EXAMINATIONS

5 April 2001 (am)

Subject 106 — Actuarial Mathematics 2

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*

2. *Mark allocations are shown in brackets.*

3. *Attempt all 9 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available Actuarial Tables and an electronic calculator.

1 The triangle below shows incremental claims for a portfolio of general insurance policies. The data have already been adjusted to take account of inflation effects. Calculate the basic chain ladder development factors, and the implied grossing-up factors.

2,541	1,029	217
2,824	790	
1,981		

[5]

2 A generalised linear model has independent Binomial responses $Z_1, \ldots, Z_k$ with $E(Z_i) = n\mu$, $\text{Var}(Z_i) = n\mu(1 - \mu)$ for $0 < \mu < 1$.

(i) Show that $Y_i = Z_i/n$ belongs to an exponential family. [2]

(ii) Identify the natural parameter and the canonical link function, and derive the variance function. [4]

[Total 6]

3 The aggregate claims process for a particular risk is a compound Poisson process with $\lambda = 20$. Individual claim amounts are £100 with probability ¼, £200 with probability ½, or £250 with probability ¼. The initial surplus is £1,000. Using a Normal approximation, calculate approximately the smallest premium loading factor θ such that the probability of ruin at time 3 is at most 0.05. [7]

4 (i) Let $S = X_1 + \ldots + X_N$, where $X_1, X_2, \ldots$ are independent, identically distributed random variables, and N is a random variable independent of the X_i's. Derive an expression for the moment generating function of S in terms of the probability generating function of N and the moment generating function of X_i. [2]

(ii) In a group of policies, the monthly number of claims for a single policy has a Poisson distribution with parameter λ, where λ is a random variable with density $f(\lambda) = 2e^{-2\lambda}$, $\lambda > 0$.

(a) Show that the probability of n claims on a policy picked at random from the group is $\dfrac{2}{3^{n+1}}$, $n = 0, 1, 2, \ldots$

(b) Find the moment generating function for the aggregate claims distribution if the claims have a gamma distribution with mean 2 and variance 2. [6]

[Total 8]

5 A company is considering setting up a new class of insurance. All risks will be at one of four possible levels of intensity, I_1, I_2, I_3 and I_4. The company has to decide at what level to set the premium, which will attract differing amounts of business as follows:

Annual premium	£85	£81	£79
No. of policies ('000s)	100	150	200

The company has annual fixed costs of £1.5 million plus annual per policy expenses of £18. Under each level of intensity the company expects to pay out an average in claims per policy of:

Level of intensity	I_1	I_2	I_3	I_4
Average cost (£)	40	45	57	60

(i) Determine the minimax solution based on annual profits. [7]

(ii) Given the probability distribution $p(I_1) = 0.1$, $p(I_2) = 0.4$, $p(I_3) = 0.3$, $p(I_4) = 0.2$, determine the Bayes criterion solution based on annual profits. [2]

[Total 9]

6 An insurance company is monitoring the length of time staff take to pick up telephones after they first ring. It is assumed that the time follows an exponential distribution with parameter θ.

10 calls are monitored at random and the average response time is calculated as 3.672 seconds.

(i) (a) Show that the gamma distribution is the conjugate prior distribution for θ .

(b) Assuming that the prior distribution for θ has mean 0.315 and standard deviation 0.251, derive the posterior distribution of θ and calculate the Bayesian estimator of θ under quadratic loss. [7]

(ii) A further 70 calls are monitored and have the same average response time of 3.672 seconds. Calculate the Bayesian estimator of θ under quadratic loss using all the data collected. [2]

(iii) Comment on your answers in (i) and (ii) [2]

[Total 11]

7 A no claims discount system has four levels, 0%, 25%, 50% and 75%. The rules for moving between levels are as follows:

> If no claims are made in one year, the policyholder moves to the next higher level, or remains at the 75% level;
>
> If one claim is made in one year, the policyholder moves down one level, or remains at the 0% level;
>
> If two or more claims are made, the policyholder moves straight down to, or remains at, the 0% level.

Policyholders at different levels are found to experience different rates of claiming. The number of claims made per year follow a Poisson distribution with parameter λ as follows:

Level	0%	25%	50%	75%
λ	0.29	0.22	0.18	0.10

(i) Derive the transition matrix. [6]

(ii) Calculate the proportions at each different level when the system reaches a steady state. [9]

[Total 15]

8 For each of m independent policies, the probability of one claim in a year is θ $(0 < \theta < 1)$ and the probability of no claims in a year is $1 - \theta$. The total number of claims in one year is a random variable X. Independent observations $x_1, \ldots, x_n$ of X are available. The prior distribution of θ has density $f(\theta) \propto \{\theta(1-\theta)\}^{\beta-1}$, $0 < \theta < 1$, for some constant $\beta > 0$.

(i) (a) Derive the posterior distribution of θ given $x_1, \ldots, x_n$.

(b) Derive the maximum likelihood estimate of θ, $g(\underset{\sim}{x})$.

(c) Derive the Bayesian estimate of θ under quadratic loss, and show that it takes the form of a credibility estimate,

$$Zg(\underset{\sim}{x}) + (1 - Z)\,\mu$$

where μ is a quantity you should specify in terms of the prior distribution of θ.

(d) Explain what happens to Z as the number of data points increases.

[11]

(ii) Calculate the Bayesian estimate of θ and the value of Z if $n = 6$, $m = 10$ and $x_1 = 1$, $x_2 = 4$, $x_3 = 2$, $x_4 = 1$, $x_5 = 1$, $x_6 = 3$, when

(a) $\beta = 1$

(b) $\beta = 4$

By considering the prior variance, comment on the effect on Z of increasing β, and relate this effect to the quality of the prior information about θ in each case. [6]

[Total 17]

9 (i) A random variable X has the lognormal distribution with density function $f(x)$ and parameters μ and σ. Show that for $a > 0$

$$\int_a^\infty xf(x)dx = \exp\left(\mu + \frac{\sigma^2}{2}\right)\left(1 - \Phi\left(\frac{\log a - \mu - \sigma^2}{\sigma}\right)\right)$$

where Φ is the cumulative distribution function of the standard normal distribution. [4]

(ii) Claims under a particular class of insurance follow a lognormal distribution with mean 9.070 and standard deviation of 10.132 (figures in £000s). In any one year 20% of policies are expected to give rise to a claim.

An insurance company has 200 policies on its books and wishes to take out individual excess of loss reinsurance to cover all the policies in the portfolio. The reinsurer has quoted premiums for two levels of reinsurance as follows (figures in £000s):

Retention Limit	*Premium*
25	48.5
30	38.2

(a) Calculate the probability, under each reinsurance arrangement, that a claim arising will involve the reinsurer.

(b) By investigating the average amount of each claim ceded to the reinsurer, calculate which of the retention levels gives the best value for money (ignoring the insurer's attitude to risk).

(c) The following year, assuming all other things equal, the insurer believes that inflation will increase the mean and standard deviation of the claims in its portfolio by 8%. If the reinsurer charges the same premiums as before, which of the retention levels will be best value for money next year?

[18]

[Total 22]