INSTITUTE AND FACULTY OF ACTUARIES

SUMMARY

May 2016

CA2: Model Documentation, Analysis and Reporting

Paper 2

FeltCo

Objective

The overall objective of this project is to determine the price that FeltCo needs to pay the farmer for three different coloured bags of wool in order to meet their profit criterion.

The project will require a projection of the gradual loss in the bags of wool, and then of felt sheets during transportation from the farmer to market.

The prices will be determined allowing for the expenses incurred in getting the felt sheets to market and the revenue expected when the felt sheets are sold at market.

An alternative scenario based on a rumour that will affect the rate of loss of different coloured felt sheets during transportation will also be investigated.

Data

The following key data items have been provided by FeltCo:

- The number of bags of coloured wool purchased from the farmer.
- The rate at which the bags of wool will be eaten by the Wool Eating Moth whilst being transported to the factory.
- The rate at which the felt sheets will be stolen by the rugby players whilst being transported from the factory to the market.
- Expenses incurred in transporting the bags of wool and the felt sheets.
- FeltCo's profit criterion and the risk discount rate to be used.

Although the background to this model is unfamiliar, the data provided does not seem unreasonable and there is nothing obvious to question its validity.

Assumptions

- Each of the 300 sheep produces exactly one bag of wool.
- The price paid for each bag of coloured wool is in the same ratio as the price paid for each coloured felt sheet, so the red and green bags of wool are the same price and the black bag of wool is half this price.
- All expenses are settled at the end of the day in which they are incurred.
- The one-off transportation expenses are paid at the start of the relevant part of the journey.

- The revenue received at the market is uniform across the two days and is deposited in a bank at the end of each day.

- No interest is earned on the revenue deposited in the bank.

- Inflation of the price of felt sheets can be ignored, as we are told that historically there has been very little inflation in Actuaria.

- Apart from the moths and the rugby players, no other decrements apply.

- **Once the moths enter the factory, the moths are removed/destroyed**

- **Under the alternative scenario, the black felt sheets are more likely to be stolen, so the daily theft decrement is assumed to be 4%, whereas the daily theft decrement for the red and green felt sheets is each assumed to be 1%.**

- **Under the alternative scenario, all the felt sheets are still assumed to be sold at market over two days.**

- **Under the alternative scenario, the prices paid for the bags of wool to the farmer are unchanged, even though the ratio of the prices of the felt sheets sold at market have changed.**

Method

The method used involved projecting the number of bags of wool that would be transported from the farmer to the factory, and then after they were processed and converted into felt sheets, from the factory to the market, where they would be sold.

Projection of bags of wool

300 bags of wool are bought from the farmer, of which 100 are coloured red, 100 are coloured green and 100 are coloured black. From then, it would take 15 days for them to be transported to the factory.

During this time, they are expected to be eaten by Wool Eating Moths, who we are told can eat 6 bags in a 24 hour period. The moths eat twice as many bags of red wool compared to the bags of green and black wool, which means that 3 bags of red coloured wool are eaten each day, and 1.5 bags each of green and black coloured wool.

The number of bags of wool is projected as follows:

Number of bags of wool at the end of the day t =
{Number of bags of wool at the end of day $t-1$} - {Number of bags eaten per day}

for any bag of coloured wool.

The number of bags of wool that remain when they reach the factory are multiplied by 10 to produce the number of felt sheets. The felt sheets are the same colour as the bags of wool, so

there will be red, green and black coloured felt sheets produced. This procedure will take 7 days.

Projection of felt sheets

The felt sheets are transported from the factory to the market and this takes 20 days.

During this time, the felt sheets will be stolen by the Actuarian rugby players at the rate of 2% of the sheets remaining on the train. As they will operate in the dark and will not be able to tell the difference between the different coloured sheets, it is assumed that it is equally likely that they will steal a felt sheet of any colour.

The number of felt sheets is projected as follows:

Number of felt sheets at the end of day t =

{Number of felt sheets at the end of day $t - 1$} $\times$ $(1 - 2\%)$

for any coloured felt sheet.

At the end of the 20 days, the remaining felt sheets are sold at the market.

Pricing calculations

To determine the price which FeltCo pays the farmer, all the cashflows need to be determined.

For the transportation of the bags of wool to the factory, the one-off cost is calculated based on the number of bags of wool at the start of the journey. The one-off cost is \$3 per bag of wool, so the one-off cost is therefore \$900. In addition, there is a fixed cost of \$150 per day for 15 days.

During the 7 days at the factory, there is a fixed cost of \$100 per day.

For the transportation of the felt sheets to the market, the one-off cost is calculated based on the number of felt sheets at the start of the journey. The one-off cost is \$0.60 per felt sheet, so the one-off cost is therefore:

\$0.60 $\times$ number of bags of wool that reach the factory $\times$ 10.

In addition, there is a fixed cost of \$250 per day for 20 days.

Once the felt sheets reach the market, the revenue received over the two days is:

(Number of red and green coloured felt sheets) $\times$ \$30 + Number of black coloured felt sheets $\times$ \$15.

It is assumed that the felt sheets are sold uniformly over the two days so FeltCo expect to receive and deposit half the revenue in the bank at the end of the first day at market, and the other half at the end of the second day.

The payment to the farmer is initially unknown but will be:

$$200 \times \text{price of red/green bag of wool} + 100 \times \text{price of black bag of wool}$$

where

$$\text{price of black bag of wool} = 50\ \% \times \text{price of red/green bag of wool.}$$

To determine this price, the present value of the cashflows at time $t = 0$ is determined using the risk discount rate of 10% per annum. As the model uses days, the risk discount rate used is $(1.10^{1/365} - 1)$.

The price per bag of red/green wool is found when the NPV of the cashflows equals the profit criterion of $2,500.

Also, setting the profit criterion to $0, we calculated an Internal Rate of Return for the project.

Alternative scenario

One of the board members at FeltCo has heard a rumour that the rugby players have bought torches and so it is more likely that black felt sheets will be stolen each night.

As already stated, it is assumed that the daily theft decrement for black felt sheets increases to 4% and the daily theft decrement for the red and green felt sheets is assumed to fall to 1%.

With fewer black felt sheets available to be sold at market, it would be expected that this would push the price of the black felt sheets up by 50%. The revenue is therefore recalculated as:

(Number of red and green coloured felt sheets) × $30 + Number of black coloured felt sheets × $22.50.

Assuming no change in the price paid to the farmer for the bags of wool, the profit is recalculated and so is the IRR.

Results

Projection of bags of wool

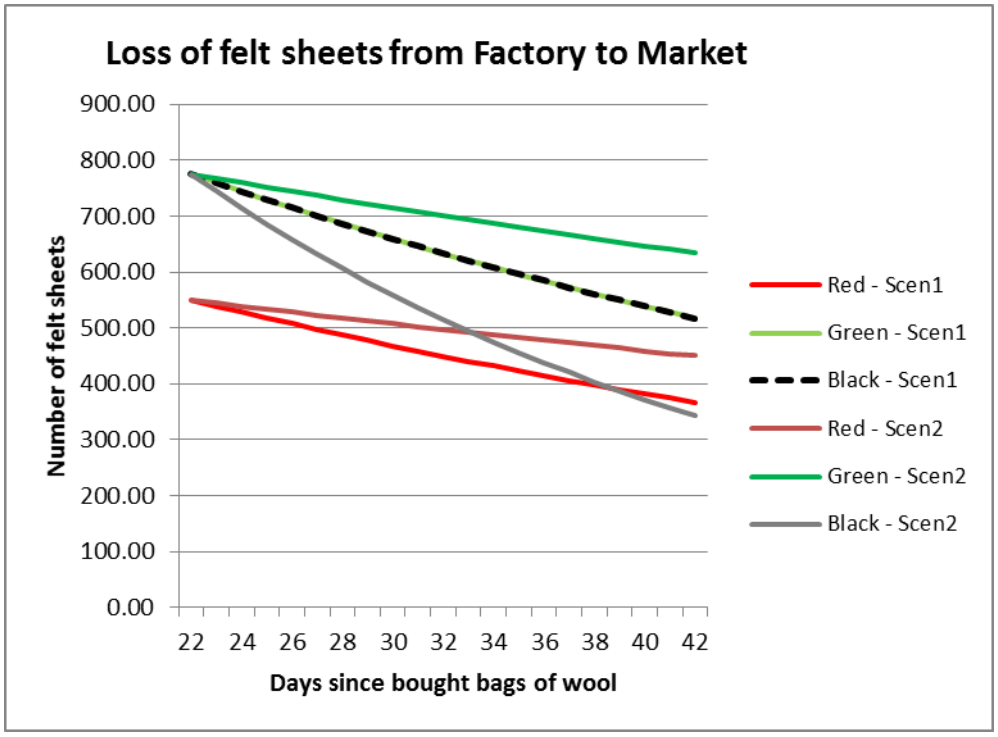
The number of bags of wool is projected from the point where they are bought from the farmer to when they arrive at the factory 15 days later. The bar chart below shows how the number of bags of wool falls during this time.

- The bar chart shows that the number of bags of wool uniformly falls each day as there is only a decrement and no increments.

- The number of red bags of wool falls more quickly during transportation as the Wool Eating Moth eats more bags of red coloured wool.

Projection of felt sheets

The number of felt sheets is projected from the point where they leave the factory to when they arrive at the market 20 days later. The line chart below shows how the number of felt sheets falls during this time.



- For both scenarios, the number of felt sheets falls each day as there is only a daily decrement and no increments.

- Also, the fall is uniform as the decrement applied is a constant percentage each day.

- Under the first scenario, the fall in the number of felt sheets is the same for any colour as the same decrement is applied, due to there being an equal chance that any of the different coloured felt sheets will be stolen.

- The number of black felt sheets falls at a faster rate under the second scenario as the decrement is larger – it falls twice as quickly.

- Conversely, the number of red and green felt sheets falls at a slower rate under the second scenario as the decrement is 50% lower.

- Comparing the two scenarios, the number of felt sheets that arrive at the market is slightly higher under the second scenario. This is because for most of the journey during transportation, the black felt sheets make up less than one third of the total sheets on the train, so the total decrement is less than 2%. With the total decrement lower for most days compared to the first scenario, fewer sheets are stolen and therefore more sheets arrive at the market.

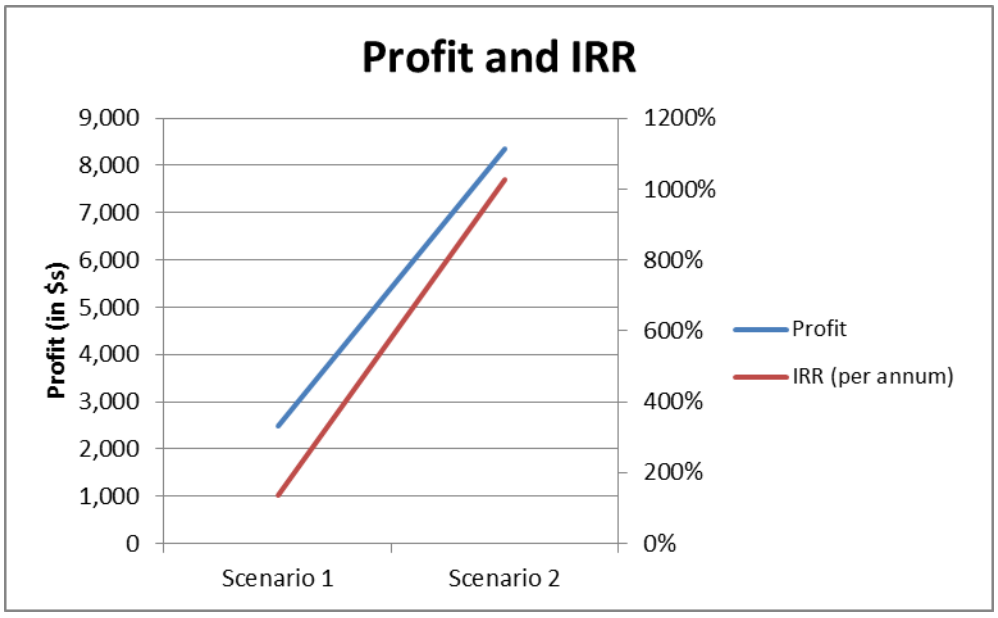
Financial results – both scenarios

Under the original scenario, the prices to pay the farmer for the three different coloured bags of wool for FeltCo to meet their required profit criterion is as follows:

Colour of felt sheet	*Price*
Red	$85.43
Green	$85.43
Black	$42.72

Using these prices, the expected net present value of profit under the original scenario is $2,500 (the required profit criterion) and the IRR is 0.235% per day (135.8% per annum). The IRR is determined by setting the profit criterion to zero, so as a profit is made with a risk discount rate of 10% per annum, the IRR will be higher than 10%.

Under the second scenario, the NPV of profit increases to $8,354 and the IRR increases to 0.666% per day (1,028% per annum). As expected, because the profit is higher under this scenario, the IRR is also higher.



The increase in revenue does not come from the 50% increase in the price of the black felt sheets, as fewer black sheets expected to get to market mean that the revenue from black felt sheets falls slightly. Instead, the reason is that more red and green felt sheets will be sold at the higher price of $30 each at the market.

Overall, the second scenario has an increase in revenue of $5,921, but no change in cashflow out. This is the reason why the NPV of profit has increased by $5,854 to $8,354 and why the IRR is much higher.

Next steps

- Validate all the information provided from independent sources.
- Are there any sources available that can confirm the eating habits of the Wool Eating Moth? And the stealing capabilities of the Actuarial rugby players?
- Verify all the assumptions made. In particular, the timing of the cashflows.
- Investigate whether the rumour about some of the rugby players buying torches and the impact of them stealing black felt sheets is valid.
- Perform sensitivity tests on key parameters, such as the decrement rates, expenses and the number of felt sheets obtained from a bag of wool.
- Re-run the original scenario with a different profit criterion and/or a different risk discount rate.
- Investigate a different pricing structure for the bags of wool, for example, have the same price for each different coloured bag of wool.
- For the second scenario, determine the prices to be paid for the bags of wool if the prices are in the same ratio as the price of the felt sheets.
- Investigate whether there are other farmers who would be prepared to sell wool at a cheaper price.
- Investigate whether there are farmers and/or markets who are closer to the factory who would save on transportation costs.
- Investigate whether there are alternative methods of transporting the wool/felt sheets that would save time and expense.
- Investigate the impact on the profit criterion/prices for the wool if different numbers of bags of coloured wool were bought.
- Investigate the impact on profit criterion/prices for the wool if different prices are charged for felt sheets at market.
- Investigate whether more felt sheets can be produced from one bag of wool at the factory.
- Introduce a probability in the model to allow for some of the felt sheets not being sold at the market.
- Model the impact that a change in the colours of Actuaria's national costume would have on the profit.

- Consider whether past experience of low inflation will continue, and if not, include an inflation assumption in the model.

- Consider how a shock scenario would affect the profit, e.g. the sheep are unable to produce as much wool as expected, or there is a lack of supply of food that the sheep feed on.

- Model the impact of the market not being open on two consecutive days.

- Update the model for experience as it occurs.

- Obtain a peer review of the work performed.

END OF SUMMARY