EXAMINATIONS

29 September 2004 (pm)

Subject 102 — Financial Mathematics

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 12 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

1 A 90-day treasury bill was bought by an investor for a price of £91 per £100 nominal. After 30 days the investor sold the bill to a second investor for a price of £93.90 per £100 nominal. The second investor held the bill to maturity when it was redeemed at par.

Determine which investor obtained the higher annual effective rate of return. [3]

2 (i) The one-year forward rate applying at a particular point in time t is defined as $f_{t,1}$.

If $f_{t,1}$ = 4%, calculate the continuous time forward rate $F_{t,1}$ applying over the same period of time. [2]

(ii) Define the instantaneous forward rate F_t. [2]

[Total 4]

3 An insurance company has a continuous payment stream of liabilities to meet over the coming 20 years. The payment stream will be at a rate of £10 million per annum throughout the period.

Calculate the duration of the continuous payment stream at a rate of interest of 4% per annum effective. [4]

4 An individual purchases a car for £15,000. The purchaser takes out a loan for this amount that involves him making 24 monthly payments in advance. The annual rate of interest is 12.36% per annum effective.

Calculate the flat rate of interest on the loan. [5]

5 (a) An investor holds a portfolio of fixed interest securities to meet future liabilities. State the conditions that need to be met if the investor is to be immunised from small, uniform changes in the rate of interest.

(b) State the relationship between "volatility" and "duration" where "volatility" is defined as:

$$\frac{-1}{A}.\frac{dA}{di}$$

where A is the value of a portfolio of investments and i is the annual effective rate of interest.

(c) A perpetuity pays annual coupons in arrears. Show that the volatility of the perpetuity, as defined in (b) above, is equal to $1/i$ where i is the rate of return.

[6]

6 An investor is considering two investment projects A and B. Both involve outlays of £1 million. Project A will provide a single incoming cash payment after 8 years of £1.7 million. Project B will provide incoming cash payments of £1 million after 8 years, £0.321 million after 9 years, £0.229 million after 10 years and £0.245 million after 11 years.

(a) Determine the rate of interest (i') at which the net present value of the two projects will be equal.

(b) By general reasoning or by illustrative calculation, show that at a positive rate of interest i^*, where $i^* < i'$, project B will have a higher net present value than Project A.

[6]

7 (i) Describe the characteristics of ordinary shares (equities). [4]

(ii) The prospective dividend yield from an ordinary share is defined as the next expected dividend divided by the current price. A particular share is expected to provide a real rate of return of 5% per annum effective. Inflation is expected to be 2% per annum. The growth rate of dividends from the share is expected to be 3% per annum compound. Dividends will be paid annually and the next dividend is expected to be paid in 6 months time.

Calculate the prospective dividend yield from the share. [4]

[Total 8]

8 The expected annual effective rate of return from an insurance company's investments is 6% and the standard deviation of annual returns is 8%. The annual effective returns are independent and $(1 + i_t)$ is lognormally distributed, where i_t is the return in the tth year.

(a) Calculate the expected value of an investment of £1 million after ten years.

(b) Calculate the probability that the accumulation of the investment will be less than 90% of the expected value.

[8]

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9 For a particular bond market, zero coupon bonds redeemable at par are priced as follows:

bonds redeemable in exactly one year are priced at 97;
bonds redeemable in exactly two years are priced at 93;
bonds redeemable in exactly three years are priced at 88; and
bonds redeemable in exactly four years are priced at 83.

(i) Assuming no arbitrage, calculate:

(a) the one-year, two-year, three-year and four-year spot interest rates

(b) the rate of return from a bond redeemable at par in four years time that pays a coupon of 4% annually in arrears

[8]

(ii) Explain why the four-year spot rate is greater than the rate of return from a bond redeemable at par in exactly four years time paying a coupon of 4% annually in arrears. [2]

(iii) Explain the shape of the yield curve indicated by the spot rates calculated in (i)(a) using the liquidity preference theory, if the expectations of future short term interest rates are constant. [2]

[Total 12]

10 An individual borrows £100,000 from a bank using a 25-year fixed-interest mortgage. The mortgage is repayable by monthly instalments paid in advance. The instalments are calculated using a rate of interest of 6% per annum effective. After ten years, the borrower has the option to repay any outstanding loan and take out a new loan, equal to the amount of the outstanding balance on the original loan, if interest rates on 15-year loans are less than 6% per annum effective at that time. The new loan will also be repaid by monthly instalments in advance.

(i) Calculate the amount of the monthly instalments for the original loan. [3]

(ii) Calculate the interest and capital components of the 25th instalment. [4]

(iii) The borrower takes advantage of the option to repay the loan after ten years. The rate of interest on mortgages of length 15 years has fallen to 2% per annum effective at that time. The first loan is repaid and a new loan is taken out, repayable over a 15-year period, for the same sum as the capital outstanding after ten years on the original loan.

(a) Calculate the revised monthly instalment for the new loan.

(b) Calculate the accumulated value of the reduction in instalments at a rate of interest of 2% per annum effective, if the borrower exercises the option. [6]

[Total 13]

11 A fixed interest security was issued on 1 January in a given year. The security pays half-yearly coupons of 4% per annum. The security is redeemable at 110% 20 years after issue. An investor who pays both income tax and capital gains tax at a rate of 25% buys the security on the date of issue. Income tax is paid on coupons at the end of the calendar year in which the coupon is received. Capital gains tax is paid immediately on sale or redemption.

(i) Calculate the price paid by the investor to give a net rate of return of 6% per annum effective. [6]

(ii) Calculate the duration of the net payments from the fixed interest security for an investor who pays income tax as described above but who does not pay capital gains tax, at a rate of interest of 6% per annum effective. [8]

[Total 14]

12 The risk-free force of interest $\delta(t)$ at time t is given by:

$\delta(t) = 0.05$ for $0 < t \leq 10$, and
$\delta(t) = 0.08 + 0.003t$ for $t > 10$.

(i) (a) Calculate the accumulation at time 15 of £100 invested at time $t = 5$.

(b) Calculate the accumulation at time 14 of £100 invested at time $t = 5$.

(c) Calculate the accumulation at time 15 of £100 invested at time $t = 14$.

(d) Calculate the equivalent constant force of interest from time $t = 5$ to time $t = 15$.

[9]

(ii) Calculate the present value at time $t = 0$ of a continuous payment stream payable at a rate of $100e^{0.01t}$ from time $t = 0$ to time $t = 5$. [4]

(iii) A one year forward contract is issued at time $t = 0$ on a share with a price of 300p at that date. A dividend of 7p per share is expected at time $t = ½$.

Calculate the forward price of the share, assuming no arbitrage. [4]

[Total 17]

END OF PAPER