INSTITUTE AND FACULTY OF ACTUARIES

AUDIT TRAIL

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CA2: Model Documentation, Analysis and Reporting

Paper 1

Investment modelling

Objective

The purpose of this work is to model the fund values of Life Co's new investment product, to understand the returns that a typical investor can achieve with this product. The spreadsheet calculates the range of annual returns and final returns that a typical investor can achieve with this product. The spreadsheet includes

- Checks on the data and corrections where necessary.
- Simulations of 100 scenarios of a typical investment in the ASE over a term of 20 years.
- Calculations of key statistics on the maturity value and on the annualised return of the investment.
- A model of the impact of a higher assumed diffusion coefficient.

NB: Input cells are shown in blue.

Data

This worksheet contains the data that has been provided.

The following data has been provided by Life Co's Economic Scenario Generator Provider:

- the values of 100 simulations of a discrete time series at annual time periods from $t = 0$ until $t = 20$

This data has been copied into the Data worksheet.

Parameters

This worksheet contains the parameters used in the model.

Life Co has also provided the following information which is set out on this sheet:

- the term of investment
- the average amount invested in such investment policies at outset
- the initial charge that is applied to the amount invested
- the drift coefficient (μ) and diffusion coefficient (σ) as calculated from the performance of the ASE 100 over the last 5 years
- a higher diffusion coefficient reflecting the view that future market volatility may be higher

Data checks

This worksheet performs a number of checks on the discrete time series simulations.

In columns A to B we determine if the discrete time series process is 0 at $t = 0$ for every scenario. This is tested by calculating the sum of the square of B(0) across all scenarios and ensuring it is 0.

We are also informed that increments of the process should be normally distributed, namely $B(t+1) - B(t) \sim N(0,1)$.

In columns G to AA we analyse the increments of the discrete time series. At each time period, from $t = 0$ to $t = 19$, we calculate the increment $B(t+1) - B(t)$.

In columns AC to AD we perform the following checks on the increments:

- Count the number of entries – we expect 2,000 as there are 100 simulations each with a term equal to 20.
- The max and min values are determined. These are reasonable for a process that should be $N(0,1)$.
- The mean is calculated and is expected to be close to 0.
- The standard deviation is calculated and is expected to be close to 1.

Below this a frequency table has been produced which uses a bucket size of 0.25 and counts the number of observations in each bucket. A check is performed in cell AE54 to ensure all observations have been counted.

Next to this the number of observations that would be expected in each bucket based on an $N(0,1)$ distribution is determined. This uses the normal cumulative distribution function from Excel.

A histogram of the frequency table together with the corresponding $N(0,1)$ distribution have been plotted. It can be seen that the simulated increments have the same shape as the $N(0,1)$ distribution.

No issues have been identified with the data, so no amendments have been made.

Assumptions

- The simulations have been calculated correctly and do not contain any errors.
- The process S_t is an appropriate model for modelling the performance of the ASE 100 over the next 20 years.

- The provided values for the drift coefficient and diffusion coefficient are appropriate for modelling the ASE over the next 20 years (i.e. the next 20 years are expected to be consistent with the last 5 years).

- It is assumed that it is appropriate to use constant drift and volatility coefficients throughout the 20 year projection period.

- It is assumed that it is appropriate to treat each increment of the index as being independent, e.g. that market corrections can be ignored.

- Nothing will happen between the date of modelling and the launch date of 2018 to invalidate the choice of model or assumed parameters.

- 100 simulations is a sufficiently large number of simulations for analysis of the potential maturity values.

- The average amount invested in Life Co's product will equal the current average amount invested in such policies – no allowance is made for inflation between now and the launch date of 2018.

- It is assumed the initial charge is applied immediately so that the amount invested in the ASE 100 at $t = 0$ is the invested amount net of the initial charge.

- The mortality of the policyholder is ignored.

- Any potential withdrawals are ignored.

- Any potential top-ups are ignored.

StockPrice – Base

In this worksheet an investment in the ASE 100 is simulated using a diffusion coefficient of 7%.

The simulations are performed in columns A to V. Each row represents a scenario and each column represents a point in time.

The value of the investment at time t for scenario i is calculated using:

$$S_{t,i} = S_0 \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma B_{t,i}\right]$$

where:

- S_0 is the initial amount invested in the ASE 100 index after initial charges have been applied.

- μ and σ are the assumed drift and diffusion coefficients which are read in from the Parameters sheet.

- $B_{t,i}$ is the value of the i^{th} simulation of the discrete time series process at time t. This is read in directly from the Data worksheet.

The above calculation is performed for each time period from $t = 0$ to $t = 20$ and for each simulation from $i = 0$ to $i = 100$.

In column X the RANK function in Excel is used to determine the rank, in ascending order, of each simulated projection based on the maturity value at $t = 20$.

In columns Z to AA the following statistics are calculated:

- The average maturity value of the simulated returns using the AVERAGE function on the values in column V.

- The standard deviation of the maturity value of the simulated returns using the STDEV function on the values in column V.

- The minimum and maximum maturity value using the MIN and MAX functions on the values in column V.

The accuracy of the simulation is checked by determining the expected maturity value and standard deviation of the maturity value according to a log normal process, namely:

$$E[S_{20}] = S_0 \exp[20\mu]$$
$$\text{StDev}[S_{20}] = E[S_{20}]\sqrt{\left\{\exp\left[20\sigma^2\right]-1\right\}}$$

It is observed that the average maturity values from the simulations is close to the mean of the corresponding log normal process.

Similarly the standard deviation of the maturity value from the simulations is close to the standard deviation of the corresponding log normal process. There is a bigger difference between the standard deviations, but this is due to the low number of simulations.

For the average, max and minimum maturity values the corresponding annualised return is determined by dividing the projected value by the Gross Initial Investment (i.e. the initial investment before deducting the initial charge of 10%) and converting to an annual return. Namely,

$$\left(\frac{\text{Projected fund value at } t = n}{\text{Gross Initial Investment}}\right)^{1/n} - 1$$

where n is the term of the investment.

The following reasonableness checks are performed:

- The maximum annual return is greater than the average annual return.

- The minimum annual return is less than the average annual return.

- The minimum maturity value is non-negative. This is reasonable as the investment cannot become negative.

- The minimum annual return is negative. This is reasonable as in the worst case scenario it would be expected that the investor loses money.

StockPrice – High Vol

In this worksheet the investment in the ASE 100 is simulated using an assumed diffusion coefficient of 9%, with all other factors remaining unchanged.

The simulations are performed in a similar way to the previous worksheet, but this time the diffusion coefficient has been updated to $\tilde{\sigma}$ in the formula:

$$S_{t,i} = S_0 exp\left[\left(\mu - \frac{\tilde{\sigma}^2}{2}\right)t + \tilde{\sigma}B_{t,i}\right]$$

This formula references the higher assumed value on the Parameters sheet.

The simulated values are in columns A to V.

In columns X to Y the simulations are again ranked and we confirm that the ranking has not changed from the Base case. (Increasing the diffusion coefficient should not change the rank of any scenario).

In columns AA to AD the same statistics as for the base case are calculated. As with the base case the mean and standard deviations from the simulations are close to the mean and standard deviation of the corresponding log normal process.

In column AH the results from the larger diffusion coefficient case are compared to the base case.

The following reasonableness checks are performed:

- It is reasonable that the expected maturity value should be unchanged as the drift coefficient is unchanged.

- It is reasonable that the standard deviation will be higher because the diffusion coefficient, which impacts volatility, has been increased.

- As the process is more volatile it is reasonable that the maximum simulated maturity value will be greater than the base case and the minimum simulated maturity value will be smaller than the base case.

As a final check the diffusion coefficient is set to the original value and it is observed that all statistics match the base case.

Charts

In this worksheet key results are illustrated.

At the top of the sheet sample paths from the base case for the simulations ranked 25th, 50th and 75th are extracted. These are plotted on a chart.

As expected the sample path ranked 75th sits above the path ranked 50th which in turn sits above the path ranked 25th

In the middle of the sheet sample paths from the higher diffusion coefficient case for the simulations ranked 25th, 50th and 75th are extracted. These are plotted on a chart.

It is noted that the sample paths for the higher diffusion coefficient case are more spread out than the base case. This is expected as the increased volatility should lead to the sample paths being more dispersed.

Furthermore the path ranked 50th is broadly similar in the base and higher diffusion case. This is expected as the mean is the same.

At the bottom of the sheet the average, minimum and maximum annual return for the base and larger diffusion coefficient case are plotted. A bar chart comparing the different cases is produced. As expected:

- The average return in both cases is very similar.
- The maximum return is higher in the larger diffusion coefficient case.
- The minimum return is lower in the larger diffusion coefficient case.

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