SOME ESTIMATES ON THE STANDARD DEVIATION OF ULTIMATE CLAIMS WHEN JUDGEMENT IS UESED

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1998 GENERAL INSURANCE CONVENTION
AND
ASTIN COLLOQUIUM

GLASGOW, SCOTLAND: 7-10 OCTOBER 1998

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1. Introduction

1.1. In Mack [1993] an estimate of the standard deviation of the ultimate claim amounts is given. This estimate is based on a pure chain ladder approach to reserving, assumes that the initial underwriting or accident year is fully developed (that is, there is no tail). The approach in that paper makes no assumptions regarding the distribution of the link ratios.

1.2. One problem with the Mack approach is that the selected ultimate loss ratio in the actuarial assessment is often different from that calculated by the chain ladder. Indeed, it may be a (significant) number of standard deviations away from the crude chain ladder approach. The reasons for this are many, including

- The potential for a tail
- Insignificant claims (or too many claims) giving unrealistic results, particularly in the more recent years
- Known (and unknown) trends
- Large losses impacting on the results, distorting the features.
- Taking account of the underwriters view.

Furthermore the Mack approach often gives estimates of the standard deviation for the more recent years of such an order that any estimate the range of the likely outcomes has no realistic commercial meaning.

1.3. The actuary arrives at a "best estimate" of the ultimate losses by applying his professional judgement and taking account of a number of factors. This "best estimate" is not usually derived from a statistical distribution, so it is difficult to say whether or not the selected amount is at the 50th percentile of the likely outcomes (or indeed, at what percentile it actually is). In this note, "best estimate" will mean neither prudent nor imprudent. However, it may also be necessary to give a range (say 1 standard deviation) around this best estimate in assessing a realistic prudent reserve for solvency, or, in the case of Lloyd's, for Reinsurance to close (RITC). The judgements made by the actuary could be considered as implicit Bayesian, and this will result in a reduction in the Mack standard deviation. The main purpose of this note is to study the reduction of the standard deviation, and to obtain some understanding of how this judgement fits into a Bayesian approach. The calculation of a specific formula for the standard deviation in this context is complex, so a simulation approach on a specific example will be used to determine the level of reduction in the normal standard deviation is achieved by allowing for judgement. It should be noted that by selecting a distribution for the link ratios, the resulting standard deviation will of necessity be lower than that calculated by the Mack formula. However, the aim is to indicate an approach, and the uniformity in the reduction in the standard deviations could be used to give a likely estimate for the reduced Mack standard deviation.

2. The Classical Approach

2.1. Th e approach taken in the assessment will be illustrated by an example, which is set out in Appendix A. The example is based on real data, which has been substantially adjusted for confidentiality. The data represents a long tailed liability account. The paid claims have been included for completeness, but the basis of the calculations is made on incurred claims. The basis of the approach is as follows.

- First, we calculate the true chain ladder amounts, and the corresponding standard deviations for each underwriting year.
- We then use a Sherman curve to fit the tail. In this case the reserves are recalculated using a tail described by a Sherman curve. The method is based on fitting a curve of the form $\{1+a^*(t+c)^{-b}\}$ to the period by period development factors, using the method of least squares. Here t = period of development, and a,b, and c are factors derived from fitting the curve to the data. This is calculation is made by non-linear interpolation. The tail is calculated assuming the (incurred) claims are reported for 40 years. The Mack standard deviation is calculated relative to this amount.
- The ultimate claim projections are exhibited based on the average, chain ladder and Sherman curve (all having the same "Sherman tail"). Initial loss ratios are then given to give the corresponding Bornheutter-Ferguson ("B-F") adjusted results.

2.2. The selected ultimate losses need not be important. What is important is that the actual selected results can be described mathematically so that a simulation can be undertaken to obtain the results. The approach in this note will be by using the B-F adjustment to the calculated rates. It can be easily shown that any actuarial selected loss ratio can be obtained by selecting an appropriate initial loss ratio (see 3.3 below).

2.3. The expected development of the business is then projected from the known data. The actual data is used to assess an expected link ratio and a standard deviation. This is then simulated into the data up to the end of the period. The whole data square (i.e. the actual data and the simulated data) are then used to calculate average and chain ladder link ratios. The chain ladder link ratios are then used to assess the tail of the distribution by fitting a Sherman curve. The c parameter of the Sherman curve is held constant at the initial value so that the a and b parameters can be assessed by normal linear regression. The calculated tail is then added to the Average and Pure Chain Ladder assessment, and the B-F calculation is made. The process is repeated 10,000 times to obtain a distribution of outcomes.

2.4. There are a number of variations to this approach that may be used other than the one selected. The purpose is to illustrate a typical approach to assess the reduction in standard deviation that may occur when an actuary uses his professional judgement.

3. Results and Observations on the Initial Calculations.

3.1. The calculations in respect of the process are set out in Appendix A. The Selection of the ULR using the B-F method gave the reductions in the unadjusted standard deviation ("SD") are set out in Table 3.1

Table 3.1

Ratio of Adjusted SD to Unadjusted SD

	Average	Chain Ladder	Sherman
1988	95.6%	95.6%	95.6%
1989	93.4%	92.8%	91.9%
1990	91.4%	91.2%	87.7%
1991	87.1%	87.0%	84.0%
1992	83.4%	84.4%	84.7%
1993	78.0%	78.6%	82.0%
1994	66.0%	65.6%	63.4%
1995	45.1%	49.4%	34.9%
1996	11.5%	18.7%	15:7%

One of the problems with the Mack approach, as previously indicated was the high standard deviation at the undeveloped years. It is these years that are the subject of the actuarial judgement. By exercising that judgement (in effect selecting an ultimate loss ratio), the actuary implicitly gives a high credibility to that selection, resulting in a (significant) decrease in the standard deviation. It should be noted that the percentage reduction is, in broad terms, the percentage developed used in the BF calculation.

3.2. What may also a little surprising is the uniformity of the adjustment. Experiments on this data set indicate the reduction percentage is robust against the distribution selected in determining the simulated link ratios (provided the mean and standard deviation were the same). Thus, it appears that a good approximation for the modified Mack standard deviation would be to take the calculate Mack SD and apply the appropriate percentage. Note that the 1988 year, in this example, that there is a reduction in the SD due to a tail factor being incorporated in the simulated model. This would need to be factored out of any direct comparable analysis.

3.3. Any actuarial judgement can be put into this framework. The selection, by the actuary, of an ultimate loss ratio (or implied ultimate loss ratio) may be considered as selecting the chain ladder model (with appropriate variance) and an appropriate (fixed-point) Ultimate loss ratio. This ultimate may be readily calculated from the B-F formula. More specifically;

Let
ILR = the initial loss ratio to be found
SLR = selected loss ratio for the reserves by the actuary
CL = Chain ladder ULR
G = percentage developed on an incurred (paid) basis of the CL ULR

Then ILR = (SLR – G* CL)/(1-G).

4. An Alternative Bayesian Approach

4.1. The approach of using a fixed ULR in assessing the reduction of variance could be readily criticised. When an underwriter selects an initial loss ratio, the uncertainty is often greater than that implicit in the reserving. Set out in Appendix B is an example of a Bayesian approach to this problem.

4.2. We first illustrate in Appendix B how **any** selected Ultimate Losses can be expressed within a chain ladder framework. However, if one considers the implicit selection of the link ratios against the calculated link ratios, and even those given by the selected initial link ratios, it is often difficult to ascertain any consistent pattern in the corresponding link ratio developments. Again there may be good judgmental reasons for this – the chain ladder may need adjusting for secular and underwriting year trends before a consistent model is deduced.

4.3. An alternative is to put the B-F model in a Bayesian framework. If this is done, then an estimate for the initial loss ratio and its standard may be derived (see Appendix B for the calculation). The process can be simulated again using a distribution (in this case Normal) for the estimate of the Initial loss ratio. The expected ultimate values derived are (with a small error) the same. However the reduction in the standard deviation is less severe, as set out in Table 4.1 below.

Table 4.1
Reduction in Standard Deviation

	Average	Chain Ladder	Sherman
1988	101.7%	102.3%	101.6%
1989	100.5%	100.3%	99.6%
1990	97.2%	99.7%	96.8%
1991	94.3%	96.6%	95.3%
1992	94.1%	95.2%	98.2%
1993	92.2%	93.2%	97.2%
1994	84.2%	84.6%	82.7%
1995	67.7%	71.3%	60.5%
1996	33.2%	38.8%	32.3%

4.4. Th actuarial model is so complex that there is no real way to assess the reduction in variance. Classical Bayesian theory has the variance given by

Variance $= E(V(x \| \sigma)) + V(E(x \| \sigma))$

Empirically, the dominant factor in the calculation of the relevant variances is the B-F factor G, so the reduction in standard deviation should, in this case, approximate to the square of the BF factor, which can be seen to be the case.

4.5. In the above approach, normality has been assumed in both the link ratios and the selected initial loss ratio. The issue of judgement could mean that the distribution of simulated chain ladder outcomes should be skewed to the higher level of losses. Against this, the initial loss ratio may be considered pessimistic, that is should skewed to the lower estimates. This estimate is often prudent. The combination of these two types of distribution would need to be considered by an actuary wishing to take this approach.

References

Mack T (1993)
Distribution- free Calculations of the Standard Error of Chain Ladder Reserve Estimates ASTIN Vol 23 Part 2

Appendix A

A1. The Incurred Claims Triangle used in this illustration was as set out in Table A1 below.

Table A1
Incurred Claims Development

Year	1	2	3	4	5	6	7	8	9
1988	1,039	2,922	5,043	5,936	6,699	7,667	7,981	8,301	8,402
1989	1,074	8,612	11,488	12,801	13,763	14,491	15,725	15,820	
1990	1,778	4,878	8,403	9,954	11,861	14,034	14,331		
1991	2,326	9,851	13,259	16,585	20,088	21,318			
1992	3,028	10,383	14,268	18,616	21,650				
1993	3,978	10,250	15,412	19,342					
1994	9,320	23,093	29,606						
1995	5,665	16,680							
1996	6,512								

For the purpose of this exercise, it has been assumed that all exceptional events have been removed from the data, and that all the data should be used in assessing the results.

A2. The corresponding Link ratios are set out in Table A2 below

Table A2
Link ratios and Other Statistics

	02:01	03:02	04:03	05:04	06:05	07:06	08:07	09:08	Tail
1988	2.8120	1.7258	1.1769	1.1286	1.1445	1.0409	1.0401	1.0122	
1989	8.0207	1.3340	1.1143	1.0752	1.0529	1.0852	1.0061		
1990	2.7427	1.7228	1.1846	1.1916	1.1831	1.0212			
1991	4.2348	1.3459	1.2509	1.2112	1.0612				
1992	3.4292	1.3741	1.3047	1.1630					
1993	2.5765	1.5037	1.2550						
1994	2.4779	1.2820							
1995	2.9447								
Average	3.6548	1.4698	1.2144	1.1539	1.1104	1.0491	1.0231	1.0122	
STD	1.8533	0.1866	0.0685	0.0539	0.0637	0.0328	0.0241	0.0000	
Chain Ladder	3.0725	1.3928	1.2263	1.1592	1.0973	1.0510	1.0175	1.0122	
Sherman	2.5350	1.6254	1.2813	1.1370	1.0712	1.0391	1.0224	1.0134	1.0254
Selected Mean	3.0725	1.3928	1.2263	1.1592	1.0973	1.0510	1.0175	1.0122	
Selected STD	1.8533	0.1866	0.0685	0.0539	0.0637	0.0637	0.0637	0.0637	

Clearly alternate standards could be used. In assessing the simulation, we have assumed a constant standard deviation in the years of development where the lack of data gives a low standard deviation. Other techniques that may be considered are fitting an appropriate curve to some of the calculated STD. The Sherman curve derivation is direct from simulated information (see below), so has an implicit standard deviation.

A3. The other information in respect of this sample business were as set out in Table A3 below

Table A3
Other Information

Year	Premiums	Ultimate Premiums	Paid	Outstanding	Incurred
1988	12,196	12,196	4,026	4,376	8,402
1989	16,804	16,804	9,639	6,181	15,820
1990	14,323	14,349	8,830	5,502	14,331
1991	19,576	19,684	10,650	10,668	21,318
1992	26,288	26,500	10,462	11,188	21,650
1993	35,555	36,127	5,022	14,320	19,342
1994	60,951	62,378	5,975	23,630	29,606
1995	56,435	59,977	3,725	12,956	16,680
1996	51,234	64,795	820	5,692	6,512
Total	293,363	312,809	59,148	94,513	153,661

A4. The Initial Loss Ratios Assumed for the B-F Adjustment are set out in Table A4 below. These are based on the underwriters judgement as to the ultimate loss ratio at the setting of the reserves (31[st] December 1996)

Table A4
Initial Loss Ratios

Underwriting Year	Initial Loss Ratios
1988	75.0%
1989	102.5%
1990	110.0%
1991	125.0%
1992	110.0%
1993	90.0%
1994	100.0%
1995	85.0%
1996	105.0%

A5. The base calculations of the data, using traditional actuarial techniques, are set out in Table A5 below

Table A5
Initial Preliminary Results

	Ultimate			B-F				
Year	Average	Chain Ladder	Sherman	Average	Chain Ladder	Sherman	Mack STD	Initial Loss ratio
1988	8,615	8,615	8,615	8,629	8,629	8,629		75.0%
1989	16,419	16,419	16,439	16,449	16,449	16,469	338	102.5%
1990	15,217	15,135	15,226	15,250	15,169	15,258	495	110.0%
1991	23,747	23,660	23,534	23,835	23,754	23,635	1,014	125.0%
1992	26,780	26,367	25,602	27,234	26,865	26,150	1,752	110.0%
1993	27,608	27,306	26,007	29,077	28,825	27,675	2,137	90.0%
1994	51,318	51,253	51,004	55,997	55,952	55,776	3,953	100.0%
1995	42,496	40,219	46,709	47,650	46,517	49,455	4,904	85.0%
1996	60,629	48,243	46,226	67,239	65,363	64,962	15,762	105.0%
Total	272,831	257,217	259,363	291,360	287,521	288,009		

A6. A simulation was then undertaken. In this case, the assumption was that the link ratios for each development period were distributed as a Normal distribution with a mean and standard deviation as set out in Table A2. A different choice of distribution could be made. In addition, selecting a distribution results in a lower standard deviation of the results than the distribution free assumption of Mack. A typical realisation is given in Table A6 below.

Table A6
Realisation of the Simulation

Year		1	2	3	4	5	6	7	8	9
	1988	1,039	2,922	5,043	5,936	6,699	7,667	7,981	8,301	8,402
	1989	1,074	8,612	11,488	12,801	13,763	14,491	15,725	15,820	15,540
	1990	1,778	4,878	8,403	9,954	11,861	14,034	14,331	14,817	14,970
	1991	2,326	9,851	13,259	16,585	20,088	21,318	22,860	22,028	23,209
	1992	3,028	10,383	14,268	18,616	21,650	23,518	24,474	25,847	24,572
	1993	3,978	10,250	15,412	19,342	20,795	24,221	24,795	27,003	25,092
	1994	9,320	23,093	29,606	37,447	43,659	47,217	51,152	53,172	50,087
	1995	5,665	16,680	20,985	26,675	32,197	38,233	38,679	42,483	41,015
	1996	6,512	41,219	63,446	86,826	92,026	97,234	107,887	126,466	120,054

Link Ratio		02:01	03:02	04:03	05:04	06:05	07:06	08:07	09:08	
	1988	2.8120	1.7258	1.1769	1.1286	1.1445	1.0409	1.0401	1.0122	
	1989	8.0182	1.3340	1.1143	1.0752	1.0529	1.0852	1.0061	0.9823	
	1990	2.7427	1.7228	1.1846	1.1916	1.1831	1.0212	1.0339	1.0103	
	1991	4.2348	1.3459	1.2509	1.2112	1.0612	1.0723	0.9636	1.0536	
	1992	3.4292	1.3741	1.3047	1.1630	1.0863	1.0407	1.0561	0.9507	
	1993	2.5765	1.5037	1.2550	1.0751	1.1648	1.0237	1.0891	0.9292	
	1994	2.4779	1.2820	1.2649	1.1659	1.0815	1.0833	1.0395	0.9420	
	1995	2.9447	1.2581	1.2711	1.2070	1.1875	1.0117	1.0983	0.9655	
	1996	6.3297	1.5392	1.3685	1.0599	1.0566	1.1096	1.1722	0.9493	
Average		3.6545	1.4433	1.2278	1.1522	1.1202	1.0474	1.0408	0.9807	
STD		1.8524	0.1883	0.0630	0.0545	0.0557	0.0292	0.0432	0.0422	
Chain Ladder		3.6834	1.4224	1.2873	1.1219	1.0959	1.0693	1.0911	0.9613	
Sherman		2.1538	1.6242	1.3614	1.2209	1.1411	1.0936	1.0641	1.0450	1.1524

	2	3	4	5	6	7	8	9	Tail
Accumulated									
Average	10.2982	2.8180	1.9525	1.5902	1.3802	1.2320	1.1763	1.1302	1.1524
Chain Ladder	10.7187	2.9100	2.0458	1.5892	1.4165	1.2925	1.2088	1.1078	1.1524
Sherman	9.2977	4.3169	2.6579	1.9524	1.5992	1.4014	1.2814	1.2043	1.1524

The chain ladder, averages and Sherman curve factors are thus derived from a combination of known data and simulated data. Also note that the development at period n for underwriting year x differs from that for year y. An alternative approach may be to consider on link ratio applying to all unknown developments. It is also possible to use distributions which restrict the development to ensure, for example, a minimum link ratio development of unity. This was not selected for this note, as the purpose is to establish the level that the standard deviation might be reduced by applying actuarial judgement.

A7. The simulation was run 10,000 times, and the mean results are set out below in Table A7

Table A7
Results of Simulation
Mean

	Unadjusted			B-F Adjusted		
Year	Average	Chain Ladder	Sherman	Average	Chain Ladder	Sherman
1988	8,780	8,780	8,780	8,787	8,787	8,787
1989	16,737	16,739	16,838	16,732	16,728	16,834
1990	15,450	15,431	15,697	15,437	15,409	15,664
1991	24,140	24,131	24,456	24,128	24,097	24,372
1992	27,063	26,891	26,852	27,404	27,221	27,104
1993	27,947	27,849	27,510	29,277	29,162	28,694
1994	52,075	52,268	54,079	56,397	56,466	57,234
1995	42,840	41,024	48,807	47,737	46,799	49,996
1996	61,119	49,195	46,942	67,271	65,368	64,840
Total	276,151	262,308	269,961	293,170	290,037	293,525

It should be noted that the above means are similar to the direct calculated results (Table A5)

A8. Set out in Table A8 is the corresponding Standard Deviations to the above means

Table A8
Corresponding Standard Deviations

	Unadjusted			B-F Adjusted		
Year	Average	Chain Ladder	Sherman	Average	Chain Ladder	Sherman
1988	285	285	285	272	272	272
1989	755	828	704	705	769	647
1990	791	896	853	723	818	748
1991	1,351	1,576	1,739	1,177	1,372	1,461
1992	1,582	1,879	2,482	1,319	1,586	2,102
1993	1,657	1,998	3,204	1,293	1,571	2,626
1994	3,128	3,862	7,222	2,066	2,534	4,577
1995	2,647	3,333	5,852	1,198	1,646	2,044
1996	3,776	6,820	9,421	436	1,273	1,479

The corresponding percentages of the B/F adjusted standard deviation to the unadjusted standard deviation are set out in Table A9

Table A9
Ratio of Adjusted SD to Unadjusted SD

	Average	Chain Ladder	Sherman
1988	95.6%	95.6%	95.6%
1989	93.4%	92.8%	91.9%
1990	91.4%	91.2%	87.7%
1991	87.1%	87.0%	84.0%
1992	83.4%	84.4%	84.7%
1993	78.0%	78.6%	82.0%
1994	66.0%	65.6%	63.4%
1995	45.1%	49.4%	34.9%
1996	11.5%	18.7%	15.7%

Appendix B
A Bayesian interpretation of the B-F method.

B1. When a selection of Ultimate Loss Ratios is made using the B-F approach, there is a natural interpretation of this into a selection of link ratios in a chain ladder calculation. Table B1 below sets out such an interpretation, based on the data and selected ULR's given in Appendix A

Table B1
An Interpretation of the selected Ultimate Loss Ratios

Year	1	2	3	4	5	6	7	8	9	Tail
1988	1,039	2,922	5,043	5,936	6,699	7,667	7,981	8,301	8,402	8,629
1989	1,074	8,612	11,488	12,801	13,763	14,491	15,725	15,820	16,036	16,469
1990	1,778	4,878	8,403	9,954	11,861	14,034	14,331	14,657	14,858	15,258
1991	2,326	9,851	13,259	16,585	20,088	21,318	22,198	22,703	23,014	23,635
1992	3,028	10,383	14,268	18,616	21,650	23,586	24,561	25,120	25,463	26,150
1993	3,978	10,250	15,412	19,342	22,912	24,962	25,993	26,584	26,948	27,675
1994	9,320	23,093	29,606	38,983	46,178	50,308	52,387	53,579	54,312	55,776
1995	5,665	16,680	26,250	34,565	40,945	44,607	46,450	47,507	48,156	49,455
1996	6,512	21,911	34,481	45,403	53,783	58,594	61,014	62,403	63,256	64,962

Link Ratio

	02:01	03:02	04:03	05:04	06:05	07:06	08:07	09:08	Tail
1988	2.8120	1.7258	1.1769	1.1286	1.1445	1.0409	1.0401	1.0122	1.0270
1989	8.0207	1.3340	1.1143	1.0752	1.0529	1.0852	1.0061	1.0137	1.0270
1990	2.7427	1.7228	1.1846	1.1916	1.1831	1.0212	1.0228	1.0137	1.0270
1991	4.2348	1.3459	1.2509	1.2112	1.0612	1.0413	1.0228	1.0137	1.0270
1992	3.4292	1.3741	1.3047	1.1630	1.0894	1.0413	1.0228	1.0137	1.0270
1993	2.5765	1.5037	1.2550	1.1846	1.0894	1.0413	1.0228	1.0137	1.0270
1994	2.4779	1.2820	1.3167	1.1846	1.0894	1.0413	1.0228	1.0137	1.0270
1995	2.9447	1.5737	1.3167	1.1846	1.0894	1.0413	1.0228	1.0137	1.0270
1996	3.3647	1.5737	1.3167	1.1846	1.0894	1.0413	1.0228	1.0137	1.0270

B2. Consider this in a Bayesian context. Although the underwriter (or actuary) appears to be making a fixed selection decision, this is not the case. If it were so, the selected *amounts* would dominate the outcome.

B3 The Bayesian estimate of the Losses is given by the formula

$$(X/S^2 + Y/T^2)/(1/S^2 + 1/T^2)$$

where
X = Estimate of Reserves using Chain Ladder
S = Estimate of Standard Deviation of Chain Ladder
Y = Estimate of Mean of Prior Distribution
T = Estimate of Standard Deviation

Using the classical BF model, we have

BF = G *CL +(1-G)* Initial Losses
Here G = percentage developed / estimated CL ultimate.
CL = Chain ladder Losses

Thus $G = 1/S^2 / (1/S^2 + 1/T^2)$

Given that we know S, then

$$T^2 = G/(1\text{-}G) * S^2$$

B4 Applying this formula to the simulated calculations, we have the estimates set out in Table B1 below.

Table B1
Estimate of the STD of the Initial estimates

Year	CL STD	Factor G	ILR STD
1988	291	0.9752	1,823
1989	842	0.9635	4,329
1990	916	0.9469	3,868
1991	1,612	0.9010	4,863
1992	1,872	0.8211	4,011
1993	2,024	0.7084	3,155
1994	3,924	0.5776	4,590
1995	3,331	0.4147	2,804
1996	6,661	0.1350	2,631

(Note these estimates were based on prior experimental runs, and so don't exactly agree with the results in Table A8)

B5 A second simulation was undertaken, this time with the ILR being normally distributed with the above Standard Deviations. This was to measure the impact of the Bayesian type hypothesis of BF on the calculations. The results are summarised in Tables B2-B4 below. Note in these simulations the standard deviations have been set to the calculated Bayesian factor based on each type.

Table B2
Simulated Expected Values with Bayesian Assumption

	Unadjusted			B-F Adjusted		
Year	Average	Chain Ladder	Sherman	Average	Chain Ladder	Sherman
1988	8,777	8,777	8,777	8,784	8,785	8,784
1989	16,731	16,734	16,831	16,728	16,726	16,829
1990	15,444	15,425	15,689	15,432	15,408	15,659
1991	24,131	24,121	24,442	24,119	24,096	24,366
1992	27,053	26,884	26,834	27,399	27,220	27,096
1993	27,937	27,841	27,488	29,272	29,160	28,685
1994	52,056	52,263	54,030	56,389	56,470	57,217
1995	42,824	41,003	48,758	47,733	46,795	49,988
1996	61,098	49,165	46,936	67,269	65,367	64,826
Total	276,051	262,213	269,785	293,125	290,027	293,450

Table B3
Simulated Standard Deviations with Bayesian Assumption

	Unadjusted			B-F Adjusted		
Year	Average	Chain Ladder	Sherman	Average	Chain Ladder	Sherman
1988	276	276	276	281	282	280
1989	734	807	683	738	810	680
1990	777	878	830	755	876	804
1991	1,323	1,540	1,693	1,248	1,487	1,613
1992	1,541	1,822	2,420	1,451	1,734	2,376
1993	1,620	1,943	3,127	1,493	1,810	3,039
1994	3,071	3,778	7,050	2,587	3,195	5,831
1995	2,606	3,246	5,714	1,766	2,316	3,457
1996	3,719	6,703	11,014	1,236	2,599	3,555

Table B4
Ratio of the Standard Deviations

	Average	Chain Ladder	Sherman
1988	101.7%	102.3%	101.6%
1989	100.5%	100.3%	99.6%
1990	97.2%	99.7%	96.8%
1991	94.3%	96.6%	95.3%
1992	94.1%	95.2%	98.2%
1993	92.2%	93.2%	97.2%
1994	84.2%	84.6%	82.7%
1995	67.7%	71.3%	60.5%
1996	33.2%	38.8%	32.3%