PROJECT CR01 – Projected Cost of Feeding a Cat throughout its 9 Lives

Purpose and Background

Mrs. Macavity is considering obtaining a male cat but is concerned it will be unaffordable. The purpose of this project is to determine whether we expect Mrs. Macavity will be able to afford to feed a cat throughout its 9 lives. This is done by comparing the lump sum available to Mrs Macavity to the present value of the costs of tinned cat food.

The project involves first determining the probability a cat is alive in each of its 9 lives at each point in time. A cash-flow model is built in order to project the future expected cost of feeding a cat and the present value of the total cost is determined.

The calculations are repeated under a number of different scenarios which allow for the impact of up-skilling and cellular senescence on a cat's mortality and changes in the level of appetite with subsequent lives.

Data

A feline life table has been provided by an expert in feline mortality from the Cattery Mortality Investigation Bureau (CMIB). The data is provided in the form of l_x figures and covers ages 0 to 20.

Basic checks that there are no gaps in the data and that the l_x figures decrease with age have been performed. A chart has been produced so that basic smoothness can be checked by eye.

The mortality data has been used without amendment.

Initial Assumptions

- The life table refers to age x exact and is applicable for male cats
- No cats survive beyond age 20
- No allowance is made for the probability of Mrs. Macavity's survival
- The cat will be acquired soon after birth so its age is taken to be 0 at the start of the projection
- Any acquisition costs and/or additional costs associated with owning a cat (e.g. vet bills, scratching post and treats) are ignored
- Mrs. Macavity has a lump sum of £3,000 and can achieve a net of tax return of 3.00% on her investment so a discount rate of 3.00% is used. A cat will be affordable if the present value of outgo is less than £3,000
- It is further assumed that Mrs. Macavity will be able to make unlimited withdrawals from her investment without incurring any charges
- Mrs. Macavity wishes to minimise supermarket visits and we are told tins of cat food last for 6 months. Consequently a time step of 6 months has been assumed in the cash-flow model and it is assumed that cat food is purchased at the start of each 6 month period
- It is assumed that cats move along by a maximum of one "life" in each half year of age (e.g. can move from Life 1 to Life 2, but not from Life 1 to Life 3)

Projected Cost of Feeding a Cat – Doctor Sylvester's Theory – Scenario 1

Initial mortality rates are obtained in the usual way from the life table and q(20) is set at 1. The probability of dying in the next year of age falls from age 0 to age 1 then increases fairly smoothly with age. A smooth increase in mortality rates with age is to be expected and a higher mortality rate at age 0 is not unreasonable. Often for humans, mortality in the first year of life is higher than the second due to various effects such as birth defects.

First, it is necessary to determine the probability that a cat will be alive in each of its lives at the start of each time period. The 6-monthly mortality rates are approximated using

$$ {}_{\frac{1}{2}}q_x = \quad {}_{\frac{1}{2}}q_{x+\frac{1}{2}} = 1 - (1 - q_x)^{\frac{1}{2}} $$

If $l_{x,n}$ denotes the number alive at age x in life n, then this can be calculated as:

$$ l_{x,0} = l_{x-\frac{1}{2},0} * \left(1 - \quad {}_{\frac{1}{2}}q_{x-\frac{1}{2}}\right) $$

$$ l_{x,n} = l_{x-\frac{1}{2},n} * \left(1 - \quad {}_{\frac{1}{2}}q_{x-1/2)}\right) + l_{x-\frac{1}{2},n-1} * \quad {}_{\frac{1}{2}}q_{x-\frac{1}{2}})) $$

Summing across the 9 lives gives the probability a cat is alive (in any life) at each age. These can be used to determine the curtate expectation of life which has been calculated to be 17 years. Based on the background information this is an acceptable value – we were expecting a value between 15 and 20.

A cash-flow model has been built which analyses the expected cash-flows for each life of the cat. At the start of each 6 month period *t* the amount that will be spent if the cat is alive is

$$ ½ * \text{Number of tins per annum} * \text{Price per tin} * (1 + \text{Price Inflation})^t $$

The expected outflow will be this amount multiplied by the probability the cat is alive. For each time period and each life the expected outflows are determined. Using the discount rate of 3.00% the present value of the expected outgo for each life is determined.

Summing the present values across all lives gives the total expected outgo. If this is less than £3,000 then it is expected Mrs. Macavity will be able to afford to feed a cat throughout its life.

Projected Cost of Feeding a Cat – Doctor Thomas's Theory – Scenario 2

Doctor Thomas has proposed that a cat's mortality changes with each life. With each subsequent life mortality increases by 1% as a result of cellular senescence but decreases by 6% as a result of upskilling.

The above is allowed for by adjusting the mortality rates with each life. Assuming the effects apply multiplicatively the variation is simply:

$$ q_{x,n} = q_{x,n-1} * (1 + 1\%) * (1 - 6\%) $$

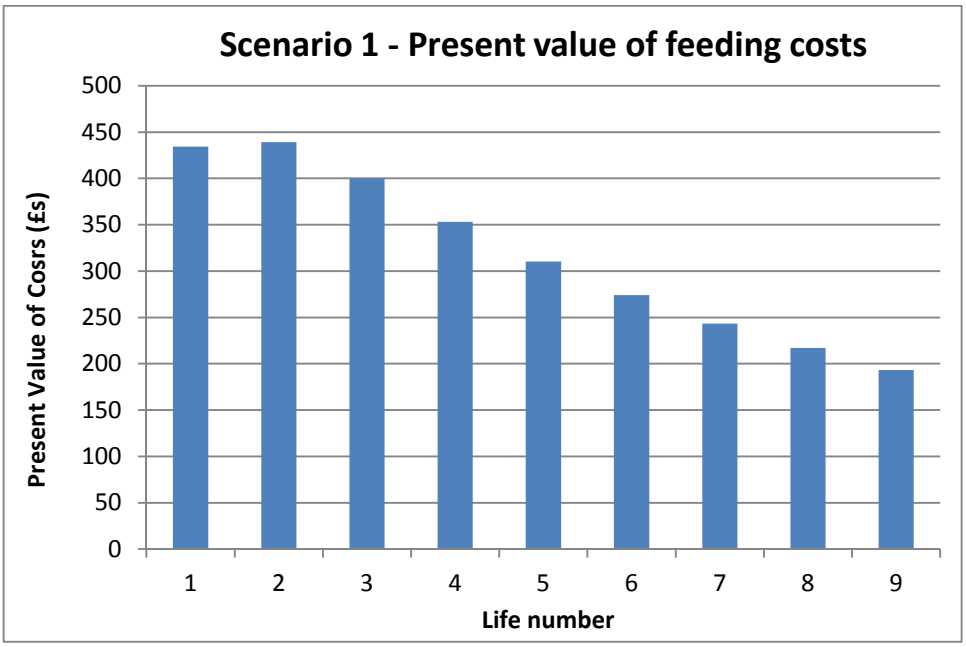
It is assumed that the effect on mortality is the same across all ages.

The revised probability that a cat is alive (in any life) at each age is then calculated.

The cashflow model is then repeated using the revised probabilities and the present value of the cat food recalculated.

Results –Scenario 1

The expected present value of feeding a cat in each of its lives under the first scenario is illustrated in the graph below:



Summing across the lives the expected present value of all feeding costs is £2,865. This is less than £3,000 so it is expected Mrs. Macavity will be able to afford to feed a cat.

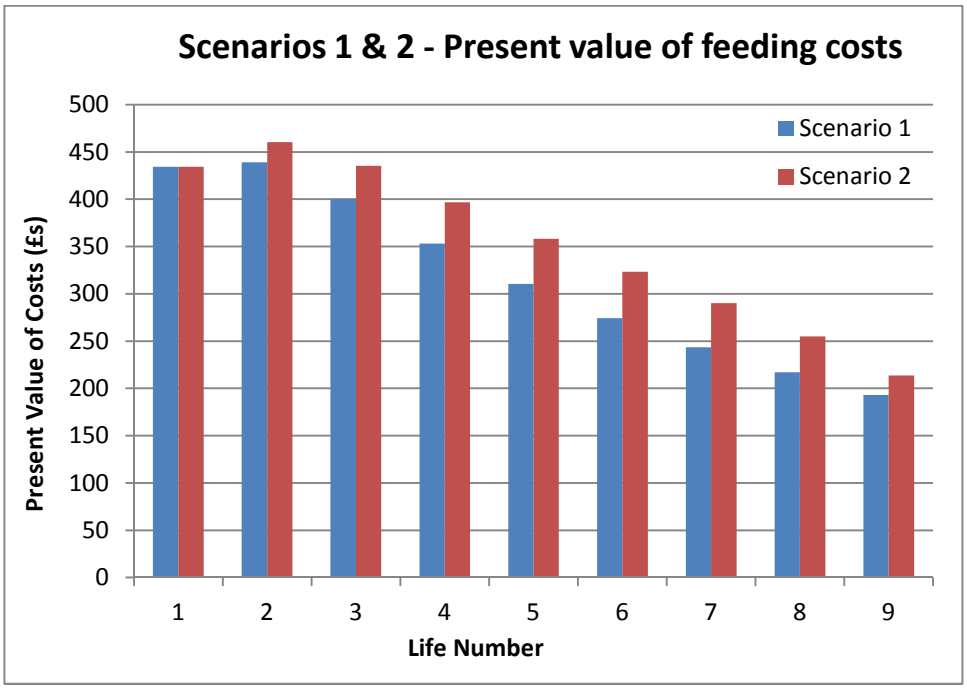
There is not a particularly large margin between the expected present value of £2,865 and the affordability threshold of £3,000. A larger margin would give a higher degree of confidence that Mrs. Macavity will be able to feed a cat.

The results are highly dependent on the assumptions made. Higher price inflation or appetite than assumed will increase the expected cost. Similarly lighter feline mortality than assumed will increase the expected cost. These will reduce the likelihood that Mrs. Macavity will be able to afford to feed a cat. The results are also sensitive to the discount rate. Though the rate is fixed at 3.00%, this could vary if Mrs Macavity is able to transfer her funds to a product offering a higher rate, or, if there is any default risk on the investment product.

It can be seen that the general trend in present value over the lives is downwards. This is not unexpected since cats are expected to live longer in their early lives than in their later lives. Additionally cats will enter later lives at later times when the impact of discounting is greater. Inflation has offset this to some extent, but not completely.

Results – Scenario 2

The expected present value of feeding a cat in each life under Doctor Thomas's theory is shown in the graph below. The base scenario has also been included for ease of comparison.



Under Doctor Thomas's theory the expected present value of the outgo is £3,167. Since this is greater than £3,000 it is not expected that Mrs. Macavity will be able to afford to feed a cat throughout its lifetime.

Under this scenario it has been determined that the maximum number of tins that could be afforded per annum is 260.47.

The overall impact of an increase of 1% in mortality (cellular senescence) and a decrease of 6% in mortality (up-skilling) is a decrease in the underlying mortality rates for each life. Apart from life 1, where up-skilling and cellular senescence have no impact, the mortality will be lighter than in the base scenario. Consequently it is expected that the expected present value for each life will be greater than the base scenario (for life 1 it will be unchanged). This is confirmed by the graph and results in an overall increase in the expected present value of the outgo.

As in the base scenario the results are highly dependent on the assumptions made. Under Doctor Thomas's theory there are also the assumptions relating to up-skilling and cellular senescence. If cellular senescence has a greater impact on mortality and/or up-skilling has a lesser impact then the likelihood Mrs. Macavity will be able to feed the cat will be increased.

Scenario 3

Background
Another expert at the CMIB, Doctor Scratchy, supports Doctor Thomas's theory but also believes that a cat's appetite will reduce by a constant amount with each subsequent life. However, Doctor Scratchy has not yet determined the amount of the reduction. The objective is to determine the minimum level of appetite reduction required such that it is expected Mrs. Macavity will be able to afford to feed a cat.

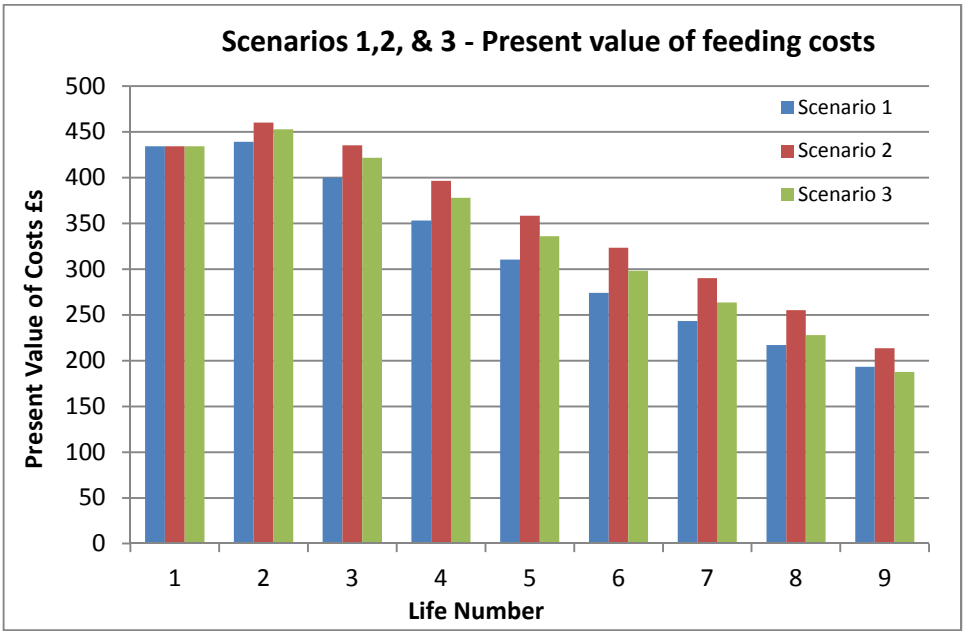
A new parameter for the appetite reduction, *f*, has been included and the cash-flow model has been adjusted such that the amount spent at each time period if the cat is alive is given by:

$$\text{Amount Spent in Life } n = \text{Amount Spent in Life } n\text{-}1 * (1 - f)$$

The amount spent in life 1 is unchanged as it is unaffected by Doctor Scratchy's theory.

The required level of appetite reduction has been found by varying the appetite reduction parameter, *f*, until the expected present value of outgo is £3,000.

Results
The expected present value of feeding a cat under Doctor Scratchy's theory is shown in the graph below. The results from scenarios 1 & 2 have been included for ease of comparison.



Using Excel goalseek it has been found that an appetite reduction of 1.6% results in an expected present value of outgo of £3,000. Consequently if the appetite reduction is at least 1.6% with each life then Mrs. Macavity will be able to afford to feed a cat throughout its lifetime.

It can be seen that the expected present value for life 1 is the same under all 3 scenarios. This is to be expected as the appetite reduction only applies on death so will not affect life 1. For all subsequent lives the expected present value under Dr Scratchy's theory is lower than under Dr Thomas's theory. This is to be expected since the effect of the appetite reduction will be to lower the expected present value.

It should be emphasised that Dr Scratchy's work is at an early stage. There are inherent dangers of relying upon the results of a partially completed study – especially if the work has not yet been subject to independent peer review. Consequently, the assumption that appetite reduction is fixed from life to life should be used with caution.

SUGGESTED NEXT STEPS

Potential next steps to refine and to expand the modelling work include:

- Results are very dependent on the assumptions made so the base case assumptions made for price inflation, appetite levels and discount rate should be verified.
- In particular, seek scientific evidence to support the assertion that cats have 9 lives.
- The mortality data has been provided from what should be a reliable source but independent verification should be worthwhile. Other studies (e.g. those performed by cat food manufacturers) should be sought to provide a comparison.
- Obtain confirmation that the data provided is for exact age x and male cats.
- It is unclear whether the mortality tables provided include any improvements in mortality over time. If not, then consider incorporating an allowance for mortality improvements over time.
- Seek opinions from other feline experts on Dr. Thomas's theories of cellular senescence and up-skilling and the impact levels on mortality being proposed.
- Only a small sample of cat food tins has been checked to determine the expiry time. Consider looking at a larger sample.

- Consider the mortality of female cats. Perhaps, unlike human females, their mortality is heavier than males. If this is the case then Mrs. Macavity could opt to obtain a female cat and have it spayed (then need to allow for lower fund available, reduced by the cost of spaying).
- Consider the effect of different breeds of cat – some may display heavier mortality and/or smaller appetites so are more easily affordable.
- Rerun the model for a different pet altogether.
- Consider using a smaller time step (e.g. monthly). This would allow for the possibility of final death during the first year of life – something that is not allowed for in the current implementation. Also, for the cash-flow calculations, it may be more realistic if Mrs Macavity is unable to carry six months' supply of tins in one trip.
- Factor in the cost of trips to the supermarket, e.g. bus fares.
- Consider the impact of using a local store on a more frequent basis or using on-line shopping. This would reduce the risk of having excess food at the end of a 6 month period. However, a local store may charge a higher price and there may be delivery charges for on-line shopping.
- Since Mrs. Macavity is purchasing 6 months' cat food in advance she may be able to secure a bulk discount on the price, or consider a different brand which costs less.

- Sensitivity test the assumptions made, in particular the level of inflation and appetite. Furthermore sensitivities to the level of mortality could be performed.
- Sensitivity test the levels of cellular senescence and up-skilling proposed by Dr. Thomas.
- Consider the impact of appetite varying with age in addition to step changes on death.
- The model could be enhanced to allow for Mrs. Macavity's mortality as there is no certainty she will survive to the end of a cat's lifetime.

- The conditions of the proposed investment should be checked, e.g. the minimum/maximum withdrawals allowed, minimum level of investment, any withdrawal penalties in place
- Consider other possible investments offering a higher return, with the same level of flexibility, and thereby increase the likelihood that a cat is affordable?
- Investigate any default risk in the proposed investment and adjust the discount rate as appropriate.

- Enhance the model to allow for expenses incurred in acquiring the cat and ongoing expenses such as cat collars, litter and vetinary expenses. A potential approach for allowing for vetinary expenses could be to incorporate a premium for pet insurance into the cash-flow model.
- Mrs. Macavity could opt to obtain the cat at a slightly older age (rather than from birth).
- Model how long Mrs Macavity would afford to feed her cat to determine the point when she'll need to look for further funds (as an alternative to rationing her cat on less tins per year)