ACTUARIAL REFERENCE No (ARN)						

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION BOOKLET

2015

CA2: Model Documentation, Analysis and Reporting

Paper 2

DO NOT OPEN UNTIL INSTRUCTED TO DO SO.

Examination instructions

1. You should periodically save all the files you are working on onto the PC's hard drive.

You will be given instructions for submitting your work at the end of the examination.

It is your responsibility to ensure your work is adequately saved.

2. Ensure that your spreadsheet, audit trail (if separate) and summary are clearly labelled and your filenames include your ARN **e.g. Summary_ARN.docx**

Please note that you should use your ARN and NOT your name on all of the material you submit for marking.

The work you submit MUST be saved in Microsoft 2007 format, i.e. using docx (Word) or xlsx (Excel) file extensions. Do not embed documents in your spreadsheet.

Each file should contain your ARN as a header or footer on at least one page.

3. You must submit your spreadsheet model, audit trail and summary by the end of the stated exam time. By submitting your files you are confirming that all material is entirely your own work and you wish this to be taken into account for this assessment.

It is your responsibility to ensure that a complete electronic copy of your work is submitted.

You must stop working after this time as failure to do so could result in your exam not being marked.

4. You must also hand in this examination booklet, together with any other materials from the examination. This includes handing in any planning or rough notes that you have made during the examination, and any print-outs that you have done of your work.

Professional behaviour is mandatory and no material relating to the exam may be taken from the exam room nor disclosed or discussed with others.

Failure to comply with this will be deemed to be a breach of examination regulations and may result in disciplinary action.

Data has been provided electronically.

You should use the first 15 minutes of the exam as reading and planning time.

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Exam Requirements

1. Read the background document, which describes the scenarios that have been modelled and documented for this project.

2. Read the audit trail which has been written by your colleague, another actuarial student, for the calculations that they performed. This will assist you in following and understanding the calculations performed in the Excel model provided.

You are not required to add to or amend the audit trail.

3. Build on the model provided to determine the maximum annual mortality improvement rate which can be experienced before the net present value of the cash flows under Long Re's contract becomes negative. **[5]**

4. Illustrate the following using suitable charts:

- the composition of the annuity portfolio
- the premium income and expected outgo under the base scenario
- a comparison of the projected net cash flows in each of the scenarios produced by your colleague and in your new scenario

When producing these charts, you should assume that your colleague's calculations have been checked and are correct. **[10]**

5. Prepare a summary document of around five to seven pages, capturing the main features and results from the entire model. You can assume that the summary is being prepared for your boss, a senior actuary, who will present the work to the sign-off committee tomorrow.

Your summary should cover the following:

- purpose of the project, data, approach and assumptions used
- results, including charts
- commentary on results, key conclusions and suggested next steps

You are not required to add to the audit trail.

Marks available for the summary:

Methodology (including purpose, data, approach and assumptions)	**[20]**
Results, including charts	**[10]**
Commentary on results and key conclusions	**[20]**
Next steps	**[25]**
Drafting	**[10]**

[Sub-total 85]

[Total 100]

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Background

In the country of Actuaria there is an active annuity market, with the majority of pensioners purchasing immediate annuities from life insurance companies at age 65.

The future cash flows that the insurance companies have to pay out are uncertain since it is not known when each pensioner (or "annuitant") will die and hence the annuity payments will stop.

Over recent years, concern has grown amongst Actuarian life insurance companies that pensioners who have purchased annuities may live for much longer than had been expected when the annuities were priced, and hence the companies may be making losses on this business.

You are an actuarial student working at Long Re, a reinsurance company based in Actuaria. Long Re specialises in providing contracts that enable a life insurance company to swap uncertain future immediate annuity cash flows for a known stream of future cash flows.

This known stream of future cash flows is calculated as the insurance company's *expected* values of its uncertain future immediate annuity cash flows **plus** a constant loading factor that is determined by Long Re. This constant loading factor is referred to as Long Re's price.

The manner in which the contract operates is detailed at the end of this booklet.

An insurance company (Ins Co) specialising in single life level immediate annuities with benefits that are payable annually has recently approached Long Re. Ins Co wishes to swap the uncertain future cash flows on its in-force annuity business for a known stream of future cash flows.

Ins Co has provided the following information:

- high level details of its annuity portfolio, including the fact that more than 95% of the annuitants are female

- a table of mortality rates for females that reflects the insurance company's best estimate view of current female annuitant mortality

The insurance company has also stated that under its best estimate assumptions, it expects mortality to improve by 1% each year in the future.

Ins Co has asked that an initial price be calculated based on the details it has provided and it is not prepared to provide further information at this time.

Long Re's approach to pricing is first to determine the loading that should be applied to the insurance company's best estimate cash flows in order to cover:

- Long Re's own best estimate future cash flows.
- Long Re's expenses.

This loading is referred to as the "break even" price. A profit margin of 2.5% is then added to this "break even" price in order to determine the total price that Long Re will charge for the contract. That is, if the "break even" price is determined as x% then the overall price is a loading of x+2.5%, which is used to uplift the insurance company's best estimate cash flows.

Whilst Long Re agrees with the insurance company's view of mortality improvements, it believes that current female annuitant mortality is lighter. Specifically, its analysis suggests using 97% of the insurance company's mortality table.

Long Re's expenses for taking on these contracts consist of annual fixed expenses of $50,000 plus variable expenses equal to 0.05% of the annuity benefit paid. Fixed expenses have increased by an average 3% per annum over the last 15 years.

Your boss, who is an actuary, has already asked your colleague to build a model to determine an indicative price for providing the contract. This involved projecting the cash flows that Long Re will receive from the insurance company and the cash flows that Long Re expects to pay.

Based on the assets Long Re intends to hold, your colleague has used a flat discount rate of 5% per annum.

Additionally, your boss requested that a stress scenario be performed with a higher mortality improvement rate, namely 1.5% per annum. This would not impact the cash flows received from the insurance company but would impact Long Re's expected outgo.

These calculations have been completed and your colleague has also produced a first draft of his audit trail.

Your colleague is currently at an actuarial conference and cannot be contacted. In his absence your boss needs you to prepare a summary document for when she returns to the office in three hours.

Additionally, your boss would also like you to determine the rate of mortality improvement that would result in the net present value of the net future cash flows being zero.

You are not expected to include this additional calculation in the audit trail, but the results should be presented in the summary.

Data

Annuity Portfolio

The details provided on the insurance company's in-force immediate annuity portfolio are:

Age Nearest	*Number of Annuitants*	*Annual Average Benefit (in $)*
75–79	2,100	25,000
80–84	3,900	19,000
85–89	4,200	17,000
90–94	3,700	18,000
95–99	1,000	23,000
100+	25	21,000

where it is known that more than 95% of the annuitants are female.

Source: Ins Co

Mortality Rates

Mortality rates have also been provided by Ins Co.

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Additional information – Cash flow swap contract

When an insurance company writes an immediate annuity, it is exposed to the risk that the annuitant lives longer than it had expected.

Long Re offers a contract that enables an insurance company to transfer this risk by "swapping" the uncertain future annuity payments relating to a portfolio of in-force immediate annuities for a known stream of future cash flows.

Hence, instead of paying the uncertain future annuity payments in respect of that portfolio, the insurance company will:

- pay (to Long Re) a known stream of future cash flows for a fixed period – these payments are known as "premiums".

- receive (from Long Re) the actual future annuity payments until the last annuitant dies.

In practice, rather than the insurance company paying known cash flows to Long Re and Long Re paying the actual annuity amount to the insurance company, only the net difference is exchanged.

The known stream of future cash flows is based on the insurance company's best estimate of future mortality, plus a constant loading factor determined by Long Re. This loading is the price that Long Re is charging for the contract and is normally expressed as a percentage.

The premium due to be paid to Long Re at time t would be:

$$\text{Premium due at time } t = \text{Expected annuity payment at time } t \times (1 + \text{price})$$

where the expected annuity payment is based on the insurance company's best estimate.

Example

Under its best estimate mortality assumption, an insurance company expects to pay \$10,000 in annuity benefits at future time t in respect of its in-force portfolio. Assume that in this case Long Re has determined that the "break even" price on this contract to be 25 basis points, i.e. 0.25%. The total price including profit loading is therefore 2.75%, and the premium due to be payable by the insurance company to Long Re at time t will therefore be:

$$\$10{,}000 \times (1 + 2.75\%) = \$10{,}275$$

This would be agreed at the outset of the swap contract.

At time t, the actual amount that needs to be paid to annuitants may turn out to be \$11,000 (i.e. if more of them survived the period to time t than had been initially expected). Long Re will then pay the difference of \$725 (i.e. \$11,000 – \$10,275) to the insurance company.

Alternatively, at time t the amount payable to the annuitants could be \$9,000 (i.e. if more of them died during the period to time t than had been expected). In this case, the insurance company will then pay the difference of \$1,275 (i.e. \$10,275 – \$9,000) to Long Re.

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Audit trail

The following audit trail should be read alongside the provided model.

Objective

The purpose of the spreadsheet is to complete the following calculations in relation to an immediate annuity portfolio:

- Determine initial rates of annuitant mortality from the insurance company and Long Re's perspective.
- Project the expected benefit cash flows arising in respect of the annuity portfolio on the insurance company's best estimate mortality assumptions and the annuity benefit and expense cash flows using Long Re's assumptions.
- Determine the loading which must be applied to the insurance company's best estimate benefit cash flows so that the net present value is equal to the net present value of the annuity benefit and expense cash flows under Long Re's assumptions.
- Include a profit margin to determine Long Re's price.
- Calculate the net present value if mortality improvement rates are higher than those under the base scenario.

NB: Input cells are shown in blue.

Data

This worksheet includes data provided by the insurance company.

The insurance company has provided details of its in-force immediate annuity portfolio split by age band, number of annuitants and the average annual benefit for each band. The insurance company has also stated that more than 95% of the annuitants are female.

A quick check by eye suggests there are no data items missing or materially misstated.

The worksheet also includes female mortality rates for ages 75 to 120. An auto check has been performed in column K to ensure that all mortality rates are in the range [0, 1].

Assumptions

- Assume that the portfolio data provided is correct and contains no errors.
- Assume that the age definition for the female mortality table is age exact.
- Assume that births are uniformly distributed over the calendar year, so that age nearest (in-force portfolio age definition) is on average equivalent to age exact.
- Assume that ages are uniformly distributed within each age group, so that the average policyholder age will be the mid-point of the age group. For example, for the age group 75–79 it is assumed that the average age of policyholders will be exact age 77.

- Assume that all annuities are level, i.e. the same amount with no increase or decrease from year to year.

- Within the age group 100+, assume that the average age is 105.

- Assume a maximum possible age of 120, namely $q(120) = 1$ under both the insurance company and Long Re's view on mortality.

- Assume that the entire portfolio will be female, given that the proportion is known to be close to 100%. This will be prudent if female mortality is lighter than male mortality.

- Assume that no-one will enter or leave the portfolio between the effective date of the portfolio data and the start date of the longevity swap.

- No allowance is made for ageing between the effective date of the portfolio data and the start date of the longevity swap. This will be prudent.

- Assume that tax and reserving costs can be ignored.

Parameters

This worksheet contains all the parameters for the model.

The following parameters need to be input to column B of this worksheet: discount rate, Long Re's base mortality adjustment factor, mortality improvement factor, fixed and variable expenses, expense inflation, profit margin and revised mortality improvement factor.

The names of each parameter are detailed in column C.

Portfolio

This worksheet determines the portfolio breakdown and the mortality rates for Long Re.

Table 1 reads in the portfolio data from the **Data** worksheet. For convenience a label A, B, …, F is provided to each age band 75–79, 80–84, …, 100+ respectively.

The average age for each age band is taken to be the mid-point (i.e. average of minimum and maximum whole age calculated in column D). For the age band 100+ it is assumed that the average age will be 105.

The total amount of benefit for each band is determined as the product of the number of pensioners and the average benefit (column G).

Table 2 reads in the initial rates of mortality q_x for each age from $x = 75$ to $x = 120$ from the **Data** worksheet (column J).

Long Re's mortality rates (column K) are determined by calculating $\tilde{q}_x = \text{mort adj factor} \times q_x$ for each age from $x = 75$ to $x = 119$, where the mortality adjustment factor is taken from the **Parameters** worksheet.

For $x = 120$, $\tilde{q}_x$ is manually set to 1.

Base Price

This worksheet determines the base price for the longevity swap.

For each age band A–F the following calculations are performed (in columns, going across the worksheet):

The age, x, at time $t = 0$ is taken to be the assumed average age of the age band.

Income and Outflow are considered from Long Re's perspective.

Time and age increase by one year in each row of the worksheet. Let $L(x,t)$ represent the number of lives at age x and time t and $q(x,t)$ the mortality rate applicable to age x exact at time t.

Income

- The number of lives at age x at time $t = 0$ is looked up from the portfolio data.
- The number of lives $L(x, t)$ for $t \geq 1$ is given by the number of lives at the previous time period multiplied by the probability of survival. Namely

$$L(x,t) = L(x-1,t-1) \times [1 - q(x-1,t-1)]$$

- The insurance company q_x is looked up from the **Portfolio** worksheet and adjusted for mortality improvements by the following method:

$$q(x,t) = q_x \times (1 - \text{mortality improvement factor})^t$$

  The base scenario mortality improvement factor is taken from the **Parameters** worksheet.

- The expected **income** for Long Re is given by

$$L(x, t) \times \text{Average Benefit} \times (1 + K)$$

  where the Average Benefit is taken from the portfolio data and K is the price required to break even. K is to be determined and can be found in cell F2.

Outflow

- The number of lives $L(x, t)$ is determined as for the Income, but using Long Re's mortality rates.
- Long Re's $\tilde{q}_x$ is looked up from the **Portfolio** worksheet and adjusted for mortality improvement, namely

$$\tilde{q}(x,t) = \tilde{q}_x \times (1 - \text{mortality improvement factor})^t$$

- The expected **outflow** for the reinsurance company is given by

$$L(x, t) \times \text{Average Benefit}$$

  where the Average Benefit is taken from the portfolio data. There is no allowance for the price of the longevity swap in this cash flow.

The above calculations are repeated for all age bands.

Net Present Value

The Total Income at each time period is determined in column AY by summing over the expected income for each age band.

Similarly the Total Benefit Payment at each time period is determined in column AZ by summing over the expected outflow.

Fixed expenses at each time period (column BA) are simply the initial fixed expense parameter value increased by inflation each year, i.e. initial expense × (1 + expense inflation)^t.

Variable expenses (column BB) are calculated as the variable expense parameter multiplied by the expected total benefit payment at each time period.

(The expense and inflation parameters are picked up from the **Parameters** worksheet.)

The net value of the cash flows is given in column BD by:

$$\text{Net Cashflow} = \text{Total Income} - \text{Total Benefit Payment} - \text{Fixed Expenses} - \text{Variable Expenses}$$

A net present value (NPV) of the cash flows is determined in column BE by discounting the cash flows recursively at the given discount rate (from the **Parameters** worksheet) and summing over each time period back to time zero.

Longevity Price

The break even price K is determined by varying the value of K until the Net Present Value of cash flows at time 0 is 0. This is achieved via a goal seek. Instructions for the use of goal seek are given at the top of the worksheet.

A check in cell I1 shows whether the goal seek needs to be re-run.

The final longevity swap price is determined by adding the appropriate profit margin to the break even price, namely

$$\text{Longevity Swap Price} = K + \text{profit margin}$$

where the profit margin parameter is taken from the **Parameters** worksheet.

Premiums

The premiums the reinsurance company expects to receive at each time period are the unloaded premiums multiplied by (1 + Longevity Swap Price). These are obtained from the values in the Total Income column as follows

$$\text{Premium} = \frac{\text{Total Income}}{(1+K)} \times (1 + \text{Longevity Swap Price})$$

The net cash flow is then:

$$\begin{aligned}\text{Net Cashflow} = {} & \text{Final Premiums} - \text{Total Benefit Payment} \\ & - \text{Fixed Expenses} - \text{Variable Expenses}\end{aligned}$$

and the NPV of these cash flows can be determined as previously.

Higher Mortality Improvement Factors

This worksheet determines the net present value of cash flows if the annual mortality improvement factor increases.

The premium is unchanged so can be read in from the "Base Price" worksheet.

For each age band A–F the revised expected outgo is calculated. This is calculated as in the base price calculation, but with

$$q(x,t) = q_x \times (1 - \text{revised mortality improvement factor})^t$$

where the new mortality improvement factor is picked up from the **Parameters** worksheet.

The expected outgo values for each age band are determined and summed to give a "Total Benefit Payment" in column AH.

Fixed expenses are unchanged from the base price calculation.

Variable expenses are calculated in the same way as for the base price calculation, using the new total benefit payment.

The net cash flow and NPV are determined as before.

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