REPORT OF THE BOARD OF EXAMINERS

April 2003

Subject 102 — Financial Mathematics

EXAMINERS' REPORT

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

J Curtis
Chairman of the Board of Examiners

3 June 2003

EXAMINERS' COMMENT

Please note that differing answers may be obtained depending on whether figures obtained from tables or from calculators are used in the calculations but candidates are not penalised for this.

However, candidates maybe penalised where excessive rounding has been used or where insufficient working is shown.

1 A preference share pays a dividend which is generally fixed. An ordinary share pays a dividend out of residual profits which is at the discretion of the company.

A preference share dividend is a prior charge so that, in general, no ordinary share dividend can be paid if a preference share dividend is outstanding.

A preference share holder may have no voting rights and is likely to get prior ranking in a winding up.

Credit was also given for any other relevant points.

2 Let Price = P

$$P = v^{\frac{1}{6}} \times 12(1 + 1.04v + 1.04^2 v^2 + \ldots) \text{ at } 7\%$$

$$= \frac{12}{(1.07)^{\frac{1}{6}}} \times \frac{1}{\left(1 - \frac{1.04}{1.07}\right)}$$

$$= 423.20$$

3 PV of outlay $= 60{,}000 + 25{,}000\, v^{\frac{8}{12}}$ at 9%

$= 83{,}604.19$

PV of income (in ‘000s)

$$v^2(5\bar{a}_{\overline{4}|} + 9v^4\bar{a}_{\overline{4}|} + 13v^8\bar{a}_{\overline{4}|} + 17v^{12}\bar{a}_{\overline{4}|} + 21v^{16}\bar{a}_{\overline{4}|})$$

$$= v^2\bar{a}_{\overline{4}|}(5 + 9v^4 + 13v^8 + 17v^{12} + 21v^{16})$$

and $\bar{a}_{\overline{4}|} = \frac{i}{\delta} \times a_{\overline{4}|} = 1.044354 \times 3.2397$

$= 3.38339$

$\Rightarrow PV$ of income

$= 0.84168 \times 3.38339 \, (5 + 9 \times 0.70843 + 13 \times 0.50187 + 17 \times 0.35553 + 21 \times 0.25187)$

$= 0.84168 \times 3.38339 \times 29.23346$

$= 83.24905$ (i.e. £83,249.05)

PV of final cash flow $= 50{,}000v^{22}$ at 9%

$= 7{,}509.09$

PV of project $= 83{,}249.05 + 7{,}509.09 - 83{,}604.19$

$= £7{,}153.95$

4 (i) A forward contract is an agreement made between two parties under which one agrees to buy from the other a specified amount of an asset at a specified price on a specified future date.

The investor agreeing to sell the asset is said to hold a "short forward position" in the asset, and the buyer is said to hold a "long forward position".

(ii) The forward price at the outset of the contract was:

$$(96 - 7\underset{4\%}{v^9}) \times (1.04)^{10} = 134.82$$

The forward price that should be offered now is:

$$(148 - 7\underset{4\%}{v^2}) \times (1.04)^3 = 159.20$$

Hence, the value of the contract now is:

$$(159.20 - 134.82)\ \underset{4\%}{v^3} = 21.67$$

This result can also be obtained directly from:

$$148 - 96 \times (1.04)^7 = 21.67$$

since the coupon of £7 is irrelevant in this calculation.

5 We will consider the three conditions necessary for immunisation.

(1) $V_A = V_L$

$$\begin{aligned} V_A &= 12.425v^{12} + 12.946v^{24} \quad \text{at 8\% p.a.} \\ &= 6.9757 \end{aligned}$$

$$\begin{aligned} V_L &= 15v^{13} + 10v^{25} \\ &= 6.9756 \end{aligned}$$

Condition (1) satisfied.

(2) $V'_A = V'_L$ where $V'_A = \dfrac{dV_A}{d\delta}$ and $V'_L = \dfrac{dV_L}{d\delta}$

$$\begin{aligned} V'_A &= 12 \times 12.425v^{12} + 24 \times 12.946v^{24} \quad \text{at 8\% p.a.} \\ &= 108.207 \end{aligned}$$

$$\begin{aligned} V'_L &= 13 \times 15v^{13} + 25 \times 10v^{25} \\ &= 108.206 \end{aligned}$$

Condition (2) satisfied.

(3) $V''_A > V''_L$

$$\begin{aligned} V''_A &= 12^2 \times 12.425v^{12} + 24^2 \times 12.946v^{24} \quad \text{at 8\% p.a.} \\ &= 1{,}886.46 \end{aligned}$$

$$\begin{aligned} V''_L &= 13^2 \times 15v^{13} + 25^2 \times 10v^{25} \\ &= 1{,}844.73 \end{aligned}$$

Condition (3) satisfied.

Thus the company is immunised against small changes in the rate of interest.

Candidates could, instead, have differentiated the expressions for V_A and V_L with respect to the interest rate (rather than the force of interest) to earn full marks.

6 (i) Let i_r = the r-year spot rate

$f_{t,r}$ = the r-year forward rate at time t years

$1000(1 + i_2)^2 = 1118$

$1000(1 + f_{1,2})^2 = 1140$

$1000(1 + f_{1,1}) = 1058$

$\Rightarrow i_2 = 5.73552\%$ p.a.

and $(1 + i_1)(1 + f_{1,1}) = (1 + i_2)^2$

$$\Rightarrow 1 + i_1 = \frac{1.118}{1.058} = 1.0567108$$

$\Rightarrow i_1 = 5.67108\%$ p.a.

and $(1 + i_1)(1 + f_{1,2})^2 = (1 + i_3)^3$

$$\Rightarrow (1 + i_3)^3 = 1.0567108 \times 1.140$$

$$= 1.20465$$

$\Rightarrow i_3 = 6.40295\%$ p.a.

(ii) Let C% be the 3-year par yield at time $t = 0$.

$$\text{Then } 100 = \frac{C}{1+i_1} + \frac{C}{(1+i_2)^2} + \frac{C}{(1+i_3)^3} + \frac{100}{(1+i_3)^3}$$

$$\Rightarrow 100 = C\left(\frac{1}{1.0567108} + \frac{1}{(1.0573552)^2} + \frac{1}{(1.0640295)^3}\right) + \frac{100}{(1.0640295)^3}$$

$$\Rightarrow 100 = C \times 2.6709035 + 83.0116$$

$$\Rightarrow C = 6.3605\%$$

7 (i) Accumulated amount at $t = 10$ is:

$$500\exp\left(\int_2^{10}\delta(s)ds\right)+800\exp\left(\int_9^{10}\delta(s)ds\right)$$

$$\int_2^{10}\delta(s)ds = \int_2^3 0.05ds+\int_3^8(0.09-0.01s)ds+\int_8^{10}(0.01s-0.03)ds$$

and $\int_2^3 0.05ds=[0.05s]_2^3=0.05$

$$\int_3^8(0.09-0.01s)ds = \left[0.09s-\frac{0.01s^2}{2}\right]_3^8$$

$$= 0.40 - 0.225 = 0.175$$

$$\int_8^{10}(0.01s-0.03)ds = \left[\frac{0.01s^2}{2}-0.03s\right]_8^{10}$$

$$= 0.20 - 0.08 = 0.12$$

Hence £500 accumulates to:

$$500e^{0.05+0.175+0.12} = 500e^{0.345}$$

$$= 705.99$$

and for 2nd term (i.e. the accumulation of £800):

$$\int_9^{10}\delta(s)ds = \int_9^{10}(0.01s-0.03)ds = \left[\frac{0.01s^2}{2}-0.03s\right]_9^{10}$$

$$= 0.20 - 0.135$$

$$= 0.065$$

So, £800 accumulates to $800e^{0.065} = 853.73$.

Therefore overall accumulation $= 705.99 + 853.73$

$$= 1{,}559.72$$

(ii) Let i = annual effective rate.

$$500(1 + i)^8 + 800(1 + i) = 1{,}559.72$$

approx: $1300(1 + i)^4 = 1{,}559.72$

$$\Rightarrow i \simeq 4.7\%$$

Try 5%, LHS = 1,578.73

Try 4%, LHS = 1,516.28

$\Rightarrow$ answer = 5% to nearest %

8 (i) (a) $(1 + i)^2 = \dfrac{43}{40} \times \dfrac{49}{43+4} \times \dfrac{53}{49+2}$

$$= 1.164695$$

$\Rightarrow i = 7.921\%$ p.a.

(b) 1st sub-interval = 1/1/2000–31/12/2000

IRR for the interval, i_1 given by

$$1 + i_1 = \frac{43}{40} \Rightarrow i_1 = 7.5\%$$

2nd sub-interval = 1/1/2001–31/12/2001

IRR for this interval, i_2, given by

$$(43 + 4)(1 + i_2) + 2(1 + i_2)^{½} = 53$$

Let $1 + i_2 = x^2$

$$x = \frac{-2 \pm \sqrt{4 + 4 \times 53 \times 47}}{2 \times 47}$$

$$= \frac{-2 \pm 99.8399}{94} = 1.04085 \text{ (taking +ve root)}$$

$\Rightarrow 1 + i_2 = x^2 = 1.08337$

i.e. $i_2 = 8.337\%$ p.a.

Hence linked IRR is i where

$$(1+i)^2 = 1.075 \times 1.08337$$

$\Rightarrow i = 7.918\%$ p.a.

(ii) LIRR = TWRR when the sub-intervals chosen for the LIRR coincide with the intervals between net cash flows being received.

In this instance, the sub-intervals would need to be:

1/1/2000–31/12/2000, 1/1/2001–30/6/2001

and 1/7/2001–31/12/2001

9 (i) $j = 0.04 \times 0.4 + 0.06 \times 0.2 + 0.08 \times 0.4$

$= 0.06$

$\Rightarrow$ mean accumulation $= 1{,}000 \times (1+j)^{10}$

$= 1{,}000 \times (1.06)^{10}$

$= £1{,}790.85$

(ii) $s^2 = 0.04^2 \times 0.4 + 0.06^2 \times 0.2 + 0.08^2 \times 0.4 - 0.06^2$

$= 0.00392 - 0.00360$

$= 0.00032$

Var(accumulation) $= 1{,}000^2\{(1 + 2j + j^2 + s^2)^{10} - (1+j)^{20}\}$

$= 1{,}000^2\{1.12392^{10} - (1.06)^{20}\}$

$= 9{,}145.60$

SD(accumulation) $= \sqrt{9145.60} = £95.63$

(iii) (a) By symmetry $j = 0.06$ (as in (i))

Hence, mean(accumulation) will be same as in (i) (i.e. £1,790.85).

The spread of the yields around the mean is lower than in (i). Hence, the standard deviation of the accumulation will be lower than £95.63.

(b) Mean(accumulation) > £1,790.85 since the investment is being accumulated over a longer period.

SD(accumulation) > £95.63 since investing over a longer term than in (i) will lead to a greater spread of possible accumulated amounts.

10 (i) $g(1 - t_1) = 0.08 \times 0.6 = 0.048 < i^{(2)}_{5\%} = 0.04939$

$\Rightarrow$ Capital gain $\Rightarrow$ so assume redeemed as late as possible

$$P = 0.6 \times 8a^{(2)}_{\overline{10|}} + 100v^{10} - 0.3(100 - P)v^{10} \quad \text{at } 5\%$$

$$P = 4.8 \times 1.012348 \times 7.7217 + 100 \times 0.61391 - 30 \times 0.61391 + 0.3P \times 0.61391$$

$$\Rightarrow P = \frac{98.9128 - 18.4173}{1 - 0.184173} = 98.67$$

(ii) $g = 0.08 > i^{(2)}_{7\%} = 0.068816$

$\Rightarrow$ assume redeemed as early as possible

Let P' = Price at which investor sold the security.

Then $P' = 8a^{(2)}_{\overline{3|}} + 100v^3 \quad$ at 7%

$= 8 \times 1.017204 \times 2.6243 + 100 \times 0.81630$

$= 102.99$

(iii) Work in half years — CGT payable

CGT = $0.3(102.99 - 98.67) = 1.30$

So $98.67 = 0.6 \times 4a_{\overline{4|}} + (102.99 - 1.30)v^4$

$= 2.4a_{\overline{4|}} + 101.69v^4]$

Try 3%, RHS $= 2.4 \times 3.7171 + 101.69 \times 0.88849$

$= 99.272$

Try 4%, RHS $= 2.4 \times 3.6299 + 101.69 \times 0.85480$

$= 95.636$

$$i = 0.03 + \frac{99.272 - 98.670}{99.272 - 95.636} \times 0.01$$

$= 3.166\%$

$\Rightarrow$ Answer $= 2 \times 3.166 = 6.33\%$ p.a. convertible half yearly

11 (i) $(Ia)_{\overline{n|}} = v + 2v^2 + 3v^3 + \ldots + nv^n$

$(1+i)(Ia)_{\overline{n|}} = 1 + 2v + 3v^2 + \ldots + nv^{n-1}$

$\Rightarrow i(Ia)_{\overline{n|}} = (1 + v + v^2 + \ldots v^{n-1}) - nv^n$

$$\Rightarrow (Ia)_{\overline{n|}} = \frac{\ddot{a}_{\overline{n|}} - nv^n}{i}$$

(ii) Loan $= 200(Ia)_{\overline{15|}} + 2800a_{\overline{15|}}$ @ 8%

$$(Ia)_{\overline{15|}} = \frac{\ddot{a}_{\overline{15|}} - 15v^{15}}{0.08} = \frac{1.08 \times 8.5595 - 15 \times 0.31524}{0.08}$$

$= 56.4458$

$\Rightarrow$ Loan $= 200 \times 56.4458 + 2{,}800 \times 8.5595$

$= 35{,}255.76$

(iii) Capital o/s after 8^{th} payment

$$= 200(Ia)_{\overline{7|}} + 4,400a_{\overline{7|}} \quad @\ 8\%$$

$$(Ia)_{\overline{7|}} = \frac{\ddot{a}_{\overline{7|}} - 7v^7}{0.08} = \frac{1.08 \times 5.2064 - 7 \times 0.58349}{0.08}$$

$$= 19.2310$$

$\Rightarrow$ Cap o/s = 200 × 19.2310 + 4,400 × 5.2064

= 26,754.36

Year	*Loan o/s at start*	*Repayment*	*Interest element*	*Capital element*
9	26,754.36	4,600	2,140.35	2,459.65
10	24,294.71	4,800	1,943.58	2,856.42

(iv) Loan o/s after 10^{th} payment

= 24,294.71 − 2,856.42 = 21,438.29

Let 11^{th} payment be X then

$$200(Ia)_{\overline{5|}} + (X - 200)a_{\overline{5|}} = 21{,}438.29 \quad @\ 6\%$$

$$(Ia)_{\overline{5|}} = \frac{1.06 \times 4.2124 - 5v^5}{0.06} = 12.1476$$

Hence 200 × (12.1476 − 4.2124) + X × 4.2124 = 21,438.29

$\Rightarrow X$ = 4,712.57