EXAMINATIONS

28 April 2004 (pm)

Subject 102 — Financial Mathematics

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1.   Enter all the candidate and examination details as requested on the front of your answer booklet.

2.   You must not start writing your answers in the booklet until instructed to do so by the supervisor.

3.   Mark allocations are shown in brackets.

4.   Attempt all 11 questions, beginning your answer to each question on a separate sheet.

Graph paper is *not* required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

> *In addition to this paper you should have available Actuarial Tables and your own electronic calculator.*

1 An individual purchases £100,000 nominal of a bond on 1 January 2003 which is redeemable at 105 in four years time and pays coupons of 4% per annum at the end of each year.

The investment manager wishes to invest the coupon payments on deposit until the bond is redeemed. It is assumed that the rate of interest at which the coupon payments can be invested is a random variable and the rate of interest in any one year is independent of that in any other year.

Deriving the necessary formulae, calculate the mean value of the total accumulated investment on 31 December 2006 if the annual effective rate of interest has an expected value of 5½% in 2004, 6% in 2005 and 4½ % in 2006.

[5]

2 The force of interest, $\delta(t)$, is a function of time and at any time t (measured in years) is given by:

$$\delta(t) = \begin{cases} 0.05 & 0 < t < 8 \\ 0.04 + 0.0004t^2 & 8 \leq t \leq 15 \end{cases}$$

Calculate the accumulated value at time $t = 15$ of a continuous payment stream of £50 per annum payable from time $t = 0$ to $t = 8$.

[6]

3 A fund had a value of £120,000 on 1 January 2001. A net cash flow of £20,000 was received on 1 November 2001 and a further net cash flow of £48,000 was received on 1 May 2002. Immediately before receipt of the first cash flow the fund had a value of £137,000, and immediately before the second cash flow the fund had a value of £173,000. The value of the fund on 31 December 2002 was £205,000.

(i) Calculate the annual effective time weighted rate of return earned on the fund for the period 1 January 2001 to 31 December 2002. [3]

(ii) Discuss the relative strengths and weaknesses of using the time weighted rate of return as opposed to the money weighted rate of return when comparing the performance of two investment managers over the same period. [3]

[Total 6]

4   In a particular bond market, the two-year par yield at time $t = 0$ is 4.15% and the issue price at time $t = 0$ of a two-year fixed interest stock, paying coupons of 8% annually in arrears and redeemed at 98, is £105.40 per £100 nominal.

Calculate:

(a)   the one-year spot rate
(b)   the two-year spot rate

[6]

5   An insurance company has liabilities of £10 million due in 10 years time and £20 million due in 15 years time, and assets consisting of two zero-coupon bonds, one paying £7.404 million in 2 years time and the other paying £31.834 million in 25 years time. The current interest rate is 7% per annum effective.

(i)   Show that Redington's first two conditions for immunisation against small changes in the rate of interest are satisfied for this insurance company.

[5]

(ii)   Determine the profit or loss, expressed as a present value, that the insurance company will make if the interest rate increases immediately to 7.5% per annum effective.   [2]

(iii) Explain how you might have anticipated, before making the calculation in (ii), whether the result would be a profit or loss.   [2]

[Total 9]

6   (i)   Explain what is meant by the "no arbitrage" assumption in financial mathematics.   [2]

(ii)   A three-year forward contract is to be issued on a particular company share. The current market value of the share is £4.50 and a dividend of £0.20 per share has just been paid. The parties to the contract assume that the future quarterly dividends will increase by 1% per quarter-year compound for the first two years and by 1½% per quarter-year compound for the final year.

Assuming a risk-free force of interest of 5% per annum, and no arbitrage, calculate the forward price.   [7]

[Total 9]

7 A government issued a number of index-linked bonds on 1 June 2000 which were redeemed on 1 June 2002. Each bond had a nominal coupon rate of 3% per annum, payable half yearly in arrears, and a nominal redemption price of 100. The actual coupon and redemption payments were indexed according to the increase in the retail price index between 6 months before the bond issue date and 6 months before the coupon or redemption payment dates.

The values of the retail price index in the relevant months were:

Date	Retail price index
December 1999	100
June 2000	102
December 2000	107
June 2001	111
December 2001	113
June 2002	118

(i) An investor purchased £100,000 nominal at the issue date, and held it until it was redeemed.

The issue price was £94 per £100 nominal.

Calculate all the investor's cash flows from this investment, before tax. [3]

(ii) The investor is subject to income tax at a rate of 25% and capital gains tax at a rate of 35%. When calculating the amount of capital gain which is subject to tax, the price paid for the investment is indexed in line with the increase in the retail price index between the month in which the investment was purchased and the month in which it was redeemed.

(a) Calculate the investor's capital gains tax liability in respect of this investment.

(b) Calculate the net effective yield per annum obtained by the investor on these bonds.

[6]
[Total 9]

8 In a particular country, income tax and capital gains tax are both collected on 1 April each year in relation to gross payments made during the previous 12 months.

A fixed interest bond is issued on 1 January 2003 with term of 25 years and is redeemable at 110%. The security pays a coupon of 8% per annum, payable half-yearly in arrears.

An investor, who is liable to tax on income at a rate of 25% and on capital gains at a rate of 30%, bought £10,000 nominal of the stock at issue for £9,900.

(i) Assuming an inflation rate of 3% per annum over the term of the bond, calculate the net real yield obtained by the investor if they hold the stock to redemption. [9]

(ii) Without doing any further calculations, explain how and why your answer to (i) would alter if tax were collected on 1 June instead of 1 April each year. [2]
[Total 11]

9 An investor borrows £120,000 at an effective interest rate of 7% per annum. The investor uses the money to purchase an annuity of £14,000 per annum payable half-yearly in arrears for 25 years. Once the loan is paid off, the investor can earn interest at an effective rate of 5% per annum on money invested from the annuity payments.

(i) Determine the discounted payback period for this investment. [5]

(ii) Determine the profit the investor will have made at the end of the term of the annuity. [7]
[Total 12]

10 An investment fund provides annual rates of return, which are independent and identically distributed, with annual accumulation factors following the log-normal distribution with mean 1.04 and variance 0.02.

(i) An investor knows that she will have to make a payment of £5,000 in five years time.

Calculate the amount of cash she should invest now in order that she has a 99% chance of having sufficient cash available in five years time from the investment to meet the payment. [9]

(ii) Comment on your answer to (i). [3]
[Total 12]

11   A loan was taken out on 1 September 1998 and was repayable by the following increasing annuity:

The first repayment was made on 1 July 1999 and was £1,000. Thereafter, payments were made on 1 November, 1 March and 1 July until 1 March 2004, inclusive. Each payment was 5% greater than its predecessor. The effective rate of interest throughout the period was 6% per annum.

(i)   Show that the amount of loan was £17,692 to the nearest pound.   [5]

(ii)   Calculate the amount of capital repaid on 1 July 1999.   [3]

(iii)   Calculate both the capital component and the interest component of the seventh repayment.   [7]

[Total 15]

END OF PAPER