INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION BOOKLET – ONLINE VERSION

February 2016

CA2: Model Documentation, Analysis and Reporting

Paper 1

This document must be destroyed after the examination has been completed

Please note that the content of this booklet is confidential and students are not to discuss or reveal the contents under any circumstances.

Examination instructions

1. You must download the exam assignment at the start of the exam time stated. All times given are UK times. Please note that it will not be available to you at any other time. Paper 1 will take place at 09.00hrs to 12.15hrs. The exam paper is three hours plus 15 minutes reading time. **It is your responsibility to ensure that all of your files are submitted within this time limit. Failure to do so will result in your assignment not being marked.** To submit your assignment please upload your documents as instructed or e-mail your files to online_exams@actuaries.org.uk. Only your first submission will be accepted and marked.
2. You may refer to any written or electronic reference material provided as part of the CA2 exam. You have been supplied with all data electronically at the start of the exam time. It is recommended that you use the first 15 minutes as reading and planning time.
3. The work you submit MUST be saved in Microsoft 2007 format, i.e. using docx (Word) or xlsx (Excel) file extensions. Do not embed documents in your spreadsheet.
4. You must build your model from scratch and not use an imported e-template.
5. You are required to work through the exam assignment without assistance from another person. The assessment regulations of the Institute and Faculty of Actuaries apply as set out in the Examination Regulations except that you may refer to reference material. By submitting your files you are confirming that all material is entirely your own work and you wish this to be taken into account for this assessment.
6. Save your work regularly. You do not have to print out your work but you may choose to do so from time to time if you prefer to check a printed copy. Saving your work is your responsibility so failure to do so will not be a significant mitigating circumstance.
7. You must not discuss or disclose the material. To do otherwise may lead to a disciplinary case.
8. You are reminded that by undertaking this exam you are bound by the Institute and Faculty of Actuaries' Examination Rules and Regulations.
9. At the end of the allotted time or when you have completed your exam, you need to submit your work.

   Your filenames must include your ARN (e.g. Summary_90XXXXX.docx) and each file should also contain your ARN as a header or footer on at least one page. You will receive an acknowledgement by email from the Online Exams Team confirming receipt. The Online Exams Team will send you an email after the exam requesting you to delete all your files relating to the exam, together with your planning notes and any print-outs. If you experience difficulties in submitting your work, you must inform the Online Exams Team immediately at online_exams@actuaries.org.uk or T. +44 (0)1865 268 255.

Professional behaviour is mandatory and no material relating to the exam may be disclosed or discussed with others, nor used in a further attempt at the exam.

Failure to comply with this will be deemed to be a breach of examination regulations and may result in disciplinary action.

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Exam requirements

1. Read the background document, which describes the scenarios that need to be modelled and documented for this project. Technical assistance for the modelling work, should you require it, can be found in the additional guidance contained in this booklet.

No marks will be deducted for the use of this guidance.

2. Construct a spreadsheet model that produces the following calculations and charts. You should ensure that your spreadsheet contains appropriate self-checks and that you have performed robust reasonableness checks at each stage of your calculations.

(i) Validate the interest rate data supplied and amend any corrupt (i.e. unusually high or low) or missing values. **[4]**

(ii) Calculate 5-year forward rates from the fixed interest forward curve *F*. **[2]**

(iii) Calculate monthly compounded interest rates, using the 5-year forward rates calculated in step (ii). **[2]**

(iv) Calculate the mortgage fees payable for the fixed rate mortgage. **[2]**

(v) Calculate the total mortgage payments made each year for the fixed rate mortgage. *[Hint: first calculate annuity factors using the interest rates calculated in steps (ii) and (iii), assuming monthly payments in arrears.]* **[5]**

(vi) Repeat step (iii) (using the variable interest rate curve *V*) and hence repeat step (v) to find the mortgage payments for the variable rate mortgage. **[4]**

(vii) Illustrate the monthly mortgage payments from both the variable and fixed mortgages, and the difference between them. **[2]**

(viii) Project the annual household income and expenditure each year for 25 years. **[5]**

[Hint: Determine which type of mortgage Mr Rae is likely to choose based on his decision criteria and allow for the mortgage payments to reflect this.]

(ix) Project the Rae family's savings fund each year for the term of the mortgage, using your results from steps (viii). **[1]**

(x) Determine the year in which the savings fund reaches the level required for Mr and Mrs Rae to be able to start the renovation work. **[1]**

(xi) Illustrate the growth in the savings fund over the mortgage term using a suitable chart. **[2]**

Marks available for spreadsheet model and checks:

Accurate completion of above modelling steps and data checks	**[30]**
Demonstration of good modelling technique and practice	**[7]**
Other (non-data) checks	**[8]**

[Sub-total 45]

3. Produce an audit trail for your spreadsheet model which includes the following aspects:

- purpose of the model
- data and assumptions used
- methodology, i.e. description of how each calculation stage in the model has been produced
- explanation of the checks performed

You should ensure that your audit trail is suitable for both a senior actuary, who has been asked to approve your work, and a fellow student, who has been asked to peer review and correct your model, or to continue work on it, or to use it again for a similar purpose in the future.

Marks available for audit trail:

Audit approach

- **Fellow student can review and check methods used in the model [8]**
- **Senior actuary can scrutinise and understand what has been done [8]**
- **Written in clear English [4]**
- **Written in a logical order [3]**

Audit content

- **All steps clearly explained [7]**
- **Clear signposting included throughout [5]**
- **Statement of assumptions made [5]**
- **All model steps accurately covered [15]**

[Sub-total 55]

[Total 100]

Background

Individuals living in a country can take out a mortgage to fund the purchase of a property. All mortgages have a 25-year term, and the mortgage interest rate can be set using one of two approaches. Either:

- a fixed rate mortgage where the interest rate is fixed for 5-year periods and re-evaluated at the end of each period, or

- a variable rate mortgage where the interest rate varies each year.

The mortgage holder chooses either the fixed or variable rate mortgage from outset for the entire term of the mortgage.

You are an actuarial student working at Counting Your Pennies, a financial advisory firm. One of the firm's clients, Mr Rae, is looking for mortgage advice as he is considering purchasing a new property. Your boss, who is an actuary, has asked you to build a model to project indicative mortgage payments for Mr Rae, so that he can decide between the variable rate mortgage and the fixed rate mortgage.

All mortgages are issued by the Central Bank (CB). Your boss has received the following information from the CB on the two approaches available:

General mortgage information

- All mortgages have a term of 25 years.

- Mortgage payments are made monthly in arrear.

- Inflation is currently 3% p.a.

- The CB produces interest rate forward curves $V(t)$ and $F(t)$ for variable and fixed rate mortgages respectively. These curves indicate what the CB anticipates interest rates F and V will be over t years (up to 25 years). These curves are in the data reproduced at the end of this document and in an in Excel file.

Information on CB's fixed rate mortgage

- Interest rates, in line with the five year forward rates, are recalculated at the start of every five years based on the curve $F(t)$.

- Fixed rate mortgages have an arrangement fee, payable when the interest rate is set at the start of every five year period.

- The arrangement fee is currently \$500 and is assumed to increase in line with inflation.

- The arrangement fee is added to the outstanding mortgage balance.

Information on CB's variable rate mortgage

- Interest rates are recalculated at the start of every year based on the curve $V(t)$.
- Variable rate mortgages do not have any arrangement fees.

Mr Rae is looking to purchase a house for $350,000, with a deposit of $50,000 (i.e. he requires a mortgage of $300,000). Mr Rae has requested details of the mortgage payments which would be due each year throughout the term of the mortgage under both a fixed rate and variable rate basis.

Household budgeting

Mrs Rae runs the Rae family's general household finances. Following the mortgage projections requested by Mr Rae, Mrs Rae has asked for some broader household budgeting advice based on her desire to save sufficient money to enable them to undertake renovations on the house they are purchasing.

To help you in projecting the family's savings, Mrs Rae has provided you with her expectations for household income and expenditure over the coming years:

	Annual amount ($)	*Comment*
Available income after tax	30,000	Expected to increase annually in line with inflation
Inflation linked expenditure	7,250	Household expenditure such as utility bills Expected to increase in line with inflation
Income linked expenditure	25%	Household expenditure such as leisure activities Varies with income

The mortgage payments will need to be added to the expenditure and will depend on whether Mr Rae has chosen the fixed or variable rate. You should assume that he will choose the fixed rate if its average monthly cost is no more than $50 above that of the average monthly cost of the variable rate mortgage. If not, he will choose the variable rate. (This is a one-time decision, made at the time of taking out the mortgage.)

It is assumed that the Raes' saving fund is currently at $0. The money left at the end of the year (i.e. income – expenditure) is added to the savings fund. It is assumed that the savings fund will not earn any interest.

The renovations Mrs Rae wishes to undertake are anticipated to cost $20,000. She would therefore like to know when the household's savings are likely to reach $20,000 to know when to the renovations can start.

Data

Forward interest rate curves

Source: Central Bank (CB)

Year	*Interest Rate F (%)*	*Interest Rate V (%)*
1	0.10	0.38
2	0.28	0.60
3	0.59	0.90
4	0.90	1.19
5	1.22	1.42
6	1.50	1.61
7	1.76	1.78
8	1.98	1.90
9	2.17	2.01
10	2.34	2.11
11	2.48	2.18
12	2.61	2.25
13	2.71	2.30
14	2.79	3.34
15	2.86	2.38
16	2.92	2.40
17	2.98	2.42
18	-3.02	2.44
19	3.06	2.45
20	3.08	2.46
21	3.11	2.46
22	3.14	2.46
23	3.16	2.46
24	3.17	2.46
25	3.19	2.46

The interest rates *F* and *V* (%) are one year forward rates, compounded annually.

- *F* relates to CB's fixed interest rate mortgage product.
- *V* relates to CB's variable interest rate mortgage product.

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Additional guidance

Five-year forward rate

The *t*-year forward rate for a year in the future, year *s*, is calculated as follows:

> Multiply together (1 + forward interest rate) over the *t* years, starting in year *s*. Take the t^{th} root of this and deduct 1.
>
> e.g. the 5 year forward rate for year 10 would be:

$$\left[\prod_{x=10}^{14}(1+i_x)\right]^{\frac{1}{5}}-1$$

Calculating the mortgage payments

Having calculated forward rates (one-year or five-year for variable and fixed rate mortgages respectively), monthly compounded interest rates need to be calculated. An annually compounded interest rate can be converted to an interest rate compounded *m* times per year using the following formula:

$$i^{(m)}=\left((1+i)^{1/m}-1\right)\times m$$

The present value of an annuity which pays amount 1/*m* in arrears *m* times per annum for *n* years is given by:

$$a_n^{(m)}=\frac{1-v^n}{i^{(m)}}$$

The outstanding mortgage at the beginning of the year is equal to, in:

Year 1: The mortgage required plus any mortgage fees payable.

Year *n* (*n* > 1): The outstanding mortgage at the end of year *n* – 1 plus any mortgage fees payable.

The payments to the bank can be found by taking the outstanding mortgage at the beginning of the year, divided by the annuity applicable to that year.

The outstanding mortgage at the end of the year is equal to:

> (Outstanding Mortgage at the BOY) × (1 + one year forward interest rate)
> – (payments to bank) × (1 + one year forward interest rate)$^{5.5/12*}$

BOY = beginning of year

*The payments to the bank are assumed to be made monthly in arrears. Therefore on average they are made 6.5 months through the year. Hence, on average, the interest on the mortgage will have been overstated to a value of 5.5 months' interest on those payments.

Fixed rate mortgage

Some elements of the mortgage repayments are to remain fixed over each five year period, or only occur once every 5 years. For example the fees are only due once every five years when the mortgage is renewed.

The **MOD()** function in Excel returns the remainder after a number is divided by a divisor. For example, MOD[number, divisor], will return the remainder once you divide "number" by the "divisor". MOD[23, 3] would return 2 (as 23 divided by 3 is 7, remainder 2).

Useful Excel functions

The **MATCH()** function in Excel can be used to return the position of a particular cell within an array. For example, MATCH [value, array] will return the position within your array where value is found. The default MATCH() function will find the largest value that is less than or equal to value.

The **AND()** function in Excel can be used to return TRUE if all of the conditions are true. It returns FALSE if any of the conditions are false. For example, AND [condition 1, condition2,…] will return TRUE if all of the conditions are true.

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