The Actuarial Profession
making financial sense of the future

Rabbit Population Projection Project – Summary

Overview

The purpose of this project is to produce projections of a rabbit population from 1985 population survey information, separately for males and females, based on assumptions regarding fertility and mortality rates.

The following sensitivity and scenario tests have also been performed:

- Adjustments to fertility rates in order to reproduce actual 2005 population survey information.
- A "shock" epidemic scenario, affecting both fertility and mortality experience.
- Analysis of whether an increase to the assumed proportion of births which are male would reproduce the actual 2005 population survey information.

Data and assumptions

Data

Data has been provided on:

- 1985 rabbit population survey, by sex and age band.
- 1985 estimate of rabbit mortality rates, by sex and age band.
- 1985 estimate of rabbit fertility rates, by age band.
- 2005 rabbit population survey, by sex and age band.

All data has been converted into single year age bands in order to facilitate calculations.

The mortality rates provided are for "age exact", whereas the population data is split by "age last". It is therefore necessary to derive mortality rates that can be used on "age last" population data. This was done as follows:

Rabbits aged x last birthday at the start of the year would on average be age $x + 0.5$ "exact" and hence would experience mortality rate $q_{x+0.5}$ for the following calendar year. It has been assumed that the following approximation can be used to convert the mortality rates as required: $q_{x+0.5} = \{q_x + q_{x+1}\} / 2$.

Each female rabbit of reproductive age has 3 babies each year if she is aged from 2 to 4 and 2 babies if she is aged from 5 to 7.

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Assumptions

- Population is evenly spread over each age band.
- All data, mortality and fertility rate estimates provided by the scientist are correct.
- Deaths occur evenly during the year.
- Births occur on average halfway through the year.

Methodology and assumptions

Methodology

The population in each age band for each sex has been projected forwards from year end 1985 to the end of each future year up to 2005.

This is done for age band x+1 by multiplying the number of rabbits in age band x at the previous year end by the appropriate survivorship factor (= $1 - q_{x+0.5}$).

To determine the projected number of rabbits in age band 0, it is necessary first to calculate the expected number of births each year. As it has been assumed that births occur on average halfway through the year, the number of births has to allow for half a year's exposure to mortality of the mother rabbit (prior to giving birth) during the first half of the calendar year. The total baby rabbits are then split into males and females. Finally, the number of baby rabbits has to be multiplied by $\{1 - q_0/2\}$ to allow for the probability of death during the first six months of life, i.e. the second half of the calendar year.

Additional assumption

- Half of the baby rabbits are male and half are female.

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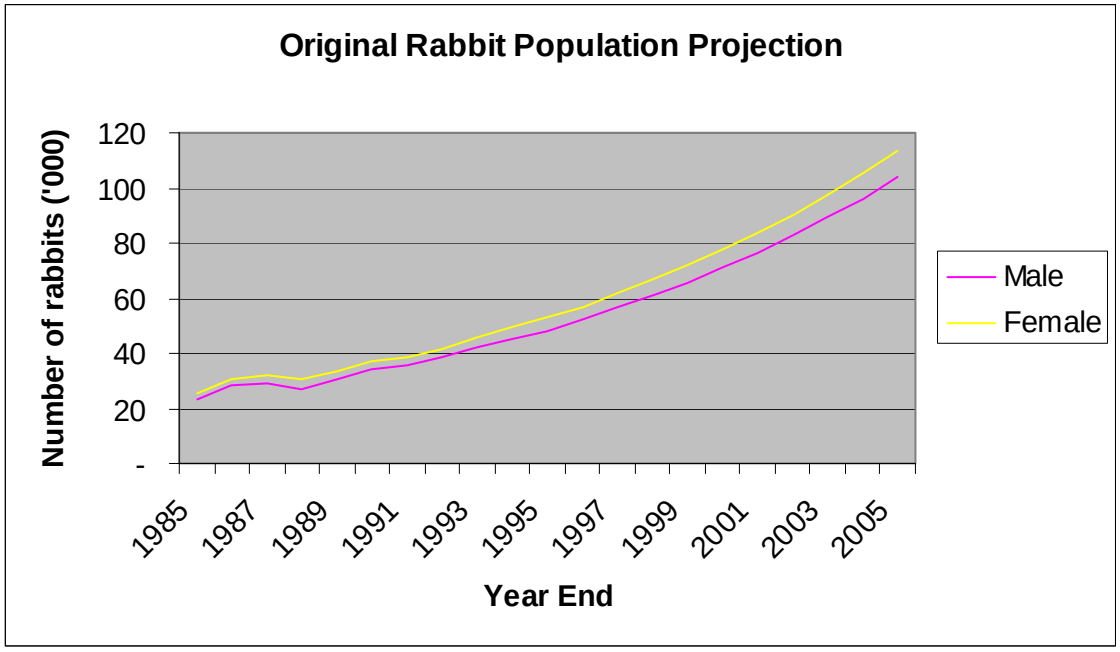
1985 data – population projection

Results

The projected populations of male and female rabbits, based on 1985 data, mortality and fertility rates are as illustrated in this graph:



Comments

- The increasing trend reflects the fact that, on average, fertility rates materially exceed mortality rates.

- The trend is similar for both males and females, with the proportionate difference between the two remaining similar. This reflects the assumption that half of baby rabbits are male and half are female, and the fact that overall the mortality rates do not differ significantly between the sexes.

2005 survey comparisons

Methodology

The 1985 data projected to 2005 was compared with actual 2005 survey data, with the latter being significantly lower than the original projections.

The following theories to explain the discrepancy were investigated:

- Incorrect original fertility rate assumption: tested by multiplying the number of births in each year by an adjustment factor. The factor that minimised the discrepancy (measured using a "sum of squares" approach) was 86%.

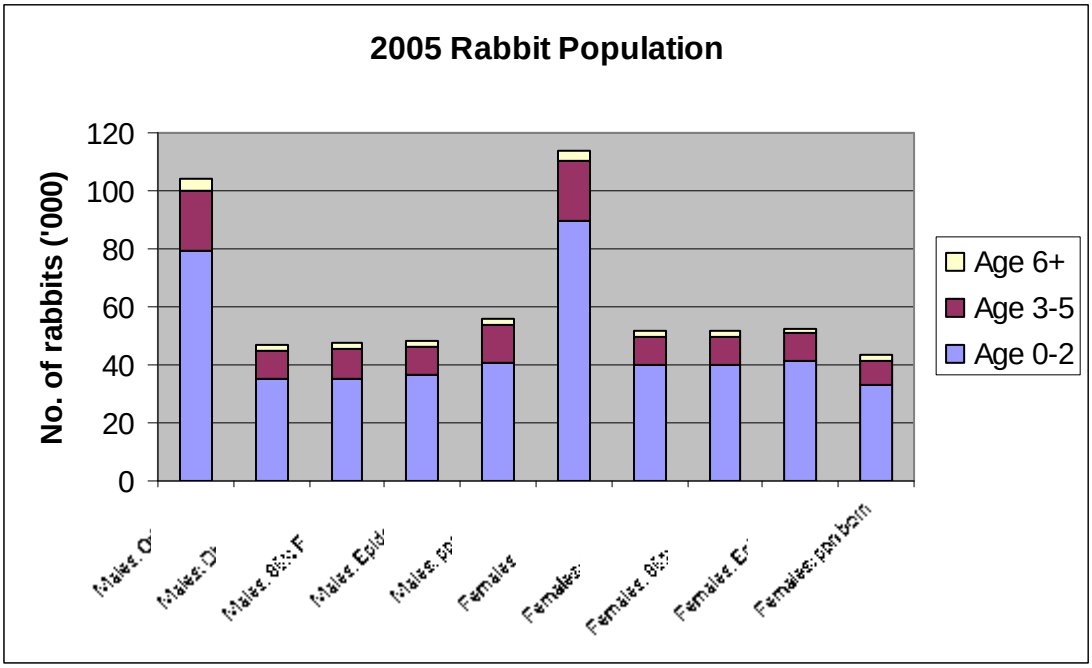
- 1995 rabbit 'flu epidemic: tested by applying a one-off mortality "shock", killing one third of all rabbits, plus a reduction in fertility to 60% of original assumed levels from 1996 to 1998.

- The proportion of male births is higher than for females: tested by solving to find the revised proportion of male births that gives a zero difference between actual and projected total population (males plus females, across all age bands).

Additional assumptions

- Fertility rates were reduced by the same proportion for all ages.
- The deaths as a result of the epidemic occurred at the end of 1995.
- The epidemic affected all rabbits equally, irrespective of age.

Results



Comments

As intended, this shows the total projected populations being the same as the observed populations separately for males and females under the adjusted fertility rate scenario.

Since the original projection was too high, it is appropriate that fertility rates need to be *reduced* (to reduce the number of births and hence also the numbers of future births) in order to match the observed populations. Similarly, the epidemic scenario reduces the projected populations due to lower fertility (hence fewer baby rabbits) and higher deaths.

The similarity of the 2005 survey data with the adjusted fertility rate projection results and the 'flu epidemic projection results suggest that either theory could justify the discrepancy between 1985 original population projections and 2005 actual population survey.

The results of just adjusting the proportions of male and female births give the same total populations but the breakdown by age and sex in the projection gives a poor fit to the observed population. This adjustment is unlikely to account for difference between the original projection and the observed population.

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However the scientist was correct to suggest a *higher* proportion of male births, because the projections needed to be reduced. A lower proportion of female births will reduce the number of births and hence overall population going forwards.

Conclusions

- The 2005 survey data implies that fertility rates could have been over-stated in the original 1985 research, and are actually nearer to 86% of their estimated values.

- Alternatively, the theory regarding the impact of the 1995 'flu epidemic could explain the difference between projections on 1985 estimates and the 2005 survey data.

- The assumption of different proportions of male and female births is unlikely to be the sole explanation of the difference.

Potential next steps include:

- Investigate the existence of other research to support either theory.

- Consider any evidence supporting the original projection assumptions, both fertility and mortality.

- Seek evidence to support the assumptions made in the projections relating to the timing of births during the year or deaths are spread evenly over the year.

- Seek information on actual ratio of male to female births.

- Consider other theories, e.g. that population growth rates will decline as the population increases due to finite food resource, emigration in search of food, any studies of potential predators in or around Actuopolis, adverse weather conditions.

- Investigate the accuracy of the surveys, both in 1985 and 2005. Have they been carried out consistently? What estimation approaches have been used and how robust are these considered to be?

- Look for more information on the number of rabbits within each age group.

- Alternatively, test a model which allocates rabbits within each age group differently.

- Model the impact of different fertility/mortality rate impacts on different age bands.

- Consider the combined impact of scenarios, e.g. adjusted male birth proportions combined with adjusted fertility rates.

- Sensitivity test the normal (i.e. non epidemic) mortality rates and see whether these could account for the discrepancy.

END