INSTITUTE AND FACULTY OF ACTUARIES

September 2017

CA2: Model Documentation, Analysis and Reporting

Paper 2

Time allowed: 3 hours + 15 minutes reading time

INSTRUCTIONS TO THE CANDIDATE

1. You have 15 minutes reading time at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have 3 hours to complete the paper.

2. You must build your model from the beginning and not use an imported e-template.

Your file names must include your ARN, the name of the document and the paper sat (e.g. 9000000-Summary-Paper1) and each file should contain your ARN as a header or footer.

Please note that the content of this booklet is confidential and students are not to discuss or reveal the contents under any circumstances nor are they to be used in a further attempt at the exam.

If you encounter any issues during the examination please contact the Online Education team at online_exams@actuaries.org.uk T. 0044 (0) 1865 268 255

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Exam requirements

1. Read the background document, which describes the scenarios that have been modelled and documented for this project, and the work which remains outstanding. Technical guidance for the modelling work, should you require it, can be found in the additional guidance contained in this booklet.

2. Read the audit trail which has been written by your colleague, another actuarial student, for the calculations that they performed. This will help you follow and understand the calculations performed in the Excel model provided.

You are not required to add to or amend the audit trail.

You should assume that your colleague's calculations have been checked and are correct.

3. Build on the model provided to produce the following additional calculations. You should ensure that the additional spreadsheet work contains appropriate self-checks.

(i) Determine the cumulative probabilities for survival and the expectations of life for females and males aged 75 for the *Get Active* campaign. **[2]**

(ii) Repeat the calculations for the probabilities for survival, expectations of life, and annuity due factors for the *Eat Healthily* campaign. **[5]**

4. Construct, for each of the following, a suitable chart to illustrate:

- the expectations of life for females and males aged 65 and 75 in the base scenario.
- the mortality rates for females only under the base scenario and both campaigns.
- the expectations of life for females only aged 65 and 75 under the base scenario and both of the campaigns.
- the annuity-due factors for females and males aged 65 exact, under the base scenario and both of the campaigns.

When producing these charts, you should assume that your colleague's calculations have been checked and are correct. **[8]**

[Sub-total 15]

5. Prepare a summary document of around five to seven pages, capturing the main features and results from the entire model. You can assume that the summary is being prepared for your boss, a senior actuary, who will present the work to the client.

Your summary should include the following:

- purpose of the project, data, method and assumptions used by you and your colleague
- results, including charts
- commentary on the results
- key conclusions

- suggested next steps

Commentary on the results should cover, but not be limited to:

- analytical comments on each stage of the results, including an explanation of any patterns and unusual features.

- an explanation of the differences between the results under the various scenarios modelled.

Next steps need to be specific to the project, with some mention of why each is a valid next step.

The summary should cover the full scope of the project, including the current approach which was modelled in the spreadsheet provided.

You are not required to add to or amend the audit trail.

Marks available for the summary:

Methodology (including purpose, data, approach and assumptions)	**[25]**
Results, including charts	**[10]**
Commentary on results and key conclusions	**[20]**
Next steps	**[20]**
Drafting	**[10]**

[Sub-total 85]
[Total 100]

Background

The Department of Health in a particular country is planning on running an information campaign to improve the health of individuals aged over 65. The two options being considered are campaigns to encourage older citizens either to G*et Active* or to *Eat Healthily*. The Department of Health believes the *Get Active* campaign will cause a mortality improvement of 2% per annum, and the *Eat Healthily* campaign would initially cause a mortality improvement of 4% per annum, but this would fall gradually to a long-term effect of 1% per annum after the first ten years. The mortality improvements are expected to have a cumulative effect over time.

You are an actuarial student working for a consultancy. The Department of Health has approached your boss, a qualified actuary, and asked him for help in performing calculations to show the impact that the reduced mortality rates will have on the expectations of life of older citizens.

The Department of Health has asked what impact the two campaigns would have on the life expectancy of citizens. Based on the expected impact the Department of Health will choose the campaign to run and will quote the results in its promotional material. To do this, separate calculations are required for males and females at both ages 65 and 75 exact.

The Department of Pensions provides pensions in retirement to the citizens of the country. They have asked the Department of Health to provide them with an estimate of the change in annuity-due factors at age 65 only, for each campaign that would arise as a result of the expected mortality improvements so they can estimate the impact on their pension liabilities. (A description of annuity-due factors is in the Additional Information section below.)

Your boss obtained mortality rates from the Actuarial Association, for females and males from age 65 onwards. He then asked your colleague to produce a model to perform the following calculations.

Base scenario

To calculate the expectations of life for females and males aged 65 and 75 exact, ignoring the expected mortality improvements from the two public health campaigns.

To calculate the annuity-due factors at age 65 for both females and males, ignoring the expected mortality improvements and assuming an interest rate of 5% per annum.

Your colleague has completed the calculations for this scenario.

Scenario: *Get Active*

To recalculate the mortality rates and expectations of life for females and males aged 65 and 75, and the annuity-due factors at age 65 under the assumption that there is a mortality improvement of 2% per annum, as per the *Get Active* campaign. This is done by reducing the mortality rates for each year by the improvement factor compounded for the relevant number of years to get an adjusted set of mortality rates.

Unfortunately, while your colleague was able to update the mortality rates she did not have time to calculate the cumulative probabilities of survival for females and males aged 75, and the resultant expectations of life at this age.

Scenario: *Eat Healthily*

To recalculate the mortality rates and expectations of life for females and males aged 65 and 75, and the annuity-due factors at age 65 under the assumption that there is a mortality improvement in the first year of 4% but which reduces linearly to a long term rate of 1% per annum after 10 years (i.e. from year 11 onwards).

Unfortunately, your colleague also did not have time to undertake the calculations for the *Eat Healthily* scenario.

Your colleague has produced a first draft of her audit trail for the calculations which have been completed for the Base scenario and the *Get Active* scenario.

She is currently at an actuarial conference and cannot be contacted. Your boss has therefore asked you to complete the calculations for the *Get Active* scenario and perform the final set of calculations for the *Eat Healthily* scenario, using the existing model as your starting point.

Finally, your boss needs you to prepare a summary document covering all elements of the work (both the original work your colleague completed and the additional modelling you undertake).

Your summary should include the following:

- purpose of the project, data, method and assumptions used by you and your colleague
- results, including charts
- commentary on the results and key conclusions
- suggested next steps.

You are not expected to include the additional calculations in the audit trail, but the results should be presented in the summary.

Additional information

Expectation of life

The expectation of life from age x can be calculated as:

$$e_x = \sum_{t=1}^{\infty} {}_t p_x$$

where ${}_t p_x$ is the cumulative probability of survivorship from age x to age $x + t$, which can be calculated as:

$${}_t p_x = p_x \times p_{x+1} \times p_{x+2} \times \ldots \times p_{x+t-1}$$

Annuity-due factors

The annuity-due factor required is the present value, at a particular age, of a regular series of payments of 1, starting immediately and continuing until the recipient dies. The payments are discounted at an assumed interest rate (i%) and the probability of the person surviving to receive the payment is also factored in.

An annuity-due factor can be calculated using an iterative process based on the following relationship:

$$\ddot{a}_x = 1 + v \times p_x \times \ddot{a}_{x+1}$$

where $v = 1 / (1+i)$ and $p_x = (1 - q_x)$

You may assume that for age $x = 105$, $\ddot{a}_{105} = 1$

Audit trail

The following audit trail should be read alongside the model provided.

Objective

The Department of Health is considering launching one of two health campaigns; either *Get Active* or *Eat Healthily*. The Department of Health believes that there will be an improvement in mortality rates of 2% each year as a result of the *Get Active* campaign, or an initial improvement of 4% in the first year, gradually falling to the long term rate of 1% per annum after 10 years as a result of the *Eat Healthily* campaign. The purpose of the model is to perform the following:

- calculation of the expectation of life at exact ages 65 and 75 for both females and males using base mortality rates, before any allowance for the improvement in mortality rates due either of the two campaigns

- calculation of the expectations of life at exact ages 65 and 75 for both females and males assuming the mortality improvement rates under the *Get Active* campaign

- estimation of the impact on annuity-due factors for the Department of Pensions of the *Get Active* campaign using an interest rate of 5% per annum

Assumptions

The following assumptions were made in the model:

- Assume that 105 is the limiting age under all scenarios.

- Assume that the mortality rate data provided is correct and contains no errors.

- Assume that the mortality improvement factors indicated by the Department of Health are valid and do not vary by age or gender.

- Assume that the mortality improvements start immediately, i.e. as soon as the health campaigns start.

"Base mortality" worksheet

This worksheet contains the raw data and performs some simple checks on it.

The data was provided by the Actuarial Association and comprises mortality rates for females and males, at age x exact from ages 65 to 105 inclusive. The cells A4 to C44 in this worksheet hold this information.

In columns F and G, checks are performed to show that the mortality rates are greater than 0 and less than 1 for each age and gender. A chart of the base mortality rates has been constructed. A visual inspection shows the rates increase smoothly over time as expected.

"Base scenario" worksheet

Base scenario: This worksheet calculates the expectations of life at ages 65 and 75 for females and males using the base mortality rates.

In columns C and D, the probabilities of survival (p_x) are determined as $1 - q_x$ for both females and males and at each age from 65 to 105, with the q_x being referenced from the worksheet "Base mortality".

In columns F and G, the cumulative probabilities of survival from age 65 to age 65 + t ($_tp_{65}$) are determined for ages 66 to 105 i.e. t = 1 to 40, for both sexes. (Note: no figure is calculated for age 65 (i.e. $_0p_{65}$) since this is not required for the expectation of life calculations.)

The $_tp_{65}$ are calculated as the product of p_x (from columns C and D) from ages x = 65 to 65 + t – 1 (using the Excel function PRODUCT to multiple the p_x's).

The calculation of the cumulative probabilities of survival from age 75 ($_tp_{75}$) are in columns I and J. To calculate these, the product of p_x from ages 75 to 75 + t is calculated for ages 76 to 105, i.e. t = 1 to 30 in this case.

In row 48, the expectations of life from age 65 and age 75 for both sexes are calculated, by summing the $_tp_{65}$ in columns F and G for age 65, and by summing the $_tp_{75}$ in columns I and J for age 75.

The annuity due factors are calculated in columns L and M using the recursive formula for annuities. The annuity-due factor for age 105 is set to 1 as only one final annuity payment will be made to a 105 year old. Then the annuity factor for age 104 is equal to the payment of 1 made at age 104 plus the present value of the annuity-due factor at age 105, discounted at the appropriate interest rate and allowing for the probability that the person will survive until age 105 (p_{104}).

Similarly the factor for age 103 can be calculated from the age 104 result above and this is repeated back to age 65.

"Get Active" worksheet

This worksheet recalculates the expectations of life from ages 65 for both sexes using mortality rates which are reduced by a mortality improvement factor of 2% per annum.

The annual mortality reduction factor is input into cell B2.

In columns C and D, the adjusted mortality rates for each sex and at each age from x = 65 to 104 are determined as the base mortality rate from worksheet "Base mortality" multiplied by (1 – mortality reduction factor) to the power of (x + 1 – 65).

At age 105, the mortality rates are set to 1 (i.e. it is assumed that age 105 is unchanged as the limit of life).

In columns F and G, the related adjusted probabilities of survival for a 65 year old are determined as (1 – adjusted mortality rate).

In columns I and J, the related cumulative probabilities of survival and expectations of life from age 65 (row 48) are determined using the adjusted probabilities of survival. The calculation approaches used are the same as those used in worksheet "Base scenario".

In columns L and M the annuity-due factors are calculated with the same approach as used previously but with the updated mortality rates.

In columns O and P the adjusted mortality rates are calculated with the improvements starting from age 75 instead of 65. So the power applied to (1 – mortality reduction factor) is $(x + 1 – 75)$ to reflect the fact that the citizens are already 75 years old before the mortality improvements start.

The adjusted probabilities of survival from age 75 are calculated in columns R and S from the adjusted mortality rates. Based on the adjusted probabilities of survival the related cumulative probabilities of survival and expectations of life from age 75 need to be determined in columns U and V.

END OF PAPER