Stochastic Made "Simple"

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Institute of Actuaries
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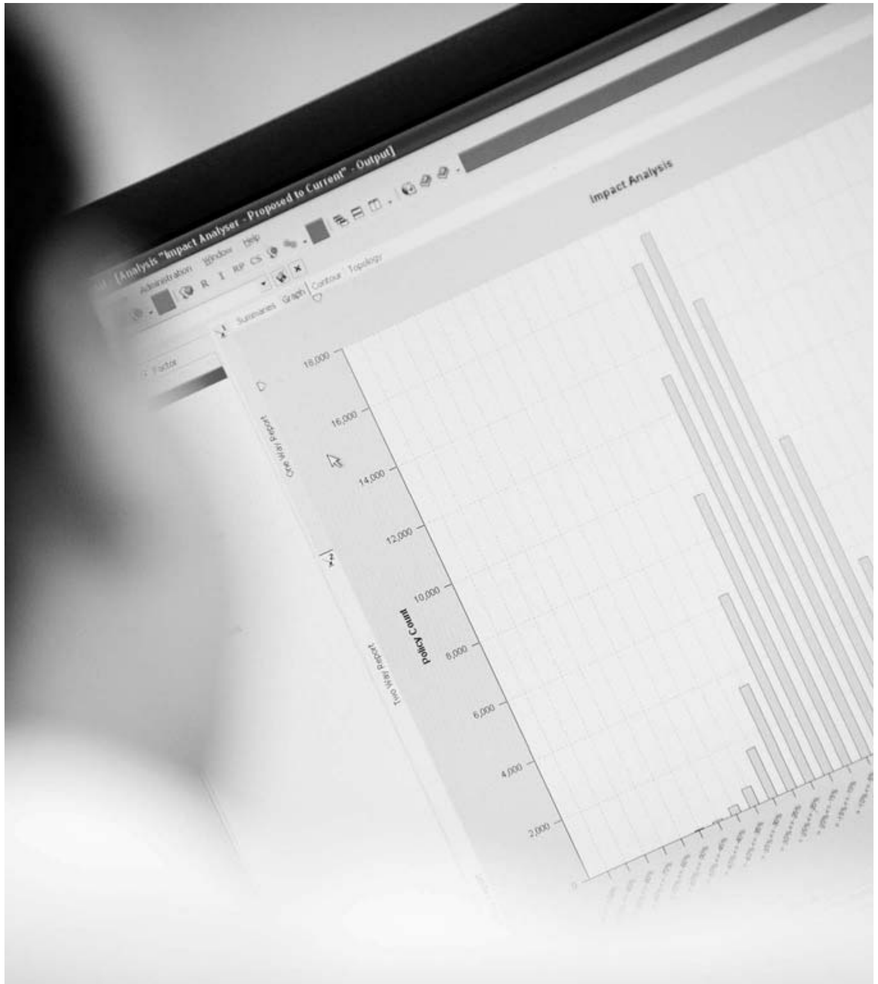
Aims and Objectives

Aims

- To explain core concepts
- To take away some of the mystique

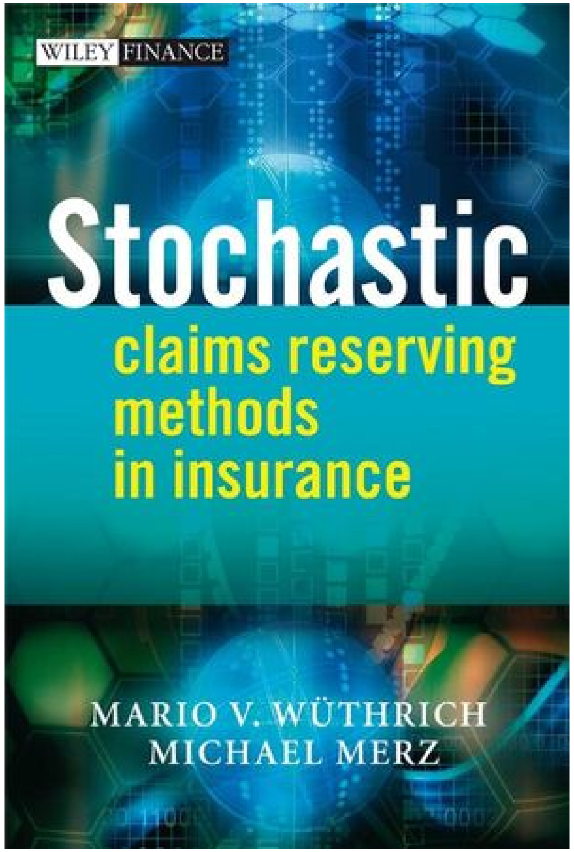
Agenda

- Conceptual framework
- A basic example
- The over-dispersed Poisson model
- Mack's model
- The 1 year view of reserve risk
- Conclusions



Stochastic claims reserving

- This has become a new academic discipline
- It has spawned several PhDs
- Numerous papers appearing in academic journals
- Presentations at every actuarial conference
- A book has appeared
- There is a Wikipedia page



Reserving Risk

- Reserving is concerned with forecasting outstanding liabilities
- There is uncertainty associated with any forecast
- Reserving risk attempts to capture that uncertainty
- We are interested in the predictive distribution of ultimate losses AND the associated cash flows
  - Don’t just focus on “Ultimates” or “Reserves”
  - We need distributions of cash flows for discounting and for capital models
- We need methods that can provide those distributions
- The methods are still evolving

Conceptual Framework

Reserve Estimate (Measure of Location)	← Traditional deterministic methods
Variability (Prediction Error)	← Statistical assumptions required ← Prediction Error = SD of Forecast ← Can be estimated analytically
Predictive Distribution	← Usually cannot be obtained analytically ← Simulation methods required

Approaches to stochastic reserving

"We can do this the easy way, or the hard way…"

- A lot of work in the academic literature has focused on specifying a model, then devising analytic formulae for the standard deviation of the forecast. This is the hard way.
  - It doesn't get us very far. A standard deviation is useful, but the formulae are specific to the model. What if we want other models, other risk measures, or a full distribution?
- More recent work has focused on using simulation techniques (bootstrap or MCMC) to provide a full distribution of cash-flows (hence reserves). This is the "easy" way.
  - We still need to specify the model, and the analytic methods are useful for checking the results
  - There are still many practical difficulties and limitations

The "ultimo" view and the one-year view
An added complication

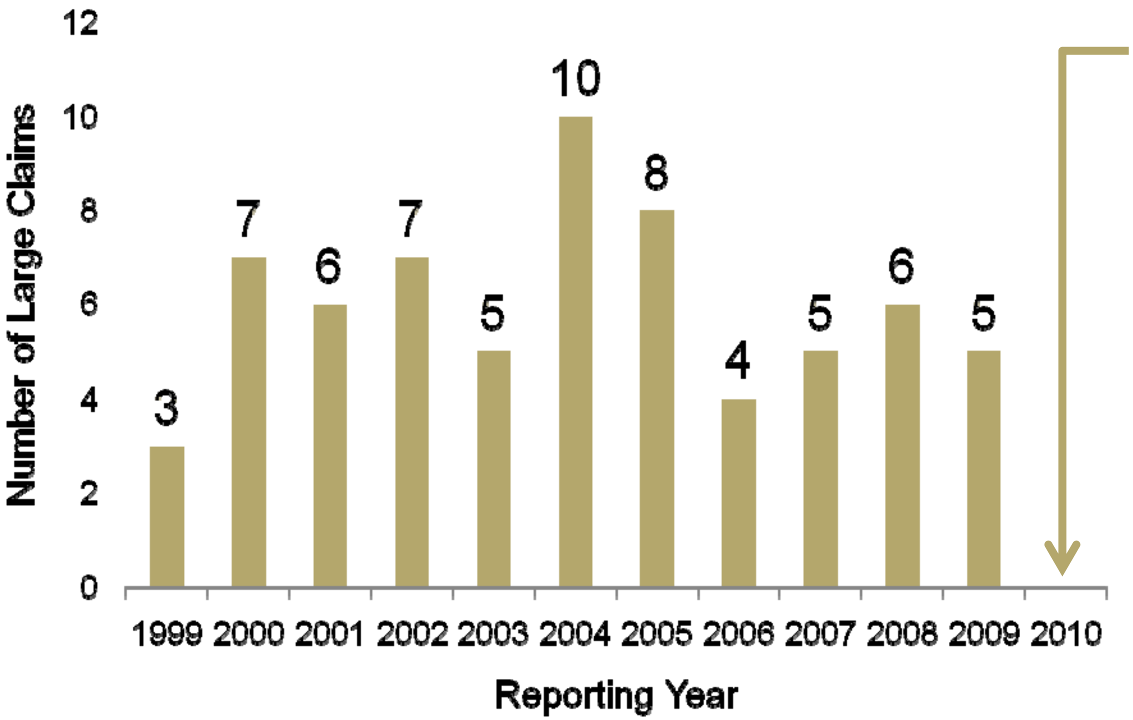
- A standard actuarial reserving analysis tries to find the expected outstanding liabilities, giving the expected ultimate cost of claims over the lifetime of the liabilities
- The traditional actuarial approach to reserving risk is to look at the uncertainty in the outcomes over the lifetime of the liabilities (the "ultimo" perspective)
- Under Solvency II, a 1 year view is taken. We need a distribution of the expected outstanding liabilities after 1 year. This is a different view of reserving risk.
  - Can the two views be reconciled in some way?

Basic Concepts

Uncertainty when Forecasting: Prediction errors and Predictive distributions

A Basic Example

- Suppose you are an established Private Motor insurer and have written the same number of policies for the last 11 years
- You have had the following number of large claims:



How many large claims do you expect next year?

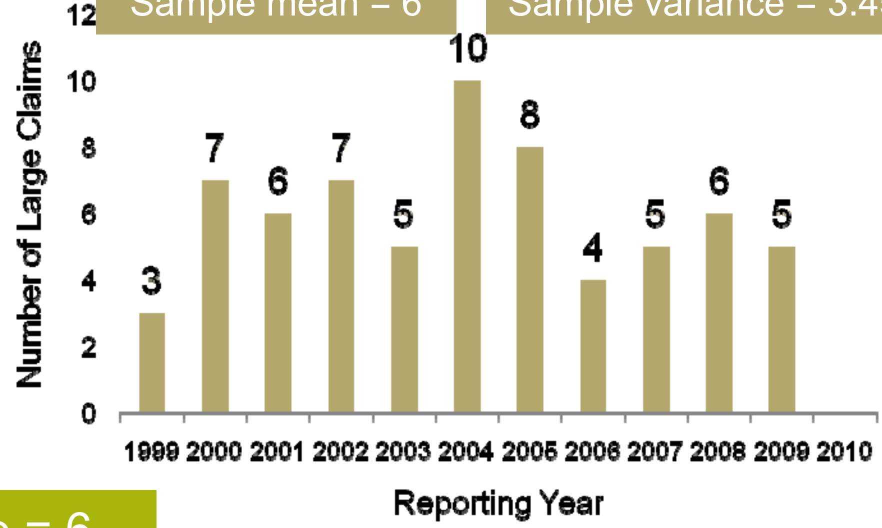
What is the uncertainty in your estimate?

What is the uncertainty in the outcome?

A Basic Example – A Solution

Sample mean = 6

Sample variance = 3.45



- Suppose that the large claims come from a Poisson distribution with mean λ.
- Can estimate the mean λ from the observed large claims
- What is the variance of a Poisson (6) distribution? Process Variance = 6
- How can we measure the uncertainty of using the sample mean?

  Sample mean = $(\sum x_i)/n$

  If x_i are independent and identically distributed, then variance of the sample mean = sample variance / n Estimation Variance = 0.314

Prediction Variance = Process Variance + Estimation Variance

The Bootstrap

An alternative way of calculating the estimation variance is the **bootstrap**

Original Sample

Data must be independent and identically distributed

Produce a pseudo data sample by re-sampling with replacement

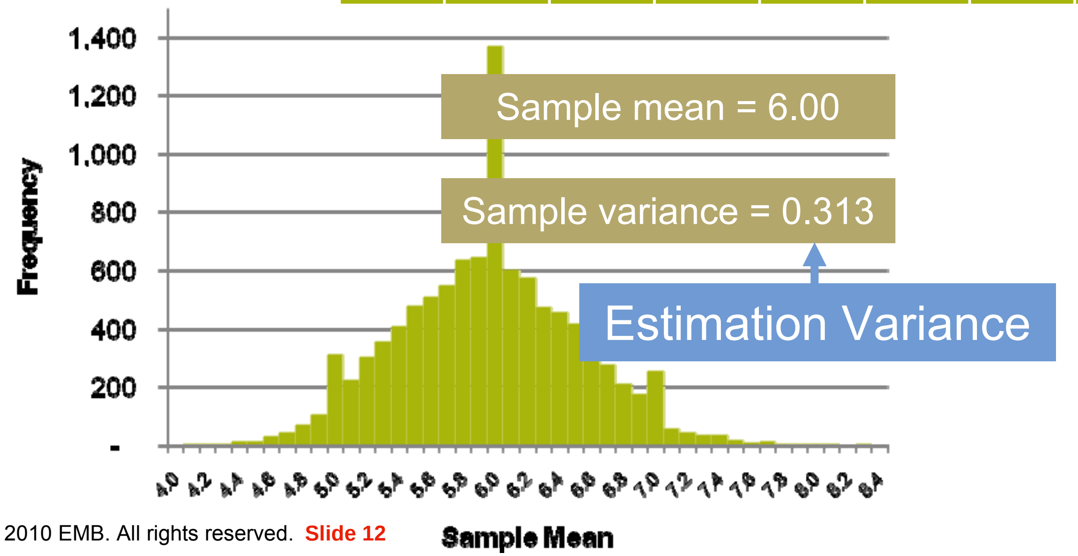
Calculate the parameters of interest

Repeat many times, giving a distribution of the parameters

The SD gives a bootstrap estimate of the standard error of the parameters

A Basic Example - Bootstrap

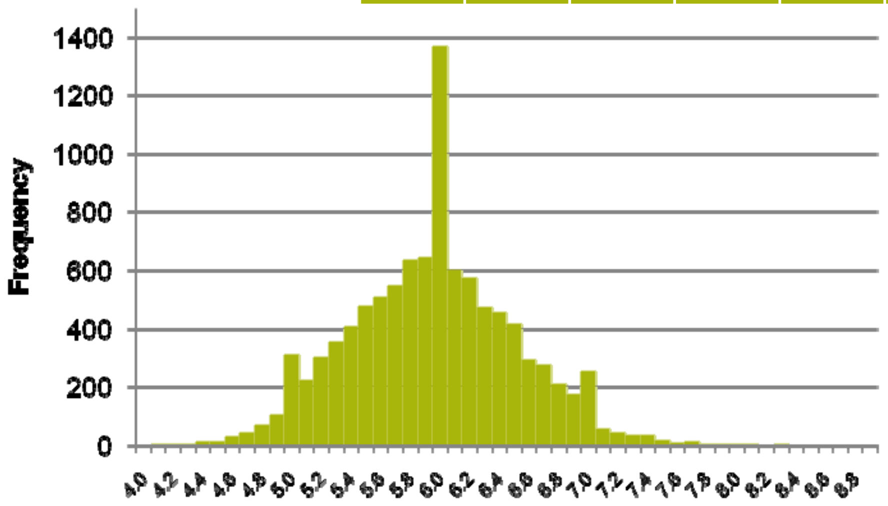
												Mean
Observed data :	3	7	6	7	5	10	8	4	5	6	5	6
Pseudo data 1												5.5
Pseudo data 2	6	8	3	10	6	6	5	5	7	5	8	6.3
…												
Pseudo data 10k	8	5	7	8	4	5	6	3	10	5	5	6.0

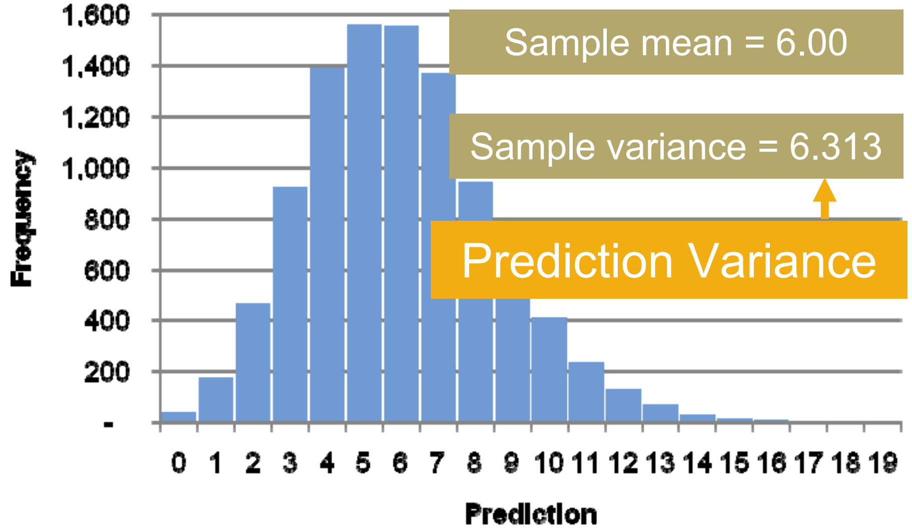


If we sample from Poisson distributions with these means, we can derive the predictive distribution including process variance.

A Basic Example - Bootstrap

												Mean	
Observed data :	3	7	6	7	5	10	8	4	5	6	5	6	
Pseudo data 1	4	10	8	7	7	5	3	5	5	4	3	5.5	7
Pseudo data 2	6	8	3	10	6	6	5	5	7	5	8	6.3	11
…													
Pseudo data 10k	8	5	7	8	4	5	6	3	10	5	5	6.0	4





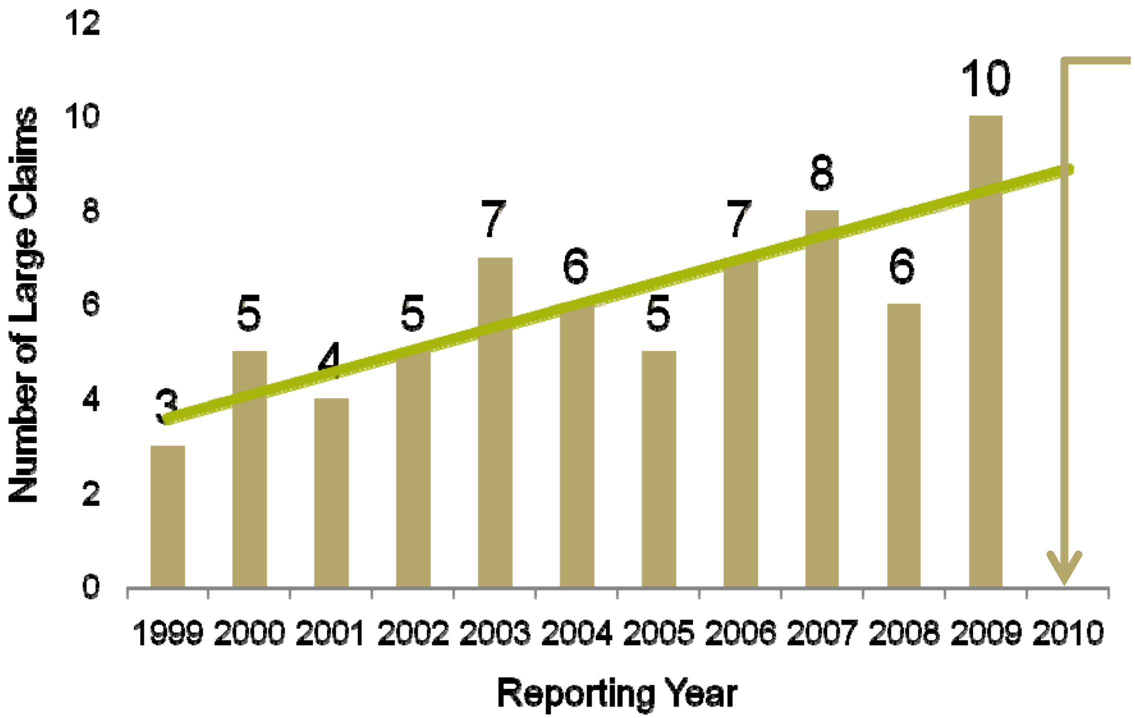
Important Lessons

- We could calculate the SD of the forecast ("prediction error") analytically, taking account of parameter uncertainty.
  - This is the HARD way.
- Bootstrapping gives a distribution of parameters, hence an estimate of the estimation error, without the hard maths
- When supplemented by a second simulation step incorporating the process error, a distribution of the forecast is generated
  - This is the EASY way

A More Complicated Example

- Suppose now that the number of large claims had been:

i.e. The same number of large claims but in a different order



How many large claims do you expect next year?

What is the uncertainty in your estimate?

Residual for linear regression = Actual - Fitted

Regression-type problems and Bootstrapping

Define and fit statistical model

Obtain residuals and pseudo data

Re-fit statistical model to pseudo data

Obtain forecast, including process error

Any model that can be clearly defined can be bootstrapped

Bootstrapping a Linear Regression Problem

- The bootstrap process for the estimation variance is then:
  - Specify a model (e.g. linear regression)
  - Define the residuals
  - Re-sample the residuals with replacement
  - Rearrange the residual definition to create new ‘pseudo’ data
  - Refit the model on the ‘pseudo’ data
  - Project forward to get a mean claim amount for the next time period
  - The variance of the trended mean gives the estimation variance
- We can still keep the Poisson assumption for the process distribution, just with a trended mean
  - Simulate from a Poisson distribution, conditional on the simulated mean
  - The variance of the forecasts gives the prediction variance
- Note: We have used standard linear regression in this example for simplicity – ideally we would fit a Poisson GLM

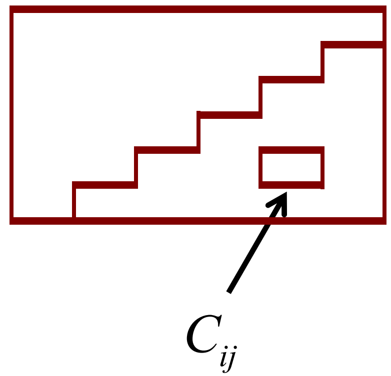
Stochastic Reserving

Over-dispersed Poisson Model

Doing it the HARD way

Over-Dispersed Poisson Model

C_{ij} = Incremental claims in origin year i and development year j



$$C_{ij} \sim ODP(\mu_{ij}, \phi_j)$$

$$E[C_{ij}] = \mu_{ij}$$

$$Var[C_{ij}] = \phi_j \mu_{ij}$$

Variance proportional to expected value

Example Predictor Structures

$$\log(\mu_{ij}) = \eta_{ij}$$ ← Log "link" function

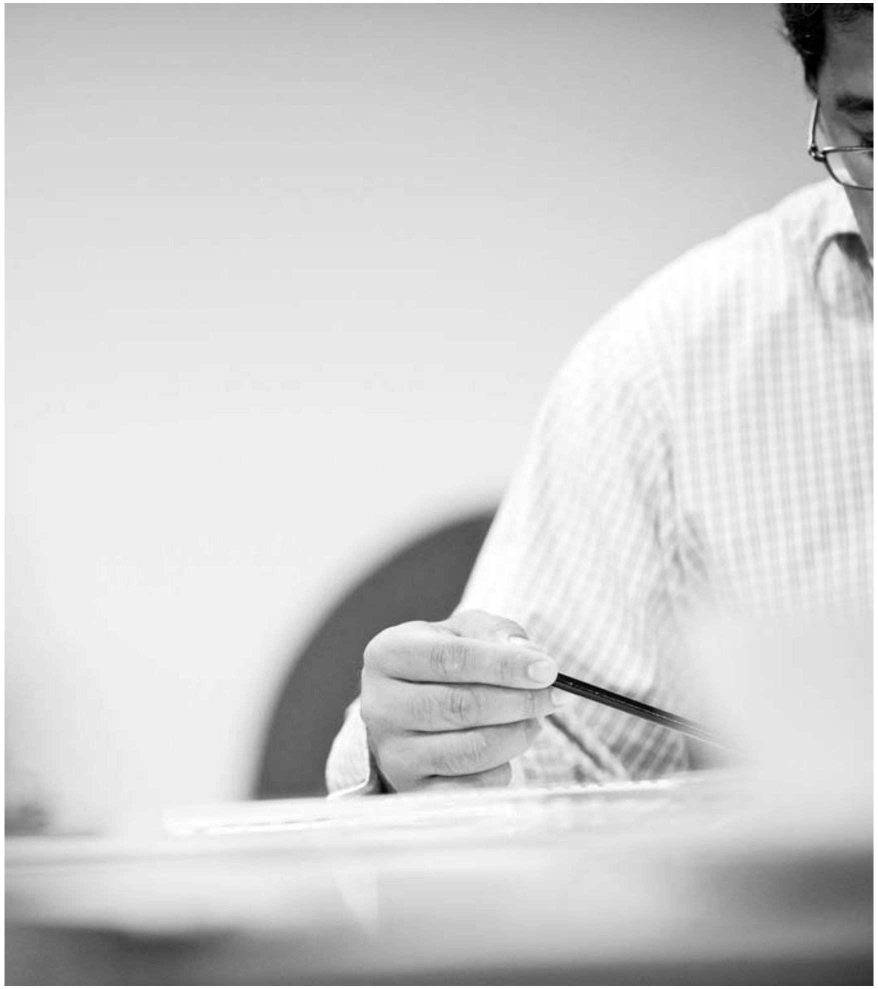
$$\eta_{ij} = c + a_i + b_j$$ Chain Ladder

$$\eta_i(t) = c + a_i + b.t + d\log(t)$$ Hoerl Curve

$$\eta_i(t) = c + a_i + s_1(t) + s_2(\log(t))$$ Smoother

Parameter estimation

- Write down joint density of the data given the parameters – the “Likelihood”
- Treat as a function of the parameters
- Maximise the (log) Likelihood with respect to the parameters



Variability in Claims Reserves

- Variability of a forecast
- Includes estimation variance and process variance

$$\text{prediction error} = (\text{process variance} + \text{estimation variance})^{1/2}$$

- Problem reduces to estimating the two components
- This is difficult analytically, but possible (see, for example, E & V 2002)
- Note: “prediction error” is also known as “root mean square error of prediction” (RMSEP)

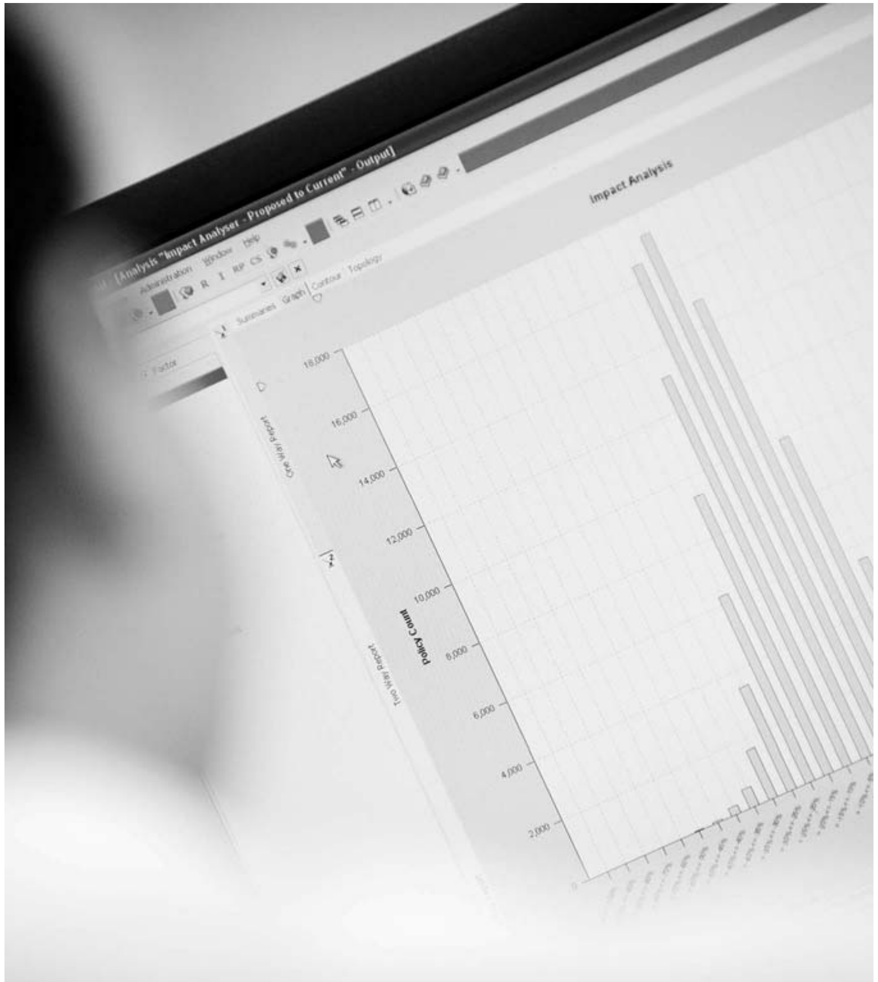
Stochastic Reserving

Over-dispersed Poisson Model

Doing it the EASY way

Stochastic Reserving: Bootstrapping

- Bootstrapping assumes the data are independent and identically distributed
- With regression type problems, the data are often assumed to be independent but are not identically distributed (the means are different for each observation)
- However, the residuals are usually *i.i.d*, or can be made so
- Therefore, with regression problems, it is common to bootstrap the residuals instead



Reserving and Bootstrapping

Define and fit statistical model

Obtain residuals and pseudo data

Re-fit statistical model to pseudo data

Obtain forecast, including process error

Any model that can be clearly defined can be bootstrapped

Bootstrapping the Chain Ladder
Over-dispersed Poisson model

1. Fit chain ladder model
2. Obtain Pearson residuals → $r_{ij} = \dfrac{C_{ij} - \mu_{ij}}{\sqrt{\phi_j \mu_{ij}}}$
3. Resample residuals
4. Obtain pseudo data, given r_{ij}^*, μ_{ij}

$$C_{ij}^* = r_{ij}^* \sqrt{\phi_j \mu_{ij}} + \mu_{ij}$$

5. Use chain ladder to re-fit model, and estimate future incremental payments

Bootstrapping the Chain Ladder

6. Simulate observation from process distribution assuming mean is incremental value obtained at Step 5
7. Repeat many times, storing the reserve estimates (this gives the predictive distribution)
8. Prediction error is then standard deviation of results

Note: Where curve fitting has been used for smoothing and extrapolation (for tail estimation), replace the chain ladder model in steps 1 and 5 by the actual model used

Excel Example

Taylor & Ashe Data
Observed incremental values

	1	2	3	4	5	6	7	8	9	10
1	357,848	766,940	610,542	482,940	527,326	574,398	146,342	139,950	227,229	67,948
2	352,118	884,021	933,894	1,183,289	445,745	320,996	527,804	266,172	425,046	
3	290,507	1,001,799	926,219	1,016,654	750,816	146,923	495,992	280,405		
4	310,608	1,108,250	776,189	1,562,400	272,482	352,053	206,286			
5	443,160	693,190	991,983	769,488	504,851	470,639				
6	396,132	937,085	847,498	805,037	705,960					
7	440,832	847,631	1,131,398	1,063,269						
8	359,480	1,061,648	1,443,370							
9	376,686	986,608								
10	344,014									
Dev Factors	**3.49061**	**1.74733**	**1.45741**	**1.17385**	**1.10382**	**1.08627**	**1.05387**	**1.07656**	**1.01772**	**1.00000**

Taylor & Ashe Data
Fitted incremental values (chain ladder model)

	1	2	3	4	5	6	7	8	9	10	Reserve
1	270,061	672,617	704,494	753,438	417,350	292,571	268,344	182,035	272,606	67,948	0
2	376,125	936,779	981,176	1,049,342	581,260	407,474	373,732	253,527	379,669	94,634	94,634
3	372,325	927,316	971,264	1,038,741	575,388	403,358	369,957	250,966	375,833	93,678	469,511
4	366,724	913,365	956,652	1,023,114	566,731	397,290	364,391	247,190	370,179	92,268	709,638
5	336,287	837,559	877,254	938,200	519,695	364,316	334,148	226,674	339,456	84,611	984,889
6	353,798	881,172	922,933	987,053	546,756	383,287	351,548	238,477	357,132	89,016	1,419,459
7	391,842	975,923	1,022,175	1,093,189	605,548	424,501	389,349	264,121	395,534	98,588	2,177,641
8	469,648	1,169,707	1,225,143	1,310,258	725,788	508,792	466,660	316,566	474,073	118,164	3,920,301
9	390,561	972,733	1,018,834	1,089,616	603,569	423,113	388,076	263,257	394,241	98,266	4,278,972
10	344,014	856,804	897,410	959,756	531,636	372,687	341,826	231,882	347,255	86,555	4,625,811

Total 18,680,856

Taylor & Ashe Data

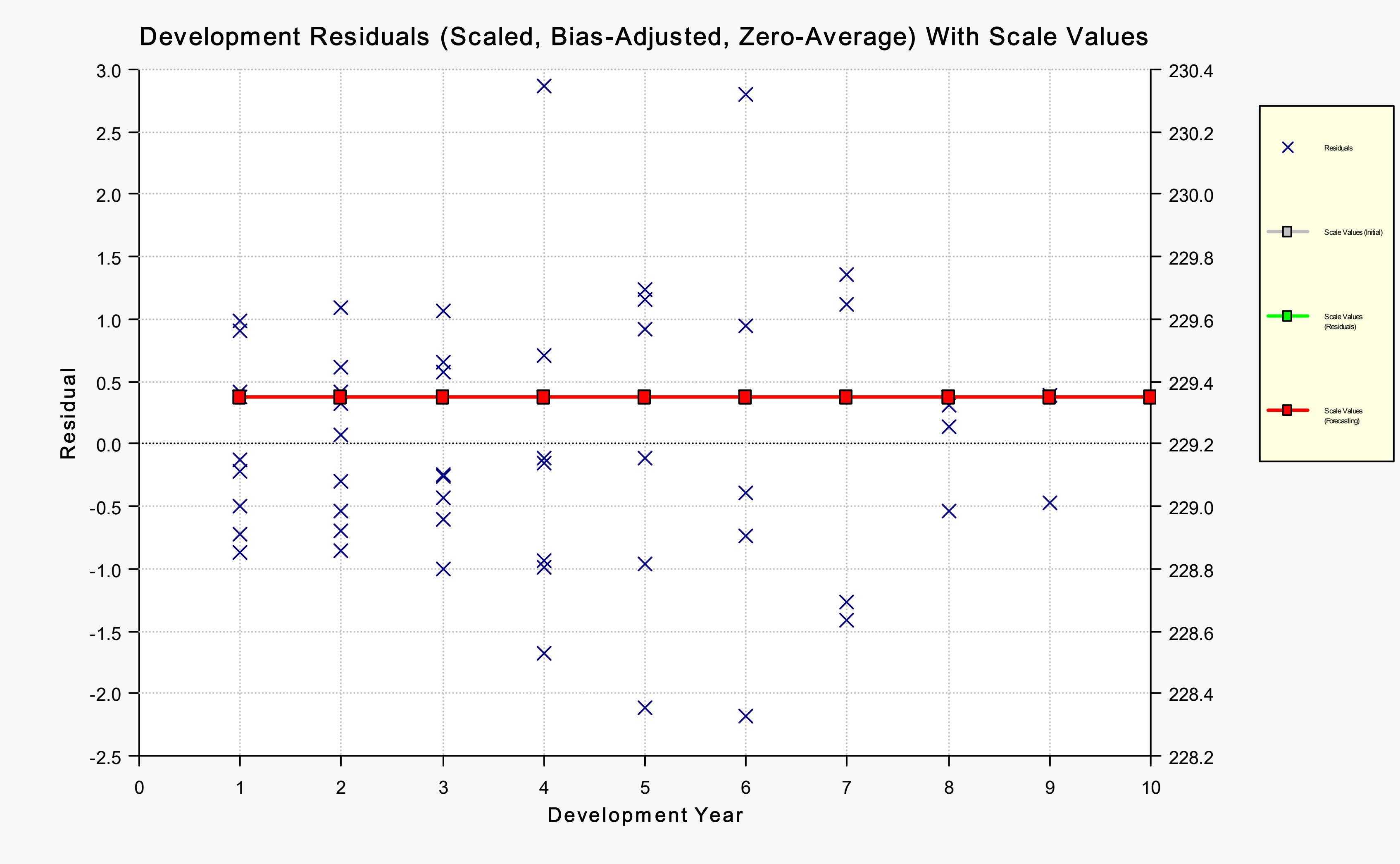
Scaled residuals : ODP with constant scale parameter

	1	2	3	4	5	6	7	8	9	10
1	0.737	0.501	-0.488	-1.359	0.742	2.272	-1.027	-0.430	-0.379	0.000
2	-0.171	-0.238	-0.208	0.570	-0.775	-0.591	1.099	0.110	0.321	
3	-0.585	0.337	-0.199	-0.094	1.008	-1.760	0.903	0.256		
4	-0.404	0.889	-0.804	2.325	-1.704	-0.313	-1.142			
5	0.804	-0.688	0.534	-0.759	-0.090	0.768				
6	0.310	0.260	-0.342	-0.799	0.939					
7	0.341	-0.566	0.471	-0.125						
8	-0.701	-0.436	0.860							
9	-0.097	0.061								
10	0.000									

Scale^0.5	**229.3**	**229.3**	**229.3**	**229.3**	**229.3**	**229.3**	**229.3**	**229.3**	**229.3**	**229.3**

Taylor & Ashe Data

Scaled residuals : ODP with constant scale parameter



Note that the volatility is lower at the earlier and later development periods

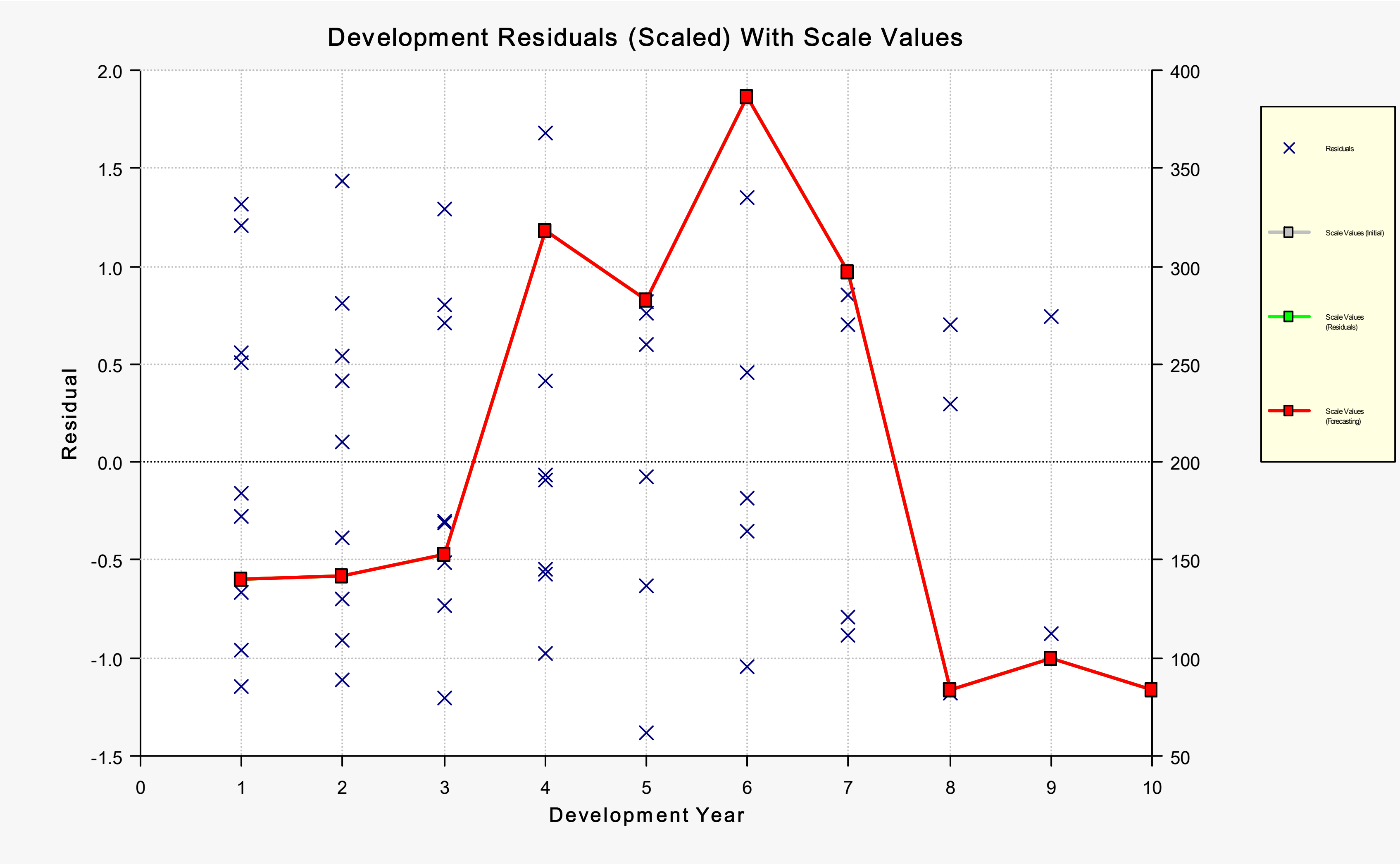
Taylor & Ashe Data

Scaled residuals : ODP with non-constant scale parameter

	1	2	3	4	5	6	7	8	9	10
1	1.207	0.808	-0.731	-0.980	0.602	1.348	-0.794	-1.176	-0.873	0.000
2	-0.280	-0.383	-0.312	0.411	-0.629	-0.350	0.849	0.299	0.740	
3	-0.958	0.544	-0.299	-0.068	0.818	-1.045	0.698	0.701		
4	-0.662	1.433	-1.206	1.676	-1.383	-0.186	-0.883			
5	1.317	-1.109	0.800	-0.548	-0.073	0.456				
6	0.509	0.419	-0.513	-0.576	0.762					
7	0.559	-0.913	0.706	-0.090						
8	-1.149	-0.702	1.288							
9	-0.159	0.099								
10	0.000									
Scale^0.5	**139.9**	**142.3**	**153.0**	**318.1**	**282.6**	**386.6**	**296.7**	**83.9**	**99.6**	**83.9**

Taylor & Ashe Data

Scaled residuals: ODP with non-constant scale parameter



Note that the residuals are standardised better when using non-constant scale parameters

Taylor & Ashe Data

ODP: Constant vs Non-Constant Scale Parameters

	Simulated		Simulated	
	Constant Scale		Non-Constant Scale	
Accident Year	Prediction Error	Prediction Error %	Prediction Error	Prediction Error %
1	0	0.0%	0	0.0%
2	112,552	119.0%	43,882	45.3%
3	217,547	46.2%	109,449	23.0%
4	262,934	36.9%	141,509	19.8%
5	306,595	31.0%	256,031	25.7%
6	375,745	26.4%	398,377	27.8%
7	500,332	22.9%	529,898	24.2%
8	791,481	20.1%	735,245	18.7%
9	1,060,473	24.7%	809,457	18.9%
10	2,025,898	43.3%	1,285,560	27.6%
Total	2,992,296	15.9%	2,228,677	11.9%

Over-dispersed Poisson model

- Note the possibility of obtaining negative pseudo incremental values when using non-parametric bootstrapping (resampling residuals), which could in turn lead to negative pseudo cumulative values.
- This is a known issue with non-parametric bootstrapping. For example:

  "Although the [non-parametric] bootstrap/ simulation procedure provides prediction errors that are consistent with their analytic counterparts, the predictive distribution produced in this way might have some undesirable properties. For example, for some origin year reserves, the minimum values of the predictive distribution could be negative." [1]

  "It [non-parametric bootstrapping] is not without its difficulties, for example: a small number of sets of pseudo data may be incompatible with the underlying model…" [2]

If $r_{ij} = \dfrac{C_{ij} - \mu_{ij}}{\sqrt{\phi_j \mu_{ij}}}$ then $C^*_{ij} = r^*_{ij}\sqrt{\phi_j \mu_{ij}} + \mu_{ij}$

$C^*_{ij} < 0$ if $r^*_{ij} < -\sqrt{\dfrac{\mu_{ij}}{\phi_j}}$

$C =$ incremental amounts

$\mu =$ expected incremental amounts

$\phi =$ scale parameter

$r =$ scaled Pearson residual

This issue disappears with parametric bootstrapping

(1) England (2002)

(2) England & Verrall (2006)

ODP Bootstrapping – Practical Issues

- Most suitable for paid amounts
- Can result in negative incrementals/reserves in some simulations when the pseudo data are generated by re-sampling residuals and inverting
  - If this is a problem, use parametric bootstrapping instead
- E&V (1999) only considered using a constant dispersion parameter: in general it is better to use non-constant scale parameters
- Choice of process distribution?
  - Ideally we want an “over-dispersed Poisson” distribution
  - In practice just use a proxy (eg Gamma) with the same mean and variance properties

ODP Model – Characteristics

- It is a model of *incremental* amounts
- It is not suitable when development factors are less than 1
- When forecasting, by using a distribution that only allows positive values (eg Gamma or Lognormal), forecast incremental values will be positive
  - That is, simulated cumulative amounts will be strictly increasing
  - Simulated reserves can never be negative
  - The ultimate claims will be at least as big as the observed cumulative paid for each origin period
  - (Although note comments on parametric vs non-parametric bootstrapping above)

Stochastic Reserving

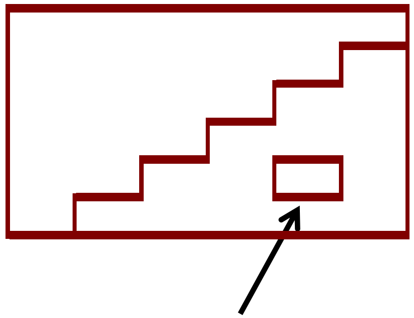
Mack's Model

Doing it the HARD way

Mack's Model

Mack, T (1993), *Distribution-free calculation of the standard error of chain-ladder reserve estimates*. ASTIN Bulletin, 22, 93-109

$$D_{ij} = \text{Cumulative claims in origin year } i \text{ and development year } j$$



D_{ij}

Specified mean and variance only:

$$E\left(D_{ij}\right) = \lambda_j D_{i,j-1}$$

Expected value proportional to previous cumulative

$$V\left(D_{ij}\right) = \sigma_j^2 D_{i,j-1}$$

Variance proportional to previous cumulative

Mack's Model

$$\hat{\lambda}_j = \frac{\sum_{i=1}^{n-j+1} w_{ij} f_{ij}}{\sum_{i=1}^{n-j+1} w_{ij}}$$

← Estimator for lambda

$$\hat{\sigma}_j^2 = \frac{1}{n-j} \sum_{i=1}^{n-j+1} w_{ij} \left(f_{ij} - \hat{\lambda}_j \right)^2$$

← Estimator for sigma squared

$$w_{ij} = D_{i,j-1} \text{ and } f_{ij} = \frac{D_{ij}}{D_{i,j-1}}$$

Variability in Claims Reserves

- Variability of a forecast
- Includes estimation variance and process variance

$$\text{prediction error} = (\text{process variance} + \text{estimation variance})^{1/2}$$

- Problem reduces to estimating the two components. For example, for the reserves in origin year *i*:

$$RMSEP\left[\hat{R}_i\right] \approx \sqrt{\hat{D}_{in}^2 \sum_{k=n-i+1}^{n-1} \frac{\hat{\sigma}_{k+1}^2}{\hat{\lambda}_{k+1}^2} \left(\frac{1}{\hat{D}_{ik}} + \frac{1}{\sum_{q=1}^{n-k} D_{qk}} \right)}$$

Mack's Model as a GLM

Step 1: Reformulate Mack's model as a model of the ratios

Step 2: Recognise that Mack's "Scale" parameters are derived from the squared residuals of a weighted normal regression model

$$E\left(\frac{D_{ij}}{D_{i,j-1}}\right) = E\left(f_{ij}\right) = \lambda_j$$

$$V\left(\frac{D_{ij}}{D_{i,j-1}}\right) = V\left(f_{ij}\right) = \frac{\sigma_j^2}{w_{ij}}$$

$$w_{ij} = D_{i,j-1} \text{ and } f_{ij} = \frac{D_{ij}}{D_{i,j-1}}$$

$$D_{ij} = \text{Cumulative claims in origin year } i \text{ and development year } j$$

Mack's Model as a GLM

$$f_{ij} \sim N\left(\lambda_j, \frac{\sigma_j^2}{w_{ij}}\right)$$

Weighted normal errors GLM with weights *w*

$$r_{ij} = \sqrt{w_{ij}}\left(f_{ij} - \hat{\lambda}_{ij}\right) \quad \text{Pearson residual}$$

(unscaled)

$$\hat{\sigma}_j^2 = \frac{1}{n-j}\sum_{i=1}^{n-j+1} w_{ij}\left(f_{ij} - \hat{\lambda}_j\right)^2$$

Note: Mack's model was not derived as a weighted normal GLM, but a weighted normal GLM gives the same estimator of sigma

Stochastic Reserving

Mack's Model

Doing it the EASY way

Reserving and Bootstrapping

Define and fit statistical model

Obtain residuals and pseudo data

Re-fit statistical model to pseudo data

Obtain forecast, including process error

Any model that can be clearly defined can be bootstrapped

Bootstrapping the Chain Ladder

1. Fit chain ladder model to the observed link ratios
2. Obtain (scaled) Pearson residuals → $r_{ij} = \dfrac{\sqrt{w_{ij}}\left(f_{ij} - \lambda_j\right)}{\sigma_j}$
3. Resample residuals
4. Obtain pseudo data, given r_{ij}^{*}, λ_j

$$f_{ij}^{*} = \frac{r_{ij}^{*}\sigma_j}{\sqrt{w_{ij}}} + \lambda_j$$

5. Use chain ladder model to re-estimate the development factors (as a weighted average of the pseudo-link ratios, using the original weights w)

Bootstrapping the Chain Ladder
Mack's model

6. Given the observed cumulative payments to date, move 1 period ahead by multiplying the previous cumulative claims by the appropriate simulated development factor obtained at Step 5
   - Include the process error by sampling a single observation from the underlying process distribution
7. Move to the next period, where the forecast cumulative amounts are now conditional on the simulated 1 period ahead forecast obtained at Step 6 (including the process error)
8. Repeat many times, storing the reserve estimates (this gives the predictive distribution)
9. Prediction error is then standard deviation of results

Note: Where curve fitting has been used for smoothing and extrapolation (for tail estimation), replace the chain ladder model in steps 1 and 5 by the actual model used

Excel Example

Taylor & Ashe Data

Scaled residuals: Mack's model

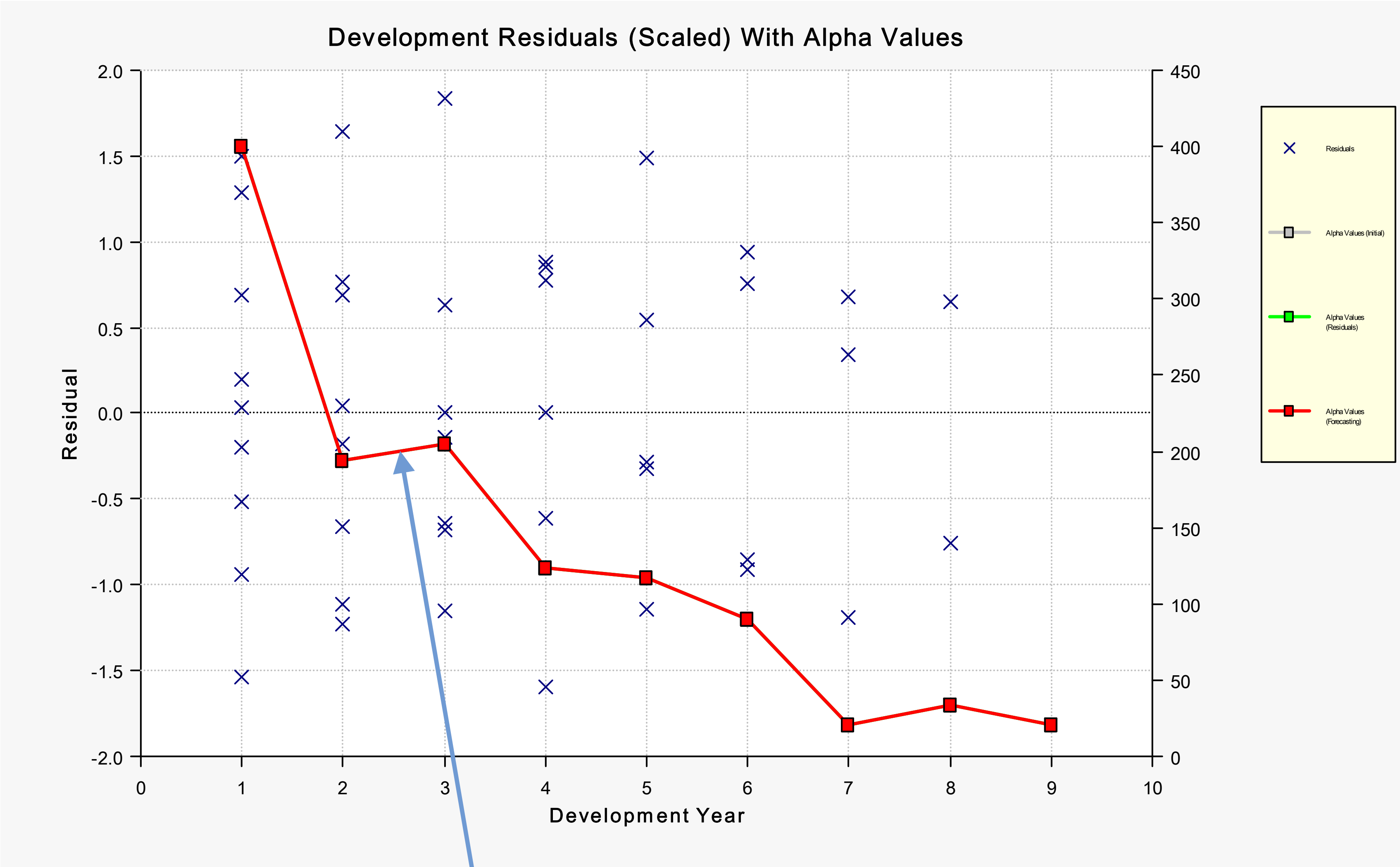
	1	2	3	4	5	6	7	8	9
1	-0.519	-1.117	-1.152	0.772	1.490	-0.850	-1.189	-0.759	0.000
2	0.030	0.047	0.632	-0.608	-0.322	0.939	0.346	0.651	
3	1.290	-0.179	0.006	0.850	-1.141	0.758	0.682		
4	1.500	-1.228	1.840	-1.594	-0.282	-0.907			
5	-1.540	0.689	-0.683	0.005	0.543				
6	-0.197	-0.664	-0.636	0.878					
7	-0.942	0.764	-0.137						
8	0.693	1.647							
9	0.197								

Note that the α parameters decrease rapidly

Mack's alpha	**400.4**	**194.3**	**204.9**	**123.2**	**117.2**	**90.5**	**21.1**	**33.9**	**21.1**

This parameter is highly influential on the overall variability

Taylor & Ashe Data
Scaled residuals: Mack's model



Note that the α parameters decrease rapidly

Taylor & Ashe Data

Bootstrapping Mack's Model

Accident Year	Analytic Prediction Error	Analytic Prediction Error %	Simulated Prediction Error	Simulated Prediction Error %
1	0	0.0%	0	0.0%
2	75,535	79.8%	75,001	78.4%
3	121,699	25.9%	121,578	26.0%
4	133,549	18.8%	132,939	18.7%
5	261,406	26.5%	261,911	26.5%
6	411,010	29.0%	414,910	29.1%
7	558,317	25.6%	558,639	25.7%
8	875,328	22.3%	880,184	22.4%
9	971,258	22.7%	979,052	22.8%
10	1,363,155	29.5%	1,368,720	29.4%
Total	2,447,095	13.1%	2,454,616	13.1%

Practical Issues

- Choice of process distribution?
  - Normal: theoretically correct, but allows negative cumulative amounts
  - Gamma: pragmatic alternative
- When used with incurred data:
  - Provides distribution of ultimate claims
  - Provides distribution of IBNR+IBNER (not outstanding claims)
  - Also provides a distribution of Ultimates
    - By subtracting the observed paid amounts gives a distribution of outstanding amounts
  - Requires (simulated) paid to incurred ratios if paid cash-flows are required

Bootstrapping Mack's Model
Characteristics

- It is a model of the *cumulative* amounts
- It will work with negative incremental observed claims where development factors are less than 1
- Although it is possible to force simulated cumulative amounts to be positive, there is nothing to stop a simulated cumulative amount being less than the previous amount. That is, negative incremental amounts are always possible.
  - This may be beneficial with incurred data, but possibly a disadvantage with paid data
- Note: bootstrapping provides predictive distributions for Mack's model (including cash-flows)

Stochastic Reserving

Lognormal Models

Lognormal Models

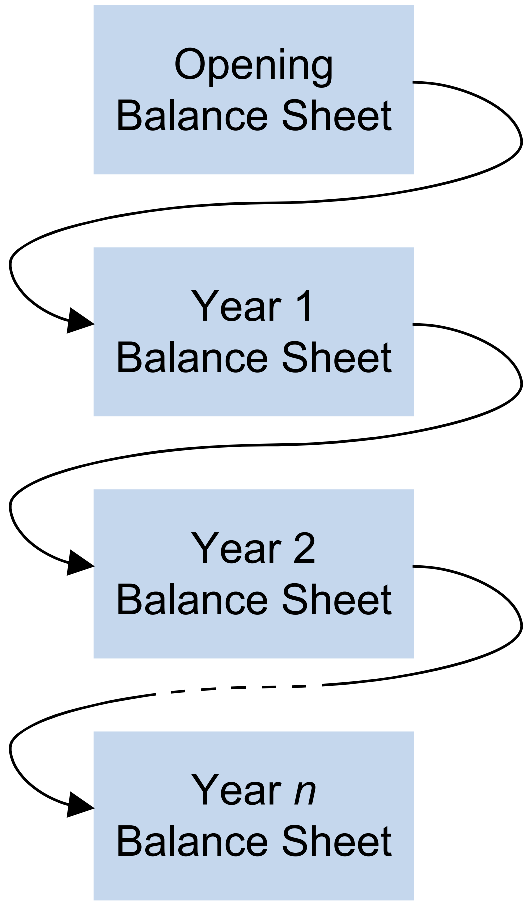
- It is also possible to fit linear regression models to the log of the incremental claims (log-normal models)
- Again, the prediction error can be calculated the hard way (analytically) or the easy way (using bootstrap or MCMC methods)
- To bootstrap the lognormal models, simply follow the steps outlined above for reserving and bootstrapping (see E&V 2006)

Stochastic Reserving

The one-yr view of risk

A Projected Balance Sheet View

- When projecting Balance Sheets for solvency, we have an opening balance sheet with expected outstanding liabilities
- We then project one year forwards, simulating the payments that emerge in the year
- We then require a closing balance sheet, with (simulated) expected outstanding liabilities conditional on the payments in the year
- In a multi-year model, the closing balance sheet after one year becomes the opening balance sheet in the second year, and so on



Solvency II

- For Solvency II, a 1 year perspective is taken, requiring a distribution of the *expected* value of the liabilities after 1 year, for the 1 year ahead balance sheet in internal capital models
- If the standard formula is used, a 1 year-ahead “reserve risk” standard deviation % is required. This could be:
  - The standard parameter for the line-of-business
  - An undertaking specific parameter
- The 1 year-ahead “reserve risk” standard deviation is the SD of the distribution of profit/loss on reserves after 1 year

The one-year run-off result (undiscounted)
(the view of profit or loss on reserves after one year)

For a particular origin year, let:

The opening reserve estimate be R_0

The reserve estimate after one year be R_1

The payments in the year be C_1

The run-off result (claims development result) be CDR_1

Then

$$CDR_1 = R_0 - C_1 - R_1 = U_0 - U_1$$

Where the opening estimate of ultimate claims and the estimate of the ultimate after one year are U_0, U_1

The one-year run-off result

(the view of profit or loss on reserves after one year)

Merz & Wuthrich (2008) derived analytic formulae for the standard deviation of the claims development result after one year assuming:

- The opening reserves were set using the pure chain ladder model (no tail)
- Claims develop in the year according to the assumptions underlying Mack's model
- Reserves are set after one year using the pure chain ladder model (no tail)
- The mathematics is quite challenging. This is the HARD way

The M&W method is gaining popularity, but has limitations. What if:

- We need a tail factor to extrapolate into the future?
- Mack's model is not used – other assumptions are used instead?
- We want another risk measure (say, VaR @ 99.5%)?
- We want a distribution of the CDR (not just a standard deviation)?

Merz & Wuthrich (2008)
Data Triangle

Accident Year	12m	24m	36m	48m	60m	72m	84m	96m	108m
0	2,202,584	3,210,449	3,468,122	3,545,070	3,621,627	3,644,636	3,669,012	3,674,511	3,678,633
1	2,350,650	3,553,023	3,783,846	3,840,067	3,865,187	3,878,744	3,898,281	3,902,425	
2	2,321,885	3,424,190	3,700,876	3,798,198	3,854,755	3,878,993	3,898,825		
3	2,171,487	3,165,274	3,395,841	3,466,453	3,515,703	3,548,422			
4	2,140,328	3,157,079	3,399,262	3,500,520	3,585,812				
5	2,290,664	3,338,197	3,550,332	3,641,036					
6	2,148,216	3,219,775	3,428,335						
7	2,143,728	3,158,581							
8	2,144,738								

Merz & Wuthrich (2008)
Prediction errors

	Analytic Prediction Errors	
Accident Year	1 Year Ahead CDR	Mack Ultimate
0	0	0
1	567	567
2	1,488	1,566
3	3,923	4,157
4	9,723	10,536
5	28,443	30,319
6	20,954	35,967
7	28,119	45,090
8	53,320	69,552
Total	$\boxed{81,080}$	108,401

Expressed as a percentage of the opening reserves, this forms a basis of the reserve risk parameter under Solvency II (QIS 5 Technical Specification)

QIS 5: Undertaking Specific Parameters

For the reserving risk parameter:

Method 2: $$\sigma_{U,lob} = \frac{\sqrt{MSEP_{lob}}}{PCO_{lob}}$$

Method 3: $$\sigma_{U,lob} = \frac{\sqrt{MSEP_{lob}}}{CLPCO_{lob}}$$

There is also a credibility weighting between this and the standard parameter:

$$\sigma_{res,lob} = c.\sigma_{U,res,lob} + (1-c).\sigma_{M,res,lob}$$

$MSEP$ is from the Merz-Wuthrich formulae. Clearly there are some inconsistencies here:

- PCO is discounted, but $MSEP$ is calculated using undiscounted amounts
- $MSEP$ is only valid under the chain-ladder model and Mack's assumptions
- What if other assumptions are used?

The one-year run-off result in a simulation model
The EASY way

For a particular origin year, let:

The opening reserve estimate be R_0

The expected reserve estimate after one year be $R_1^{(i)}$

The payments in the year be $C_1^{(i)}$

The run-off result (claims development result) be $CDR_1^{(i)}$

Then

$$CDR_1^{(i)} = R_0 - C_1^{(i)} - R_1^{(i)} = U_0 - U_1^{(i)}$$

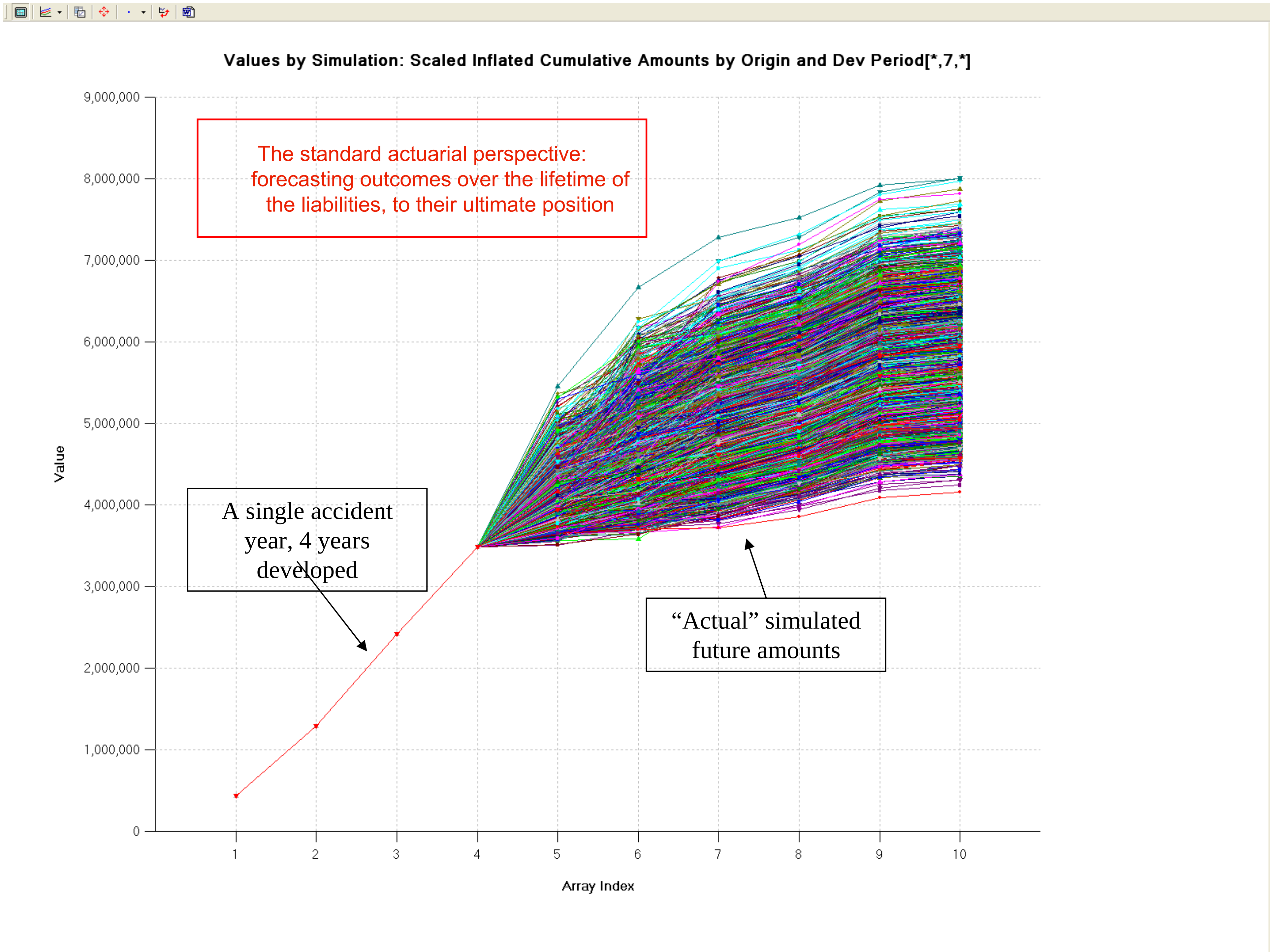
Where the opening estimate of ultimate claims and the expected ultimate after one year are $U_0, U_1^{(i)}$

for each simulation *i*

The one-year run-off result in a simulation model
The EASY way

1. Given the opening reserve triangle, simulate all future claim payments to ultimate using bootstrap (or Bayesian MCMC) techniques.
2. Now forget that we have already simulated what the future holds.
3. Move one year ahead. Augment the opening reserve triangle by one diagonal, that is, by the simulated payments from step 1 in the next calendar year only. An actuary only sees what emerges in the year.
4. For each simulation, estimate the outstanding liabilities, conditional only on what has emerged to date. (The future is still "unknown").
5. A reserving methodology is required for each simulation – an "actuary-in-the-box" is required*. We call this re-reserving.
6. For a one-year model, this will underestimate the true volatility at the end of that year (even if the mean across all simulations is correct).

** The term "actuary-in-the-box" was coined by Esbjörn Ohlsson*



Values by Simulation: Scaled Inflated Cumulative Amounts by Origin and Dev Period[*,7,*]
The standard actuarial perspective: forecasting outcomes over the lifetime of the liabilities, to their ultimate position
A single accident year, 4 years developed
"Actual" simulated future amounts
Value
Array Index
9,000,000
8,000,000
7,000,000
6,000,000
5,000,000
4,000,000
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2,000,000
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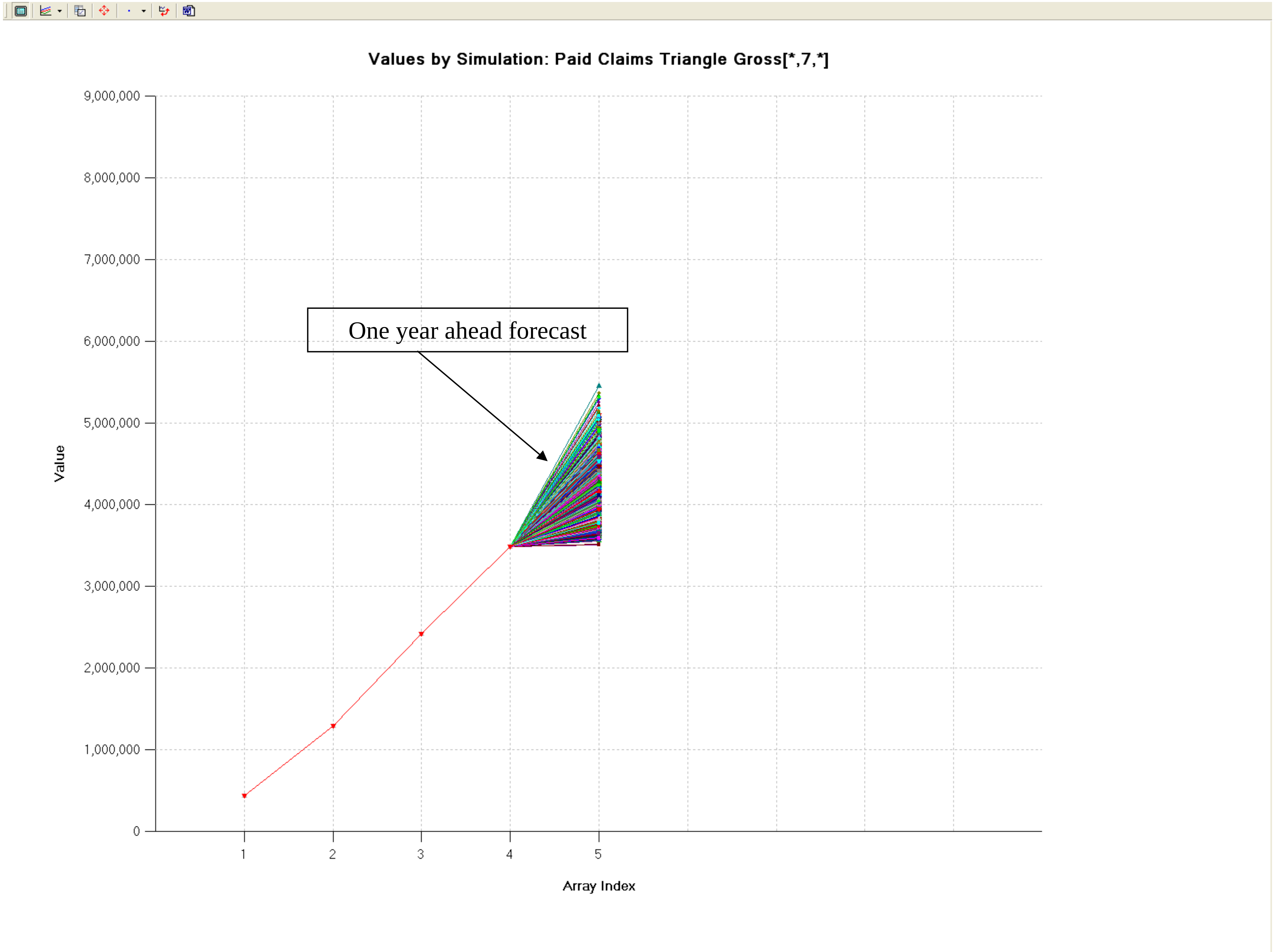
6
7
8
9
10



Values by Simulation: Paid Claims Triangle Gross[*,7,*]
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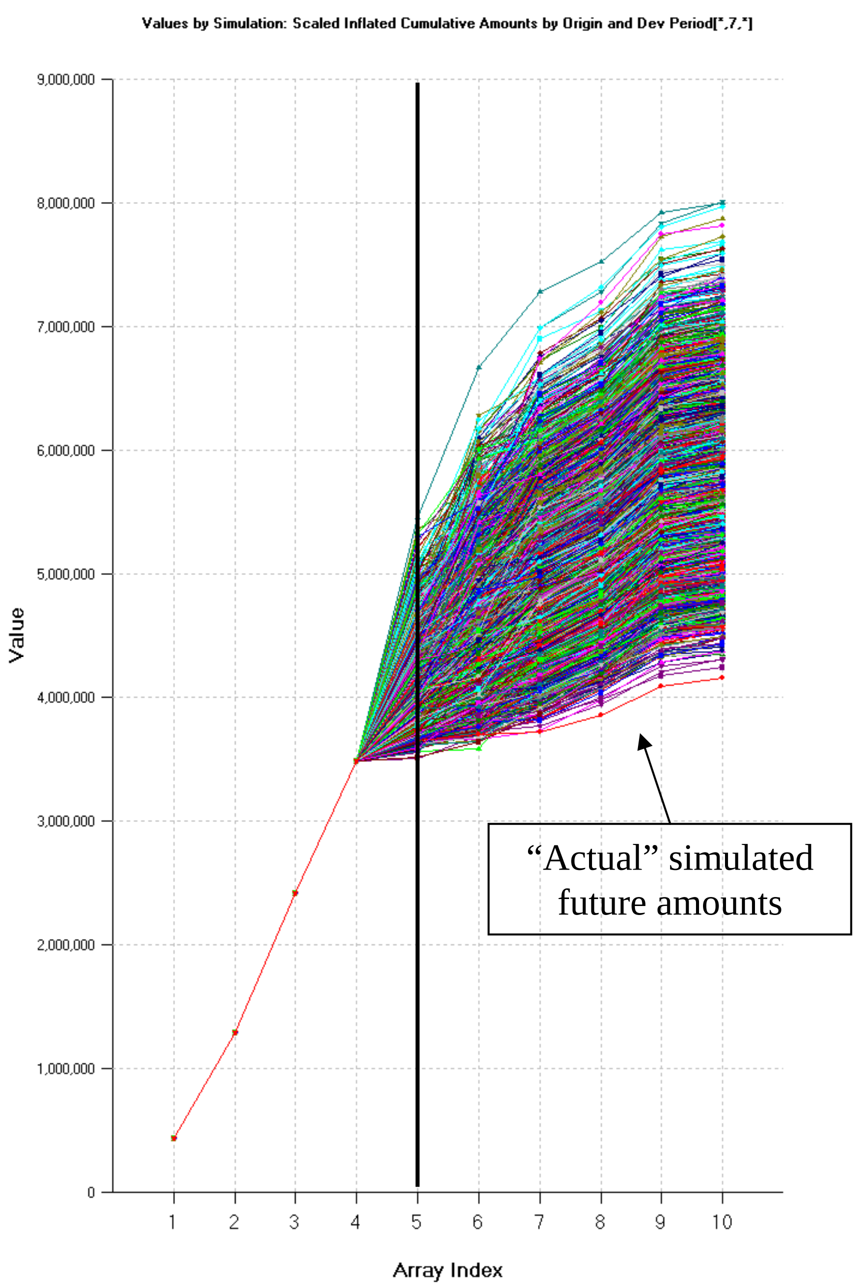
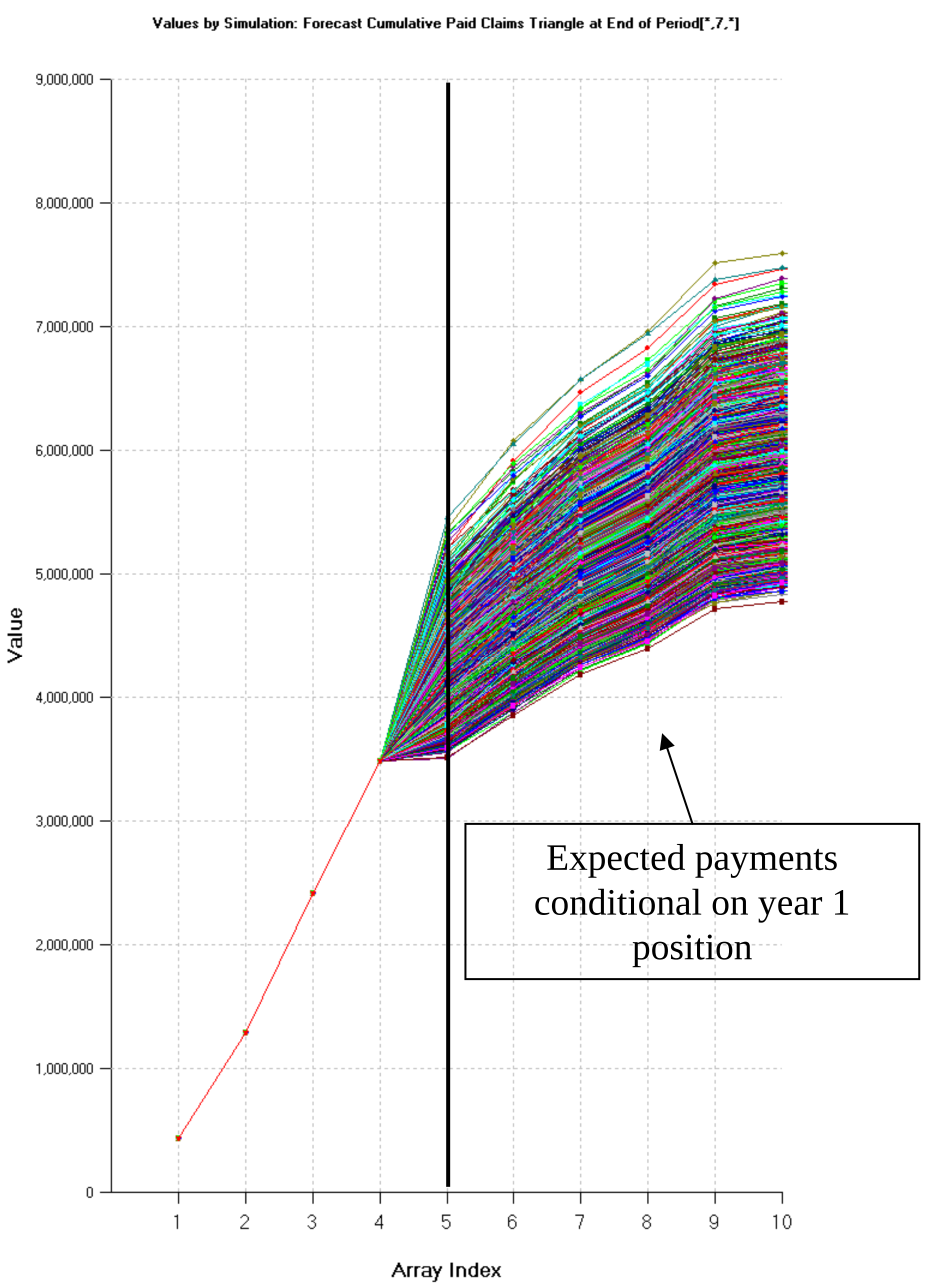
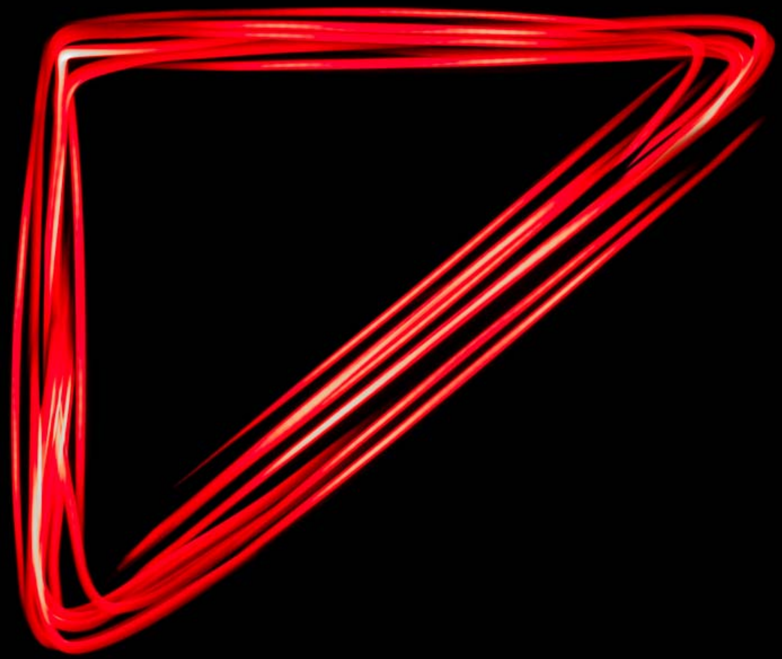
8,000,000
7,000,000
6,000,000
5,000,000
4,000,000
3,000,000
2,000,000
1,000,000
0
Value
One year ahead forecast
1
2
3
4
5
Array Index


Values by Simulation: Scaled Inflated Cumulative Amounts by Origin and Dev Period[*,7,*]
9,000,000
8,000,000
7,000,000
6,000,000
5,000,000
4,000,000
3,000,000
2,000,000
1,000,000
0
Value
1 2 3 4 5 6 7 8 9 10
Array Index
"Actual" simulated future amounts


Values by Simulation: Forecast Cumulative Paid Claims Triangle at End of Period[*,7,*]
9,000,000
8,000,000
7,000,000
6,000,000
5,000,000
4,000,000
3,000,000
2,000,000
1,000,000
0
Value
1 2 3 4 5 6 7 8 9 10
Array Index
Expected payments conditional on year 1 position

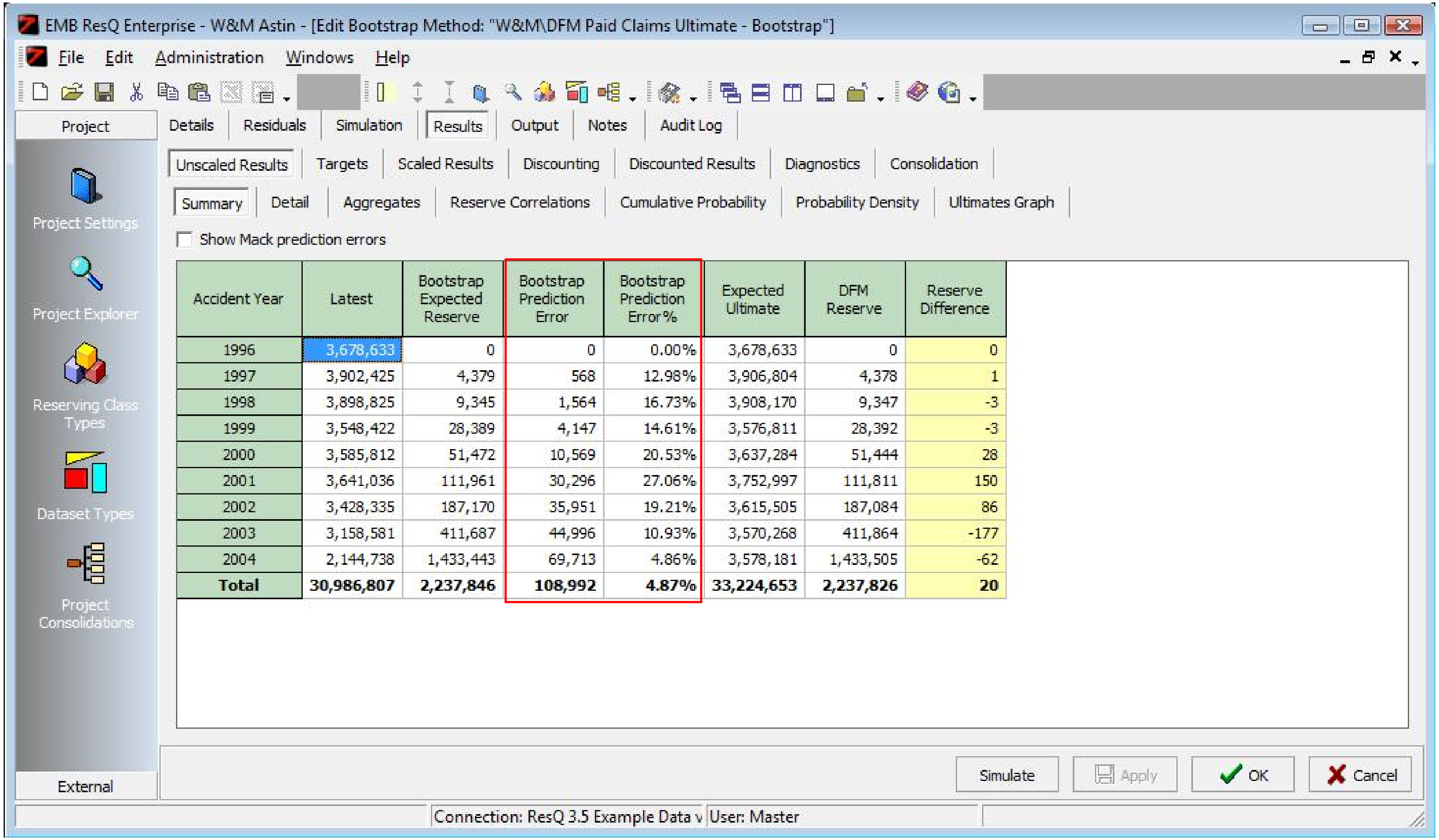
EMB ResQ Example



 Slide 70

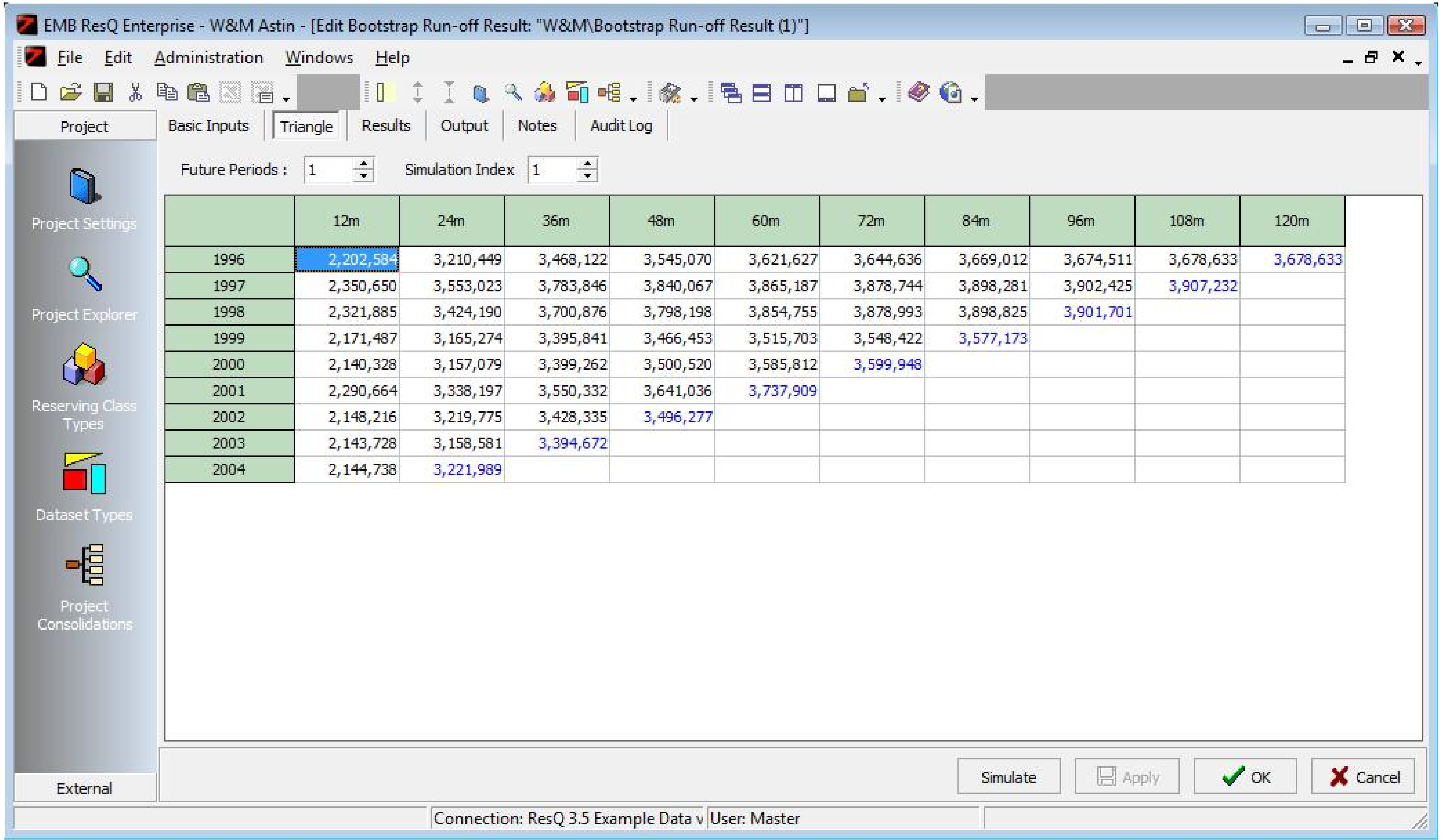
ResQ Example

Bootstrap Results Summary – “Ultimo” perspective



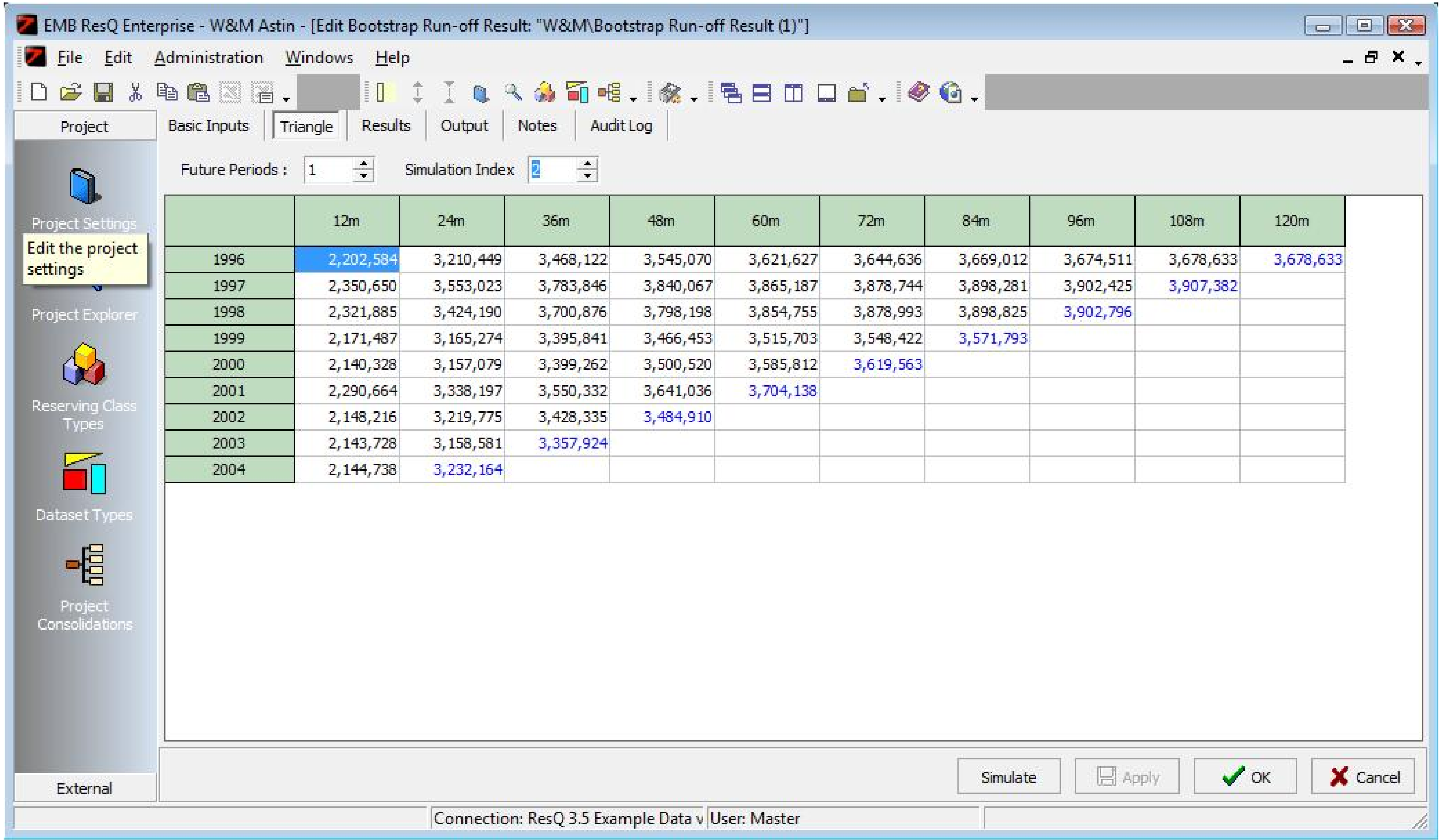
Accident Year	Latest	Bootstrap Expected Reserve	Bootstrap Prediction Error	Bootstrap Prediction Error %	Expected Ultimate	DFM Reserve	Reserve Difference
1996	3,678,633	0	0	0.00%	3,678,633	0	0
1997	3,902,425	4,379	568	12.98%	3,906,804	4,378	1
1998	3,898,825	9,345	1,564	16.73%	3,908,170	9,347	-3
1999	3,548,422	28,389	4,147	14.61%	3,576,811	28,392	-3
2000	3,585,812	51,472	10,569	20.53%	3,637,284	51,444	28
2001	3,641,036	111,961	30,296	27.06%	3,752,997	111,811	150
2002	3,428,335	187,170	35,951	19.21%	3,615,505	187,084	86
2003	3,158,581	411,687	44,996	10.93%	3,570,268	411,864	-177
2004	2,144,738	1,433,443	69,713	4.86%	3,578,181	1,433,505	-62
Total	**30,986,807**	**2,237,846**	**108,992**	**4.87%**	**33,224,653**	**2,237,826**	**20**

ResQ Example
1 Year ahead – Simulation 1



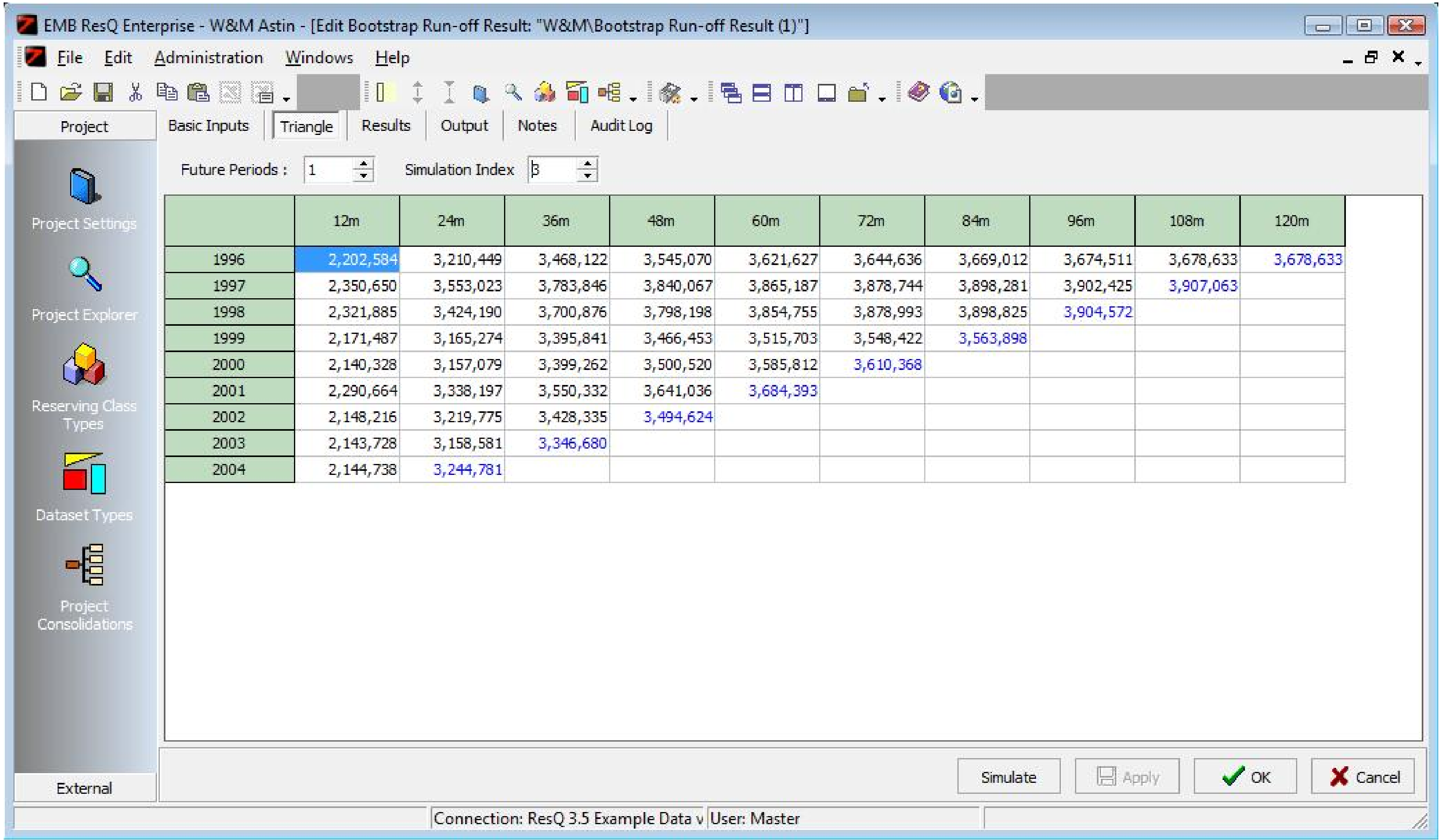
	12m	24m	36m	48m	60m	72m	84m	96m	108m	120m
1996	2,202,584	3,210,449	3,468,122	3,545,070	3,621,627	3,644,636	3,669,012	3,674,511	3,678,633	3,678,633
1997	2,350,650	3,553,023	3,783,846	3,840,067	3,865,187	3,878,744	3,898,281	3,902,425	3,907,232	
1998	2,321,885	3,424,190	3,700,876	3,798,198	3,854,755	3,878,993	3,898,825	3,901,701		
1999	2,171,487	3,165,274	3,395,841	3,466,453	3,515,703	3,548,422	3,577,173			
2000	2,140,328	3,157,079	3,399,262	3,500,520	3,585,812	3,599,948				
2001	2,290,664	3,338,197	3,550,332	3,641,036	3,737,909					
2002	2,148,216	3,219,775	3,428,335	3,496,277						
2003	2,143,728	3,158,581	3,394,672							
2004	2,144,738	3,221,989								

ResQ Example
1 Year ahead – Simulation 2



	12m	24m	36m	48m	60m	72m	84m	96m	108m	120m
1996	2,202,584	3,210,449	3,468,122	3,545,070	3,621,627	3,644,636	3,669,012	3,674,511	3,678,633	3,678,633
1997	2,350,650	3,553,023	3,783,846	3,840,067	3,865,187	3,878,744	3,898,281	3,902,425	3,907,382	
1998	2,321,885	3,424,190	3,700,876	3,798,198	3,854,755	3,878,993	3,898,825	3,902,796		
1999	2,171,487	3,165,274	3,395,841	3,466,453	3,515,703	3,548,422	3,571,793			
2000	2,140,328	3,157,079	3,399,262	3,500,520	3,585,812	3,619,563				
2001	2,290,664	3,338,197	3,550,332	3,641,036	3,704,138					
2002	2,148,216	3,219,775	3,428,335	3,484,910						
2003	2,143,728	3,158,581	3,357,924							
2004	2,144,738	3,232,164								

ResQ Example
1 Year ahead – Simulation 3



	12m	24m	36m	48m	60m	72m	84m	96m	108m	120m
1996	2,202,584	3,210,449	3,468,122	3,545,070	3,621,627	3,644,636	3,669,012	3,674,511	3,678,633	3,678,633
1997	2,350,650	3,553,023	3,783,846	3,840,067	3,865,187	3,878,744	3,898,281	3,902,425	3,907,063	
1998	2,321,885	3,424,190	3,700,876	3,798,198	3,854,755	3,878,993	3,898,825	3,904,572		
1999	2,171,487	3,165,274	3,395,841	3,466,453	3,515,703	3,548,422	3,563,898			
2000	2,140,328	3,157,079	3,399,262	3,500,520	3,585,812	3,610,368				
2001	2,290,664	3,338,197	3,550,332	3,641,036	3,684,393					
2002	2,148,216	3,219,775	3,428,335	3,494,624						
2003	2,143,728	3,158,581	3,346,680							
2004	2,144,738	3,244,781								

ResQ Example

Bootstrap Run-off Results Summary – 1 year perspective

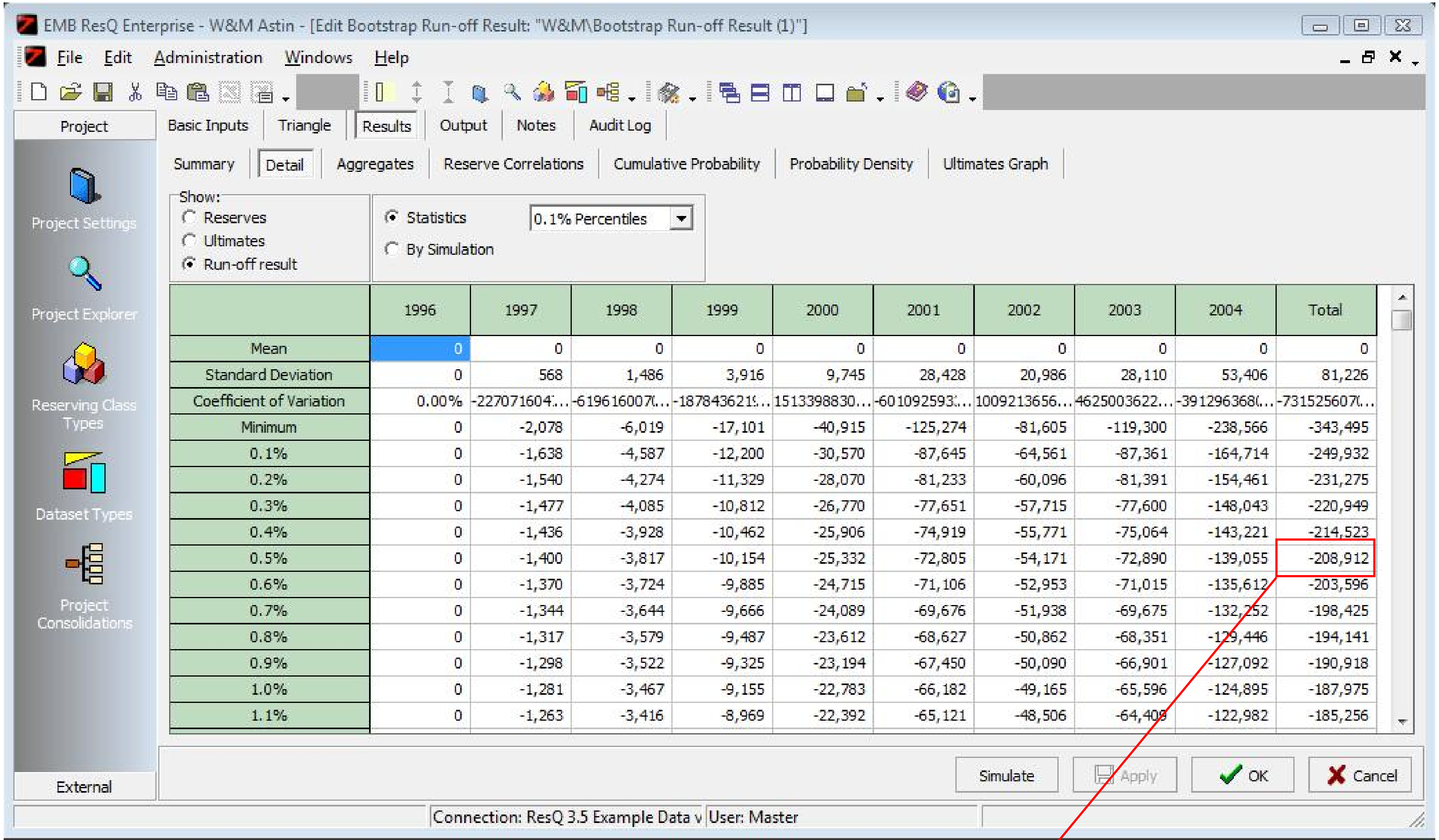
EMB ResQ Enterprise - W&M Astin - [Edit Bootstrap Run-off Result: "W&M\Bootstrap Run-off Result (1)"]

Accident Year	Avg Latest Cumulative Amount	Avg Closing Expected Reserve	StDev Closing Expected Reserve	StDev %	Avg Closing Expected Ultimate	Avg Opening Expected Reserve	Expected Run-Off Result	StDev Run-Off Result	StDev Run-off Result Ratio	Expected Payment	Avg Opening Expected Ultimate
1996	3,678,633	0	0	0.00%	3,678,633	0	0	0	0.00%	0	3,678,633
1997	3,906,804	0	0	0.00%	3,906,804	4,379	0	568	12.98%	4,379	3,906,804
1998	3,903,790	4,380	293	6.69%	3,908,170	9,345	0	1,486	15.91%	4,965	3,908,170
1999	3,568,257	8,554	527	6.16%	3,576,811	28,389	0	3,916	13.80%	19,835	3,576,811
2000	3,608,419	28,864	1,081	3.74%	3,637,284	51,472	0	9,745	18.93%	22,607	3,637,284
2001	3,699,870	53,127	2,272	4.28%	3,752,997	111,961	0	28,428	25.39%	58,834	3,752,997
2002	3,507,728	107,777	5,081	4.71%	3,615,505	187,170	0	20,986	11.21%	79,393	3,615,505
2003	3,385,653	184,615	5,884	3.19%	3,570,268	411,687	0	28,110	6.83%	227,072	3,570,268
2004	3,165,496	412,685	9,019	2.19%	3,578,181	1,433,443	0	53,406	3.73%	1,020,758	3,578,181
Total	**32,424,651**	**800,002**	**19,608**	**2.45%**	**33,224,653**	**2,237,846**	**0**	**81,226**	**3.63%**	**1,437,844**	**33,224,653**

Connection: ResQ 3.5 Example Data v User: Master

ResQ Example

99.5th percentile of the Bootstrap Run-off Result



	1996	1997	1998	1999	2000	2001	2002	2003	2004	Total
Mean	0	0	0	0	0	0	0	0	0	0
Standard Deviation	0	568	1,486	3,916	9,745	28,428	20,986	28,110	53,406	81,226
Coefficient of Variation	0.00%	-227071604…	-619616007…	-187843621…	1513398830…	-601092593…	1009213656…	4625003622…	-391296368…	-731525607…
Minimum	0	-2,078	-6,019	-17,101	-40,915	-125,274	-81,605	-119,300	-238,566	-343,495
0.1%	0	-1,638	-4,587	-12,200	-30,570	-87,645	-64,561	-87,361	-164,714	-249,932
0.2%	0	-1,540	-4,274	-11,329	-28,070	-81,233	-60,096	-81,391	-154,461	-231,275
0.3%	0	-1,477	-4,085	-10,812	-26,770	-77,651	-57,715	-77,600	-148,043	-220,949
0.4%	0	-1,436	-3,928	-10,462	-25,906	-74,919	-55,771	-75,064	-143,221	-214,523
0.5%	0	-1,400	-3,817	-10,154	-25,332	-72,805	-54,171	-72,890	-139,055	-208,912
0.6%	0	-1,370	-3,724	-9,885	-24,715	-71,106	-52,953	-71,015	-135,612	-203,596
0.7%	0	-1,344	-3,644	-9,666	-24,089	-69,676	-51,938	-69,675	-132,252	-198,425
0.8%	0	-1,317	-3,579	-9,487	-23,612	-68,627	-50,862	-68,351	-129,446	-194,141
0.9%	0	-1,298	-3,522	-9,325	-23,194	-67,450	-50,090	-66,901	-127,092	-190,918
1.0%	0	-1,281	-3,467	-9,155	-22,783	-66,182	-49,165	-65,596	-124,895	-187,975
1.1%	0	-1,263	-3,416	-8,969	-22,392	-65,121	-48,506	-64,409	-122,982	-185,256

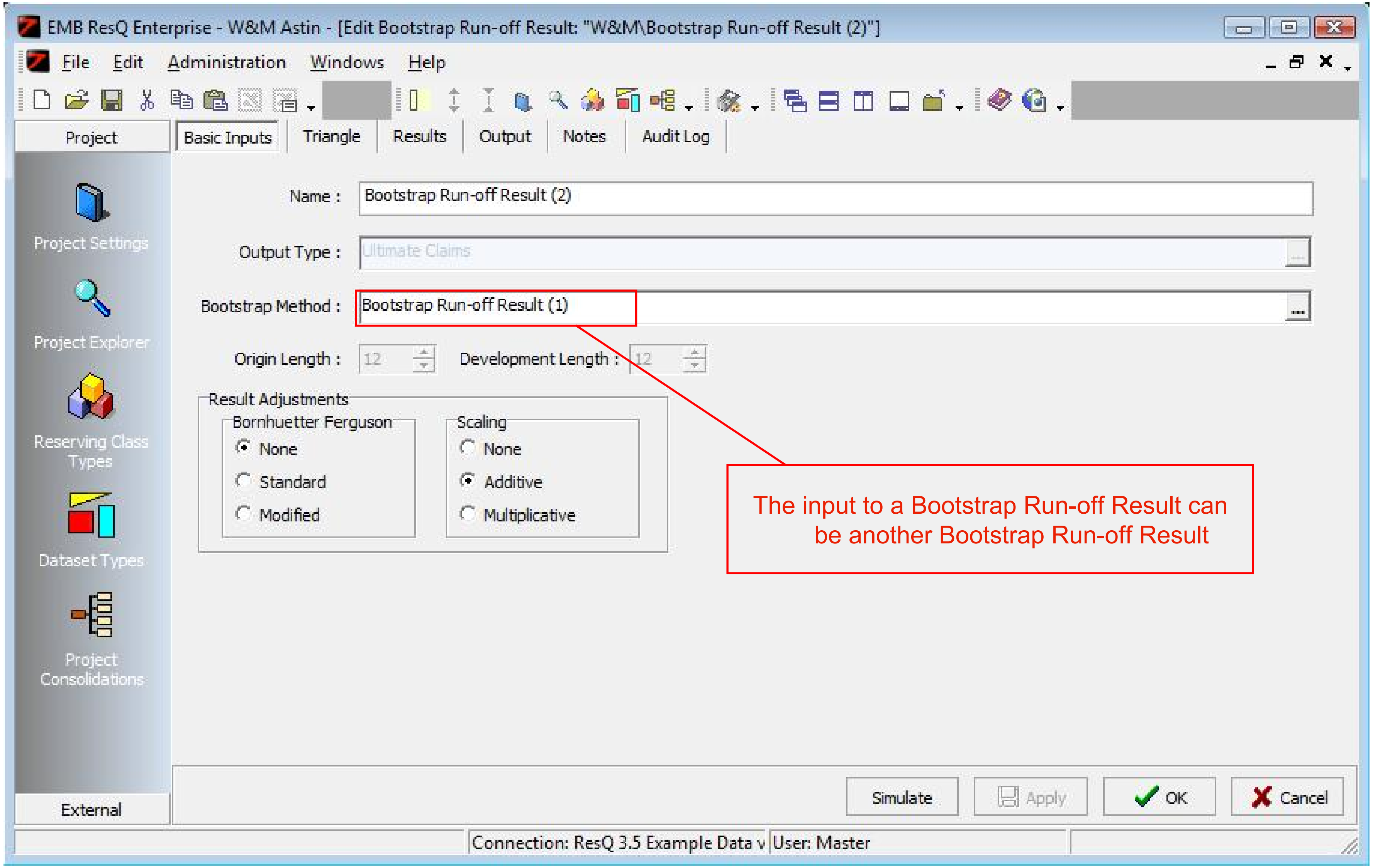
Var @ 99.5% = - (0.5^{th} percentile) = 208,912

Merz & Wuthrich (2008)

Analytic vs Simulated: Summary

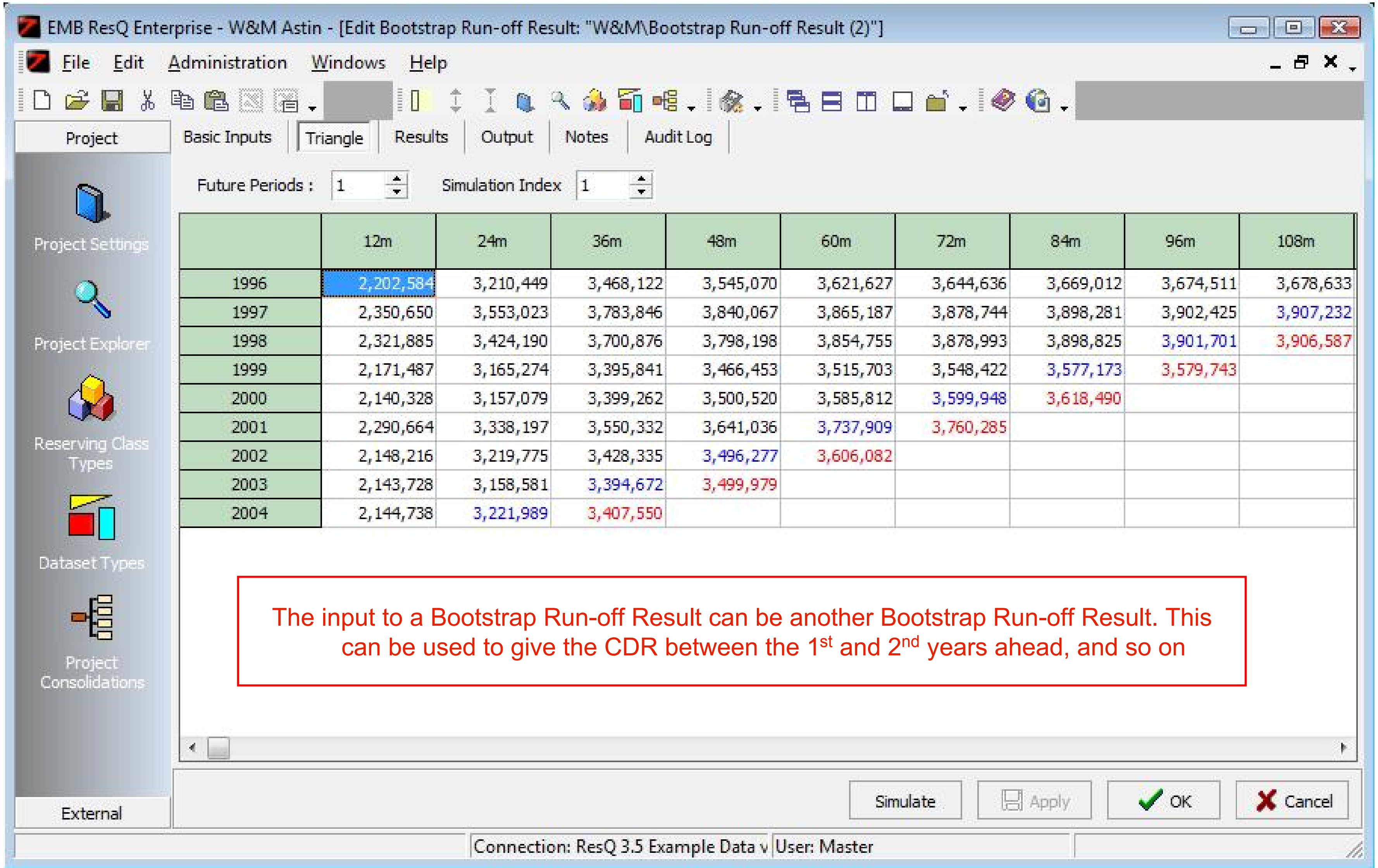
	Analytic		Simulated	
	Prediction Errors		Prediction Errors	
Accident Year	1 Year Ahead CDR	Mack Ultimate	1 Year Ahead CDR	Mack Ultimate
0	0	0	0	0
1	567	567	568	568
2	1,488	1,566	1,486	1,564
3	3,923	4,157	3,916	4,147
4	9,723	10,536	9,745	10,569
5	28,443	30,319	28,428	30,296
6	20,954	35,967	20,986	35,951
7	28,119	45,090	28,110	44,996
8	53,320	69,552	53,406	69,713
Total	81,080	108,401	81,226	108,992

ResQ Example
Cascading Bootstrap Run-off Results



ResQ Example

Cascading Bootstrap Run-off Results

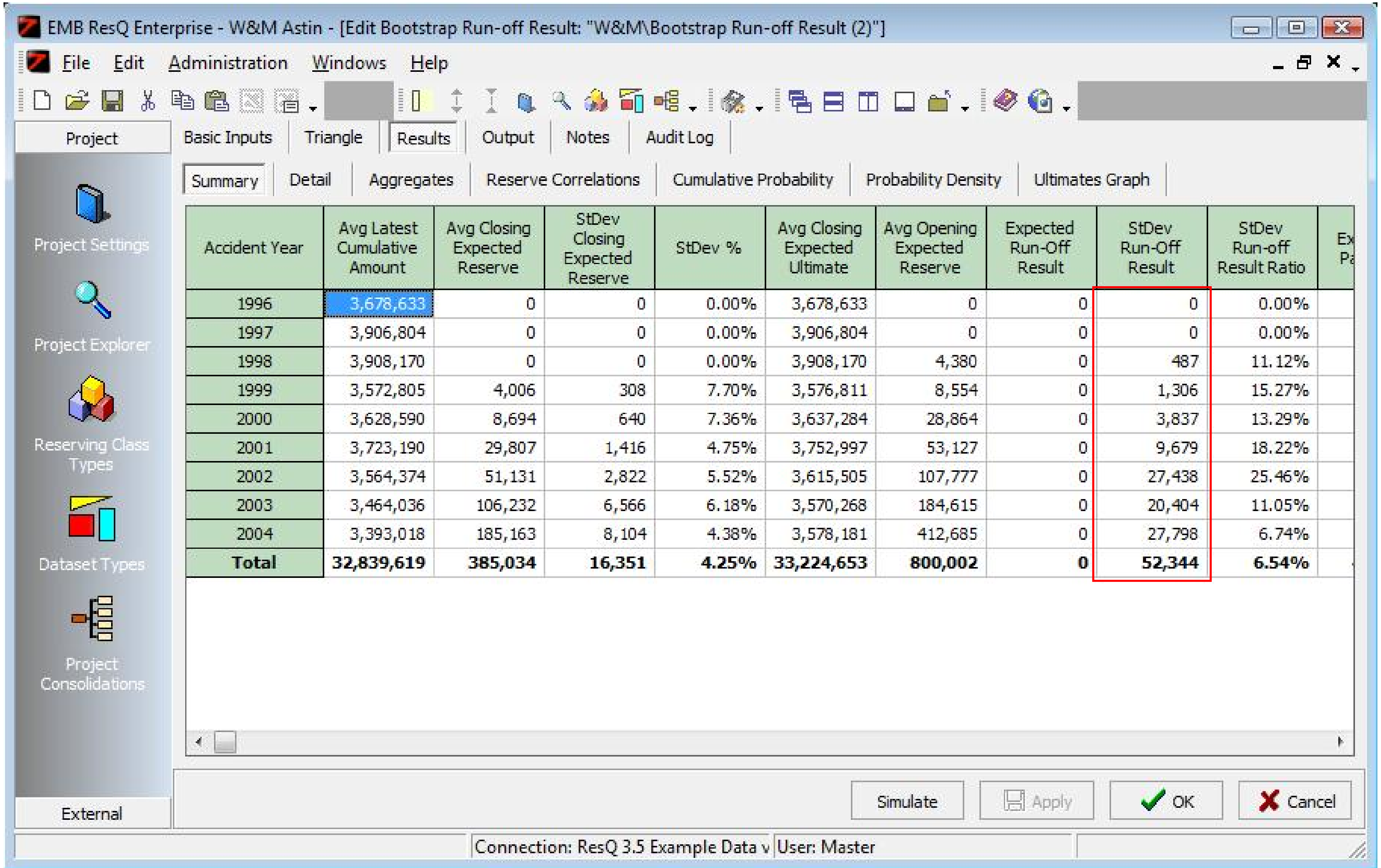


	12m	24m	36m	48m	60m	72m	84m	96m	108m
1996	2,202,584	3,210,449	3,468,122	3,545,070	3,621,627	3,644,636	3,669,012	3,674,511	3,678,633
1997	2,350,650	3,553,023	3,783,846	3,840,067	3,865,187	3,878,744	3,898,281	3,902,425	3,907,232
1998	2,321,885	3,424,190	3,700,876	3,798,198	3,854,755	3,878,993	3,898,825	3,901,701	3,906,587
1999	2,171,487	3,165,274	3,395,841	3,466,453	3,515,703	3,548,422	3,577,173	3,579,743	
2000	2,140,328	3,157,079	3,399,262	3,500,520	3,585,812	3,599,948	3,618,490		
2001	2,290,664	3,338,197	3,550,332	3,641,036	3,737,909	3,760,285			
2002	2,148,216	3,219,775	3,428,335	3,496,277	3,606,082				
2003	2,143,728	3,158,581	3,394,672	3,499,979					
2004	2,144,738	3,221,989	3,407,550						

The input to a Bootstrap Run-off Result can be another Bootstrap Run-off Result. This can be used to give the CDR between the 1st and 2nd years ahead, and so on

ResQ Example

Cascading Bootstrap Run-off Results



Accident Year	Avg Latest Cumulative Amount	Avg Closing Expected Reserve	StDev Closing Expected Reserve	StDev %	Avg Closing Expected Ultimate	Avg Opening Expected Reserve	Expected Run-Off Result	StDev Run-Off Result	StDev Run-off Result Ratio
1996	3,678,633	0	0	0.00%	3,678,633	0	0	0	0.00%
1997	3,906,804	0	0	0.00%	3,906,804	0	0	0	0.00%
1998	3,908,170	0	0	0.00%	3,908,170	4,380	0	487	11.12%
1999	3,572,805	4,006	308	7.70%	3,576,811	8,554	0	1,306	15.27%
2000	3,628,590	8,694	640	7.36%	3,637,284	28,864	0	3,837	13.29%
2001	3,723,190	29,807	1,416	4.75%	3,752,997	53,127	0	9,679	18.22%
2002	3,564,374	51,131	2,822	5.52%	3,615,505	107,777	0	27,438	25.46%
2003	3,464,036	106,232	6,566	6.18%	3,570,268	184,615	0	20,404	11.05%
2004	3,393,018	185,163	8,104	4.38%	3,578,181	412,685	0	27,798	6.74%
Total	**32,839,619**	**385,034**	**16,351**	**4.25%**	**33,224,653**	**800,002**	**0**	**52,344**	**6.54%**

Multiple 1 yr ahead CDRs
An interesting result

- Creating cascading CDRs over all years gives the following results:

Accident Year	Number of years ahead								Sqrt(Sum of Squares)	Mack Ultimate
	1 Yr	2 Yrs	3 Yrs	4 Yrs	5 Yrs	6 Yrs	7 Yrs	8 Yrs		
1	0	0	0	0	0	0	0	0	-	0
2	568	0	0	0	0	0	0	0	568	568
3	1,486	487	0	0	0	0	0	0	1,564	1,564
4	3,916	1,306	431	0	0	0	0	0	4,151	4,147
5	9,745	3,837	1,277	425	0	0	0	0	10,560	10,569
6	28,428	9,679	3,824	1,272	425	0	0	0	30,303	30,296
7	20,986	27,438	9,343	3,693	1,226	409	0	0	35,998	35,951
8	28,110	20,404	26,922	9,162	3,613	1,208	402	0	45,055	44,996
9	53,406	27,798	20,236	26,687	9,111	3,600	1,203	402	69,600	69,713
Total	81,226	52,344	38,513	29,010	10,120	3,879	1,285	402	108,543	108,992

- The sum of the variances of the repeated 1 yr ahead CDRs (over all years) equals the variance over the lifetime of the liabilities
  - Under Mack's assumptions/chain ladder, this can be proved
  - (The simulation based results are approximate due to numerical error)
- This means that we expect the risk under the 1 year view to be lower than the standard “ultimo” perspective

Re-reserving in Simulation-based Capital Models

The advantage of investigating the claims development result (using re-reserving) in a simulation environment is that the procedure can be generalised:

- Not just the chain ladder model
- Not just Mack's assumptions
- Can include curve fitting and extrapolation for tail estimation
- Can incorporate a Bornhuetter-Ferguson step
- Can be extended beyond the 1 year horizon to look at multi-year forecasts
- Provides a *distribution* of the CDR, not just a standard deviation
- Can be used to help calibrate Solvency II internal models

Conclusions

- Stochastic reserving has a reputation for being difficult
  - Attempted analytically, it can be, and the limitations of the formulae should be recognised
- Simulation techniques can simplify the modelling enormously, giving results that are analogous to the analytic results (when applied correctly), as well as providing additional information and allowing the models to be generalised
- An understanding of both the analytic and simulation based approaches can be obtained by following the key principles in each case
- The characteristics of the models and the effect of their key parameters should be understood. This will help with interpretation of outputs, especially when things go wrong.
- A reconciliation between the 1 year view and the "ultimo" view can be obtained by understanding the differences between the perspectives.

Risk Theory and Simulation

"Modern computer simulation techniques open up a wide field of practical applications for risk theory concepts, without the restrictive assumptions, and sophisticated mathematics, of many traditional aspects of risk theory".

- Daykin, Pentikainen and Pesonen, 1996

"I believe that stochastic modelling is fundamental to our profession. How else can we seriously advise our clients and our wider public on the consequences of managing uncertainty in the different areas in which we work?"

- Chris Daykin, 1995

Recent developments in stochastic reserving

- Wuthrich & Merz (book & papers)
- Esbjorn Ohlsson (one yr view)
- Dorothea Diers (one yr view)
- Tom Wright (LMAG presentation one yr view)
  - They [M&W] use a completely different approach to the one described here, but the two methods give exactly the same variance for the one-year CDR
- Schnieper/Liu (splitting IBNR into new reported claims and IBNER)
- Quarg & Mack/Liu (combining information in paid and incurred)
- Stochastic BF – Mack, Verrall, Alai, Wuthrich
- Greg Taylor et al (individual claims)
- Magda Schiegl (3D)
- Verrall & Brydon (calendar year trends)
- Jens Perch-Nielsen (calendar year trends)
- Susanna Bjorkwall (parametric bootstrap, separation technique)
- Pavel Shevchenko (Bayesian)
- Fabrizio Restione (Bayesian)
- Anders Jessen (Bayesian - incorporating claim numbers)
- Piet de Jong (estimating correlations for multiple triangles)
- Murphy and McLennan (projecting individual open large claims and netting down)
- + many others

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Biography
Peter England BSc PhD CStat

Peter graduated from City University with a BSc and PhD in Actuarial Science. After completing his PhD, entitled “Statistical Modelling of Excess Mortality of Medically Impaired Insured Lives”, Peter worked as a medical statistician at the London School of Hygiene and Tropical Medicine, conducting research into risk factors associated with Sudden Infant Death Syndrome (SIDS/Cot Death) and lecturing to post-graduate students in Medical Statistics.

Peter then returned to actuarial work, within the Group Non-Life Technical Department at Commercial Union (now Aviva), supporting the Executive Directors in worldwide reserve monitoring, business plan monitoring, and outwards catastrophe reinsurance modelling, amongst other activities.

Peter then moved to Lloyd’s as “Manager, Capital Modelling” in the Market Risk Unit, where he was jointly responsible for the Risk Based Capital system used for setting member capital requirements at Lloyd's.

After working at Lloyd's, Peter joined EMB in November 1999, specialising in research and statistical modelling, particularly financial risk modelling using simulation techniques. His main areas of work are:

- Risk based capital modelling
- Reserve variability methodologies
- Liability model parameterisation, including parameter uncertainty
- Catastrophe risk aggregation and reinsurance modelling
- Asbestos liability modelling
- Pricing using simulation techniques
- Generalised linear and non-linear modelling techniques

Peter is also involved in the development of EMB software, staff and client training, and is a regular speaker at seminars. He is a Chartered Statistician, a Senior Visiting Fellow at the Cass Business School, London, and is the author (or co-author) of numerous papers, including the prize-winning Institute of Actuaries paper "Stochastic Claims Reserving in General Insurance".

