INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2019 Examinations

Subject CM1A – Actuarial Mathematics

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Mike Hammer
Chair of the Board of Examiners
July 2019

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Actuarial Mathematics subject is to provide a grounding in the principles of modelling as applied to actuarial work – focusing particularly on deterministic models which can be used to model and value known cashflows as well as those which are dependent on death, survival, or other uncertain risks.

2. Candidates may have concluded to different answers than what is shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations but candidates are not penalised for this. However, candidates may lose marks where excessive rounding has been used or where insufficient working is shown.

B. General comments on *student performance in this diet of the examination*

1. The comments that follow the questions concentrate on areas where candidates could have improved their performance. Where no comment is made, the question was generally answered well by most candidates. The examiners look most closely at the performance of the candidates close to the pass mark and the comments therefore often relate to those candidates.

2. This was a new large subject which was broadly a merging of the old CT1 and CT5 subjects. There appeared to be a large number of ill-prepared candidates who had underestimated the quantity of study required for the new larger subject. However, given this is a new subject, it is difficult to compare the performance of candidates in this diet with those in previous years.

C. Pass Mark

The Pass Mark for this exam in combination with CM1B was 58.

Solutions

Q1

${}_{10|4}q_{[27]+2}$ is the probability:

- that a life aged 29 [½]
- who joined the select population ... [1]
- .... at age 27 [½]
- will survive for 10 years [½]
- but then die in the subsequent 4 years. [½]

[Total 3]

This question was answered well. An answer defining the periods in terms of the relevant ages was given full credit. The most common error was to omit mention of select mortality.

Q2

$${}_{2.75}q_{84.5} = 1 - {}_{2.75}p_{84.5} = 1 - {}_{0.5}p_{84.5} \times {}_{2}p_{85} \times {}_{0.25}p_{87}$$

[1]

using UDD $\quad {}_{t-s}p_{x+s} = 1 - \dfrac{(t-s)q_x}{1-sq_x}$ for $0 \le s < t < 1$ and ${}_tq_x = tq_x$ for $0 \le t \le 1$

$${}_{0.5}p_{84.5} = 1 - \frac{0.5q_{84}}{1-0.5q_{84}} = 1 - \frac{0.5\times(0.08757)}{1-0.5\times(0.08757)} = 0.95421$$

[½]

$${}_{2}p_{85} = \frac{l_{87}}{l_{85}} = \frac{30,651}{38,081} = 0.80489$$ [½]

$${}_{0.25}p_{87} = 1 - 0.25q_{87} = 1 - 0.25\times(0.11859) = 0.97035$$

[½]

$${}_{2.75}p_{84.5} = 1 - 0.95421\times 0.80489 \times 0.97035 = 0.74526$$

$${}_{2.75}q_{84.5} = 1 - 0.74526 = 0.25474$$ [½]

[Total 3]

<u>Alternatively :</u>

$${}_{2.75}q_{84.5} = 1 - {}_{2.75}p_{84.5} = 1 - \frac{l_{87.25}}{l_{84.5}}$$

$$=1-\frac{\left(\frac{1}{4}l_{88}+\frac{3}{4}l_{87}\right)}{\left(\frac{1}{2}l_{84}+\frac{1}{2}l_{85}\right)}=1-\frac{\left(\frac{1}{4}\times 27,017+\frac{3}{4}\times 30,651\right)}{\frac{1}{2}\left(41,736+38,081\right)}$$

$$=1-\frac{29,742.5}{39,908.5}=0.25473$$

Generally well-answered although some candidates multiplied the q factors together in an attempt to calculate the overall probability of death rather than multiplying the p factors together to get an overall probability of survival.

Q3

An endowment assurance provides a survival benefit at the end of the term, but it also provides a lump sum benefit on death before the end of the term. [1½]

The benefits are provided in return for a series of regular premiums (or a single premium). [1]

The sum assured payable on death or survival need not be the same, although they often are. [½]

[Total 3]

A knowledge based question generally well-answered although weaker candidates tended to only make the first of the points above.

Q4

The value of the policyholders annuity benefit is given by:

$$v^{15}\times\frac{l_{65}^{f}}{l_{50}^{f}}\times\left(15,000\ddot{a}_{65}^{(12)}\right)$$

$$=v^{15}\times\frac{l_{65}^{f}}{l_{50}^{f}}\times\left(15,000\left(\ddot{a}_{65}-\frac{11}{24}\right)\right)=0.555265\times\left(\frac{9,703.708}{9,952.697}\right)\times 15,000\times\left(14.871-\frac{11}{24}\right)$$

$$=15,000\times 7.80363=117,039.50$$

[2½]

The value of the spouse's annuity benefit is given by:

$$v^{15}\times\frac{l_{65}^{f}}{l_{50}^{f}}\times\frac{l_{68}^{m}}{l_{53}^{m}}\times\left(8,000\left(\ddot{a}_{68}^{(12)}-\ddot{a}_{68:65}^{(12)}\right)\right)$$

$$= 0.555265 \times \left(\frac{9,703.708}{9,952.697}\right) \times \left(\frac{9,440.717}{9,922.995}\right) \times \left(8,000 \times \left(12.412 - 11.112\right)\right)$$

$$= 0.555265 \times \left(0.974983\right) \times \left(0.951398\right) \times \left(8,000 \times 1.3\right)$$

$$= 5,356.64$$

[3]

The total value of benefits is therefore $117,039.50 + 5,356.64 = 122,396.14$ [½]

[Total 6]

> *A more challenging question that distinguished well between stronger and weaker candidates. Many candidates did not identify the reversionary element of the benefit. Another common error was not to include the survival probability of the male life during the deferred period of the reversionary annuity.*

Q5

(i) With $d = 0.005$ per month, equivalent nominal rate of interest per annum convertible half-yearly is $i^{(2)}$ given by:

$$1 = \left(1 + \frac{i^{(2)}}{2}\right)^2 \times (1-d)^{12} = \left(1 + \frac{i^{(2)}}{2}\right)^2 \times (1 - 0.005)^{12} \Rightarrow i^{(2)} = 0.061064$$ [2]

(ii) With $d^{(0.5)} = 0.06$, equivalent nominal rate of interest per annum convertible half-yearly is $i^{(2)}$ given by:

$$1 = \left(1 + \frac{i^{(2)}}{2}\right)^4 \times \left(1 - \frac{d^{(0.5)}}{0.5}\right) = \left(1 + \frac{i^{(2)}}{2}\right)^4 \times (1 - 0.12) \Rightarrow i^{(2)} = 0.064949$$ [2]

(iii) With $i^{(4)} = 0.06$, equivalent nominal rate of interest per annum convertible half-yearly is $i^{(2)}$ given by:

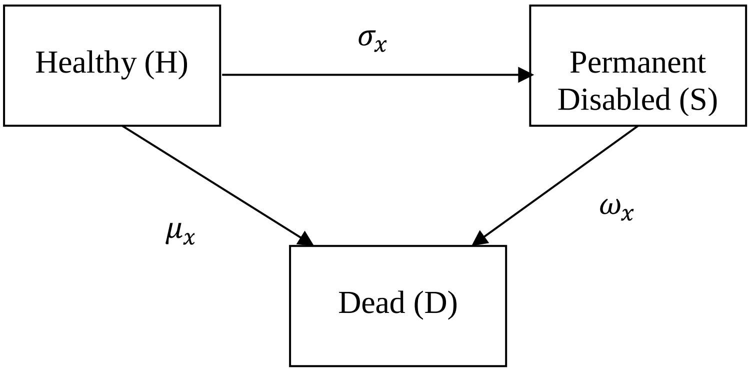
$$\left(1 + \frac{i^{(4)}}{4}\right)^4 = \left(1 + \frac{i^{(2)}}{2}\right)^2 \Rightarrow \left(1 + \frac{0.06}{4}\right)^4 = \left(1 + \frac{i^{(2)}}{2}\right)^2 \Rightarrow i^{(2)} = 0.060450$$ [2]

[Total 6]

> *Parts (ii) and (iii) were generally done well although many candidates failed to follow the rounding instructions.*

Q6

(i) The transition state model is shown by:



[2]

(ii) The expected present value of death benefits arising from the state H is given by:

$$\int_{t=0}^{20} 150,000\mu e^{-(\delta+\mu+\sigma)t}dt = 150,000\times(0.03)\int_{t=0}^{20} e^{-0.081t}dt = 4,500\times\left[\frac{-e^{-0.081t}}{0.081}\right]_{t=0}^{20}$$

$$= 55,555.56\times(1-0.197899) = 44,561.17$$

[2½]

The expected present value of the permanent disability benefit given by:

$$\int_{t=0}^{20} 75,000\sigma e^{-(\delta+\mu+\sigma)t}dt$$

$$= 75,000\times(0.001)\int_{t=0}^{20} e^{-(\delta+\mu+\sigma)t}dt = 75\int_{t=0}^{20} e^{-0.081t}dt = 75\times\left[\frac{-e^{-0.081t}}{0.081}\right]_{t=0}^{20}$$

$$= 925.93\times(1-0.197899) = 742.69$$

[2]

Therefore, EPV of death benefit from sick state is:

$$\int_{t=0}^{20} 150000\omega_t\, p_x^{HS} e^{-\delta t}dt$$

where

$$ {}_t p_x^{HS} = \int_0^t {}_s p_x^{\overline{HH}} \times \sigma_{x+s} \times {}_{t-s} p_{x+s}^{\overline{SS}} ds = \int_0^t e^{-0.031s} \times 0.001 \times e^{-0.08(t-s)} ds $$

$$ = \int_0^t 0.001 \times e^{-0.031s+0.08s-0.08t} ds = \int_0^t 0.001 \times e^{+0.049s-0.08t} ds $$

$$ = 0.001 \times e^{-0.08t} \times \left[\frac{e^{0.049s}}{0.049} \right]_0^t = \frac{0.001}{0.049} \times \left(e^{-0.031t} - e^{-0.08t} \right) $$

Thus EPV

$$ = \int_{t=0}^{20} 150000 \times 0.08 \times \left[\frac{0.001}{0.049} \left(e^{-0.031t} - e^{-0.08t} \right) \right] e^{-0.05t} dt $$ [2]

$$ = 244.90 \int_{t=0}^{20} \left(e^{-0.081t} - e^{-0.13t} \right) dt $$

$$ = 244.90 \times \left[\frac{e^{-0.081t}}{-0.081} - \frac{e^{-0.13t}}{-0.13} \right]_0^{20} = 681.19 $$ [1]

Therefore the total value of the expected benefits is:

$44,561.17 + 742.69 + 681.19 = 45,985.05$

i.e. Approximately £45,985 [½]

[Total 10]

Part (i) was done well. In part (ii), candidates found the calculation of the various probabilities to be challenging. The determination of ${}_t p_x^{HH}$ that was needed to calculate the EPV of death benefits arising from the healthy state should have been straightforward but most marginal candidates struggled with this. The ${}_t p_x^{HS}$ probability that was needed to calculate the EPV of death benefits arising from the sick state was considerably more difficult and candidates were given credit for any reasonable approach.

Q7

(i) (a) Duration of the annuity is $\dfrac{10,000(Ia)_{\overline{15|}}}{10,000\, a_{\overline{15|}}}$ at 5%

$$ = \frac{(Ia)_{\overline{15|}}}{a_{\overline{15|}}} = \frac{73.6677}{10.3797} = 7.0973 \text{ years} $$

[2]

(b) Duration of bond is $\dfrac{6(Ia)_{\overline{9|}}+900V^9}{6a_{\overline{9|}}+100V^9}$ at 5%

$$=\frac{6\times 33.2347+900\times 0.64461}{6\times 7.1078+100\times 0.64461}$$

$$= =\frac{779.5572}{107.1078}=7.2782 \text{ years}$$ [3]

(ii) The duration of the assets (the bond) is greater than the duration of the liabilities (the annuity). [1]
Therefore, if there is a small decrease in interest rates then the present value of the assets increases by more than the present value of the liabilities. [1½]
Therefore, the insurance company would make a profit. [½]
[Total 8]

Part (i) was done well. It is much more straightforward to calculate the duration/DMT of the bond directly than via the calculation of the volatility which involves some relatively complex differentiation. Part (ii) was very poorly done with many candidates assuming that because the company was not immunised, it must follow that it would make a loss.

Q8

(i) We have the accumulated amount

$$\begin{aligned}
&= 15,000\times\exp\left(\int_1^9 \delta(t)\,dt\right)\\
&= 15,000\times\exp\left(\int_1^2 (0.03+0.005t)\,dt+\int_2^9 (0.045-0.0025t)\,dt\right)\\
&= 15,000\times\exp\left(\left[0.03t+0.0025t^2\right]_{t=1}^{t=2}+\left[0.045t-0.00125t^2\right]_{t=2}^{t=9}\right)\\
&= 15,000\times\exp(0.0375+0.21875)\\
&= 19,381.14
\end{aligned}$$

[1 for formula + 3 for solution]

(ii) The PV of the payment stream is $PV=\int_{10}^{12}\rho(t)v(t)\,dt$

where $v(t)=\exp\left(-\int_0^t \delta(s)\,ds\right)$. [1]

Then, for $t\geq 10$, we have:

$$
\begin{aligned}
v(t) &= \exp\left[-\left(\int_0^2 (0.03+0.005s)\,ds + \int_2^{10}(0.045-0.0025s)\,ds + \int_{10}^{t} 0.02ds\right)\right] \\
&= \exp\left[-\left(\left[0.03s+0.0025s^2\right]_{s=0}^{s=2} + \left[0.045s-0.00125s^2\right]_{s=2}^{s=10} + \left[0.02s\right]_{s=10}^{s=t}\right)\right] \\
&= \exp\left[-\left(0.07+0.24+\left[0.02t-0.20\right]\right)\right] \\
&= \exp\left[-\left(0.02t+0.11\right)\right]
\end{aligned}
$$

[3]

Thus, the PV of the payment stream is:

$$
\begin{aligned}
&= \int_{10}^{12} 60e^{0.02t} \times e^{-(0.02t+0.11)}dt \\
&= 60e^{-0.11} \times \int_{10}^{12} dt \\
&= 120e^{-0.11} \\
&= 107.50
\end{aligned}
$$

[2]

[Total 10]

<u>Alternatively:</u>

The value of the payment stream at $t = 10$

$$= \int_{10}^{12} \rho(t)\exp\left(-\int_{10}^{t}\delta(s)\,ds\right)dt$$

with

$$\exp\left(-\int_{10}^{t}\delta(s)\,ds\right) = \exp\left(-\int_{10}^{t}0.02ds\right)$$

$$= \exp\left(-\left[0.02s\right]_{10}^{t}\right) = \exp\left(0.2-0.02t\right)$$

So value at $t = 10$

$$= \int_{10}^{12} 60e^{0.02t}e^{0.2-0.02t}dt = 60e^{0.2}\int_{10}^{12}dt = 60e^{0.2}\left[t\right]_{t=10}^{t=12} = 120e^{0.2}$$

PV $= 60e^{0.2}v(10)$ where

$$
\begin{aligned}
v(10) &= \exp\left[-\left(\int_0^2 (0.03+0.005s)\,ds + \int_2^{10}(0.045-0.0025s)\,ds\right)\right] \\
&= \exp\left[-\left(\left[0.03s+0.0025s^2\right]_{s=0}^{s=2} + \left[0.045s-0.00125s^2\right]_{s=2}^{s=10}\right)\right] \\
&= \exp\left[-(0.07+0.24)\right] \\
&= \exp\left[-0.31\right]
\end{aligned}
$$

and so PV $=120e^{-0.11}=107.50$

Generally well done although some candidates in part (ii), discounted using a fixed discount factor, e.g. $v(10)$, rather than a time-dependent discount factor.

Q9

Costs [½]

- Model development requires a considerable investment of time, and expertise.
- An example i.e. financial costs of development can be quite large given the need to check the validity of the model's assumptions, the computer code, the reasonableness of results and the way in which results can be interpreted in plain language by the target audience. [1½]

Multiple Runs [½]

- In a stochastic model, for any given set of inputs each run gives only estimates of a model's outputs.
- So, to study the outputs for any given set of inputs, several independent runs of the model are needed. [1½]

Input vs Output [½]

- As a rule, models are more useful for comparing the results of input variations than for optimising outputs. [1½]

Real world relevance [½]

- Models can look impressive when run on a computer so that there is a danger that one gets lulled into a false sense of confidence.
- If a model has not passed the tests of validity and verification its impressive output is a poor substitute for its ability to imitate its corresponding real-world system [1½]

Quality of data [½]

- Models rely heavily on the data input.
- if the data quality is poor or lacks credibility, then the output from the model is likely to be flawed
- parameter error [1½]

Black Box effect [½]

- users of the model must understand the model and the uses to which it can be safely put
- danger of using a model as a 'black box' from which it is assumed that all results are valid without considering the appropriateness of using that model for the data input and the output expected. [1½]

Predictability [½]

- It is not possible to include all future events in a model.
- For example, a change in legislation could invalidate the results of a model, but may be impossible to predict when the model is constructed. [1½]

Interpretation of results [½]

- It may be difficult to interpret some of the outputs of the model.
- They may only be valid in relative rather than absolute terms, as when, for example, comparing the level of risk of the outputs associated with different inputs. [1½]

[max 8]
[Total 8]

It was pleasing to see many candidates make a reasonable attempt at this question which was taken from part of the syllabus not previously included in CT1 or CT5.

Q10

(i) The prospective reserve is the expected present value of the future outgo less the expected present value of the future income. [2]

(ii) If

- the retrospective and prospective reserves are calculated on the same basis; and
- this basis is the same as the basis used to calculate the premiums used in the reserve calculation, using the equivalence principle

then the retrospective reserve will be equal to the prospective reserve [2]

(iii) The premium is given by $P = \dfrac{S\bar{A}_x}{\ddot{a}_x}$ [½]

The prospective reserve at time t is

$${}_tV_x^{\,prosp} = S\bar{A}_{x+t} - P\ddot{a}_{x+t}$$ [½]

The retrospective reserve at time t is

$${}_tV_x^{\,retro} = \frac{l_x}{l_{x+t}}(1+i)^t\left(P\ddot{a}_{x:\overline{t}|} - S\bar{A}^{1}_{x:\overline{t}|}\right)$$ [½]

From above

$$P\ddot{a}_x - S\bar{A}_x = 0$$

Multiplying by $\frac{l_x}{l_{x+t}}(1+i)^t$ gives

$$\frac{l_x}{l_{x+t}}(1+i)^t\left(P\ddot{a}_x - S\bar{A}_x\right) = 0$$

Adding this to ${}_tV_x^{prosp}$

$${}_tV_x^{prosp} = S\bar{A}_{x+t} - P\ddot{a}_{x+t} + \frac{l_x}{l_{x+t}}(1+i)^t\left(P\ddot{a}_x - S\bar{A}_x\right)$$

$$= S\left(\bar{A}_{x+t} - \frac{l_x}{l_{x+t}}(1+i)^t\,\bar{A}_x\right) - P\left(\ddot{a}_{x+t} - \frac{l_x}{l_{x+t}}(1+i)\,\ddot{a}_x\right)$$

$$= -S\frac{l_x}{l_{x+t}}(1+i)^t\,\bar{A}_{x:\overline{t|}} + P\frac{l_x}{l_{x+t}}(1+i)\,\ddot{a}_{x:\overline{t|}}$$

$$= \frac{l_x}{l_{x+t}}(1+i)^t\left(P\ddot{a}_{x:\overline{t|}} - S\bar{A}_{x:\overline{t|}}\right)$$

${}_tV_x^{retro}$ [2½]

[Total 8]

Alternatively:

$$P\ddot{a}_x = S\bar{A}_x$$

$$P\left(\ddot{a}_{x:\overline{t|}} + {}_tp_x \times v^t \times \ddot{a}_{x+t}\right) = S\left(\bar{A}^1_{x:\overline{t|}} + {}_tp_x \times v^t \times \bar{A}_{x+t}\right)$$

$$P\ddot{a}_{x:\overline{t|}} - S\bar{A}^1_{x:\overline{t|}} = \left({}_tp_x \times v^t\right)S\bar{A}_{x+t} - \left({}_tp_x \times v^t\right)P\ddot{a}_{x+t}$$

$$P\ddot{a}_{x:\overline{t|}} - S\bar{A}^1_{x:\overline{t|}} = \left({}_tp_x \times v^t\right)\left(S\bar{A}_{x+t} - P\ddot{a}_{x+t}\right)$$

$$\frac{1}{{}_tp_x \times v^t}\left(P\ddot{a}_{x:\overline{t|}} - S\bar{A}^1_{x:\overline{t|}}\right) = S\bar{A}_{x+t} - P\ddot{a}_{x+t}$$

$$\frac{l_x}{l_{x+t}}(1+i)^t\left(P\ddot{a}_{x:\overline{t|}} - S\bar{A}^1_{x:\overline{t|}}\right) = S\bar{A}_{x+t} - P\ddot{a}_{x+t}$$

$${}_tV_x^{\,retro} = {}_tV_x^{\,prosp}$$

In part (i), weaker candidates tended to miss out 'expected' and/or present value' from their definitions.
Part (iii) was answered poorly with many candidates unable to state the equation for the retrospective reserve.

Q11

(i) At 1/2/17, PV of future dividends

$$= 0.40 \times \left(1.05v + 1.04 \times 1.05v^2 + \sum_{k=1}^{\infty} 1.05 \times 1.04 \times 1.03^k v^{k+2} \right)$$

[2]

$$= 0.40 \times \left(\frac{1.05}{1.09} + \frac{1.05 \times 1.04}{(1.09)^2} + \frac{1.05 \times 1.04}{(1.09)^2} \times \sum_{k=1}^{\infty} \left(\frac{1.03}{1.09} \right)^k \right)$$

$$= 0.40 \times \left(\frac{1.05}{1.09} + \frac{1.05 \times 1.04}{(1.09)^2} + \frac{1.05 \times 1.04}{(1.09)^2} \times a_{\overline{\infty}|}^{i'\%} \right)$$

where $\dfrac{1}{1+i'} = \dfrac{1.03}{1.09} \Rightarrow i' = 0.058252427$

$$\Rightarrow a_{\overline{\infty}|}^{i'} = \frac{1}{i'} = 17.1\dot{6}$$

Hence $PV = 0.40 \times \left(0.9633028 + 0.9191146 + 0.9191146 \times 17.1\dot{6} \right)$

$= 7.0642 \quad (ie\ £7.06)$

[4]

(ii) Let i denote real return achieved:

$$7.00 \times \frac{221.2}{211.0} \times (1+i)^2 = 0.428 \times \frac{221.2}{215.7} \times (1+i) + (0.449 + 7.50)$$

[2]

$$\Rightarrow 7.33839(1+i)^2 - 0.43891(1+i) - 7.949 = 0$$

$$\Rightarrow 1+i = \frac{0.43891 \pm \sqrt{\left(0.43891^2 + 4\times 7.33839\times 7.949\right)}}{2\times 7.33839}$$

=1.0711 (+ ve root)

$$\Rightarrow i \ = 7.11\%\, pa$$

[3]
[Total 11]

<u>Alternatively</u>

$$7.00 = 0.428\times\frac{211}{215.7}\times v + \left(0.449+7.50\right)\times\frac{211}{221.2}\times v^2$$

$$0 = 7.5825\times v^2 + 0.41867\times v - 7.00$$

$$\Rightarrow v = \frac{-0.41867 \pm \sqrt{\left(\left(-0.41867\right)^2 + 4\times 7.5825\times 7\right)}}{2\times 7.5825}$$

$v = 0.933613058$ (+'ve root)

$$\Rightarrow i = 7.11\%\, pa$$

Generally well-done. Common errors were:

- *not to include the 5%/4% growth rates in the valuation of the dividends from year 3 onwards*
- *to include the dividend that had just been paid*

Many candidates used a trial and error/interpolation approach to find the yield in part (ii). Full credit was given for this approach.

Q12

(i) Let R denote the level monthly instalment.

Then, we have:

$$\begin{aligned} & 12R\times a^{(12)}_{\overline{10}|8\%} &=& \ 80{,}000 \\ \Rightarrow\ & 12R\times 1.036157\times 6.7101 &=& \ 80{,}000 \\ \Rightarrow\ & R &=& \ 958.86 \end{aligned}$$

[2]

(ii) On 1st November 2018, remaining term is 7 years and 2 months (i.e. $7\frac{2}{12}$ years). [½]

Then, outstanding loan is:

$$L = 12R\times a^{(12)}_{\overline{7\frac{2}{12}}|8\%} = 12\times 958.86\times\frac{1-v_{8\%}^{7\frac{2}{12}}}{0.077208} = 63{,}180.76$$ [2½]

Or alternatively, working in months and using effective interest rate of 0.6434% per month, we have:

$$L = 958.86 \times a_{\overline{86}|0.6434\%} = 958.86 \times \frac{1 - v_{0.6434\%}^{86}}{0.006434} = 63{,}180.54$$

(iii) (a) Work in months, where 9% per annum convertible monthly $\Rightarrow$ 0.75% per month.

Let n denote remaining number of months, given by:

$$\begin{aligned} 900 \times a_{\overline{n}|0.75\%} &\geq 63180.76 + 250 \\ \Rightarrow \frac{1 - v_{0.75\%}^{n}}{0.0075} &\geq 70.47862 \\ \Rightarrow v_{0.75\%}^{n} &\leq 0.471410 \\ \Rightarrow n \times \ln\left(v_{0.75\%}\right) &\leq \ln(0.471410) \\ \Rightarrow n &\geq 100.646 \end{aligned}$$

[3]

Thus, loan will be repaid in 101 months (or 8 years and 5 months) from 1st November 2018 $\Rightarrow$ final payment will now be made on 1st April 2027. [½]

(b) Let X denote amount of final instalment. Then, we have:

$$63180.76 + 250 = 900 \times a_{\overline{100}|0.75\%} + Xv_{0.75\%}^{101} \Rightarrow X = \frac{63430.76 - 900 \times 70.1746}{0.470164} = 581.97$$

[2½]

[Total 11]

This question was done well apart from part (iii)(b). A common error for that part was to not discount the final payment by the correct number of months (or indeed to discount it at all).

Q13

(i) (a) Total Reserve for the Endowment Assurance portfolio at 1st January 2019, ${}_{17}V^{EA}$ is:

$${}_{17}V^{EA} = 15{,}203 \times 200{,}000 A_{52:\overline{8}|} - 82{,}774{,}000 \times \ddot{a}_{52:\overline{8}|}$$

$$= 15{,}203 \times 200{,}000(0.73424) - 82{,}774{,}000 \times 6.910 = 1{,}660{,}561{,}804$$

(Reserve is 109,226 per policy in force on 1 Jan 2018) [2]

Death Strain at Risk :

$DSAR^{EA} = 15,203 \times 200,000 - 1,660,561,804 = 1,380,038,196$ (or 90,774 per policy in force on 1 Jan 2018)

[1]

Mortality Profit is given by:

$$MP^{EA} = q_{51} \times 1{,}380{,}038{,}196 - \left(\frac{46}{15{,}203}\right) \times 1{,}380{,}038{,}196$$

$$= 0.002809 \times 1,380,038,196 - \left(\frac{46}{15,203}\right) 1,380,038,196 = 3,876,527 - 4,175,607$$

$= -299{,}080$

[1½]

(b) Total Reserve for the Annuity portfolio as at 1st January 2019, ${}_{17}V^{ann}$,

$${}_{17}V^{ann} = 12,352 \times 10,000 \ddot{a}_{82}$$

$= 12,352 \times 10,000(6.801) = 840,059,520$ (or 68,010 per policy in force on 1 Jan 2018)

[1]

Death Strain at Risk

$DSAR^{ann} = 0 - (840,059,520 + 0) = -840,059,520$ (or -68,010 per policy in force on 1 Jan 2018)

[1]

Mortality Profit is given by:

$$MP^{ann} = q_{81} \times -840,059,520 - \left(\frac{746}{12,352}\right)(-840,059,520)$$

$$= 0.059952 \times -840,059,520 - \left(\frac{746}{12,352}\right)(-840,059,520) = -50,363,248 - (-50,735,460)$$

$=372{,}212$

[1½]

(ii) Endowment Assurance Policies

- With endowment assurances earlier than expected deaths lead to an earlier payment of the benefit - the benefit is paid as a death benefit rather than as a maturity benefit. This implies earlier than expected deaths leads to a mortality loss [1½]

- The company expected approximately 42.7 deaths, whereas 46 deaths actually occurred. So actual mortality was heavier than expected. [½]
- Here more deaths occurred than was expected and so the company suffers a mortality loss of £299,080. [½]

Annuity Policies

- With annuities there is no death benefit, however when a death occurs it leads to the release of the reserve being held to cover the future annuity payments. [1½]
- The company expected approximately 740.5 deaths, whereas 746 deaths actually occurred. So actual mortality was heavier than expected. [½]
- Here more deaths occurred than expected and so the company has a greater release of reserves than expected. Hence the company sees a mortality profit of £372,212 for these annuities. [½]

Mortality profit/loss from the 2 products cancel each other out to an extent. [1]

Mortality profit relatively small compared to book of business, especially for annuity business. [1]

[max 5]

The intention of the question was that the endowment assurance reserve should be calculated using the premium data given in the question. Instead some candidates calculated a net premium reserve.
Part (ii) was poorly answered despite the points required being comparatively straightforward.

END OF EXAMINERS' REPORT