EXAMINATIONS

12 September 2003 (am)

Subject 106 — Actuarial Mathematics 2

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

1 When rating general insurance business, such as motor insurance, state the main issues that have to be considered to convert a pure risk premium into an office premium. [4]

2 A portfolio consists of a total of 120 independent risks. On each risk, no more than one event can occur each year, and the probability of an event occurring is 0.02. When such an event does occur, the number of claims, N, has the following distribution.

$$P(N = x) = 0.4 \times 0.6^{x-1} \qquad (x = 1, 2, \ldots)$$

Determine the mean and variance of the distribution of the number of claims which arise from this portfolio in a year. [4]

3 Claims occur in a Poisson process rate 20. Individual claims are independent random variables with density

$$f(x) = \frac{3}{(1+x)^4}, \qquad x > 0,$$

independent of the arrivals process.

(i) Calculate the mean and variance of the total amount claimed by time $t = 2$. [3]

(ii) Using a normal approximation, derive approximately the probability of ruin at $t = 2$ if the premium loading factor is 30% and the initial surplus is $u = 10$. [2]
[Total 5]

4 The claims process is a compound Poisson process with rate λ, and individual claim amounts have a gamma distribution with mean $2\alpha^{-1}$ and variance $2\alpha^{-2}$. A premium loading factor of 50% is used.

(i) Determine the adjustment coefficient. [4]

(ii) (a) Derive an upper bound on the probability of ruin if the initial surplus is u.

(b) State how this upper bound depends on α, and explain this dependence using general reasoning. [3]
[Total 7]

5 (i) The distribution of claims on a portfolio of general insurance policies is a Weibull distribution, with density function $f_1(x)$ where

$$f_1(x) = 2cxe^{-cx^2} \qquad (x > 0)$$

It is expected that one claim out of every 100 will exceed £1,000. Use this information to estimate c. [2]

(ii) An alternative suggestion is that the density function is $f_2(x)$, where

$$f_2(x) = \lambda e^{-\lambda x} \qquad (x > 0)$$

Use the same information as in part (i) to estimate λ. [2]

(iii) (a) For each of $f_1(x)$ and $f_2(x)$ calculate the value of M such that

$$P(X > M) = 0.001$$

(b) Comment on these results. [3]

[Total 7]

6 Cumulative claims incurred on a motor insurance account are as follows:

Figures in £000's

	Development year			
Policy year	*0*	*1*	*2*	*3*
1998	1,417	1,923	2,101	2,209
1999	1,701	2,140	2,840	
2000	1,582	1,740		
2001	2,014			

The data have already been adjusted for inflation. Annual premiums written in 2001 were £3,912,000 and the ultimate loss ratio has been estimated as 92%. Claims paid to date for policy year 2001 are £561,000, and claims are assumed to be fully run-off by the end of development year 3.

Estimate the outstanding claims to be paid arising from policies written in 2001 **only**, using the Bornhuetter-Ferguson technique. [9]

7 In a portfolio of property insurance policies, let θ denote the proportion of policies on which claims are made in the year. The value of θ is unknown and is assumed to have a Beta prior distribution with parameters α and β. A claims analyst estimates that the mean and standard deviation of θ are 0.20 and 0.25 respectively.

From a random sample of 50 policies, a claim is made on 24% of them during the year.

(i) Determine the values of the parameters, α and β, of the prior distribution. [3]

(ii) Determine the posterior distribution and hence the posterior mean of θ. [2]

(iii) For the general case where x is the number of claims arising from a sample size n and μ is the mean of the Beta prior distribution, show that the posterior mean of θ can be expressed as:

$$Z.(x / n) + (1 - Z).\mu$$

and express Z as a function of α, β and n. [3]

(iv) (a) Calculate the value of Z for the situation in part (ii) and explain what it represents.

(b) Without performing any further calculations, explain how you would expect the value of Z to change if:

(1) The analyst now believes the standard deviation, σ, of the prior distribution to be 0.50.

(2) The sample size, n, was 400.

(c) State the limiting value of Z as σ and n increase and explain what this means. [4]

[Total 12]

8 There are m male drivers in each of three age groups, and data on the number of claims made during the last year are available. Assume that the numbers of claims are independent Poisson random variables. If Y_{ij} is the number of claims for the jth male driver in group i ($i = 1, 2, 3; j = 1, \ldots, m$), let $E(Y_{ij}) = \mu_{ij}$, and suppose $\log(\mu_{ij}) = \alpha_i$.

(i) Show that this is a generalised linear model, identifying the link function and the linear predictor. [3]

(ii) Determine the log-likelihood, and the maximum-likelihood estimators of α_1, α_2 and α_3. [4]

For a particular data set with 20 observations in each group, several models are fitted, with deviances as shown:

		Deviance
Model 1	$\log(\mu_{ij}) = \alpha_i$	60.40
Model 2	$\log(\mu_{ij}) = \begin{cases} \alpha & \text{if } i = 1,2 \\ \beta & \text{if } i = 3 \end{cases}$	61.64
Model 3	$\log(\mu_{ij}) = \alpha$	72.53

(iii) Determine whether or not model 2 is a significant improvement over model 3, and whether or not model 1 is a significant improvement over model 2. [5]

(iv) Interpret these three models. [3]

[Total 15]

9 An insurance company, C, models the loss amount for each claim from its portfolio of policies as having a mean and standard deviation of £500. A claims analyst assumes that the loss amount follows an exponential distribution. There are 200 policies in the portfolio and C expects 30% of these policies to make a claim each year.

C is considering a reinsurance policy under which a reinsurer will pay the excess, if any, over £2,500 for each individual claim made in the portfolio. A reinsurance company, R, has offered to provide cover for the next year for an annual premium of £300. C has asked you for your advice on whether or not to accept this offer.

(i) Calculate the expected total claim amount ceded to R over the whole portfolio [5]

(ii) Comment on your answer to (i) and indicate the main points you would make in advising C on whether to reinsure with R. [3]

A second claims analyst suggests that the claims follow a lognormal distribution.

(iii) Calculate the expected total claim amount ceded to R under the second analyst's assumption.

(Note that for the lognormal distribution:

$$\int_M^\infty xf(x)\cdot dx = e^{\mu+½\sigma^2}\left[1-\Phi\left[\frac{\ln M-\mu-\sigma^2}{\sigma}\right]\right]$$

where Φ is the cumulative distribution function of the standard Normal distribution.) [6]

(iv) Comment on your answer to (iii). [4]

[Total 18]

10 An insurance company has arranged a policy with a large organisation to cover a particular type of disastrous event. Under the terms of the policy the insurer will cover the total cost of the claim for up to three disasters in any year. The insurer is considering taking out reinsurance for the policy whereby the reinsurer will cover the excess, if any, of the total cost over £2 million per disaster. The reinsurer has quoted the premiums for two different types of cover as follows:

	Cover provided by reinsurer	**Premium**
D_1	claims over £2m on the first disaster only	£500,000
D_2	claims over £2m on each of the first two disasters	£1,000,000

The insurer needs to decide whether to opt for cover levels D_1, D_2 or not to reinsure (D_0).

The insurer models the total loss from a single disaster as having a Pareto distribution with mean £1.5 million and standard deviation £2.5 million.

(i) Determine the expected total loss per individual disaster, with and without reinsurance. [7]

(ii) Complete the decision matrix below based on total outgoings (you may ignore expenses):

	no disasters Θ_0	*1 disaster* Θ_1	*2 disasters* Θ_2	*3 or more disasters* Θ_3
D_0				
D_1				
D_2				

[6]

(iii) (a) Determine the minimax solution.

(b) Explain your answer using general reasoning. [2]

(iv) The insurer believes that the number of disasters each year follows a Poisson distribution, parameter 0.9. Determine the Bayes Criterion solution. [4]

[Total 19]