EXAMINATIONS

29 April 2004 (am)

Subject 106 — Actuarial Mathematics 2

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

1 The loss severity distribution for a portfolio of household insurance policies is assumed to be Pareto with parameters $\alpha = 3.5$, $\lambda = 1{,}000$.

Next year, losses are expected to increase by 5%, and the insurer has decided to introduce a policyholder excess of £100.

Calculate the probability that a loss next year is borne entirely by the policyholder. [3]

2 The loss function under a decision problem is given by:

	θ_1	θ_2	θ_3
d_1	120	97	131
d_2	132	74	89
d_3	117	141	37

(i) Determine the minimax solution to this problem. [2]

(ii) Given the probability distribution $p(\theta_1) = 0.3$, $p(\theta_2) = 0.3$, $p(\theta_3) = 0.4$, determine the Bayes criterion solution. [2]

[Total 4]

3 A risk consists of 5 policies. On each policy in one month there is exactly one claim with probability θ and there is negligible probability of more than one claim in one month. The prior distribution for θ is uniform on (0, 1). There are a total of 10 claims on this risk over a 12 month period.

(i) Derive the posterior distribution for θ. [2]

(ii) Determine the Bayesian estimate of θ under:

(a) quadratic loss
(b) all-or-nothing loss

[3]

[Total 5]

4 A portfolio consists of two types of policies. For type 1, the number of claims in a year has a Poisson distribution with mean 1.5 and the claim sizes are exponentially distributed with mean 5. For type 2, the number of claims in a year has a Poisson distribution with mean 2 and the claim sizes are exponentially distributed with mean 4. Let S be the total amount claimed on the whole portfolio in one year. All policies are assumed to be independent.

(i) Determine the mean and variance of S. [2]

(ii) Derive the moment generating function of S and show that S has a compound Poisson distribution. [4]

[Total 6]

5 Claims arrive in a Poisson process rate λ and they are exponentially distributed with mean μ. The premium loading factor is θ.

(i) Derive the formula for the adjustment coefficient, and state an upper bound on the probability of ruin if the initial capital is u. [4]

(ii) Suppose $\mu = 1$.

(a) Determine a value of θ such that the probability of ruin is at most 0.01 when the initial capital is 20.

(b) State how this value of θ changes if the initial capital is increased. [2]

[Total 6]

6 The reserving department of a general insurance company has obtained the following incremental claims data (in £'000s). You may assume that all claims are paid at the end of the year.

	Development Year			
Accident Year	*0*	*1*	*2*	*3*
2000	210	95	40	10
2001	225	105	45	
2002	215	95		
2003	220			

Underlying claims inflation rates over the twelve months to the middle of each year were as follows:

2001	3.0%
2002	2.5%
2003	2.5%

Claims inflation from the middle of 2003 onwards is assumed to be 3.0% per annum.

(i) Calculate the outstanding claims reserve at 31 December 2003 using the inflation adjusted chain ladder, without adjusting forecast claims for inflation. [6]

(ii) State the assumptions made. [2]
[Total 8]

7 The total amount claimed for a particular risk in a portfolio is observed for each of 5 consecutive years.

(i) From past knowledge of similar portfolios, an insurer believes that the claims are normally distributed with mean θ and variance 25, and that the prior distribution of θ is normal with mean 125 and variance 36.

(a) Derive the Bayesian estimate for θ under quadratic loss, and show that it can be written in the form of a credibility estimate combining the mean observed claim size for this risk with the prior mean for θ.

(b) State the credibility factor, and calculate the credibility premium if the mean claim size over the 5 years is 122.

(c) Comment on how the credibility factor and the credibility estimate change if the variance of 25 is increased.

[6]

(ii) A second insurer does not believe that this is an appropriate prior distribution for risks in this portfolio, and decides to use Empirical Bayes Credibility, Model 1, where the credibility premium combines the mean for the particular risk with the estimated value of $E(m(\theta))$. Data from 3 risks in this portfolio over 5 years are available. Let X_{ij} be the claim for risk i in year j. The table shows various summary statistics for the observed data.

	$\bar{X}_i$	$\sum_{j=1}^{5}(X_{ij} - \bar{X}_i)^2$
Risk 1 (i = 1)	122	2,848
Risk 2 (i = 2)	164	1,628
Risk 3 (i = 3)	106	1,887

(a) Calculate the estimated credibility factor, and calculate the credibility premium for risk 1.

(b) Compare your answer to that obtained in (i)(b).

[6]

[Total 12]

8 (i) Show that

$$\int_0^d x^m f(x)dx = e^{m\mu + ½ m^2\sigma^2}\,\Phi\left(\frac{\log d - \mu - m\sigma^2}{\sigma}\right)$$

where $f(x) = \dfrac{1}{x}\dfrac{1}{\sigma\sqrt{2\pi}}\exp\left\{-½\left(\dfrac{\log x - \mu}{\sigma}\right)^2\right\}$ [4]

(ii) The loss amount, X, from a portfolio of non-life insurance policies is assumed to be independently distributed with mean £800 and standard deviation £1,200.

Calculate the values of the parameters of a lognormal distribution with this mean and standard deviation. [3]

(iii) The company is considering purchasing reinsurance cover, and has to decide whether to purchase excess-of-loss or proportional reinsurance.

The amounts paid by the direct insurer and reinsurer respectively, are given by

$$X_I^{(\text{Prop})} = (1-k)\,X$$
$$X_R^{(\text{Prop})} = kX$$

and
$$X_I^{(XL)} = \min\{X, d\}$$
$$X_R^{(XL)} = \max\{0, X-d\}$$

where X denotes the loss amount.

Using the loss distribution from (ii), calculate the value of k such that

$$E[X_I^{(\text{Prop})}] = 0.7E[X]$$

and show that if $d = 1189.4$, $E[X_I^{(XL)}] = 0.7E[X]$ [4]

(iv) Using the values of k and d from (iii), calculate the values of $\text{Var}[X_I^{(\text{Prop})}]$ and $\text{Var}[X_I^{(XL)}]$. [4]

(v) Comment on the results in (iii) and (iv). [2]
[Total 17]

9 Let Y_{ij} be the number of accidents on a particular motorway in the jth quarter of year i, $i = 1, 2, 3$, $j = 1, \ldots, 4$. Suppose that Y_{ij} has a Poisson distribution with mean μ_{ij}.

(i) (a) Derive the log-likelihood function as a function of μ_{ij} and determine the maximum likelihood estimate of μ_{ij}.

(b) If $\log(\mu_{ij}) = \mu$, determine the maximum likelihood estimate of μ.

(c) Define the scaled deviance, and derive an expression for the scaled deviance for the model in (i)(b).

[8]

(ii) Three models are shown below.

		Deviance	*Degrees of Freedom*
Model 1	$\log(\mu_{ij}) = \mu$	266.35	11
Model 2	$\log(\mu_{ij}) = \alpha_i$	202.19	9
Model 3	$\log(\mu_{ij}) = \alpha_i + \beta_j$	10.68	6

(a) Interpret each of these models.

(b) Determine which model you would recommend, giving your reasons.

[7]

(iii) It is found that the model $\log(\mu_{ij}) = \alpha \times i + \beta_j$ provides a reasonable fit to the data, with the estimate of α given as 0.34.

Interpret this model. [2]

[Total 17]

10 A marine insurer offers policies to boat owners to protect against collision damage. Cover is provided to coincide with the calendar year. For each policyholder the probability of having an accident during the year is 0.2. The policy meets the cost of repairs which, regardless of the timing of the accident, are always carried out at the end of the year. The insurer operates a no claims discount system. In the event of a claim free year, the policyholder moves to the next higher level of discount. In the event of a claim during the year, the policyholder moves in the next year to the next lower level of discount, unless the claim was as a result of drunken behaviour. In this case, the policyholder moves in the next year to the lowest level of discount.

The discount levels are 0%, 20% and 50%. 25% of claims are due to drunken behaviour.

The insurer charges a premium of £2,500 for policyholders at the 0% discount level.

A policyholder makes a claim following an accident if the cost of repairs is greater than the additional premiums payable at the next two renewals, assuming no further claims are made.

(i) Calculate the cost of a repair below which the policyholder will not claim, for each level of discount. [7]

(ii) Assuming that the cost of each repair follows a lognormal distribution with parameters $\mu = 6.5$ and $\sigma^2 = 3.5$, calculate the probability that a policyholder makes a claim in the event of an accident, at each level of discount. [6]

(iii) Write down the transition matrix. [5]

(iv) Derive the proportion of policyholders at each level of discount, once a steady state has been reached. [4]

[Total 22]

END OF PAPER