EXAMINATIONS

7 September 2001 (pm)

Subject 102 — Financial Mathematics

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
2. *Mark allocations are shown in brackets.*
3. *Attempt all 12 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available Actuarial Tables and an electronic calculator.

1 A 91-day government bill provides the purchaser with an annual effective rate of return of 5%. Determine the annual simple discount rate at which the bill is discounted. [2]

2 A particular share is expected to pay a dividend of d_1 in exactly one year. Dividends are expected to grow by g per annum effective every year thereafter. The share pays annual dividends. Let V_0 be the present value of the share and r be the investor's required annual effective rate of return.

Show that $V_0 = \dfrac{d_1}{r - g}$. [3]

3 An asset has a current price of 100p. It will pay an income of 5p in 20 days' time. Given a risk-free rate of interest of 6% per annum convertible half-yearly and assuming no arbitrage, calculate the forward price to be paid in 40 days. [4]

4 An annuity is paid half-yearly in arrears at a rate of £1,000 per annum, for 20 years. The rate of interest is 5% per annum effective in the first 12 years and 6% per annum convertible quarterly for the remaining 8 years.

Calculate the accumulation of the annuity at the end of 20 years. [4]

5 An investor purchases a bond, redeemable at par, which pays half-yearly coupons at a rate of 8% per annum. There are 8 days until the next coupon payment and the bond is ex-dividend. The bond has 7 years to maturity after the next coupon payment.

Calculate the purchase price to provide a yield to maturity of 6% per annum effective. [4]

6 $(1 + i_t)$ follows a log normal distribution where i_t is the rate of interest over a given time period beginning at time t. The parameters of the distribution are $\mu = 0.06$ and $\sigma^2 = 0.0009$.

Calculate the inter-quartile range for the accumulation of 100 units of money over the given time period, beginning at time t. [6]

7 (i) The annual effective forward rate applicable over the period from t to $t + r$ is defined as $f_{t,r}$ where t and r are measured in years. If $f_{0,1} = 8\%$, $f_{1,1} = 7\%$, $f_{2,1} = 6\%$ and $f_{3,1} = 5\%$, calculate the gross redemption yield at the issue date from a 4-year bond, redeemable at par, with a 5% coupon payable annually in arrears. [7]

(ii) Explain why the gross redemption yield from the 4-year bond is higher than the 4-year forward rate $f_{3,1}$. [2]

[Total 9]

8 A fast food company is considering opening a new sales outlet. The initial cost of the outlet would be £1,000,000 incurred at the outset of the project. It is expected that rents of £40,000 per annum would have to be paid quarterly in advance for 10 years, increasing after ten years to £48,000 per annum. The net revenue (sales minus costs, other than rent) from the venture is expected to be £100,000 for the first year and £200,000 for the second year. Thereafter, the net revenue is expected to grow at 3% per annum compound so that it is £206,000 in the third year, £212,180 in the fourth year and so on. The revenue would be received continuously throughout each year. Twenty years after the outset of the project, the revenue and costs stop and the project has no further value.

Calculate the internal rate of return from the project. [11]

9 (i) Prove that $Da_{\overline{n|}} = \dfrac{n - a_{\overline{n|}}}{i}$. [3]

(ii) A bank makes a loan to be repaid by instalments paid annually in arrears. The first instalment is 20, the second is 19 with the payments reducing by 1 per annum until the end of the 10^{th} year after which there are no further payments. The rate of interest charged by the lender is 6% per annum effective.

(a) Calculate the amount of the loan.

(b) Calculate the interest and capital components of the first payment.

(c) Calculate the amount of capital repaid in the instalment at the end of the 8^{th} year.

[8]

[Total 11]

10 An investor purchased a bond with exactly 20 years to redemption. The bond, redeemable at par, has a gross redemption yield of 6%. It pays annual coupons, in arrears, of 5%. The investor does not pay tax.

(i) Calculate the purchase price paid for the bond. [3]

(ii) After exactly ten years, immediately after payment of the coupon then due, this investor sells the bond to another investor. That investor pays income and capital gains tax at a rate of 30%. The bond is purchased by the second investor to provide a net rate of return of 6.5% per annum.

(a) Calculate the price paid by the second investor.

(b) Calculate the annual effective rate of return earned by the first investor during the period for which the bond was held. [10]

[Total 13]

11 The force of interest, $\delta(t)$, is:

$$\begin{aligned}\delta(t) &= 0.05 \text{ for } 0 < t \le 10,\\ &= 0.006t \text{ for } 10 < t \le 20\\ &= 0.003t + 0.0002t^2 \text{ for } 20 < t\end{aligned}$$

(i) Calculate the present value of a unit sum of money due at time $t = 25$. [7]

(ii) Calculate the effective rate of interest per unit time from time $t = 19$ to time $t = 20$. [3]

(iii) A continuous payment stream is paid at the rate of $e^{-0.03t}$ per unit time between time $t = 0$ and time $t = 5$. Calculate the present value of that payment stream. [4]

[Total 14]

12 (i) (a) In the context of a stream of future receipts paid at discrete times, let volatility be defined as the proportionate change in the present value of a payment stream per unit change in the force of interest, for small changes in the force of interest. Prove that the discounted mean term is equal to the volatility.

(b) If volatility is now defined as the proportionate change in the present value of a payment stream per unit change in the annual effective rate of interest, for small changes in the annual effective rate of interest, find the relationship between the discounted mean term and volatility. [5]

(ii) A life insurance company manages a small annuity fund. Payments are expected to be made from the fund of £1,000,000 per annum at the end of years 1 to 10 and £1,500,000 at the end of each of the following 10 years. Assets are held in two types of bonds. The first is a zero coupon bond redeemable in 10 years' time. The second is a bond which pays an annual coupon of g% per annum in arrears and is redeemable at par at the end of 19 years. £10,000,000 nominal of the zero coupon bond have been purchased.

Find the nominal amount of the coupon paying bond which must be purchased and the rate of coupon which is received from the bond if the insurance company is to equalise the present values and discounted mean terms of its assets and liabilities at an effective rate of interest of 5% per annum. [12]

(iii) If the present value and discounted mean term of the assets and liabilities are equalised, state the third condition which is necessary for the insurance company to be immunised from small, uniform changes in the rate of interest. [2]

[Total 19]