THE SOLVENCY OF GENERAL INSURANCE COMPANIES REVISITED

Following the GISG Seminar at Stratford in November 1982, a Working Party on Solvency was established with the following terms of reference:

(a) to review the lessons to be learnt from the Finnish report on Solvency of Insurers and Equalization Reserves, and to suggest specific investigations which might be carried out in the UK in order to develop the Finnish work.

(b) to consider the extent to which the variability of a company's results should be reflected in the methods and bases used for the valuation of the assets and liabilities.

An initial report was presented to the Bristol Seminar of the GISG in November 1983, and that report was subsequently reworked into a paper which was presented to the Institute of Actuaries on 27 February 1984[1]. However, it was clear that the Working Party had done little more than describe the problems and throw out a few possible ideas for their solution. In particular, no new work had been carried out along the lines of the Finnish report, and it was felt desirable that the Working Party should continue its studies. With a changed membership, the Working Party has continued to meet and is now able to report on further progress.

With the invaluable assistance of Geoffrey Bernstein of the Centre for Research in Insurance and Investment at The City University, the Working Party has established a simulation model for investigating the behaviour of the run-off of an insurance portfolio under various conditions. Initial work has been confined to the study of the run-off of the assets and liabilities of a closed portfolio, but the model is in the course of being extended to incorporate the possibility of continuing new business,in the first instance for a year or 18 months, but possibly eventually for longer periods, bringing our work closer to the work described by the Finns[2].

The simulation model has been described in a paper[3] which two of the members of the Working Party are presenting to the ASTIN Colloquium in Biarritz in early October.The main features of the model are summarized in Appendix 1 to this paper, whilst Appendix 2 sets out some of the initial results which were presented in the ASTIN paper. Appendix 3 takes this a stage further and shows the results in the form of the required solvency margins to produce given probabilities of ruin under specified simulation conditions.

Whilst the simulation model provides a new and valuable tool for examining the behaviour of an insurance portfolio with due regard to the stochastic nature of both asset and liability values, it has to be remembered that a simulation will give results that are wholly dependent on the models of variability adopted and the parameters selected. If the conclusions resulting from such an exercise are to attract general acceptance, it will need to be demonstrated that they are broadly robust and do not depend too critically on the particular models which are chosen. The Working Party is expecting to devote more attention in the months ahead to testing the sensitivity of the results to the assets and liabilities models adopted and the various parameters involved. In the meantime, the results should be taken as giving only a first indication of the solvency margins required under certain circumstances to achieve what might be described as acceptable probabilities of ruin.

What has emerged from the simulation exercise is something which will be of no surprise to general insurance practitioners: the solvency margin which is required depends on the precise nature of the liabilities and also on the type of investments held. Leaving aside the rather more extreme cases investigated, the results point towards a solvency margin of at least 30% of outstanding claims provisions in order to achieve a probability of ruin of around 1 per 1000, given a normal mixture of liabilities and a not too extreme investment policy. It is furthermore evident that a solvency margin of 10% or less of outstanding claims provisions produces a risk of insolvency which might be considered unacceptably high on a run-off basis.

However, a major philosophical difficulty is that the results of a simulation exercise depend critically on assumptions about the position from which the company starts. It might be regarded as unreasonably stringent to require solvency margins to be established which imply the possibility of substantial falls in asset values at a time when, for example, equity prices are already severely depressed. Can we really envisage that in a 1974 situation, solvency margins would have to be established to take into account the possibility of a further fall of similar proportions? But who is to say whether the market has reached a trough, or is still on the way down? For example, at the end of 1973 the F.T.500 share index had fallen some 35% from its high point in the summer of 1972, already a substantial fall. Many people would have assumed then that the market was close to a trough. In fact the index fell by more than 50% in 1974.

Similarly with the liabilities, much depends upon the strength of the provisions held to meet outstanding claims. Additional strength in the reserving basis is equivalent to an implicit solvency margin, whilst inadequate provisions could mean that the solvency margin that is shown is in reality far less of a buffer against adverse circumstances than it might appear. As part of the simulation exercise, we examined solvency in the context of a range of assumptions which might be used to set up provisions for outstanding claims, including our standard basis, which employed an assumption of 5% a year inflation and no discounting. The initial solvency margin was set at 15% of whatever the outstanding claims provisions were in each case. However, the effect of the different reserving bases is to give very different levels of total initial assets, to meet the same underlying liabilities. The excess of the total initial assets for each case over the liabilities calculated on the standard reserving basis can be expressed in terms of an equivalent solvency margin on the standard reserving basis as follows:

Reserving Basis	Effective solvency margin related to liabilities on standard basis
Inflation 0%; no discounting	2%
Inflation 10%; discount rate 7%	9%
Inflation 5%; no discounting (standard)	15%
Inflation 10%; no discounting	48%

There is no way of establishing with any degree of precision whether the provisions of an insurer are on a cautious basis or not. Even the insurer may not know. We can examine whether provisions set up in the past have subsequently proved to be adequate or not, but we cannot be sure that the conclusions have any relevance in relation to the provisions now held in respect of outstanding claims. Various methods of analysing triangles of claims paid may be used to deduce whether provisions appear weak or strong, but all such methods have their shortcomings, and none could reasonably be regarded as sufficiently reliable for use in setting a solvency standard, even if they are of some assistance in identifying weak companies.

In seeking to point the way to more appropriate control methods, we find ourselves increasingly driven back to the need for each company to have a professional person with responsibility for establishing proper provisions and for monitoring the adequacy of the company's free reserves. Whilst this would place the supervisory authorities in a position of heavy reliance on this person for ensuring the continued solvency of the company, such reliance already exists in the case of certification of returns by the auditors and, in the case of long-term business, valuation of the liabilities and calculation of solvency margin by the appointed actuary.

BREAK UP OR GOING CONCERN?

The Nature of Solvency

Traditionally the solvency of a general insurance company is demonstrated at a point of time by the position shown in the balance sheet. An excess of assets over liabilities is taken to mean that the company is solvent. However, the balance sheet represents no more than an opinion about the financial position of the company, since there is considerable uncertainty about the true amount of the liabilities and the realizable value of the assets. There is no single correct value that can be ascribed to either assets or liabilities and different values may be appropriate according to one's perspective. In reality it will not usually be possible to make an unequivocal statement about a company's solvency, but only to express a view as to the likelihood or otherwise that the assets will prove adequate to meet the liabilities. A balance sheet which shows a solvent position should reflect an expectation of adequacy, but it may, either deliberately or inadvertently, present a misleading picture, and it does not give any idea of the (almost certainly non zero) probability that the assets may be inadequate to meet the liabilities.

In most countries a general insurance company is only permitted to carry on writing business if it has some specified excess of the value of the assets over the liabilities. The object of this solvency margin is two-fold. It reduces the probability that the assets will prove inadequate to meet the liabilities and it provides a buffer against further deterioration in a company's financial position which can occur in the period before its authorization to write new business can be withdrawn.

Although such a margin is not required of other trading companies, this can be said to reflect not only the nature of the business, but also the extent of the insured's interest in the continued viability of the company. In many cases the insured can be exposed to quite serious liabilities in the event of the insurer failing to meet a claim. He cannot limit his liability in the way that he can with a trading company.

In practice, therefore, a company can only carry on writing business if the supervisory authority says that it is solvent and the way in which they may define solvency for this purpose may differ from the criterion which might by used by a Court in determining whether a company should be wound up. Indeed, it is relatively rare for insurance companies to be wound up by the Courts, a more normal procedure being for the existing business to be run off to extinction, or be transferred to another company.

This concern with safeguarding the position should a company cease trading is peculiarly the preserve of the supervisory authority. The objectives of the management of an insurance company will normally be to ensure that it does not have to cease trading. This leads to a theoretical distinction between what a company should show in its accounts, prepared as is the normal convention, on a going concern basis, and what it should show in its returns to the supervisory authority, designed to present the financial position of the company on a break up basis. By 'break up' in this context is meant a test assuming no further business is written, but existing business is run off to extinction. This clearly differs from a winding-up basis in which the assets are divided up on the basis of an assessment of the liabilities.

It may, of course, be questioned whether it is appropriate to test whether an insurer may be permitted to continue in business by examining his ability to do just the opposite. The principle is that a company clearly ought to be able to meet the liabilities in respect of the business it has already taken on before it takes on any further liabilities, particularly since the business of taking on further risks may need additional capital to finance strains resulting from the expenses of running a full commercial operation and the need to make prudent provision for possible liabilities arising.

On the other hand, continuing to write business may enable the costs of meeting the claims in respect of the liabilities to date to be spread more widely, suggesting that the liabilities could be run off for a lesser sum with the company on a going concern basis than on a break up basis. Even more questionable is the suggestion implicit in the current format of presentation of returns to the supervisory authority as at a particular date that the then current market value of the investments has any direct relevance to the solvency of the company even in the context of the break up basis. As indicated above, a balance sheet statement of the financial position of the company has severe limitations because of the difficulty of placing values on the assets and liabilities. What is important is whether the proceeds of the assets, both capital and income, will prove sufficient to meet all the liabilities as they emerge.

A change in philosophy which we would like to see, therefore, would be to seek to relate solvency to the ability of the assets to meet the liabilities, rather than to their strict arithmetical relationship on particular definitions (usually unspecified in the case of liabilities) at a point in time. It would follow from this that a simulated run-off could be a helpful way of looking at the problem. It is not without its own difficulties, since one has to form a view as to whether the value of the investments is more likely to rise than fall, or vice versa, and assumptions need to be made in a number of other areas. However, it would focus attention on the true uncertainties and the parameters which are important in establishing whether the company is solvent or not, rather than on a purely notional comparison between an assessed value of the liabilities and a value of the assets which may be of little more than academic interest in relation to their value when they have to be realized to meet the liabilities. In 1975 the supervisory authorities were able to have regard to the recovery of market values which had taken place subsequently when looking at the technical solvency position on a market value break up basis as at 31 December 1974. If large numbers of companies were to appear technically insolvent on a particular date because of a dramatic fall in the market, it would raise the question as to whether it would be politically possible to require a substantial segment of the market to stop writing business.

Some would also call into question the concept of adequacy adopted by the supervisory authorities. From their point of view, outstanding claims provisions should be sufficient to enable all claims to be met with a reasonably high degree of probability. Implicit in this would be that on average adequate provisions would be expected to give rise to a surplus of assets once the liabilities had run off. If this were always so, successful supervisory policy would ensure that authorization was withdrawn from companies sufficiently early for them to be able to run off with an ultimate excess of assets over liabilities, ie before they ever became insolvent in any absolute sense. This is not likely to be considered satisfactory by the shareholders, although they clearly have the option, before the supervisor withdraws authorization, of putting additional capital into the company to ensure that it is able to continue in business. Some may regard this system as being rather loaded against the shareholders, but in terms of providing adequate protection for policy holders it is difficult to see what alternative there is.

One possible conclusion would be that it would normally be better to allow an insurer to continue in business so long as he can reasonably be expected to trade at a profit. It may be argued that if the insurer has got himself into a position where his solvency is in doubt, it is probably a result of trading at a loss, so that there is a presumption that he may not be able to mend his ways. Unfortunately, the unprofitability might well be hidden in the reporting of the results in such a case. But much may also depend on what has been happening in the market generally. In practice the supervisory authorities work with a two stage solvency margin trigger provided by the EEC solvency margin and guarantee fund. If an insurer fails to maintain its solvency margin it has to provide a plan for the restoration of a sound financial position, which may include demonstration that on a properly drawn up business plan, and with realistic assumptions about profitability, the solvency margin will be restored within a reasonably short space of time. Only if the company fails to maintain the guarantee fund, set at one third of the solvency margin, with a specified minimum in absolute terms, would immediate action to inject additional capital be required in order to stave off withdrawal of authorization.

An alternative to running off the existing business is to merge with a more successful insurer, and this has certainly happened in a number of cases in the UK, although perhaps less frequently than in other markets. The reason for this difference may be the diversity of companies in the market, with no particular feeling that the imprudent or mismanaged company ought to be bailed out by the rest of the industry in order to save face, combined with a reluctance on the part of the supervisory authorities to twist the arms of the larger companies to absorb smaller unsuccessful companies for no perceptible financial advantage.

Solvency Margins Required

Given the limitations of an approach to solvency based purely on balance sheet values of assets and liabilities, we have suggested that an alternative would be to use a simulation approach to test solvency against the criterion of being able to run off the existing business satisfactorily. This is what we have sought to do in our work so far using the simulation model described in Appendix 1. We have had to stipulate an expected run-off, together with a stochastic process to define the actual payments made. We have assumed that the variability can be satisfactorily described in terms of a distribution of claims sizes, and we have assumed that there is such a distribution for the claims payments in each year of the run-off for each year of origin and each class of business. We have not sought to take account of any correlation between differences in expected and actual claims from one year to the next. Nor have we assumed that the claims size distribution will vary as the run-off proceeds. Clearly there is room for making the model more sophisticated and perhaps more realistic in these areas but increasing the model's complexity increases the difficulty of understanding what is going on. The DTI returns do not require any information to be given about claim amount distributions although it is clear that this is important for assessing the risk profile of a portfolio.

We also need models of the investment scene and the related parameter, the rate of inflation. There is plenty of room for differences of opinion over the precise nature of the stochastic investment model, although the development of such a model would fall rather outside our main remit. We have accordingly made use of the models already suggested by Professor Wilkie[4] and have not as yet sought to change these models, or to test the sensitivity of our results to alternative parameters other than to a limited degree. We see this, however, as a necessary next step, if our results are to command general acceptance. In particular, there may be scope for examining alternative inflation models. However, a fundamental question is what one assumes as the starting position. We have taken the neutral starting position of the Wilkie models, in order not to build in any automatic bias towards asset appreciation as the model tends towards the trend line from a current set of starting parameters. Assuming a sudden change in inflation produces some interesting results, showing fairly clearly the danger of changes in inflation as compared with the relatively benign effects of continued high inflation, at least with the asset relationships implied by these models. The Finnish study[2] also showed that "shock inflation" was more dangerous than steady inflation.

It is of the nature of these models that they can be only a crude representation of uncertainty in relation to either assets or liabilities. We are, of course, open to criticism in respect of our assumption that all uncertainty can be taken care of by stochastic variables. Some may cite significant changes in the economy, or changes in law affecting Court settlements, such as the recent change in Court rules regarding liability compensation, as instances of uncertainty which is not stochastic in nature. In practice, we have assumed that such changes can be taken care of by a pure stochastic approach, albeit in a rather crude manner, since the past data which may be used to calibrate the parameters will undoubtedly include other changes which may have taken place in the past, and future changes are implicitly allowed for by setting an appropriate standard deviation. The problem, of course, is deciding what is appropriate.

So far, the approach of the Working Party has been to examine solvency margin requirements as a percentage of outstanding claims provisions, since this appears to reflect most closely the shape of the uncertainty. At present, however, statutory solvency margins are defined in terms of a percentage of premiums, with a back up provision relating to incurred claims. Both of these relate effectively to only one year's business. The premiums basis may have some rationale in relation to the risks associated with remaining open to business, and the supervisory authorities are concerned with this to some extent, because of the lags between accounting dates, reporting dates and the time at which effective action can begin to be taken, but the risks associated with the uncertainty of the run-off itself would in our view be better covered by a solvency margin based on the outstanding claims provisions. Interestingly, the Australian Insurance Commissioner put forward the view in his 1983-84 Annual Report that a solvency margin requirement ought perhaps to have regard to both premium income and outstanding claims. Other determinants of necessary solvency margin might also be considered, such as rate of growth and changes in type of business written, although they might be difficult to apply in a statutory standard.

Further discussion is necessary on how to allow for size and type of insurer, the extent to which variability should be assumed to differ between different classes of liabilities, how to take into account uncertainties relating to currencies, and what to do about reinsurance, both business accepted and ceded. A comprehensive picture on the asset side would need to take fuller account of non-income-bearing assets, such as agents' balances, which are not only non-income-bearing, but also have a potential for default, and investment in subsidiaries, often representing the insurance business of the company carried on in other territories. There are, of course, severe limitations in treating the overseas business of an international insurance group in this way. It may in fact represent a major part of the group's insurance liabilities and it would be unrealistic to suppose that such a company would always wish to limit its losses in overseas markets to the extent of its investment in its dependants. It would be reasonable for as much attention to be paid to the supervision of major subsidiaries as to the parent company itself.

It is clear that the appropriate solvency margin requirements for different companies could differ considerably, according to the nature of the business written and the investments held. Even if only the surplus is assumed to be invested in equities, a very substantially higher level of solvency margin is required than if all the investments are in cash. A solvency system which does not take this into account must be regarded as defective. However, a full application of a simulation approach to establishing solvency standards would imply that each company's solvency margin was individually assessed, and the difficulties of doing so would necessitate a suitably qualified professional person within each company who could perform this function, both as a contribution towards the management of the company and as an essential part in the supervisory process.

Adequacy of Provisions

As we have already highlighted, discussion of a company's solvency depends critically on the adequacy or otherwise of the outstanding claims provisions. Part of the process of establishing a suitable solvency standard, therefore, ought to be to ensure some consistency and reliability in the estimation of outstanding claims. Much effort has been expended in seeking new methods of analysing claims payment data in order to project outstanding claims, but it remains and is likely to remain an imprecise science. The recent article[5] by Chris Daykin and William Hewitson in the GIRO bulletin illustrates vividly the extent to which companies got their estimates wrong in the late 1970s. It will be interesting to see the results of the GISG investigation into methods of estimating outstanding claims which are currently used by companies. However, it is unlikely that there will ever be a generally accepted method or even methods and it seems essential to rely on professional expertise and judgement. This would need to be accompanied by appropriate professional guidance, such as has been issued by the Australian Institute to non-life actuaries working there.

What we are envisaging goes rather further than professional certification of loss reserves, since the professional person would be responsible not only for the provisions, but also for assessing the proper level of solvency margin which the Directors would be required to demonstrate that they had attained. The inter-relationship between the solvency margin requirements and the asset portfolio would necessitate the direct involvement of the professional in the company's investment policy, and this would also be relevant to discounting considerations in respect of outstanding claims. The position would be not unlike that of appointed actuary in a long term business company and the person in question would have to fulfil responsibilities not only towards shareholders and management, but also towards policy holders and supervisory authorities.

Reinsurance

Reinsurance business accepted may be regarded as simply another class of business, which, depending on its nature, may be particularly volatile and unpredictable. Appropriate reserving levels for such business are bound to present problems, since it can take many years for the liabilities (including IBNR) to develop fully. Solvency margins ought certainly to have regard to this uncertainty. In principle there seems no reason why the simulation approach should not provide valuable insights in this area of an insurer's portfolio as well as for direct business.

Much more difficult to handle in the context of solvency assessment is the security of reinsurance cessions. Many insurers are critically dependent on their ability to recover from reinsurers, since the size of the risks they write is such as to bankrupt or cripple them if they had to bear the liability alone. One safeguard against reinsurance failure is to spread reinsurance cessions widely, so that there is not any great dependence on any particular reinsurer. However, this does not remove the need to look carefully at the security of individual reinsurers chosen for the programme.

From the reserving point of view, a decision has to be made on the extent to which reinsurance recoveries can be relied on Extreme caution might point to reserving for the full gross liability but this is not a practical commercial possibility in most cases. Clearly recoveries from reinsurance companies already known to be in trouble should be ignored or heavily discounted, but it is more difficult to know what should be done when there are no specific known problems. In accounting terms it may be difficult to set up a provision against an unseen and unquantifiable possibility of reinsurance failure. On the other hand the accountancy concept of prudence would preclude taking credit in advance for receipts which are uncertain, so it would be possible to justify taking only partial credit for reinsurance recoveries, depending on an assessment of the viability of the reinsurers.

A detailed examination of the reinsurance programme can hardly be practicable for the supervisory authorities and here again it seems there is a task of considerable technical complexity, requiring an expert within the company to carry it out and report on it. No general solvency margin requirement can be a substitute for this. The present practice of adopting a fraction with a minimum of ½ but based on actual claims in the past 3 years is a very rough and ready solution and it does not have any regard to the actual dependence on reinsurers for future recoveries. With excess of loss reinsurance the premium is very small in relation to the potential liability, so no percentage of premium is likely to make sense as a solvency margin. A percentage of anticipated recoveries from reinsurers would have a stronger rationale, but what percentage? The result would also depend heavily on the reserving basis adopted (both gross and net of reinsurance).

With a reasonable spread of reinsurers it might be possible to envisage a probability of failure by individual reinsurers, so that the solvency margin could in principle be set at a level to achieve a given probability of survival of the primary insurer, as under the simulation approach we have discussed.

In practical terms, however, would all reinsurers be ascribed similar probabilities? How would the probabilities be assessed? How would one deal with reinsurers on which there was a substantial degree of dependence? Such an approach would also require an adequate model of the variability of the gross outstanding claims, taking full account of catastrophe loss possibilities. We have not yet sought to develop anything along these lines but it is clear that such an approach to reinsurance security assessment would be complex, highly dependent on the assumptions and possible only for an expert with an intimate knowledge of the company's reinsurance programme and the business being reinsured.

There are no easy answers to the problem of ceded reinsurance but here again we see a more promising way forward being through placing responsibility on a statutorily defined expert for each company.

FUTURE DEVELOPMENTS

Apart from dealing with the question of reinsurance, the model now needs to be developed to incorporate the risks of remaining open to new business. A realistic model to deal with this over a period of some years would need to incorporate reasonably sophisticated feedback and decision-making mechanisms. However, this is not so necessary if the horizon is limited to one or two years and such a short period will still be of interest in the context of statutory solvency margins. Further development of the model to allow for a longer period and to incorporate concepts such as the rate of return on capital will be of interest commercially and will provide a further, final stage.

With the inclusion of new business rather more care will need to be devoted to unexpired risks than has been the case up to now, and further attention needs to be paid to overheads and expenses of administration.

Work has also begun on analysis of the causes of some previous insolvencies of general insurance companies, although detailed information on many of the cases is not available. Although fraudulent activities have sometimes been at the root of companies' difficulties, mismanagement is a more common cause of insolvency. The effects of mismanagement will be seen in business written, inadequate reserving, poor quality reinsurance, etc, factors which we are considering as part of the stochastic approach. The effect of mismanagement is thus not so much to add new risk factors as to greatly increase the probability of adverse outcomes on the normal risks.

The question arises as to whether there should be a further loading on the solvency margin for this risk, whether the parameters for the main risk factors should be set more stringently in recognition of the possibility, or whether the issue should be ignored as simply increasing the probability of ruin in companies that experience mismanagement. Other aspects of supervision are, of course, intended to minimise the risk of mismanagement, eg the fit and proper screening arrangements.

It remains clear that there are no simple answers to the problem of setting solvency standards. The simulation model seems to provide a useful tool for examining the question further, both in the general theoretical context and in the circumstances of a particular company. However, there will still be other aspects which cannot be explored analytically and a decision on the appropriate level for statutory solvency margins will always in the end be partly a political one, reflecting a balance between a desire for security for policyholders and the cost to the policyholder of increased security, because of the return on capital required by the owners of insurance enterprises.

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APPENDIX 1

DESCRIPTION OF SIMULATION MODEL

A model has been established to simulate the run-off of a general insurance portfolio, using stochastic processes to describe uncertainty in both assets and liabilities. A particular set of parameters and assumptions was taken as the standard basis and the effects of varying each parameter or assumption were examined separately. Details of the results are given in Appendix 2. Appendix 3 shows how the simulation model may be used to determine the necessary initial margin of solvency in order to achieve a desired level of probability of ruin under given conditions. These include not only the variations considered in Appendix 2, but also a few further combinations of adverse circumstances.

Variability of Assets

The variability inherent in the asset portfolio of a company depends on the nature and distribution of the assets. The realizable value of many assets will vary from day to day as market conditions change. In our model, market movements are represented by a series of inter-related stochastic processes, suggested by Wilkie, which generate future scenarios for the values of different types of asset and the income therefrom. Although Wilkie's models were not originally intended to be used for relatively short-term simulations such as those with which we are concerned, we consider that they can validly be used for our purposes. We would, however, expect to carry our more testing of the sensitivity of our results to the particular models assumed.

The initial distribution of the assets by category is specified and the model analyses separately the development of the various components of the asset distribution, taken as cash, irredeemable Government securities and ordinary shares, simulating the income generated and the capital value of each type of asset for each future year.

A more realistic model would replace investment in irredeemable Government securities with short, medium and long-dated securities. In practice, however, short-dated securities behave somewhat like cash and long-dated securities like irredeemables, so our model represents a tolerable proxy.

For the standard basis we have used the neutral parameters suggested by Wilkie. For a variant we have examined the effect of doubling the standard deviation of the yield, but further investigations are necessary into the sensitivity of the results to the particular model employed.

In considering initial asset distributions we have related the investment policy separately to assets covering the liabilities (L) and to assets representing the margin of assets over liabilities, or surplus (S). In the standard basis the liabilities are assumed to be covered half by cash and half by Government securities, whilst the surplus is assumed to be invested in ordinary shares. The investment strategy adopted is to sell equities first, then gilts, before using the cash, and to assume that any excess of income over expenditure is kept in cash. Allowance is made for taxation if the run-off proceeds profitably, although where taxable losses arise these are assumed to create an offset against any tax liability. In the simulations shown here no taxable losses have been assumed at the start of the run-off, although the tax situation can be imput as an initial parameter to investigate the effect of brought forward losses.

Variability of liabilities

Six classes of business were considered, three short-tailed and three long-tailed. The volume of business in each class was assumed to be such as would require equal reserves at the end of the year in which the business was written. The claims were assumed to be settled in accordance with the run off patterns suggested for illustrative purposes by Abbott et al (JIA108, 143-145).

On the standard basis reserves were assumed to be set up using the expected claims settlement pattern with 5% inflation and no discounting. In each realisation "actual" claims payments in real terms each year were generated using a series of claims payment distributions, one for each combination of year of origin and year of settlement. The distributions were taken as log-normal, with mean equal to the claims expected to be settled in real terms in that year of development on the reserving basis. The standard deviation was specified as 30% of the mean for the standard basis, with variant assumptions of 0%, 10% and 50%.

"Actual" inflation was then allowed for, as generated by the model suggested by Wilkie to link with his asset models. For the standard basis the mean expected level of inflation in Wilkie's model was taken as 5%. As variants we examined models with mean expected levels of inflation of 10% and 15%, and also investigated the effect of changing the mean expected level of inflation in the model from 5% to 10% or 15% as at the date of the solvency assessment.

Solvency Margin

The initial excess of assets over liabilities, or solvency margin, was specified as a percentage of the liabilities at the start of the run off. For the standard basis we took the solvency margin to be 15% of the liabilities. This was intended to be broadly equivalent, for the mix of business, to the EEC solvency margin requirement for a small company of 18% of premiums or 26% of average incurred claims. For short-tailed business the EEC solvency margin requirement would represent a much higher proportion of liabilities outstanding at the end of the year of origin (over 30% for the short-tailed business considered here) whereas for long-tailed business the requirement would be lower (less than 10% for the long-tailed business considered here). This simply reflects the fact that a premium-based solvency margin requirement is not very well targetted as far as the run-off risks are concerned.

The Simulation Model

For each set of criteria, the model generates a large number of run-off scenarios, incorporating claims paid as generated from the distributions and based on the simulated level of inflation. Asset values vary in accordance with the investment models and, where investment income falls short of simulated outgo, assets are realized as necessary, in accordance with the investment strategy, in order to meet the liabilities as they fall due. However, because the model generates asset values at annual intervals, we have assumed that claims are paid and assets realised at the end of the year in question. This process continues until either all the liabilities have been met or until the assets are exhausted. If the assets are exhausted before all the liabilities have been met, this is defined as an insolvency. The simulation is, however, continued in these cases with negative net assets in order to illustrate "how insolvent" the company really is in these cases.

APPENDIX 2

RESULTS OF SIMULATIONS ON VARIOUS ASSUMPTIONS

Figures 1 and 2 show, for illustrative purposes, the results of 100 simulations on the standard basis, producing no insolvencies. Figure 1 shows the assets of the model company year by year throughout the run-off. Figure 2 shows the solvency margin (expressed as a percentage of the initial liabilities) at each point of the run off. We carried out 5000 simulations on the standard basis, producing insolvencies in 29 cases. The distribution of assets at the end of the run-off can be summarized as follows;

Remaining Assets as % of initial outstanding claims	Number of cases out of 5000 simulations
Less than 0	29
0 - 10	65
10 - 20	309
20 - 30	992
30 - 40	1576
40 - 50	1438
50 and over	591
	5000

The mean level of remaining assets for the 5000 simulations was 36.6% of the initial value of outstanding claims.

These figures, of course, only provide estimates of the probability of ruin, since the process effectively involves sampling 5000 sets of random numbers from an infinite series to generate the simulations. Subsequent to completing the ASTIN paper we have repeated this exercise using very different sequences of random numbers and have shown that the number of insolvencies is reasonably sensitive to the random number stream, even with 5000 simulations. A simulation involving 100,000 runs produced 444 insolvencies (equivalent to 22 with 5000 runs) but the mean level of remaining assets was still 36.6% of the initial value of outstanding claims. In looking at the results shown in this Appendix it is important to bear in mind that they are subject to sampling error and that it is the order of magnitude of the number of insolvencies which is significant rather than the precise figures given. However, all are based on the same set of 5000 random numbers.

Results for alternative assumptions are given in Table 1. In each case all the assumptions are as for the standard basis, except for the variant described. Figures 3 to 12 show the development of the assets from year to year on 10 of the variant bases, based on 100 simulations. In each case the principal parameters are shown, and the number of insolvencies and the asset distribution at the end of the run-off using 5000 simulations. To complete the picture, realisations which result in insolvency are allowed to continue "in the red" on the diagrams, in order to show "how insolvent" they were.

The importance of the reserving basis can be seen clearly. The assumed rate of inflation is vitally important, as, of course, is the rate of discount used. Although the probability of ruin is only 6 per 1000 on the standard basis, with a 5% allowance for inflation, no allowance at all for inflation in setting the initial reserves raises the probability of ruin to 55 per 1000, even if a 15% solvency margin is still held. Comparing the different bases, the total assets held in order to cover the technical reserves on the no inflation, no discounting basis, and show a 15% solvency margin on that basis, would in fact represent only a 2% solvency margin if the liabilities were restated using 5% inflation and no discounting, as on the standard basis. Conversely, allowing for 10% inflation with no discounting and a 15% solvency margin is equivalent to having a 48% solvency margin if the liabilities were reassessed on the standard basis.

The results shown for only short-tail or only long-tail business illustrate a related point. The initial liabilities are assessed in each case using 5% inflation and no discounting. This provides a larger effective margin in the case of long-tail business than in the case of short-tail.

The results in respect of different asset distributions are much as one would expect, except that the relatively small size of the solvency margin assumed as compared with the liabilities means that the results are not very sensitive to the way in which the solvency margin is invested. The model shows the greater risks resulting from investing in long dated Government securities and equities, but also the higher expected rewards.

Increasing the mean expected level of inflation in the model appears to raise the mean assets remaining at the end of the run-off and reduce the risk of insolvency. It is in fact implicit in these variants that the mean level of inflation not only is now at the higher level, but has been there for some time, so that the asset model has already adjusted to the high inflation conditions. Alternative scenarios involving a switch at the time the assessment is being made from mean inflation of 5% to a higher level of 10% or 15% produced probabilities of ruin 31 and 129 per 1000 respectively. This suggests that it is not high inflation as such which is dangerous, but periods when inflation changes relatively suddenly to a higher general level.

However, it is too stringent in general in assessing probabilities of ruin to assume a sudden shift in the underlying pattern of inflation with a probability of one. A more realistic estimate of the probability of ruin, given the possibility that the mean expected level of inflation could change sharply, might be obtained by taking a weighted mean of these bases with the standard basis.

TABLE 1 SUMMARY OF RESULTS OF SIMULATIONS

		Number of insolvencies out of 5000 simulations	Mean assets remaining as % of initial outstanding claims
	Standard basis	29	36.6
	Variant bases		
1.	Type of business		
	(a) All long-tail	19	38.9
	(b) All short-tail	29	32.5
2.	Reserving basis		
	(a) No inflation; no discounting	275	23.3
	(b) 10% inflation; no discounting	0	47.9
	(c) 10% inflation; 7% discount	71	30.9
3.	Solvency margin		
	(a) 10%	55	30.2
	(b) 20%	5	42.6
4.	Variability of outstanding claims		
	(a) Standard deviation 0%	13	36.4
	(b) Standard deviation 10%	15	36.4
	(c) Standard deviation 50%	41	36.4
5.	Inflation model		
	(a) Mean expected inflation 10%	18	51.3
	(b) Mean expected inflation 15%	21	71.3
	(c) Mean expected inflation 10% (from starting point only)	157	28.4
	(d) Mean expected inflation 15% (from starting point only)	646	18.0
	(e) Mean expected inflation 5% (standard deviation doubled)	491	32.7
6.	Social inflation		
	(a) 5% addition for long tail	21	37.4
7.	Asset model		
	(a) Double standard deviation of yield	39	37.8
8.	Initial asset distribution		

	Cash	Gilts	Equities	Number of insolvencies	Mean assets remaining
(a)	L+S	-	-	2	34.8
(b)	-	L+S	-	113	39.8
(c)	L	$\frac{1}{2}$S	$\frac{1}{2}$S	7	35.2
(d)	$\frac{1}{2}$L+$\frac{1}{2}$S	$\frac{1}{2}$L	$\frac{1}{2}$S	13	35.9
(e)	$\frac{1}{2}$L	$\frac{1}{2}$L+$\frac{1}{2}$S	$\frac{1}{2}$S	15	36.1
(f)	L	-	S	8	35.5
(g)	-	L	S	117	40.4
(h)	$\frac{1}{2}$L	-	$\frac{1}{2}$L+S	103	39.1
(i)	-	-	L+S	349	59.0

FIGURE 1

ASSETS REMAINING AT EACH POINT DURING RUN-OFF

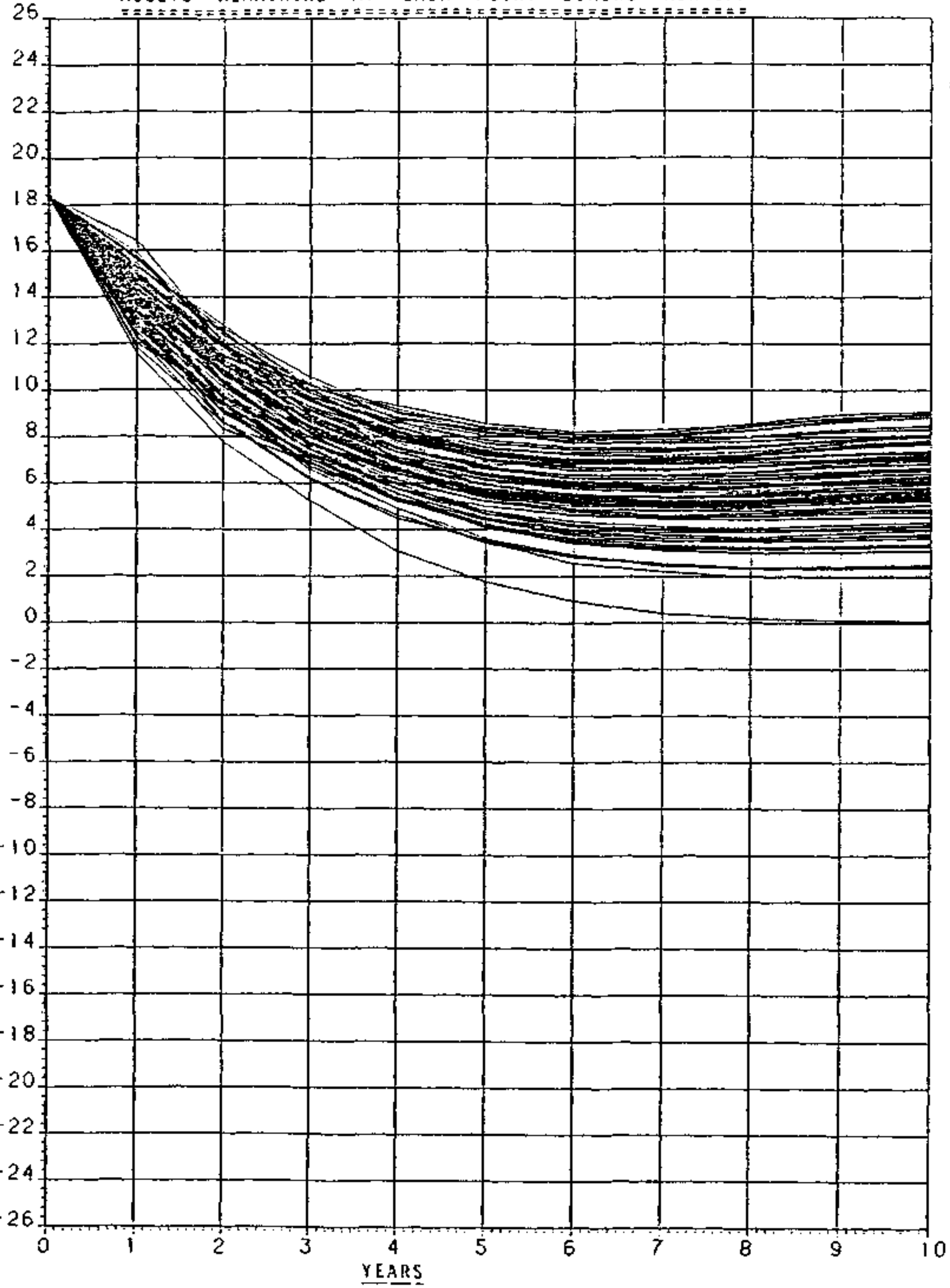


FIGURE 2

SOLVENCY MARGIN DURING RUN-OFF AS PROPORTION OF THE INITIAL LIABILITIES

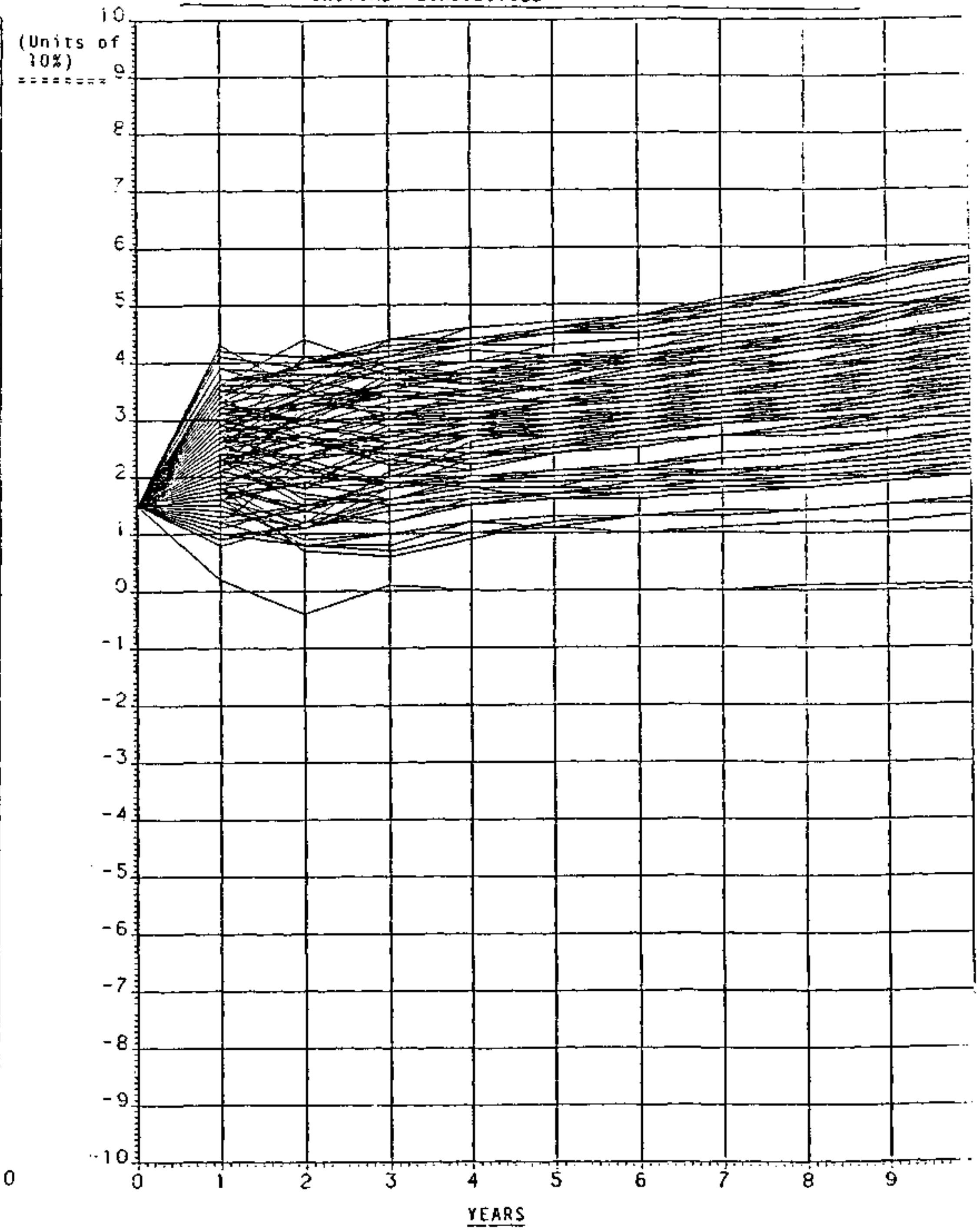
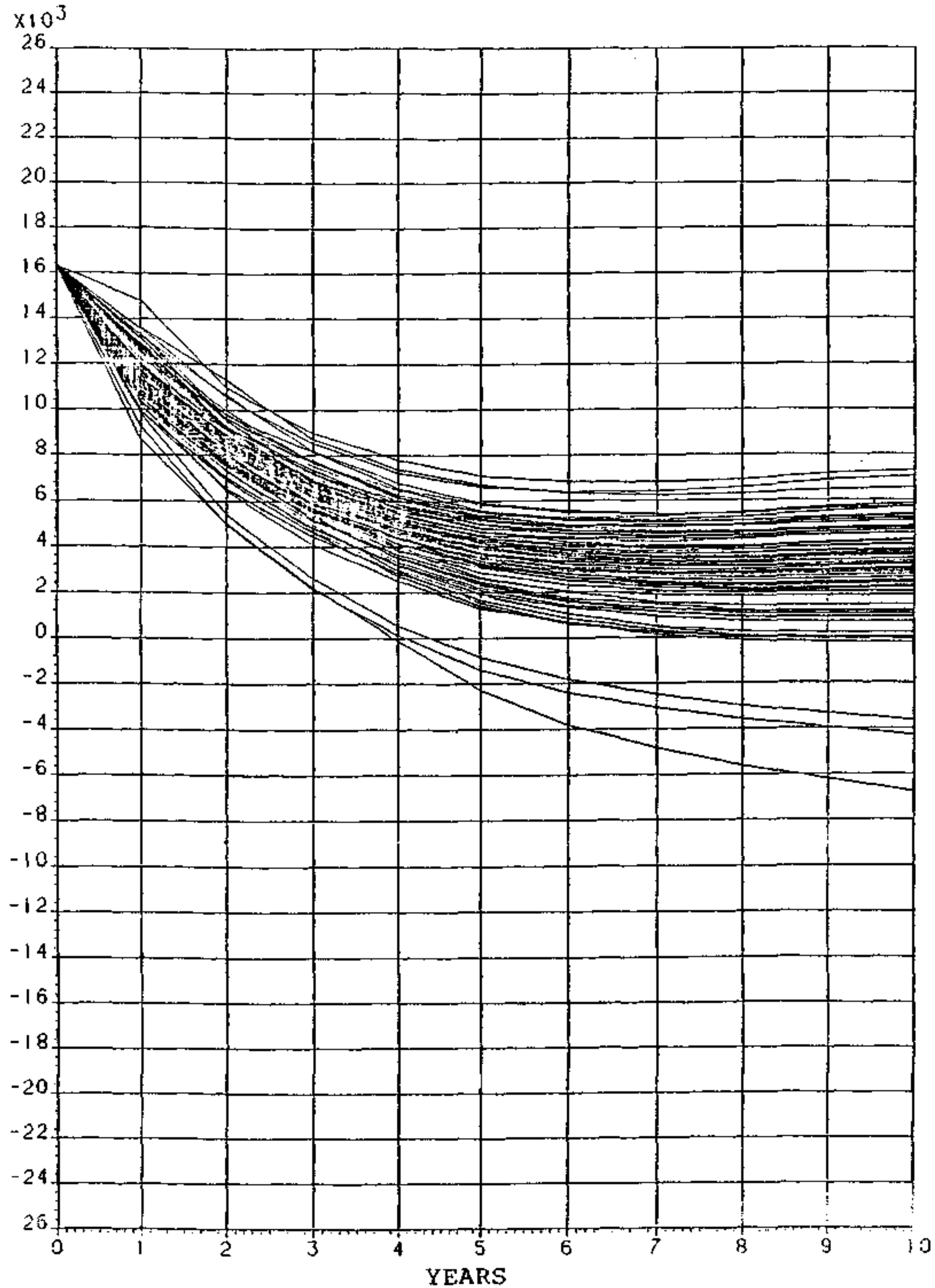


FIGURE 3 ASSETS REMAINING AT EACH POINT DURING RUN-OFF: VARIANT 2(a)



Variant 2(a)

Standard basis except for outstanding claims reserves which are assessed using

Inflation : 0%
Discount factor: 0%

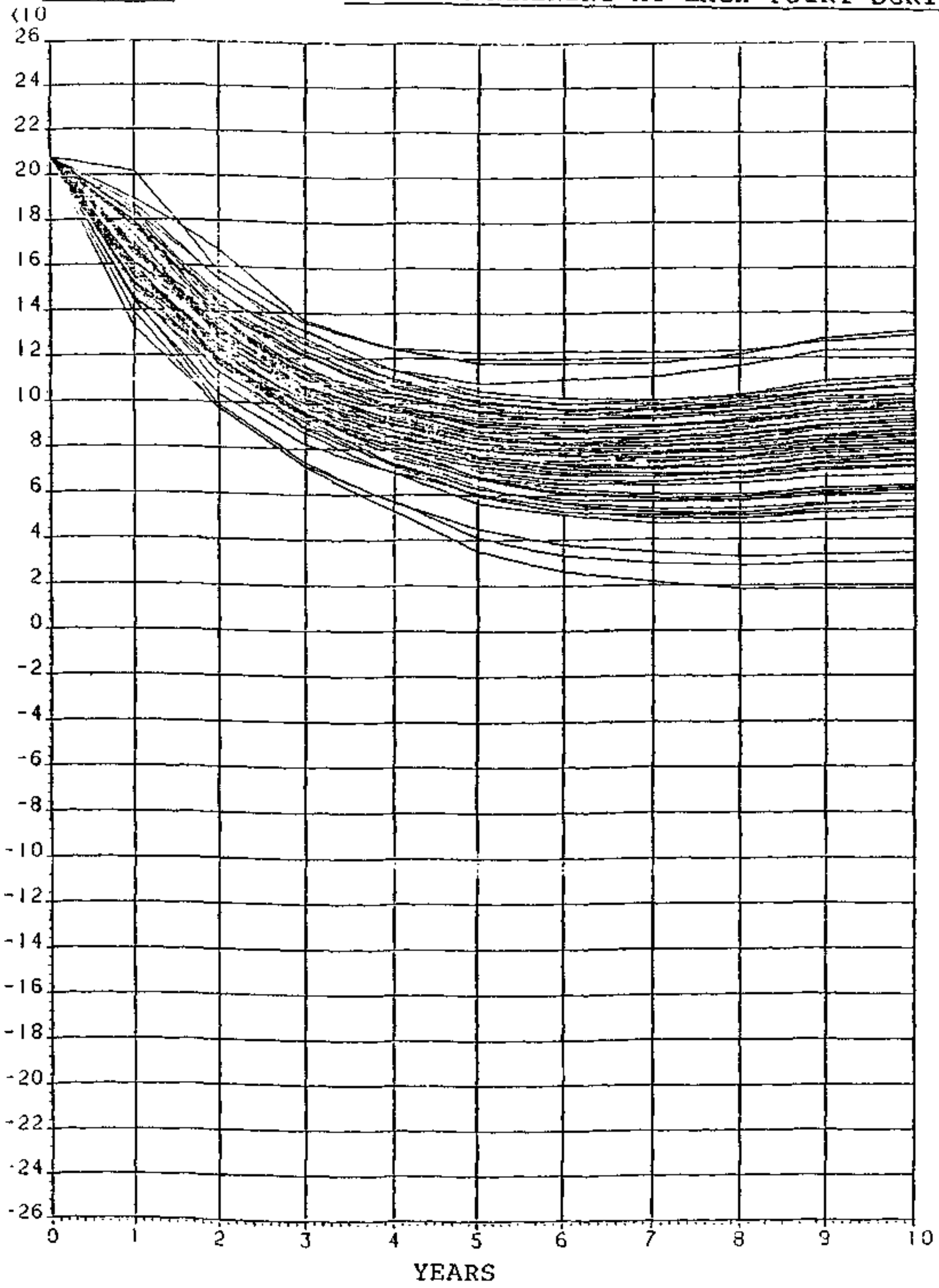
With 5000 simulations:

Number of insolvencies : 275
Mean assets remaining : 23.3%

Remaining assets as % of initial outstanding claims	Number of cases out of 5000 simulations
less than 0	275
0 - 10	461
10 - 20	1099
20 - 30	1509
30 - 40	1179
40 - 50	397
50 and over	80
	5000

FIGURE 4

ASSETS REMAINING AT EACH POINT DURING RUN-OFF: VARIANT 2(b)



Variant 2(b)

Standard basis except for outstanding claims reserves which are assessed using

Inflation : 10%
Discount factor: 0%

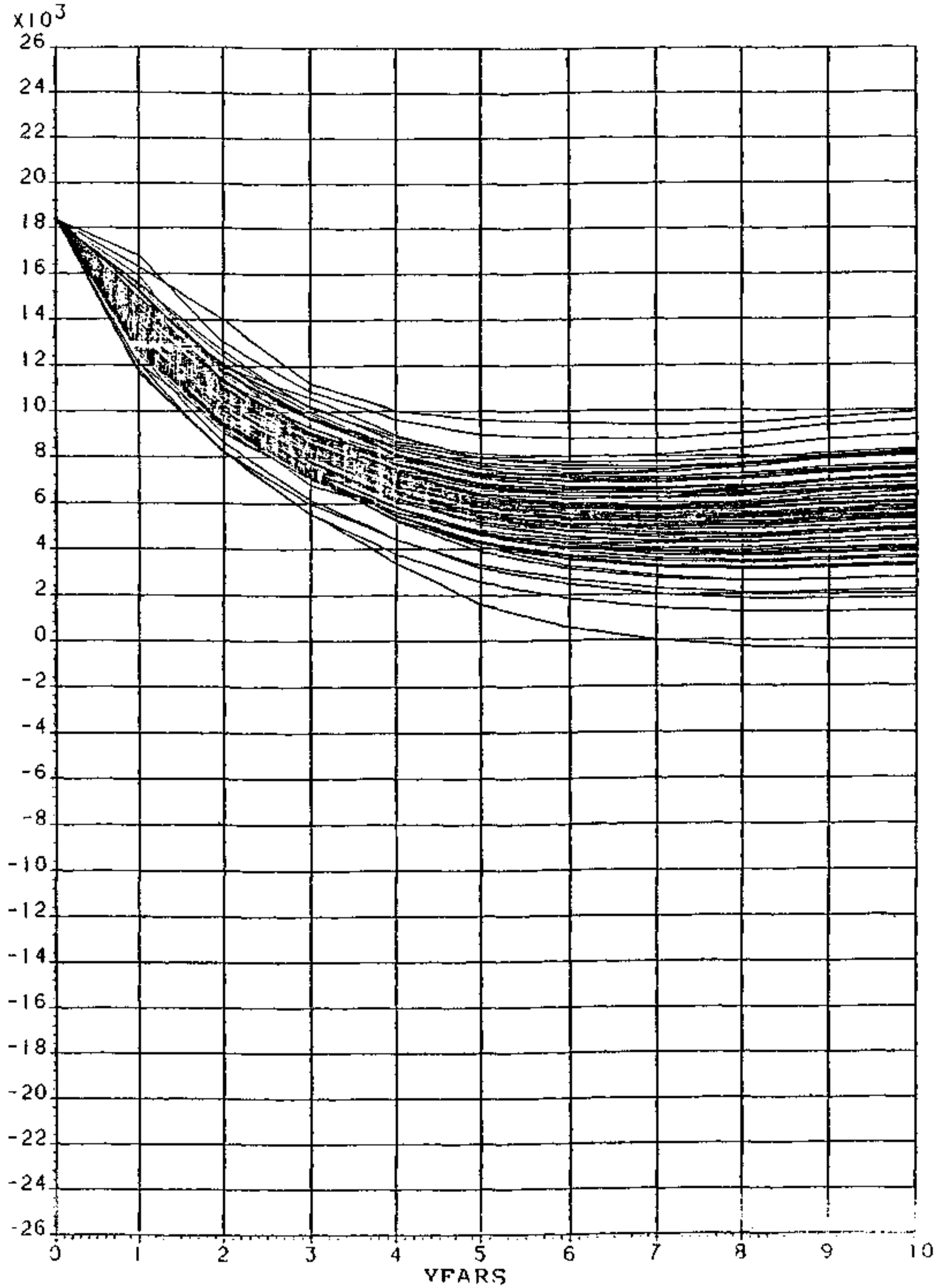
With 5000 simulations:

Number of Insolvencies : 0
Mean assets remaining : 47.9%

Remaining assets as % of initial outstanding claims	Number of cases out of 5000 simulations
less than 0	0
0 - 10	2
10 - 20	43
20 - 30	235
30 - 40	875
40 - 50	1699
50 and over	2146
	5000

FIGURE 5

ASSETS REMAINING AT EACH POINT DURING RUN-OFF: VARIANT 4(b)



Variant 4(b)

Standard basis except for variability of outstanding claims which is estimated using

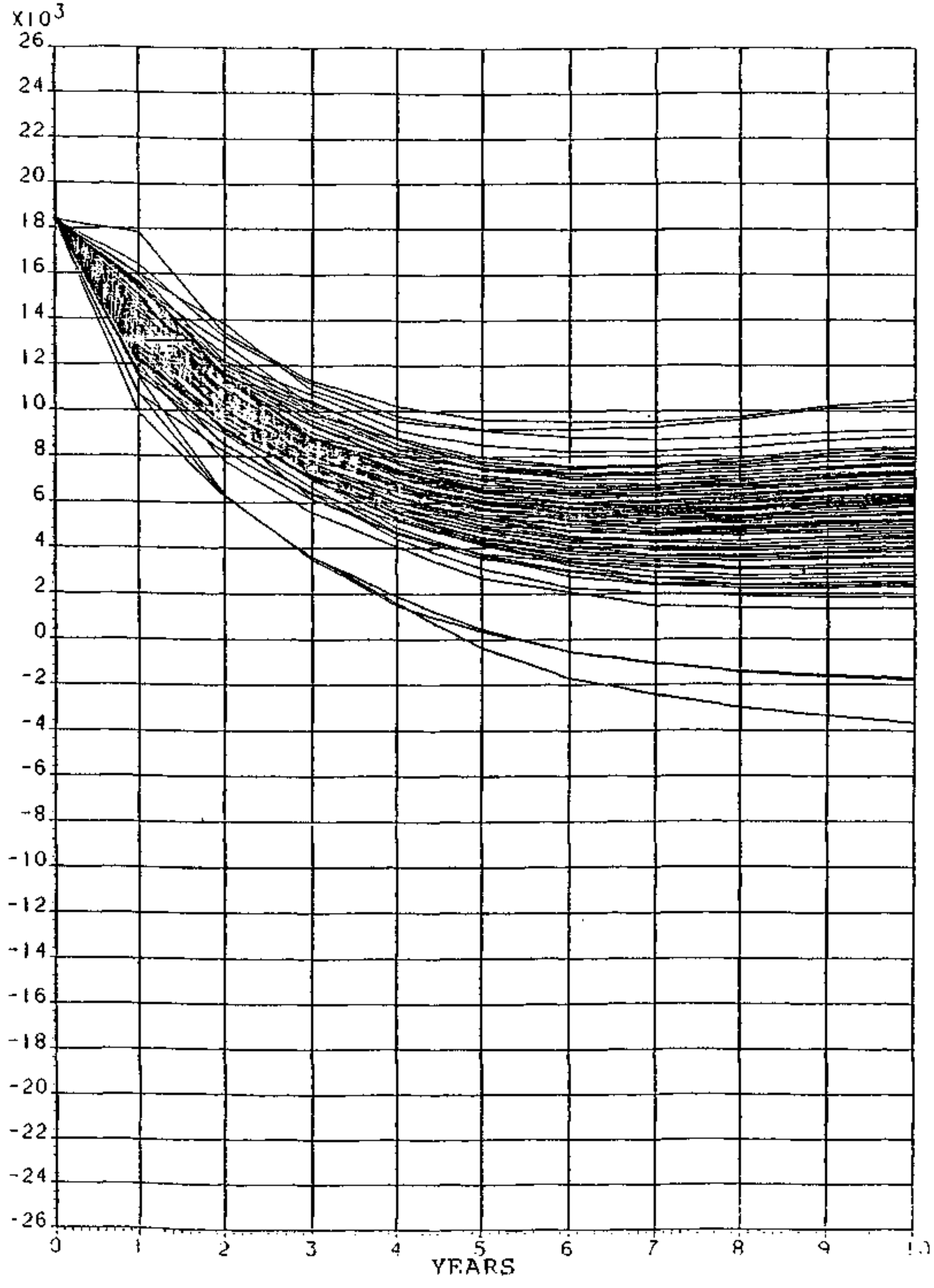
Standard deviation : 10%

With 5000 simulations:

Number of insolvencies : 15
Mean assets remaining : 36.4%

Remaining assets as % of initial outstanding claims	Number of cases out of 5000 simulations
less than 0	15
0 - 10	71
10 - 20	322
20 - 30	980
30 - 40	1667
40 - 50	1398
50 and over	547
	5000

FIGURE 6 ASSETS REMAINING AT EACH POINT DURING RUN-OFF: VARIANT 4(c)



Variant 4(c)

Standard basis except for variability of outstanding claims which is estimated using

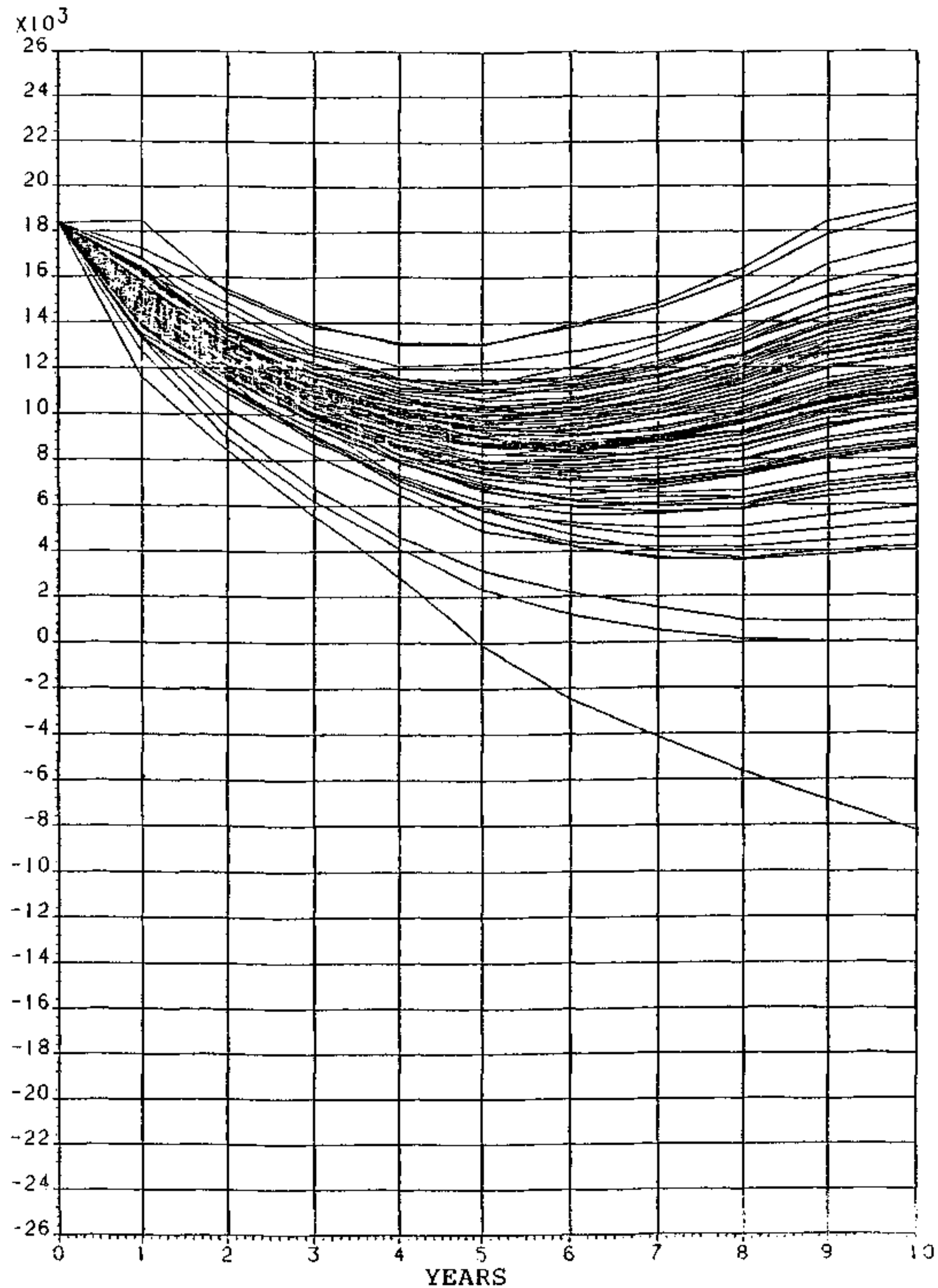
Standard deviation : 50%

With 5000 simulations:

Number of Insolvencies : 41
Mean assets remaining : 36.4%

Remaining assets as % of initial outstanding claims	Number of cases out of 5000 simulations
less than 0	41
0 - 10	102
10 - 20	370
20 - 30	936
30 - 40	1523
40 - 50	1357
50 and over	671
	5000

FIGURE 7 ASSETS REMAINING AT EACH POINT DURING RUN-OFF: VARIANT 5(b)



Variant 5(b)

Standard basis except for inflation model which is based on:

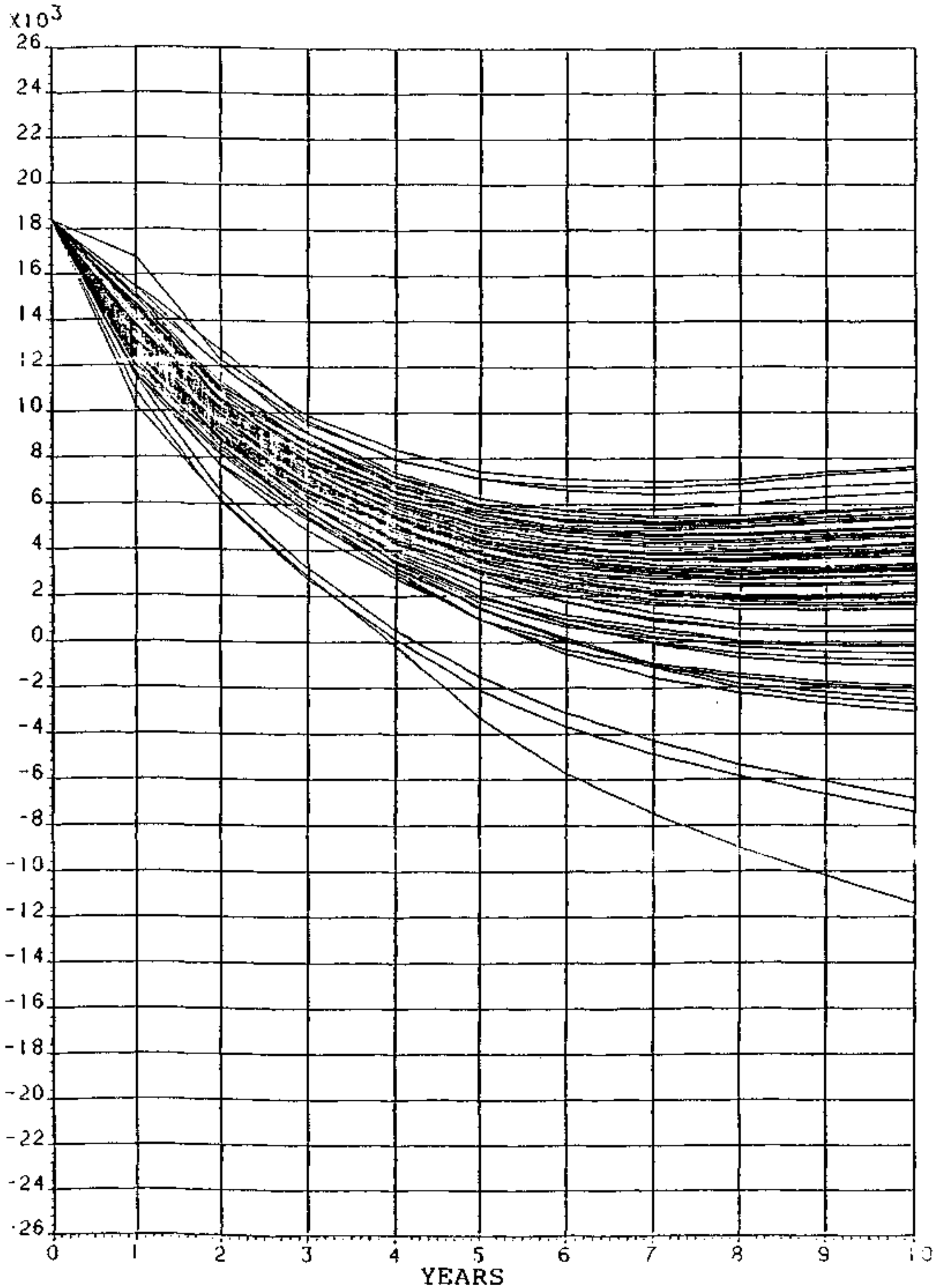
Mean expected inflation : 15%

With 5000 simulations:

Number of Insolvencies : 21
Mean assets remaining : 71.3%

Remaining assets as % of initial outstanding claims	Number of cases out of 5000 simulations
less than 0	21
0 - 10	27
10 - 20	61
20 - 30	126
30 - 40	220
40 - 50	391
50 and over	4154
	5000

FIGURE 8 ASSETS REMAINING AT EACH POINT DURING RUN-OFF: VARIANT 5(c)



Variant 5(c)

Standard basis except for inflation model which is based on:

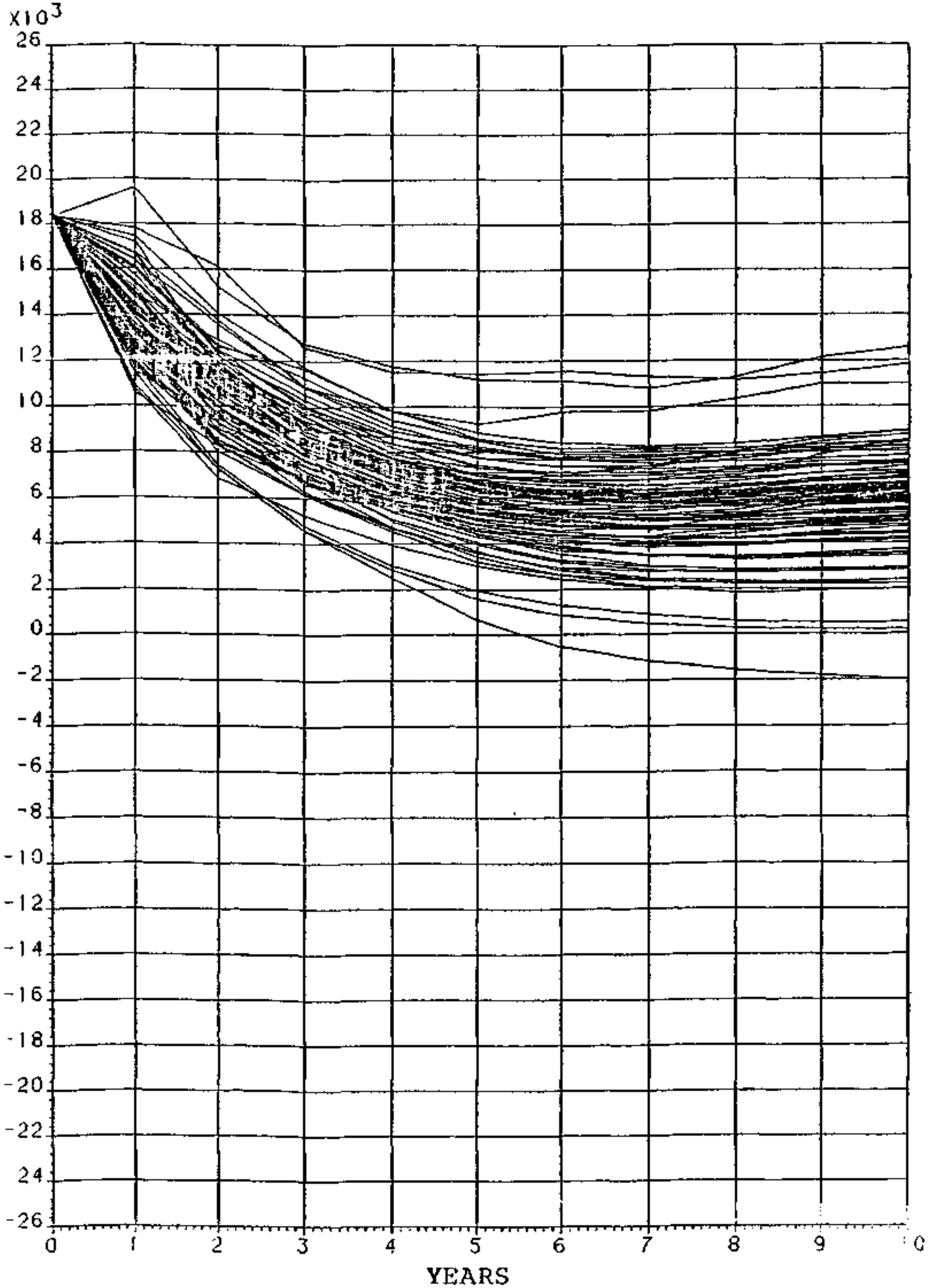
Mean expected inflation to starting point: 5%
Mean expected inflation thereafter : 15%

With 5000 simulations:

Number of Insolvencies : 646
Mean assets remaining : 18.0%

Remaining assets as % of initial outstanding claims	Number of cases out of 5000 simulations
less than 0	646
0 - 10	642
10 - 20	1125
20 - 30	1374
30 - 40	865
40 - 50	292
50 and over	56
	5000

FIGURE 9 ASSETS REMAINING AT EACH POINT DURING RUN-OFF: VARIANT 7(a)



Variant 7(a)

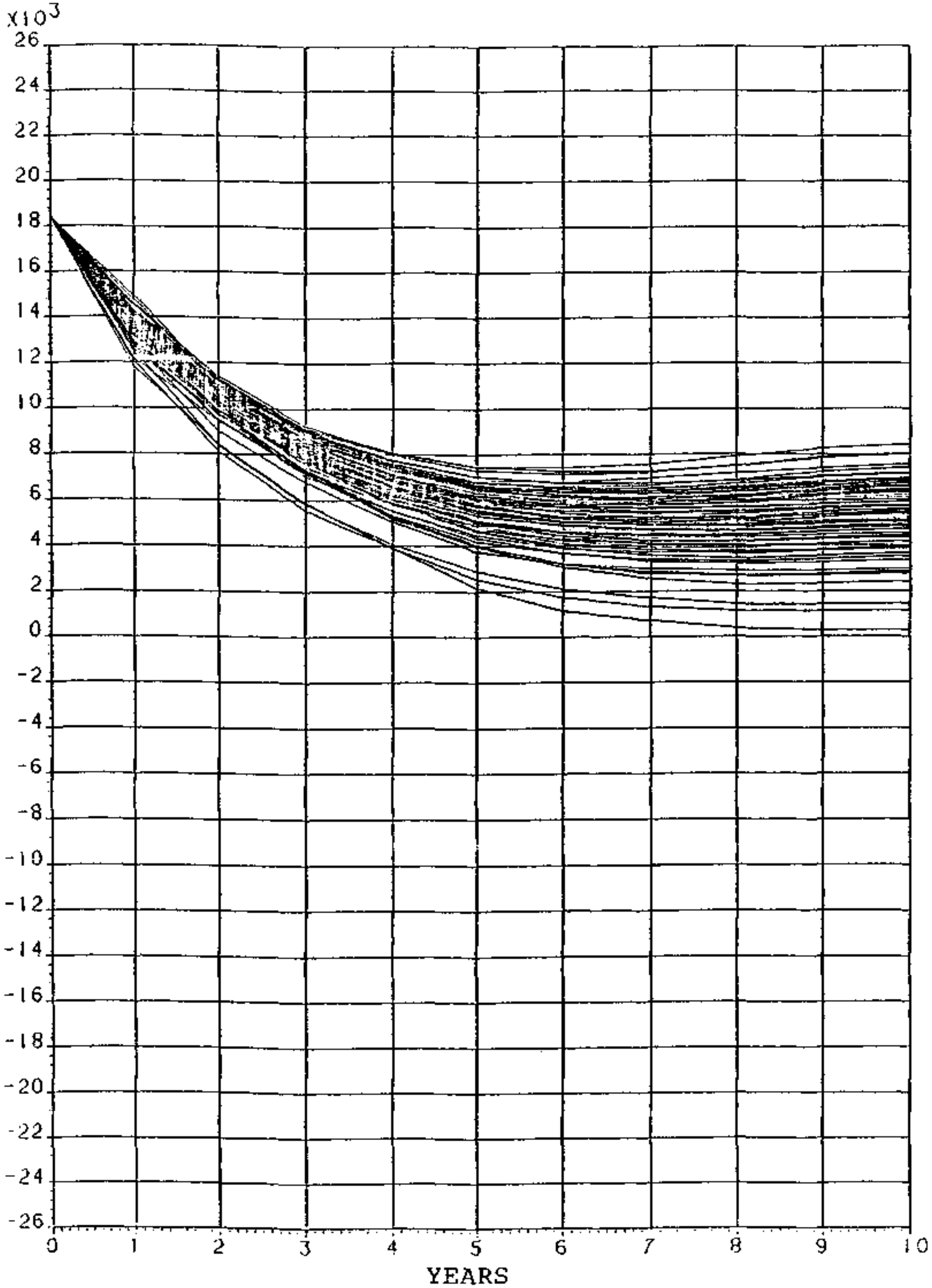
Standard basis except for asset model which is based on double the standard deviation of the yield

With 5000 simulations:

Number of Insolvencies : 39
Mean assets remaining : 37.8%

Remaining assets as % of initial outstanding claims	Number of cases out of 5000 simulations
less than 0	39
0 - 10	114
10 - 20	346
20 - 30	928
30 - 40	1418
40 - 50	1211
50 and over	944
	5000

FIGURE 10 ASSETS REMAINING AT EACH POINT DURING RUN-OFF: VARIANT 8(a)



Variant 8(a)

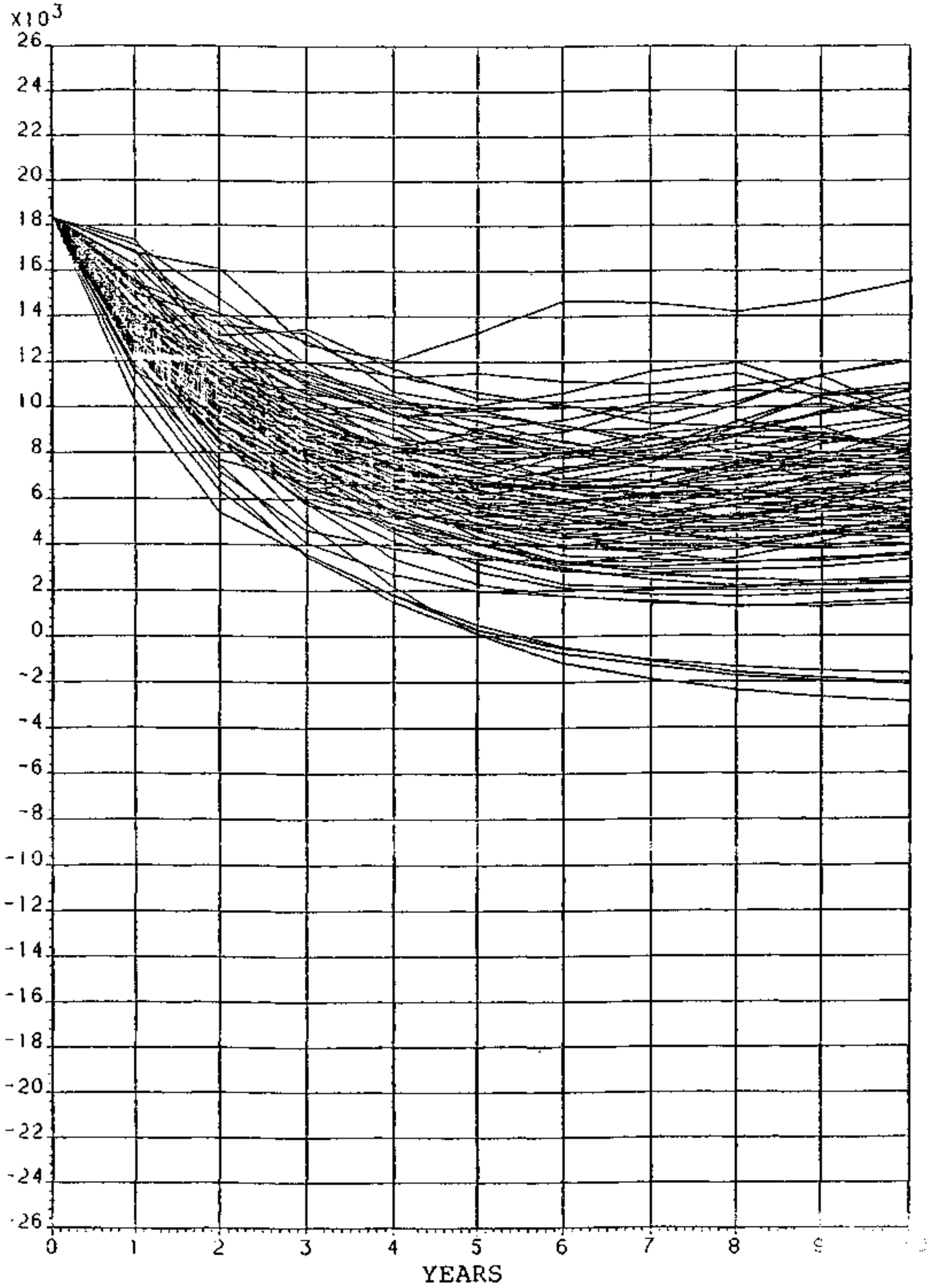
Standard basis except for initial asset distribution which is assumed to be 100% in cash

With 5000 simulations:

Number of insolvencies : 2
Mean assets remaining : 34.8%

Remaining assets as % of initial outstanding claims	Number of cases out of 5000 simulations
less than 0	2
0 - 10	29
10 - 20	227
20 - 30	1054
30 - 40	2286
40 - 50	1291
50 and over	111
	5000

FIGURE 11 ASSETS REMAINING AT EACH POINT DURING RUN-OFF: VARIANT 8(b)



Variant 8(b)

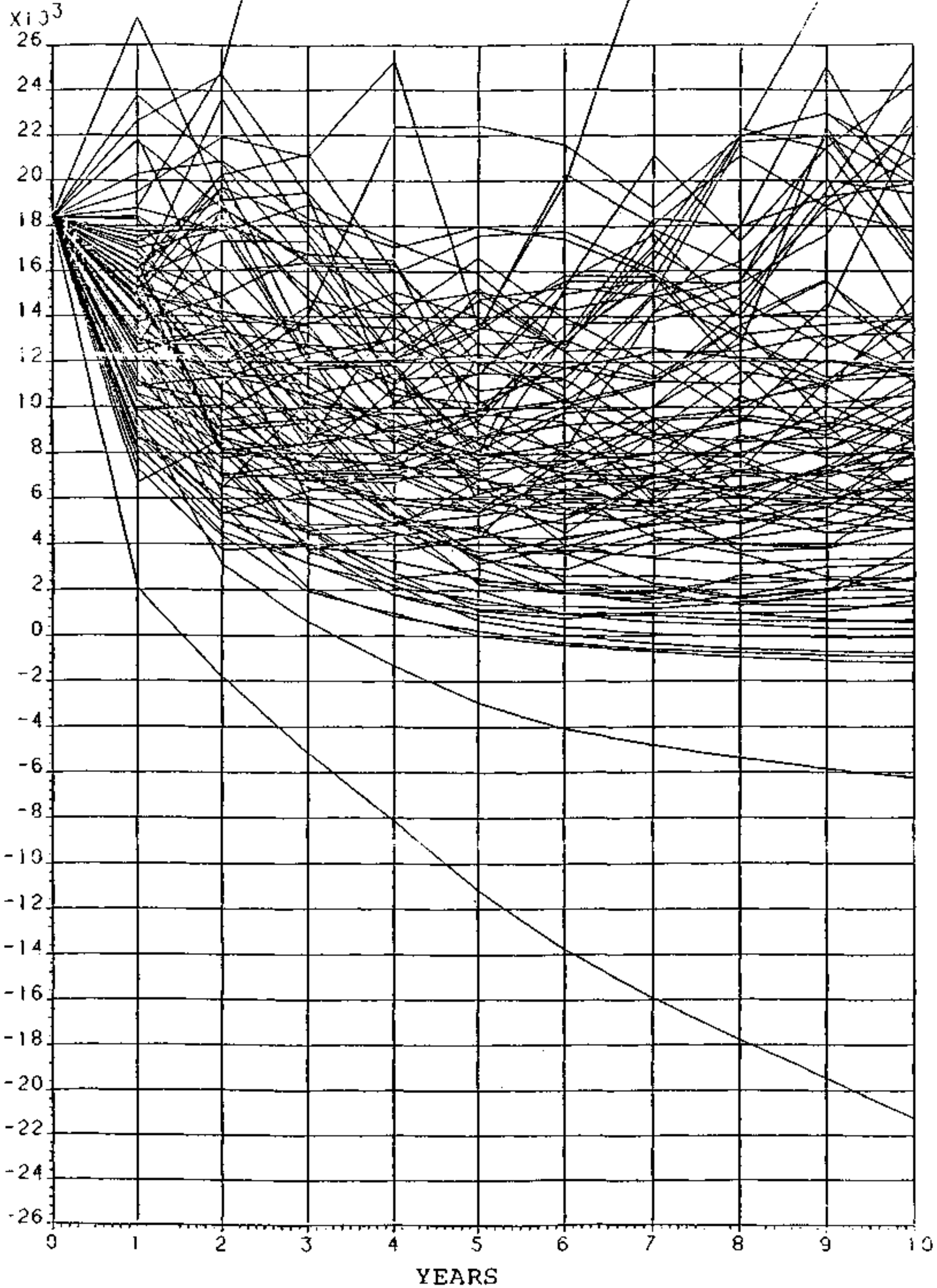
Standard basis except for initial asset distribution which is assumed to be 100% in government securities

With 5000 simulations:

Number of insolvencies : 113
Mean assets remaining : 39.8%

Remaining assets as % of initial outstanding claims	Number of cases out of 5000 simulations
less than 0	113
0 - 10	195
10 - 20	463
20 - 30	817
30 - 40	1014
40 - 50	988
50 and over	1410
	5000

FIGURE 12 ASSETS REMAINING AT EACH POINT DURING RUN-OFF: VARIANT 8(i)



Variant 8(i)

Standard basis except for initial asset distribution which is assumed to be 100% in ordinary shares

With 5000 simulations:

Number of insolvencies : 349
Mean assets remaining : 59.0%

Remaining assets as % of initial outstanding claims	Number of cases out of 5000 simulations
less than 0	349
0 - 10	205
10 - 20	384
20 - 30	449
30 - 40	498
40 - 50	476
50 and over	2639
	5000

APPENDIX 3

SOLVENCY MARGINS REQUIRED

We have carried out simulations on a variety of different bases using a range of initial solvency margins. This enables us to interpolate back to deduce what initial solvency margin would be necessary in each case to achieve a desired level of probability of ruin. However, without carrying out a very large number of simulations in each case, the probabilities of ruin in each case are subject to significant sampling error, so that only approximate values can be deduced for the necessary solvency margins. These results are given in Table 2.

On the standard basis we estimate that a solvency margin of around 10% of outstanding claims provisions would give a probability of ruin of 1 in 100, whereas a margin of 25% would reduce the probability of ruin to 1 in 1000. This compares with the EEC requirements which we estimate might be equivalent to 30% for short-tail business and 10% for long-tail business, although it should be noted that these are also intended to provide cover for the risk of remaining open to new business, and for risks arising from fraud, negligence, mismanagement, reinsurance failure, etc.

As we have already indicated on page 3, the strength of the reserves is critical to the determination of the adequacy of the solvency margin. It can be seen from the results shown there that a range of effective solvency margins from 10% to 25% relative to our standard reserving basis (inflation 5%; no discounting) is equivalent to changing the reserving basis from an assumed rate of inflation of just over 3% to one of around 7% (no discounting in either case) with a 15% solvency margin throughout.

It is of interest to see how much of the required solvency margin is due to the variability of the liabilities (apart from the uncertainty due to inflation). Setting the stochasticity factor to zero for the liabilities reduces the required solvency margins to 10% and 20% of outstanding claims provisions for probabilities of ruin of 1 in 100 and 1 in 1000 respectively. Increasing the standard deviation to 50% in the distributions of claims payment amounts (if 30% in the standard basis) increases the required solvency margins to 15% and 30% respectively.

The solvency margin requirement is clearly sensitive to the way in which the assets are invested, ranging from 20% of outstanding claims provisions for 100% invested in cash (basis otherwise as for standard) to 35% for 100% in gilts and 75% for 100% in equities, in each case with a view to producing a probability of ruin of 1 in 1000.

Increasing the mean expected value of inflation in the inflation model does not in itself increase the solvency margin required to a significant degree. If the model is assumed to jump suddenly to one with a higher mean expected value from the date of the assessment, the required solvency margin is increased markedly. However, for consistency, allowance should be made for the probability of such a change, with the result that allowing for this factor would probably only increase the required solvency margins to some 15% and 30% for probabilities of ruin of 1 in 100 and 1 in 1000 respectively.

With 100% investment in equities, as well as liabilities with a standard deviation of 50% in the distributions of claims payments amounts, the respective margins become 45% and 85%, which could be shaded up to 50% and 90% to allow for the possibility of a change in the underlying model of inflation.

Assuming that the standard deviation of the yield in the asset model rises to double the level implied by asset movements over the last 50 years or so raises the required margin to 125%, even for 1 in 100 probability of ruin, with 100% investment in equities.

The figures quoted here for required solvency margins all relate to the run-off scenario and reflect in part the structure of the portfolio (type of assets and type of liabilities) and in part one's view of the appropriate models for representing the variability of the assets, the liabilities and inflation. We have shown a variety of results to show the sensitivity to different parameters, but in practice the initial distribution of the assets and the type of liabilities would be known. It would then probably be reasonable to take the Wilkie models for assets and inflation in their entirety as representing a coherent and justifiable model of the underlying variable. We hope to be investigating further whether the results are particularly sensitive to this model, or whether similar results are obtained using other equally justifiable models.

It may be that for this particular application one would want to use a model which gave more emphasis to the possibility of sudden jumps in the underlying level of inflation, and we have shown how to make a crude adjustment for this. However, it ought to be possible to deal with this in a more satisfactory way through the structure of the model itself.

This provides a procedure for analysing the required solvency margin in any particular case. The concepts will need to be further developed to include the risks associated with continued new business for a period and the risks of reinsurance failure. Other risks are not susceptible to treatment in this way (eg fraud) and it has to be considered whether to add a further loading, or assume that the other factors provide solvency margin enough and that all the factors are very unlikely to operate at the same time.

TABLE 2 SOLVENCY MARGINS REQUIRED

	Required solvency margin (±5%) to achieve probability of ruin standard of:	
	1/100	1/1000
Standard basis	10%	25%
Variant bases		
4. Variability of outstanding claims		
(a) Standard deviation 0%	10%	20%
(b) Standard deviation 10%	10%	25%
(c) Standard deviation 50%	15%	30%
5. Inflation model		
(a) Mean expected inflation 10%	10%	25%
(b) Mean expected inflation 15%	10%	25%
(d) Mean expected inflation 15% (from startihg point only)	35%	55%
8. Initial asset distribution		
(a) 100% cash	5%	20%
(b) 100% gilts	20%	35%
(g) L: 100% gilts; S: 100% equities	20%	40%
(i) 100% equities	45%	75%
9. 8(g) with s.d of o/s claims 50%	25%	40%
10. 8(i) with s.d of o/s claims 50%	45%	85%
11. 8(i) with double standard deviation of yield	125%	240%

APPENDIX 4

MEMBERSHIP OF THE WORKING PARTY

The current members of the Solvency Working Party are:

C D Daykin (Chairman)
S M Coutts
E R Devitt
G B Hey
D I W Reynolds
P D Smith
G D Bernstein (Research assistant)