ACTUARIAL REFERENCE No (ARN)						

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION BOOKLET

September 2015

CA2: Model Documentation, Analysis and Reporting

Paper 2

DO NOT OPEN UNTIL INSTRUCTED TO DO SO.

Examination instructions

1. You should periodically save all the files you are working on onto the PC's hard drive.

You will be given instructions for submitting your work at the end of the examination.

It is your responsibility to ensure your work is adequately saved.

2. At the end of the allotted time, or when you have completed your exam, you need to submit your work.

Your filenames must include your ARN (**e.g. Summary_ARN.docx)**. Ensure that your spreadsheet model and summary are clearly labelled and also contain your ARN as a header or footer on at least one page.

Please note that you should use your ARN and NOT your name on all of the material you submit for marking.

The work you submit MUST be saved in Microsoft 2007 format, i.e. using docx (Word) or xlsx (Excel) file extensions. Do not embed documents in your spreadsheet.

3. You must submit your spreadsheet model and summary by the end of the stated exam time. By submitting your files you are confirming that all material is entirely your own work and you wish this to be taken into account for this assessment.

It is your responsibility to ensure that a complete electronic copy of your work is submitted.

You must stop working after this time as failure to do so could result in your exam not being marked.

4. You must also hand in this examination booklet, together with any other materials from the examination. This includes handing in any planning or rough notes that you have made during the examination, and any print-outs that you have done of your work.

Professional behaviour is mandatory and no material relating to the exam may be taken from the exam room nor disclosed or discussed with others.

Failure to comply with this will be deemed to be a breach of examination regulations and may result in disciplinary action.

A spreadsheet model has been provided electronically.

You should use the first 15 minutes of the exam as reading and planning time.

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Exam requirements

1. Read the background document, which describes the scenarios that have been (or need to be) modelled and documented for this project. Additional information, to explain some of the calculations done in the modelling work, can be found on page 7 of this booklet.

2. Read the audit trail which has been written by your colleague, another actuarial student, for the calculations that they performed. This will help you follow and understand the calculations performed in the Excel model provided.

You are not required to add to or amend the audit trail.

3. Build on the model provided to determine the compound percentage change in the mortality rates that is needed to give an expectation of life from birth of 10 years for the Gamma hound. [5]

4. Illustrate the following using three suitable charts:

(i) The base mortality rates for the three types of dog.
(ii) The expectations of life from birth under the base scenario for the three types of dog.
(iii) The expectations of life from birth under the base scenario, Scenario 2 and Scenario 3 for the three types of dog.

When producing these charts, you should assume that your colleague's calculations have been checked and are correct. [10]

5. Prepare a summary document of around five to seven pages, capturing the main features and results from the entire model. You can assume that the summary is being prepared for your boss, a senior actuary, who will present the work to the client.

Your summary should include the following:

- purpose of the project, data, approach and assumptions used by you and your colleague
- results, including charts
- commentary on results, key conclusions and suggested next steps

The summary should cover the full scope of the project, including the scenarios which were modelled in the spreadsheet provided.

You are not required to add to or amend the audit trail.

Marks available for the summary:

Methodology (including purpose, data, approach and assumptions)	**[20]**
Results, including charts	**[10]**
Commentary on results and key conclusions	**[20]**
Next steps	**[25]**
Drafting	**[10]**

[Sub-total 85]

[Total 100]

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Background

In the country of Actuaria there are three popular breeds of dog which are owned as pets: the Weibull terrier, Poisson poodle and Gamma hound.

PetCo is a producer of dog food which has recently developed a new product: Fitter Food For Fido, which is appropriate for dogs of age 2 and above.

PetCo claims that this new brand of dog food will improve dog longevity, reducing mortality rates by 20% throughout life, from age 2 onwards.

You are an actuarial student working for a consultancy in Actuaria. PetCo has approached your boss, a qualified actuary, and asked him for help in performing calculations to show what impact the reduced mortality rates will have on the expectations of life of pet dogs.

As the Gamma hound is the national dog of Actuaria, PetCo has also asked what percentage change in mortality rates would be needed in order for that breed of dog to have an expectation of life from birth of 10 years.

In order to meet the client's requests, your boss obtained mortality rates from the Canine Research Association of Actuaria, for male dogs of each breed and for each age exact. He then asked your colleague to produce a model to perform the following calculations.

Base scenario

Your boss first asked your colleague to calculate the expectation of life from birth for each of the three dog breeds, ignoring the mortality improvements.

Scenario 2

Secondly, he asked your colleague to recalculate the three expectations of life from birth under the reduced mortality rates assumption.

However, he realised that he was not sure whether PetCo meant a 20% reduction on a simple basis or on a compound basis. As his contact at the company was not available to check this information with, he decided to ask your colleague to calculate the adjusted expectations of life on both interpretations.

Your colleague was therefore asked to recalculate the three expectations of life from birth, assuming a **simple** 20% reduction in mortality rates. In this scenario, the mortality rates at each age at which the food will have an effect (i.e. ages 2 and over) are multiplied by 0.8 and then used to determine the expectations of life in the same way as under the base scenario.

Scenario 3

Similarly, your colleague was asked to recalculate the three expectations of life from birth assuming a **compound** 20% reduction in mortality rates. In this scenario, the mortality rate is multiplied by 0.8 at the first age at which the food will have an effect (i.e. age 2), but at the next year of age it is multiplied by 0.8^2 to reflect the compounding effect, and so on.

Scenario 4

Finally, your colleague was asked to determine the **simple** percentage change in mortality rates that would result in an expectation of life from birth for the Gamma hound of 10 years.

The above calculations have been completed and your colleague has also produced a first draft of her audit trail.

Scenario 5

Unfortunately, your colleague did not have time to determine the **compound** percentage change in mortality rates that is needed to give an expectation of life from birth for the Gamma hound of 10 years.

She is currently at an actuarial conference and cannot be contacted.

Your boss has therefore asked you to perform this final set of calculations, using the existing model as your starting point.

He has also asked you to prepare a summary document covering all of the scenarios, and has requested that this should be ready for when he returns to the office in three hours.

You are not expected to include the additional calculation (Scenario 5) in the audit trail, but the results should be presented in the summary.

Additional information: Expectation of life

The expectation of life from age x can be calculated as:

$$e_x = \sum_{t=1}^{\infty} {}_t p_x$$

where ${}_t p_x$ is the cumulative probability of survivorship from age x to age $x + t$, which can be calculated as:

$${}_t p_x = p_x \times p_{x+1} \times p_{x+2} \times \ldots \times p_{x+t-1}$$

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Audit trail

Objective

The purpose of the model is to perform the following:

- Calculate the expectation of life from birth for each of three dog breeds, using base mortality rates.
- Recalculate these expectations of life from birth assuming a **simple** % reduction in mortality rates from a given age onwards.
- Recalculate these expectations of life from birth assuming a **compound** % reduction in mortality rates from a given age onwards.
- Determine the **simple** % change in mortality rates from the given age onwards that would result in a stated target expectation of life from birth for one of the dog breeds.

Base mortality

This worksheet contains the raw data and performs some simple checks on it.

The data was provided by the Canine Research Association of Actuaria and comprises mortality rates for male dogs of each of the three breeds of interest (Weibull terrier, Poisson poodle, Gamma hound), at age x exact from ages 0 to 20 inclusive.

In rows 27 and 28, checks are performed that the mortality rates are greater than 0 and less than 1 for each breed in turn.

Base expn of life

Base scenario: This worksheet calculates the expectation of life from birth for each of the three dog breeds using the base mortality rates.

In columns C to E, the probabilities of survival (p_x) are determined as $1 - q_x$ for each breed of dog and at each age from 0 to 20, with the q_x being referenced from the worksheet "Base mortality".

In columns G to I, the cumulative probabilities of survival from age 0 to age t ($_tp_o$) are determined for ages $t = 1$ to 20, for each breed of dog. [Note: no figure is calculated for age $t = 0$ (i.e. $_op_o$) since this is not required for the expectation of life calculations.]

The $_tp_o$ are calculated as the product of p_x (from columns C to E) from ages $x = 0$ to $t - 1$ (using the Excel function PRODUCT).

In row 28, the expectation of life from birth for each of the three breeds of dog is calculated, by summing the $_tp_o$ in columns G to I, over the range $t = 1$ to 20.

Adj mort simple

*Scenario 2: This worksheet recalculates the expectations of life from birth for the three dog breeds using mortality rates which are reduced by a given factor, on a **simple** basis, from a given age.*

The mortality reduction factor (f) is input to cell G3.

The age from which it applies (z) is input to cell M3.

In columns C to E, the adjusted mortality rates for each breed and at each age from $x = 0$ to 19 are determined as the base mortality rate from worksheet "Base mortality" multiplied by:

- 1 if age $x < z$.
- $(1 - f)$ otherwise.

At age 20, the mortality rates are set to 1 (i.e. it is assumed that age 21 is unchanged as the limit of life).

In columns G to I, the related adjusted probabilities of survival are determined as 1 – adjusted mortality rate.

In columns K to M, the related cumulative probabilities of survival and expectations of life from birth (row 30) are determined using the adjusted probabilities of survival. The calculation approaches used are the same as those used in worksheet "Base expn of life".

Adj mort compound

*Scenario 3: This worksheet recalculates the expectations of life from birth for the three dog breeds using mortality rates which are reduced by a given factor, on a **compound** basis, from a given age.*

These calculations are the same as for the worksheet "Adj mort simple", with the exception of the following:

- The figures in cells G3 (mortality reduction factor) and M3 (age from which reduction applies) are referenced from the previous worksheet rather than being input again.

- For ages $x \geq z$, the base mortality rates are now adjusted by the compounded factor $(1 - f)^{(x - z + 1)}$ [so that, for example, at age $x = z$ the adjustment is $(1 - f)$].

Target expn simple

*Scenario 4: This worksheet determines the **simple** mortality reduction factor F at which the expectation of life from birth for the Gamma hound is equal to a given target.*

The target expectation of life (Z) is input to cell E3.

The simple mortality reduction factor (F) which gives the required target result is found in cell E4.

Cell E5 contains the age (z) from which this mortality reduction factor is applied, referenced from its use in the previous worksheets (as it is unchanged).

Column C contains the base mortality rates (from worksheet “Base mortality”, Gamma hound only) adjusted in the same way as the mortality rates in worksheet “Adj mort simple” but now using the factor F from this worksheet.

Columns D and E calculate the probabilities of survival, cumulative probabilities of survival and expectation of life from birth (cell E30) in the same way as for worksheet “Adj mort simple”, but using the column C adjusted mortality rates.

Cell E31 calculates the difference between the expectation of life calculated in this worksheet and the target expectation of life *Z*.

Goalseek is run by varying cell E4 (*F*) until cell E31 equals zero. [Cell C31 flags whether goalseek has been successful or needs to be re-run.]

This gives the required value of *F*, the simple mortality reduction factor which meets the target expectation of life from birth for the Gamma hound.

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