INSTITUTE AND FACULTY OF ACTUARIES

SUMMARY

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CA2: Model Documentation, Analysis and Reporting

Paper 2 (L21 P2)

Summary

Dog food mortality improvements project

Objective

The purpose of this project is to assess for the client, PetCo, the potential impact on the expectations of life from birth of three popular pet dog breeds (Weibull terrier, Poisson poodle and Gamma hound) of a new dog food product: Fitter Food For Fido.

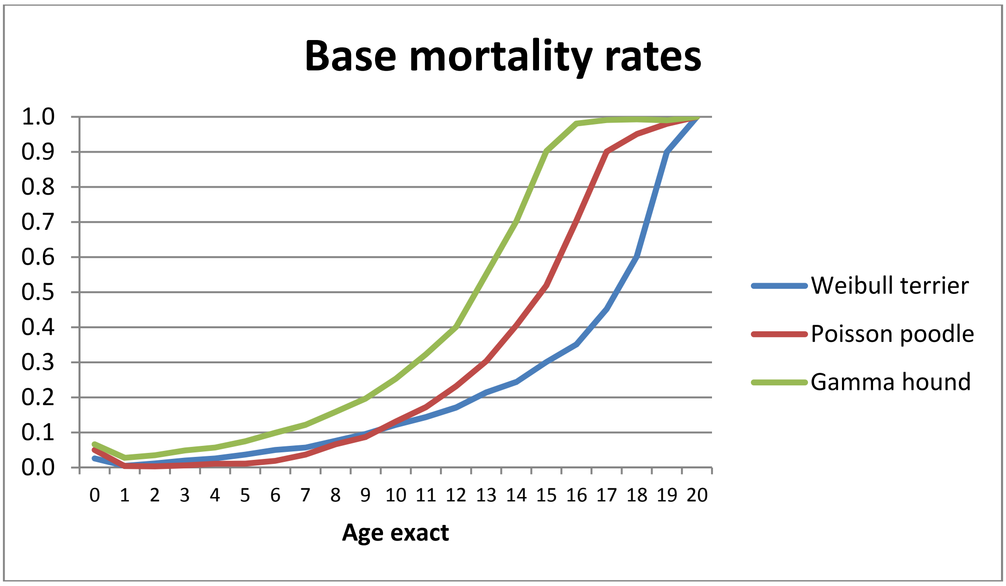
The project assesses the expectations of life from birth of each of these dog breeds using the base mortality provided, and then the expectations of life from birth if the new dog food results in a simple mortality rate improvement of 20% throughout life, from age 2 onwards. It also assesses the expectations of life from birth if the mortality rates improve by 20% on a compounded basis, from age 2 onwards.

Finally, the simple and compound mortality rate reduction factors that would be required in order for the Gamma hound (the national dog of Actuaria) to have an expectation of life of 10 years are determined.

Data

Mortality rate data for each breed, for male dogs, at each age exact from age 0 to 20 inclusive have been provided by the Canine Research Association of Actuaria.

These mortality rates are illustrated in the chart below:



A high level review by eye suggests that the pattern of mortality rates is reasonably smooth for each breed. The shape (high initially, reducing at age 1 but then increasing more rapidly with age) is also appropriate. This suggests that no data items are missing or materially misstated.

Auto checks have also been performed on the mortality rates to ensure they are always non-negative and don't exceed 1.

Assumptions

- Assume that 21 is the limiting age under all scenarios.
- **Assume that the mortality rate data provided is correct and contains no errors.**
- **Assume that the analysis would show similar results for female dogs.**
- **Assume that the mortality improvement factor of 20% indicated by PetCo is valid.**
- **Assume that the mortality improvements start immediately, i.e. as soon as the food is eaten.**
- **Assume that the mortality improvement factor is constant over time.**
- **Assume that the mortality improvement factor is the same for all ages.**
- **Assume that the same mortality improvement factor is applicable to each breed of dog.**
- **Assume that the dogs do not eat the food at ages lower than 2.**

Methodology

Base scenario: Expectations of life from birth

The probabilities of survival (p_x) are determined as $1 - q_x$ for each breed of dog and at each age from 0 to 20, and these are used to determine the cumulative probabilities of survival from age 0 to age t (${}_t p_o$), for t = 1 to 20, again for each breed of dog. These ${}_t p_o$ are calculated as the product of p_x from ages x = 0 to $t - 1$.

The expectation of life from birth for each of the three breeds of dog is calculated by summing the ${}_t p_o$ over the range t = 1 to 20.

Scenario 2: Simple 20% mortality reduction factor

The mortality rates for each breed and at each age from x = 2 to 19 inclusive are determined as the base mortality rate multiplied by 0.8. The mortality rates at ages 0 and 1 are unchanged from the base rates, as the food is assumed not to be eaten at these youngest ages.

The mortality rate at age 20 has been left at 1 (which is consistent with the assumption about the limiting age of 21).

The cumulative probabilities and expectations of life from birth are then recalculated in the same way as under the base scenario, using these adjusted mortality rates.

Scenario 3: Compound 20% mortality reduction factor

These calculations are the same as for the simple 20% reduction factor, except that the base mortality rates are now adjusted by the compounded factor $0.8^{(x-1)}$ for ages x = 2 to 19.

Scenario 4: Gamma hound target expectation of life using simple mortality reduction factor

The expectation of life calculations are performed again for the Gamma hound only, using the simple mortality rate improvement approach. The improvement rate is varied from the original 20% simple adjustment, until the expectation of life from birth equals 10 years.

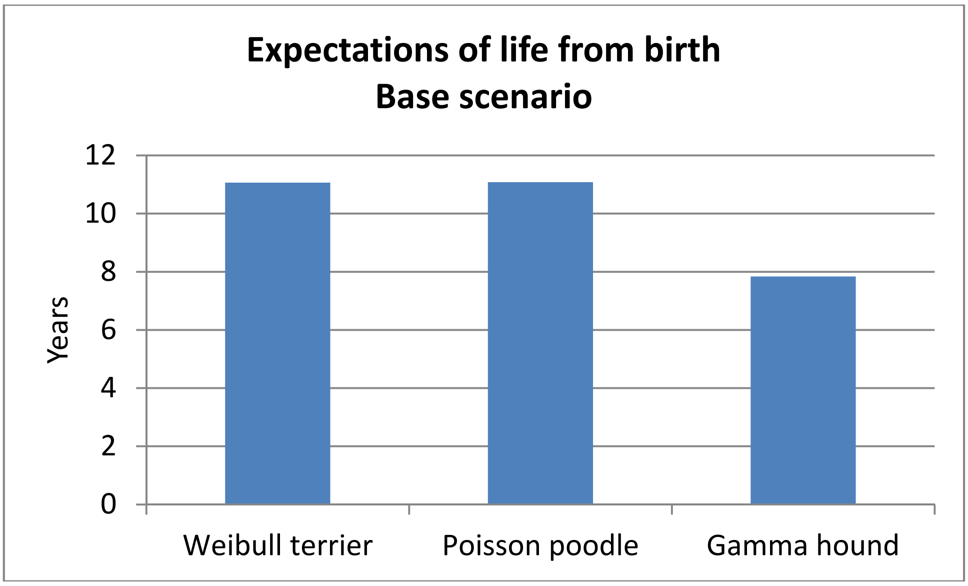
Scenario 5: Gamma hound target expectation of life using compound mortality reduction factor

The target expectation of life calculations are performed again for the Gamma hound only, this time using the compound mortality rate improvement approach. The improvement rate is again varied from the original 20% compound adjustment, until the expectation of life from birth equals 10 years.

Results

Base scenario

The base scenario expectations of life from birth for each of the three breeds are illustrated on the following chart:



The expectations of life from birth are the same for the Weibull terrier and Poisson poodle (to 1 dp) at 11.1 years. That for the Gamma hound is materially lower, at 7.8 years.

This is consistent with the patterns shown in the base mortality rates chart:

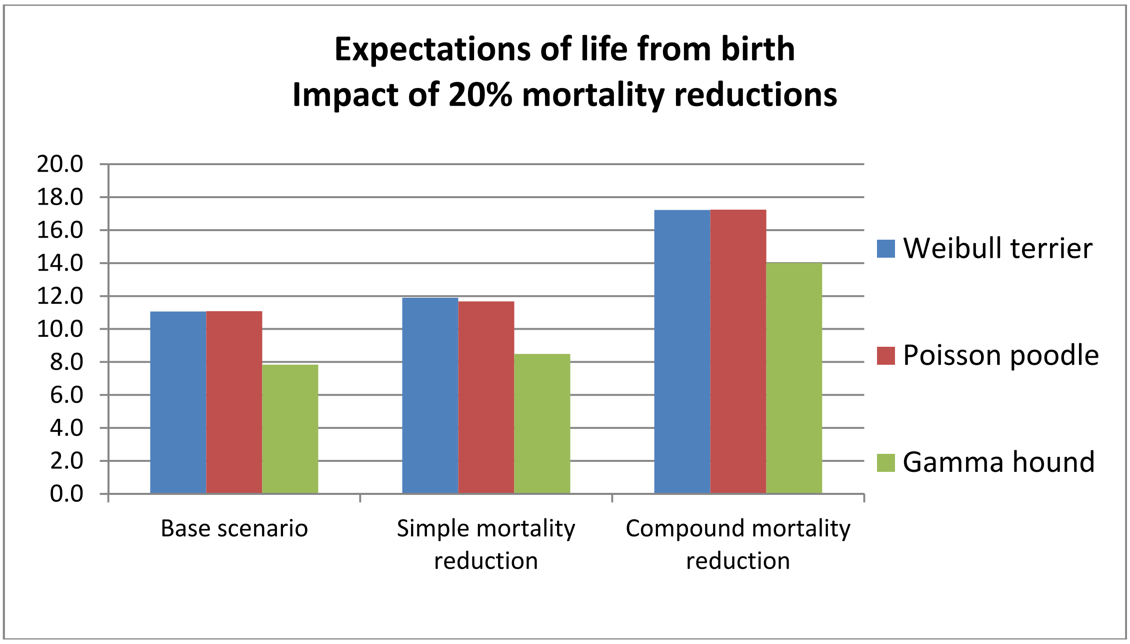
- Mortality rates for the Gamma hound are higher than for the other two breeds at all ages, therefore it would be expected that the expectation of life is lower (as expected to die sooner on average).

- Although mortality rates are higher for the Weibull terrier than for the Poisson poodle at the lower ages, they are lower for the terrier than for the poodle from age 10 onwards. Hence, overall, it is not unreasonable that the expectations of life from birth are broadly the same for these two breeds.

- The difference between the mortality rates for these two breeds is much greater at the higher ages, which might imply that overall the Poisson poodle should have the lower expectation of life. However, as a reducing proportion of dogs are expected to reach these higher ages, the relative weighting will be lower for this range.

Scenarios 2 and 3

The expectations of life from birth for each of the three breeds under the base scenario, simple mortality and compound mortality reduction scenarios are illustrated on the following chart:



For clarity, the expectations of life from birth illustrated are as follows (in years, to 1 dp):

	Weibull terrier	*Poisson poodle*	*Gamma hound*
Base scenario	11.1	11.1	7.8
Simple mortality reduction	11.9	11.7	8.5
Compound mortality reduction	17.2	17.2	14.0

Under the simple mortality reduction:

- The expectations of life increase for all breeds, due to the lighter mortality rates and hence greater probabilities of surviving to higher ages.

- The expectation of life for the Weibull terrier is now higher than for the Poisson poodle. This is because the impact of the lower mortality rates will be most significant at early ages (on a simple basis), due to a reducing proportion of dogs being expected to reach the higher ages. Therefore since the mortality rates at the lower ages are higher for the Weibull terrier than for the Poisson poodle, the impact of the reduction in mortality is more significant for the terrier.

Under the compound mortality reduction:

- The expectations of life are further increased for all breeds. This is because the compounding effect results in higher mortality reductions (after age 2) than under the simple approach.

- The expectation of life for the Weibull terrier is again the same as for the Poisson poodle (to 1 dp) and in fact the chart indicates that it is actually a little lower to a greater degree of accuracy. This is because the compounded mortality reductions will have a proportionately greater impact at later ages (once the compounding effect accelerates), which is where the Poisson poodle exhibits the higher mortality rates. Hence the Poisson poodle is benefited relative to the Weibull terrier under the compounded mortality reduction approach.

Scenarios 4 and 5

Under the simple model, the mortality reduction required to target an expectation of life from birth of 10 years for the Gamma hound is **51%**.

This is higher than 20% because the expectation of life under Scenario 2 is less than 10 years, so the improvement rate needs to be increased in order to model a greater number of hounds reaching higher ages.

The magnitude is broadly as would be expected, since a 20% reduction factor resulted in an increase of 0.7 years to 8.5 years, and therefore approximately a further 30% would be required in order to increase it by another 1.5 years to 10.0 years.

Under the compound model, the mortality reduction required to target an expectation of life from birth of 10 years for the Gamma hound is **9%**.

This is lower than 20% because the expectation of life under Scenario 3 is higher than 10 years, so the improvement rate needs to be reduced in order to model a lower number of hounds reaching higher ages.

The magnitude is not unreasonable: a 20% reduction factor resulted in an increase of 6.2 years to 14.0 years, and therefore between a third and a half of this would be appropriate to increase it by the 2.2 years required to achieve the target of 10.0 years.

Conclusions

The calculations are very sensitive to whether the improvement factors are allowed for on a compound or simple basis.

It is most likely that the intention was a simple mortality adjustment, as the compound mortality rates become very low at the highest ages, which is unlikely to be realistic.

Next steps

- Obtain confirmation as to whether the 20% improvement factor was intended to apply on a simple or compound basis.
- Validate the mortality rate data e.g. against another source.
- Check that the mortality rate data obtained is sufficiently up-to-date.
- If necessary, adjust the base mortality rate data for possible changes between the date at which the data was collected and the valuation date.
- If possible, validate the appropriateness of the 20% improvement factor claims.
- Obtain mortality data for female dogs and redo the calculations.
- Extend the mortality improvements beyond age 20 / remove or increase the limiting age.
- Adjust the model to allow for an improvement factor which varies over time.
- Allow for other (i.e. not pet food related) mortality changes in the model, e.g. due to expected improvements in veterinary care.
- Adjust the model to allow for an improvement factor which varies by dog breed.
- Adjust the model to allow for an improvement factor which varies by age.
- Test the impact if dogs eat the food at ages lower than 2.
- Extend the model to cover other breeds of dogs.
- Test the impact on expectations of life for dogs aged > 0.
- Enhance the model to test implications of the pet food for morbidity (i.e. health) as well as mortality.
- Solve for the improvement factor for the Gamma hound for different target expectations of life.
- Solve for improvement factors for a target expectation of life for the other two dog breeds.

- Sensitivity test the results by using different base mortality rates.
- Sensitivity test the results by using different improvement factors.
- Model the mortality rates and improvement factor stochastically so that a probability distribution of potential results can be produced.
- Test the model against emerging experience going forwards.

END OF SUMMARY