

# EXAMINATIONS

20 September 2000 (pm)

## Subject 102 — Financial Mathematics

*Time allowed: Three hours*

### ***INSTRUCTIONS TO THE CANDIDATE***

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
2. *Mark allocations are shown in brackets.*
3. *Attempt all 13 questions, beginning your answer to each question on a separate sheet.*

***Graph paper is not required for this paper.***

*AT THE END OF THE EXAMINATION*

*Hand in BOTH your answer booklet and this question paper.*

*In addition to this paper you should have available Actuarial Tables and an electronic calculator.*

- 1** Describe how cash flows are exchanged in an “interest rate swap”. [2]
- 2** A Government issues a 90-day Treasury Bill at a simple rate of discount of 5% per annum. Calculate the rate of return per annum convertible half yearly received by an investor who purchases the Bill and holds it to maturity. [3]
- 3** An asset has a current price of £1.20. Given a risk free rate of interest of 5% per annum effective and assuming no arbitrage, calculate the forward price to be paid in 91 days. [3]
- 4** State the main differences between a preference share and an ordinary share. [3]
- 5** An investor purchases a bond on the issue date at a price of £96 per £100 nominal. Coupons at an annual rate of 4% are paid annually in arrears. The bond will be redeemed at par twenty years after the issue date.  
Calculate the gross redemption yield from the bond. [4]
- 6** An insurance company calculates the single premium for a contract paying £10,000 in ten years’ time as the present value of the benefit payable, at the expected rate of interest it will earn on its funds. The annual effective rate of interest over the whole of the next ten years will be 7%, 8% or 10% with probabilities 0.3, 0.5 and 0.2 respectively.  
(a) Calculate the single premium.  
(b) Calculate the expected profit at the end of the term of the contract. [4]
- 7** An ordinary share pays annual dividends. The next dividend is expected to be 10p per share and is due in exactly 9 months time. It is expected that subsequent dividends will grow at a rate of 5% per annum compound and that inflation will be 3% per annum. The price of the share is 250p and dividends are expected to continue in perpetuity.  
Calculate the expected effective real rate of return per annum for an investor who purchases the share. [5]
- 8** An investment bank models the expected performance of its assets over a five-day period. Over that period, the return on the bank’s portfolio,  $i$ , has a mean value of 0.1% and standard deviation 0.2%.  $(1 + i)$  is lognormally distributed.  
Calculate the value of  $j$  such that the probability that  $i$  is less than or equal to  $j$  is 0.05. [5]

**9** (i) Show that  $(Ia)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$ . [3]

(ii) (a) Calculate the present value, at a rate of interest of 3% per annum effective, of an annuity where 10 is paid at the end of the first year, 12 is paid at the end of the second year, 14 is paid at the end of the third year and so on, with payments increasing by 2 per annum until the payment stream ends at the end of 20 years.

(b) Calculate the capital outstanding in the above annuity at the end of the 15<sup>th</sup> year, after the payment then due has been received.

(c) Determine the interest and capital components in the 16th payment.

[9]

[Total 12]

**10** The force of interest,  $\delta(t)$ , is:

$$\begin{aligned} \delta(t) &= 0.04 && \text{for } 0 < t \leq 5, \\ &= 0.01(t^2 - t) && \text{for } 5 < t \end{aligned}$$

(i) Calculate the present value of a unit sum of money due at time  $t = 10$ . [5]

(ii) Calculate the effective rate of interest over the period  $t = 9$  to  $t = 10$ . [3]

(iii) (a) In terms of  $t$ , determine an expression for  $v(t)$ , the present value of a unit sum of money due during the period  $0 < t \leq 5$ .

(b) Calculate the present value of a payment stream paid continuously for the period  $0 < t \leq 5$ , where the rate of payment,  $\rho(t)$ , at time  $t$ , is  $e^{0.04t}$ . [4]

[Total 12]

**11**  $f_{t,r}$  is the forward rate applicable over the period  $t$  to  $t + r$ .  $i_t$  is the spot rate over the period 0 to  $t$ . The gross redemption yield from a one year bond with a 6% annual coupon is 6% per annum effective; the gross redemption yield from a two year bond with a 6% annual coupon is 6.3% per annum effective; and the gross redemption yield from a three year bond with a 6% annual coupon is 6.6% per annum effective. All the bonds are redeemed at par and are exactly one year from the next coupon payment.

(i) (a) Calculate  $i_1$ ,  $i_2$  and  $i_3$  assuming no arbitrage.

(b) Calculate  $f_{0,1}$ ,  $f_{1,1}$  and  $f_{2,1}$  assuming no arbitrage. [10]

(ii) Explain why the forward rates increase more rapidly with term than the spot rates. [2]

[Total 12]

**12** An investment project gives rise to the following cash flows. At the beginning of each of the first three years £180,000 will be invested in the project. From the beginning of the first year until the end of the twenty-fifth year, net revenue will be received continuously. The initial rate of payment of net revenue will begin at £25,000 per annum. The rate of payment is assumed to grow continuously at a rate of 6% per annum effective.

- (i) Calculate the net present value of the project at an effective rate of interest of 7% per annum. [6]
  - (ii) Calculate the discounted payback period of the project at an effective rate of interest of 7% per annum. [5]
  - (iii) Calculate the annual effective rate of growth of net revenue which would be required if the project is to have a zero net present value at an effective rate of interest of 7% per annum. [6]
- [Total 17]

**13** A pension fund expects to make payments of £100,000 per annum at the end of each of the next five years. It wishes to immunise these liabilities by investing in two zero coupon bonds which mature in five years and in one year respectively. The rate of interest is 5% per annum effective.

- (i) (a) Show that the present value of the liabilities is £432,948.  
(b) Show that the duration of the liabilities is 2.9 years. [6]
  - (ii) Calculate the nominal amounts of the two zero coupon bonds which must be purchased if the pension fund is to equate the present value and duration of assets and liabilities. [6]
  - (iii) (a) Calculate the convexity of the assets.  
(b) Without calculating the convexity of the liabilities, comment on whether you think Redington's immunisation has been achieved. [6]
- [Total 18]