

# **INSTITUTE AND FACULTY OF ACTUARIES**

## **AUDIT TRAIL**

February 2016

**CA2: Model Documentation, Analysis and Reporting**

**Paper 1**

## Mortgage and household budget model

### Objective

In the country where we live there are two interest rate approaches which can be selected between when taking out a mortgage, resulting in individuals having either a five-year fixed rate mortgage or a variable rate mortgage. Our client, Mr Rae, is considering purchasing a new house and will need a mortgage to do so. He would like some advice in this area. In addition, his wife, Mrs Rae, looks after their household budget and would like some advice regarding the affordability of house renovations.

The purpose of this spreadsheet is to complete the following calculations:

- Calculate the interest rate applicable to the mortgage repayments.
- Project the annual mortgage repayments required for the two types of mortgages on offer.
- Determine which mortgage Mr Rae would prefer based on his criteria.
- Calculate the Raes' annual household income and expenditure and therefore how much they can save each year.
- Project their savings to determine when they will have reached the level required to undertake renovation work.

*NB: Input cells are shown in blue.*

### Data

The following data has been provided by the Central Bank (CB):

- A forward curve of interest rate  $F$  – the interest rate applicable to fixed rate mortgage.
- A forward curve of interest rate  $V$  – the interest rate applicable to variable rate mortgages.
- Features of all mortgages:
  - They have a term of 25 years.
  - Mortgage payments are paid monthly in arrears.
- The current level of inflation (3% p.a.).
- Details on the features of fixed rate mortgages including renewal frequency (5 years) and renewal fees (\$500).

In addition, the following household budgeting information has been provided by Mrs Rae:

	<i>Annual amount (\$)</i>	<i>Comment</i>
Net income	30,000	Expected to increase in line with inflation
Inflation linked expenditure	7,250	Household expenditure such as utility bills Expected to increase in line with inflation
Income linked expenditure	25% of income	Household expenditure such as leisure activities Varies with income

The mortgage payments also need to be added to the expenditure and will depend on whether Mr Rae has chosen the fixed or variable rate. It has been stated that Mr Rae will choose the fixed rate if its average monthly cost is no more than \$50 above that of the average monthly cost of the variable rate mortgage. If not, he will choose the variable rate. (This is a one-time decision at the time of taking out the mortgage.)

- The desired household renovation work is anticipated to cost \$20,000.

## Raw data

**This worksheet performs a number of checks on the curve data.**

The raw data is validated as follows:

- Count of data provided. This should equal 25 for each column of data.
- Calculation of the minimum, maximum yield in each column.
- A line graph plotted of the two interest rate curves to spot anomalies.
- Check that the interest rates are all positive.
- Check that the curve is upward sloping.

These checks flagged the following data errors:

- The minimum interest rate  $F$  was negative. This was for year 18. It was also identified that this yield was an outlier in the graph.

The yield for year 18 in the interest rate  $F$  curve was updated to +3.02. (On the “Curves” sheet it can be seen that this value gives rise to a smooth curve for interest rate  $F$ ).

- The interest rate for year 14 in the interest rate  $V$  curve was identified as an outlier in the graph.

The yield for year 14 in the interest rate V curve was updated to be 2.34. This is the average of the yield figures in year 13 and 15, and a minimal change from the existing entry, suggesting the original data contained a typo. (On the “Curves” sheet, it can be seen that this value gives rise to a smooth curve for interest rate V).

All checks are re-applied on the corrected data (found in the “Curves” worksheet). No errors are indicated.

In addition, the data provided by Mrs Rae was checked by eye. The values she provided seem reasonable. There is no reason to doubt the accuracy of the household budgeting information, however this should be verified.

## **Assumptions**

- Other than identified errors, the data provided is correct.
- Current levels of inflation are a good guide to future inflation.
- The interest rate curves provided by CB are a good guide to future interest rates.
- The CB will not change the structure of the mortgages on offer in the next 25 years.
- Mr Rae will be able to purchase the property he would like for \$350,000.
- There are no additional costs when purchasing the property which might affect the level of mortgage Mr Rae requires.
- There are no additional outgoings which will affect the level of the Rae’s savings.
- Mr and Mrs Rae will remain in their jobs, earning a consistent level of income, not achieving any significant promotions to increase the income above annual inflation.
- Income tax rates will not change.
- The savings fund will have an opening balance of \$0.
- No interest will be earned on the savings fund.
- The cost of renovations will not change with inflation

## **Parameters sheet**

This sheet sets out all parameters used in the spreadsheet model, including:

- Mortgage variables – details of the mortgage required.

- Criteria – the difference between the monthly mortgage repayment amounts for the fixed and variable approaches which would result in Mr Rae selecting a fixed rate mortgage approach.
- Household budget variables – First year income and expenditure variables, taken from the household budgeting information provided by Mrs Rae, plus the level of inflation to be assumed.
- The target value of the savings fund.

Where named ranges have been defined these are detailed in red italic adjacent to the relevant cells.

## **Method**

### **Curves**

**In this worksheet the interest rates applicable to the mortgage repayments are calculated.**

The raw data is linked through from “Raw data” worksheet in columns B and E and the amendments identified above are made.

The interest rate required for the fixed rate mortgage is calculated as follows:

- Column C calculates  $1 + \text{interest rate F provided} / 100$
- Column D calculates the five year forwards as follows:

Take the product of  $(1 + \text{forward interest rate})$  over the next five years. Take the fifth root of this and deduct 1.

### *Checks*

The five-year average of the one-year forward rates is calculated and compared to the five year forward rates. A tolerance of 1% is allowed as a difference between the two before the results are flagged. All results are ok. The test only runs as far as year 20 as we do not have the one-year forward rates beyond year 25 therefore the five year forward rates beyond 20 years cannot be calculated.

## **Mortgage calcs**

**The aim of the worksheet is to calculate the annual mortgage repayment amounts under the two mortgage options.**

### **Fixed rate mortgage**

Under the fixed rate mortgage option the repayment amounts are fixed every five years. This term is included in the parameters worksheet.

In row 5, the five-year forward rates are referenced from the curves worksheet for each year of the mortgage (which has a term of 25 years, as specified by CB). As the mortgage is renewed every 5 years, the rate applicable for the fixed term of the mortgage is picked up i.e. years 1–5 are fixed and therefore the year 1 yield is picked up and years 6–10 are fixed so the year 6 yield is picked up etc.

The mortgage payments are made monthly in arrears therefore the monthly compounded interest rates are required. The interest rates in the “curves” worksheet are annual compound rates. Therefore to calculate the monthly compounded rates the following formula was used in row 6:

$$((1 + \text{annually compounded rate})^{\frac{1}{12}} - 1) \times 12$$

Using the annually compounded and monthly compounded interest rates the annuity for the remaining term of the mortgage is calculated using the following formula, in row 7:

$$a_{25}^{(12)} = \frac{1 - v^{25}}{i^{(12)}}$$

Where  $v$  is calculated using the annually compounded interest rate and  $i^{(12)}$  is the monthly compounded rate.

The fees applicable each year are calculated in row 8. The first year has a fee of \$500 as specified in the parameters worksheet. For subsequent years it is assumed that the fee increases in line with inflation so each year the fee is increased by 3% (the rate of inflation is specified in the parameters worksheet). Inflation is compounded year on year. The fee is only applicable in years in which the mortgage is renewed i.e. years 1, 6 etc. The fee is set to 0 in the intermediate years.

The outstanding mortgage at the beginning of the first year is the mortgage required plus the fee applicable to the first year. In subsequent years the outstanding mortgage at the beginning of the year is the outstanding mortgage at the end of the previous year plus any applicable fees.

The annual mortgage repayments are fixed for a five-year period. These are set at the beginning of each five-year period i.e. in years 1, 6 etc. In these years the payment to bank is calculated as the outstanding mortgage divided by the annuity. In the intermediate years the annual repayments are set equal to the previous year in order to keep them constant over the fixed term.

The outstanding mortgage at the end of the year (row 11) is calculated as:

$$\begin{aligned} & (\text{Outstanding Mortgage at the BOY}) \times (1 + \text{one year forward interest rate}) \\ & - (\text{payments to bank}) \times (1 + \text{one year forward interest rate})^{5.5/12*} \end{aligned}$$

BOY = beginning of year

\*The payments to the bank are assumed to be made in arrear. Therefore on average they are made 6.5 months through the year. Hence, on average, the interest on the mortgage will have been overstated to a value of 5.5 months interest on those payments.

Summary statistics have also been calculated (B15–B17) – maximum, minimum and average monthly repayment. I.e. they have been divided by 12 to find the monthly rather than annual amounts

### *Checks*

The monthly compounded interest rate has been compared to the annually compounded interest rate to check the monthly one is lower (monthly compounded rates should be lower as interest is earned on the interest received and therefore the interest rate needs to be lower to earn the same level of return). This is true at all terms.

### *Reasonableness checks*

Given the upward sloping shape of the interest rate F curve we would expect the annual mortgage repayments to increase over the term of the mortgage.

### **Variable mortgage**

Under the variable rate mortgage option the repayment amounts are updated every year.

The one year interest rates are referenced, in row 23, from the “Curves” sheet for each year of the mortgage (which also has a term of 25 years, as specified by CB).

In row 24, the monthly compounded interest rates are derived in a consistent manner to that used in the fixed mortgage calculations.

There are no fees for the variable mortgage.

The remaining rows are calculated in a consistent manner to the fixed rate mortgage.

Summary statistics have also been calculated (B32–B34) – maximum, minimum and average monthly repayment. i.e. they have been divided by 12 to find the monthly rather than annual amounts

### *Checks*

The monthly compounded interest rate has been compared to the annually compounded interest rate to check the monthly one is lower. This is true at all terms.

*Reasonableness checks*

Given the upward sloping shape of the interest rate  $V$  curve it is expected that the annual mortgage repayments would increase over the term of the mortgage.

Given that the interest rate  $F$  curve is lower than the interest rate  $V$  curve in the early years and higher in the later years, it is expected that the fixed rate mortgage would be cheaper than the variable rate mortgage in some of the early years.

It is expected that the 5-year fixed rate mortgage would be more costly on average as, in general, individuals are willing to pay more for the certainty offered by a fixed rate mortgage.

## **Mortgage graph**

**The worksheet summarises results and draws charts.**

This spreadsheet references the mortgage repayment amounts from each of the two mortgage worksheets and divides them by 12 to get the monthly repayment amounts.

The difference between the two mortgage options is found.

A line graph is drawn showing:

- Primary axis:
  - fixed mortgage monthly repayment
  - variable mortgage monthly repayment
- Secondary axis:
  - difference between mortgage options

The average difference between the mortgage options is found (fixed interest – variable rate). This can be found in cell B11.

*Reasonableness checks*

Given that the interest rate for the variable rate mortgage changes each year, and the mortgage payments are recalculated accordingly we would expect the variable rate payment line to be smooth.

Given the upward sloping shape of the interest rate  $V$  curve we would expect the variable mortgage repayments to also reflect an upward sloping curve.

As the fixed rate repayments remain constant for 5 years we would expect the fixed rate repayment line to show a step change every 5 years.

Given the upward sloping shape of the interest rate  $F$  curve we would expect the fixed mortgage repayments to also trend upwards.



As we expect that the fixed rate mortgage would be cheaper than the variable rate mortgage in some of the early years, the difference between the two repayment streams should be negative in the early years and positive in the latter years.

## **Household budget**

### **Income**

The household income is equal to \$30,000 in the first year. Each of the following years it increases by inflation i.e. last year's income  $\times (1 + \text{inflation})$ .

### **Expenditure**

The expenses incurred are:

- Inflation linked:
  - Year 1 = \$7,250
  - Year 2 onwards = previous years amount  $\times (1 + \text{inflation})$
- Income linked:  $25\% \times \text{income}$ .
- Mortgage payments: If the average difference between the fixed rate monthly payments and the variable rate monthly payments is less than \$50 the fixed rate annual repayment amount is pulled through, else the variable rate annual repayment is pulled through.

These elements are summed to give the total expenditure.

### **Balance**

The balance is equal to income minus total expenditure.

### **Savings fund**

- Year 1 = year 1's balance
- Year 2 onwards = last year's savings fund + this year's balance

The year in which the renovations budget target is reached is found in cell B20 by looking for the entry of the savings fund which is first over the \$20,000 budget.

### *Checks*

All income and expenditure elements should increase year on year. This is the case.

### *Reasonableness checks*

Expect the addition to the savings fund to increase year on year, which is the case.

## **Budget graph**

**The worksheet summarises results and draws charts.**

This spreadsheet brings through the savings fund amounts from each of the two budget worksheets.

A line graph is drawn showing:

- household budget savings fund.
- renovation savings target (\$20,000).

### *Reasonableness checks*

Expect the savings fund line to show a smooth upward sloping line, which is the case.

Expect the savings fund line to cross above \$20,000 just before year 7 when the fund exceeds the renovation target.

**END OF AUDIT TRAIL**