INSTITUTE AND FACULTY OF ACTUARIES

AUDIT TRAIL

May 2016

CA2: Model Documentation, Analysis and Reporting

Paper 1

Small Motor Insurance Company – Reinsurance

Purpose

The purpose of the model is to complete the following calculations:

- Use a set of $U[0,1]$ random numbers to simulate a set of individual claim amounts on SMIC's portfolio of motor car policies, based on a lognormal distribution with pre-calculated parameters.
- Combine these simulated individual claim amounts with the simulated numbers (which represent the number of claims occurring during the year) to obtain 250 simulations of the aggregate claim amounts on each policy over the forthcoming year.
- Calculate the retention and recovery under each simulation for two different reinsurance arrangements.

Data

The following data has been provided the claim department of SMIC:

- 250 random numbers from $U[0,1]$ to use for simulating the claim numbers
- the empirical distribution for the number of claims
- 250×7 random numbers from $U[0,1]$ to use for simulating the individual claim amounts

The data above is included in the "RawData" worksheet.

Assumptions

- The probabilities for the claims provided for modelling individual claim numbers are appropriate and complete.
- Claim amounts follow the lognormal distribution as specified.
- No allowance for inflation is required i.e. the claim amount figures provided have already have been inflation-adjusted to an appropriate level for the forthcoming year, or alternatively inflation is low enough to be ignored.
- Individual claim amounts are independent, and so correlations between the amounts paid under claim events arising during the year can be ignored.
- All claim events are covered by the reinsurance arrangements.
- All motorists are equally likely to make a claim.

- The reinsurance company does not default on the payment of any recoveries.

- There is no delay between SMIC incurring a claim and recovering the amount from the reinsurer i.e. no reporting delays are modelled.

"SimNos" worksheet

This worksheet simulates 250 claim numbers (one for each simulation of SMIC's portfolio) and performs a number of checks on the simulated data.

Empirical distribution of claim numbers:

- The probability of the number of claims occurring on the portfolio per year are assumed to follow the empirical distribution outlined in columns E and F of the "Rawdata" worksheet. That is it is assumed that there will be 0 claims on the portfolio per year with a probability of 18%. 1 claim with a probability of 31.0%, …, up to 7 claims with a probability of 1.2%.

- The cumulative probability for the number of claims (using the empirical data above) is assumed to follow a continuous $U[0,1]$ distribution and is calculated in column C. That is it is assumed that there will be 0 claims on the portfolio per year with a probability of 18%. 1 claim or less with a probability of 49.0% (18% + 31%), 2 claims or less with a probability of 70% (18% + 31% + 21%),…, 7 claims of less with a probability of 100% (18% + 31% + 21% +…). This is summarised in columns A:D of the "SimNos" worksheet.

- The data in columns B of the "RawData" worksheet gives 250 random numbers from the $U[0,1]$ distribution.

- The number of claims for each of 250 simulations of the portfolio is determined by working out where the random number for each simulation equates to on the $U[0,1]$ distribution, and equating that to the relevant number of claims. This is done in column I using a VLOOKUP function on the claims empirical distribution table and using cells C5:D12. For example if the random number is 0.965 then this equates to an expected number of claims of 6.

- Note that the number of claims from column A of the "SimNos" worksheet is repeated in column D (offset by one row) to assist with the simulation calculation.

- The squared deviation from the mean is calculated in column E on the "SimNos" sheet, and the mean is calculated in cell L8

Summary statistics are calculated alongside, including:

- count of values – this should be 250.

- minimum value – this should be 0.

- maximum value – this should be 7.

- average value, vs expected. This is within a reasonable tolerance figure (set at 10%, in cell N8).

- standard deviation value, vs expected. This is within a reasonable tolerance figure (set at 5%, in cell N9). The expected standard deviation calculation uses the squared deviation as calculated in column E. The standard deviation of the simulated numbers uses the STDEV Excel function.

As an additional check that the claim numbers simulated have been correctly taken from the empirical distribution provide, a Chi-Squared test is done in columns Q:V.

- Actual claim numbers from the simulation are calculated in column R – using COUNTIF.

- Expected claim numbers are calculated in column T, as the probability of each claim number, multiplied by the total number of simulations (250).

- The $(A - E)^2/E$ quantity for the Chi-Squared test is calculated in column V.

- The Chi-Squared test is carried out in cells V15:V21 and the test is passed at a test level of 95% confidence.

Reasonableness check: the claim number distributions are broadly Normal but skewed towards the higher claim numbers. This is expected given the law of large numbers and the occasional occurrence of higher claim numbers.

"SimAmounts" worksheet

The simulated aggregate claim amounts (S) are determined as follows:

- In cells F8:L257 (a 250 × 7 table consistent with the format of the random numbers), the $U[0,1]$ random numbers are used to simulate an individual lognormal random variable with the specified parameters (in cells J2 and J3). This uses the LOGINV Excel function.

- The resultant simulated amounts are in £s. This is noted in cell A3. A scaling factor is included in cell B4 and the simulated amounts are scaled to £s from £000s by multiplying the LOGINV simulated value by the scaling factor (1000).

- To the left of this table, column B includes the 250 claim numbers as simulated in the "SimNos" worksheet.

- Within the amounts table, the simulated individual claim amounts are set to zero for all columns that are more than f columns across the table, where f is the simulated claim number for that particular row. This results in there being exactly f non-zero simulated individual claim amounts in each row of the 250 × 7 table. These represent the expected f individual claims simulated to occur during the following year.

- Column N calculates the total of these individual simulated claim amounts for each row, and hence represents the aggregate claim amount (S) for each simulation.

The minimum and maximum value in column N is calculated is cells N2 and N3 respectively.

The annual premium for SMIC's motor car insurance portfolio is set out in cell R3. This is £250,000.

The required statistics are calculated in cells R4:R6 (mean, standard deviation and the mean as a percentage of the premium).

Reasonableness check: the aggregate claim amount distribution is broadly Normal but skewed towards the higher claim amounts. This is expected given the law of large numbers and the occasional occurrence of higher claim amounts.

"Reinsurance" worksheet

This worksheet calculates, for each simulation, the retention and recovery amounts for the two reinsurance arrangements that SMIC have received quotes for.

Assumptions

- All claim events are covered by the reinsurance arrangement.

In rows 4:6, the characteristics of the reinsurance arrangements are entered: i.e. the retained percentage (p), the upper limit (Y) and the price quoted for the reinsurance.

Column B brings in the aggregate claim amount S for each simulation, from the "SimAmounts" worksheet and recalculates the summary statistics in cells B9:B12.

For each simulation, SMIC's reinsurance recovery amounts R are calculated for the two reinsurance arrangements in columns F and K as:

$$R = \min [(1 - p) \times S, Y]$$

The MIN() Excel function is used in the above formula to ensure that R can never be more than the upper limit Y.

For each simulation, SMIC's retained amount is calculated as $S - R$.

There are checks in columns G and L to show Retention + Recovery = Total aggregate claim amount within a very small error tolerance (set in cell B7).

Reasonableness check: the mean of the reinsurer recoveries are higher for the first arrangement compared to the second arrangement. This is expected because the reinsurer is paying a higher proportion of the claims (50% vs 40%).

Reasonableness check: the standard deviation of the reinsurer recoveries are lower for the first arrangement compared to the second arrangement. This is expected because the reinsurer is paying a higher proportion of the claims (50% vs 40%) and so the volatility of its claims experience under arrangement 1 will be lower cf arrangement 2.

Reasonableness check: the mean recoveries as a percentage of reinsurance premium quoted are lower for arrangement 2. This is reasonable given the reinsurer is taking on greater volatility (and so should be keeping more of the premium they charge).

"RIStats" worksheet

This worksheet calculates, for each of the two reinsurance arrangements, every tenth percentile of the following distributions:

- *the aggregate claims distribution with no reinsurance*
- *the claims retained by SMIC after reinsurance, for each of the two arrangements*

The 0^{th} percentile (the minimum value) of the distributions are calculated first, then the 10^{th}, 20^{th} (etc.) percentile, up to the 100^{th} percentile (the maximum value).

The PERCENTILE() Excel function is used for this calculation.

"Chart1" worksheet

This chart sheet illustrates the aggregate claims distribution with no reinsurance against the claims retained by SMIC after reinsurance, for each of the two arrangements. The chart is a line graph with the percentiles along the x-axis, and the claim amounts along the y-axis.

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