EXAMINATIONS

12 September 2002 (am)

Subject 106 — Actuarial Mathematics 2

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 9 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

1 Claims on a portfolio of insurance policies are exponentially distributed with mean $1/\lambda$, where previous experience with similar portfolios suggests that the prior distribution of λ is gamma with mean 1 and variance ½. Twenty claims are observed with average value 1.2.

Determine the posterior distribution of λ. [4]

2 A sample of claims from 50 individuals are independent normal random variables Y_{ij}, ($i = 1, 2;\ j = 1, \ldots, 25$) with variance σ^2 and

$$E(Y_{ij}) = \alpha_i + \beta x_{ij},$$

where group 1 consists of males ($i = 1$) and group 2 of females ($i = 2$) and x_{ij} is the age of the jth individual in group i.

(i) Show that this is a generalised linear model, by writing it in the form of an exponential family, and by stating the link function and the linear predictor. [3]

(ii) Sketch on the same diagram the dependence specified by the model of expected claim size on age for males and females. [1]

[Total 4]

3 (i) Derive the moment generating function of the total amount, T, claimed if the number of claims, N, has a Poisson distribution with mean $\lambda > 0$ and the claim severity distribution has moment generating function $M(t)$. [2]

(ii) A portfolio consists of 210 risks each of which gives rise to claims as a Poisson process. The claim severity distribution is exponential. The portfolio is divided into 3 groups, as follows:

Group	*Number of risks*	*Poisson rate per risk*	*Mean of claim severity distribution*
1	40	1	400
2	120	2	500
3	50	2.5	600

(a) Derive the moment generating function of the total claim amount S from all 210 independent risks in one time unit.

(b) Show that S has a compound Poisson distribution and determine the corresponding Poisson parameter and the claim severity density. [6]

[Total 8]

4 A market trader has the option for one day of selling either ice-cream (d_1), hot food (d_2) or umbrellas (d_3) at an outdoor festival. He believes that the weather is equally likely to be fine (θ_1), overcast (θ_2) or wet (θ_3) and estimates his profits under each possible scenario to be:

	θ_1	θ_2	θ_3
d_1	25	19	7
d_2	10	30	8
d_3	0	2	34

(i) Determine the minimax solution to this problem. [2]

(ii) The trader's partner is very optimistic and believes that the criterion to adopt in deciding which product to sell should be to maximise the maximum profit. What decision would the trader's partner make based on these predicted profits? [1]

(iii) Determine the Bayes criterion solution to this problem. [2]

(iv) The trader's partner agrees that it is equally likely to be either fine or wet but believes that there is more than an evens chance of it being overcast. By sketching a graph of the Bayes risk for each of the three possible decisions against the probability of it being overcast (p), or otherwise, determine the revised Bayes criterion solution. [4]

[Total 9]

5 (i) Claims arrive as a Poisson process with rate λ. The claim sizes are independent, identically distributed random variables $X_1, X_2, \ldots$ with

$$P(X_i = k) = p_k, \quad k = 1, \ldots, M, \quad \sum_{k=1}^{M} p_k = 1.$$

If the premium loading factor is θ, show that the adjustment coefficient R satisfies:

$$\frac{1}{M}\log(1+\theta) < R < \frac{2\theta m_1}{m_2},$$

where $m_i = E(X_1^i)$, $i = 1, 2$. [7]

[The inequality $e^{Rx} \le \frac{x}{M}e^{RM} + 1 - \frac{x}{M}$ $(0 \le x \le M)$ may be used without proof.]

(ii) If $\theta = 0.2$ and X_i is equally likely to be 1 or 2, determine an upper and a lower bound for R, and hence derive an upper bound on the probability of ruin when the initial surplus is u. [3]

[Total 10]

6 The no claims discount system for a particular class of annual insurance policy has three categories with discount levels of 0%, 30% and 50%. If a policyholder makes any claims during the year he or she moves down a single category (or stays at the 0% discount level). If no claims are made, then the policyholder moves to the next higher category (or stays at the 50% discount level).

The probability that a policyholder will make at least one claim in any one year is:

p	in the 0% discount category
$0.8p$	in the 30% discount category
$0.6p$	in the 50% discount category

The premium charged at the 0% discount category is c.

(i) Write down the transition matrix for this system in terms of p. [2]

(ii) Derive the steady state distribution of policyholders in each discount category in terms of p. [5]

(iii) Calculate the average premium paid, $A(p, c)$, in the steady state in terms of p and c. [3]

(iv) Suppose that the insurance company calculates premiums for this class of insurance by grouping policyholders into two types. The first type is known to have $p = 0.1$ and the second type has $p = 0.15$, while the premium charged for the first type of policyholder at the 0% discount level is 1,000. Ignoring expenses and profit loadings and assuming that all other characteristics of these risks are the same, calculate the value c, the premium in the 0% discount category for the second type of policyholder, in order that $A(0.15, c) = 1.5A(0.1, 1{,}000)$. [2]

[Total 12]

7 The cumulative claims paid each year under a certain cohort of insurance policies are recorded in the table below, for accident years 1998, 1999, 2000 and 2001.

	Development year			
Accident year	*0*	*1*	*2*	*3*
1998	2,457	4,196	4,969	5,010
1999	2,648	4,715	5,561	
2000	3,084	5,315		
2001	3,341			

(i) Calculate the development factors under the basic chain ladder technique and state the assumptions underlying the use of this method. [4]

(ii) The rate of claims inflation over these years, measured over the 12 months to the middle of each year, is given in the table below.

1999	2.1%
2000	10.5%
2001	3.2%

Calculate the development factors under the inflation-adjusted chain ladder technique and state the assumptions underlying the use of this method. [6]

(iii) Based on the development factors calculated in parts (i) and (ii), calculate the fitted values under these two models and comment on how these compare with the actual values. [7]

[Total 17]

8 (i) Show that

$$\int_a^b \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(\ln x-\mu)^2} dx = e^{\mu+\frac{1}{2}\sigma^2}\left(\Phi\left(\frac{\ln b-\mu-\sigma^2}{\sigma}\right)-\Phi\left(\frac{\ln a-\mu-\sigma^2}{\sigma}\right)\right)$$

[4]

(ii) Individual claim amounts on a certain type of general insurance policy have a log-normal distribution, with mean 264 and standard deviation 346. A policyholder excess of 100 is a standard condition on each policy, so that the insurance company only covers the loss amount in excess of 100.

(a) Calculate the expected claim size payable by the insurance company.

(b) Next year, claims are expected to increase by 10%. Also, a new condition will be introduced on all policies so that the maximum amount that the insurance company will pay on any claim will be 1,000. The policyholder excess will remain unchanged at 100.

Calculate the expected claim size payable by the insurance company.

[14]

[Total 18]

9 An insurance company has insured a fleet of cars for the last four years. For year j ($j = 1, \ldots, 4$), let Y_j and P_j be the total amount claimed and the number of cars in the fleet, respectively. Let $X_j = Y_j/P_j$ be the average amount claimed per car in year j. Assume that the distribution of X_j depends on a risk parameter θ and that the conditions of Empirical Bayes Credibility Theory Model 2 are satisfied. Let

$$m(\theta) = E(X_j \mid \theta),$$

$$s^2(\theta) = P_j V(X_j \mid \theta),$$

$$m = E(m(\theta))$$

and $c = V(m(\theta)) > 0$.

(i) (a) Derive $E(X_j)$.

(b) Derive $E(X_j X_k)$, for $j \neq k$.

(c) Determine whether X_j and X_k are independent ($j \neq k$). [3]

(ii) The credibility premium per car is obtained by minimising

$$E\left(\left(m(\theta) - a_0 - \sum_{j=1}^{4} a_j X_j\right)^2\right).$$

Derive the credibility premium per car, and state the credibility factor. [10]

(iii) The company has insured 10 similar fleets over the last four years. Using the data from these years, m, $E(s^2(\theta))$ and $V(m(\theta))$ are estimated to be 62.8, 106.32 and 5.8 respectively.

Calculate next year's credibility premium for a fleet of cars with claims over the last four years given below, if the fleet will have 16 cars next year.

	Year			
	1	*2*	*3*	*4*
Total amount claimed	1,000	1,200	1,500	1,400
Number of cars	15	16	18	15

Explain how and why the credibility factor would be affected if the estimate of V($m(\theta)$) increases, and comment on the effect on the credibility premium. [5]

[Total 18]