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Stochastic mortality models for actuarial work

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Overview

1. Deterministic model risk
2. Stochastic model risk
3. Advanced ages
4. Conclusions

1. Deterministic model risk

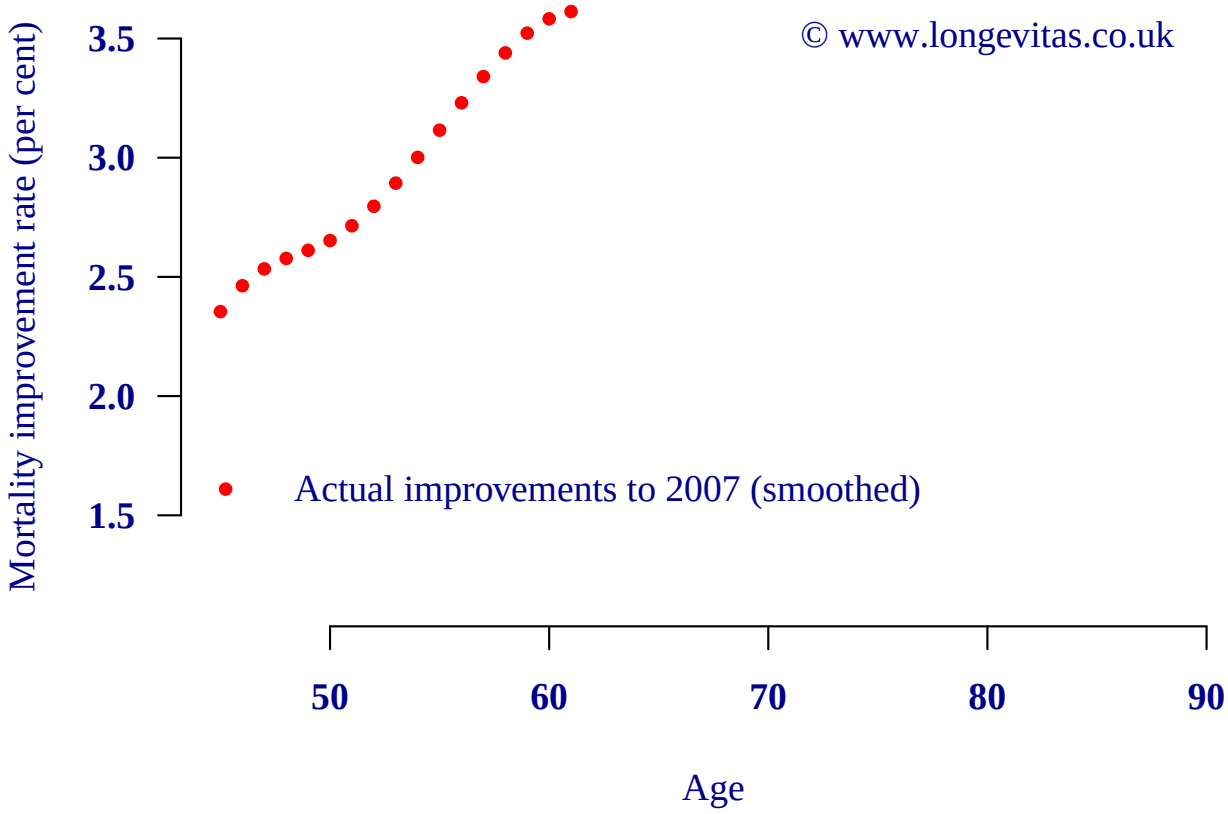
1. Deterministic model risk

- How do you know your model is correct?
- What are the consequences if it is not?

1. Deterministic model risk — CMI 2010

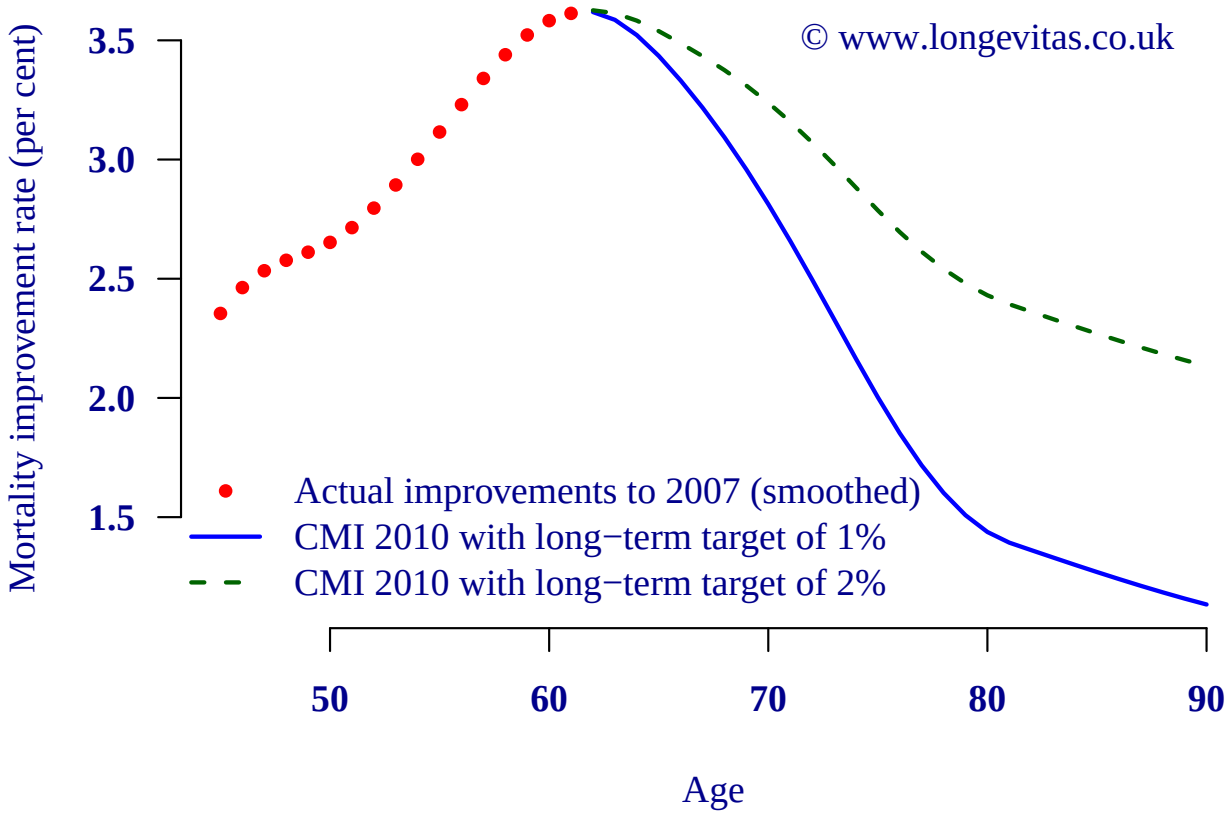
- Deterministic
- Defaults to projecting *decelerating* mortality improvements. . .

1. Deterministic model risk — CMI 2010



Source: Improvement rates labelled "actual" in CMI 2010.

1. Deterministic model risk — CMI 2010



Source: Smoothed actual mortality-improvement rates for males born in 1946, together with projected rates according to CMI 2010 model using default parameters and a long-term target of 1% or 2% improvement per annum.

2. Stochastic model risk

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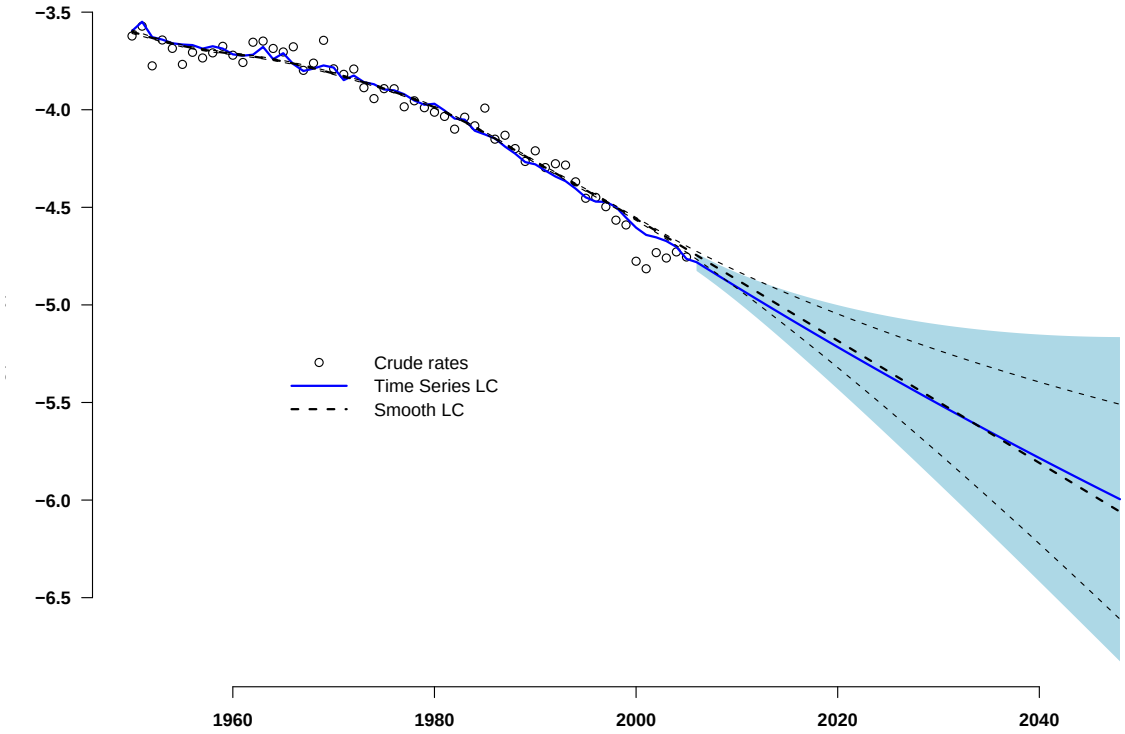
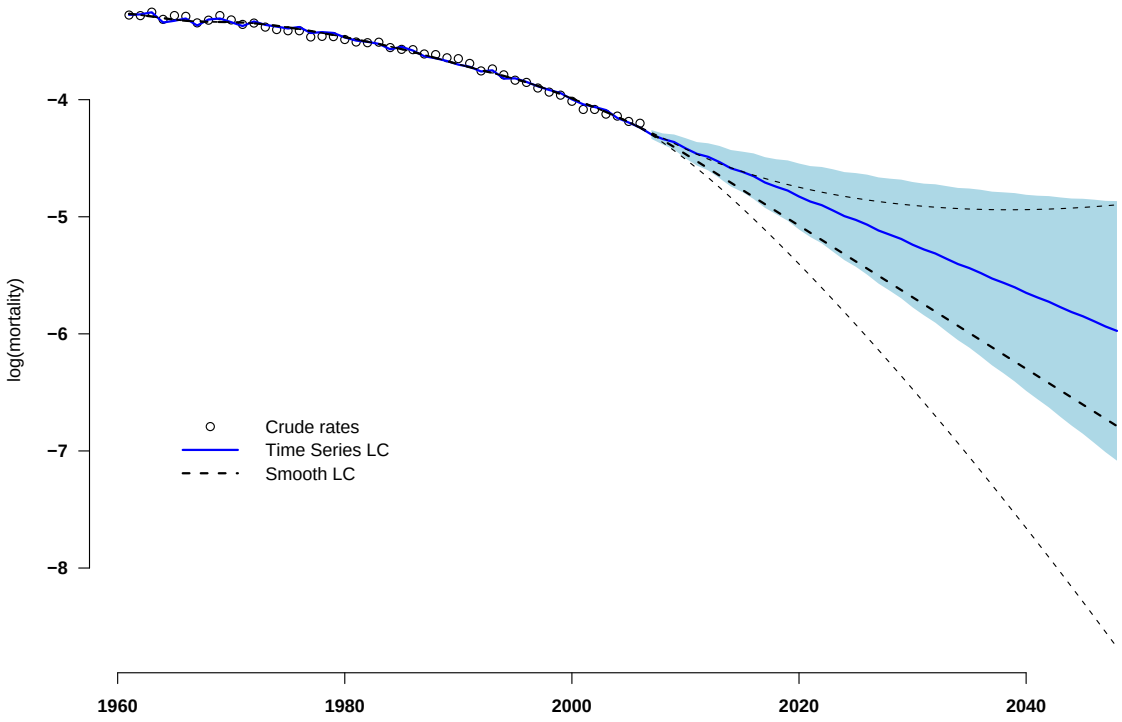
- Model risk most obviously applies to deterministic models...
- ...but applies to *all* models, including stochastic ones

2. Stochastic model risk



Source: Richards and Currie (2009). CMI assured-lives data, fitted and forecast rates for males at age 65 under two Lee-Carter variants: DDE model (time series) and Currie-Richards model (smooth).

2. Stochastic model risk



Source: Richards and Currie (2009). ONS population data, fitted and forecast rates for males at age 65 under two Lee-Carter variants: DDE model (time series) and Currie-Richards model (smooth).

2. Stochastic model risk

Model	Best-estimate reserve	99.5% reserve	Capital required
DDE	13.92	14.42	3.6%
CBD5	13.96	15.04	8.0%

Source: Richards (2011). Best-estimate and stressed annuity values for a male aged 65 following population mortality in England and Wales. Continuous temporary annuities to age 105, valued at 3% per annum. DDE model is that of Delwarde, Denuit and Eilers (2007), while CBD5 model is that of Cairns, Blake and Dowd (2006), as modified by Currie (2010) to allow for a non-linear effect by age. The DDE model projects a single constant drift term, while the CBD5 model projects a bivariate drift term.

2. Stochastic model risk

- DDE model says a reserve of 14.42 suffices with 99.5% probability
- CBD5 model says the same reserve suffices with 87.3% probability

2. Model risk

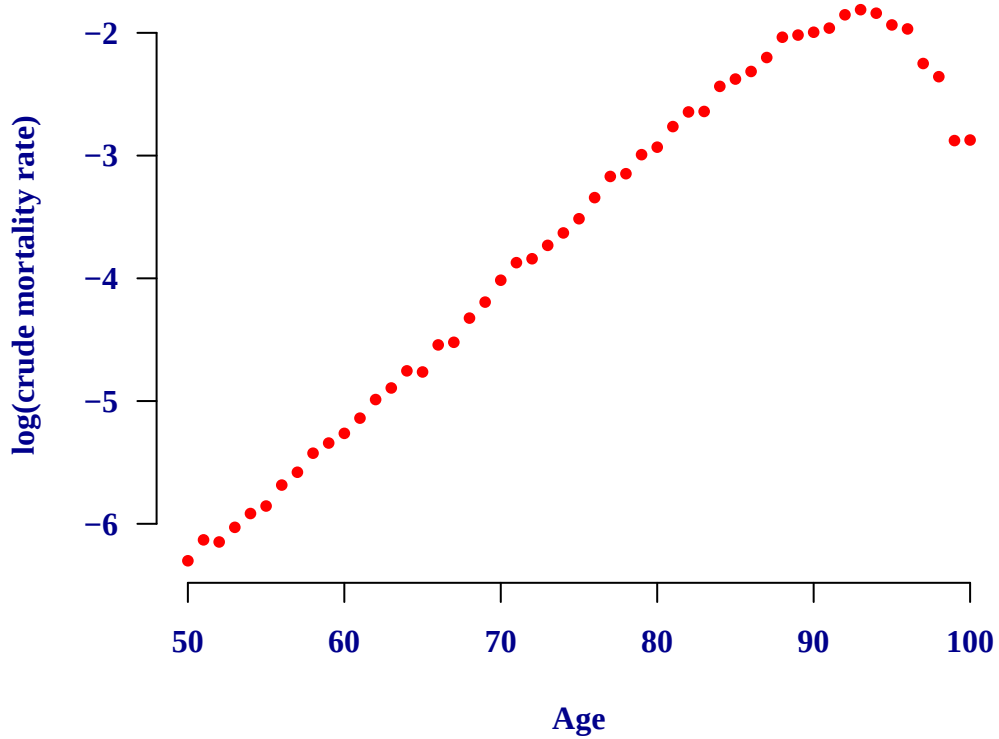
- How do you know your model is correct?
- You don't
- Must use a *variety* of models to explore model risk

3. Advanced ages

3. Advanced ages

- ONS data only goes up to age 105
- Usable CMI data stops even earlier...

3. Advanced ages

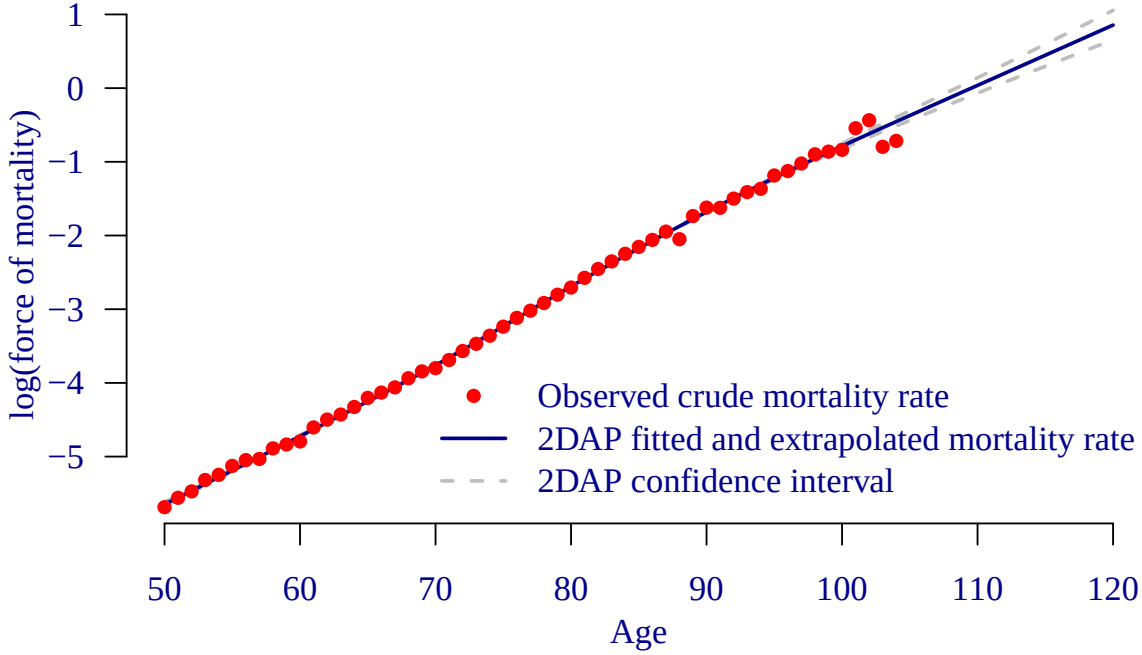


Source: CMI assured-lives data. Logarithm of crude force of mortality for CMI data at ages 50–100 aggregated over the years 2001–2005. The data above age 95 are unreliable and cannot be included in any projection model. However, actuarial calculations for annuities and pensions typically require mortality rates up age 120, so some form of extrapolation is required from age 95 to age 120.

3. Advanced ages

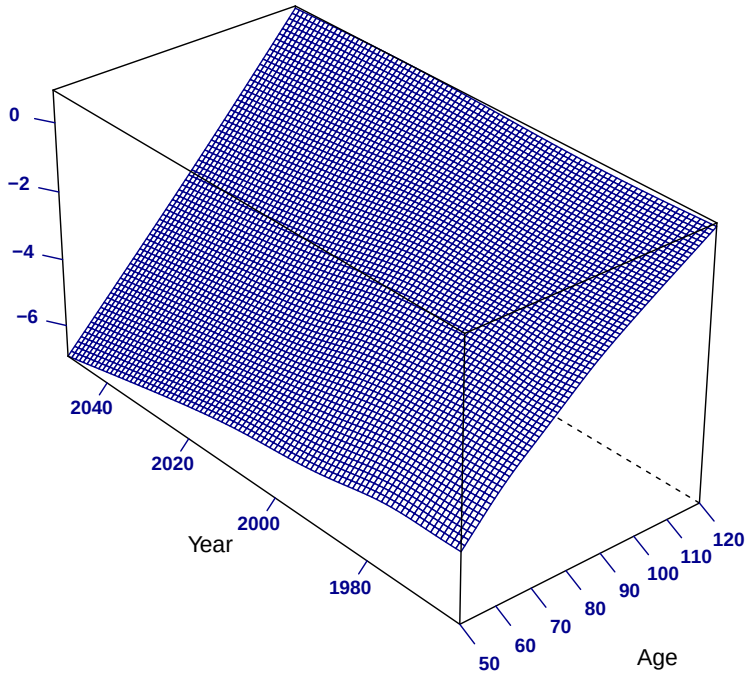
- How to get rates *and* projections at advanced ages?
- 2D P-spline models can extrapolate by age as well as project in time

3. Advanced ages — 2D P-spline models



Source: CMI assured-lives data fitted to 2D age-period P-spline model. Logarithm of crude force of mortality for ONS data for males in 2007 at ages 50–104, together with the fitted and extrapolated rates to age 120 under the 2D age-period model (Currie, Durban and Eilers, 2004). As with a projection in time, the uncertainty over the extrapolated rates increases the greater the distance from the actual data, i.e. there is an expanding "funnel of doubt". In this case the funnel is rather narrow because of the strength of the age signal.

3. Advanced ages — 2D P-spline models



Source: Longevitas Ltd. Fitted and projected logarithm of force of mortality for ONS data for males. Mortality rates are simultaneously extrapolated to age 120 and projected to 2050 using the two-dimensional P-spline model of Currie, Durban and Eilers (2004).

3. Advanced ages — model risk

- Model risk demands we use more than one model
- Need other models capable of extrapolation by age
- Cairns-Blake-Dowd (2006) model can also extrapolate by age

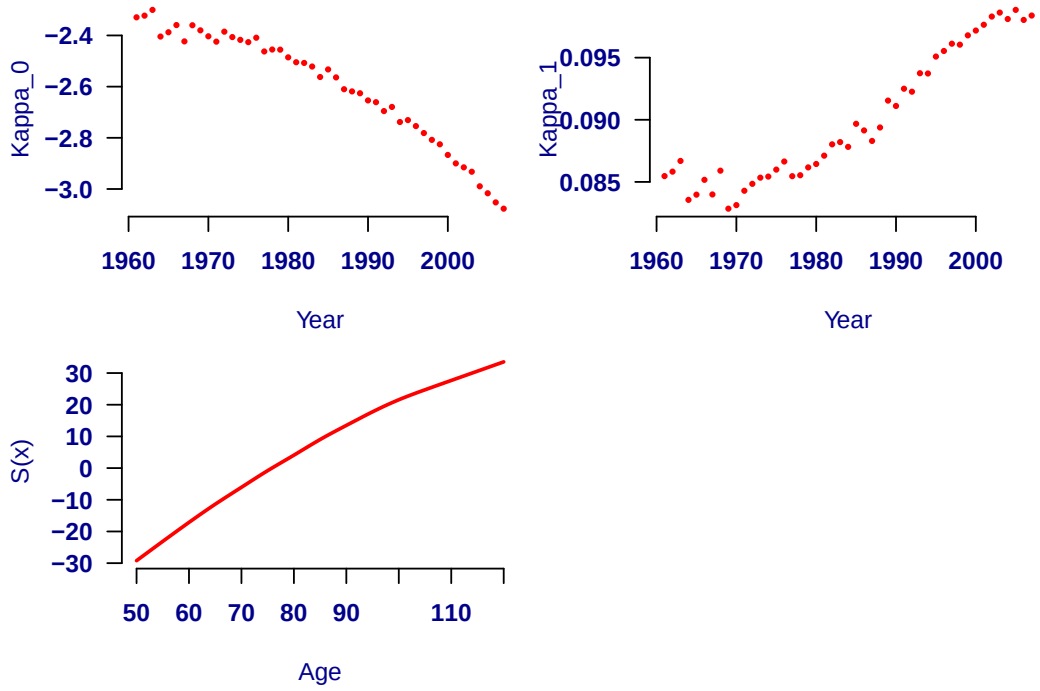
3. Advanced ages — CBD 5

$$\log \mu_{x,y} = \kappa_{0,y} + \kappa_{1,y} S\left(x\right)$$

- κ_0 and κ_1 form a bivariate time series for projecting future rates
- $S(x)$ is a smooth function of age

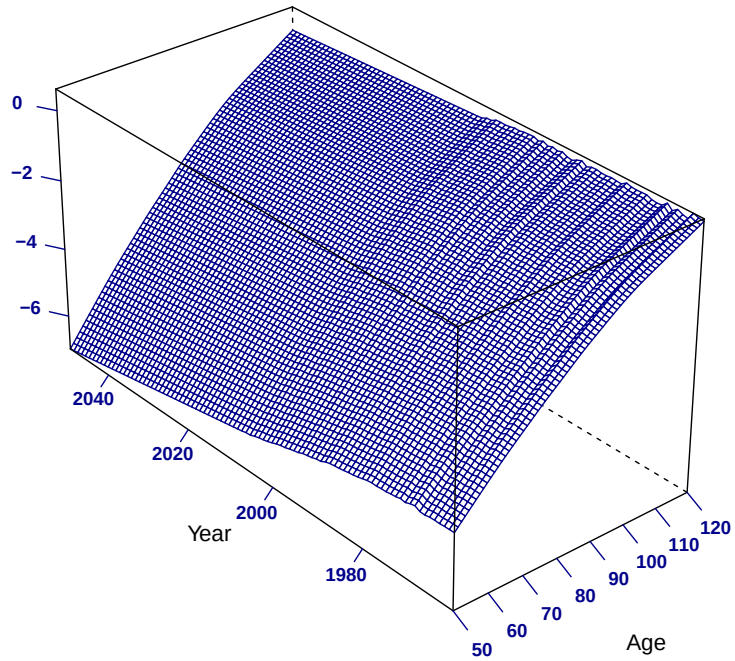
Source: Cairns, Blake and Dowd (2006) as modified by Currie (2010).

3. Advanced ages — CBD 5



Source: Longevitas Ltd. Parameters for the Cairns-Blake-Dowd model number 5, as modified by Currie (2010). The function S(x) is smoothed by splines with a five-year knot spacing, which enables age extrapolation as in the 2D P-spline models.

3. Advanced ages — CBD 5



Source: Logarithm of force of mortality for ONS data for males. Mortality rates are extrapolated to age 120 using the S(x) function, and projected to 2050 using the bivariate time series for κ_0 and κ_1. The fitted rates have ridges due to period effects.

4. Conclusions

4. Conclusions

- CMI 2010 model defaults to decelerating mortality improvements
- Model risk is very substantial for mortality projections
- Don't rely on a single projection model or methodology
- Stochastic projection models can project by age as well as time

References I

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