

SKELETON TABLES DERIVED FROM THE EXPERIENCE OF THE CONTINUOUS MORTALITY INVESTIGATION

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THE method adopted in the calculation of the tables appended to this note was essentially that of the abridged mortality tables described by George King in *J.I.A.* Vol. XLVIII, pp. 294-300. Special formulae were used to suit the form in which the statistics were presented and to deal with the select data of durations 1-4.

FORMULAE FOR CENTRAL TERMS

King's first step was to obtain values of q_x at central ages of quinquennial groups by applying the formula $u_0 = \cdot 2w_{-2} - \cdot 008\Delta^2w_{-7}$ to both deaths and exposed to risk. In this formula $w_x = \sum_0^x u_{x+t}$ and differences are taken quinquennially. It is the particular case, for $n=5$, of the general formula for the central term of three consecutive groups of n terms each:

$$u_0 = \frac{1}{n} w_{-\frac{1}{2}(n-1)} - \frac{n^2-1}{24n^3} \Delta^2 w_{-\frac{1}{2}(3n-1)},$$

where $w_x = \sum_0^{n-1} u_{x+t}$ and differences are taken over successive n th values of w . The assumption underlying the formula is that u_x is of the form

$$a + bx + cx^2 + dx^3$$

for $3n$ consecutive values of x .

The corresponding formula for the central term of four groups is

$$u_0 = \frac{1}{2n} (w_{\frac{1}{2}-n} + w_{\frac{1}{2}}) - \frac{4n^2-1}{48n^3} (\Delta^2 w_{\frac{1}{2}-2n} + \Delta^2 w_{\frac{1}{2}-n}),$$

or, when $n=5$,

$$u_0 = \cdot 1 (w_{-4\frac{1}{2}} + w_{\frac{1}{2}}) - \cdot 0165 (\Delta^2 w_{-9\frac{1}{2}} + \Delta^2 w_{-4\frac{1}{2}}),$$

which may also be expressed in the form

$$u_0 = \frac{1}{60} \{7 (w_{-4\frac{1}{2}} + w_{\frac{1}{2}}) - (w_{-9\frac{1}{2}} + w_{5\frac{1}{2}})\} + \frac{1}{6000} (\Delta^2 w_{-9\frac{1}{2}} + \Delta^2 w_{-4\frac{1}{2}}).$$

If the last term is neglected as being insignificant and the fraction $\frac{1}{60}$ be ignored (since it will cancel out in obtaining q), the operation may be stated in summation form as 8 [2]-[4]. This has the advantage of avoiding differences of changing sign and is fairly simple to use. It was applied to the data for duration 0 and durations 5 and over respectively, summed in quinquennial age-groups $10\frac{1}{2}$ - $14\frac{1}{2}$, $15\frac{1}{2}$ - $19\frac{1}{2}$, $20\frac{1}{2}$ - $24\frac{1}{2}$, etc., and gave q 's at the quinquennial points 20, 25, etc.

King's formula for the central term of three groups was applied in summation form, viz. 28 [1]-[3], to the data for durations 1-4 added together. This produced q 's at ages $22\frac{1}{2}$, $27\frac{1}{2}$, etc., which were assumed for the purposes of this investigation to relate to an average duration of $2\frac{1}{2}$ and therefore to ages 20, 25, etc., at entry.

ULTIMATE TABLES

The ultimate tables were completed on King's plan, obtaining successively:

(1) $\log p_x$.

(2) $\log {}_5p_x$ by the formula $w_5 = 5u_0 + 7\Delta u_0 + 1.6\Delta^2 u_0 - .2\Delta^3 u_0$. Differences were avoided, however, by transforming the formula into

$$w_5 = -.2u_0 + 3.2u_5 + 2.2u_{10} - .2u_{15}.$$

For the earliest age King's formula was

$$w_0 = 5u_0 + 2\Delta u_0 - .4\Delta^2 u_0 + .2\Delta^3 u_0,$$

but it is sufficient at this end of the table to stop at first differences, taking

$$w_0 = 3u_0 + 2u_5.$$

At ages 95 and 100, the graduated values of $\log p_x$ and $\log {}_5p_x$ from the A 1924-29 table were used.

(3) $\log l_x$ by summing $\log {}_5p_x$, taking $\log l_{20}$ as 5.00000. At this stage $x \log v$ at 2% was introduced to give $\log D_x$.

(4) l_x and D_x .

(5) $\sum_1^5 l_{x+t}$ and $\sum_1^5 D_{x+t}$ by the formula

$$w_6 = 5u_0 + 8\Delta u_0 + 2.6\Delta^2 u_0 - .2\Delta^3 u_0$$

which was transformed into

$$w_6 = -.2u_0 + 2.2u_5 + 3.2u_{10} - .2u_{15}.$$

For the earliest age

$$w_1 = 2u_0 + 3u_5$$

was used instead of King's

$$w_1 = 5u_0 + 3\Delta u_0 - .4\Delta^2 u_0 + .2\Delta^3 u_0.$$

At age 100, $\sum_1^5 l_{x+t}$ was taken as $1.175l_{100}$, where $1.175 = \frac{\sum_1^5 l_{100+t}}{l_{100}}$ by the A 1924-29 ultimate table. Similarly $\sum_1^5 D_{95+t}$ was taken as $1.55D_{95}$.

(6) $\sum_1^{\omega-x} l_{x+t}$ and N_{x+1} .

(7) e_x and a_x .

SELECT TABLES

For the select portions of the tables the steps were as follows:

(1) $\log p_{[x]}$ and $\log p_{[x]+2\frac{1}{2}}$.

(2) $\log l_{[x]} = \log l_{x+5} - (\log p_{[x]} + 4 \log p_{[x]+2\frac{1}{2}})$.

(3) $\log l_{[x]+2\frac{1}{2}} = \log l_{x+5} - 2.5 \log p_{[x]+2\frac{1}{2}}$.

(4) $l_{[x]}$, $l_{[x]+2\frac{1}{2}}$ and l_{x+5} .

(5) $\sum_1^5 l_{[x]+t}$ from (4) by the formula

$$w_1 = .4u_0 + 3.2u_{2\frac{1}{2}} + 1.4u_5.$$

(6) $\sum_1^{\omega-x} l_{[x]+t} = \sum_1^5 l_{[x]+t} + \sum_6^{\omega-x} l_{x+t}$.

(7) $e_{[x]}$.

The above was the process actually used in the construction of the tables, values of $\log v^x$ being introduced at appropriate points in order to proceed to monetary functions.

The following alternative for steps (3), (4) and (5) was later tested, but was found to make very little difference to the final results:

$$(3) \log l_{[x]+1} = \log l_{[x]} + \log p_{[x]}.$$

$$(4) l_{[x]}, l_{[x]+1} \text{ and } l_{x+5}.$$

$$(5) \sum_1^5 l_{[x]+t} \text{ from (4) by the formula}$$

$$w_1 = -2u_0 + 5u_1 + 2u_5.$$

APPLICATION TO THE EXPERIENCE OF THE CONTINUOUS MORTALITY INVESTIGATION

The method outlined above was applied to the 1924-38 statistics of the Continuous Mortality Investigation in the following sections:

- (1) Whole-life with profits (medical and non-medical combined), 1924-38.
- (2) All classes, 1924-38.
- (3) All classes, 1934-38.

It is not possible to make a direct comparison of the results obtained with the graduated A 1924-29 table, either for the select or ultimate table, since the A 1924-29 ultimate table was constructed from the data of durations 3 and over, while the select portion, in addition to being restricted to the first three years' duration, was based on the statistics of 1927-29 only.

An attempt was therefore made to apply the method to the A 1924-29 data. Two ultimate tables were constructed, one from the data for durations 5 and over for comparison with the 1924-38 experiences and the other for durations 3 and over to compare with the graduated table. The small differences in the latter case are probably attributable to the overstatement of mortality in the graduated table which is apparent from Table VII on p. xxii of the Introduction to Vol. 1 of the *Monetary Tables*.

In calculating the select portion of the table the deaths and exposed for durations 1 and 2, which contained only 3 years' material, were doubled as a correction for the effect of aggregating them with those of durations 3 and 4. A further table was constructed without doubling the data for durations 1 and 2, by way of control, but the differences were insignificant, as will be seen from the extracts given below.

Age	Ultimate Table			Select Table		
	Values of e_x			Values of $e_{[x]}$		
	A 1924-29 graduated table (durations 3 and over)	A 1924-29 data by abridged method		A 1924-29 graduated table	A 1924-29 data by abridged method (5-year select period)	
		(durations 3 and over)	(durations 5 and over)		(durations 1 and 2 doubled)	(durations 1 and 2 flat)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
20	49·23	49·24	49·08	49·29	49·19	49·19
30	40·30	40·30	40·20	40·35	40·28	40·29
40	31·34	31·34	31·27	31·42	31·44	31·43
50	22·76	22·77	22·72	22·89	22·92	22·91
60	15·00	15·01	14·98	15·23	15·30	15·29
70	8·81	8·82	8·82	—	—	—
80	4·73	4·73	4·73	—	—	—

The differences between the figures shown in cols. (3) and (4) and between those in cols. (5) and (6) are to be expected if selection persists into durations 3 and 4. The relative weights of the data for durations 3 and 4 in the different age groups are shown in the following table where the A 1924-29 exposed to risk for durations 3 and 4 are given as percentages of the total exposed for durations 3 and over.

Age-group	Percentage
15½-24½	56
25½-34½	29
35½-44½	15
45½-54½	11
55½-64½	7

A comparison of col. (6) with col. (5) shows the effect of transferring this data from the ultimate part to the select part of the table. Part of the difference may however be due to the method employed and part to the fact that the *graduation of the A 1924-29 select table on the whole overstated the mortality* (see Table VIII on p. xxii of the Introduction to Vol. I of the *Monetary Tables*).

All comparisons have therefore been made with the abridged table constructed from the A 1924-29 data. The conclusion to be drawn is that there is a small margin in current premiums on the ground of improved mortality judged by the experience of 1934-38 or even by that of the whole 15 years 1924-38. At the present time, however, any such margin may be more than absorbed by the insufficiency of other factors.

SKELETON MORTALITY TABLES

Age x	Select				Ultimate (durations 5 and over)			
	Abridged table con- structed from A 1924-29 data	Life with profits 1924-38	All classes 1924-38	All classes 1934-38	Abridged table con- structed from A 1924-29 data	Life with profits 1924-38	All classes 1924-38	All classes 1934-38
	100,000 q_x				100,000 q_x			
20	145	156	145	138	262	232	224	196
25	149	110	138	133	239	226	218	188
30	175	121	134	126	251	222	222	199
35	187	179	160	146	296	253	263	239
40	234	235	225	207	401	373	358	319
45	321	420	312	301	541	547	504	461
50	468	564	474	449	769	806	747	716
55	706	778	713	652	1,214	1,291	1,182	1,162
60	1,110	1,147	1,009	890	1,963	2,076	1,903	1,849
65	—	—	—	—	3,242	3,281	3,163	3,015
70	—	—	—	—	5,340	5,259	5,228	4,948
75	—	—	—	—	8,485	8,332	8,332	8,137
80	—	—	—	—	12,748	12,655	12,644	12,480
	e_x				e_x			
20	49·19	49·30	49·68	50·25	49·08	49·18	49·59	50·19
25	44·79	44·87	45·22	45·73	44·68	44·73	45·13	45·65
30	40·28	40·31	40·68	41·15	40·20	40·21	40·60	41·07
35	35·81	35·74	36·14	36·58	35·71	35·65	36·05	36·49
40	31·44	31·22	31·66	32·01	31·27	31·15	31·56	31·95
45	27·10	26·90	27·31	27·62	26·94	26·79	27·17	27·51
50	22·92	22·71	23·09	23·39	22·72	22·59	22·92	23·22
55	18·92	18·90	19·11	19·43	18·70	18·62	18·88	19·17
60	15·30	15·41	15·51	15·77	14·98	14·98	15·14	15·42
65	—	—	—	—	11·66	11·72	11·78	12·03
70	—	—	—	—	8·82	8·90	8·92	9·09
	a_x 2 %				a_x 2 %			
20	30·26	30·33	30·47	30·72	30·19	30·26	30·42	30·68
25	28·59	28·65	28·80	29·03	28·52	28·56	28·74	28·98
30	26·70	26·73	26·90	27·13	26·64	26·66	26·85	27·08
35	24·64	24·61	24·82	25·04	24·57	24·55	24·76	24·99
40	22·46	22·32	22·59	22·77	22·35	22·27	22·51	22·73
45	20·10	19·96	20·22	20·39	19·98	19·88	20·12	20·32
50	17·62	17·47	17·74	17·91	17·48	17·38	17·61	17·79
55	15·07	15·06	15·21	15·43	14·91	14·84	15·03	15·22
60	12·61	12·69	12·76	12·95	12·36	12·34	12·47	12·67
65	—	—	—	—	9·92	9·96	10·01	10·20
70	—	—	—	—	7·72	7·78	7·80	7·94

Age	Select net premiums % at 2%				Increase (+) or decrease (-) compared with A 1924-29 abridged table		
	Abridged table con- structed from A 1924-29 data	Life with profits 1924-38	All classes 1924-38	All classes 1934-38	Life with profits 1924-38	All classes 1924-38	All classes 1934-38
Whole-life Assurance, premiums throughout life							
20	1.238	1.231	1.217	1.192	-.007	-.021	-.046
30	1.649	1.645	1.623	1.594	-.004	-.026	-.055
40	2.302	2.327	2.278	2.246	+.025	-.024	-.056
50	3.410	3.453	3.375	3.327	+.043	-.035	-.083
60	5.387	5.344	5.307	5.208	-.043	-.080	-.179
Whole-life Assurance, limited premiums							
10 years' payments							
20	4.263	4.244	4.215	4.157	-.019	-.048	-.106
40	5.993	6.032	5.962	5.919	+.039	-.031	-.074
60	8.810	8.771	8.729	8.656	-.039	-.081	-.154
20 years' payments							
20	2.368	2.355	2.339	2.305	-.013	-.029	-.063
40	3.402	3.430	3.381	3.353	+.028	-.021	-.049
50	4.278	4.334	4.251	4.215	+.056	-.027	-.063
30 years' payments							
20	1.753	1.741	1.729	1.702	-.012	-.024	-.051
40	2.630	2.659	2.610	2.584	+.029	-.020	-.046
Endowment Assurance							
Term 10 years							
20	9.053	9.043	9.044	9.034	-.010	-.009	-.019
40	9.137	9.155	9.133	9.126	+.018	-.004	-.011
50	9.363	9.417	9.358	9.350	+.054	-.005	-.013
60	10.055	10.028	9.992	9.954	-.027	-.063	-.101
Term 20 years							
20	4.158	4.145	4.146	4.134	-.013	-.012	-.024
40	4.339	4.361	4.331	4.319	+.022	-.008	-.020
50	4.776	4.834	4.760	4.739	+.058	-.016	-.037
Term 30 years							
20	2.569	2.554	2.554	2.540	-.015	-.015	-.029
30	2.652	2.640	2.634	2.616	-.012	-.018	-.036
40	2.909	2.939	2.896	2.879	+.030	-.013	-.030
Term 40 years							
20	1.822	1.810	1.805	1.789	-.012	-.017	-.033