

EXAMINATIONS

10 September 2003 (pm)

Subject 102 — Financial Mathematics

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 13 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

1 A government issues a 90-day Treasury Bill at a simple rate of discount of 6% per annum. Calculate the effective rate of return per annum received by an investor who purchases the Bill at issue and holds it to maturity. [3]

2 The following table gives information concerning an investment fund:

Calendar year	2000 <i>£ million</i>	2001 <i>£ million</i>	2002 <i>£ million</i>
Value of fund on 1 January before cash flow	100	80	200
Net cash flow received on 1 January	20	30	10
Value of fund on 31 December	80	200	200

Calculate the effective time weighted rate of return per annum over the three year period. [3]

3 (i) Define the characteristics of a government index-linked bond. [2]

(ii) Explain why most index-linked securities issued carry some inflation risk, in practice. [2]

[Total 4]

4 A one-year forward contract is issued on 1 April 2003 on a share with a price of 600p at that date. Dividends of 30p per share are expected on 30 September 2003 and 31 March 2004. The 6-month and 12-month spot risk-free interest rates are 4% and 4.5% per annum effective respectively on 1 April 2003.

Calculate the forward price at issue, assuming no arbitrage. [4]

5 An investment bank models the expected performance of its assets over a 5-day period. Over that period, the return on the bank's portfolio, i , has a mean value of 0.15% and standard deviation 0.3%. $(1 + i)$ is lognormally distributed.

Calculate the value of j such that the probability that i is greater than or equal to j is 0.9. [6]

6 An ordinary share pays annual dividends. The next dividend is expected to be 5p per share and is due in exactly 3 months time. It is expected that subsequent dividends will grow at a rate of 4% per annum compound and that inflation will be 1.5% per annum. The price of the share is 125p and dividends are expected to continue in perpetuity.

Calculate the effective real rate of return per annum for an investor who purchases the share. [7]

- 7 A bank makes a loan of £100,000 to an individual. The loan is to be repaid by level monthly instalments in arrears over a period of 25 years. The instalments are such that the borrower pays interest at an effective rate of 5% per annum on the loan.
- (i) Calculate the amount of the instalments. [3]
- (ii) (a) Calculate the capital outstanding after 12 years, immediately after payment of the instalment then due.
- (b) Determine the split of the 145th instalment between capital and interest. [4]
- [Total 7]

- 8 (i) Describe the risk characteristics of a government-issued, conventional, fixed-interest bond. [2]
- (ii) A particular government bond is structured as follows:
- Annual coupons are paid in arrears of 8% of the nominal value of the bond. After five years, a capital payment is made, equal to half of the nominal value of the bond. The capital is repaid at par. The repayment takes place immediately after the payment of the coupon due at the end of the fifth year. After the end of the fifth year, coupons are only paid on that part of the capital that has not been repaid. At the end of the tenth year, all the remaining capital is repaid.
- Calculate the purchase price of the bond per £100 nominal, at issue, to provide a purchaser with an effective net rate of return of 6% per annum. The purchaser pays tax at a rate of 30% on coupon payments only. [5]
- [Total 7]

- 9 In any year t , the yield on a fund of investments has mean j_t and standard deviation s_t . In any year, the yield is independent of the value in any other year. The accumulated value, after n years, of a unit sum of money invested at time 0 is S_n .
- (i) Derive formulae for the mean and variance of S_n if $j_t = j$ and $s_t = s$ for all years t . [5]
- (ii) (a) Calculate the expected value of S_8 if $j = 0.06$.
- (b) Calculate the standard deviation of S_8 if $j = 0.06$ and $s = 0.08$. [3]
- [Total 8]

- 10** The force of interest $\delta(t)$ is a function of time and at any time, measured in years, is given by the formula:

$$\delta(t) = 0.04 + 0.001t$$

- (i) (a) Calculate the accumulated value of a unit sum of money accumulated from time $t = 0$ to time $t = 8$.
- (b) Calculate the accumulated value of a unit sum of money accumulated from time $t = 0$ to time $t = 9$.
- (c) Calculate the accumulated value of a unit sum of money accumulated from time $t = 8$ to time $t = 9$. [5]
- (ii) Using your results from (i), or otherwise, calculate:
- (a) The eight-year spot rate of interest from time $t = 0$ to time $t = 8$.
- (b) The nine-year spot rate of interest from time $t = 0$ to time $t = 9$.
- (c) $f_{8,1}$, where $f_{8,1}$ is the one-year forward rate of interest from time $t = 8$. [3]
- [Total 8]

- 11** An insurance company has a portfolio of annuity contracts under which it expects to pay £1 million at the end of each of the next 20 years, followed by £0.5 million at the end of each of the following 20 years. The government bond with the longest duration in which it can invest its funds pays a coupon of 10% per annum in arrears and is redeemed at par in 15 years time. The yield to maturity of the government bond is 6% per annum effective and a coupon payment has just been made.

- (i) (a) Calculate the duration of the insurance company's liabilities at a rate of interest of 6% per annum effective.
- (b) Calculate the duration of the insurance company's assets at a rate of interest of 6% per annum effective, if all the insurance company's funds are invested in the government bond with the longest duration. [8]
- (ii) (a) Explain why the insurance company cannot immunise its liabilities by purchasing government bonds.
- (b) Without any further calculations, state the circumstances under which the insurance company would make a loss if there were a uniform change in interest rates. Explain why a loss would be made. [4]
- [Total 12]

- 12** (i) (a) Explain what is meant by the “expectations theory” explanation for the shape of the yield curve.
- (b) Explain how expectations theory can be modified by both “liquidity preference” and “market segmentation” theories. [6]
- (ii) Short-term, one-year annual effective interest rates are currently 10%; they are expected to be 9% in one year’s time, 8% in two years’ time, 7% in three years’ time and to remain at that level thereafter indefinitely.
- (a) If bond yields over all terms to maturity are assumed only to reflect expectations of future short-term interest rates, calculate the gross redemption yields from 1-year, 3-year, 5-year and 10-year zero coupon bonds.
- (b) Draw a rough plot of the yield curve for zero coupon bonds using the data from part (ii)(a). (Graph paper is not required.)
- (c) Explain why the gross redemption yield curve for coupon paying bonds will slope down with a less steep gradient than the zero coupon bond yield curve. [8]
- [Total 14]

- 13** (i) In respect of an investment project, define:
- (a) the discounted payback period
- (b) the payback period [3]
- (ii) Discuss why both the discounted payback period and the payback period are inferior measures compared with the net present value for determining whether to proceed with an investment project. [3]

- (iii) A consortium of investors is considering bidding to host a major athletics event. The project will be regarded as viable if it provides a positive net present value at a rate of interest of 10% per annum effective. The consortium estimates that the following cash flows will be generated by the event (all figures in £100 million):

Costs

Initial costs of building:	To be incurred continuously at a rate of 1 per annum for five years starting on 1 January 2006.
Running costs of the event:	To be incurred continuously at a rate of 1 per annum for 3 months starting on 1 January 2011.
Cost of making bid:	0.2 to be incurred on 1 January 2004.

Revenue

Sale of television rights:	To be received continuously at a rate of 0.3 per annum for 3 months starting on 1 January 2011.
Other revenue from sale of merchandise, marketing rights, tickets, etc.:	Assumed to be received in the middle of each year from 2004 to 2015 inclusive. The revenue from this source is expected to start at 0.1 per annum and increase each year by 0.1 up to and including 2011. The same revenue is expected in 2012 as in 2011. After 2012 revenue is expected to decline by 0.2 per annum until 2015, after which year no further revenue will be received from this source.
Revenue from sale of the stadium and other infrastructure:	This will be received on 1 January 2015.

Determine the sale price of the stadium and other infrastructure that would have to be achieved for the project to be considered viable.

[11]
[Total 17]