

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION BOOKLET – ONLINE VERSION

May 2016

CA2: Model Documentation, Analysis and Reporting

Paper 1

This document must be destroyed after the examination has been completed

Please note that the content of this booklet is confidential and students are not to discuss or reveal the contents under any circumstances.

Examination instructions

1. You must download the exam assignment at the start of the exam time stated. All times given are UK times. Please note that it will not be available to you at any other time. The exam period commences at 09.00hrs and ends at 12.15hrs. The exam paper is three hours plus 15 minutes reading time. **It is your responsibility to ensure that all of your files are submitted within this time limit. Failure to do so will result in your assignment not being marked.** To submit your assignment please upload your documents as instructed or e-mail your files to online_exams@actuaries.org.uk. Only your first submission will be accepted and marked.
2. You may refer to any written or electronic reference material provided as part of the CA2 exam. You have been supplied with all data electronically at the start of the exam time. It is recommended that you use the first 15 minutes as reading and planning time.
3. The work you submit **MUST** be saved in Microsoft 2007 format, i.e. using docx (Word) orxlsx (Excel) file extensions. Do not embed documents in your spreadsheet.
4. You must build your model from scratch and not use an imported e-template.
5. You are required to work through the exam assignment without assistance from another person. The assessment regulations of the Institute and Faculty of Actuaries apply as set out in the Examination Regulations except that you may refer to reference material. By submitting your files you are confirming that all material is entirely your own work and you wish this to be taken into account for this assessment.
6. Save your work regularly. You do not have to print out your work but you may choose to do so from time to time if you prefer to check a printed copy. Saving your work is your responsibility so failure to do so will not be a significant mitigating circumstance.
7. You must not discuss or disclose the material. To do otherwise may lead to a disciplinary case.
8. You are reminded that by undertaking this exam you are bound by the Institute and Faculty of Actuaries' Examination Rules and Regulations.
9. At the end of the allotted time or when you have completed your exam, you need to submit your work.

Your filenames must include your ARN, the name of the document and the paper sat. (e.g. 90XXXXXX – Summary-Paper 1) and each file should also contain your ARN as a header or footer on at least one page. If you experience difficulties in submitting your work, you must inform the Online Exams Team immediately at online_exams@actuaries.org.uk or T. +44 (0)1865 268 255.

Professional behaviour is mandatory and no material relating to the exam may be disclosed or discussed with others, nor used in a further attempt at the exam.

Failure to comply with this will be deemed to be a breach of examination regulations and may result in disciplinary action.

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Exam requirements

1. Read the background document, which describes the calculations that need to be modelled and documented for this project. Technical assistance for the modelling work, should you require it, can be found in the additional guidance contained in this booklet.

No marks will be deducted for the use of this guidance.

2. Construct a spreadsheet model that produces the following calculations and charts. You should ensure that your spreadsheet contains appropriate self-checks and that you have performed robust reasonableness checks at each stage of your calculations.
 - (i) Perform 250 simulations of claim numbers for the motor portfolio, using the 250 random numbers from the continuous Uniform distribution data in the file provided. [3]
 - (ii)
 - (a) Calculate the mean and sample standard deviation of the simulated claim numbers from step (i).
 - (b) Calculate the expected mean and standard deviation of the empirical distribution.
 - (c) Compare your answers for part (a) with part (b) using a chi squared test. [6]
 - (iii) Use the Uniform random numbers to generate simulations of the individual claim amounts, and combine these with the claim numbers to produce 250 aggregate claim amounts for the forthcoming year. *[Hint: use the Excel function LOGINV to generate individual claim amounts – see Additional guidance.]* [5]
 - (iv) Calculate the following statistics for the aggregate claim amounts, before reinsurance, across the simulations:
 - (a) the mean
 - (b) the standard deviation
 - (c) the mean as a percentage of SMIC's total annual premiums received [2]
 - (v) Calculate the amounts that would be paid by the insurance company and the amounts that would be paid from the reinsurer under the two reinsurance arrangements, for each simulation. [6]
 - (vi) Calculate the following statistics for the reinsurance recovery amounts across the simulations, separately for each reinsurance arrangement:
 - (a) the mean
 - (b) the standard deviation
 - (c) the mean as a percentage of the reinsurance price quoted [2]

- (vii) Calculate every tenth percentile of the following distributions:
- (a) the aggregate claim distribution with no reinsurance
 - (b) the claims retained by SMIC after reinsurance, for each of the two arrangements
- [3]
- (viii) Illustrate the results of step (vii) using a suitable chart. [3]

Marks available for spreadsheet model and checks:

Accurate completion of above modelling steps and data checks	[30]
Demonstration of good modelling technique and practice	[7]
Other (non-data) checks	[8]

[Sub-total 45]

3. Produce an audit trail for your spreadsheet model which includes the following aspects:

- purpose of the model
- data and assumptions used
- methodology, i.e. description of how each calculation stage in the model has been produced
- explanation of the checks performed

You should ensure that your audit trail is suitable for both a senior actuary, who has been asked to approve your work, and a fellow student, who has been asked to peer review and correct your model, or to continue work on it, or to use it again for a similar purpose in the future.

Marks available for audit trail:

Audit approach

- **Fellow student can review and check methods used in the model** **[8]**
- **Senior actuary can scrutinise and understand what has been done** **[8]**
- **Written in clear English** **[4]**
- **Written in a logical order** **[3]**

Audit content

- **All steps clearly explained** **[7]**
- **Clear signposting included throughout** **[5]**
- **Statement of assumptions made** **[5]**
- **All model steps accurately covered** **[15]**

[Sub-total 55]

[Total 100]

Background

You are an actuarial student working for the Small Motor Insurance Company (SMIC).

SMIC provides motor insurance to motor car drivers in a small region. The total insurance premiums received for these drivers is £250,000 for the forthcoming year. Most claims that arise on SMIC's insurance portfolio relate to own damage, third party liability and theft.

SMIC is currently looking at renewing its reinsurance arrangement for next year. The reinsurance company that currently reinsures SMIC's private car insurance portfolio has offered two alternative reinsurance arrangements, and has quoted a price for each.

The two reinsurance arrangements are as follows:

Reinsurance 1: percentage retained: 50%, upper limit £200,000, price £110,000

Reinsurance 2: percentage retained: 60%, upper limit £250,000, price £105,000

Under these two arrangements:

- The total individual claims incurred over the year on the portfolio are added together to produce the "aggregate claim amount" (S).
- Subject to the condition below, SMIC receives a recovery payment of $(1 - p) \times S$ from the reinsurer, where p is the percentage retained by the insurer.
- SMIC cannot receive more than the upper limit from the reinsurer. The recovery amount is capped at this amount.

Your boss, a senior actuary, has asked you to complete a modelling exercise to help determine which of the two reinsurance arrangements SMIC should purchase. Your boss would like you to build a simulation model to illustrate 250 possible aggregate claim amount scenarios from SMIC's motor insurance portfolio, over the next year.

In order to assist you in your simulations, a colleague in your office has provided you with the following information:

- For each simulation, the number of claims in the year can be 0, 1, 2, ..., 7 with the number of claims following the distribution shown below:

<i>Number of claims</i>	<i>Probability</i>
0	18.0%
1	31.0%
2	21.0%
3	14.4%
4	7.3%
5	4.8%
6	2.3%
7	1.2%

- To simulate claim numbers on the portfolio, 250 random numbers have been supplied, taken from a continuous Uniform distribution on $[0,1]$ (these are provided in an Excel file).
- The individual claim amounts (in £000s) i.e. the amount paid out in respect of an individual claim, ignoring reinsurance, are assumed to follow a lognormal distribution with parameters $\mu = 3$ and $\sigma = 1.44$.

You can assume that the information provided has been checked and is accurate.

To model individual claim amounts for each of the 250 simulations, an array of random numbers has been supplied, taken from a continuous Uniform distribution (these are also provided in an Excel file, allowing for up to seven claims for each simulation, i.e. 250×7).

Your boss requires the individual claim amount simulations to be combined with the simulated numbers of claims to produce a set of 250 aggregate claim amount simulations (before reinsurance is taken into account).

You are also required to produce the following summary statistics across the simulations (before reinsurance):

- the mean
- the standard deviation
- the mean as a percentage of SMIC's total annual premiums received

Your boss then requires calculation of the amount that would be paid by SMIC and the amount that would be paid by the reinsurer, for each simulation and for each arrangement.

He has then asked you to produce the following summary statistics across the simulations, separately for each arrangement, for the reinsurance recovery amounts:

- the mean
- the standard deviation
- the mean as a percentage of the reinsurance price quoted

He would also like to see every tenth percentile of the following distributions:

- the aggregate claim distribution with no reinsurance
- the claims retained by SMIC after reinsurance, for each of the two arrangements

Based on these calculations, SMIC will consider which reinsurance arrangement to purchase.

Additional guidance

Excel hints

1. To use COUNTIF to determine the number of items in a range which are less than a particular value, the syntax required is:

COUNTIF(range, "<X") if X is included as a numerical figure

OR:

COUNTIF(range, "<"&X) if X is included as a cell reference.

2. The function LOGINV(probability, μ , σ) returns the inverse of the lognormal cumulative distribution function of x , where x follows a lognormal distribution with parameters μ and σ .

The values of μ and σ are specified as 3 and 1.44 respectively. The "probability" value are the random $U[0,1]$ values. These can be put straight into the LOGINV() function to generate the individual claim amounts in thousands of pounds. The amounts need to be multiplied by 1,000 to get the claim amounts in pounds.

3. The function PERCENTILE(range, p) determines the p th percentile value of a range of cells containing figures. For instance, the 40th percentile x is the value where the cumulative density function $F(x)$ satisfies the equation $F(x) = 0.4$.

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