INSTITUTE AND FACULTY OF ACTUARIES

SUMMARY

September 2016

CA2: Model Documentation, Analysis and Reporting

Paper 2

Residential property investment

Objective

The purpose of this project is to determine if HouseCo's required rate of return is met on a proposed residential property investment.

Part of the investment will be financed by a mortgage. Two different mortgage products are considered and a loan schedule is produced for each.

A cash flow model of the expected cash flows from the investment is produced so that it can be determined if the required rate of return is met. This is done for both mortgage products and the product which offers the best rate of return is identified.

For the option with the best rate of return the maximum level of expense inflation which can be experienced before the rate of return falls below the required level is also calculated.

Data

The following data has been provided by HouseCo:

- The selling price of the property HouseCo intends to purchase is $300,000.
- The term of the investment is 20 years.
- The local rental market suggest a rent of $1,700 per month.
- Expenses on the property are expected to be $400 per month.
- Details of the two mortgage products.

In addition the Office for National Statistics has provided a report that house price inflation is currently at 3% per annum and is not expected to change.

Assumptions

- Assume that the interest rate on the mortgage will remain fixed throughout the life of the mortgage (with the exception of the known change on Product 2).
- Assume that rental inflation remains at 0% per annum throughout the duration of the investment.
- Assume that monthly mortgage repayments occur at the end of each month.
- Assume that this particular property's value increases at 3% per annum over the term of the investment.
- Assume that the property's true value can be achieved in a sale.

- Any tax on rental income has been ignored.
- The data and parameters provided are all assumed to be correct.
- Assume that a tenant willing to pay $1,700 per month can be secured.
- Assume that all other monthly cash flows occur at the start of each month.
- Assume that the identified property will be available to purchase and that the price paid will be $300,000.
- Assume that tenants can be secured immediately and there will be no periods during the investment term when there is no tenant.
- Assume that the planned efficiencies are achieved and expenses remain at $400 per month throughout the investment.
- Assume that there are no exceptional expenses incurred over and above the $400 per month.
- Assume that no overpayments are made on the mortgage.
- Assume that no payment holidays are taken on the mortgage.
- Assume that proceeds from the property's sale will be received immediately at the end of the investment.
- Purchase costs such as stamp duty and solicitor fees have been ignored.
- Any costs associated with selling the property have been ignored.
- Any arrangement fees on the mortgage have been ignored.
- For the additional scenario it is assumed that expense inflation will be constant throughout the term of the investment.
- For the additional scenario it is assumed that expenses will inflate on the yearly anniversary of the investment.

Loan schedules

The monthly mortgage payment for each mortgage product is calculated using standard actuarial formulae.

For Product 1 the nominal rate of interest payable 12 times each year, $i^{(12)}$, is calculated and the corresponding discount factor v is calculated. The monthly payment is then calculated as:

$$\text{Monthly Payment} = \frac{\text{initial loan}}{12 \times a_n^{(12)}}$$

where for a term of n years

$$a_n^{(12)} = \frac{1 - v^n}{i^{(12)}}$$

is the present value of making monthly payments in arrear at a rate of 1 per year.

The loan schedule simply tracks the progress of the outstanding loan over time. At each time period we determine:

- The loan outstanding at the start of the month – this will either be the initial loan at outset or the loan outstanding at the end of the preceding month.
- The interest due, which is the product of the loan outstanding and the effective monthly interest rate.
- The mortgage payment made is the payment previously calculated.
- The amount of capital paid back on the loan is the difference between the mortgage payment made and the interest due.
- The loan outstanding at the end of the month is the loan outstanding at the start of the month less the capital repaid.

This is repeated for the term of the mortgage.

A similar approach is used for Product 2, but it is necessary to allow for the change in interest after the end of the introductory period. The mortgage payment during the introductory period is calculated using:

$$\text{Monthly Payment during Introductory Period} = \frac{\text{initial loan}}{12 \times a_n^{(12)}}$$

where $a_n^{(12)}$ is calculated using the interest rate applicable during the introductory period. Note that n here is the term of complete mortgage.

The mortgage payment after the introductory period is calculated using:

$$\text{Monthly Payment after Intro Period} = \frac{\text{Loan outstanding after Intro Period}}{12 \times a_{\tilde{n}}^{(12)}}$$

where $a_{\tilde{n}}^{(12)}$ is calculated using the interest rate applicable after the introductory period and $\tilde{n}$ is the term of the mortgage in years of the mortgage **less the number of years that the introductory rate applied for**.

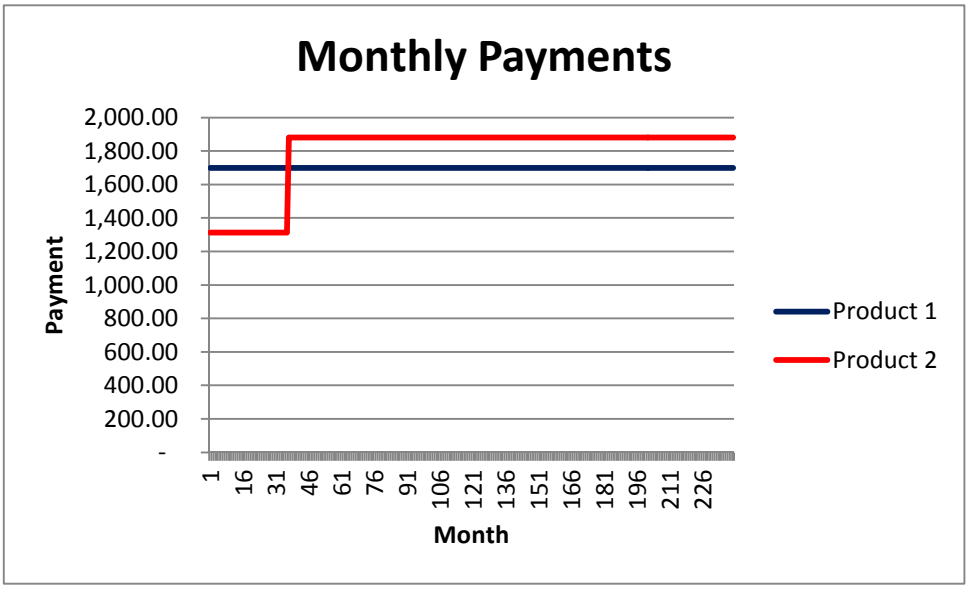
The loan schedule is similar to Product 1, but it is necessary to ensure the mortgage payment and interest rate change after the introductory period is complete.

Results

The monthly mortgage payment under Product 1 is $1699.97 per month.

The monthly mortgage payment under Product 2 is $1,313.07 during the introductory period and $1,880.11 once it has expired.

The chart below shows the monthly mortgage payments throughout the term of the mortgage on both mortgage products.

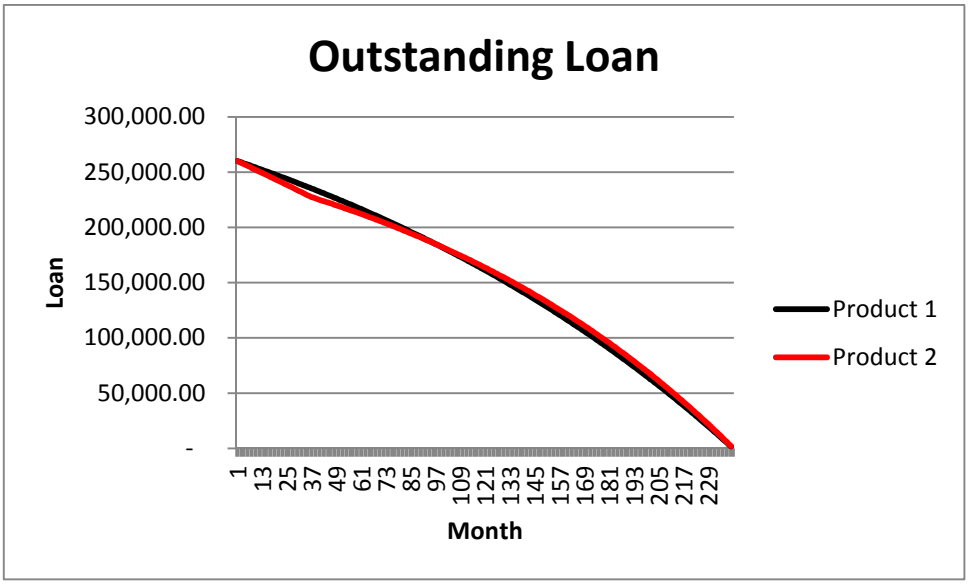


The following observations can be made:

- As expected the monthly payment on Product 1 remains the same throughout the life of the mortgage.
- It is observed that the monthly payment on Product 2 increases at the end of the introductory period.
- The introductory monthly payment on Product 2 is lower than on Product 1. This is expected as the interest rate applying is lower.

- The monthly payment on Product 2 after the end of the introductory period is higher than the monthly payment on Product 1. This is expected as the interest rate is higher on Product 2 (and the introductory period has not been long enough to reduce the outstanding loan significantly).

The following chart shows the outstanding loan at each point in time for both mortgage products:



The following observations can be made:

- The outstanding loan reduced over time for both Product 1 and Product 2. This is expected as the total mortgage payment amount always exceeds the interest accruing over a month.

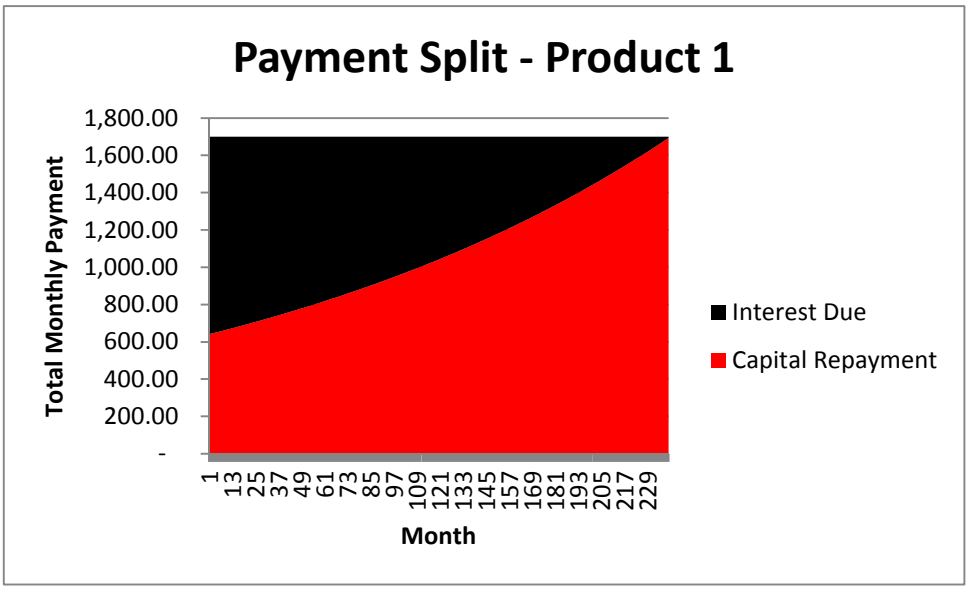
- At the end of the mortgage term the outstanding loan is zero for both mortgage products.

- There is an inflexion point on the curve for Product 2 at the end of the introductory period. This makes sense as the rate at which the loan is decreasing changes.

- During the introductory period the outstanding loan on Product 2 falls quicker than on Product 1. This is expected as the interest rate applicable is lower so it is expected that the outstanding loan on Product 1 will be higher.

- After the introductory period the rate at which the outstanding loan is decreasing on Product 2 falls below the rate on Product 1. Hence it is expected that the curves will cross and the outstanding loan on Product 1 falls below that of Product 2.

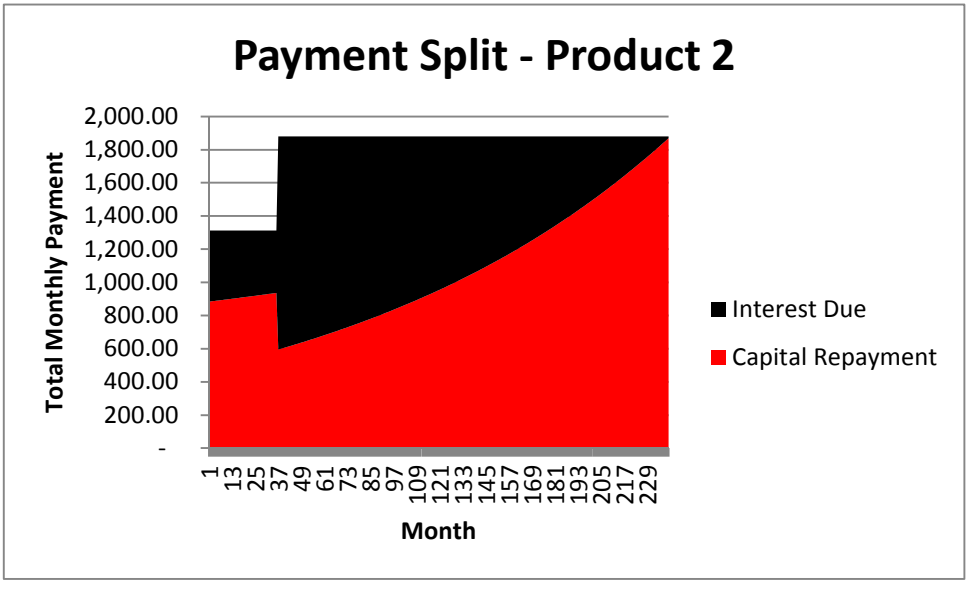
The following chart shows the split of the mortgage payment on Product 1 between interest and capital repayment.



The following observations can be made:

- The total level of payment remains fixed throughout the term of the mortgage.
- The amount of capital repaid with each monthly payment increases over time. This makes sense since as the outstanding loan falls the amount of interest due will be lower. Consequently an increasing proportion of the monthly payment will be available to repay capital.

The following chart shows the split of the mortgage payment on Product 1 between interest and capital repayment.



The following observations can be made:

- The total level of payment increases after the introductory period.

- In the introductory period the proportion of payment that is interest is much smaller than immediately after the end of the introductory period. This is expected because a lower interest rate applies during the introductory period.

- Similarly in the introductory period the proportion of payment that is used to repay capital is higher than immediately after the end of the introductory period. Again this is because a lower interest rate applies during the introductory period so that more of the monthly payment is available to repay capital.

Cash flow modelling

The expected cash flows from the residential investment are modelled under both mortgage products.

For each time period the following are read in to the model:

- The monthly rent – which is a given parameter.
- The mortgage payment - which has been calculated.
- The monthly expenses – which is a given parameter.

At $t = 1$ there is an additional cash flow item which is the initial amount that HouseCo will invest.

At $t = 240$ there is an additional cash flow item which is the amount that HouseCo will receive from selling the property in 20 years. This is calculated as the entire purchase price of the property inflated by house price inflation for the appropriate number of years.

The net cash-flow at the start of each time period is given by:

Monthly Rent – Monthly Expense – Initial Investment ($t = 1$ only)
+ [Property Sale Proceeds ($t = 240$ only) – Mortgage Payment] × Monthly Discount Factor

Note that the mortgage payment and sale of the property occur at the end of the month, so have been discounted to the start of the month in order to determine a net cash-flow effective at the start of the month.

The net present value of the cash flows is calculated using:

$$\text{NPV}(t) = \text{Net Cash Flow}(t) + \text{NPV}(t + 1) \times \text{Monthly Discount Factor}$$

where the discount factor used is HouseCo's required rate of return of 8% per annum.

Results

The net present values at $t = 0$ are:

Mortgage	NPV @ 8%
Product 1	$28,442
Product 2	$24, 622

It is observed that:

- The net present value on both Products is positive so the required rate of return is achieved with the investment.

- The net present value is higher with Product 1, so a higher rate of return is achieved if mortgage product 1 is selected.

Maximum level of expense inflation

A sensitivity test has been performed in order to determine the maximum level of expense inflation which could be experienced before the rate of return drops below the required rate. This is only performed for Product 1.

The cash flows for Product 1 have been reproduced, but now there is a parameter for annual expense inflation. Each year the level of monthly expenses is inflated by this percentage using:

$$\text{Monthly Expenses (in year } X) = \text{Initial Monthly Expenses} \times \left(1 + \text{ Expense Inflation}\right)^{X-1}$$

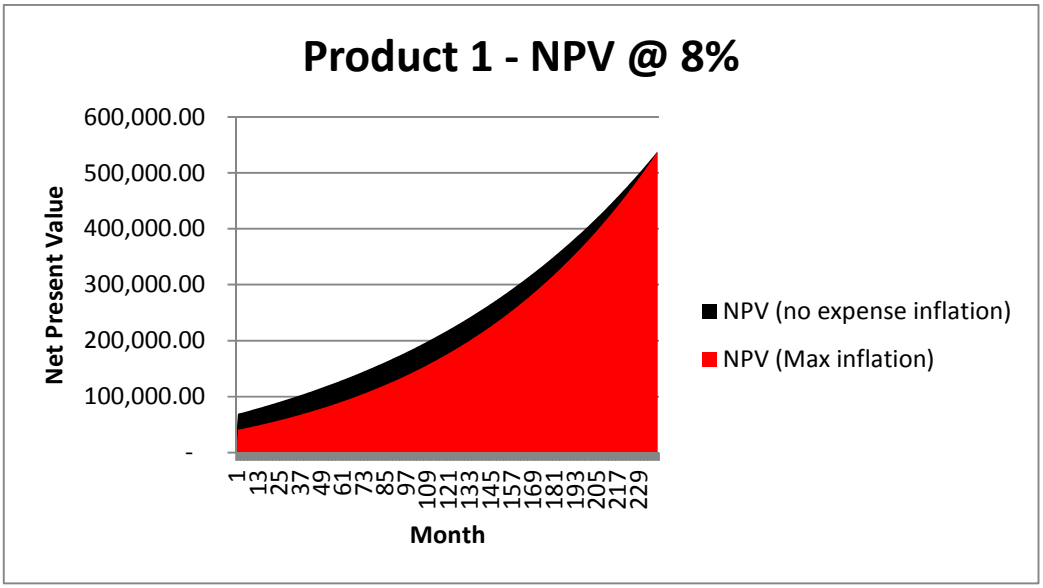
The parameter for expense inflation is varied until the net present value is 0. This is the maximum level of expense inflation that can be experienced. If it is higher then the net present value will be negative and the rate of return will have fallen below the required level.

Results

The maximum level of expense inflation is 5.93% per annum.

It is expected that this is greater than 0 because the scenario with zero expense inflation gave a return in excess of the required rate of return.

The chart below shows the net present value of the investment at each point in time for Product 1 with and without expense inflation:



It is expected that the net present value with no expense inflation will be greater than the net present value with expense inflation, since the present value of expenses in the latter case will be higher and expenses are a negative cashflow component in the net present value calculation.

It is observed that the two sets of NPVs are reasonably close. This is because expense inflation is a relatively small part of the overall cash flows, which are dominated by the rental and mortgage payments.

The NPVs converge towards the end of the projection because they are dominated by the net present value at $t = 240$ which is the sale price of the property and the same in both scenarios.

Conclusions

A rate of return in excess of 8% per annum is achieved on the residential property investment regardless of whether mortgage product 1 or product 2 is used.

The net present value using Product 1 is $28,442 and the net present value using Product 2 is $24,622.

It is recommended that Product 1 is chosen as it offers a better rate of return.

Under Product 1 monthly expenses can experience annual inflation up to 5.93% per annum before the rate of return drops below 8% per annum.

Next steps

- Validate that the parameters and information provided are correct.
- Allow for any change in the purchase price between now and the expected date of investment.
- Confirm that the assumed level of rental income can be obtained for the actual property being purchased.
- Confirm the timing of the expenses and rental income cash flows.
- Model the underlying interest rate stochastically in order to obtain a range of possible outcomes.
- Determine the actual rate of return for Product 1 and Product 2.
- Determine the level of expense inflation which can be experienced under Product 2 before the rate of return drops below 8%.
- Vary the level of HouseCo's initial investment to see if a higher rate of return can be obtained.
- Sensitivity test the level of monthly rental income to see how sensitive the rate of return is to this assumption.
- Allow for the possibility of voids – periods where there are no tenants – in the cash flow model.
- Confirm that the annual rate of house price inflation is applicable for the property being purchased.
- Obtain another source of data for house price inflation (e.g. a building society).
- Model future house price inflation stochastically so that a range of future sale prices can be obtained.
- Sensitivity test the level of house price inflation or the sale price that can be achieved on the property.
- Amend the model so that it can allow for variable interest rates over time.
- Amend the model so that it can allow for variable expense inflation rates over time.
- Allow for the impact of taxation in the cash flow projection.

- Allow for the impact of costs associated with the purchase and sale of a residential property.

- Consider some extreme scenario tests – e.g. large crash in the property market, imposition of rental controls, etc.

- Explore if there are alternative mortgage products available including variable interest products.

- Consider alternative sources of capital for the proposed investment.

- Update the model in the future to reflect actual experience to date and/or a change in mortgage interest rates.

- Model over a different time period / consider a different term mortgage.

- Obtain a peer review of the work performed.

[Note: some students assumed that expenses would inflate monthly, rather than at the end of each year. The examiners determined that this was a valid approach given the wording in the question and therefore credit was given to students who took this approach. The model_for_solution in worksheet 'Cashflows – Max Information' illustrates both approaches].

END OF SUMMARY