EXAMINATIONS

19 April 2000 (pm)

Subject 102 — Financial Mathematics

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1.   Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.

2.   Mark allocations are shown in brackets.

3.   Attempt all 14 questions, beginning your answer to each question on a separate sheet.

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

> *In addition to this paper you should have available Actuarial Tables and an electronic calculator.*

1   The following *n*-year spot rates apply at time $t = 0$.

1 year spot rate of interest: 4½% per annum effective
2 year spot rate of interest: 5% per annum effective
3 year spot rate of interest: 5½% per annum effective

Calculate the two year forward rate of interest from time $t = 1$ expressed as an annual effective rate of interest.   [2]

2   Describe what is meant by a "currency swap".   [2]

3   For a rate of interest of 7% per annum, convertible monthly, calculate:

(i)   the equivalent rate of interest per annum convertible half yearly, and [2]

(ii)   the equivalent rate of discount per annum convertible monthly   [2]
[Total 4]

4   A certain company issues both debenture stocks and unsecured loan stocks. Describe the differences between these two types of asset.   [4]

5   You are given the following information in respect of a pension fund:

Calendar Year	Value of fund at 1 January	Value of fund at 30 June	Net cash flow received on 1 July
1997	£180,000	£212,000	£25,000
1998	£261,000	£230,000	£18,000
1999	£273,000	£295,000	£16,000
2000	£309,000		

Calculate the annual effective time weighted rate of return earned on the fund over the period from 1 January 1997 to 1 January 2000.   [4]

6   (i)   Calculate $\ddot{s}^{(12)}_{\overline{5.5}|}$ at an effective rate of interest of 13% per annum.   [3]

(ii)   Explain what your answer to (i) represents.   [2]
[Total 5]

7   An investor is considering the purchase of 100 ordinary shares in a company. Dividends from the share will be paid annually. The next dividend is due in one year and is expected to be 8p per share. The second dividend is expected to be 8% greater than the first dividend and the third dividend is expected to be 7% greater than the second dividend. Thereafter, dividends are expected to grow at 5% per annum compound in perpetuity.

Calculate the present value of this dividend stream at a rate of interest of 7% per annum effective.   [5]

8   The force of interest $\delta(t)$ at time t is given by

$\delta(t) = 0.06$ $0 < t \le 6$

$\delta(t) = 0.05 + 0.0002t^2$ $6 < t \le 12$

Calculate the accumulated value at time $t = 12$ of a continuous payment stream of £100 per annum payable from time $t = 0$ to time $t = 6$.   [5]

9   (i)   Explain what is meant by the "no arbitrage" assumption in financial mathematics.   [2]

(ii)   A 7-month forward contract is issued on 1 January 2000 on a stock with a price of £60 per share. Dividends of £2 per share are expected after 3 and 6 months.

Assuming a risk-free force of interest of 7% per annum and no arbitrage, calculate the forward price.   [4]

[Total 6]

10   A fixed interest stock bears a coupon of 7% per annum payable half yearly on 1 April and 1 October. It is redeemable at par on any 1 April between 1 April 2004 and 1 April 2010 inclusive at the option of the borrower.

On 1 July 1991 an investor purchased £10,000 nominal of the stock at a price to give a net yield of 6% per annum effective after allowing for tax at 25% on the coupon payments.

On 1 April 1999 the investor sold the holding at a price which gave a net yield of 5% per annum effective to another purchaser who is also taxed at a rate of 25% on the coupon payments.

(i)   Calculate the price at which the stock was bought by the original investor.   [4]

(ii)   Calculate the price at which the stock was sold by the original investor.   [3]

[Total 7]

11 In any year, the rate of interest on funds invested with a given insurance company is independent of the rates of interest in all previous years.

Each year the value of $(1 + i_t)$, where i_t is the rate of interest earned in the tth year, is lognormally distributed. The mean and standard deviation of i_t are 0.07 and 0.20 respectively.

(i) Determine the parameters μ and σ^2 of the lognormal distribution of $1 + i_t$.

[5]

(ii) (a) Determine the distribution of S_{15}, where S_{15} denotes the accumulation of one unit of money over 15 years.

(b) Determine the probability that $S_{15} > 2.5$. [4]

[Total 9]

12 A motor manufacturer is to develop a new car model to be produced from 1 January 2002 for six years until 31 December 2007. The development cost will be £33 million, of which £18 million will be incurred on 1 January 2000, £10 million on 1 July 2000 and £5 million on 1 January 2001.

The production cost of each car is assumed to be incurred at the beginning of the calendar year of production and will be £9,000 during 2002. The sale price of each car is assumed to be received at the end of the calendar year of production. Both the production costs and the sale prices are assumed to increase by 5% on each 1 January, the first increase occurring on 1 January 2003. It is also assumed that 5,000 cars will be produced each year and that all will be sold.

The sale price of each car produced in 2002 is £12,100.

(i) Calculate the discounted payback period at an effective rate of interest of 9% per annum. [9]

(ii) Without doing any further calculations, explain whether the discounted payback period would be greater than, equal to, or less than the period calculated in (i) if the effective rate interest were substantially less than 9% per annum. [3]

[Total 12]

13 A loan is repayable by a decreasing annuity payable annually in arrears for 20 years. The repayment at the end of the first year is £6,000 and subsequent repayments reduce by £200 each year. The repayments were calculated using a rate of interest of 9% per annum effective.

(i) Calculate the original amount of the loan. [4]

(ii) Construct the schedule of repayment for years eight (after the seventh payment) and nine, showing the outstanding capital at the beginning of the year, and the interest element and the capital repayment in each year. [5]

(iii) Immediately after the ninth payment of interest and capital, the interest rate on the outstanding loan is reduced to 7% per annum effective.

Calculate the amount of the tenth payment if subsequent payments continue to reduce by £200 each year, and the loan is to be repaid by the original date, i.e. 20 years from commencement. [6]

[Total 15]

14 (i) A variable annuity, payable annually in arrears for n years, is such that the payment at the end of the year t is t^2. Show that the present value of this annuity can be expressed as

$$\frac{2(Ia)_{\overline{n}|} - a_{\overline{n}|} - n^2\, v^{n+1}}{1 - v}$$ [4]

(ii) A pension fund has a liability to pay £100,000 at the end of one year, £105,000 at the end of two years, and so on, the amount increasing by £5,000 each year to £195,000 at the end of 20 years, this being the last payment. The fund values these payments using an effective interest rate of 7% per annum. This is also the interest rate at which the current prices of all bonds are calculated.

The fund invests an amount equal to the present value of these liabilities in the following two assets:

(A) a zero coupon bond redeemable in 25 years, and

(B) a fixed interest bond redeemable at par in 12 years' time which pays a coupon of 8% per annum annually in arrears

(a) Calculate the present value and the duration of the liabilities.

(b) Calculate the amount of cash that should be invested in each asset if the duration of the assets is to equal that of the liabilities.

[16]

[Total 20]