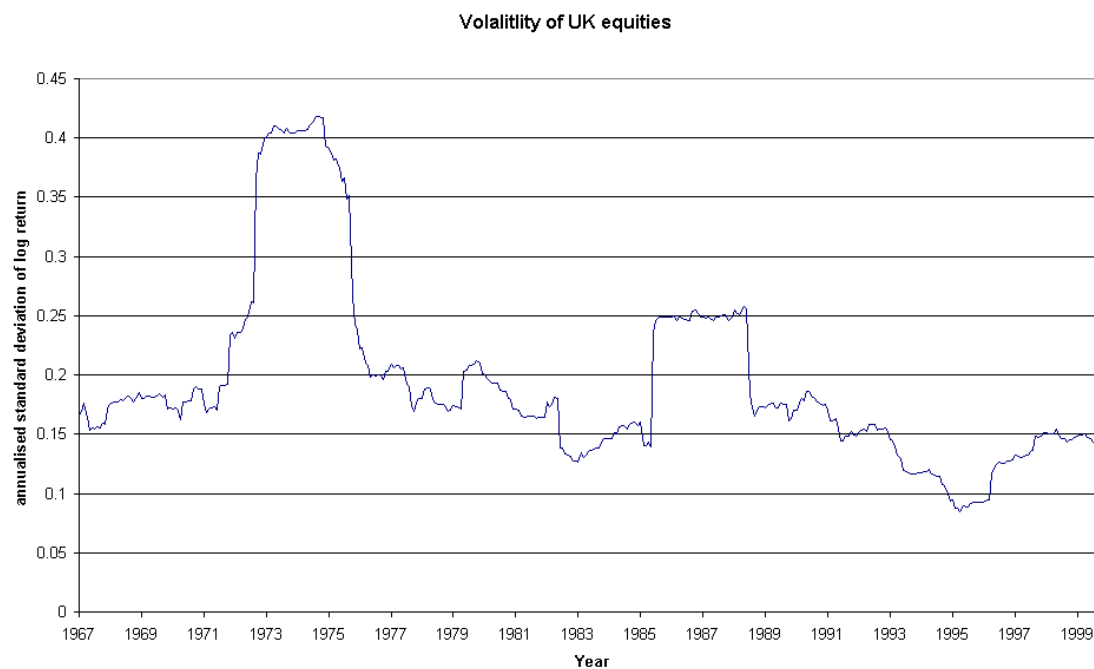
2.15 The fact that volatility is not constant over time neither supports nor disproves time diversification, as time-varying risk premiums, rather than time-varying risk, is the basis for time diversification. It is possible to construct a heteroscedastic (non-constant variance) model that exhibits time diversification, such as the ARCH (auto-regressive conditional heteroscedastic) effects that were considered by Geoghegan et al. (1992) as an adaptation of the Wilkie (1986) model. Such a model exhibits the decay of equity risk, and because variances are non-constant conditional on the past but constant unconditionally, exhibit a constant variance over the long term. Similarly, it is possible to construct a heteroscedastic model that does not exhibit time diversification, such as a modified random walk where the error term has non-constant variance.

2.16 Exley, Mehta, Smith and van Bezooyen (1998) dismiss time diversification as a "persistent fallacy" on theoretical grounds. This is based on the argument that diversification in general is based on the premise of independence (or at least lack of perfect correlation). This means that even if share price movements in each period are independent, the share price at the end of a period depends on the share price at the start of the period. It follows that if the assumption of independence does not hold, so time diversification does not hold. Instead risk "compounds through time" rather than decays.

2.17 However, this theoretical dismissal is based on the notion that returns in non-overlapping periods are independent of each other. The simplest (but not only) model which incorporates this is the random walk. The random walk model is based on the Efficient Market Hypothesis (EMH), which in its weakest form states that "there is no information in historic price movements which enables a trader to predict future movements profitably" (Smith (1996) p 1075). If there is evidence that returns in non-overlapping periods are not independent of each other, then there is evidence that diversification exists.

2.18 Campbell & Viceira (2002) argue that the notion that equity risk decays is mathematically equivalent to the notion that excess stock returns mean-revert (i.e. that there is time variation

8

in the equity risk premium). It follows that if time diversification does exist, a buy-and-hold strategy is sub-optimal. If mean-reversion exists, then it is optimal to reduce equity exposure after a period of high returns and visa versa.

Mean Reversion

2.19 Mean reversion in share prices (or share indices) is described by Balvers, Wu & Gilliland (2000) as the "tendency of asset prices to return to a trend path". If mean reversion exists, a consequence is that if equities have performed badly (or very well) then there is an expectation (at least in the long term) that future returns will be higher (or lower) so that the trend path is maintained. Put another way, by Campbell & Viceira (2002), it means "an unexpectedly high return today reduces expected returns in the future, and thus high short-term returns tend to be offset by lower returns over the long-term" (p 95).

2.20 The concept can be illustrated to two simple models: the random walk and the autoregressive process. Defining P(t) as a total return equity index at time t, and R(t) as the total log return over the period (t-1) to t:

$$P(t) = P(t\text{-}1)e^{R(t)}$$

$$R(t) = \alpha + \beta R(t\text{-}1) + e(t)$$

Where $e(t) \sim N(0, \sigma)$

2.21 In this model α is the trend. If β is zero, the process is a modified random walk and is not mean reverting. If $-1 < \beta < 0$, then the returns are autoregressive and the process is mean reverting (we are only concerned with range $-1 < \beta < 0$ since β must be negative if "good returns follow bad" and visa versa). The unconditional expectation and variance of R(t) are $\alpha / (1 - \beta)$ and $\sigma^2 / (1 - \beta^2)$ respectively.

2.22 The difference between a mean-reverting and non mean-reverting processes can be illustrated by way of example. If the annual log equity return had a mean of 0.1 and a standard deviation of 0.2, the values of the parameters for this model for different values of β would be as follows:

| β	α	σ
0	0.1	0.2
-0.5	0.15	0.17
-0.9	0.19	0.09

2.23 Under each of the above 3 parameterisations, the annual log return has the same mean and standard deviation (0.1 and 0.2) but the pattern of returns over periods longer than one year is substantially different, as is illustrated by the 100 simulations of this model shown below for each parameterisation:

UK equity return index
100 simulations with β = 0

UK equity return index
### 100 simulations with β = -0.5

10
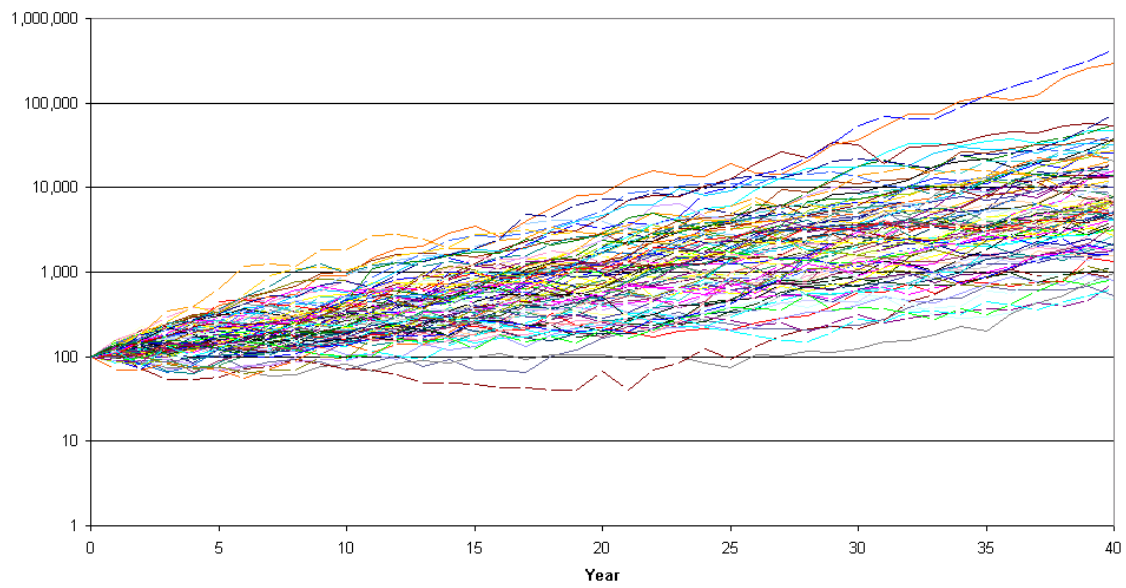
UK equity return index
**100 simulations with β = -0.9**
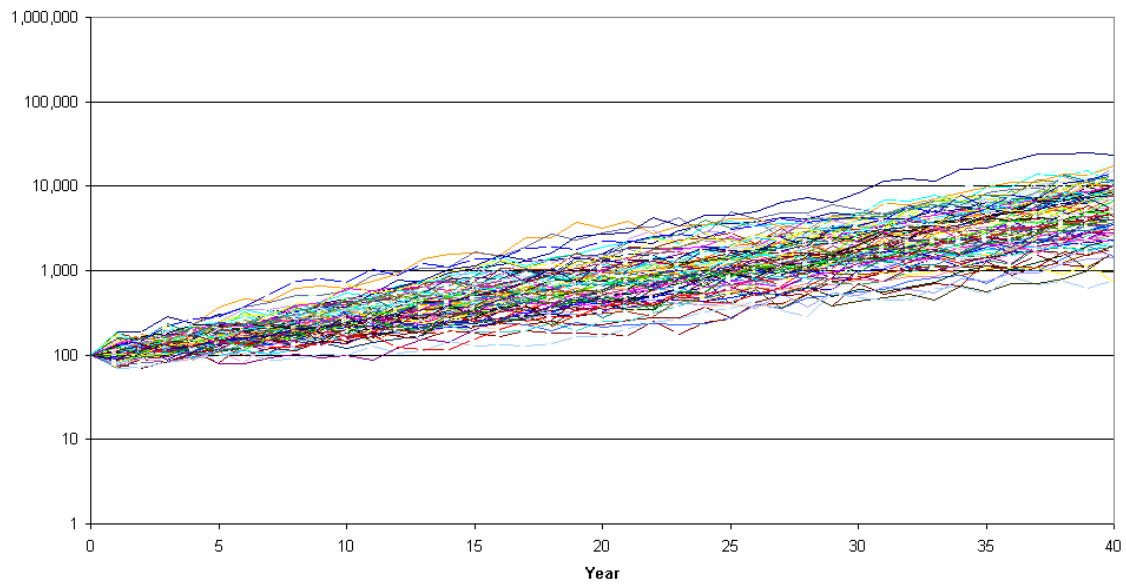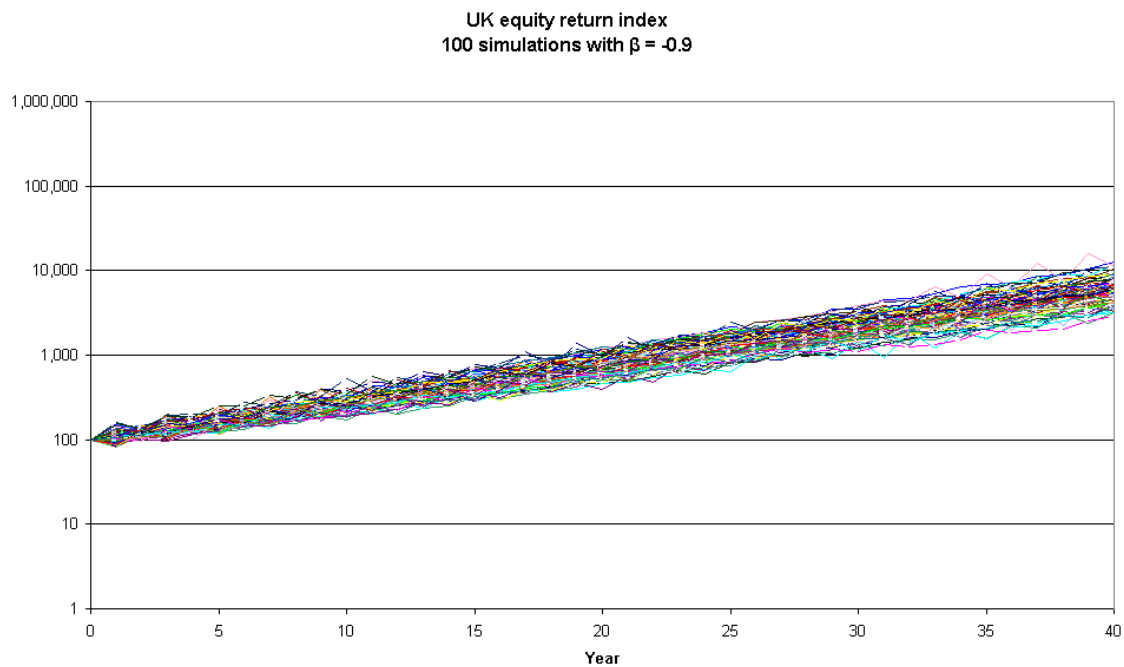
2.24 Wilkie (1994) argues that the autoregressive parameter (β in the model above) is close to zero but not equal to zero, so that share prices appear to follow a random walk over the short term, but are actually slowly mean-reverting over the long term. For this to be the case, share prices should exhibit negative autocorrelation over longer periods. The graph below shows the autocorrelation of UK equity returns over the 36 year period ended 20/04/2002:

Autocorrelation of UK equity returns over different periods
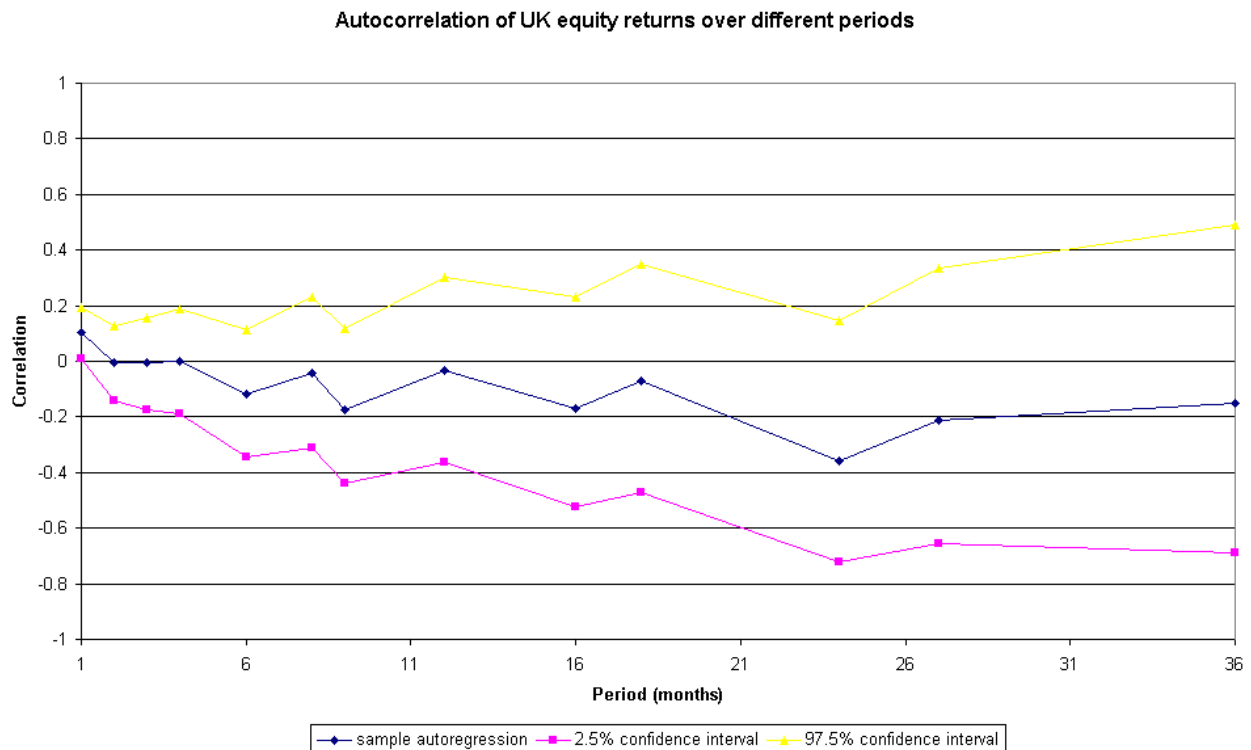
2.25 As does the analysis in section 2.11, this provides some (although not conclusive) evidence

that the UK equity market is mean-reverting. The observed mean-reversion is weak, and only observable for periods 6 months and longer.

2.26   There have been numerous academic studies into mean-reversion in equity markets. Notably Fama & French (1988) and Poterba & Summers (1988) found evidence that markets do mean-revert, but their work has been criticised on statistical grounds (by Kim, Nelson & Startz (1988) and Richardson (1989)). By contrast, Lo & MacKinlay (1988) did not find evidence for mean-reversion.   More recently, Jedadeesh (1991) found evidence for mean-reversion in the US and UK markets (but noted it was a seasonal phenomenon entirely concentrated in January) and Balvers, Wu & Gilliland (2000) also found evidence for mean-reversion using data from 18 countries. In summary the evidence is mixed, and the debate and study will no doubt continue.

2.27   At most there is limited evidence that share prices are weakly and slowly mean-reverting. By weakly, we mean that the autoregression is small, and by slowly, we mean that the effect is only noticeable over the longer term. What does this imply for investment strategy when time horizon is very long? Firstly, it implies that a static investment strategy is sub-optimal, and that a long-term investor should vary their allocation to equity over time to capture the mean-reversion.

2.28   Secondly, it means that the allocation to equity of a long-term investor can be higher than under the assumption of no mean-reversion. But how much? The short answer is very little. If the autoregressive parameter is close to zero, risk compounds only slightly slower than under a non mean-reverting process. Estimating the autoregressive parameter for the model in section 2.20 based on annual UK data for the period April 1967 to April 2002, we find $\beta =$ -0.03.   The parameter is not statistically significant, but even if it was the reduction in volatility due to mean reversion would be only about 5% over 40 years. As illustrated in the projection below, the confidence bounds for a random walk (RW) are very similar to those of the fitted autoregressive (AR) process:

Projected equity index
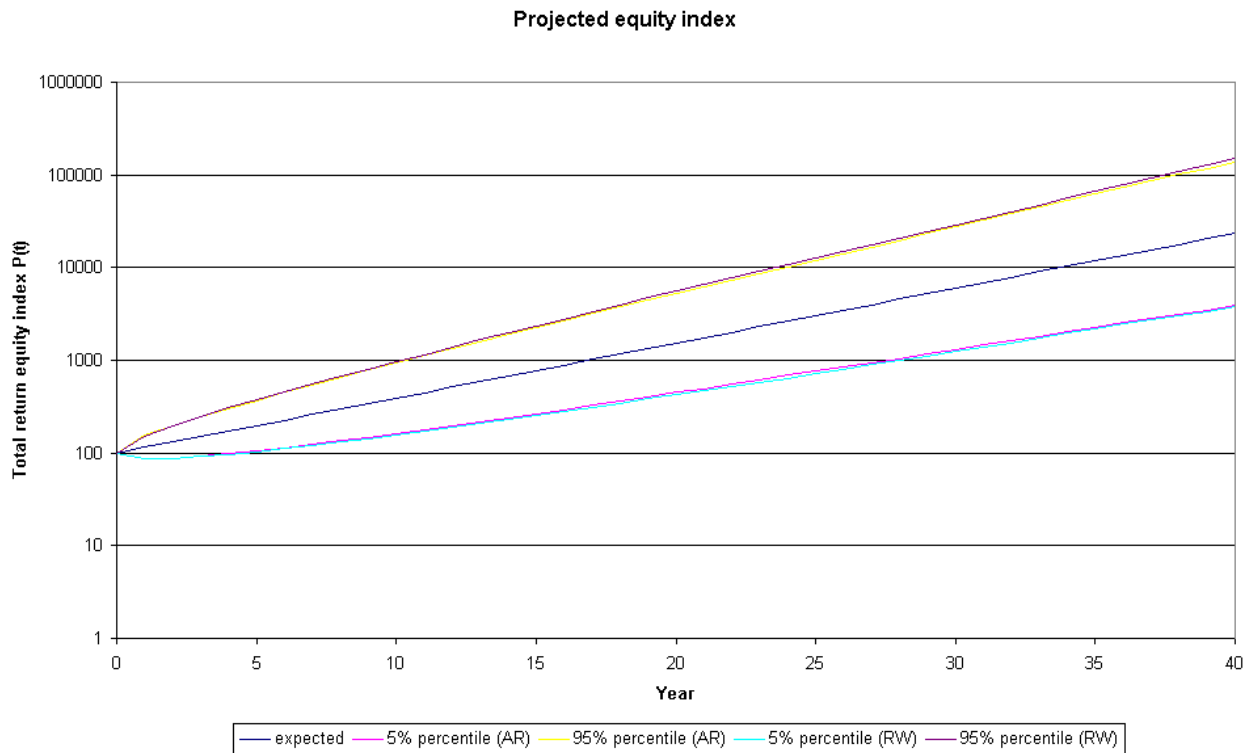
Reinvestment Rates

2.29 For long-term investors, cash and money market investments are not risk-free, because they must be rolled over at uncertain future real interest rates. In the same way as short-term borrowers have to take the risk of having to refinance at high rates in a financial crisis, long-term investors must realise that short-term investments carry the risk of having to reinvest at low rates in the future. The graph below shows the same analysis for cash returns as was shown for equity returns in section 2.11.

Annualised standard deviation of UK cash log returns
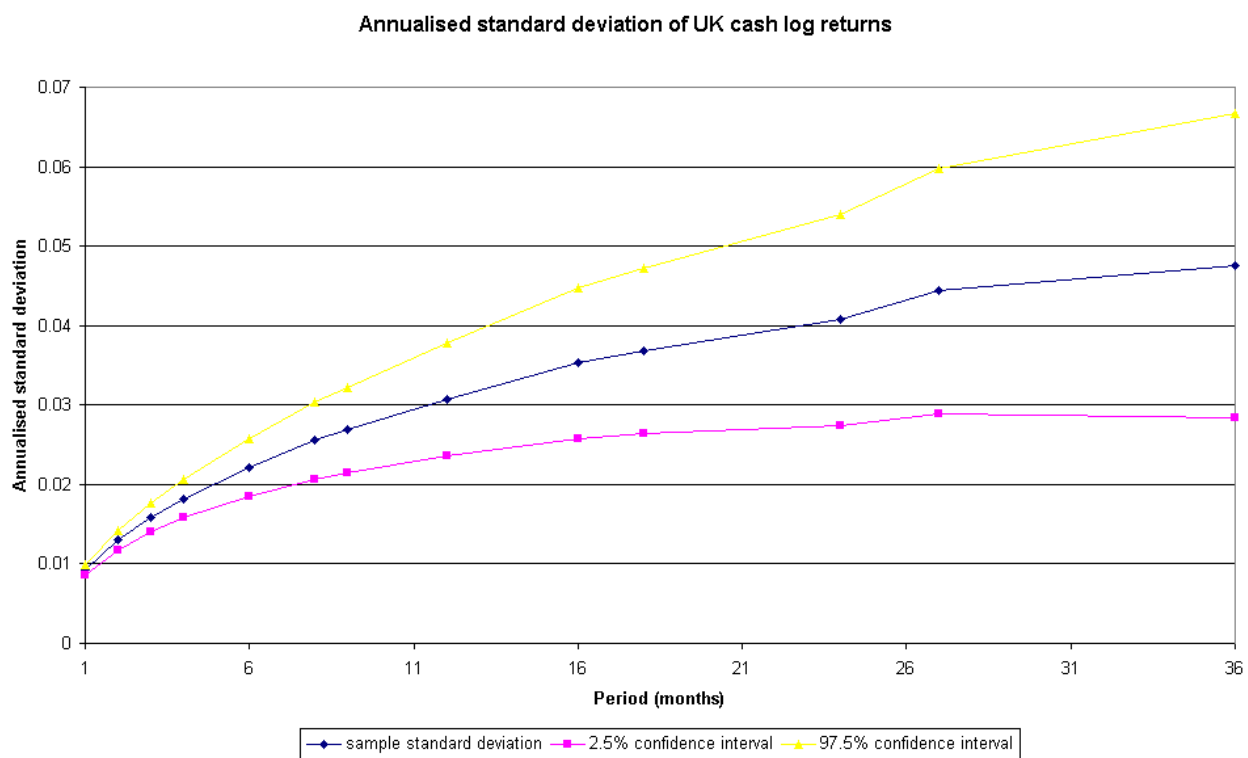
2.30 Unlike equity returns, the risk of holding cash clearly expands over time, illustrating the reinvestment risk.

3.   "Lifestyling" in Defined Contribution Pension Schemes

3.1   A lifestyle investment strategy is often the default option for defined contribution pension schemes. Under such a strategy, equities are held until a certain period before retirement, after which they are gradually switched into bonds (and sometimes cash) so that on retirement, assets are entirely invested in bonds (and cash).

Objectives and Rationale

3.2   Lifestyling aims to provide protection against fluctuating annuity prices (and capital loss if there is also a switch into cash) in the years prior to retirement. Exley, Metha, Smith and van Bezooyen (1998) suggest that this is based on the rationale that markets mean-revert, risk aversion changes near retirement, equity returns match salaries over the long-term, and younger members have a long time horizon.

3.3   We have already shown that mean-reversion, even if it genuinely exists, has little impact on investment strategy because the effect is second-order.

3.4   In section 2, we saw that equity risk reduced over time if risk was measured as the probability of underperforming bonds. Similarly, if we define risk as the probability of achieving a pension below some target, equities will appear less risky over longer time horizons if the "hedge portfolio" (the portfolio that most closely matches the pension) is index-linked bonds.

Hedge Portfolio

3.5   Defined contribution pensions plans are designed to provide pensions on retirement for members. As such the liability is the pension on retirement, although the risk of not achieving a target pension is transferred to the member and hence the accounting liability of the pension plan is simply the market value of accumulated contributions.

3.6   As was shown by Dyson & Exley (1995) the hedge or minimum risk portfolio for pension fund liabilities comprises bonds of suitable term and nature. If we make the assumption that the target pension is linked to price (or wage) inflation, then hedge portfolio comprises index-linked bonds of suitable term.

3.7   Based on studies such as Exley, Mehta & Smith (1997), it is now largely accepted that there is little if any link between equities and pension liabilities of active members. This thinking is confirmed by the plot below, which shows the large volatility of equity relative to wages, compared to the relatively constant differential between price and wage inflation.

Comparison of equity returns to price and wage inflation
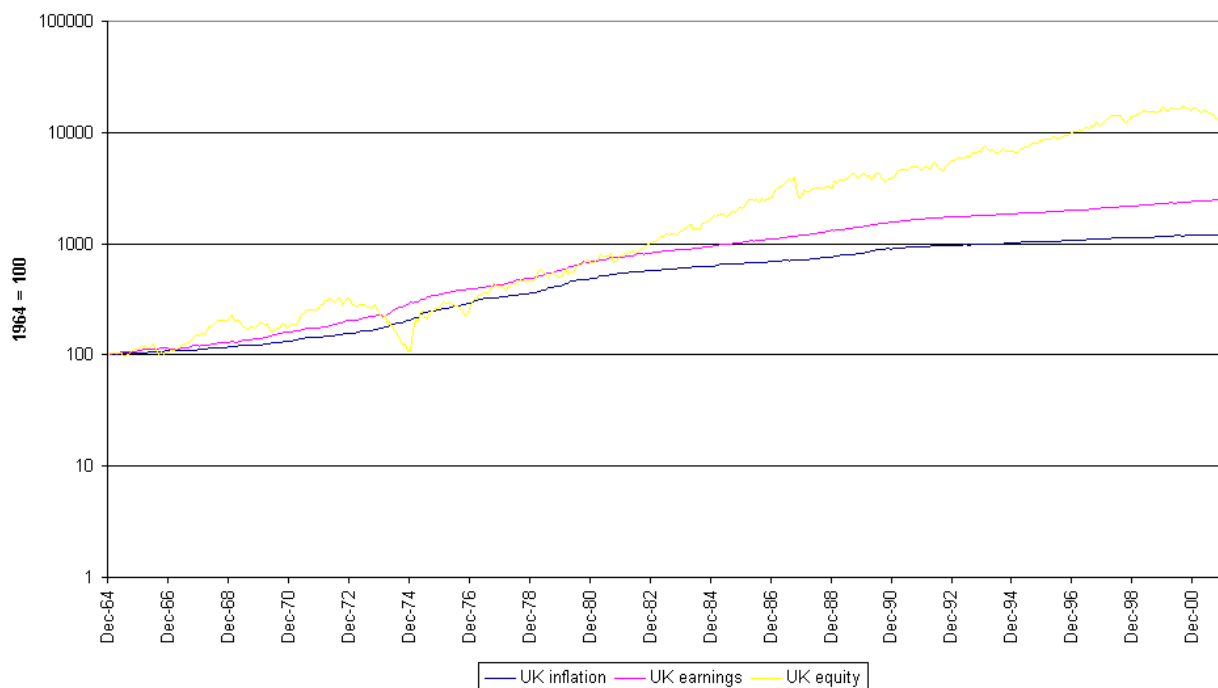
2.31 Campbell & Viceira (2002) also argue that the least-risk asset class for a long term investor is inflation-linked bonds. They also note that, contrary to standard analysis over shorter time horizons, index-linked bonds are less risky than cash. These do not have a stable market value in the short term, but they deliver a predictable stream of real income which can support a stable standard of living in the long term. However, when there is little uncertainty about inflation, nominal bonds behave like inflation-indexed bonds and are acceptable substitutes.

Asset-Liability Modelling

3.8 It is relatively uncommon to conduct an asset-liability modelling (ALM) study for a single defined contribution pension plan. More likely, consulting actuaries use ALM to determine the basis of generic advice on lifestyling. However, in contrast to the work on defined benefit pension plans, there is limited research on the applications of ALM in a defined contribution context. If the trend from defined benefit to defined contribution continues, no doubt the lack of (published) research will be addressed.

3.9 One application of ALM is documented by Booth & Yakoubov (2000), who apply the Wilkie (1995) model to find the distributions of the accumulated cash fund, fixed nominal annuities and fixed real annuities for different investment strategies in a defined contribution pension plan.

3.10 Booth & Yakoubov's (2000) findings, both from the ALM study and empirical analysis of past data, did not support the lifestyling approach and suggested that maintaining an allocation to equity was preferred.

16

4.   Asset Allocation Models

4.1   The mean-variance analysis of Markowitz (1952) concentrates purely on one-period returns and risk, differentiating between the different types of investor (aggressive, conservative etc.). If the period is 40 years, rather one, does the methodology still apply?

Mean-variance analysis

4.2   If we assume returns are log-normal and can ignore any mean reversion, then based on monthly returns over the period 31 January 1986 to 30 April 2002, the inputs to a traditional mean-variance optimisation would be as follows:

Asset class	1 month		1 year		40 years	
	Expected return	Standard deviation	Expected return	Standard deviation	Expected return	Standard deviation
UK cash	0.7%	0.3%	8.7%	1.0%	2698.0%	162.5%
UK long index-linked bonds	0.7%	2.1%	8.9%	7.9%	2908.4%	1463.4%
UK long bonds	0.9%	2.7%	11.7%	10.5%	8112.9%	5365.0%
Global equity	0.9%	4.8%	12.0%	18.7%	9220.3%	13154.0%
UK equity	1.1%	4.9%	14.1%	19.4%	19258.2%	28241.1%

4.3   Apart from the adjustments that would normally take place to reflect current market conditions or those suggested by van Bezooyen et al. (2001) to ensure consistent results, the only difference between the 1 month, 1 year and 40 year assumptions are scale. The graphs below plot these assumptions.

1 year time horizon

40 year time horizon
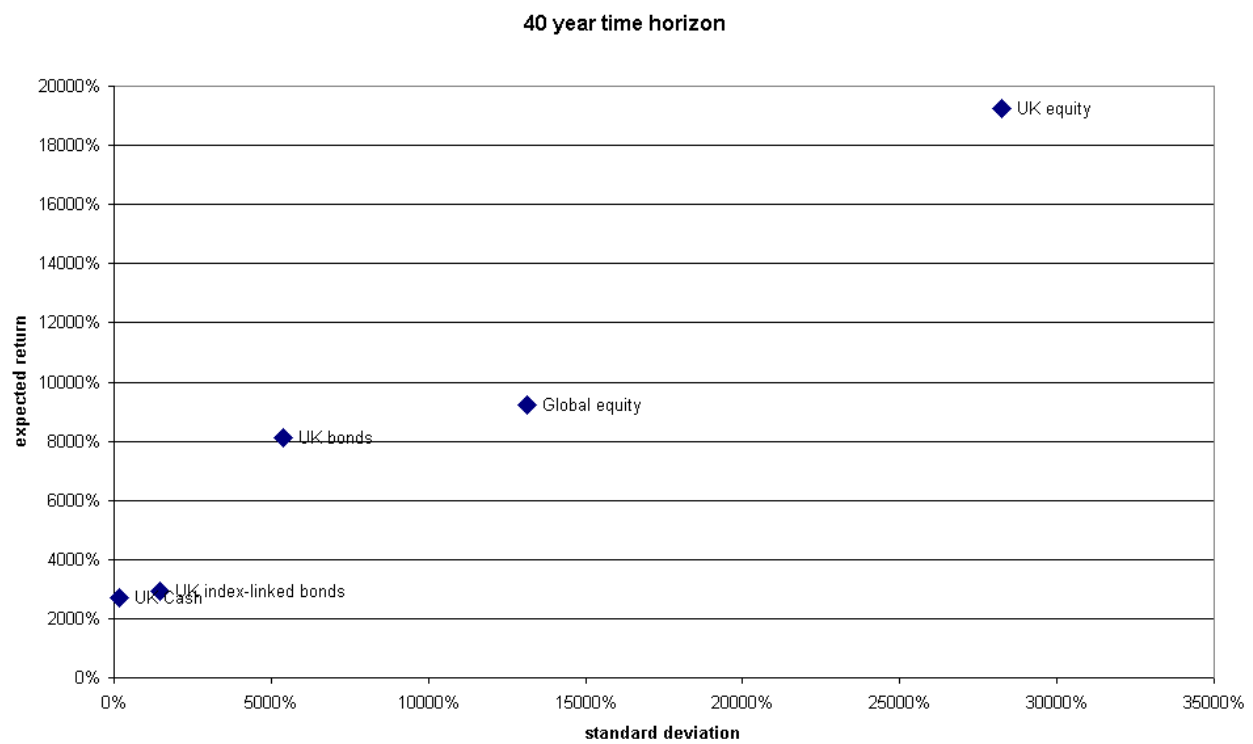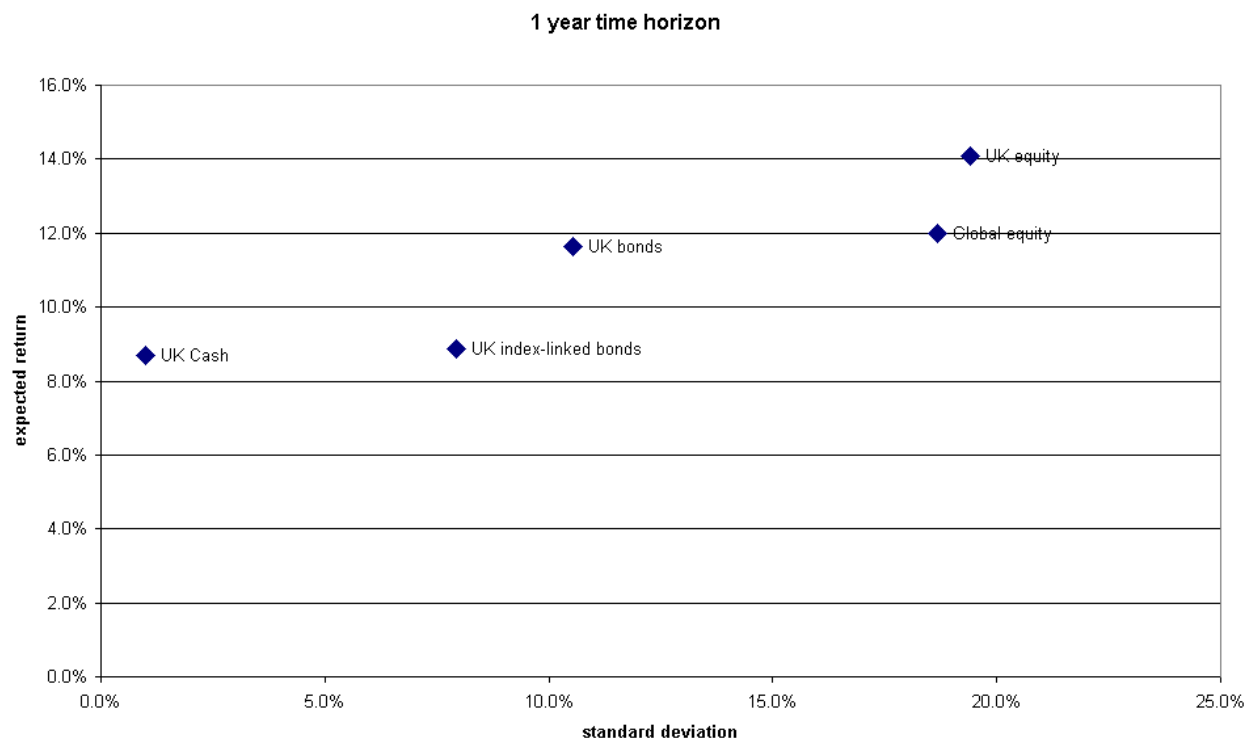
4.4 As we discussed in section 3.5, the hedge portfolio comprises index-linked bonds, so it is worthwhile comparing returns relative to the hedge portfolio, which would result in the

following set of assumptions:

Asset class	1 month		1 year		40 years	
	Relative expected return	**Relative standard deviation**	**Relative expected return**	**Relative standard deviation**	**Relative expected return**	**Relative standard deviation**
UK cash	0.0%	2.1%	0.3%*	7.3%	14.8%*	55.8%
UK long index-linked bonds	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
UK long bonds	0.2%	2.3%	2.7%	8.2%	188.5%	155.0%
Global equity	0.3%	4.7%	3.1%	17.0%	240.6%	472.4%
UK equity	0.4%	4.7%	4.9%	17.0%	580.6%	920.8%

* it is interesting to note that the log transformations mean that cash has an expected relative outperformance of index-linked bonds

4.5　Despite the difficulties of setting consistent assumptions described by van Bezooyen et al. (2001), if we can make the assumption that log returns are normally distributed, mean-variance analysis can be applied over both long and short periods. The change of basis from absolute to relative returns is useful as it allows the analysis of risk relative to a hedge portfolio using the same methodologies.

4.6　One notable weakness of mean-variance analysis is that it does not consider labour income in the asset allocation process. The work of Merton (1969, 1971, 1973) considers this problem.
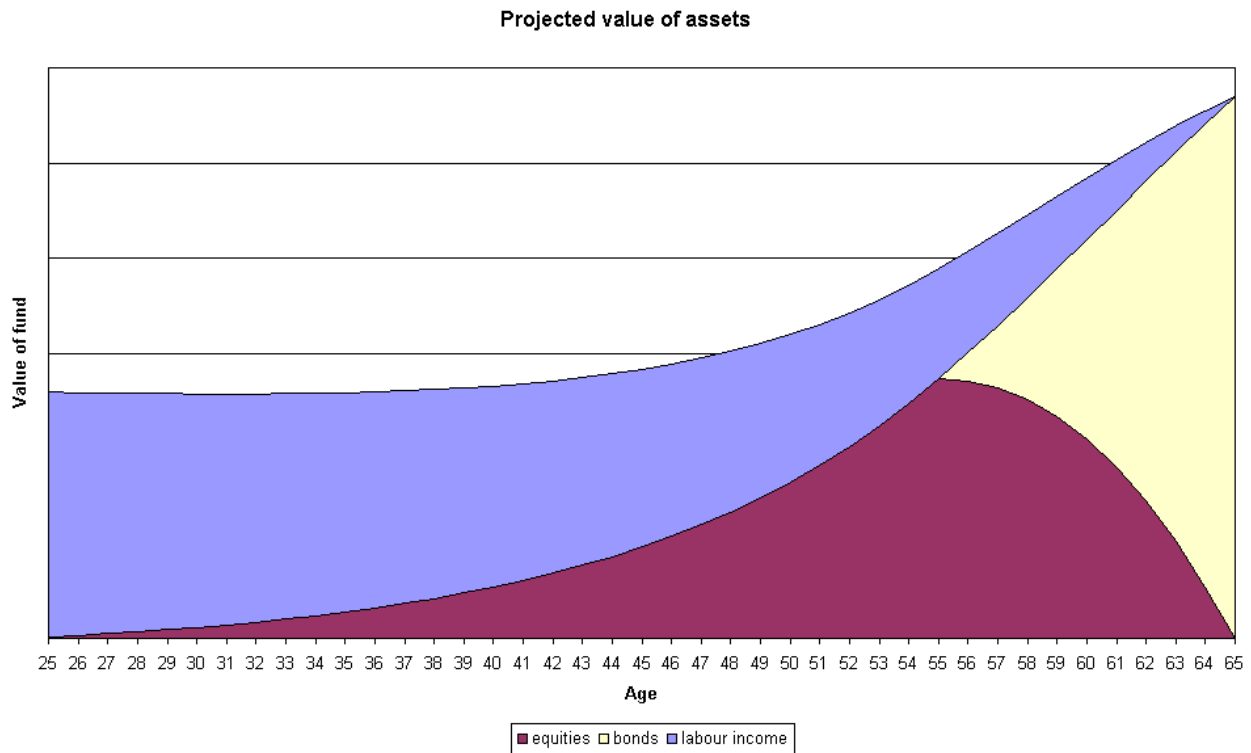
Labour Income

4.7　Under typical analysis, assuming the only assets of an individual are the defined contribution pension assets, the projection of the individual's assets, allowing for a lifestyling programme incorporating switching 10 years before retirement, would look something like this:

Projected value of assets
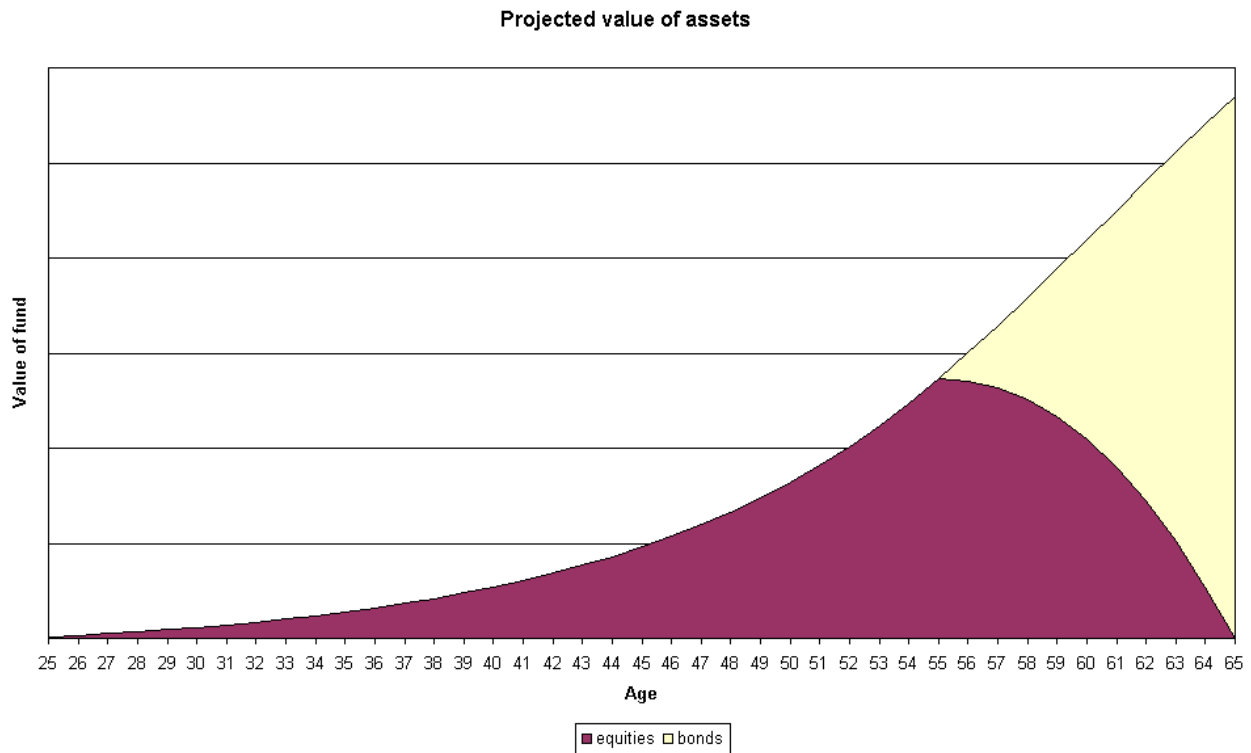
4.8 Firstly, it would be necessary to incorporate non-pension assets when setting overall investment strategy. More importantly, it is useful to consider the present value of future income as an asset of the individual. Campbell & Viceira (2002) argue that this asset can be either bond-like or equity-like, although in the majority of cases it is more bond-like. Examples of equity-like income are when a large proportion of pay is determined by the performance of the employer's share price, or when the individual is a self-employed entrepreneur.

4.9 It we take this asset into account, the projection might looking something like this:

Projected value of assets

4.10 If labour income is substantially bond-like, then the asset allocation of young individuals automatically has a high weighting to bonds. This is supportive of a the notion the investment strategy of pension assets can be more aggressive (equity based) for younger members. Exley, Mehta, Smith and van Bezooyen (1998) describe how the results of the Merton (1969, 1971, 1973) model, when labour income is incorporated, supports the switching from equity into bonds in the years prior to retirement, assuming that the overall asset allocation has an allocation to equities. However, it also shows that the amount allocated to equities at retirement does not optimally reduce to zero, as is typically the case in lifestyling strategies.

5. Conclusions

Equity risk over long time horizons

5.1 Ignoring any mean-reversion, the standard deviation of returns compound over time at the root of t whereas returns compound at t. This means that if equity risk is measured as the probability of underperforming another asset class (e.g. bonds), this risk reduces as time horizon increases.

5.2 Mathematically, the decay of equity risk over time (time diversification) and mean-reversion are equivalent. If mean-reversion existed, then equity risk (when measured by the standard deviation of returns) would compound at a rate smaller than the root of t, and in the extreme case would cease to compound after a certain period.

5.3 There is limited evidence that equity markets do exhibit mean-reversion, but if they do, the effect is weak and slow. We can conclude that mean-reversion has little impact on equity risk, even for very long time horizons.

Defined contribution lifestyling strategies

5.4 As for defined benefit pension liabilities, the hedge portfolio which most closely matches the target pension is a portfolio of index-linked bonds of suitable duration. As such, lifestyling strategies are risky, in that they are equity-based for most of the working lifetime of members. Even over long time horizons, equity-based strategies show a significant probability of underperforming the hedge portfolio (we estimate there is just under a 10% probability of equities underperforming index-linked bonds over 40 years). However, if it can be assumed that members are willing to accept some risk, equity-based strategies can be justified.

5.5 If risk is measured as the probability of underperforming the hedge portfolio, then this risk increases as members approach retirement, which would lead to a strategy of switching into bonds, assuming members want to maintain a constant probability of underperforming the hedge portfolio over time. Exley, Mehta, Smith and van Bezooyen (1998) also describe how the results of the Merton (1969, 1971, 1973) model, when labour income is incorporated (which can typically be treated as a bond-like asset), support the switching from equity into bonds.

5.6 If the time diversification effect was genuine, it would mean that lifestyling was sub-optimal as it would require a dynamic approach to asset allocation. However, since there is limited evidence that the effect exists, and if it does it is very small, so this is neither supportive nor dismissive of the lifestyling approach.

5.7 The studies of Booth & Yakoubov (2000) and implications of the Merton (1969, 1971, 1973) model show that switching 100% into bonds before retirement is sub-optimal.

5.8 In summary, there is some support for the lifestyle strategy of switching equity into bonds as members approach retirement. However, it might be preferable to maintain some allocation to equity right up until retirement. This is caveated by the assumption that members have some risk tolerance, as any equity allocation at any time horizon is risky relative to the hedge portfolio of index-linked bonds.

6. References

Balvers R, Wu Y & Gilliland E (2000), "Mean Reversion across National Stock Markets and Parametric Contrarian Investment Strategies", Journal of Finance, Vol 55

Booth P and Yakoubov Y (2000), "Investment Policy for Defined-Contribution Scheme Members Close to Retirement: An Analysis of the 'Lifestyle' Concept", North American Actuarial Journal, Vol 4

Campbell JY & Viceira LM (2002), "Strategic Asset Allocation, Portfolio Choice for Long-Term Investors", Oxford University Press

Dyson ACL & Exley CJ (1995), "Pension Fund Asset Valuation and Investment", British Actuarial Journal, Vol 1

Exley CJ, Mehta S & Smith A (1997), "The Financial Theory of Defined Benefit Pension Schemes", British Actuarial Journal, Vol 3

Exley CJ, Mehta S, Smith A & van Bezooyen J (1998), "Life Style Strategies for Defined Contribution Pension Schemes", Institute of Actuaries

Fama EF & French KR (1988), "Permanent and Temporary Components of Stock Prices", Journal of Political Economy, Vol 96

Geoghegan TJ, et al. (1992), "Report on the Wilkie Stochastic Investment Model", Journal of the Institute of Actuaries

Howie RJ (1997), "Investment Risk and Asset Modelling - Can Financial Economics Be of Use?", Transactions of the Actuarial Society of South Africa, Vol 11

Jedadeesh N (1991), "Seasonality in Stock Price mean Reversion: Evidence from the US and UK", Journal of Finance, Vol 46

Kim M, Nelson CR & Startz R (1988), "Mean Reversion in Stock Prices? A Reappraisal of the Empirical Evidence", University of Washington (Unpublished)

Lo AW & MacKinlay AC (1988), "Stock Market Prices Do Not Follow Random Walks: Evidence from a Simple Specification Test", Review of Financial Studies, Vol 1

Markowitz HM (1952), "Portfolio Selection", Journal of Finance, Vol 7

Merton RC (1969), "Lifetime Portfolio Selection under Uncertainty: The Continuous Time Case", Review of Economic Statistics, Vol 51

Merton RC (1971), "Optimum Consumption and Portfolio Rules in a Continuous Time Model", Journal of Economic Theory, Vol 3

Merton RC (1973), "An Intertemporal Capital Asset Pricing Model", Econometrica, Vol 41

Poterba J & Summers L (1988), "Temporary Components of Stock Prices: Evidence and Implications", Journal of Financial Economics, Vol 22

Richardson M (1989), "Temporary Components of Stock Prices: A Sceptic's View", Stanford University (Unpublished)

Roy AD (1952), "Safety-First and the Holding of Assets", Econometrica, Vol 20

Smith AD (1996), "How Actuaries can Use Financial Economics", British Actuarial Journal, Vol 1

Siegel J (1994), "Stocks for the Long Run", McGraw-Hill, New York

Van Bezooyen J, Guthrie K, Howie R, Ludvik P, Mehta S & Smith A (2001), "Consistent Assumptions for Multinational Models", Institute of Actuaries

Wilkie AD (1986), A Stochastic Investment Model for Actuarial Use, Transactions of Faculty of Actuaries, Vol 39

Wilkie AD (1994), Presentation to the Actuarial Society of South Africa, Unpublished

Wilkie AD (1995), More on a Stochastic Asset Model for Actuarial Use, British Actuarial Journal, Vol 1

Yakoubov YH, Teeger MH & Duval DB (1999), "The TY Model: A Stochastic Investment Model for Asset and Liability Management", Staple Inn Actuarial Society