INSTITUTE AND FACULTY OF ACTUARIES

21 September 2016

CA2: Model Documentation, Analysis and Reporting

Paper 1

Time allowed: 3 hours + 15 minutes reading time

INSTRUCTIONS TO THE CANDIDATE

1. You have 15 minutes reading time at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have 3 hours to complete the paper.

2. You must build your model from the beginning and not use an imported e-template.

Your file names must include your ARN, the name of the document and the paper sat (e.g. 9000000-Summary-Paper1) and each file should contain your ARN as a header or footer.

Please note that the content of this booklet is confidential and students are not to discuss or reveal the contents under any circumstances nor are they to be used in a further attempt at the exam.

If you encounter any issues during the examination please contact the Online Education team at online_exams@actuaries.org.uk T. 0044 (0) 1865 268 255

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Exam requirements

1. Read the background document, which describes the scenarios that need to be modelled and documented for this project. Technical assistance for the modelling work, should you require it, can be found in the additional guidance contained in this booklet. *No marks will be deducted for the use of this guidance.*

2. Construct a spreadsheet model that produces the following calculations and charts. You should ensure that your spreadsheet contains appropriate self-checks and that you have performed robust reasonableness checks at each stage of your calculations.

(i) Perform appropriate checks on the data that has been provided and adjust the data if necessary. **[5]**

(ii) Simulate 100 scenarios of the investment fund values from $t = 0$ to $t = 20$, using the given formula for S_t and the coefficient values provided. You should assume an appropriate value for S_0, the initial investment amount. **[5]**

(iii) Calculate the mean, maximum and minimum simulated maturity values at $t = 20$ and the corresponding annualised returns, respectively. **[4]**

(iv) Calculate the standard deviation of the simulated maturity values at $t = 20$. **[2]**

(v) Calculate the mean and standard deviation of the maturity values at $t = 20$ using the given mathematical formulae for the mean and standard deviation of the process S_t. **[4]**

(vi) Determine the rank in ascending order of each simulation based on the maturity value at $t = 20$. **[2]**

(vii) Illustrate the requested sample paths using an appropriate chart. **[2]**

(viii) Repeat steps (ii)–(vii) with the higher diffusion coefficient. **[5]**

(ix) Illustrate the following using an appropriate chart: **[2]**

(a) the mean returns for the different diffusion coefficient assumptions

(b) the maximum returns for the different diffusion coefficient assumptions

(c) the minimum returns for the different diffusion coefficient assumptions

Marks available for spreadsheet model and checks:

Accurate completion of above modelling steps and data checks [31]
Demonstration of good modelling technique and practice [7]
Other (non-data) checks [7]

[Sub-total 45]

3. Produce an audit trail for your spreadsheet model which includes the following aspects:

(i) purpose of the model

(ii) data and assumptions used

(iii) methodology, i.e. description of how each calculation stage in the model has been produced

(iv) explanation of any checks performed

You should ensure that your audit trail is suitable for both a senior actuary, who has been asked to approve your work, and a fellow student, who has been asked to peer review and correct your model, or to continue work on it, or to use it again for a similar purpose in the future.

Marks available for audit trail:

Audit approach

- **Fellow student can review and check methods used in the model [8]**
- **Senior actuary can scrutinise and understand what has been done [8]**
- **Written in clear English [4]**
- **Written in a logical order [3]**

Audit content

- **All steps clearly explained [7]**
- **Clear signposting included throughout [5]**
- **Statement of assumptions made [5]**
- **All model steps accurately covered [15]**

[Sub-total 55]

[Total 100]

Background

You are an actuarial student working for the insurance company Life Co.

Life Co is a growing life insurance company that plans to launch a new equity based investment product in 2018. The investment product that Life Co is developing will work as follows:

- The policyholder will invest an initial amount at the start of the contract.
- An initial charge of 10% will be applied to the amount invested at t=0.
- This money will be invested in a fund, which can be assumed to exactly match the performance of an index known as the ASE 100.
- Each year the value of the fund will increase or decrease in line with the ASE 100 Index.
- The contract will have a term of 20 years.

The marketing department at Life Co has conducted market research showing that the average amount invested in such policies is $150,000.

Life Co would like to model the investment product in order to understand the range of annual returns and absolute final values that a typical investor could achieve.

Modelling the ASE 100 Index

An external company specialising in Economic Scenario Generators has provided 100 simulations of a discrete time series $\{B_t, t \geq 0\}$. For each simulation they have provided annual values of B_t starting at $t = 0$ and ending at $t = 20$. These simulations can be found in the Data file provided.

The company has not provided any evidence of checks performed on the simulations. However, it is expected that the simulations will satisfy the following properties:

(i) B_t has Gaussian increments, i.e. the distribution of $B_{t+1} - B_t$ is $N(0,1)$.

(ii) $B_0 = 0$.

Life Co has decided to use these discrete time series simulations to model the ASE 100 Index using the formula:

$$S_t = S_0 \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma B_t\right]$$

where:

- S_t is the value of the ASE 100 Index at time t.
- S_0 is the value of the ASE 100 Index at time 0.
- B_t is the discrete time series.
- μ is the drift coefficient, and
- σ is the diffusion coefficient.

Furthermore, the mean and standard deviation of the process are given by:

$$E[S_t] = S_0 \exp[\mu t] \quad \text{and} \quad \text{StDev}[S_t] = E[S_t]\sqrt{\exp[\sigma^2 t] - 1}$$

Analysis of the ASE 100 over the last 5 years suggests that the annual drift coefficient μ is 2.0% whilst the diffusion coefficient σ is 7.0%.

Your boss, a qualified actuary, has asked you to perform appropriate checks on the discrete time series simulations that have been provided and to adjust them if necessary.

For each of the 100 simulations, this data should then be used to simulate a typical investment in the ASE 100 from $t = 0$ until $t = 20$, using the given formula for S_t.

Your boss would like to know the mean and standard deviation of the maturity values of the modelled investment at $t = 20$. The corresponding mean annualised return of the modelled investment should also be calculated, in relation to the $150,000 initial investment amount.

In order to understand the range of potential maturity values, she would also like you to calculate the maximum and minimum modelled maturity values at $t = 20$ and the corresponding annualised returns.

As a check on the modelling, she has requested that you determine the actual expectation and standard deviation of S_{20} using the standard mathematical formulae for the mean and standard deviation for the process S_t and the given coefficient values.

Finally, she has requested that you rank the simulations in ascending order according to the projected maturity value at $t = 20$ and illustrate sample paths for those simulations ranked 25th, 50th and 75th (ie set out the projected fund values for each year from t=0 to t=20).

Higher diffusion coefficient

Your boss is concerned that investment markets may be more volatile in the future. She has therefore requested that you re-run your simulations, but with a larger diffusion coefficient, namely $\sigma = 9.0\%$.

The same statistics as for the base case should be calculated.

You should also illustrate the simulations ranked 25th, 50th and 75th on an appropriate chart.

Additional guidance

Useful Excel functions

The **RANK()** function in Excel can be used to return the ranking of a cell within an unsorted array of numbers. For example, RANK [cell, array, 0] will return the ranking that the number in the given cell would have if the (unsorted) array was sorted into descending order. And RANK[cell, array, 1] will take a similar approach but assuming it was sorted into ascending order.

The **AVERAGE()** function in Excel returns the average (arithmetic mean) of the arguments. For example, if the range A1:A20 contains numbers, the formula AVERAGE(A1:A20) returns the average of those numbers.

The **STDEV()** function in Excel estimates the standard deviation based on the provided sample. For example, if the range A1:A20 contains numbers, the formula STDEV(A1:A20) will estimate the standard deviation of the distribution using the numbers provided.

The **EXP()** function in Excel returns *e* raised to the power specified, where the constant *e* is the base of the natural logarithm. For example, the formula EXP(2) will return the base of the natural logarithm *e* raised to the power of 2.

The **MATCH()** function searches for a specified item in a range of cells, and then returns the relative position of that item in the range. For example, if the range A1:A3 contains the values 5, 25, and 38, then the formula MATCH(25,A1:A3,0) returns the number 2, because 25 is the second item in the range.

The **NORMDIST()** function returns the normal distribution for the specified mean and standard deviation.

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