INSTITUTE AND FACULTY OF ACTUARIES

Summary

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Subject CA2
Model Documentation, Analysis and Reporting

Paper Two

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Immediate annuity portfolio cash flow swap arrangement

Objective

The purpose of this project is to determine an indicative price for Long Re's contract which swaps uncertain future annuity cash flows for a known stream of future cash flows.

For a specific immediate annuity portfolio, the expected cash flows (both inflows and outflows) are determined, including both annuity benefits and the reinsurance company's expenses.

A "break even" base price is first calculated by determining the loading to apply to expected income such that the net present value of cash flows is zero.

The "break even" base price is then itself loaded for profit to determine the overall price for the longevity swap.

The impact on the net present value of cash flows in the event of higher than expected mortality improvements is also considered, including finding the highest improvement rate that can be withstood before going into loss.

Data

Data has all been provided by the insurance company, and includes:

- An overview of the insurance company's annuity portfolio split by age band, number of pensioners and the average annual benefit. The insurance company has also stated that more than 95% of the annuitants in the portfolio are female.

- A table of mortality rates for females covering the age range 75 to 120.

High level checks (performed by eye) on the annuity portfolio confirm that no data items are missing or materially misstated.

Auto checks have been performed on the mortality rates to ensure they are always non-negative and don't exceed 1.

Assumptions

- Assume that the portfolio data provided is correct and contains no errors.

- Assume that the age definition for the female mortality table is age exact.

- Assume that births are uniformly distributed over the calendar year, so that age nearest (in-force portfolio age definition) is on average equivalent to age exact.

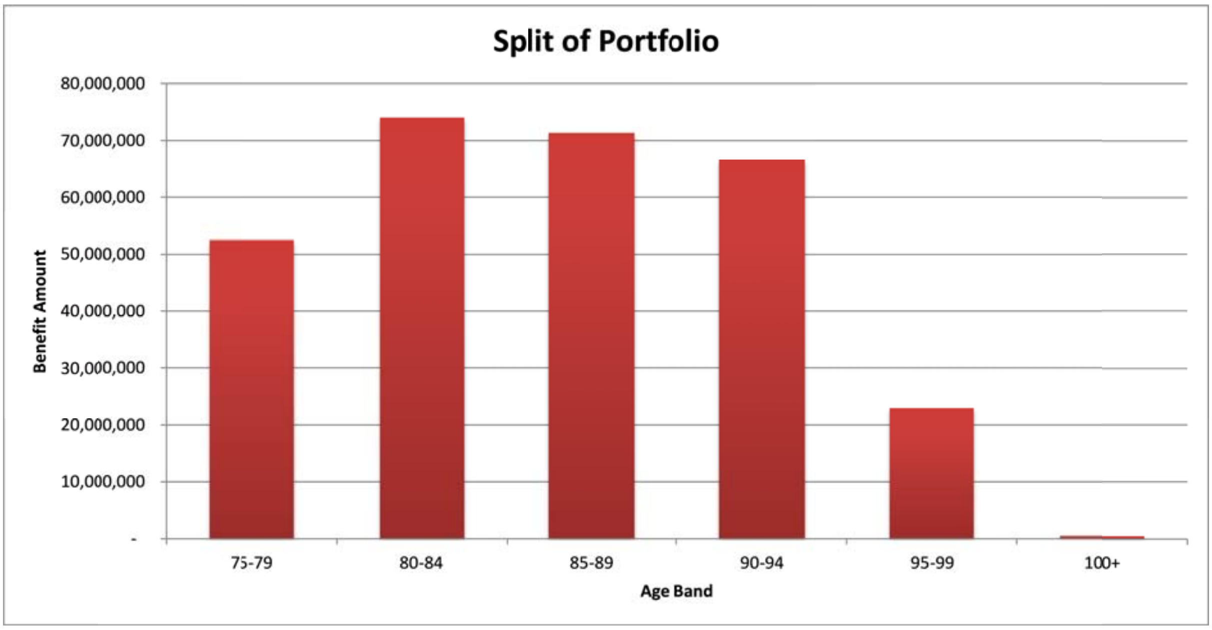
- Assume that ages are uniformly distributed within each age group, so that the average policyholder age will be the mid-point of the age group.

- Assume that all annuities are level, i.e. the same amount with no increase or decrease from year to year.

- Within the age group 100+, assume that the average age is 105.
- Assume a maximum possible age of 120, namely $q(120) = 1$.
- Assume that the entire portfolio will be female. Over 95% of the portfolio is female and this will be prudent if female mortality is lighter than male mortality.
- Assume that no-one will enter or leave the portfolio between the effective date of the portfolio data and the start date of the longevity swap.
- No allowance is made for ageing between the effective date of the portfolio data and the start date of the longevity swap. This will be prudent.
- Assume that tax and reserving costs can be ignored.
- **Assume that the insurance company's mortality and improvement assumptions are appropriate for the portfolio under consideration.**
- **Assume that the mortality improvement rates are unchanged throughout the projection period (i.e. improvements do not accelerate or decelerate).**
- **Assume that the mortality improvement rate is the same for all ages.**
- **Assume that the longevity swap cash flows (including expenses) occur at the start of each year.**
- **Assume that the level of variable expenses remains appropriate.**
- **Assume that fixed expenses increase by 3% each year (i.e. past inflation is a good guide to future inflation, and that the inflation rate is constant in each future year).**
- **Assume that there are no guarantees or additional benefits payable on death.**

Annuity portfolio

For each age band the total number of pensioners has been multiplied by the average benefit to determine the total amount for each age band. The split by total amount is illustrated below:



It can be seen that there is very little exposure in the 100+ age band. This is expected as there are limited pensioners who will survive to over 100.

The biggest exposure is in the age band 80–84. Even though this age band doesn't contain the largest number of pensioners, the high number of pensioners combined with high average benefit yields the largest total benefit amount.

The profile suggests that the new annuitant business volumes sold by the insurance company peaked around 15–20 years ago and has fallen since then.

Mortality Rates

For each age 75 to 120 the insurance company's view of the initial rate of mortality is taken directly from the provided mortality table.

Long Re's view of mortality is determined by taking 97% of the insurance company's initial rate of mortality.

The only exception is $q(120)$ which is 1 for both the insurance company and Long Re.

Cash Flow Modelling

Each age band in the portfolio is considered in turn. Inflows and outflows are considered separately.

Inflows

The initial mortality rates from the insurance company's perspective are used and adjusted for mortality improvement using the formula

$$q(x,t) = q_x \times (1 - \text{mortality improvement factor})^t$$

where $q(x,t)$ is the mortality rate applicable to age x at time t. For this base scenario, the mortality improvement factor is 1%.

The number of pensioners alive at time 0 is taken from the portfolio. The number alive at subsequent time periods t is determined by multiplying those alive in the previous time period by [1 – $q(x-1, t-1)$].

The expected inflow at each time t for Long Re will be:

$$\text{Average Benefit} \times \text{Number of Pensioners}(t) \times (1 + K)$$

where K is the base price which must be determined.

Outflows

The initial mortality rates from Long Re's perspective are used and adjusted for mortality improvement (as described above).

The number of pensioners alive at time 0 is taken from the portfolio. The number alive at subsequent time periods is determined by allowing for mortality (as above).

The expected outflow at each time t for the reinsurer will be:

$$\text{Average Benefit} \times \text{Number of Pensioners}(t)$$

The above is repeated for each age band and then summed to provide the total annuity related inflows and outflows.

Fixed expenses are increased by 3% per annum and together with variable expenses (calculated as 0.05% multiplied by the expected annuity benefit outflow) are added to the annuity outflows to give the total outflow.

The total outflow is then subtracted from the total inflow to yield a net cash flow. The net cash flows are discounted at the risk discount rate of 5% p.a. and summed to provide a total net present value.

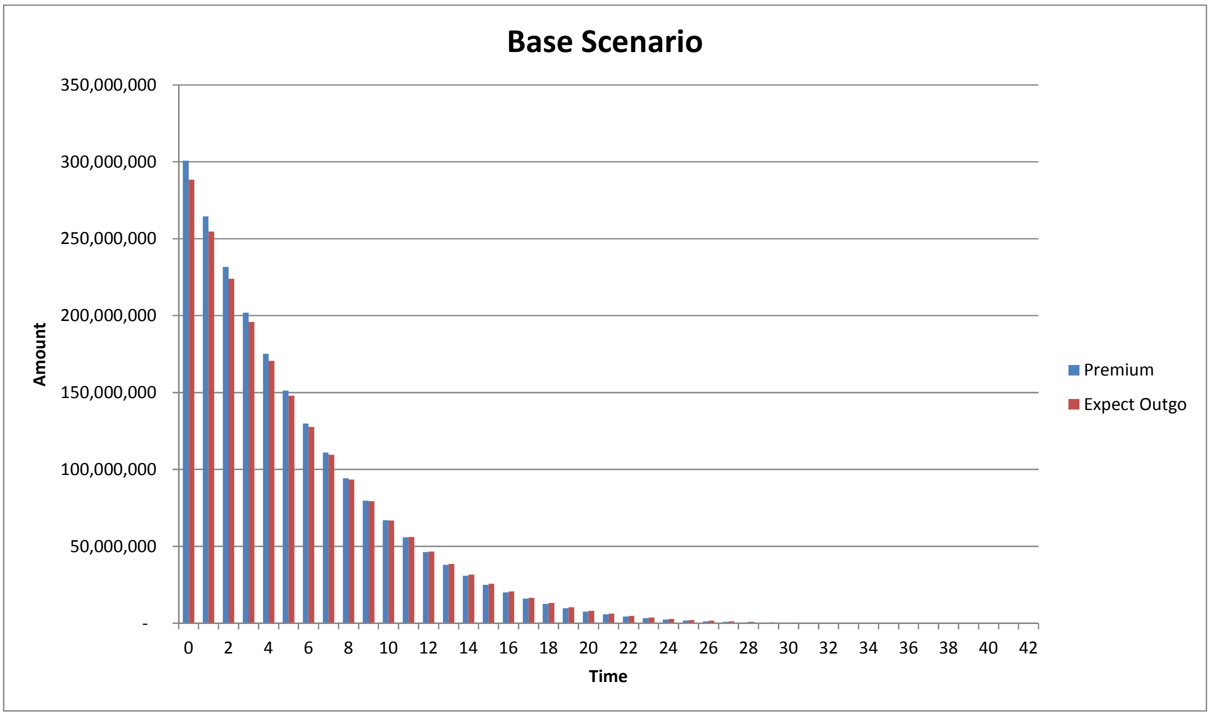
The break even price is obtained by varying K until the net present value is 0. A profit margin of 2.5% is added to this base price to give the total price of Long Re's contract (i.e. overall price = K + 2.5%).

The premiums receivable are then calculated as the best estimate benefit cashflows loaded up by multiplying by (1 + total price) rather than by (1 + K), and the net present value is then recalculated using these final premiums.

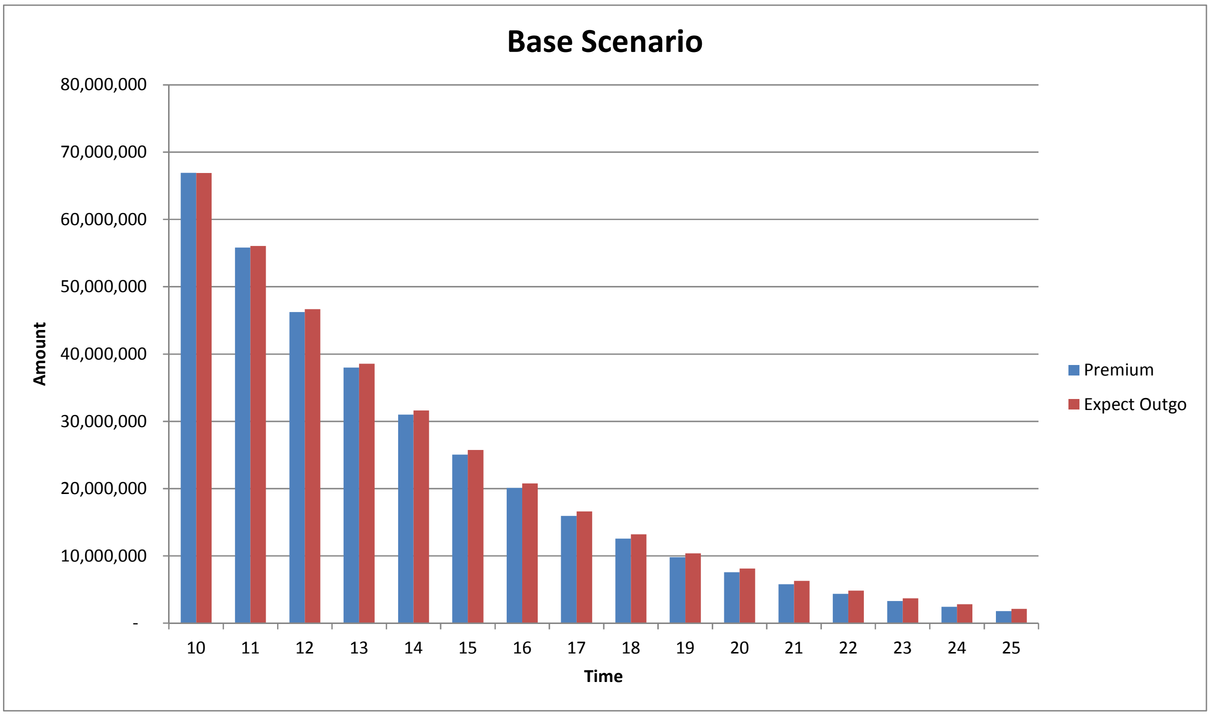
Results – Long Re Contract

The break even price for the portfolio (K) is calculated as 1.87%. Loading for profit gives an overall contract price of 4.37%. The net present value of the contract will be $40.1m.

The premium and expected outgo for the contract are shown below:



It can be seen that the premiums initially exceed the expected outgo. However, if we consider a chart showing time periods from 10 to 25 we can more clearly see that the expected outgo exceeds the premium from t = 11.



Since the premium includes a profit margin it should initially exceed the expected outgo. However, the premium is based on the insurance company's view of mortality (which is heavier) so the premium will fall faster than the expected outgo.

Furthermore the expected outgo contains a fixed expenses element. There will be a point where very few lives remain and the premium received is small. However, the fixed expenses will still be incurred, so that the expected outgo will exceed the premium.

Higher Mortality Improvement

A scenario with a higher level of mortality improvements (1.5% p.a. instead of 1.0% p.a.) is considered. The premium remains unchanged but the expected outflow is recalculated allowing for this new level of mortality improvement.

As in the base scenario, expenses are added to obtain the total outflow and this is subtracted from the premium to give a net cash flow and the net present value is again determined.

The NPV under this scenario is $18.5m.

As expected this is lower than in the base scenario. This is because with a higher level of mortality improvement the reinsurer will need to pay out the actual annuity benefits for longer but the premium received remains unchanged. Hence the net present value of the contract will reduce.

It is observed that even with this higher level of mortality improvement, the NPV of the contract remains positive.

Maximum Mortality Improvement

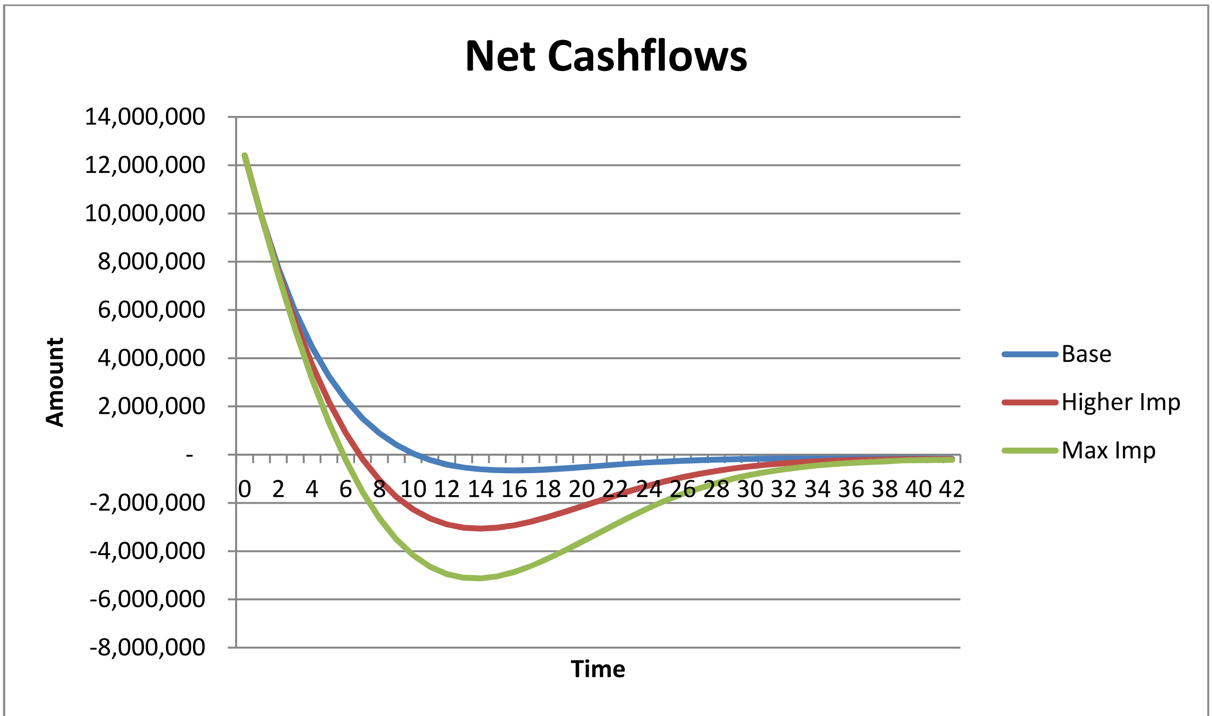
A sensitivity test has been performed in order to determine the level of mortality improvement that will result in an NPV of 0. The improvement rate has been varied until the NPV is 0. This gave an improvement rate of 1.91% per annum.

The NPV with improvement rates of 1% p.a. and 1.5% p.a. was positive. As higher improvement rates reduce the NPV, it is anticipated that the rate of improvement required to give an NPV of 0 will be greater than 1.5% – as is the case.

It is observed that the 50bps increase in improvement rate from 1.0% p.a. to 1.5% p.a. caused a drop in NPV in the order of $20m (a reduction from $40.1m to $18.5m). A further increase in the order of 50 bps (1.5% p.a. to 2.0% p.a.) in the improvement rate would be expected to cause the NPV to drop another $20m to approximately 0. Hence the derived rate of 1.91% per annum has the right order of magnitude,

Results – Net Cash Flows

The chart below shows the net cash flows for each scenario.



The base scenario is highest. This is reasonable since net cash flows are lower in the other scenarios as the expected outgo will be greater under lower mortality rates.

The net cash flow at the start of the projection is the same for all scenarios. This is expected as the initial portfolio is the same in all cases and the cashflows are assumed to be incurred at the start of the year before making any allowance for mortality, therefore any differences in mortality will have no impact.

Over time they diverge. However, at later time periods when few lives remain the net cash flow will be dominated by the fixed expenses. As these are the same in each scenario the net cash flows converge at later time periods.

Conclusions

The price of Long Re's contract is 4.37% which achieves an NPV of $40.1m. Should mortality improvements increase from 1% to 1.5% the NPV will fall but it will still be positive. Mortality improvement rates can be as high as 1.91% per annum before the NPV becomes negative.

The calculated price will be very sensitive to the mortality assumptions made – both the current level and assumed level of improvement.

The price will also be impacted by the assumed level of expenses and expense inflation.

The actual level of NPV achieved will depend upon the level of mortality experienced in reality. This could be either higher or lower than that which has been modelled.

Next steps

- Validate that the portfolio information provided is correct.
- Obtain average ages for each age band. In particular confirm the average age for the final age band.
- Confirm that the mortality table is "age exact".
- Explore whether the insurance company is able to provide individual policy data for the portfolio.
- Obtain portfolio data including gender as a factor.
- Obtain male mortality rates also and allow for them explicitly.
- Perform more sophisticated splits of the data and mortality e.g. smoker v. non-smoker.
- Confirm whether all annuities in the portfolio are level or whether some are increasing.
- Confirm whether any additional death benefits are paid or whether there are minimum guaranteed payment periods.
- Confirm when the portfolio data extract was taken and whether there have been any exits or entries since this date.
- Allow for ageing between the data and valuation dates.
- Sensitivity test the level of expenses.
- In particular vary the assumed level of fixed expense inflation.
- Enhance the model to allow for inflation rates that can vary in future years (i.e. non-level inflation).
- Sensitivity test Long Re's view of initial mortality, by assuming a lighter rate of base mortality. Analyse the impact on the NPV.

- Combine this lighter initial mortality sensitivity with the higher improvement rate sensitivity to see the impact on the NPV.

- Obtain historic portfolio data and perform an experience investigation in order to validate the mortality assumptions.

- Compare the mortality assumptions against similar portfolios that Long Re may have written contracts for.

- Confirm the timing of the contract cash flows (e.g. mid or end year instead of start year).

- If necessary enhance the model to allow for monthly cash flows.

- Confirm that a discount rate of 5% p.a. is an appropriate level to use.

- Sensitivity test the results on a different risk discount rate.

- Discount using a yield curve (i.e. interest rates varying over time).

- Model the mortality stochastically so that a probability distribution of potential results can be produced.

- Enhance the model to allow for varying rates of mortality improvement over time.

- Enhance the model to allow for different rates of mortality improvement by age or cohort.

- Allow for any reserving requirements for such a contract.

- Allow for the impact of taxation in the cash flow projection.

- Calculate the price that should be charged using the higher mortality improvement rate.

- Calculate the NPV under a particular "shock" scenario, e.g. cure for cancer.

- Obtain a peer review of the work performed.