EXAMINATIONS

4 April 2001 (pm)

Subject 102 — Financial Mathematics

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*

2. *Mark allocations are shown in brackets.*

3. *Attempt all 12 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available Actuarial Tables and an electronic calculator.

1 Explain what is meant by a certificate of deposit. [3]

2 The following table gives information concerning an investment fund:

Calendar Year	*1997* £ *millions*	*1998* £ *millions*	*1999* £ *millions*	*2000* £ *millions*
Value of fund at 30 June	—	460	500	650
Net cash flow received on 1 July	—	50	40	60
Value of fund at 31 December	400	550	600	X

If the time weighted rate of return earned on the fund during the period from 31 December 1997 to 31 December 2000 is 11% per annum effective, calculate X, the value of the fund on 31 December 2000. [3]

3 The rate of discount per annum convertible quarterly is 8%.

Calculate:

(i) The equivalent rate of interest per annum convertible half-yearly. [2]

(ii) The equivalent rate of discount per annum convertible monthly. [2]

[Total 4]

4 (i) State what is meant by a "forward contract". Your answer should include reference to the terms "short forward position" and "long forward position". [3]

(ii) A 3-month forward contract is issued on 1 February 2001 on a stock with a price of £150 per share. Dividends are received continuously and the dividend yield is 3% per annum. In addition, it is anticipated that a special dividend of £30 per share will be paid on 1 April 2001.

Assuming a risk-free force of interest of 5% per annum and no arbitrage, calculate the forward price per share of the contract. [3]

[Total 6]

5 An investor is considering investing in the shares of a particular company.

The shares pay dividends every 6 months, with the next dividend being due in exactly 4 months' time. The next dividend is expected to be d_1, the purchase price of a single share is P and the annual effective rate of return expected from the investment is $100i\%$. Dividends are expected to grow at a rate of $100g\%$ per annum from the level of d_1 where $g < i$. Dividends are expected to be paid in perpetuity.

(i) Show that $P = \dfrac{d_1(1+i)^{1/6}}{(1+i)^{1/2} - (1+g)^{1/2}}$ [4]

(ii) The investor delays purchasing the shares for exactly 2 months at which time the price of a single share is £18, $100g\% = 4\%$ and $d_1 = £0.50$. Calculate the annual effective rate of return expected by the investor to the nearest 1%. [3]

[Total 7]

6 The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula:

$$\begin{aligned} \delta(t) &= 0.04 + 0.01t && 0 \le t < 8 \\ &= 0.07 && 8 \le t \end{aligned}$$

(i) Derive, and simplify as far as possible, expressions for $A(t)$ where $A(t)$ is the accumulated amount at time t of an investment of £1 invested at time $t = 0$. [5]

(ii) Calculate the present value at time $t = 0$ of £100 due at time $t = 10$. [2]

[Total 7]

7 A small technology company set up a new venture on 1 January 2001. The initial investment on that date was £2 million with a further £1.5 million required on 1 August 2001.

It is expected that on 1 January 2002, net income (i.e. income less running costs) will begin at the rate of £0.3 million per annum and that the rate will increase by £0.1m per annum on 1 January of each subsequent year. It is assumed that the net income will be received continuously throughout the project.

The company expects to sell the business on 31 December 2011 for £3 million.

Calculate the net present value of the venture on 1 January 2001 at a rate of interest of 6% per annum, convertible half-yearly. [8]

8 The n-year spot rate of interest, y_n, is given by:

$$y_n = 0.04 + \frac{n}{1000} \qquad \text{for } n = 1, 2 \text{ and } 3$$

(i) Calculate the implied one-year forward rates applicable at times $t = 1$ and $t = 2$. [3]

(ii) Assuming that coupon and capital payments may be discounted using the same discount factors, and that no arbitrage applies, calculate:

(a) The price at time $t = 0$ per £100 nominal of a bond which pays annual coupons of 3% in arrears and is redeemed at 110% after 3 years.

(b) The 2-year par yield. [6]

[Total 9]

9 The annual yields from a particular fund are independent and identically distributed. Each year, the distribution of $1 + i$ is log-normal with parameters $\mu = 0.07$ and $\sigma^2 = 0.006$, where i denotes the annual yield on the fund.

(i) Find the mean accumulation in ten years' time of an investment in the fund of £20,000 at the end of each of the next ten years, together with £150,000 invested immediately. [5]

(ii) Find the single amount which should be invested in the fund immediately to give an accumulation of at least £600,000 in ten years' time with probability 0.99. [7]

[Total 12]

10 (i) An investment provides income of £1 million payable at the end of each year for the next ten years. There is no capital repayment. If the interest rate is 7% per annum effective, show that the "discounted mean term" (or "Macauley duration") of the investment is 4.946 years. [4]

(ii) An investment company has liabilities of £7 million due in 5 years' time and £8 million due in 8 years' time. The company holds two investments, A and B. Investment A is the investment described in part (i) and Investment B is a zero coupon bond which pays £X at the end of n years (where n is not necessarily an integer).

The interest rate is 7% per annum effective. Investigate whether values of £X and n can be found which ensure that the investment company is immunized against small changes in the interest rate.

You are given that $\sum_{t=1}^{10} t^2 v^t = 228.451$ at 7%. [8]

[Total 12]

11 A loan of £80,000 is repayable over 25 years by level monthly instalments in arrears of capital and interest. The repayments are calculated using an effective rate of interest of 8% per annum.

Calculate:

(i) (a) The capital repaid in the first monthly instalment.

(b) The total amount of interest paid during the last six years of the loan.

(c) The interest included in the final monthly payment. [9]

(ii) Explain how your answer to (i)(b) would alter if, under the original terms of the loan, repayments had been made less frequently than monthly. [3]

[Total 12]

12 On 15 March 1996 the government of a country issued an index-linked bond of term 6 years. Coupons are payable half-yearly in arrears, and the annual nominal coupon rate is 3%.

Interest and capital payments are indexed by reference to the value of an inflation index with a time lag of 8 months.

A tax-exempt investor purchased the stock at £111 per £100 nominal on 16 September 1999, just after the coupon payment had been made.

You are given the following values of the inflation index:

Date	*Inflation Index*
July 1995	110.5
March 1996	112.1
July 1999	126.7
September 1999	127.4

(i) Calculate the amount of the coupon payment per £100 nominal stock on 15 March 2000. [3]

(ii) Calculate the effective real annual yield to the investor on 16 September 1999. You should assume that the inflation index will increase continuously from its value in September 1999 at the rate of 4% per annum effective. [11]

(iii) Without doing any further calculations, explain how your answer to (ii) would alter, if at all, if the inflation index for July 1995 had been more than 110.5. [3]

[Total 17]