

# EXAMINATIONS

17 September 2002 (pm)

## Subject 102 — Financial Mathematics

*Time allowed: Three hours*

### ***INSTRUCTIONS TO THE CANDIDATE***

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*

***Graph paper is not required for this paper.***

### ***AT THE END OF THE EXAMINATION***

*Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.*

*In addition to this paper you should have available Actuarial Tables and your own electronic calculator.*

- 1 Describe the features and risk characteristics of a “Government bill”. [4]
  
- 2 A 10-month forward contract is issued on 1 March 2002 on a stock with a price of £12 per share at that date. Dividends of £1.50 per share are expected on 1 June, 1 September and 1 December 2002.  
  
Calculate the forward price, assuming a risk-free rate of interest of 6% per annum convertible half-yearly and no arbitrage. [4]
  
- 3 Calculate the nominal rate of interest per annum convertible quarterly which is equivalent to:
  - (i) An effective rate of interest of 0.5% per month. [2]
  - (ii) A nominal rate of interest of 6% per annum convertible every 2 years. [2]

[Total 4]
  
- 4 An equity pays annual dividends and the next dividend, payable in 10 months time, is expected to be 5p. Thereafter, dividends are expected to grow by 3% per annum compound. Inflation is expected to be 2% per annum throughout. Calculate the value of the equity assuming that the real yield obtained is 2.5% per annum, convertible half-yearly. [5]
  
- 5 Three bonds paying annual coupons in arrears of 7% and redeemable at 105 per £100 nominal reach their redemption dates in exactly one, two and three years time, respectively. The price of each of the bonds is £98 per £100 nominal.
  - (i) Determine the gross redemption yield of the 3-year bond. [3]
  - (ii) Calculate all possible spot rates implied by the information given. [5]

[Total 8]

- 6** A company has agreed to build and operate a toll bridge for a regional government. The company will invest £10 million per annum for the first two years of the project, the investment being made continuously during this period. The bridge will then come into operation and the company will start to receive payments at the end of each year, the first payment occurring at the end of year 3 of the project. The amount of payment at the end of year 3 will be £8 million, reducing by £0.5 million in each of the subsequent years until the annual amount is £3 million, after which the annual reduction will be £1 million. When the payments have reduced to zero, the company's involvement in the project will end.

(i) Calculate the net present value of the project at a rate of interest of 10% per annum effective. [7]

(ii) Explain whether the internal rate of return achieved on this project will be greater or less than 10% per annum effective. [2]

[Total 9]

- 7** A loan of nominal amount £100,000 is to be issued bearing interest payable half-yearly in arrears at a rate of 7% per annum. The loan is to be redeemed with a capital payment of 110 per £100 nominal on a coupon date between 10 and 15 years after the date of issue, inclusive, the date of redemption being at the option of the borrower.

An investor who is liable to income tax at 25% and capital gains tax of 30% wishes to purchase the entire loan at the date of issue at a price which ensures that the investor achieves a net effective yield of at least 5% per annum.

(i) Determine whether the investor would make a capital gain if the investment is held until redemption. [3]

(ii) Explain how your answer to (i) influences the assumptions made in calculating the price the investor should pay. [2]

(iii) Calculate the maximum price which the investor should pay. [5]

[Total 10]

- 8 £10,000 is invested in a bank account which pays interest at the end of each year.

The rate of interest is fixed randomly at the beginning of each year and remains unchanged until the beginning of the next year. The rate of interest applicable in any one year is independent of the rate applicable in any other year.

During the first year the rate of interest per annum effective will be one of 3%, 4% or 6% with equal probability.

During the second year, the rate of interest per annum effective will be either 5% with probability 0.7 or 4% with probability 0.3.

- (i) Assuming that interest is always reinvested in the account, calculate the expected accumulated amount in the bank account at the end of two years. [4]
  - (ii) Calculate the variance of the accumulated amount in the bank account at the end of two years. [7]
- [Total 11]

- 9 The force of interest  $\delta(t)$  is a function of time and at any time, measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.03 + 0.01t & 0 \leq t \leq 8 \\ 0.05 & 8 < t \end{cases}$$

- (i) Derive, and simplify as far as possible, expressions for  $v(t)$  where  $v(t)$  is the present value of a unit sum of money due at time  $t$ . [5]
  - (ii)
    - (a) Calculate the present value of £500 due at the end of fifteen years.
    - (b) Calculate the rate of discount per annum convertible quarterly implied by the transaction in (a). [4]
  - (iii) A continuous payment stream is received at rate  $10e^{-0.02t}$  units per annum between  $t = 10$  and  $t = 14$ . Calculate the present value of the payment stream. [4]
- [Total 13]

- 10** A company incurs a liability to pay  $\pounds 1,000(1 + 0.4t)$  at the end of year  $t$ , for  $t$  equal to 5, 10, 15, 20 and 25. It values these liabilities assuming that in the future there will be a constant effective interest rate of 7% per annum. An amount equal to the total present value of the liabilities is immediately invested in two stocks:

Stock A pays coupons of 5% per annum annually in arrears and is redeemable in 26 years at par.

Stock B pays coupons of 4% per annum annually in arrears and is redeemable in 32 years at par.

The gross redemption yield on both stocks is the same as that used to value the liabilities.

- (i) Calculate the present value of the liabilities. [3]
  - (ii) Calculate the discounted mean term of the liabilities. [3]
  - (iii) If the discounted mean term of the assets is the same as the discounted mean term of the liabilities, calculate the nominal amount of each stock which should be purchased. [9]
- [Total 15]

- 11** A loan of  $\pounds 10,000$  was granted on 1 April 1994. The loan is repayable by an annuity payable monthly in arrears for fifteen years. The amount of the monthly repayment increases by  $\pounds 10$  after five years and by a further  $\pounds 10$  after ten years, and was calculated on the basis of a nominal rate of interest of 8 % per annum convertible quarterly.

- (i) Calculate the initial amount of monthly repayment. [7]
  - (ii) Calculate the amount of capital which was repaid on 1 May 1994. [2]
  - (iii) Calculate the amount of loan which will remain outstanding after the monthly repayment due on 1 April 2002 has been made. [5]
  - (iv) The borrower requests that after the 1 April 2002 payment has been made, future repayments be for a fixed amount payable on each 1 April, the last repayment being on 1 April 2006. Calculate the amount of the revised annual payment on this basis if the interest rate remains unchanged. [3]
- [Total 17]