EXAMINATIONS

13 April 2000 (am)

Subject 106 — Actuarial Mathematics 2

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
2. *Mark allocations are shown in brackets.*
3. *Attempt all 9 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available Actuarial Tables and an electronic calculator.

1 The profit per client-day made by a privately owned health centre depends on the variable costs involved. Variable costs, over which the owner of the health centre has no control, take one of three levels θ_1 = high, θ_2 = most likely, θ_3 = low. The owner has to decide at what level to set the number of client-days for the coming year. Client-days can be either $d_1 = 16$, $d_2 = 13.4$ or $d_3 = 10$ (each in 000s). The profit (£) per client-day is as follows:

	θ_1	θ_2	θ_3
d_1	85	95	110
d_2	105	115	130
d_3	125	135	150

(i) Determine the Bayes criterion solution based on the annual profits, given the probability distribution $p(\theta_1) = 0.1$, $p(\theta_2) = 0.6$, $p(\theta_3) = 0.3$. [3]

(ii) Determine both the minimax solution and the maximin solution to this problem. [2]

[Total 5]

2 An insurance company operates a No Claims Discount system for its motor insurance business, with discount levels 0%, 15%, 30% and 50%. The full annual premium is £500. The rules for moving between discount levels are:

- If no claims are made during a year, the policyholder moves to the next higher level of discount or remains at the maximum discount level.
- If one or more claims are made during a year, a policyholder at the 30% or 50% discount level moves to the 15% discount level and a policyholder at the 0% or 15% discount level moves to, or remains at, the 0% discount level.

When an accident occurs, the distribution of the loss is exponential with mean £1,000. In the event of an accident, a policyholder will claim only if the loss is greater than the total extra premiums that would have to be paid over the next three years, assuming that no further accidents occur.

For each discount level, calculate:

(i) the smallest loss for which a policyholder will make a claim. [3]

(ii) the probability of a claim being made in the event of an accident occurring. [3]

[Total 6]

3 (i) Loss amounts from a particular type of insurance have a Pareto distribution with parameters α and λ. If the company applies a policy excess, E, find the distribution of claim amounts paid by the insurer. [3]

(ii) Assuming that $\alpha = 4$ and $\lambda = 15$, calculate the mean claim amount paid by the insurer

(a) with no policy excess ($E = 0$),

(b) with an excess of 10 ($E = 10$). [2]

(iii) Using your answers to (ii), comment on the effect of introducing a policy excess. [2]
[Total 7]

4 The proportion, θ, of staff working in a particular office who have access to the internet at home is to be estimated. Of a sample of 50 people questioned, 29 have access to the internet at home.

(i) Using a suitable uniform distribution as the prior distribution, calculate an estimate of θ under the quadratic loss function. [3]

(ii) Using instead a beta distribution, with parameters $\alpha = 4$ and $\beta = 4$, as the prior distribution for θ, calculate the Bayesian estimator for θ under the "all-or-nothing" loss function. [4]
[Total 7]

5 The following table shows incremental claims relating to the accident years 1997, 1998 and 1999. It is assumed that claims are fully run-off by the end of development year 2. Estimate total outstanding claims using the chain-ladder technique, ignoring inflation.

Incremental Claims

	Development Year		
Accident Year	*0*	*1*	*2*
1997	2587	1091	251
1998	2053	1298	
1999	3190		

[7]

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6 A generalised linear model (GLM) has independent Poisson responses $\{Y_{ix}\}$, with

$$E(Y_{ix}) = m_{ix}\ ,\ \mathrm{Var}(Y_{ix}) = m_{ix}\ .$$

The linear parameterised predictor η_{ix} is linked to the mean response by the log function, such that

$$\log m_{ix} = \eta_{ix}\ .$$

(i) (a) Write down an expression for the variance function $V(.)$ for this GLM.

(b) Evaluate the integral expression

$$d_{ix} = 2\int_{\hat{m}_{ix}}^{y_{ix}} \frac{y_{ix} - t}{V(t)}\, dt$$

to determine an expression for the general component d_{ix} of the deviance of this GLM in term of the observed responses y_{ix} and fitted values $\hat{m}_{ix}$.

(c) Write down expressions for the deviance residual and Pearson residual for this model. [5]

(ii) The actual deaths a_{ix}, with matching exposures r_{ix} to the risk of death, based on policy counts for UK female assured lives in 1987–90, are modelled as the independent responses A_{ix} of a Poisson GLM with

$$E(A_{ix}) = m_{ix} = r_{ix}\,\mu_{ix}$$

where μ_{ix} is the targeted unknown force of mortality, and

$$\log m_{ix} = \eta_{ix} = \log r_{ix} + \log \mu_{ix}\ .$$

The structure of the linear predictor is generated by the two covariates:

policy duration, denoted by the factor $D(i)$, $i = 1, 2, 3$, which represent the three levels 0, 1, or 2+ years respectively; and

age, denoted by the variable x, coded in 5 yearly bands, x = (mid age band – 17.5) / 5.

The term log r_{ix} is a known offset and log μ_{ix} takes various linear parameterised forms as implied by the following computer output:

Fit 1:
scaled deviance = 16829, residual df = 41

Add in straight-line age effects:
scaled deviance = 338.83 (change = 16400),
residual df = 40 (change = 1)

Add in quadratic age effects:
scaled deviance = 306.71 (change = 32.12),
residual df = 39 (change = 1)

Add in cubic age effects:
scaled deviance = 306.51 (change = 0.1976),
residual df = 38 (change = 1)

Add in duration effects:
scaled deviance = 46.055 (change = 260.5),
residual df = 36 (change = 2)

estimate	*s.e.*	*parameter*
–9.504	0.1382	1
0.3370	0.05627	x
0.01011	0.007445	x^2
–0.0001376	0.0003055	x^3
0.3663	0.06365	$D(2)$
0.6682	0.04979	$D(3)$

(a) List the various fitted parameterised formulae.

(b) Interpret the output and state your conclusions.

(c) What additional relevant computer output would be useful?

[8]
[Total 13]

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7 (i) Claims on a group of policies of a certain type arise as a Poisson process with parameter λ_1. Claims on a second, independent, group of policies arise as a Poisson process with parameter λ_2. The aggregate claim amounts on the respective groups are denoted S_1 and S_2.

Using moment generating functions (or otherwise), show that S (the sum of S_1 and S_2) also has a compound Poisson distribution and hence derive the Poisson parameter for S. [4]

(ii) An insurance company offers accident insurance for employees. A total of 650 policies have been issued split between two categories of employees. The first category contains 400 policies, and claims occur on each policy according to a Poisson process at a rate of one claim per 20 years, on average. In this category all claim amounts are £3,000. In the second category, claims occur on each policy according to a Poisson process at a rate of one claim per 10 years, on average. In this category, the claim amount is either £2,000 or £3,000 with probabilities 0.4 and 0.6, respectively. All policies are assumed to be independent.

Let S denote the aggregate annual claims from the portfolio.

(a) Calculate the mean, variance and coefficient of skewness of S. [4]

(b) Using the normal distribution as an approximation to the distribution of S, calculate Y such that the probability of S exceeding Y is 10%. [3]

(c) The insurance company decides to effect reinsurance cover with aggregate retention £100,000, so that the insurance company then pays no more than this amount out in claims each year. In the year following the inception of this reinsurance, the numbers of policies in each of the two groups remains the same but, because of changes in the employment conditions of which the company was unaware, the probability of a claim in group 2 falls to zero. Using the normal distribution as an approximation to the distribution of S, calculate the probability of a claim being made on the reinsurance treaty. [3]

[Total 14]

8 The aggregate claims process for a risk is a compound Poisson process with parameter λ. Individual claim amounts X are independent and identically distributed with density function

$$f(x) = \frac{1+2x}{3}\, e^{-x}, \qquad x > 0.$$

The insurer's premium is paid continuously at a constant rate and is calculated so that the premium loading factor $\theta = \frac{3}{8}$.

(i) Define the adjustment coefficient for such a process and calculate its value. [11]

(ii) Define the probability of ruin $\psi(u)$ with initial capital of $u(> 0)$. [1]

(iii) For the detailed process described above it is possible to show that

$$\psi(u) = \tfrac{3}{4}e^{-u/5} - \tfrac{1}{14}e^{-15u/11}.$$

(a) Sketch this function.

(b) State Lundberg's inequality and hence find an upper bound for $\psi(u)$. Comment on the result.

[3]

(iv) A company insuring the above risk is offered proportional reinsurance from another company which charges a premium loading factor of 50%. If the reinsurer pays 16% of each claim, derive the equation which determines the adjustment coefficient for the direct insurer, but DO NOT solve it.

[5]

[Total 20]

9 The aggregate claims X each year, from a portfolio of insurance policies, are assumed to have the normal distribution with unknown mean θ and known variance τ^2. Prior information is such that θ is assumed to have a normal distribution with known mean μ and known variance σ^2. Independent aggregate claims over the last n years are denoted by $x_1, x_2, ..., x_n$.

(i) (a) Derive the posterior distribution of θ.

(b) Using the answer in (a), write down the Bayesian point estimate of θ under a quadratic loss function.

(c) Show that the estimate in (b) can be expressed in the form of a credibility estimate, and derive the form of the credibility factor.

(d) Determine the limiting form for this credibility estimate as the number of years of data increases. [9]

(ii) The following denote the annual aggregate claims for two companies over five years

Company:		*A*	*B*
Year:	*1*	217	196
	2	250	239
	3	249	233
	4	239	222
	5	265	244

(a) Determine the Bayes credibility estimate of the risk premium based on the modelling assumptions of part (i) above, in the two separate cases:

Case (1)

Company:	*A*	*B*
τ^2	400	400
μ	270	260
σ^2	2500	2500

Case (2)

Company:	*A*	*B*
τ^2	400	400
μ	270	260
σ^2	225	225

(b) Using the answer to (ii)(a), comment on the effect of the value of σ^2.

(c) Determine the empirical Bayes credibility estimate of the risk premium for each of the two companies.

[12]
[Total 21]