The Actuarial Profession
making financial sense of the future

Project CR01

Instructions to CA2 candidate

(i) Read the attached document, which describes the background to this project. You will also be given an additional piece of information later on in the day.

(ii) Use the provided data to determine mortality rates at each age.

(iii) By constructing a life table, or otherwise, and by choosing a suitable interval between points in time, determine the probability of a cat being alive in each of its lives at each point in time.

(iv) Using a cash-flow model, or otherwise, determine the expected cost of feeding a cat in each of its lives and the total expected cost. Use your results to answer your boss's questions and illustrate your results using a suitable chart.

(v) Repeat your projections allowing for Doctor Thomas's theory on cellular senescence and up-skilling. Determine the revised cash-flows and the total expected cost. Use your results to determine whether, and how, the answers to your boss's questions would change and illustrate the results using a suitable chart.

(vi) Repeat the above as appropriate using the additional information given to you later in the day.

(vii) Prepare a summary of five or six pages capturing the main features and results. You can assume that the summary is being prepared for your boss, a qualified actuary, who will present the findings to Mrs. Macavity's financial adviser. You should cover the following:

- Data, approach and assumptions used
- Expected cost of feeding a cat in each of its lives and total expected cost
- The validity of Mrs. Macavity's concerns
- Amended results allowing for Doctor Thomas's theory on cellular senescence and up-skilling
- Results using the new information
- Conclusions, including suggested next steps

Hints:
Partial year mortality rates may be approximated using

$$ {}_{\frac{1}{n}}q_x = {}_{\frac{1}{n}}q_{x+\frac{1}{n}} = {}_{\frac{1}{n}}q_{x+\frac{2}{n}} = \ldots = {}_{\frac{1}{n}}q_{x+\frac{n-1}{n}} = 1-(1-q_x)^{\frac{1}{n}} $$

for n ≥ 1

END

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Background for CA2 candidate

Mrs. Macavity is considering acquiring a kitten for company during her retirement. However, she is deeply concerned as to whether she will be able to afford to feed the cat throughout its life.

She has little in the way of disposable income but does have a lump sum of £3,000 to invest. Her independent financial adviser has advised that she can invest the £3,000 at a fixed rate of 3.00% per annum (payable daily) over a period of 25 years.

You are an actuarial student at an actuarial consultancy. Mrs. Macavity's financial adviser has approached your firm to investigate whether her £3,000 investment will enable her to feed a cat throughout its lifetime. She has made it clear that she's only interested in acquiring a male kitten in order to avoid the risk of any unplanned pregnancies and subsequent kittens.

Your boss has asked you to build a simple model in order to determine the **expected present value** of feeding a cat throughout its lifetime.

As everyone knows, cats have 9 lives, so that, unlike humans, they can die multiple times. When a cat dies it immediately moves onto its next life but does not get any younger. Cats have a total of 9 lives – death in their 9^{th} life is final.

Being a kind hearted lady, and not wanting to incur the wrath of the animal protection society, Mrs. Macavity will wish to feed the cat throughout its entire lifetime, not just one of its 9 lives.

A selection of information has been provided to assist you with your calculations:

Number of tins consumed per cat per annum	275
Current price of a tin	52p
Price inflation per annum	5%

Mrs. Macavity has stated that she dislikes modern supermarkets and wants to limit the number of visits she makes. A study of a small sample of cat food tins suggests they have a best before date just over 6 months from the date of purchase.

An expert in feline mortality, Doctor Sylvester, from the Cattery Mortality Investigation Bureau has provided you with the latest life table for cats in their first life. Doctor Sylvester believes that the mortality rates experienced in other lives will be the same. You know little about the mortality of cats, apart from the fact that a typical cat is expected to live between 15 to 20 years (i.e. it is expected the cat will have used up all its lives somewhere between 15 to 20 years from birth).

Using the provided information your boss wants you to determine whether you expect Mrs. Macavity will be able to feed the cat throughout its lifetime. If she is unable to afford to feed the cat then your boss would like to know the maximum number of tins per annum that Mrs. Macavity could afford to purchase.

Another expert at the CMIB, Doctor Thomas, has a theory that when a cat dies there will be a change to their mortality in future lives. Specifically, Doctor Thomas believes there are two competing processes which occur upon death:

- Cellular senescence – when a cat dies (and subsequently lives again) the cellular regeneration is not perfect. As a result, with each death a cat's mortality will increase by 1%.

- Up-skilling – with each death the cat will become a little wiser and know to avoid certain hazards in future. As a result, with each death a cat's mortality will decrease by 6%.

Your boss would like you to repeat your analysis allowing for Doctor Thomas's theory.

A further piece of information will be provided later on in the day.

END

Data (this will be available electronically)

Feline Life Table 2010

Age	*lx*
0	100000
1	65779.6
2	46219.74
3	33465.15
4	23989.61
5	16827.51
6	11581.12
7	7770.249
8	5027.242
9	3110.698
10	1822.067
11	1007.667
12	518.2197
13	243.6708
14	104.1639
15	39.1215
16	12.43289
17	3.199506
18	0.642271
19	0.094174
20	0.006599

END

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Additional information for CA2 candidate

To be provided later on in the day

A third mortality expert at the CMIB, Doctor Scratchy, supports Doctor Thomas's theory on cellular senescence and up-skilling, but also believes that a cat's appetite will reduce with each subsequent life.

Doctor Scratchy's work is in an early stage and she has been unable to ascertain the level of appetite reduction expected to occur. However, she is confident that there will be a reduction and that this will apply at a constant rate.

Your boss wants you to repeat the analysis allowing for both Doctor Scratchy and Doctor Thomas's theories. He wants you to determine the minimum level of decrease in appetite under which Mrs. Macavity is expected to be able to afford to feed the cat throughout its lifetime.

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