INSTITUTE AND FACULTY OF ACTUARIES

22 September 2016

CA2: Model Documentation, Analysis and Reporting

Paper 2

Time allowed: 3 hours + 15 minutes reading time

INSTRUCTIONS TO THE CANDIDATE

1. You have 15 minutes reading time at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have 3 hours to complete the paper.

2. You must build your model from the beginning and not use an imported e-template.

Your file names must include your ARN, the name of the document and the paper sat (e.g. 9000000-Summary-Paper1) and each file should contain your ARN as a header or footer.

Please note that the content of this booklet is confidential and students are not to discuss or reveal the contents under any circumstances nor are they to be used in a further attempt at the exam.

If you encounter any issues during the examination please contact the Online Education team at online_exams@actuaries.org.uk T. 0044 (0) 1865 268 255

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Exam requirements

1. Read the background document, which describes the scenarios that have been modelled and documented for this project.

2. Read the audit trail which has been written by your colleague, another actuarial student, for the calculations that they performed. This will assist you in following and understanding the calculations performed in the Excel model provided.

You are not required to add to or amend the audit trail.

You can assume that your colleague's calculations have been checked and are correct.

3. Illustrate the following using four suitable charts, based on the model provided:

- a comparison of the monthly payments over time on both mortgage products
- a comparison of the outstanding loan over time on both mortgage products
- the split of each mortgage payment between interest and capital repayment over time, separately for each mortgage product **[8]**

4. Build on the model provided to determine the maximum level of annual expense inflation which can be experienced before the internal rate of return on the investment drops below 8% per annum. This is only required for the mortgage product offering the better rate of return under the existing model. **[5]**

5. Illustrate the net present values of the preferred mortgage product over time, with and without allowance for the maximum level of expense inflation, using an appropriate chart. **[2]**

[Sub-total 15]

6. Prepare a summary document of around five to seven pages, capturing the main features and results from the entire model. You can assume that the summary is being prepared for your boss, a senior actuary, who will present the work to the sign-off committee tomorrow.

Your summary should cover the following:

- purpose of the project, data, approach and assumptions used
- results, including charts
- commentary on results, key conclusions and suggested next steps

Marks available for the summary:

Methodology (including purpose, data, approach and assumptions)	**[20]**
Results, including charts	**[10]**
Commentary on results and key conclusions	**[20]**
Next steps	**[25]**
Drafting	**[10]**
	[Sub-total 85]
	[Total 100]

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Background

The investment company HouseCo is considering making an investment in residential property. You are an actuarial student working at HouseCo.

A suitable property has been identified which has a current selling price of $300,000. HouseCo wishes to purchase the property and hold the investment for approximately 20 years.

HouseCo intends to liquidate assets that will provide a cash lump sum of $40,000 towards the investment. The remainder of the cost of purchasing the property will be financed via a commercial mortgage.

Whilst owning the property, HouseCo intends to rent it out to a third party. HouseCo's market research into the local rental market suggests that a rent of $1,700 per month would be achievable. The research also shows that property rental has not increased during the previous 15 years.

During the rental period, HouseCo expects to incur general management and maintenance expenses on the property. These have been estimated at $400 per month. HouseCo believes that efficiencies can be achieved over time which will enable it to maintain the level of expenses at $400 per month over the full term of the mortgage.

At the end of the 20 years, HouseCo will sell the property. The National Statistics Office has recently published a report showing that house price inflation across the country has remained at 3% per annum for several years. Furthermore, the report shows that no change in the level of house price inflation is expected in the near future.

HouseCo is currently considering two different mortgage offers:

Mortgage Offer 1 **Term = 240 months**
Effective annual rate of interest = 5% (for the full term of the mortgage)

Mortgage Offer 2 **Term = 240 months**
Introductory effective annual rate of interest = 2% (for first 36 months)
then
Effective annual rate of interest = 7% (for the remaining 204 months of the mortgage)

Both mortgages are capital repayment mortgages. Where the mortgage is paid on a monthly basis, with each payment repaying the monthly interest costs on the outstanding balance of the loan, and part of the outstanding balance on the loan. The monthly repayment is calculated such that the loan is paid off after 240 months.

Under the first offer, the mortgage repayment will remain unchanged over the full term of the mortgage. Under the second offer, the mortgage repayment will change after 36 months. Namely, the mortgage payment during the first 36 months is calculated assuming that the interest rate will be 2% per annum throughout the life of the mortgage. After the first 36 months, the mortgage payment is recalculated based on the new interest rate of 7% per annum and the outstanding mortgage at that point in time.

Your boss, who is an actuary, wishes to know which mortgage offer will provide HouseCo with the best return on its investment. In particular, he wants to ensure that the investment meets HouseCo's target rate of return of 8% per annum on residential investments.

Your boss has already asked your colleague to produce the following:

- a loan schedule to illustrate the loan outstanding each month for both mortgage offers
- a monthly cashflow model to illustrate House Co's expected cashflows over the life of the property for each mortgage product
- determination of the net present value of the cashflows to and from HouseCo under each mortgage offer, at each projected time period, using a discount rate of 8% per annum

These calculations have been completed and your colleague has also produced a first draft of her audit trail, which is included later in this document. However, your colleague is currently at an actuarial conference and cannot be contacted.

In her absence, your boss needs you to perform some additional tasks.

He would first like you to illustrate the existing results using four appropriate charts:

- a comparison of the monthly payments over time on both mortgage products
- a comparison of the outstanding loan over time on both mortgage products
- the split of each mortgage payment between interest and capital repayment over time, for mortgage offer 1
- the split of each mortgage payment between interest and capital repayment over time, for mortgage offer 2

Secondly, he is concerned that it will be difficult to maintain expenses at a level of $400 per month, over the full term of the mortgage. He would therefore like you to determine the maximum annual rate of expense inflation that could occur before the rate of return drops below 8% per annum. As part of this additional modelling, he would like you to calculate the net present value of the revised cashflows, using a discount rate of 8% per annum, for each month of the projected time period. The additional calculations only need to be done for the mortgage offer that provides the better return under the existing model.

He would also like you to illustrate the net present values over time, using an appropriate chart which compares the scenario where expense inflation is ignored and the scenario where it is assumed to be at the maximum level determined in your model. Again, this only needs to be done for the mortgage offer that provides the better return.

Finally, he would like you to prepare a summary document, covering both the original work performed by your colleague and the additional work that you have been asked to complete.

You are not expected to include the additional calculations in the audit trail, but the results should be presented in the summary.

Additional guidance

Excel Charts – Changing the Interval between Tick Marks and Labels on an Axis

If there are too many tick marks or labels on an axis it is possible to alter the number shown. This can be done as follows:

1. Selecting the appropriate chart and right click the axis that you want to change.

2. Choose **Format Axis** from the popup menu.

3. In the **Format Axis** dialog box, under **Axis Options**, do one or more of the following:

    - To change the interval between tick marks, in the **Interval between tick marks** box, type the number that you want. The number that you type determines how many categories are displayed between the tick marks.

    - To change the interval between axis labels, under **Interval between labels**, click Specify interval unit, and then in the text box, type the number that you want.

END OF PAPER

Audit Trail

Residential property investment

Objective

The purpose of the spreadsheet is to complete the following calculations in relation to an investment in residential property:

- Calculate the monthly mortgage payment for two different mortgage products.
- Produce a loan schedule for each of the mortgage products.
- Project the expected cash flows over the life of the property for each of the mortgage products.
- Determine the net present value of cash flows for each mortgage product and hence determine whether the required rate of return is met and identify which mortgage product offers the better return.

NB: Input cells are shown in blue.

Assumptions

- Assume that the interest rate on the mortgage will remain fixed throughout the life of the mortgage (with the exception of the known change on Product 2).
- Assume that rental inflation remains at 0% per annum throughout the duration of the investment.
- Assume that monthly mortgage repayments occur at the end of each month.
- Assume that this particular property's value increases at 3% per annum over the term of the investment.
- Assume that the property's true value can be achieved in a sale.
- Any tax on rental income has been ignored.

"Parameters" worksheet

This worksheet contains all the parameters for the model.

The following parameters need to be input to column C of this worksheet: the initial investment by the company, the purchase price of the property, the term of the investment in years, the monthly rental income, the required internal rate of return (per annum), the level of house inflation (per annum), the monthly expenses, the annual interest rate for Product 1.

The parameters for Product 2 are also entered in column C of this worksheet: the introductory period, the introductory annual interest rate, the annual interest rate after the introductory period.

The names of each parameter are detailed in column D.

For simplicity the initial loan required is calculated on the Parameters sheet as:

- Initial Loan = Purchase Price – Initial Investment

"Loan schedules" worksheet

This worksheet determines the monthly mortgage payment for each product and produces a loan schedule.

Product 1 is considered in columns B to H.

Above the main table the following calculations are performed in order to determine the monthly mortgage payment:

- The nominal rate of interest payable 12 times each year is calculated as:

$$i^{(12)} = 12 \times \left((1+i)^{1/12} - 1 \right)$$

  where i is the effective annual rate of interest from the Parameter sheet.

- The corresponding discount factor is $v = 1/(1+i)$.

- The present value of making monthly payments in arrear at a rate of 1 per year is then given by the annuity factor

$$a_n^{(12)} = \frac{1 - v^n}{i^{(12)}}$$

  where n is the number of years of the mortgage.

- Hence the monthly payment required for the initial loan is calculated as:

$$\text{Monthly Payment} = \frac{\text{initial loan}}{12 \times a_n^{(12)}}$$

The loan schedule is calculated from row 9 onwards for $t = 1$ to $t = 240$ (time in months).

The corresponding year (column C) is calculated as:

$$\text{Integer Part of} \left[\frac{t-1}{12} \right] + 1$$

At time $t = 1$ the loan outstanding at the start of the month (column D) is the initial loan amount. At all other time periods it is the loan outstanding at the end of the preceding month (column H).

The Interest Due (column E) is calculated as Loan Outsanding at start of month $\times \frac{i^{(12)}}{12}$.

The Payment (column F) is simply the monthly payment that was calculated in cell C6.

The capital that is repaid (column G) on the outstanding loan is given by the monthly payment less the interest due.

Finally the loan outstanding at the end of the month (column H) is calculated as the loan outstanding at the start of the month less the capital repaid.

A check is performed in Cell G3 to ensure that the loan outstanding at the end of the month for $t = 240$ is zero. This ensures that the loan schedule and monthly annuity payment have been calculated correctly.

Product 2 is considered in columns J to P. A similar approach is used to Product 1 but there are some key differences.

Cells K3 to K6 work out the monthly mortgage payment assuming that the initial rate of interest is applicable throughout the entire length of the mortgage. These calculations are the same as for Product 1, but with a different interest rate.

In cells N2 to N6 the monthly payment required after the end of the initial period is determined as follows:

- The loan outstanding after the end of the introductory period is looked up from the loan schedule for Product 2.
- The nominal rate of interest payable 12 times each year is calculated using the effective rate of interest applicable after the end of the introductory period.
- The corresponding discount factor v is calculated.
- The present value of making monthly payments in arrear at a rate of 1 per year is then given by the annuity factor

$$a_{\tilde{n}}^{(12)} = \frac{1 - v^{\tilde{n}}}{i^{(12)}}$$

where $\tilde{n}$ is the number of years of the mortgage **less the number of years that the introductory rate applied for**.

- Hence the monthly mortgage payment required is:

$$\text{Monthly Payment} = \frac{\text{Loan Outstanding after Introductory Period}}{12 \times a_{\tilde{n}}^{(12)}}$$

The loan schedule for Product 2 is calculated from row 9 onwards for $t = 1$ to $t = 240$. This is the same as for Product 1 with the exception that:

- For $t \leq 36$ the interest rate and monthly payment applicable for the introductory period are used;

- For $t > 36$ the interest rate and monthly payment applicable after the introductory period has expired are used.

As with Product 1, a check is performed in Cell R3 to ensure that the loan outstanding at the end of the month for $t = 240$ is zero.

"Cashflows" worksheet

This worksheet projects the cash flows for the residential property investment for both Product 1 and Product 2. The net present value in both cases is determined.

For convenience the required annual rate of return is read in from the Parameters sheet to cell C2 and the corresponding monthly discount factor is calculated in cell C4.

Product 1 is considered in columns B to J.

The projection runs from $t = 1$ to $t = 240$ (column B).

The corresponding year (column C) is calculated as:

$$\text{Integer Part of} \left[\frac{t-1}{12} \right] + 1$$

The monthly rent (column D) and expenses (column F) are read in from the Parameters sheet and the monthly mortgage payment (column E) is read in from the Product 1 loan schedule on the Loan Schedules worksheet.

The initial investment (column G) contains the amount invested by the company at $t = 0$ and is read in from the Initial Investment parameters on the Parameters sheet. At all other times it is zero.

The property sale (column H) contains the amount received from selling the property at $t = 240$. At $t = 240$ it is the entire purchase price of the property inflated by house price inflation for the appropriate number of years:

$$\text{Purchase Price} \times \left(1 + \text{House Price Inflation}\right)^{t/12}$$

At all other times it is zero.

The net cash flow (column I) at the start of each month is given by:

$$\text{Rent} - \text{Initial Investment} - \text{Expenses} + (\text{Property Sale Proceeds} - \text{Mortgage Payment}) \times \text{Monthly Discount Factor}$$

where the property sale and mortgage payment are multiplied by the monthly discount factor because they are assumed to occur at the end of the month.

The net present value (column J) at the start of each month is calculated as the sum of the net cash flow at the start of that month plus the net present value at the start of the next month discounted by a month:

$$\text{NPV}(t) = \text{Net Cash Flow}(t) + \text{NPV}(t + 1) \times \text{Monthly Discount Factor}$$

Product 2 is considered in columns L to T. The calculation is identical to Product 1 with the exception that the mortgage payment (column O) is read in from the Product 2 loan schedule on the Loan Schedules worksheet.

END OF AUDIT TRAIL