EXAMINATIONS

3 April 2003 (pm)

Subject 102 — Financial Mathematics

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

1 State the main differences between a preference share and an ordinary share. [3]

2 An investor purchased a holding of ordinary shares two months before payment of the next dividend was due. Dividends are paid annually and it is expected that the next dividend will be a net amount of 12p per share. The investor anticipates that dividends will grow at a constant rate of 4% per annum in perpetuity.

Calculate the price per share that the investor should pay to obtain a net return of 7% per annum effective. [4]

3 A businessman is considering an investment which requires an initial outlay of £60,000 and a further outlay of £25,000 in eight months time.

Starting two years after the initial outlay, it is estimated that income will be received continuously for four years at a rate of £5,000 per annum, increasing to £9,000 per annum for the next four years, then increasing to £13,000 per annum for the following four years and so on, increasing by £4,000 per annum every four years until the payment stream stops after income has been received for 20 years (i.e. 22 years after the initial outlay). At the point when the income ceases, the investment can be sold for £50,000.

Calculate the net present value of the project at a rate of interest of 9% per annum effective. [7]

4 (i) Explain what is meant by a "forward contract". Your answer should include reference to the terms "short forward position" and "long forward position". [3]

(ii) An investor entered into a long forward contract for £100 nominal of a security seven years ago and the contract is due to mature in three years time. The price per £100 nominal of the security was £96 seven years ago and is now £148. The risk-free rate of interest can be assumed to be 4% per annum effective during the contract.

Calculate the value of the contract now if the security will pay a single coupon of £7 in two years time and this was known from the outset. You should assume no arbitrage. [5]

[Total 8]

5 A new management team has just taken over the running of a finance company. They discover that the company has liabilities of £15 million due in 13 years time and £10 million due in 25 years time. The assets consist of two zero-coupon bonds, one paying £12.425 million in 12 years time and the other paying £12.946 million in 24 years time. The current interest rate is 8% per annum effective.

Determine whether the necessary conditions are satisfied for the finance company to be immunised against small changes in the rate of interest. [8]

6 An individual is investing in a market in which a variety of spot rates and forward contracts are available.

If at time $t = 0$ he invests £1,000 for two years, he will receive £1,118 at time $t = 2$. Alternatively, if at time $t = 0$ he agrees to invest £1,000 at time $t = 1$ for two years, he will receive £1,140 at time $t = 3$. However, if at time $t = 0$ he agrees to invest £1,000 at time $t = 1$ for one year, he will receive £1,058 at time $t = 2$.

(i) Calculate the following rates per annum effective, implied by this data:

(a) The one-year spot rate at time $t = 0$.
(b) The two-year spot rate at time $t = 0$.
(c) The three-year spot rate at time $t = 0$. [5]

(ii) Calculate the three-year par yield at time $t = 0$ in this market. [3]
[Total 8]

7 The force of interest $\delta(t)$ is a function of time and at any time *t*, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.05 & 0 \le t \le 3 \\ 0.09 - 0.01t & 3 < t \le 8 \\ 0.01t - 0.03 & 8 < t \end{cases}$$

(i) If £500 is invested at $t = 2$ and a further £800 is invested at $t = 9$, calculate the accumulated amount at $t = 10$. [7]

(ii) Determine the constant effective rate of interest per annum, to the nearest 1%, which would lead to the same result as in (i) being obtained. [3]
[Total 10]

8 A pension fund had assets totalling £40 million on 1 January 2000. It received net income of £4 million on 1 January 2001 and £2 million on 1 July 2001. The value of the fund totalled:

£43 million on 31 December 2000
£49 million on 30 June 2001
£53 million on 31 December 2001

(i) Calculate for the period 1 January 2000 to 31 December 2001, to 3 decimal places:

(a) the time weighted rate of return per annum [3]

(b) the linked internal rate of return, using sub-intervals of a calendar year [5]

(ii) State both in general, and in this particular case, when the linked internal rate of return will be identical to the time weighted rate of return. [2]

[Total 10]

9 £1,000 is invested for 10 years. In any year the yield on the investment will be 4% with probability 0.4, 6% with probability 0.2 and 8% with probability 0.4 and is independent of the yield in any other year.

(i) Calculate the mean accumulation at the end of 10 years. [2]

(ii) Calculate the standard deviation of the accumulation at the end of 10 years. [5]

(iii) Without carrying out any further calculations, explain how your answers to (i) and (ii) would change (if at all) if:

(a) the yields had been 5%, 6% and 7% instead of 4%, 6% and 8% per annum, respectively; or

(b) the investment had been made for 12 years instead of 10 years [4]

[Total 11]

10 A fixed interest security pays coupons of 8% per annum half yearly on 1 January and 1 July. The security will be redeemed at par on any 1 January between 1 January 2006 and 1 January 2011 inclusive, at the option of the borrower.

An investor purchased a holding of the security on 1 January 2001, immediately after the payment of the coupon then due, at a price which gave him a net yield of at least 5% per annum effective. The investor pays tax at 40% on interest income and 30% on capital gains. On 1 January 2003 the investor sold the holding, immediately after the payment of the coupon then due, to a fund which pays no tax at a price to give the fund a gross yield of at least 7% per annum effective.

(i) Calculate the price per £100 nominal at which the investor bought the security. [5]

(ii) Calculate the price per £100 nominal at which the investor sold the security. [3]

(iii) Calculate the net yield per annum convertible half yearly which the investor actually received over the two years the investor held the security. [6]

[Total 14]

11 (i) Prove

$$(\mathrm{I}a)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}.$$ [3]

A loan is repayable by an increasing annuity payable annually in arrears for 15 years. The repayment at the end of the first year is £3,000 and subsequent payments increase by £200 each year. The repayments were calculated using a rate of interest of 8% per annum effective.

(ii) Calculate the original amount of the loan. [3]

(iii) Construct the capital/interest schedule for years nine (after the eighth payment) and ten, showing the outstanding capital at the beginning of the year, the interest element and the capital repayment. [6]

(iv) Immediately after the tenth payment of interest and capital, the interest rate on the outstanding loan is reduced to 6% per annum effective.

Calculate the amount of the eleventh payment if subsequent payments continue to increase by £200 each year, and the loan is to be repaid by the original date, i.e. 15 years from commencement. [5]

[Total 17]