Stochastic Asset Models Update 2001

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Session Objectives

- A hands-on session for existing model users
- Some models need updating to allow for recent developments. Show how to:
  - use our toy model: Timbuk1
  - calibrate models to market information
  - use fat-tailed distributions
  - add deflators to your existing model

Part I

Using the Timbuk 1 Model

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Steps to use Timbuk1

- You need Excel (Office 97 or 2000) running under some version of Windows
- Download Timbuk1.xls and Timbuk1SimTool.xla from http://www.actuaries.org.uk/cpd/conf_papers.html into the same directory.
- Load Timbuk1.xls into Excel (Do Enable Macros!) …
  - yield curves, σ, λ *etc*: you can edit these
  - can produce simulated data (including deflators)
  - and percentiles of distributions
- Advanced users: edit code, to output in other formats

Timbuk1 Demo

Run Timbuk1

Others Looking at these Issues

	Full yield curves	Fat Tails	Deflators
Barrie & Hibbert	✗	✗	(✓)
Cairns	(✓)	✗	(✓)
Chapman, Gordon, Speed	✗	✗	✓
Random Walk*	✗	✗	(✓)
Smith Jump-Equilibrium*	(✓)	✓	(✓)
Teeger & Якубов	✗	✗	✗
Timbuk 1*	✓	✗	✓
Whitten & Thomas	✗	✓	✗
Wilkie*	✗	✗	✗

For industrial strength work, many actuaries prefer proprietary models with additional features. Eg The Smith Model, Falcon Asset Model, CAP:Link etc.

** working implementations in Excel freely released*

Part II

Using Market Information,
the Whole Market Information
and Nothing but Market Information

What is a market-calibrated model?

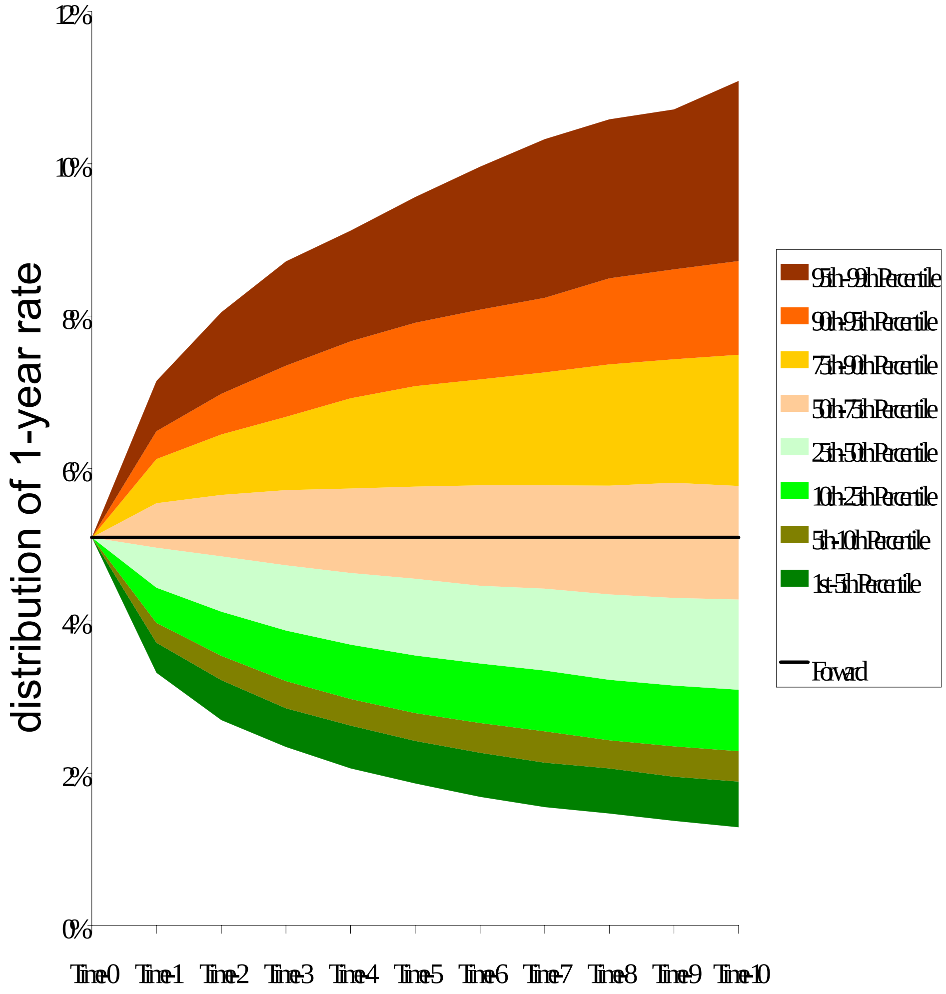
- Initial starting point fits a large number of observed market prices on the run date
  - yield curves, credit spreads, option prices
  - in this talk, we focus on yield curves
  - other aspects harder; same principles apply
- Subsequent evolution reflects initial market views
  - minimal influence of model builder’s opinions
- Necessary for fair values – and useful wherever objectivity is important

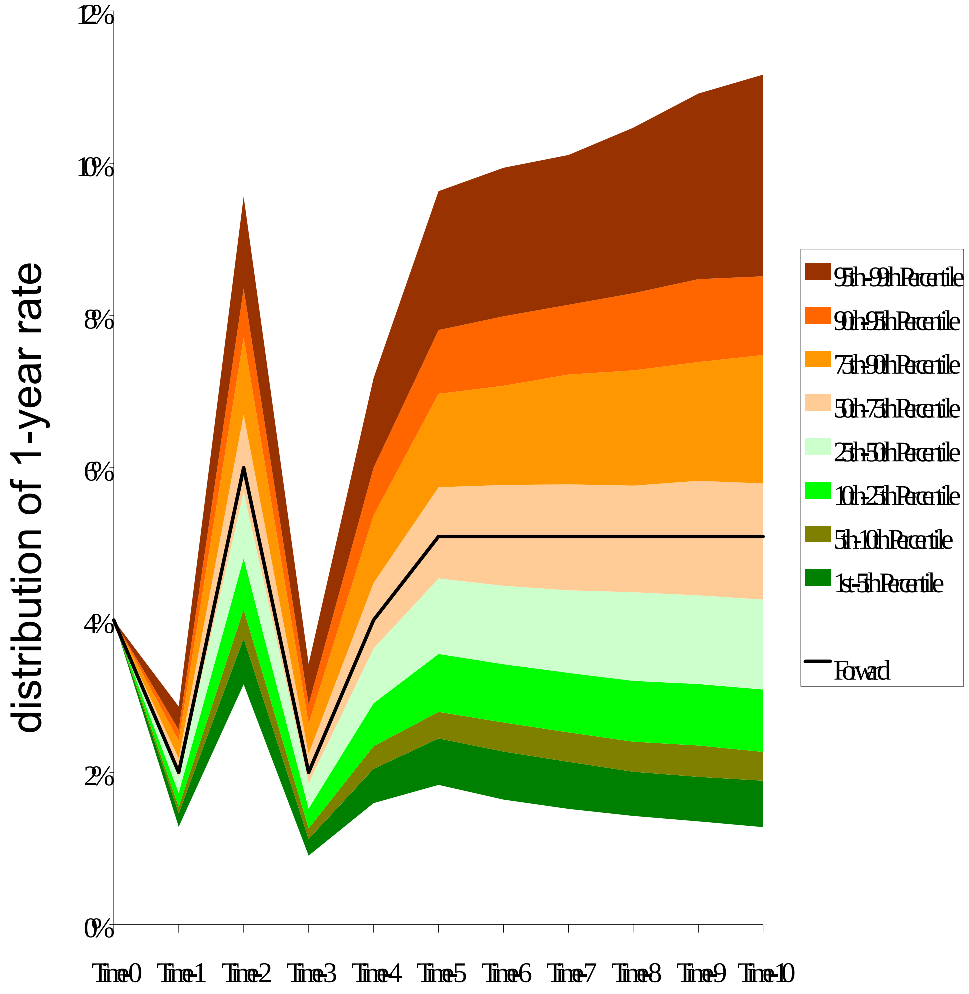
Structure of a Market Calibration

Forecasts at 2005 = Forecasts at 2004 + Error Terms + Risk Premiums

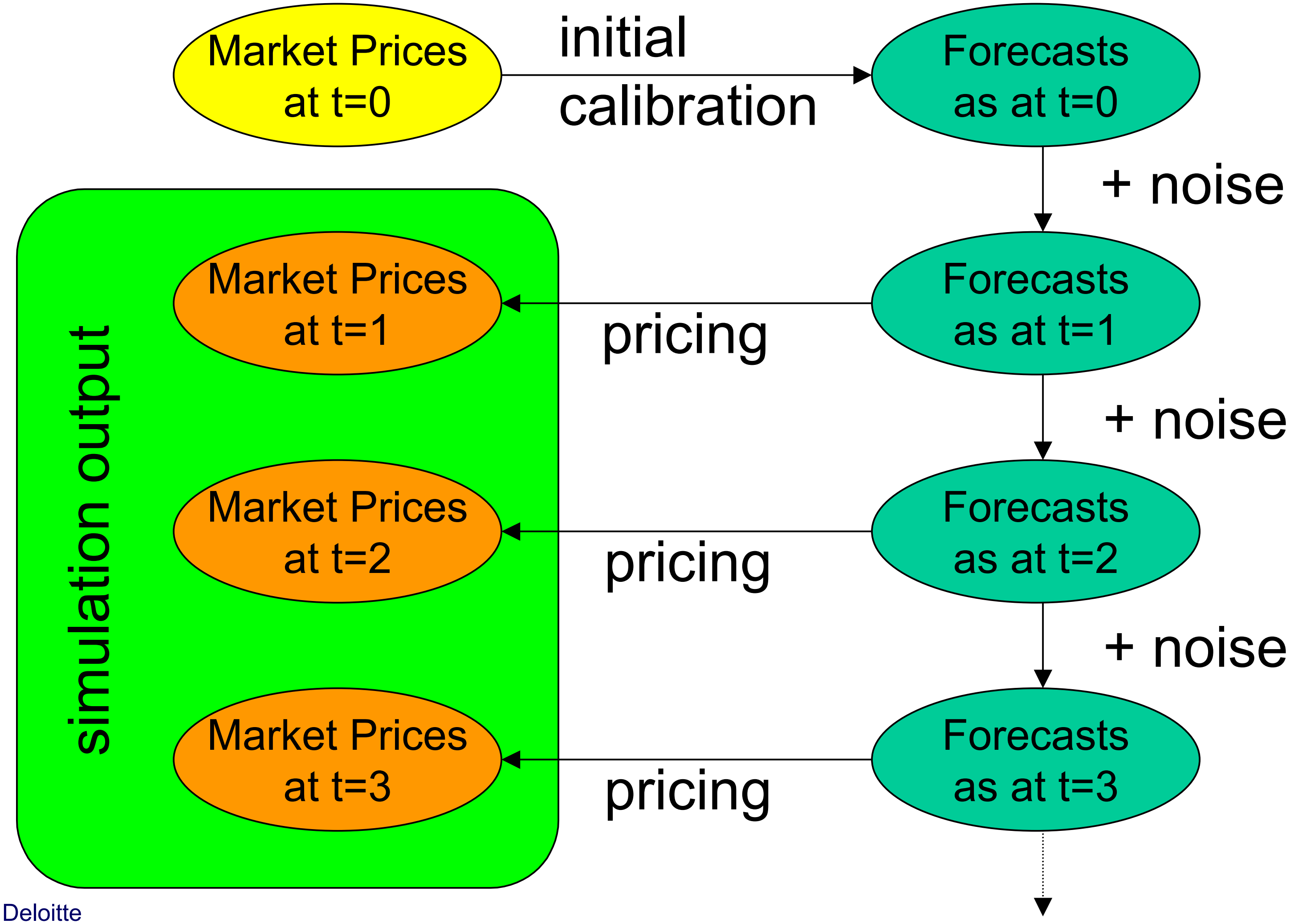
Forecasts at 2004	Error Terms	Risk Premiums
We Look At This Now		
Part II	Part III	Part IV

Initial Forward Curve drives Projection





Market Information and Projection



Time Series Approach Comparison

Table of Simulated Forward rates

Forward Date

Valuation Date	2002	2003	2004	2005	2006	2007	2008
2001	5.0%	5.5%	5.8%	5.9%	6.0%	6.0%	6.0%
2002		6.1%	6.4%	6.5%	6.6%	6.6%	6.6%
2003			6.0%	6.1%	6.2%	6.2%	6.2%
2004				5.8%	5.9%	5.9%	5.9%
2005					6.6%	6.6%	6.6%

Time-series approach looks
at statistical properties
of a 2-year rate.

Time Series Approach Comparison

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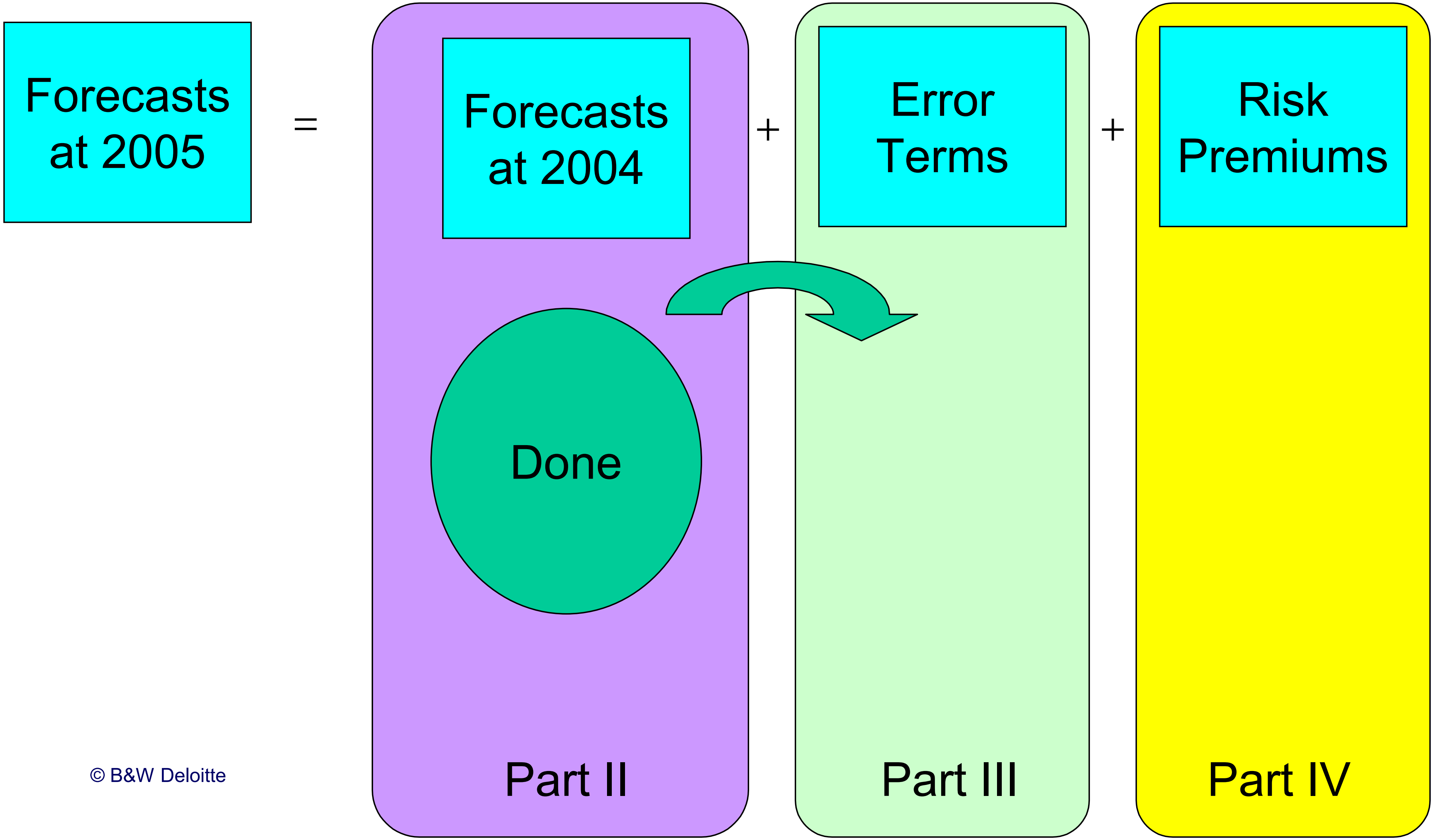
Forward Date

Valuation Date

	2002	2003	2004	2005	2006	2007	2008
2001	5.0%	5.5%	5.8%	5.9%	6.0%	6.0%	6.0%
2002		6.1%	6.4%	6.5%	6.6%	6.6%	6.6%
2003			6.0%	6.1%	6.2%	6.2%	6.2%
2004				5.8%	5.9%	5.9%	5.9%
2005					6.6%	6.6%	6.6%

Market-driven approach looks at economic evolution of a forward rate on fixed forward date.

Structure of a Market Calibration



Part III

Error distributions: fat tails, jumps and other non-normal phenomena

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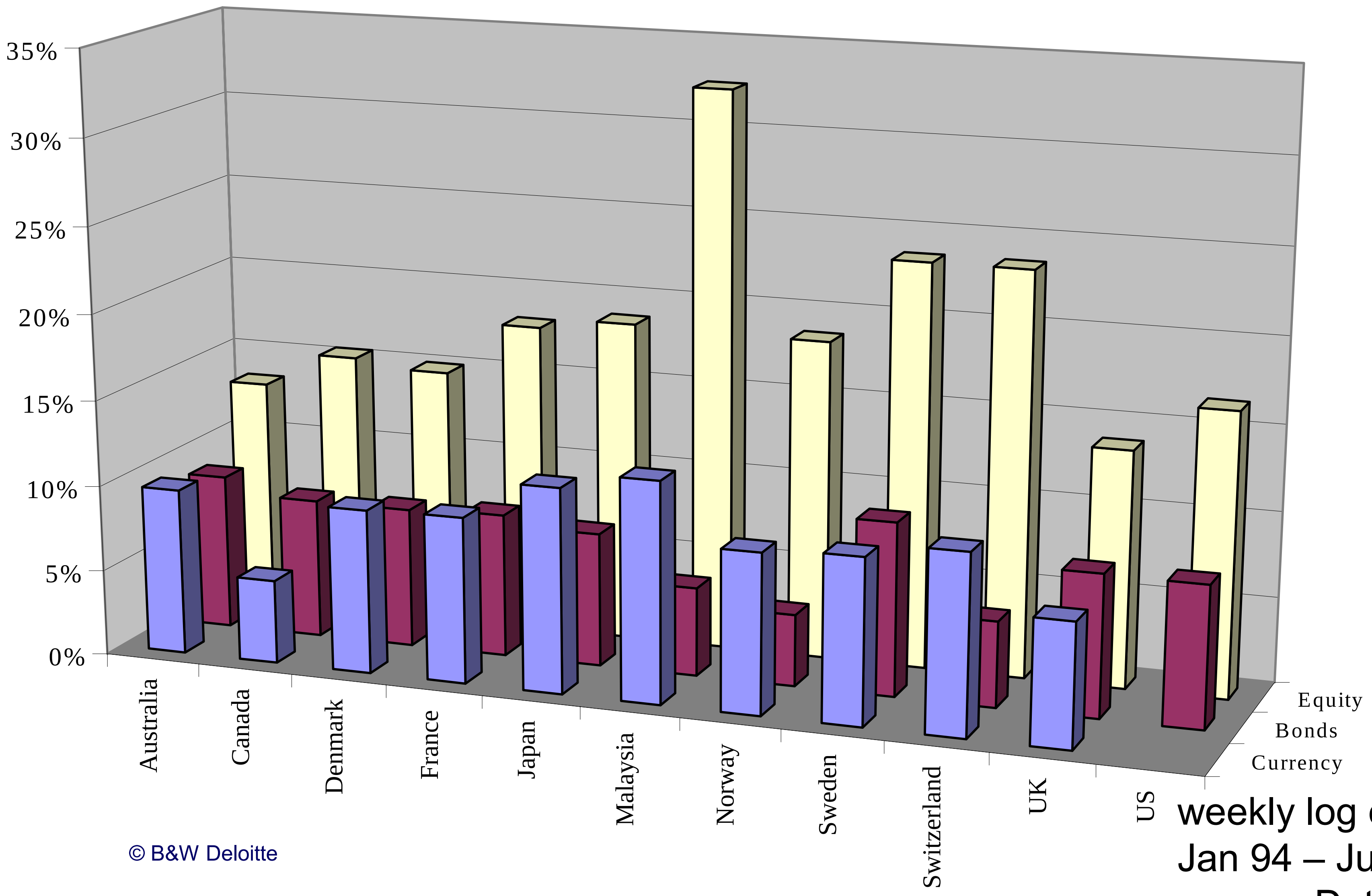
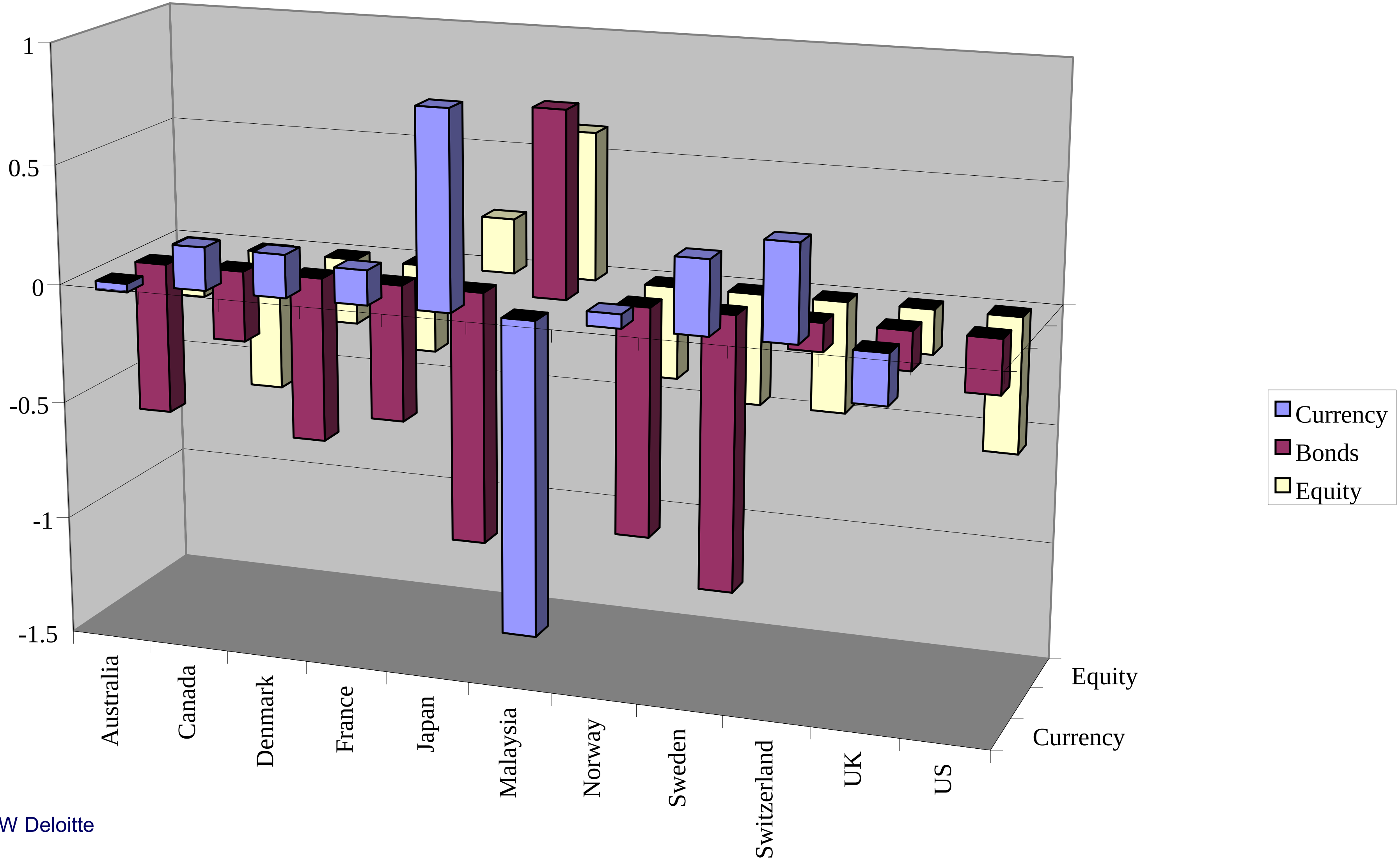
Historic Volatilities (Annualised)



weekly log excess returns
Jan 94 – June 01
source: Datastream

Historic Sample Skewness

Zero = normal



Two Approaches to Non-Normality

<u>Cascade Structure</u>

<u>Conditional Normal</u>

Strengths and Weaknesses
to be Investigated

Modelling Errors: Cascade Form

log(1+ excess return) on:

		$*Z_1$	$*Z_2$	$*Z_3$	$*Z_4$	$*Z_5$	$*Z_6$	$*Z_7$	…
Aus currency	$= RP_1 +$	0.01344							
Aus bonds	$= RP_2 +$	-0.00104	0.01250						
Aus equity	$= RP_3 +$	0.00038	0.00535	0.01769					
Can currency	$= RP_4 +$	0.00203	0.00090	0.00065	0.00634				
Can bonds	$= RP_5 +$	-0.00035	0.00688	0.00036	0.00130	0.00882			
Can equity	$= RP_6 +$	0.00195	0.00257	0.01108	0.00258	0.00215	0.01764		
Den currency	$= RP_7 +$	0.00324	-0.00033	-0.00222	-0.00063	0.00090	-0.00146	0.01254	
…									

Choose these coefficients so that $Z_1 \cdots Z_{32}$ are uncorrelated, with zero mean and unit variance. This is an iterative regression process called "Cholesky decomposition"

Error Distributions

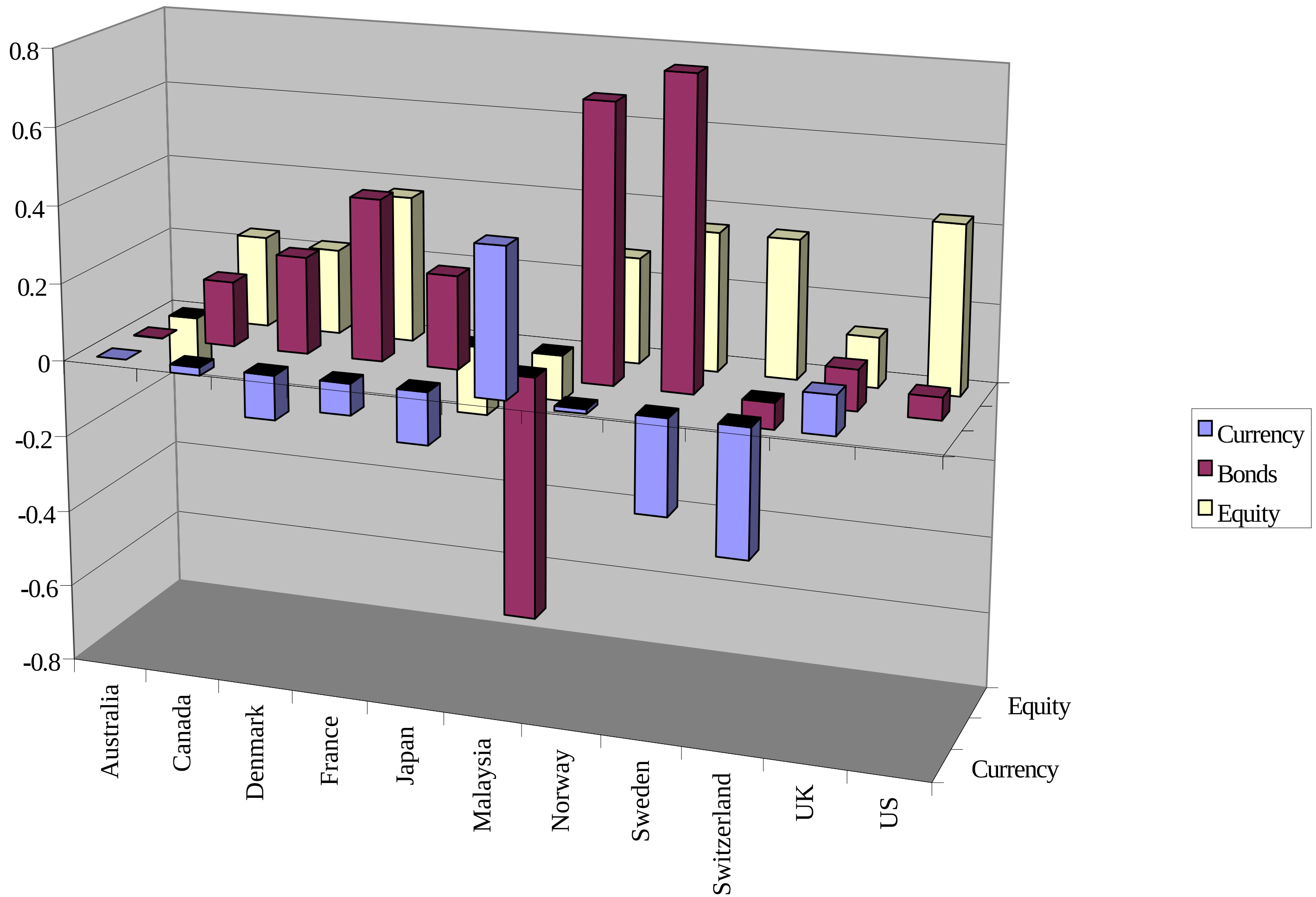
- We have now re-expressed the excess returns in terms of uncorrelated noise terms *Z*
- Simplest to assume they are independent
  - remember, independent implies uncorrelated
  - but uncorrelated does not imply independent
- Analyse error terms, choosing a distribution to fit skewness and kurtosis (eg difference of two Gamma distributions in Jump-Equilibrium model)

Putting the Model Back Together

- We decomposed error terms into independent pieces
- Analysed each piece
- And put them back together again
- Question: do we recover the historic properties of the original series?
    - Intuitively we should
    - after all, we've only manipulated historic data
    - where else could projected skewness come from?

Skewness: Projected minus Historic (Cascade Model) – Oh Dear!

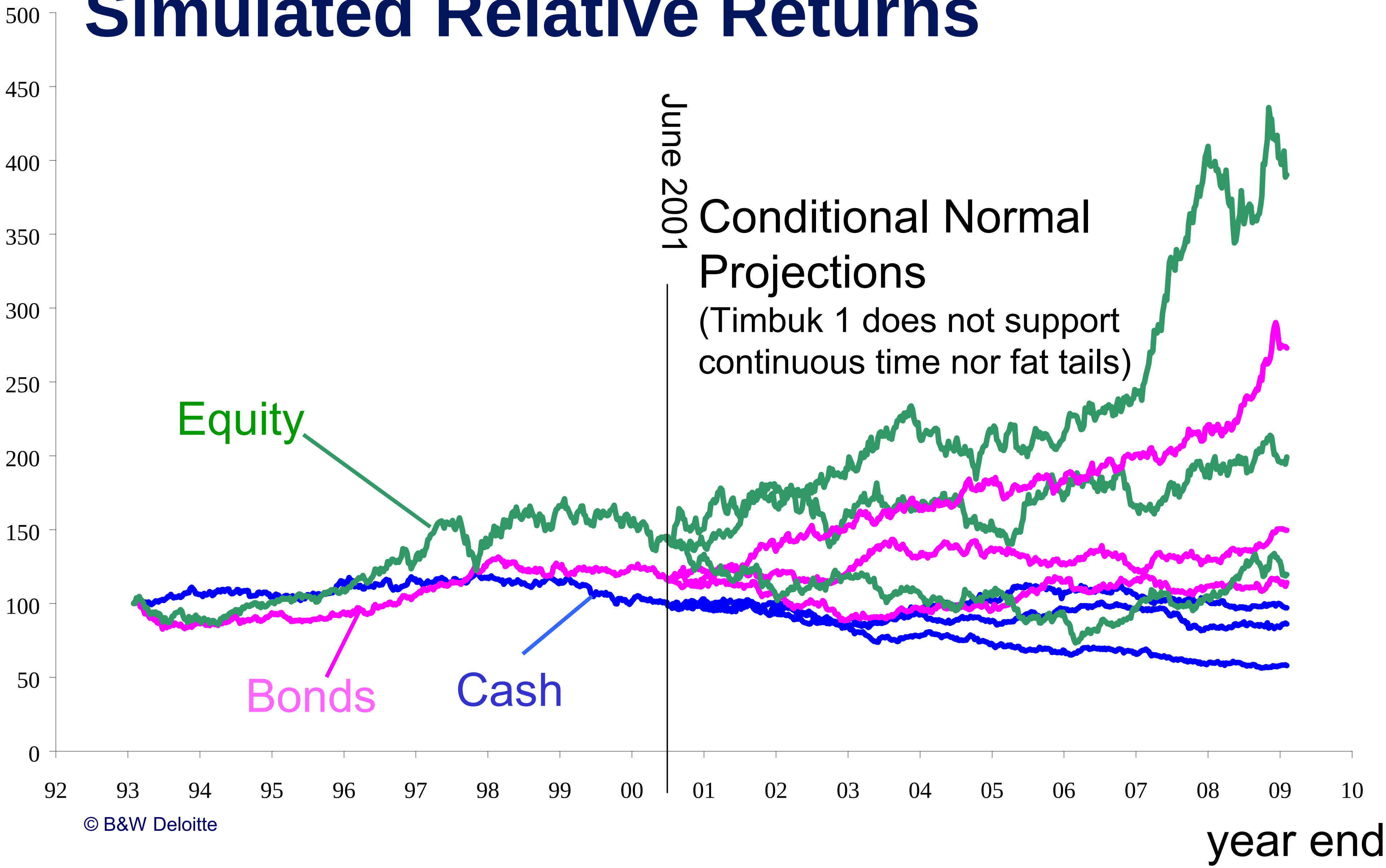
If the process worked, these should all be zero. Note deterioration from left to right.



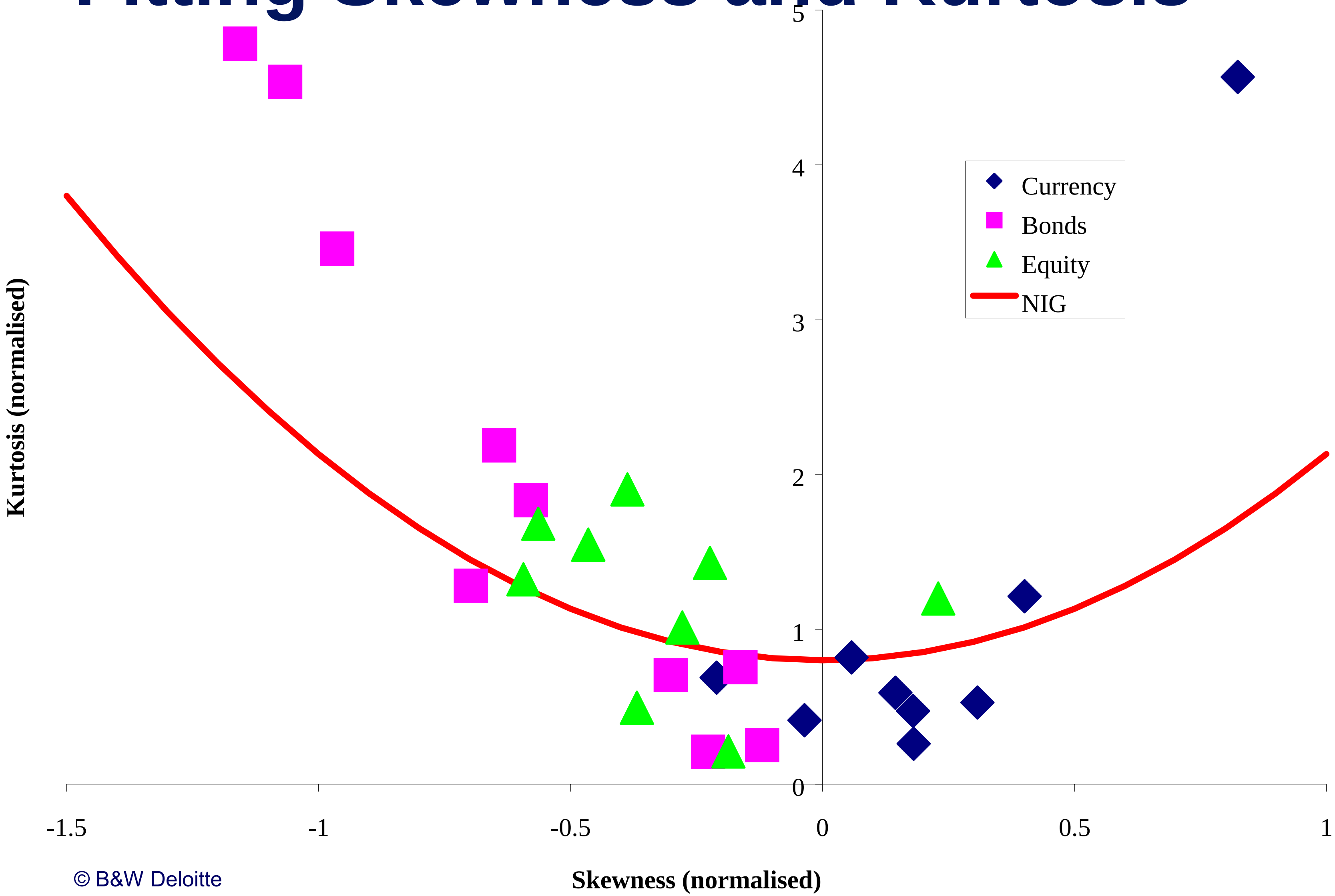
Conditional Normal Approach

- Want to simulate a vector X of noise terms
- Use the following algorithm:
  - $X \sim \mathrm{N}[(T\text{-}1)m\ ,\ TV\ ]$
  - T is a positive random variable, $\mathbf{E}(T) = 1$.
  - vector m, matrix V
- Calibration involves finding the distribution of T and the parameters m, V.
  - for example, proceed by the method of moments, based on an inverse Gaussian distribution for T

Simulated Relative Returns



Fitting Skewness and Kurtosis



Comparison of Approaches

Cascade Structure

- Can fit skewness and kurtosis of residuals
- May not replicate properties of historic price series
- Special case: independent components
- Depends on the sequence of analysing the data

Conditional Normal

- Kurtosis constrained to be a function of skewness
- Can replicate historic skewness, within limits
- Special case: rotationally symmetric distributions
- Can add / remove data series without upsetting other series

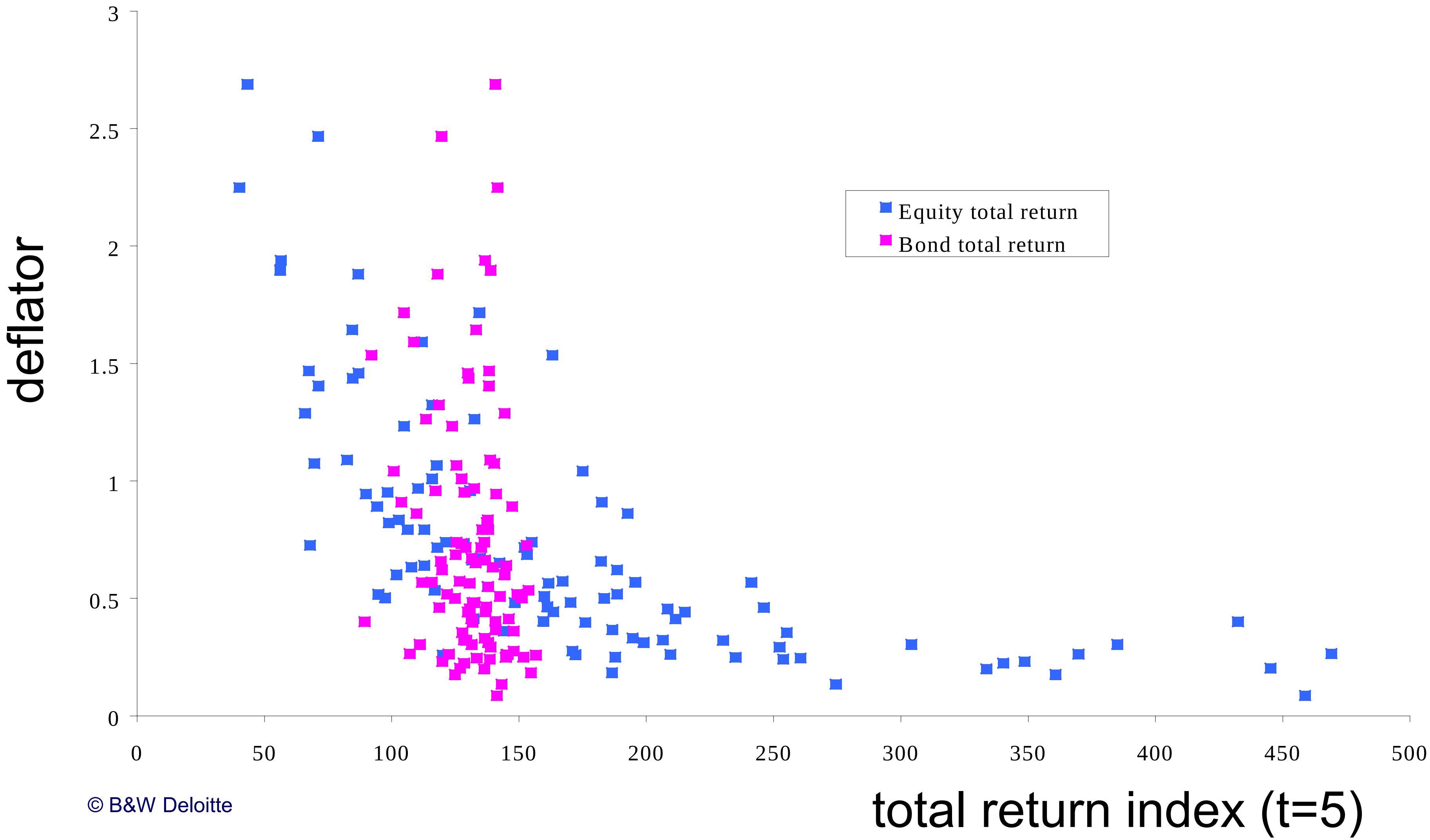
Part IV

Valuing cash flows: risk premiums, arbitrage, efficiency, consistency and deflators.

Issues in valuing cash flows

- Stochastic simulation gives probabilities
- Estimate how the market would price cash flows
  - allowing for risk
- In many cases, we know the answer (important check)
  - when cash flows arise from a traded investment
- One of several techniques proposed is “deflators”
  - essentially a multiperiod, multi-asset CAPM
  - today we assume you’ve seen these before

Timbuk 1: Returns and Deflators



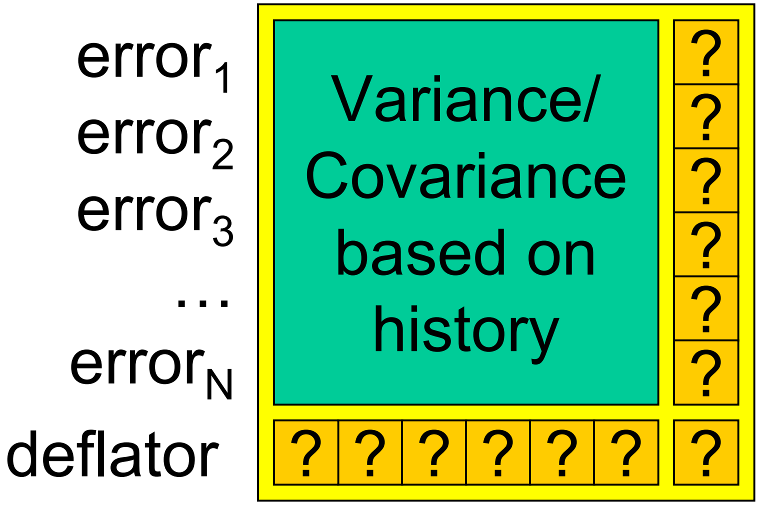
Model dynamics specified by:

$error_1$
$error_2$
$error_3$
…
$error_N$

Variance/ Covariance based on history

- Mean
- Variance-covariance matrix
- Higher moments

To generate deflators, we need:



- Mean deflator (discount factor)
- Variance-covariance matrix
- Higher moments

Question: Where do the covariances come from?

Deflator Covariance Formulas

Array of Forecasts at 2005	=	Array of Forecasts at 2004	+	Error Terms	+	Risk Premiums

Forward rates:

$$\frac{P^{USD}_{2005:2009}}{P^{USD}_{2005:2010}} - 1 \quad = \quad + \quad - \quad \mathbf{Cov}_{2004}\left(\frac{P^{USD}_{2005:2010}}{P^{USD}_{2004:2010}} \frac{D^{USD}_{2005}}{D^{USD}_{2004}}, \frac{P^{USD}_{2005:2009}}{P^{USD}_{2005:2010}} - 1 \right)$$

Exchange rates:

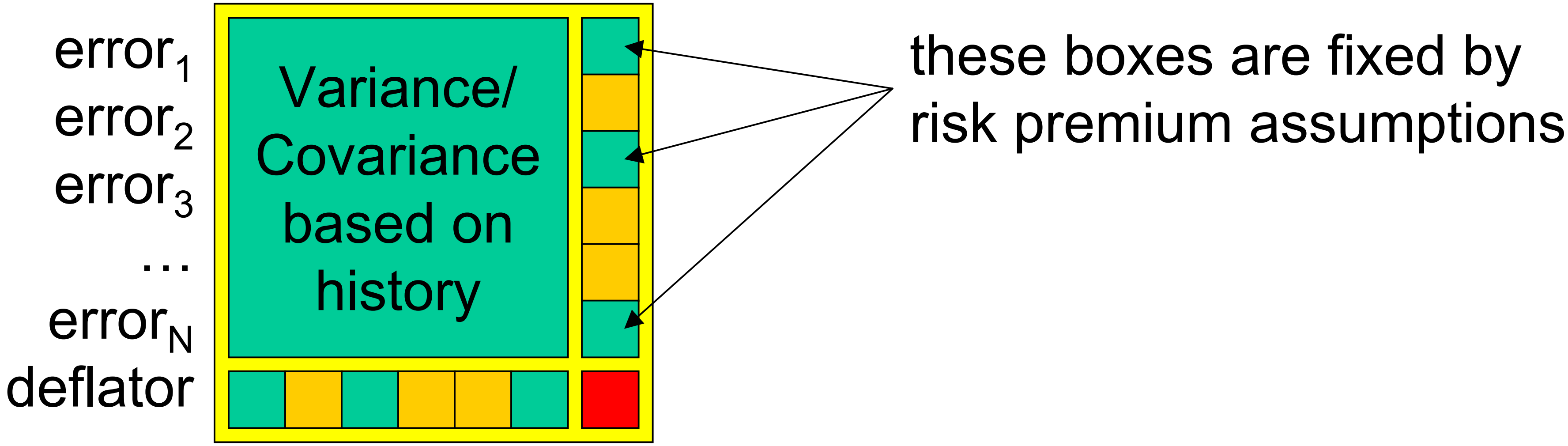
$$X^{JPY/USD}_{2005} \quad = \quad + \quad - \quad \mathbf{Cov}_{2004}\left(\frac{X^{JPY/USD}_{2004}}{P^{JPY}_{2004:2005} X^{JPY/USD}_{2005}} \frac{D^{USD}_{2005}}{D^{USD}_{2004}}, X^{JPY/USD}_{2005} \right)$$

observable scalar

deflator ratio

projected quantity

Filling the Covariance Matrices



Calibration problem: minimise by varying , subject to given values of and keeping the matrix positive definite.

Testing Deflators

Statistics from Timbuk1 Output (5 years):

	E(X)	**Stdev**(X)	**E**(D)	**Stdev**(D)	**Corr**(D,X)
100	100.0	0.0	0.780	0.641	
Equity total return	174.6	99.2	0.780	0.641	-57%
Bond total return	130.6	13.0	0.780	0.641	-23%

Deflator Verification:

	E(D)**E**(X)	**Cov**(D,X)	**E**(DX)
100	78.0	0.0	78.0
Equity total return	136.2	-36.2	100.0
Bond total return	101.8	-1.9	100.0

ZCB Price

Initial Investment

Points for Discussion

- Is market calibration necessary and/or desirable for insurance ALM and valuation applications?
- Are you happy using “black box” asset models?
    - What tests would make you more comfortable?
- Relation to other modelling challenges
    - fair value, capital allocation, risk premium project

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