

EXAMINATIONS

15 April 2002 (pm)

Subject 102 — Financial Mathematics

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

1 The price of a given share is 80p. The risk-free rate of interest is 5% per annum convertible quarterly. Assuming no arbitrage and that the share will not pay any income, calculate the forward price for the share, for settlement in exactly one quarter of a year. [2]

2 On the assumption of constant future price inflation of 2.5% per annum, an investor has purchased a UK government index-linked bond at a price to provide a real rate of return of 2% per annum effective if held to redemption. Explain why the real yield to redemption will be lower if the actual constant rate of inflation is higher than the assumed 2.5% per annum. [2]

3 An insurance company has liabilities to meet which involve a series of fixed cash payments, evenly spread over the next ten years. All its investments are currently held in cash. Describe the form of a fixed-interest swap contract which could help the insurance company immunise its liabilities. [3]

4 A particular floating rate note pays interest linked to an index of local floating interest rates. The note was purchased for a price of 99 on 1 January 2000 and sold for a price of 101 on 31 December 2000. The nominal value of the note is 100 and the note paid interest of 6.5% of the nominal value on 30 June 2000 and 6.6% of the nominal value on 31 December 2000. Calculate the nominal rate of return per annum convertible half-yearly earned by the investor. [5]

5 The force of interest $\delta(t)$ is:

$$\delta(t) = 0.005t + 0.0001t^2 \text{ for all } t$$

(i) At $t = 8$, calculate the accumulated value of an investment of £100 made at time $t = 0$. [3]

(ii) Calculate the constant annual effective rate of interest over the eight year period. [2]

[Total 5]

6 An investor purchases a bond 3 months after issue. The bond will be redeemed at par ten years after issue and pays coupons of 6% per annum annually in arrears. The investor pays tax of 25% on both income and capital gains (with no relief for indexation).

(i) Calculate the purchase price of the bond per £100 nominal to provide the investor with a rate of return of 8% per annum effective. [6]

(ii) The real rate of return expected by the investor from the bond is 3% per annum effective. Calculate the annual rate of inflation expected by the investor. [2]

[Total 8]

7 The forward rate from time $t - 1$ to time t , $f_{t-1,t}$, has the following values:

$$f_{0,1} = 4.0\%, f_{1,2} = 4.5\%, f_{2,3} = 4.8\%.$$

- (i) Assuming no arbitrage, calculate:
- (a) the price per £100 nominal of a 3-year bond paying an annual coupon in arrears of 5%, redeemed at par in exactly three years, and
- (b) the gross redemption yield from the bond [7]
- (ii) Explain why a bond with a higher coupon would have a lower gross redemption yield, for the same term to redemption. [2]
- [Total 9]

8 (i) Prove that $(Ia)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$ [3]

A government bond pays a coupon half-yearly in arrears of £10 per annum. It is to be redeemed at par in exactly ten years. The gross redemption yield from the bond is 6% per annum convertible half-yearly.

- (ii) Calculate the duration of the bond in years. [8]
- (iii) Explain why the duration of the bond would be longer if the coupon rate were £8 per annum instead of £10 per annum. [2]
- [Total 13]

9 An analyst is valuing two companies, Cyber plc and Boring plc. Cyber plc is assumed to pay its first annual dividend in exactly 6 years. It is assumed that the dividend will be 6p per share. After the sixth year, annual dividends are assumed to grow at 10% per annum compound for a further 6 years. Dividends are assumed to grow at 3% per annum compound in perpetuity thereafter. Boring plc is expected to pay an annual dividend of 4p per share in exactly one year. Annual dividends are then expected to grow thereafter by 0.5% per annum compound in perpetuity. The analyst values dividends from both shares at a rate of interest of 6% per annum effective in a discounted dividend model.

- (i) (a) Calculate the value of Cyber plc.
(b) Calculate the value of Boring plc. [8]

There is a general rise in interest rates and the analyst decides it is appropriate to increase the valuation rate of interest to 7% per annum effective.

- (ii) Show that the percentage fall in the value of Cyber plc is greater than that in the value of Boring plc. [6]
- (iii) Explain your answer to part (ii) in terms of duration. [2]
- [Total 16]

- 10** (i) An investor is deciding whether to invest in a project. Explain why the discounted payback period is a poorer decision criterion than net present value assuming that the investor is not short of capital. [2]

An investor is considering two projects A and B. Project A involves the investment of £1 million at the outset. The only income to be received will be a payment of £3.5 million after ten years. Project B also involves the investment of £1 million at the outset. Income will be received from this project continuously. In the first year the rate of payment will be £0.08 million, in the second year £0.09 million, in the third year £0.10 million, with the rate increasing by £0.01 million each year thereafter until the tenth year, after the end of which no further income will be received.

- (ii) Calculate the net present value of both investment projects at a rate of interest of 4% per annum effective. [10]
- (iii) Show that the discounted payback period of project A is after that of project B (no further calculations are necessary). [3]
- (iv) In the light of your answer to (i) above, explain which project is the most desirable to an investor with unlimited capital, and why. [2]

[Total 17]

11 A company is adopting a particular investment strategy such that the expected annual effective rate of return from investments is 7% and the standard deviation of annual returns is 9%. Annual returns are independent and $(1 + i_t)$ is lognormally distributed where i_t is the return in the t th year. The company has received a premium of £1,000 and will pay the policyholder £1,400 after 10 years.

- (i) Calculate the expected value and standard deviation of an investment of £1,000 over 10 years, deriving all formulae that you use. [9]
- (ii) Calculate the probability that the accumulation of the investment will be less than 50% of its expected value in ten years' time. [8]
- (iii) The company has invested £1,200 to meet its liability in 10 years time. Calculate the probability that it will have insufficient funds to meet its liability. [3]

[Total 20]