

Stochastic Asset Liability Modelling Working Party

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From the outset the working party assumed that it would not be able to complete its task within a one year time horizon. It would require three years to conclude targets. Consequently this is an interim report. We however provide some food for thought and we would very much welcome any input.

Terms of Reference

The following terms of reference were agreed:

- To produce a framework so that an actuary working in the area of Asset Liability Modelling (ALM) can judge their work. Hence the actuary can determine whether their work is in line with current thinking on ALM.

The following target work products were agreed:

- Investment group paper - June 96
- GISG paper - end July 96
- Possibility of Institute paper - November / December 97

Working Party Approach

Four sub groups of the working party were set up to undertake the more detailed work.

- (a) **Investment models.** These include currency, inflation as well as various asset class models.
- What time horizon e.g. 1 year, 5 years, 10 years.
 - What asset mixes e.g. equity, gilts.
 - What key parameters e.g. inflation, currency.
 - What tail type e.g. centralising or random walk.
 - Look at time series or econometric models.
 - Aim to produce an overview of the models.

- (b) **Liability models.** These include the outstanding claims reserves using conventional approaches as well as the impact of reinsurance, reinsurance security, gross and net models and correlations between classes of business.
- Look at frequency / severity models.
 - Should models be net or gross? What allowance for bad debts.
 - Correlation by: class of business, territories.
 - Consider the implications of the underwriting cycle.
 - How to allow for the key parameters e.g. inflation, discount rates, currencies.
 - How to deal with potential aggregations.
 - Different models may be required for runoff of current and future business.
- (c) **Correlations between asset variables and liabilities variables.** This is set up as a separate group to avoid the distractions of the complexity of either investment or liability models.
- There is a need to understand the factors affecting the variables e.g. inflation could be driven by legal awards or RPI.
 - How to allow for underwriting cycle.
 - Key parameters e.g. inflation, interest rates.
 - Correlation by business class e.g. interest rates and mortgage indemnity, bankers bond and economic recession.
 - There is a need to work closely with other sub groups.
- (d) **Asset liability modelling techniques and uses.** This considers how the two are brought together in a practical sense. It also considers presentation of results and the uses of such techniques.
- Aim to provide an overview of uses.
 - Document strengths and weaknesses of each model.
 - What are the practical applications.
 - Optimisation techniques (Julie Griffiths is chairing a working party on this which may be relevant).
 - Problems with computer processing time.
 - How ALM could help with capitalisation.

It was originally intended to set up a fifth group to deal with insurance futures and hybrid securities. Although they are not directly related to insurance asset liability modelling there is an overlap in interest and technique and the working party had agreed to keep a watching brief on the topic. However there has not been much in the way of significant development in this area and we have not undertaken any further work in this field.

One purpose in setting up a separate asset model working party was so that it could report to the FIMAG Conference in July. It also served the purpose of providing a focus for actuaries who are principally involved in asset and FIMAG members rather than GISG members. A presentation of some of the working parties work was made at the FIMAG Convention.

In addition to the four sub groups, the main working party met to oversee the process and to form a holistic view. The group felt that it was important to look at the issues as a whole as well as undertake some detailed work. This ties to the objective of the group to consider the inter relationships of these topics in a general insurance context. It was also designed to reduce the extent of any overlap with other parts of the profession. In particular there is much activity in other areas with asset models.

To date the whole working party has acted more as a review body providing direction rather than actually pulling each individual pieces of work together. This largely reflects the fact that much of the detailed work has not yet been completed.

We now describe the results of each of the four sub working parties:-

Assets

In part, due to a number of the asset specialists being involved in other areas of Institute activity but also because of the work being done elsewhere in the profession, it was not felt to be high priority to actually undertake asset modelling directly. This will have however to be reconsidered but it was felt more important to get the framework correct rather than replicate work being carried on elsewhere.

Type of Model

One of the key issues identified was that the actual choice of the model may have a very significant impact on the results of the work e.g. a model with a model with a centralising tendency such as the Wilkie model could produce very different results to a pure random walk model in certain circumstances. Econometric type models, which are often used for shorter term forecasting rather than the more longer term models of either Random Walk or Wilkie type models, allow the relationships with the liability side to be modelled more easily. The choice of investment model would to some extent be reflected by the nature of the study that the actuary is undertaking as well as individual actuaries views of specific investment models.

Length of Liabilities: an appropriate time-frame over which the model should be useable might be two years.

Currency: We decided that the model should not include foreign currencies at first. This would limit its application and some thought was given as to how currencies could be modelled. Purchasing power parity is an attractive long term assumption but it is well documented that currency movements differ substantially from relative purchasing power parity in the short term (our own horizon). Portfolio trades; speculation; monetary shocks; and saving and investment flows have important short and medium term effects on exchange rates. It may be possible to model some of these at a later stage. Alternatively, a random walk style model for currencies could be used. However, on balance we felt we would ignore currencies for the moment. If and when we do model currencies, the Bank of England or investment banks may have some useful ideas.

Inefficient Markets? The Wilkie model assumes that investment markets are inefficient. Regardless of the reasonableness of this assumption for long term modelling, we had to decide whether to incorporate an efficient market assumption in a shorter term general insurance model. On balance, we would not wish to do this or at least it would not be a requirement of the model. Modelling inefficiency itself is difficult. It would be very dangerous to model the return to an efficient position over a short time period.

Variable and Covariance Structure. Given the aspects of investment markets that we have decided not to model, it will be important to ensure that the investment model appropriately captures the short term variance structure of and covariance structure between asset classes. If it is not possible to do this, it will be important to be aware of the limitations of model. For example does the covariance structure collapse when it most inconvenient? Are variances of and covariances between asset classes stable? Are normal residuals reasonable? The work done by those investigating banking market risk was recognised as being of likely importance.

Investment Categories to be Modelled. It was decided that it would be reasonable to try to model domestic equities, cash, short term bonds and possibly longer term bonds. As noted, overseas investments would be a long term goal.

The working party also considered some of the key characteristics of the different types of models that were important from the perspective of general insurance.

Random Walk Type Models

They normally have the following characteristics:

- 1 Use of "naive" correlation structure between asset types.
- 2 Have normal residuals.
- 3 Based on historic data.
- 4 No economic input.
- 5 No explicit feed through of inflation.
- 6 Could incorporate shocks.

The Wilkie Model

- 1 Inherently a long term model.
- 2 Worth looking at Daykin & Hey paper.
- 3 Allow investors to take advantage of underpriced markets.
- 4 It deals with the feed through of inflation but in a way which may not be appropriate for the short term.

- 5 The Wilkie model can be adjusted to remove some of the characteristics which may be more appropriate for the long term.

Econometric Models

- 1 Only interested in investment variables and the way in which economic variables affect investment variables.
- 2 Could include useful medium term relationships such as the effects of monetary shocks, interest rates etc on inflation, currencies and asset values.
- 3 Well established literature on the value and validity of econometric models.
- 4 Little consensus on the appropriateness of different models.

At this stage the working party has made no attempt to reconcile the results of different types of asset modelling or indeed make any recommendations as to which one to specifically use. As stated above, there are a number of Institute working parties operating in this area and that is something that the working party should monitor in the future. It should also be recognised that there are no "right answers". The correct model for one circumstance may not be the most appropriate in others.

Liability Modelling

Liability Modelling

This is an area that comes more naturally to general insurance actuaries than asset modelling. It is an area where much work has been undertaken in the past. Time pressures restricted the amount of work we were able to do in this area and at this stage it should be regarded very much as an overview.

It is also worth noting that as with asset models there is not necessarily one "right answer" as much will depend on the nature of the liabilities being considered. For many companies a broad brush approach using a central limit therein may be all that is necessary as the asset risk is more important. This might be true of a number of large UK composite groups. However for many London Market operations, the reverse would be true and detailed liability modelling would be required and the assets movements may not be material as the company would often be taking very little asset risk.

Chairman - Catherine Cresswell.

Asset Liability modelling

Liability Models.

We aim to review and develop both liability models which have applications in isolation (these will be the focus of our breakout workshop - liability modelling for beginners) and those which (we hope) will be readily and usefully integrated with an asset model. The vast bulk of this work remains to be done. Here we sketch out an approach to the problem.

Frequency Severity models

Frequency severity models aim to split claims experience into two components, the frequency with which claims occur, and the severity of those claims. Both components are normally allowed to vary stochastically. Different cohorts of claims may be modelled by using different frequency and severity distributions for each component and summing the results, or by using a severity distribution which reflects several components. Both frequency and severity distributions can be either empirical distributions based on past experience, or theoretical distributions. The theoretical distributions can either be chosen by fitting past data, or on a “what if” basis.

The basic model

The basic model is just a compound of the frequency and severity distributions. We pick a random number and use the inverse of the frequency function to generate a simulation of the number of claims made in a period. For each of these claims we pick another random number and use the inverse of the severity distribution to generate a claim amount for each claim.

Distributions that might be chosen for the frequency component included Poisson, normal, negative binomial, chi-squared or an empirical distribution based on past experience.

[pictures]

Distributions that might be used for the severity distribution include the gamma, log-normal, pareto, or an empirical distribution based on past experience.

[pictures]

A simple example: Change in deductible. [o/s - CCr]

An extended example: Efficacy of reinsurance [o/s - SR]

Frequency Severity models: Linkage.

Once a basis frequency severity model has been established we need to extend it to model more complex processes and interactions. These might include:

- * Models of how the valuation of a set of claims evolves as the claims mature to ultimate. This could reflect changes over time of the economic and social environment. E.g. to reflect the impact of inflation on claims settlements.
- * Models that couple claims to economic / investment / social variables.
- * Models which allow for correlation between classes.

Asset liability applications are likely to need at a minimum the first two features, model offices all three. In addition model office applications will require simulations of premium rates and exposure levels.

[This is for information for the asset modelling & correlation subgroups, rather than for publication].

Models that couple claims to economic / investment variables

We propose examining frequency severity models that are responsive to the economic / investment / social environment by allowing the distributions for frequency and severity to change over time according to changes in social, economic or investment factors. We therefore let the frequency & severity distributions be characterised by the following parameters.

Frequency

mean of the frequency distribution to be given by:

$$\frac{\text{exposures}(t = 0)}{\text{exposures}(t = -1)} * \\ [a * \{ \text{observed_freq}(t = -1) \} \\ + b * \{ \text{observed_freq}(t = -1) * f(\text{economic/investment indices}) \} \\ + c * \{ g(\text{economic/investment indices}) \}]$$

where,

f and g are functions of current economic, social or investment indices, or lagged economic, social or investment indices or movements in economic, social or investment indices.

a,b,c are constants.

Setting one or more of a,b,or c to zero will produce a simpler model.

Clearly, other formulations are possible, e.g. weighted averages of past exposure and frequency could be used.

We propose only to consider one parameter distributions, or a normal distribution approximating a Poisson with its variance set equal to its mean. That is, we are assuming the spread and shape of the frequency distribution remains unchanged. This assumption may not hold in practice, but we would expect it to be a reasonable approximation in most circumstances.

Severity

mean of the severity distribution to be given by:

$$\begin{aligned} & [l * \{ \text{observed severity}(t = -1) * \text{severity trend} \} \\ & + m * \{ \text{observed severity}(t = -1) * s(\text{economic/investment indices}) \} \\ & + n * \{ t(\text{economic/investment indices}) \}] \end{aligned}$$

where,

s and t are functions of current economic, social or investment indices, or lagged economic, social or investment indices or movements in economic, social or investment indices.

l, m and n are constants.

What severity is being modelled, and hence what "severity" means in the above formulation depends on the application. The simplest case would be where the ultimate severity of claims is known in the year of origin, and the average severity simply the average of these claim amounts. However there are alternatives, such as the severity of claims settled in the year.

Again one or more of the constants l, m and n may be set to zero, or the severity trend set to 1 to yield simpler models.

Again, we propose to consider essentially one parameter distributions, and so propose assuming that the spread of the distribution changes in line with its mean. We thus demand a constant coefficient of variation, and will set

standard deviation of severity distribution to be equal to

$$\text{base severity s.d.} * \frac{\text{severity mean} (t = 0)}{\text{base severity mean}}$$

This is probably reasonable for from the ground up losses, but may be inappropriate for excess or reinsurance losses.

These relationships, together with forecasts, or simulations of the exposure levels, severity trend and economic, social or investment variables can then be used recurrently to generate simulations for multiple future periods.

The next step will be to derive a payment algorithm so that the outstanding liability at the end of each period can be simulated from the ultimate claim amounts found above.

Liability Models.

We aimed to review and develop both liability models which have applications in isolation (these will be the focus of our breakout workshop - liability modelling for beginners) and those which (we hope) will be readily and usefully integrated with an asset model. The vast bulk of this work remains to be done. Here we sketch out an approach to the problem.

Frequency Severity models

Frequency severity models aim to split claims experience into two components, the frequency with which claims occur, and the severity of those claims. Both components are normally allowed to vary stochastically. Different cohorts of claims may be modelled by using different frequency and severity distributions for each component and summing the results, or by using a severity distribution which reflects several components. Both frequency and severity distributions can be either empirical distributions based on past experience, or theoretical distributions. The theoretical distributions can either be chosen by fitting past data, or on a "what if" basis.

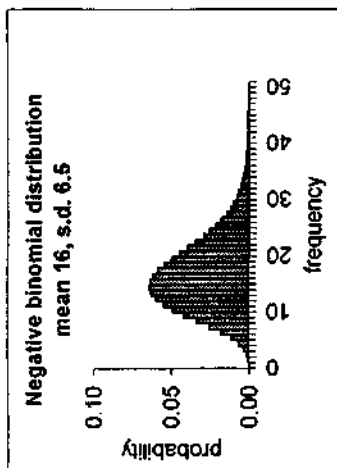
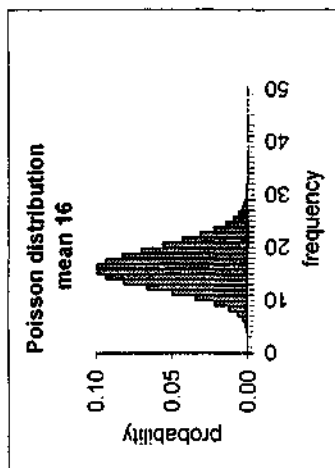
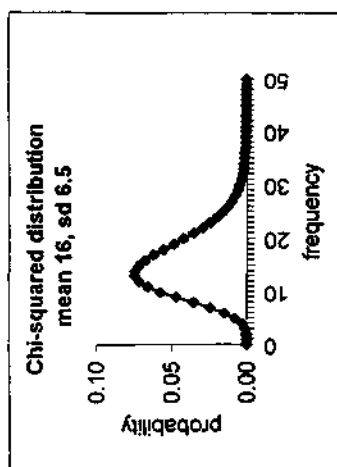
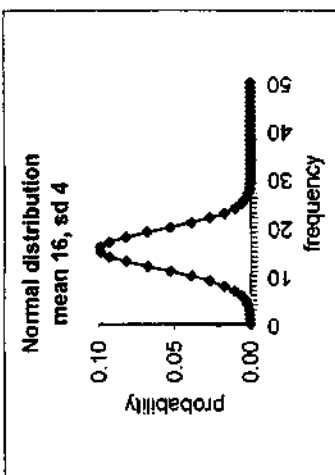
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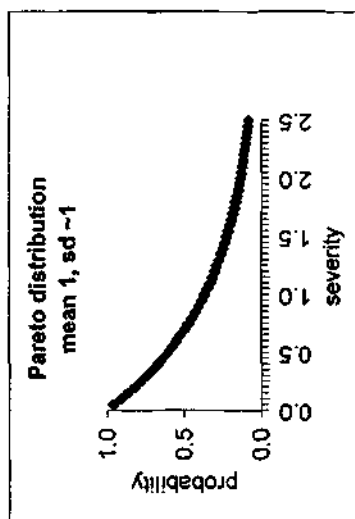
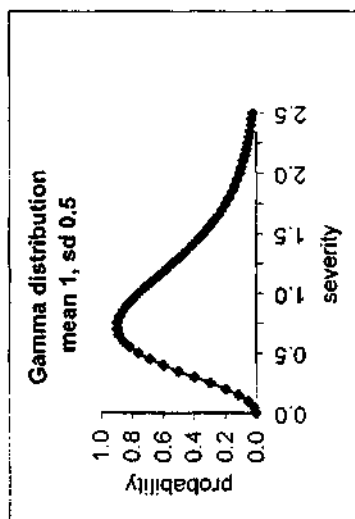
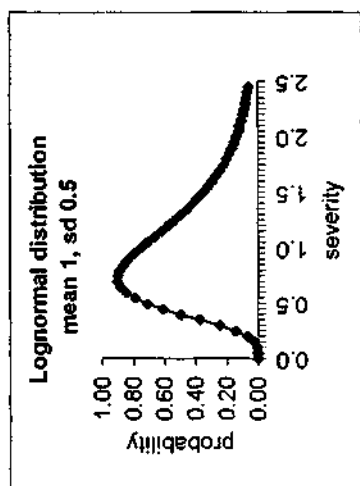
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Examples of these distributions are shown below.





At the workshop we will present some simple applications of frequency severity models to estimate the impact of changing deductibles and to assess the efficacy of a reinsurance program.

Frequency Severity models: Linkage.

Once a basis frequency severity model has been established we need to extend it to model more complex processes and interactions. These might include:

- * Models that couple claims to economic / investment / social variables.
- * Models of how the valuation of a set of claims evolves as the claims mature to ultimate. This could reflect changes over time of the economic and social environment. E.g. to reflect the impact of inflation on claims settlements.
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