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A SKETCH OF THE HISTORY

OF THE

SCIENCE OF LIFE CONTINGENCIES,

WITH SPECIAL REFERENCE TO THE

ORIGIN AND CONSTRUCTION

OF

MORTALITY TABLES.

BY

J. J. W. DEUCHAR, F.F.A. AND A.I.A.,

Assistant-Actuary of the City of Glasgow Life Assurance Company.

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A SKETCH OF THE HISTORY OF THE SCIENCE OF LIFE CONTINGENCIES, WITH SPECIAL REFERENCE TO THE ORIGIN AND CONSTRUCTION OF MORTALITY TABLES. By J. J. W. DEUCHAR, F.F.A. and A.I.A., Assistant-Actuary of the City of Glasgow Life Assurance Company.

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Mr. Chairman and Gentlemen,

When drawing up a syllabus for our first Session, the Committee decided to place before the members, as an introduction to a study in detail of such points of interest or difficulty as present themselves to us in the Theory and Practice of Insurance, two Papers, giving, in concise form, the main features in the past history of our profession, believing that therein lie at once the record of the past and the hope and promise of the future. At our last meeting we had the first of these (on the History of Fire Insurance), and I have been asked to take up the companion subject—the History of the Theory of Life Contingencies—and to make my paper, as far as possible, an introduction to the Study of Mortality Tables.

The Science of Life Contingencies may not inaptly be likened to a silken cord of double strand—the one strand representing theoretical method, and the other practical material—and these two are, in our day, so closely intertwined that we cannot very well trace their histories apart. That is to say, a notice of the development of the practical rules and mathematical formulas for calculating the duration of life and the risk of death, without some account of the origin and progress of the labours of the statistician in recording the *facts* of life and of death, and so producing Tables of Mortality, would be a very incomplete and inadequate sketch of the history of the Science of Life Contingencies.

Although the two strands of the silken cord are so closely interwoven *now*, this was not always the case, for we can find traces of what I have called theoretical method at a period very much earlier than that at which we get our first record of practical material. At the same time it is at least questionable if this theoretical method can be dignified by the name of *Science* at any time prior to its being based on recorded experience. Nevertheless, for a very long time before the world possessed anything which can properly be called a Mortality Table, estimates of a more or less scientific nature appear to have been made of the average duration of life. Indeed, so very early does some sort of attention appear to have been directed to one or other of the many phases of the subject that we are somewhat at a loss when we seek to lay our finger on *the most rudimentary* beginning of the science. Without doubt, however, the earliest records which can, even remotely, be connected with our subject are those which refer to *longevity*, or the limit of man's life. From the beginning of the world's history it would seem to have been the practice to record the ages at which the old died; and this was, no doubt, mainly due to the fact that Nature has implanted in the breast of man the almost universal desire of long life, and to the wish of each man in his own growing age to fortify himself with the hope of attaining some greater, if not *the* greatest recorded age. The plaintive words of the Patriarch Jacob, who felt the infirmities of age creeping fast upon him, while yet he "had not attained unto the days of the years of the life of his fathers in the days of *their* pilgrimage," are only an instance of the constant reference made by the individual, in this respect, to the records of the race. This habit of recording ages at death could not long be observed without at least suggesting that there was a law at work *limiting the duration of man's life;* and in our very oldest records we find references to this limitation.*

It may not be out of place to notice here the explanation which has been given by certain writers of the sudden decrease in the *recorded age* of man from the time of Abraham, and the further sudden decrease from the time of Joseph. It has been affirmed by Hensler, Hufeland, and others, that the *year* up to Abraham's time consisted of *three months* only, that it was then increased to *eight months*, and afterwards, in the time of Joseph, to *twelve*; and in support of this statement they say that there are still to

* Job vii. 1, "Is there not an appointed time to man upon earth?" &c.; Job xiv. 5, "Seeing his days are determined, the number of his months are with Thee, Thou hast appointed his bounds that he cannot pass," &c.; and numerous other passages.

be found in the East nations which reckon only *three months* to *the year*. If this explanation be the true one—and we make due allowance for the exceptionally favourable conditions, as regards longevity, under which the Patriarchs lived—Hufeland's contention that, under equally favourable conditions, man might still attain to as great an age as ever, seems not altogether unreasonable.

The noting of the ages at death which we have referred to, although it might not indicate an *average age at death*—from the younger ages probably not being recorded—would certainly render conspicuous those lives which reached an unusual age, and the desire of men to live long would naturally suggest an inquiry as to the cause of the exceptional longevity. Thus we learn from Eusebius of Plato's noting (some 400 years B.C.) that many persons lived to a great age in Egypt and Syria, *owing to the serene and equable nature of the climate*—the historian adding that the experience of many years had led to an examination of the facts, with the result that these had been handed down to his own time (A.D. 230) with greater exactness. Pliny, also, in his observations on the Census of Vespasian, carefully notes the number of persons living at ages above 100, and numerous other early writers make similar observations, as well as remark on the effect of climate and nationality on longevity. None of these observations, however, had reference to the *average duration* of life in the past, or the *probable duration* of it in the future; indeed, all such speculations would most probably have been forbidden by the religious teachers of that age, had any one thought of indulging in them. In spite of this, a need for some estimate of the value of life arose at a very early period in connection with the Falcidian Law, which, in B.C. 44, accomplished a long ineffectual intention of Roman legislation, viz.—the preventing of a testator from leaving more than three-fourths of his property in legacies, thus securing one-fourth to the legal heir. It became, therefore, absolutely necessary to devise some means of valuing such legacies as were charged on the succession by way of life annuity; and, no doubt, some estimate of the value of life would be made about this time. We do not hear of anything of the kind, however, till about the beginning of the third century of our era, when Æmilius Macer gives the rule which was then in common use, viz.—"From the earliest age until 30, compute 30 years, and from age 30 so many years as are wanting until 60; therefore, never more than 30 years are computed." To take all lives of the age of *30 or under* as of the same value, and to consider lives of the age of 60 or upwards as of no value at all, was certainly to make a tolerably rough estimate, and no doubt it would very soon be improved upon. The eminent jurist Ulpian seems to have had his attention drawn to the matter—not improbably by the faulty nature of the method described—and supplied a table of the estimated worth of Annuities on Lives.* In many countries, and at periods long prior to this, there appear to have existed wide-spread provisions for the destitute—particularly for widows and orphans; but none of these—not even the famous one, over which seven deacons were appointed to preside, which we read of in Scripture—seem to have done more than dispense charity to those who might be in want of it, and we have no evidence of any calculated provision, or any estimate based on the probable duration of life. To Ulpian, therefore, must be accorded the honour of presenting the world with—as far as we know—its *first measure of life* (graduated according to age)—*its first Life Table.* The table seems to indicate considerable skill on the part of its framer, as well as that it had actual observations of some kind for its basis. We have no definite knowledge of the materials used by Ulpian in framing his estimate, but, as the Falcidian Law had then been in operation for nearly 300 years, he may have had opportunities of examining the results of the valuation of lives by the methods previously employed. It is said, also, that there was at that time a kind of registration of deaths observed, and, if so, Ulpian would most likely turn it to account in framing his table. It is pretty clear that the element of *interest* was not introduced into the calculation, and that the table is to be looked upon more as exhibiting the *expectation of life* than the value of annuities; but in this aspect it is worthy of notice that Ulpian's table gives a far better view of the value of life than afterwards obtained till about *the close of the seventeenth century!* And it is a curious fact that this very table, after having fallen aside for more than 1500 years, was, in *December, 1814,* authorised by the Tuscan Government for the valuation of Life Annuities, in spite of the great advance of the Science of Life Contingencies at that time, and the numerous trustworthy tables in existence.

Leaving ancient times and Ulpian's table we come upon no further reference to Life Contingencies till we reach the 16th

* See Appendix.

century. There is, however, good evidence to show that at that time, if not earlier, *Temporary Assurances on Lives* were very frequently effected. These seem to have originated in an extension of the Practice of Marine Insurance. This is brought out in an interesting chapter of *Le Guidon*, a treatise compiled for the benefit of "merchants trading in the noble city of Rouen," about the beginning of the 16th century. The treatise tells how—owing to fear (on the part of those who were in the habit of undertaking *distant* voyages to the coast of Italy, Constantinople, &c.) of the galleys and frigates of the armies of the Turk, or the Corsair, who made a traffic of the sale of Christians—it had become customary for masters and captains when undertaking voyages to stipulate with their merchant-freighters, or others, for the restitution of their persons in case of capture. In such a case the master stated in the policy his own ransom and that of his companions at so much per head, as well as the name of the ship and of the person to whom the ransom was to be payable, &c. The insurer was bound to pay the sum insured for the ransom fifteen days after production of the attestation of capture and the policy. The author of *Le Guidon* goes on to tell how, in *other* countries than France, insurances were even made *upon the life of men* in case of their decease upon their voyage, and that those having rents or pensions might insure, so as, on their decease, *to continue to their heirs such pension or rent as might be due to them;* "which," he adds, "are all stipulations forbidden as against good morals and customs." The correctness of this last statement, as well as the extent to which the Insurance of Lives seems even then to have spread, are both evidenced by the numerous prohibitory ordinances issued by the different European States in the 16th and 17th centuries. One of these shows, by a reservation in favour of the kind of Insurance we have been describing, by what means the system may have gradually won its way into licensed existence. The Ordinance referred to* says:—"We forbid the making of any Insurance on the life of men. . . . . . . Nevertheless, those who shall redeem captives may have the price of the redemption assured upon the persons whom they withdraw from slavery, which the assurers are bound to pay, if the redeemed on his way back is re-taken, killed, drowned, or if he perish by other means than natural death." We have evidence also that in the 16th

* The great Marine Ordinance of Louis XIV. of France (1681).

century Life Annuities and Children's Endowments were frequently effected *with the view of evading the usury laws* which made it penal to exact interest. Dr. Thomas Wilson,* writing in 1569, says—"A merchant lendeth to a corporation or companie an hundred pound, which corporation hath by statute a grant that whosoever lendeth such a summe of money and hath a childe of one yeere, shall have for his childe, if the same childe doo live till he be full fifteene yeeres of age, 500 li. (£) of money; but if the childe die before that time the father to loose his principall for ever: whether is this merchant an usurer or no? The law saith, If I lend purposely for gaine, notwithstanding the perill or hazard, I am an usurer." He thus seems to be of opinion that the purchasers of Children's Endowments were putting themselves beyond the law; not so, however, in his opinion, those who, with the same object in view, purchased Life Annuities, for he says again—"A corporation taketh a 100 li. of a man to give him 8 in the 100 li. during his life, without restitution of the principall. It is no usurie, for that here is no lending, but a sale for ever of so much rent for so much monie." He also refers favourably to the practice of purchasing Joint Life Annuities for the same purpose. Later on Malynes, an English merchant (writing in 1622), gives a curious account of the annuity transactions of his day, which were "much used in divers cities beyond the seas to draw monies into their hands," and he gives numerous examples of the kind of terms obtained, for example,—"One hundreth pounds is delivered to have two hundreth pounds for it at the yeare's end upon eight lives, if they all live." He also gives a table of the terms charged by the city of Amsterdam for annuities of 100 guelders on different numbers of lives.

Nothing is known of the data on which the rates for any of these assurance or annuity transactions were based, and, indeed, it may safely be assumed that they were not based on mortality observations at all, but were merely very rough guesses. Interest

* Dr. (or Sir) Thomas Wilson died in 1581. The work referred to—"A Discourse upon Usurie, by waie of Dialogue and Oracions, for the better varietie and more delight of all those that shall read this Treatise; by Thomas Wilson, doctor of the Civil lawes, one of the maisters of His maiesties honourable Courte of requests. Imprinted at *London* by Roger Warde, dwelling neere Holburne Conduit, at the signe of the Talbot"—is dated 1584; but it bears as Preface, an Epistle dedicated to "his most especiall and singular deere Lord, the Earle of Leicester," which is dated "thys twenty of July, 1569," so that the work was written at least as early as the latter date.

does not appear to have been taken into account in any of them (which we know is as important an element as mortality in annuity values), and, with the exception of the endowment transaction mentioned by Dr. Thos. Wilson, there is *no reference to the ages of the lives*. Without going the length of assuming the people of that day to be so ignorant as to imagine that all lives were of the same value, we may safely assert that they were quite unaware of the steady increase in the liability to death which occurs with increase in age; and in this respect Ulpian's table, compiled so many centuries before, is incalculably superior.

While the historical facts which have been narrated are no doubt interesting, as exhibiting the bulk of what we know of the progress of Life Insurance up to the middle of the 17th century, it may be objected that they display little or nothing of the *Science of Life Contingencies;* and this is true. Without an appreciation of the fundamental proposition of the Doctrine of Probabilities (or Doctrine of *Chances* as it was originally called) and its application to the subject, there could not, strictly speaking, be a Science of Life Contingencies, and up to the middle of the 17th century there was no such thing as a Doctrine of Chances. In the year, 1654, however, Pascal, the great French mathematician, was led to give his attention to the subject by a series of questions, put to him by the Chevalier de Méré, regarding the relative chances in certain conditions of games of hazard, and, in solving the questions proposed, he established the first principles of the Doctrine of Chances. The inquiries of Pascal were continued a couple of years later by the celebrated astronomer Christian Huygens, who wrote a treatise on the subject in Dutch, and also by Francis Schooten, who translated Huygens' treatise into Latin, and published it, along with a work of his own, in 1657. It is sufficient for our present purpose to say that one of the results of the labours of these men was to establish the fundamental proposition of the Doctrine of Chances, viz., that the probability of the happening of any event may be expressed by the ratio of the number of chances for its *happening* to the total number of chances for its *happening and not happening;* and similarly that the probability of the *failing* of any event is expressed by the ratio of the number of chances for its *not happening* to the total number of chances for its *happening and not happening*—or, more generally, if an event must either happen or fail, and there are a chances for its happening and b chances for its failing (that is to say, a ways in which it may happen, and b ways in which it may fail), then the probability of its happening may be expressed by the fraction $\frac{a}{a+b}$, and the probability of its failing by $\frac{b}{a+b}$. Upon this fundamental proposition the entire fabric of the Theory of Life Contingencies may be said to rest, and a thorough appreciation of it in this connection is absolutely necessary at the outset to the Actuarial student. We need not pause to examine this further at present, as it will receive illustration as we proceed.

It was not long before the applicability of this discovery to the solution of questions connected with the duration of life was observed. In April, 1671, it was resolved by the States General of Holland (then in rather a needy condition) to raise funds by means of Life Annuities—a practice which, as we have already seen, was not altogether new to them. Their then prime minister, John de Wit, himself an eminent mathematician and pupil of Descartes, thereupon gave his attention to the subject, and prepared what has been called* "the first known production of any age treating in a formal manner on the valuation of Life Annuities." On examining the past annuity experience of his government, and perhaps other records of mortality, De Wit framed certain hypotheses which appeared to agree very well with the facts he had observed. First, he divided life into four periods:—the most vigorous period from the 3rd to the 53rd year, and periods of gradually decreasing vigour from the 53rd to the 63rd, from the 63rd to the 73rd, and from the 73rd to the 80th year—which last age he assumed to be the limit of life. Then he assumed that during the most vigorous period (the first 100 half years) each half year is equally destructive or mortal, or, in other words, that it is not more likely that death shall occur in one of these half years than in another; that each half year of the period from 53–63 is more mortal than those of the preceding period in the ratio of 3 : 2; that each half year of the 3rd period is more destructive to life than those of the first period in the ratio of 2 : 1; and that each half year of the 4th period is more mortal than those in the first period in the ratio of 3 : 1. These hypotheses may be stated to be equivalent to the assumption that, out

* Mr. Hendrik's Assce. Magne., vol. ii.

of *128* lives 3 years old, the deaths will be at the rate of 1 every *half year* for the first 100 half years, leaving 28 lives alive aged 53 at the end of the 50th year; out of these 1 will die every *nine months* for the succeeding ten years, leaving 14·66 alive aged 63 at the end of the 60th year; that out of these 1 will die every *year* for the succeeding ten years, leaving 4·66 aged 73 alive at the end of the 70th year; and that out of these 1 will die every *year and a-half* during the succeeding seven years, leaving none alive at the end of the 77th year to complete the age of 80,—the last theoretical fraction of a man (·33) having died in the previous half year.* No doubt these suppositions very nearly represented the facts as far as De Wit was able to observe them, and the hypotheses greatly simplified the attainment of his object, which was to calculate the average present value of annuities on lives three years old. His subsequent method was as follows:—Since out of 128 lives 1 will die in each of the first 100 half years, the chance of death at the outset to each of the 128 lives (as regards each of the first 100 half years) is $\frac{1}{128}$; but as annuities are payable by instalments at the end of each half year, the life which fails in the *first* half year will receive 0, the life which fails in the *second* half year will receive 1 half year's payment (the present value of which at the outset is that of an annuity certain for *half-a-year*), the life which fails in the *tenth* half year will have received 9 half years' payments (the present value of which at the outset is that of an annuity certain for four and a-half years), and so on. Therefore, each life has, at the outset, one chance in 128 to receive nothing, one chance in 128 to receive one half payment, and so on, for each of the first 100 half years. The process for the other periods is quite the same, with the exception that instead of *one chance* out of 128 for each half year, there is only $\frac{2}{3}$ds, $\frac{1}{2}$, and $\frac{1}{3}$d, respectively. Working out in this way the value of the chance for each half year up to the assumed limit of life, and adding them all together, De Wit obtained, for the first time in the world's history, the scientifically calculated value of a Life Annuity. [Perhaps De Wit's method would be clearer to some of you if I were to explain it in this way:—By hypothesis he knew

* In Mr. Hendrik's description (Assce. Magne., vol. ii., p. 246)—which is quoted in Mr. Walford's Cyclopœdia, and elsewhere—15·66 is given as the number alive at 63, 5·66 at 73, and 1 at 80, which is manifestly wrong, as De Wit's own figures, although not in the same form, clearly prove.

the exact number that would die in each year, out of 128 lives, till all were extinct. He could thus tell *the number of annuity payments each would have received* up to death. Therefore, taking the total present value of all these payments, according to an interest table, and dividing by 128, he obtained the *average present value of all the payments which would be afterwards received by the 128 lives*—or, in other words, the present value, at the outset, of an Annuity to each.] The arithmetical result of De Wit's calculation (interest being taken at 4 per cent.) was to indicate 16 years' purchase as the value of an Annuity on a life 3 years old, and his object was to show that this was a more favourable bargain for the purchaser than a Redeemable Annuity at 25 years' purchase. In the course of his argument he showed a very clear appreciation of the effect of *Selection*—speculations upon which are occasionally stated to be altogether of modern growth. It is important to notice that De Wit's treatise does *not* contain (although it has sometimes been said to do so) an estimate of the value of Life Annuities *at each age*; but only, as it were, a sample calculation for *one* age. Indeed, De Wit, and also the Burgomaster Hudde (who appended a certificate to De Wit's treatise), in stating their belief that the true value of "a Life Annuity" is at least 16 years' purchase, *never refer to the age of the supposed annuitant at all;* and one does not always feel certain that they quite realised 16 years' purchase to be only the correct value at the age for which they had made the calculations, or that the value would be different at every other age. The explanation, both of De Wit's having only made his calculations for one age, and of his expressing the result of that one calculation generally as the value of "a Life Annuity," is, I fancy, in the following considerations:—First, De Wit's object was to find the *highest value* of a Life Annuity, or the number of years' purchase which his government could with perfect safety fix as a *uniform selling price;* and, second, a Table of Annuity Values, graduated according to age, was not only not known in these days, but would have been of no use for the purpose in hand, as it was not customary for those who purchased annuities to rest them on their own lives, but to select, as *nominee*, the life which seemed to them likely to endure longer than any other. The only way, therefore, in which the granter of annuities could protect himself, against this invariable selection of nominees from among the choicest lives, was by charg-

ing the highest annuity rate shown at any age. As De Wit enabled his government to do this, he supplied all that was necessary for the purpose in hand, and it probably never occurred to him to extend his calculations to other ages. The one fault of his invention, in the eyes of the States General of Holland, was, it would appear, that it demanded much higher rates than had been previously charged—the majority of Life Annuities at that time being sold at nine years' purchase. It was not likely that the terms now shown to be necessary would tempt the public, and as it was MONEY rather than correct annuity estimates that the government wanted, *the Report and Calculations were suppressed.* In this way, we have good reason to believe, the highly original and eminently useful invention of De Wit was practically lost to the world. John de Wit and his brother Cornelius came to a melancholy end in the following year, being murdered by an infatuated mob; and although Leibnitz was in Holland two or three years later, and made every effort to obtain a copy of the treatise, he failed. As we know that on this visit Leibnitz met and conversed with the Burgomaster Hudde, who, as previously mentioned, was associated with De Wit in the scheme, and, as we also know that by this time the States of Holland had published new annuity terms much more favourable than poor De Wit had proved was consistent with safety—the reason of Leibnitz' failure to secure a copy of the grand pensionary's treatise is only too plain. Leibnitz referred to the treatise in his works, and several other writers followed his example, but no very clear idea of its contents prevailed, and the great invention may be said to have had little or no influence on the subsequent progress of the Science of Life Contingencies. To Mr. Frederick Hendriks we are indebted for the recovery of the document itself after the lapse of 180 years! He found it inserted in the "Resolutions of the States of Holland and West Friesland" for the year 1671. I have thought it well to give a tolerably minute account of this first treatise on Life Contingencies, because, besides its historical interest—which alone entitles it to particular attention in a paper like the present—it exhibits very clearly one of the most elementary forms of Life Contingency calculations, and shows both the first application of the Doctrine of Probabilities, and the permanent necessity for it in such calculations.

That De Wit's invention was immensely a-head of what existed elsewhere in his own time, or for twenty years afterwards, is at once apparent when we glance at the method of calculating annuity values then commonly adopted. This is described in a work on *College Leases*, &c., by Mr. Mabbot, of King's College, Cambridge, published in 1686, and originally supposed to have been written by Sir Isaac Newton, from his having certified his belief in the accuracy of the tables. The way of purchasing leases by lives, the author says, was commonly to reckon one life as a lease of seven years, two lives as a lease of fourteen years, three as of twenty-one years, and so on; but, *this way seeming unequal*, there is another way which is more agreeable to reason, viz., *for every life to decrease one year*; so that if one life be reckoned a lease of *ten* years, two will be a lease of nineteen years, and three a lease of twenty-seven years, &c.; if, on the other hand, one life be reckoned as a lease of *twelve* years, two will be as a lease of twenty-three years, and so on. Although this quaint old rule for calculating the value of annuities on the longest of several lives may have the merit of simplicity, it certainly has no other merit, especially when we consider that it leaves the calculator to select whatever initial value he likes for a single life, and as it suggests no restriction or limitation on account of differences of age in the lives. Such was the position of matters here twenty years after the suppression of De Wit's treatise, but the dawn of the Science of Life Contingencies in our country was at hand. In 1692 the English Government made its first attempt to raise money by means of Life Annuities, and passed the *Million Act** with the view of raising £1,000,000 stg. for carrying on the war against France. The scheme was to include 10,000 nominees at £100 each, and was *Tontine* in its nature, with an alternative, should the whole million not be subscribed, to give subscribers an annuity of *£14 per cent.* during their nominees' lives. Even on these ridiculously favourable terms the whole million was not subscribed. What is specially noticeable to us about the scheme is, that a somewhat remarkable table was published in connection with it, and apparently by authority, "designed for the encouragement of contributors," and purporting to show the expected mortality year by year, for a period of ninety-nine years, among the 10,000 nominees. The table is *in form* a Mortality Table, showing the number alive and the number dead each year till all are extinct; but as nothing is said about the data, if any,

* 4 Will. and Mary, c. 3.

on which it is based, and, particularly, as there was no restriction in the scheme as to the ages of the nominees, and consequently no estimate could be made beforehand on the point, I am afraid this table must have been little better than a hoax. There is a suspicious look of *exactness* about it, in addition, which, taken in connection with the universal ignorance of the subject at the time, as indicated in the work recently quoted, has quite an amusing look. Thus the table shows that at the end of the 99th year 9999 of the nominees would be dead and 1 alive; it also shows that the fortunate subscriber who selected this nominee (and his heirs to perhaps the fifth generation) would receive during the 99 years, for the original investment of £100 sterling, an annual payment *commencing* at £10 6s., and increasing every year till in the last or 99th year it reached the not inconsiderable sum of £70,000! If this table, then, was an *ignis fatuus*, real light was not long in being let in upon the scene; for, early in the following year (1693), Dr. Halley,* the Astronomer Royal, read to the Royal Society a paper, entitled, "An Estimate of the Degrees of Mortality of Mankind, drawn from curious Tables of the Births and Funerals at the City of Breslaw; with an attempt to ascertain the price of Annuities upon Lives." It is interesting to learn how Dr. Halley came to turn his attention to the subject and to write this paper. The Royal Society, it would appear, after 30 years of vigorous existence, had begun to show signs of feebleness, and had even for some years suspended the publication of its "Transactions." A meeting was held, and a great effort made, in December 1692, to secure the renewal and the regular publication of these; and at that meeting Halley enthusiastically undertook to supply 5 out of the 20 sheets himself. It was in casting about for material with which to begin the fulfilment of his promise that he came across Dr. Neumann's Accounts of the Births and Deaths occurring at Breslaw, in Silesia, during the 5 years 1687–1691, recently contributed to the Society, and selected them for his first paper to the revived "Transactions"—perhaps being led to regard the subject as one of special interest, on account of the recent Government Annuity Scheme. It was in this way that the *First Table of Mortality* came to be constructed, and also that the foundation of the Doctrine of Life Contingencies came to be laid; for we can hardly say that De Wit's treatise laid

* Then Mr. Halley.

the foundation of the science, seeing that its contents were not allowed to transpire, and it was thus unconnected with the subsequent progress of this branch of knowledge.

We cannot do better than pause here for a little to examine carefully how the first Table of Mortality was constructed, and what its construction and Halley's treatise meant to the science. First, I ought to say, it must not be imagined that Bills of Mortality were unknown in these days, or that the Breslaw returns referred to were the first that had appeared, or even the first that the Royal Society had received and recorded. Bills of Mortality are known to have been kept in some of the country parishes of England as early as 1538, and in London they were prepared *irregularly*, for short intervals, in connection with the great plagues of 1562 and 1592, and *regularly* from and after the great plague of 1603. The peculiar feature about these Breslaw returns, and what made them available for Halley's purpose, was that they recorded for the first time *the ages at death*. Without this information, it will be readily seen, a Mortality Table cannot be constructed from actual observations. The total number of deaths during the five years, for which returns had been provided, amounted to 5,869, or 1,174 per annum; and arranging these according to age, Halley found that, *after childhood*, the list showed considerable regularity, and he was thus led to believe that, in a *perfect* series of observations, the deaths at adjacent ages would be found to be the same, or only slightly different in number. Applying this assumption to his own figures (which gave a yearly average of about 12 deaths per age from birth to 100), he decided that the yearly deaths at any two adjacent ages after childhood, ought either to be *equal*, or at least should not differ from each other by more than a unit. In this way Halley *graduated* the rough material of the returns, from age 8 onwards. From 1 to 8 years of age he arranged the deaths (guided probably by his materials) as 145, 57, 38, 28, 22, 18, 12, and 10 respectively. Having thus arranged and graduated his list of deaths according to age, it remained for him to ascertain the *number living at each age*, so that he might determine the relation between the deaths at each age and the lives out of which they occurred, or, in other words, the rate of mortality at each age.

It may be as well to remark here, generally, that while there are various ways (as we shall afterwards see) in which a Mortality Table may be constructed, the way in which the table we are

examining (and indeed every Mortality Table previous to the Carlisle Table) was constructed, was from a Record of Deaths,* supplemented only by the yearly births, and without any knowledge of the population out of which the deaths occurred. Now a Mortality Table constructed from these materials will only give correct results *if the population is a stationary one*—that is to say, if the births are exactly equal to the deaths, and there is neither emigration nor immigration. It is, in fact, only on this assumption that a Mortality Table can be prepared from such data. The utility of the assumption is that the number living at any age consists solely of the survivors from the preceding age, and may be derived from the number living at the preceding age, simply by deducting the deaths occurring at that age. If, therefore, we have a list of the yearly deaths at each age, we are provided with a table of differences, the continual addition of which, from the oldest age downwards, or the continual deduction of which (from the number born) upwards, will give us the number alive at any age.

The number of births in Breslaw was about 1,238 per annum, being 6,193 in the five years as against 5,869 deaths, numbers tolerably near each other, and which, after allowing for the emperor's war levies and the natives of the town who might thus die abroad, Halley thought warranted him in adopting the assumption of a stationary population. He had also reasons for assuming that there was little or no emigration or immigration. Of the 1,238 children *born throughout the year*, Halley found that 348 died at different intervals in their first year of age, only 890 thus surviving a year; he, therefore, thought he might fairly assume that the population contained at any given time 1,000 children in their first year. He, therefore, took 1,000 as the number alive at age 1, and by the successive subtraction of the numbers in his graduated list of deaths obtained the number alive at each of the other ages.† In this way then the first Table of Mortality was framed, and it may be remarked that, while Tables of Mortality

* It was in this way that the term "Mortality Table" came to be used for what is in reality a *Life* Table.

† Halley's table thus represents the numbers living in each year of age, and not the numbers completing each year of age, as in most Tables of Mortality. Halley's figures correspond to the mean between the figures at the same age and the next younger age in other tables.

are not now constructed from bills of mortality alone by the aid of the assumption of a stationary population, the *form of table* used by Halley, and also the assumption he made use of in graduating his deaths (viz., that at any two adjacent ages after childhood these should vary little) continue to be made use of at the present

HALLEY'S BRESLAW TABLE.

Age Current.	Persons.	Age Current.	Persons.	Age Current.	Persons.	Age Current.	Persons.	Age Current.	Persons.	Age Current.	Persons.	Age.	Persons.
1	1,000	8	680	15	628	22	586	29	539	36	481	7	5,547
2	855	9	670	16	622	23	579	30	531	37	472	14	4,584
3	798	10	661	17	616	24	573	31	523	38	463	21	4,270
4	760	11	653	18	610	25	567	32	515	39	454	28	3,964
5	732	12	646	19	604	26	560	33	507	40	445	35	3,604
6	710	13	640	20	598	27	553	34	499	41	436	42	3,178
7	692	14	634	21	592	28	546	35	490	42	427	49	2,789
Age Current.	Persons.	Age Current.	Persons.	Age Current.	Persons.	Age Current.	Persons.	Age Current.	Persons.	Age Current.	Persons.	56	2,194
43	417	50	346	57	272	64	202	71	131	78	58	63	1,694
44	407	51	335	58	262	65	192	72	120	79	49	70	1,204
45	397	52	324	59	252	66	182	73	109	80	41	77	692
46	387	53	313	60	242	67	172	74	98	81	34	84	253
47	377	54	302	61	232	68	162	75	88	82	28	100	107
48	367	55	292	62	222	69	152	76	78	83	23		34,000
49	357	56	282	63	212	70	142	77	68	84	20		Sum total.

day. Halley was well aware that bills of mortality alone were not the best basis for a Life Table, and in his paper he distinctly says— "The only thing wanting is the number of the whole people, which, in some measure, I have endeavoured to supply by the comparison of the mortality of the people of all ages." On the other hand, it is only historical justice to say that the *form* of table had been employed more than once prior to 1693 for those imaginary tables to which allusion has been made.

Before dismissing Halley's table and taking a rapid survey over the main features of the subsequent progress of the science, it remains to indicate what was the actual gain in the possession of the Mortality Table and Treatise which Halley contributed. In the first place you will notice that the table, from its mode of construction, gives a comparative view of the whole population distributed according to age, and it would be interesting to know if the sum of the number alive at all ages, according to Halley (viz., 34,000), came near the actual population of the town. This we cannot now say, but if his *assumptions* were *facts* this should have been the case. On the assumption of a similar rate of mortality and a stationary population in any other community the same proportion of the lives would be alive at the different ages, and this, as Halley pointed out, gave a means of estimating the proportion of *men able to bear arms* in any multitude. The insurance transactions of these days were mainly temporary, and, to a considerable extent, wagering insurances, and Halley's table at once gave absolute data on which to base these.* If it were wanted to know, for instance,

* It must not be imagined by any one that Halley's table was of too elementary a nature to furnish properly-calculated Premiums for Assurance, had the methods for arriving at these been then known. His table, it is true, does not give the Annual Deaths in a separate column; but these (as we have seen) are simply the differences between the numbers alive at the consecutive ages, and could be ascertained by inspection. Thus, if 598 persons in their 20th year combined to assure their lives for £1 each, to be paid at the end of the year in which death should occur, the table shows us that 6 persons would die (and £6 would require to be provided) the first year, 7 the second, 6 the third, and so on. By taking, therefore, from an Interest Table, the present value of £6 due 1 year hence, £7 due 2 years hence, £6 due 3 years hence, and so on up to the oldest age in the table,—summing the whole, and dividing by 598, we get a correctly-calculated Single Premium for the Assurance of £1 at age 20.

the chance of a man of 30 dying within five years, the table readily gave the solution: the number alive at 30 by the table is 531, the number alive at 35, 490, the difference or 41, having died within the five years; therefore, the chance is represented by the fraction $\frac{41}{531}$, or the odds of 490 to 41. Again, if it were inquired what was the *probable duration* of any life—that is, the number of years that it is an *even chance* the life will survive—the answer is found with equal simplicity from the table. An even chance is represented by $\frac{1}{2}$—that is, the happening of one event out of two which are equally likely. Suppose the age of the life about which the question is asked is 58. At that age we find by the table that there are 262 alive, and we look down the table till we come to an age where that number is reduced to one-half or 131, and we find it at age 71. We can thus say that out of the 262 alive at age 58, 131 will reach age 71, and that the probability of a life aged 58 reaching 71—or living 13 years—is $\frac{131}{262}$, or $\frac{1}{2}$; or, in other words, that it is an *even chance*, and that therefore 13 years is the *probable duration* of a life aged 58. Simple though the table is, therefore, you will observe that, by its form alone, it enables one to solve all such problems as the foregoing *by mere inspection*. Again, numerous questions in political arithmetic, or of importance to the Friendly or Benefit Societies of the time, could have been solved very simply by it. Suppose, for instance, 1,000 persons, each with 1 child 1 year old, wished to combine for the purpose of providing £100 to every child on its attaining age 21, the table shows that out of 1,000 children 1 year old 592 will reach 21; therefore, such a sum would require to be contributed as would accumulate to £59,200 in 20 years, and $\frac{1}{1000}$th part of this would be the sum which each parent should subscribe at the outset. Or suppose a given number of marriages to be contracted between males aged 30 and females 25, and it is required to find the proportion remaining *undissolved by death* at the end of 15 years. Here, if the number of marriages be 531 (the number in table at age 30), we see at a glance that the number of *husbands* alive at 45 would be 397; and, out of the same number of wives alive at age 25, the number which may be expected to reach 40 is found by proportion to be 567:531::445=417. That is to say, at the end of 15 years there will be 397 husbands and 417 wives alive. But among these (assuming there to have been no re-marriages) there would manifestly be some widowers and some widows, while the question

was to find the number of *pairs* undissolved by death. Let us suppose, therefore, that the 397 pairs in which the *husband survives* are originally separated from the rest, and we find by proportion, as before, how many of the 397 wives will survive—thus 567:397::445:$\frac{445 \times 397}{567}$ = 311—therefore the number of *pairs* remaining undissolved by death, after 15 years, is shown by the table to be 311. This solution is merely given to show what could be done with a simple record of the number living at each age, such as Halley's table; and you will observe that in solving the question we have obtained several results which were not asked. For instance, it has been shown that 397 husbands and 417 wives will survive the period, while only 311 *wedded pairs* will remain. This is equivalent to saying that there will be 86 widowers and 106 widows at the end of the term; and, as there were 531 couples to start with, 134 husbands and 114 wives will have died in the interval; and in 28 cases *both* the husband and wife will have died.

So much for the table. The chief point of interest in the treatise is that it taught, for the first time in this country, what we have seen that De Wit showed 22 years before in Holland, viz., that the true value of a Life Annuity is to be found by taking the present value (from an interest table) of each yearly payment, which, according to the Mortality Table, *can ever be received*, and by reducing each of those by the respective chance (as shown by the Mortality Table) of the life surviving to receive it—the present value of the annuity being the sum of all these separate calculations. This is certainly the true theoretical value of a Life Annuity; but as the method is extremely laborious, even Halley himself had not perseverance enough to calculate a complete table; he did, however, make the calculations for every fifth age at one rate of interest (6 per cent.). He also investigated the chances of survivance, &c., among two or more lives, and showed that the same principles applied. He frankly owned that after repeated trials he had failed to devise any simpler method of calculating the single life annuities, and this was therefore left for later writers. Partly, no doubt, owing to this difficulty, but mainly, it would appear, from a prevailing notion that the results of mortality observations in Breslaw were quite inapplicable in London, this extremely valuable table and treatise gave very little real, or at least immediate, impetus to the science. Annuity Societies (including one announced as "The Lucky 70, or the Longest Livers take all"), and particularly the government, continued to grant annuities on ruinous terms, and for long without any restriction as to age—thus, in addition, laying themselves open to the full force of *selection* as well as fraud, till the day of reckoning came to them all.

It was in these days (24th January, 1705) that the first Life Assurance Company was established—the "Amicable Society for a Perpetual Assurance Office." It obtained a charter from Queen Anne in July, 1706, and it is interesting, from an Assurance point of view, to notice that, while the rise and spread of *Life Annuity* business in our country was mainly due to a desire on the part of investors to evade the Usury Laws, *Life Assurance* at once, or at a very early stage, struck a higher note. In the preamble to the charter the Society is said to be "a voluntary Society for the mutual benefit and interest of any person that shall at any time be a member thereof, in order to provide for their wives, children, and other relations, after a more easy, certain, and advantageous method than any that hath yet been thought of;" and the preamble goes on to say—"And it has been humbly certified unto us that their design will be of singular Use and Relief to many Families, by providing for great numbers of Widows and Orphans, who might probably be otherwise left wholly destitute of a maintenance by the sudden death of those on whom they depend."

The method referred to was as follows:—The number of members was not to exceed 2000. The subscription was to consist of an entrance fee of 5s., a contribution to the Society's stock of 5s., 1s. for the stamp, and a yearly payment of £6 4s. In return for this a certain sum was to be laid aside every year for division among the representatives of the members dying in that year; thus £2,000 was to be divided the first year, £4,000 the second, £6,000 the third, £8,000 the fourth, and £10,000 the fifth year, which sum was to be the maximum, unless increased by decision of a General Court of Members. The only restriction as to age was that *the lives must not be under 12 nor above 55.* There was no medical examination of lives, but there was a very definite method of selection adopted. Each proposer appeared at a meeting of the Board, and each Director was at liberty to ask him any questions he liked; the applicant then withdrew, and his eligibility was discussed! For the first few years, at any rate, the sum resulting to the representatives of each deceased member was

considerably under £100. It is thought that the founders of the Society believed that £100 would come to be the average share at death, and that they based their belief on the prevailing idea that the deaths in London were about 5 per cent, or 1 in 20, and on the fact that £5 per cent. was the usual charge at that time for a *one year's assurance.* Even supposing that £100 had been attained to as the average share at death, it is difficult when reading about the "more easy, certain, and advantageous method than hath yet been thought of" to prevent one's mind from reverting to the scientific table of Halley, published 13 years before, and from contrasting the clearness and soundness of the views expressed in his treatise with a system which charged upwards of £6 per cent. per annum for all lives, without distinction, between the ages of 12 and 55.

No further progress can be observed in the science till the year 1725, when Abraham de Moivre published his famous tract on the valuation of Annuities and Reversions. He had previously published an ingenious tract on the Doctrine of Chances, but his tract on Annuities is his great work. It is mainly famous for two things—(1) the propounding of what is known as De Moivre's Hypothesis, and (2) the pointing out of the method of deriving one annuity value from another. These may be taken as the first fruits of Halley's labours, as we shall see. De Moivre's famous hypothesis was suggested to him by a careful examination, year by year, of the deaths in Halley's table. There was a regularity about them which suggested the operation of some natural law, and in speculating upon this he made the wise and frequently-quoted remark, "We may consider that whatever be that law which is observed by nature in the perpetual decrements of human life, that law must, conformably to all the other laws of nature, be such as to proceed regularly, at least for some short intervals of time." In continuing his investigation into the subject, he considers 1st, What would be the result of an *hypothesis* which makes the probability of life decrease in *arithmetical progression*; 2nd, How far the calculations deduced from it agree with the tables; and, 3rd, What corrections are necessary to be made to it where it varies from the tables. The hypothesis which De Moivre evolved from this investigation was that the numbers expressing the living at the different ages in a Mortality Table are in arithmetical progression, *the annual decrements of life being equal*; or, that out of a given number of persons living at any age, an equal number will die every year until all are extinct. Applied to Halley's table the hypothesis meant that out of 86 persons born alive 1 would die each year till all were dead. This ingenious hypothesis and the theorem with which De Moivre accompanied it were extremely useful in the then condition of the science, as simplifying the various Life Contingency calculations to a very great degree; and the differences in the results obtained by his simple method and Halley's laborious calculations were really very slight, and certainly in these days of no moment, as the following annuity values (calculated at 5 per cent.) show:—

Age.	Halley.	De Moivre.
20	12·78	12·30
30	11·72	11·61
40	10·57	10·70
50	9·21	9·49
60	7·60	7·83
70	5·32	5·50

You will remember that Halley's method was to take each age by itself and calculate the value of *each annuity payment in succession*, summing the series to get the value of the whole annuity, and that he confessed to have failed to find any means of simplifying this labour. The second notable feature of De Moivre's work, however, is that he showed how, if the annuity value at any age is obtained, the value at the next younger age may be very simply deduced from it. Probably no single suggestion in the history of the science has been so eminently useful or so frequently employed as this. It may be explained simply in this way: the value of an annuity of £1 at a given age having been ascertained, it is manifest that the value of a similar annuity on a life a year younger will be the same a year hence, *after the younger life has lived a year and drawn a payment of £1 at the end of it;* the only difference between the two values, therefore, is that the ascertained value must be increased by unity and discounted for one year, both as regards *interest* and the *probability of survivance.* Hence, from an ascertained annuity value, to obtain the value at the next younger age, the rule is: add 1 to the ascertained value, and multiply by the present value of £1 due a year hence, and by the probability of the younger life surviving a year.

In the same year (1725) a Mr. Richard Hayes, "Teacher of

Merchants' Accounts," published a work, which is interesting, as indicating the slow advance towards an appreciation of what is so common in our days that it is hard to imagine a time when it could not be comprehended, viz., the assurance of a sum at death. He suggests that a man might make provision for those dependent on him by *selling the "surplusage of his income"* as an annuity on his life, and by investing the price at compound interest,—rather a more roundabout process than simply paying the said surplusage to an Assurance Company and taking a policy for the amount it will assure! This work also contained the most complete Table of Annuity Values published up to that time, viz., from age 30 to age 73 inclusive.*

The year 1737 has some interest for students of Life Contingencies, inasmuch as in that year a Mr. Weyman Lee published an essay, with the view of proving that a *Life Annuity* ought to be exactly equal to an *Annuity Certain for the number of years the life has an equal chance of living* according to the tables; and, finding that the calculations of Halley and De Moivre did not agree with his dictum, he at once assumed that *they* were wrong, and devoted 340 8vo pages to proving it, but did not come much speed. It used to be a frequent question in examination papers to prove the fallacy of this somewhat plausible theory, and this might form an interesting exercise for those of our Members who are beginning their Actuarial studies.

From this time forward the science advanced in strides. Mortality Tables and Treatises on the subject began to multiply both in England and on the Continent. The London Bills of Mortality were at last in such a form† that they could be used for the purpose, and they were frequently turned to good account. It will be impossible within the limits of this paper to do more now than briefly chronicle the leading names and events in this progress. The foremost of the names we next come to, and one of the most prominent of all names in the progress of the Science of Life Contingencies, is that of *Thomas Simpson.* His great work, *The*

* It was not till some years later that the *first complete Table of Annuity Values* was published. It appeared in the 2nd edition of De Moivre's *Doctrine of Chances*, 1738.

† For which we are indebted to a suggestion of John Smart (author of the famous Interest Tables), about the year 1728.

Doctrine of Annuities and Reversions, &c., published in 1742, was a great advance on anything that had preceded it. It included a Table of Mortality based on the London observations, and four others calculated from it, and treated of the whole subject, not only with great clearness and ingenuity, but in a manner so general that all his solutions, formulas, and rules were, to use his own language, "universal according to any table of observations or degree of probability of life whatsoever."

In 1746 Antoine Deparcieux published his *Essai sûr les Probabilités de la Durée de la Vie Humaine*, containing twenty-two tables, of which Milne said, in 1815, he considered them "some of the most valuable that have ever been published, and they must have cost him great labour." There is no doubt this work did more than any other of its time to advance the Science of Life Contingencies on the Continent; and to Deparcieux is due the credit of first demonstrating, by an actual table, the superiority of female life.

In 1755 was published the 3rd edition of Dodson's *Mathematical Repository*, which is noteworthy as being the first book which distinctly treated of *Life Assurance*. Seven years later the *Equitable Society* was established—the first Life Assurance Company conducted on the modern system; and in 1771 we reach the illustrious name of Dr. Price. He had been heard of some years previously, along with one William Dale, gallantly attacking certain annuity societies which were based on wrong principles, and in 1771 he published the first edition of his famous *Observations on Reversionary Payments, &c.*, which contained the *Northampton Table*. On the construction and history of the Northampton Table I shall not say anything at present. You are probably aware that 40 or 50 years ago nearly every Life Assurance Company in the Kingdom based its rates on this table, while to-day it is almost universally discarded.

In 1778 Mr. Charles Brand, registrar of the *Amicable Society*, published the experience of that institution for upwards of 72 years. His table possesses considerable historical interest as being the first table compiled from the experience of *assured lives* —the earnest of our most valuable tables of the present day. Mr. Morgan, of the *Equitable Society*, and Dr. Price did good service in advancing the science about this time; the latter gentleman, in the 4th edition of his work on Reversionary Payment, &c. (1783), publishing the first set of tables of money values

deduced from the Northampton Table, also a table based on Swedish returns, in which *female* mortality was distinguished from that of males.

At this point it is proper to say something of one of the most important improvements which ever took place in the system of Life Contingency calculations, viz., the invention of what is variously known as the *Columnar Method*, the *Commutation Method*, and the D and N system. I do not propose at this time to give any description of the columnar method further than to say that it consists of a combination of an interest table and a mortality table in such a way as immensely to facilitate almost every form of actuarial calculation. Of the great improvement which this invention introduced Professor de Morgan says:—"In the old method we are presented with a table of the value of annuities at all ages, which of themselves are rarely wanted, but from which, by operations more or less complex, the values of benefits of all other kinds may be computed. In the new method, on the other hand, we are presented with a table which by the mere inspection tells us nothing, but from which, while the values of the ordinary benefits can be found by a simple division, those of benefits of the the most complex description are found by operations usually consisting of nothing more than one or two subtractions and one division. In point of simplicity, moreover, in the deduction of the various formulæ, the methods admit of no comparison."

In tracing out the actual inventor of this system it is almost necessary to go backwards! For a considerable time the method was spoken and written of here as *Mr. Davies' Method*, owing to the great improvements made by him on the original idea of George Barrett, then thought to have been the inventor. Barrett, after twenty-five years' work at his tables on this plan, failed to get them published on their completion in 1811. Prior to this, Duvillard (in 1806) used the columnar method in his French table, and probably invented it for himself. But even supposing that Barrett had perfected his invention at the *commencement* of his twenty-five years' labours in 1786, it has been clearly proved by Mr. Hendriks that this is not sufficient to secure the honour of the invention to Barrett and to England; for in 1785 Professor Tetens, of Kiel, published the method in a much more complete form. Again, it has been shown that Mr. Morgan used a columnar method in his work published in 1779, but this is to some extent qualified by the fact that he appears to have been almost entirely ignorant of the value and uses of his discovery. Finally, however, the invention appears to have been traced to William Dale, an Englishman of humble origin, whose work, published in 1772, has been already alluded to, but is little known.

After Francis Baily had given to the world his great work on "The Doctrine of Life Annuities and Assurances," in 1810—which is perhaps, still, more of the nature of a standard work than any other—it must have seemed that there was little to improve in the Science of Life Contingencies; and yet there was room, five years later, for the famous treatise of Milne, in which he published the renowned *Carlisle Table*—a table which has been more extensively used, and on which there has been a greater mass of monetary and other tables based than on any other, and which is only now tardily giving place to the great table of to-day. A valuable work by Griffith Davies appeared in 1825, accompanied by a Table of Mortality, deduced from the experience of the "Equitable" Office, which was at one time a good deal used.

It may be appropriate to mention that, in 1829, Mr. James J. Duncan, Manager of the West of Scotland Life Insurance Company, published an interesting set of tables of "The probability and expectation of Male and Female Life in Glasgow, and of the value of Annuities on single lives at all ages," deduced from the Glasgow Population and Mortality Bills, on an average of six years, 1821 to 1827. The value of these tables was somewhat impaired through the estimate of the population on which they were based being merely approximate; they were, however, highly thought of at the time, and appear to have been constructed with great care.

About this time what is known as "Gompertz' Law of Mortality" began to attract much attention, and it was at one time believed that it would revolutionise the entire computation of Life Contingencies—as far at least as regarded those depending on more than one life. At that time, and even more distinctly a few years further on (1840), when the well-known work of David Jones was published by the "Society for the Diffusion of Useful Knowledge," it began to be felt that, while with immense labour tables for all the combinations of *two* joint lives might be formed, it was quite out of the question to look forward to the construction in the ordinary way of complete tables for *three* or more lives, so that recourse to more or less rough approximations was necessary

in almost every case. But Gompertz' Law, and the labours of Messrs. Woolhouse, Sprague, Makeham, &c., made it possible to perform calculations which involve any number of lives, with beautiful simplicity and a very near approach to absolute accuracy. Mr. Gompertz' Law is contained in three papers contributed by him to the Transactions of the Royal Society. The first of these was read to the Society in the year 1820, and in it he pointed out that if, in a Mortality Table, we make the intervals of age very small we shall get a series which is, for a short period of time, very nearly, if not actually, in geometrical progression. In his own words, "we may take an interval *m* from *n* so small, whatever be the real constitution of the function, that the number of the living during that interval shall decrease so as to form a geometrical progression very nearly, whilst the portions of time increase in an arithmetical progression, and that the decrements of living are also in consequence very nearly in geometrical progression; and we may, moreover, . . . accommodate the function so as to be accurate at the two extremes." In his second paper, which was published in 1825, he continued his investigations into the subject, and showed that the law which he had previously proved to hold for short periods of time was likewise true for equal intervals of long periods. "This law of geometrical progression," he says, "pervades, in an approximate degree, large portions of different Tables of Mortality, during which portions the number of persons living at a series of ages in arithmetical progression will be nearly in geometrical progression." He then suggests with great ingenuity that death may be the result of *two co-existing causes*—the one chance, which may be considered constant at all ages—the other a continually increasing deterioration or loss of power to avoid destruction. He shows that if the causes of death were restricted to those which affect old and young alike, the deaths among the young and old would be in the exact proportion of the young to the old, and that the number of living and dying (from a certain number alive at a given age) would decrease in geometrical progression. But that, if mankind be continually acquiring an increased liability to death—"which," he adds, "appears not to be an unlikely supposition with respect to a great part of life (though the contrary appears to take place at certain periods)"—then the number living would decrease in a greater ratio than the geometrical progression. If this were absolutely true for all periods of life, the Law of Mortality might be expressed by a mathematical formula, the simple expansion of which would give a perfect Mortality Table. The difficulty, however, in applying the formula to existing tables lay in the fact that experience showed a uniform ratio did not prevail throughout life; and it was to adapt his formula to the variations indicated by Tables based on actual observations, that Mr. Gompertz mainly addressed himself in his third paper, which he read to the Royal Society in 1861. To this third paper he added a short supplement in 1864, which, however, was not published. Of the formula by which the law is expressed, and the contrivances by which it is adapted to Mortality Tables it will be out of place to refer in a paper like the present. It may be mentioned, however, that according to Mr. Makeham, who suggested a modification of Gompertz' formula, the probabilities of living a year, increased (or diminished) in a constant ratio, form a series whose *logarithms* are in geometrical progression.

This brings us to about the date of the establishment of the Institute of Actuaries and of the *Assurance Magazine*, and to the labours of many men still among us. It is not the place, at the conclusion of a somewhat lengthy paper, to refer to the brilliant achievements, in the way of the Science of Life Contingencies, of the past 30 years, or to the enormous number of auxiliary tables produced in these years which have so much contributed to lighten the labours of the Actuary. I shall only allude to that most ingenious little book, published in 1850 by William Orchard, which enables one to pass *by inspection* from the value of an *Annuity* on any status to the value of an *Assurance* on the same status—whether the status be the duration of one or of several lives—whether for the full term or for a short period—indeed, any kind of status except that of Contingent Annuities. The amount of labour which has been saved by the ingenuity of this clever young Actuary (who died much too soon for the good of the profession) is incalculable, and should stimulate us to further efforts for the simplification of labour. We are apt to consider that the Science of Life Contingencies has now attained to perfection, and that nothing remains for us but thankfully to reap the fruit of the labours of others. It is, however, the province of such a Society as ours to show by its works that this is not the case, and that however near the Science may be to perfection there are still fresh developments of it, and fresh applications of it, and even additional improvements of it which can yet be achieved by the

energy and perseverance and single-mindedness of those who labour to that end.

To complete my sketch I must very briefly refer to the five great Mortality Tables which have been constructed during the past 40 years. In 1843 the first English Life Table was published, based on the Registrar-General's Returns for the whole country. In the same year a table was published, based on the combined experience of 17 Assurance Companies. In 1853 and 1864 the English Life Tables Nos. II. and III. were published; and finally, in 1869, the "Experience of 20 Offices," compiled under the care of the Institute of Actuaries, and certainly the best table extant for many purposes. All these tables were based, not on records of death alone, but on a comparison of the ascertained number living at each age with the corresponding deaths—a method altogether superior to the old.

We have now sketched, very hurriedly and imperfectly I am afraid, the main features in the history of the Science of Life Contingencies. As far as regards Mortality Tables, we have at least seen that there are several ways in which it is possible to construct them. We have seen, for example, that they may be constructed by something very nearly akin to guesswork, and we have noted that the results of such a method are distinctly not to be relied on! We have seen that they may be framed by the development of certain hypotheses or assumed laws suggested by a consideration of more or less perfect observations, and we have noticed specimens of these in the results based on De Wit's "Presuppositions," De Moivre's "Hypothesis," and Gompertz' "Law." We have also seen that tables may be framed from records of death alone, where the ages at death are given, and that numerous tables, from Halley's Breslaw Table onwards, have been constructed in that way. And, finally, we have seen that tables may be framed, and are best framed, from records of the numbers alive at each age contrasted with the corresponding deaths, and that such a method has been employed in framing the English Life Tables, as well as the Tables of the Institute of Actuaries. The former, based on the Census Returns and Death Registers of the entire Kingdom, possess the distinct advantage of an ample number of observations on which the average is based, while they have the drawback that great accuracy in such vast returns is not to be looked for. On the other hand, the Institute Tables, though the number of observations (especially at the extreme ages) is comparatively small, have the advantage of almost perfect accuracy in detail.

I have already detained you much longer than I had any intention of doing, and I shall merely add that, before handing my paper to the Secretary, in terms of our Rules, I purpose adding a short description of each of the principal Life Tables, as well as references to the best sources of information, in the hope of making it useful to some of the younger students.

APPENDIX.

De Wit's Presuppositions (described at pages 42-45).—Although De Wit did not construct any Mortality Table from his Presuppositions, they are quite capable of being thrown into this form. For the sake of comparison this has been roughly done in Table I. of this Appendix. A complete translation of De Wit's Treatise, with his calculations, is given in Mr. Hendrik's valuable paper in vol. ii. of the *Assurance Magazine.*

Breslaw Table.—This has been pretty fully described already (pages 47-53), but those who desire further information on the subject will find Halley's "Estimate of the Degrees of the Mortality of Mankind" reprinted in full in the 18th vol. of the *Assurance Magazine.* In the 6th Report of the Registrar-General also, there are some valuable remarks on the table by Dr. Farr, who there *reconstructs* it on a different plan—taking the *number born alive* as the basis, and deducing the number who *complete* each subsequent year, instead of starting as Halley did with the number *living in their first year* and deducing the number living in their second, third, &c., year (see note to page 49). Mr. E. J. Farren also devotes an interesting paper to this table alone, in vol. i. of the *Assurance Magazine.*

Deparcieux's Tables.—These were framed in 1742, and published along with the author's essay in 1746 (see page 58). The table which usually goes by his name was based on the lists of nominees in the French Tontines of 1689, 1696, and 1734, containing quite a sufficient number of observations to secure trustworthy results. The table, however, cannot be looked upon as a fair measure of French mortality. Deparcieux's work also contained tables compiled from the records of certain monasteries and convents in Paris. For further information see Milne's *Treatise on Annuities, &c.,* also his article in the *Encyclopædia Britannica.*

Northampton Table.—This was constructed by Dr. Price, his data being the registers kept in the parish of All Saints (which constituted the greater part of the town of Northampton) for the 46 years, 1735-80. In the first edition of his *Observations on Reversionary Payments, &c.,* published in 1771, and also in the second and third editions, Dr. Price gave Tables of Mortality constructed from the Northampton bills. But it was in the *fourth* edition (1783) that what we now call the Northampton Table was first published, Dr. Price saying in his preface of it that it was "now given more correctly." The Northampton Table is the most important of all those which have been based upon a record of deaths alone; but owing to certain faults in its construction, and mainly to the assumption of a stationary population having turned out to be inconsistent with the facts, the Mortality it shows is too great, and the Expectation of Life too small, at the early ages, while at the later ages the converse is the case. Dr. Farr, in the 8th Report of the Registrar-General, goes somewhat fully into the basis on which the table was constructed, and, having proved the assumption of a stationary population to have been incorrect, he designates Dr. Price's Table *the false Northampton Table*, and proceeds to calculate a *true Northampton Table* from corrected data. His remarks are well worth perusal. The student should also study Mr. Sutton's able and exhaustive paper on the subject in the 18th vol. of the *Assurance Magazine*, and the discussion which followed the reading of it.

Carlisle Table.—This was the first important table constructed on the modern principle of comparing the deaths at each age with the number of lives out of which they occurred. It was constructed by Mr. Joshua Milne, and appeared in his *Treatise on the Valuation of Annuities, &c.*, in 1815. The materials on which it was based were collected by Dr. John Heysham, and originally published by him in 1797 in a quarto tract entitled "An Abridgement of Observations on the Bills of Mortality in Carlisle from the year 1779-1787 inclusive," and also "A Catalogue of Cumberland Animals, by John Heysham, M.D." These materials comprised (1) two enumerations of the population of St. Mary's and St. Cuthbert's (two parishes in Carlisle) of date January, 1780, and December, 1787, the numbers in 1780 being 7677, and in 1787, 8677; and (2) the Abridged Bills of Mortality of the same parishes for the nine years, 1779-1787, the total number of deaths recorded being 1840. Great care was expended on the construction of the table, with the result that, although the data were so limited, it fairly represents the general mortality of the country, and has been more extensively employed than any other. Its chief defect is in its faulty graduation, which shows anomalous results at certain ages, and renders it unsuitable for short period or contingent assurances. There are also objections to it arising from the peculiar constitution of the population on which it was based. At the enumeration in 1787 the *females* exceeded the *males* in the abnormal proportion of 4813 to 3864; the population is also known to have been abnormally increased at the younger ages (20-30) by immigration. The effects of these peculiarities are traceable in the resulting mortality, and detract from the value of the table as applicable to a normal population or to assured life. For further particulars see Milne's remarks (article 704), and the Registrar-General's 12th Report. See also

the Report of a Parliamentary Committee on Assurance Companies (1853), and the various Encyclopædia articles.

Equitable Experience.—Though not by any means the earliest, the most important of the tables constructed from the experience of a single Assurance Company have been those founded on the experience of the *Equitable Society*, and two of these in particular have been a good deal used by other offices in their periodical valuations.

The first of these was published in 1825, and is known as *Davies' Equitable Experience*, having been constructed by Mr. Griffith Davies from the accounts annually given to the Members of the Society by their Actuary, Mr. Wm. Morgan. These accounts stated the *ratio* which the death-rates, experienced at different ages by the Society, bore to those indicated by Halley's *Breslaw Table*, the *Northampton Table*, and also to a table of *Observations for London*. Although *nominally* based on the experience of the Equitable Society, it can only now be looked upon (owing to various erroneous assumptions) as a hypothetical table.

The other table referred to was published in 1834. It was constructed by Mr. Arthur Morgan, then Actuary of the Society, from actual statistics of the office, from its commencement (1762) to the year 1829—the total number of the lives observed upon being 21,398, and the deaths 5,144.

Seventeen Offices' Experience.—Not long after the publication of the last-mentioned table, it became apparent that much greater value would attach to the *combined experience* of several Assurance Companies; and, Seventeen Offices having agreed to contribute their experience (58 offices uniting to defray the expense), a table was constructed under the superintendence of a Committee of London Actuaries, and published in 1843. The material consisted of 83,905 policies—of which 44,877 were in force, 25,247 had been discontinued, and 13,781 had failed by death. In constructing the table it was decided by the Committee to count each *policy* as a *life*, notwithstanding that many persons might have *several* Assurances on their lives, in one or more of the Seventeen Offices. It was believed to be impossible to eliminate all the duplicates, and that, in any case, the duplication would not sensibly affect the results. (It has now been ascertained that the Committee were correct in assuming that the duplication would have no material effect on the results.) Two interesting points were brought out by the tables—(1) that *Irish* lives showed a much heavier mortality than the average; and (2) that female *insured life* is considerably worse than that of males, in spite of the well-known superiority of female lives in connection with Tontines and Annuity Societies.

In the same year (1843) Mr. Jenkyn Jones published a Series of Tables of Annuities and Assurances based on the new Experience Table.

English Life Tables.—As already mentioned, three of these have been constructed at intervals by Dr. Farr from the Registrar-General's Returns for England and Wales. It is hardly necessary to say more of these here, than that the *First* is to be found in the 5th Report of the Registrar-General, and is based on the Census Returns of 1841 and the Deaths of that year; the *Second* was published in the 12th Report of the Registrar-General, and is based on the same Census Return (1841) and on the Deaths for the 7 years 1838-1844; and the *Third* was published as a separate work in 1864, and is based on *two* Census Returns (1841 and 1851) and the Deaths for the 17 years 1838-1854. It may be interesting to mention that this last table, and an enormous mass of tables deduced from it, were constructed by the aid of Scheutz's Calculating Machine, which not only performed the mechanical work of the calculations but also set up the tables in type and printed them off. The results of the three tables agree very closely with each other.

Institute of Actuaries' Tables.—In 1862 a further movement of the different offices took place (under the auspices of the Institute of Actuaries, and Committees of the Faculty of Actuaries and of the Managers of the Associated Scotch Offices) to combine their experience for the formation of a fresh Table of Mortality. Ten English and ten Scotch Offices contributed the following material—88,329 lives on the books, 26,721 deaths, and 45,376 discontinuances—the total lives observed upon being 160,426. All duplicate policies were eliminated, and the classification of the lives was so carefully carried out that the result has been a series of the most interesting and valuable Tables of Mortality in existence, embracing tables of male, female, healthy, and diseased lives. It is unnecessary here to refer to them more in detail. The mortality experience was published in 1869, but it was not till 1872 that the *Institute of Actuaries' Life Tables* proper appeared. The results of the tables substantially agree with those of the "17 offices' experience." The volume published in 1872 contains an important Article on the Graduation of the Table by Mr. Woolhouse; it also contains a system of notation recommended by the Institute of Actuaries, with which no Actuary can afford to be unacquainted, as it is almost universally adopted in all modern Actuarial literature.

The following tables afford a rough means of comparing many of the tables referred to, and it may be mentioned that, in the 18th volume of the *Assurance Magazine* there is a paper by Mr. James Valentine, on "A Comparison of Reserves brought out by the use of different data," in which some of the Tables I have described are more thoroughly contrasted:—

TABLE I.

Showing the number Alive at the end of each Tenth Year, up to age 90, out of 1000 Lives, according to various Estimates of Mortality.

Age.	De Wit (1671).	Breslaw (1693).	De Moivre (1725).	Deparcieux (1746).	Northampton (1771).	Carlisle (1815).	Equitable (Davies) 1825.	Glasgow (Males) 1829.	Equitable (Morgan) 1834.	17 Offices' Experience (1843).	Eng. No. 3 (Males) 1864.	Institute Hm. 1869.
10	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
20	824	905	868	925	904	943	951	928	928	933	943	962
30	649	803	737	834	773	873	879	812	861	863	863	899
40	474	673	605	747	641	786	786	704	784	787	771	823
50	298	523	474	660	503	681	681	584	692	695	661	727
60	164	366	342	526	359	564	536	422	559	560	517	589
70	67	215	211	352	217	372	361	223	360	358	324	381
80	00	62	79	134	83	148	169	51	140	133	116	139
90	00	...	00	125	8	22	23	3	13	13	14	15

TABLE II.

Showing the complete "Expectation of Life" at decennial ages, according to various Estimates.

Age.	Ulpian's Estimate (Circ. A.D. 250).	Breslaw (1693).	De Moivre (1725).	Deparcieux (1746).	Northampton (1771).	Carlisle (1815).	Equitable (1825).	Glasgow (1829).	Equitable (1834).	17 Offices (1843).	Eng. No. 3 (1864).	Institute (1869).
10	30·00	40·50	38·00	46·83	39·78	48·82	48·83	42·27	48·32	48·36	47·05	50·29
20	30·00	34·26	33·00	40·25	33·43	41·46	41·06	35·13	41·37	41·49	39·48	42·06
30	25·00	27·93	28·00	34·08	28·27	34·34	33.98	29·40	34·53	34·43	32·76	34·68
40	20·00	22·33	20·00	27·50	23·08	27·61	27·40	23·16	27·40	27·28	26·06	27·40
50	10·00	17·03	18·00	20·42	17·99	21·11	20·83	16·86	20·36	20·18	19·54	20·31
60	7·00	12·23	13·00	14·25	13·21	14·34	15·06	11·29	13·91	13·77	13·53	13·83
70	5·00	7·35	8·00	8·67	8·60	9·18	9·84	6·75	8·70	8·54	8·45	8·50
80	...	3·72	3·00	4·67	4·75	5·51	5·38	4·16	4·75	4·78	4·93	4·72
90	...	...	...	1·75	2·41	3·28	2·65	1·50	2·56	2·11	2·84	2·36

With regard to Table II., it may be explained that the complete "Expectation of Life" (or more correctly, the *Mean after lifetime*) is obtained by taking the sum of the numbers shown by the table to *complete each year* after the given age, and dividing by the number alive at the given age, adding ·5 to the result;—that is to say, taking the whole number of years afterwards to be completed by the persons alive at the given age, and dividing by the number alive at the given age, to ascertain the average share to each, ·5 being added because each of the persons, *besides completing a certain number of years*, may be expected to live a fraction of a year more—some living a very small fraction more, and others very nearly completing another year,—the result on the average being represented by the addition of one half-year (·5) to the "Expectation." The same result might be obtained from the *Deaths* of each year—persons dying in the *first* year (after the given age) being assumed to have lived on the average *half-a-year* each, those dying in the *second* year to have lived $1\frac{1}{2}$ years each, those dying in the *third* year to have lived $2\frac{1}{2}$ years each, and so on. The sum of the years thus obtained, divided by the number alive at the given age, being the "Complete Expectation" as before. When the additional half-year is excluded from the "Expectation," the result is called the *Curtate* "Expectation of Life."