

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

11 April 2019 (pm)

Subject CM1A – Actuarial Mathematics Core Principles

Time allowed: Three hours and fifteen minutes

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all questions, begin your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** Define in words ${}_{10|4}q_{[27]+2}$ [3]
- 2** Calculate ${}_{2.75}q_{84.5}$ using the method of uniform distribution of deaths.
Basis:
Mortality ELT15(Females) [3]
- 3** Describe the main features of an endowment assurance contract. [3]
- 4** A life insurance company provides the following benefits:
- an annuity, on survival to age 65, of £15,000 per annum payable monthly in advance
 - a spouse's annuity of £8,000 per annum payable monthly in advance on the death of the policyholder, provided that the policyholder survives to age 65.
- No benefit is payable if the policyholder dies before age 65.
- Calculate the single premium in respect of a female policyholder currently aged exactly 50 who has a male spouse currently aged exactly 53.
- Basis:
- | | | |
|---------------|--------------|----------|
| Mortality | Female | PFA92C20 |
| | Male | PMA92C20 |
| Interest rate | 4% per annum | |
| Expenses | Ignore | |
- [6]
- 5** Calculate, as a percentage to four decimal places, the nominal rate of interest per annum convertible half-yearly which is equivalent to:
- (i) an effective rate of discount of 0.5% per month. [2]
 - (ii) a nominal rate of discount of 6% per annum convertible every two years. [2]
 - (iii) a nominal rate of interest of 6% per annum convertible quarterly. [2]
- [Total 6]

6 A life insurance company issues a 20-year term assurance with additional permanent disability benefit. The benefits provided are:

- on death (whether the life was previously healthy or permanently disabled) a lump sum payment of £150,000 payable immediately
- on permanent disability a lump sum of £75,000 payable immediately.

(i) Draw a transition state model for this policy, labelling your diagram. [2]

(ii) Calculate the total expected present value of the benefits. [8]

Basis:

Force of mortality from healthy	0.03 for all ages
Force of mortality from permanent disability	0.08 for all ages
Force of permanent disability	0.001 for all ages
Force of interest	5% per annum

[Total 10]

7 An individual buys an annuity from an insurance company for a single lump sum premium. The annuity will pay £10,000 annually in arrears for 15 years. The insurance company invests the premium in a fixed-interest bond which pays coupons at the rate of 6% per annum annually in arrears and is redeemable at par in exactly nine years.

- (i) (a) Calculate the duration of the annuity at an interest rate of 5% per annum effective.
- (b) Calculate the duration of the bond at an interest rate of 5% per annum effective.

[5]

(ii) Explain whether the insurance company will make a profit or a loss if interest rates decrease slightly at all terms. [3]

[Total 8]

8 The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.03 + 0.005t & 0 \leq t < 2 \\ 0.045 - 0.0025t & 2 \leq t < 10 \\ 0.02 & t \geq 10 \end{cases}$$

(i) Calculate the accumulated amount at time $t = 9$ of an investment of £15,000 made at time $t = 1$. [4]

(ii) Calculate the present value at time $t = 0$ of a payment stream paid continuously from time $t = 10$ to time $t = 12$, under which the rate of payment at time t is $p(t) = 60e^{0.02t}$. [6]

[Total 10]

9 Describe four limitations of using models in actuarial work. [8]

10 (i) Define the term “prospective reserve” when used for a life insurance contract. [2]

(ii) State the conditions necessary for the prospective reserve to equal the retrospective reserve. [2]

A life insurance company issues a whole life assurance with sum assured S to a life aged exactly x . Annual premiums, payable annually in advance, are paid throughout the policy term. The benefit is payable immediately on death and there are no expenses.

(iii) Demonstrate that the prospective reserve is equal to the retrospective reserve at time t , assuming that the conditions referred to in part (ii) are met. [4]
[Total 8]

11 On 1 February 2017, an investor was considering purchasing ordinary shares in Actuarial PLC.

Dividends are payable annually, and a dividend of £0.40 per share had just been paid.

At the date of purchase, dividends were expected to grow each year on a compound basis. The rate of growth was expected to be 5% in the first year, 4% in the second year and 3% per annum thereafter.

The investor was not entitled to the dividend which had just been paid.

(i) Calculate the maximum price per share the investor would have been prepared to pay at this date to give a rate of return of 9% per annum effective, assuming the investor holds the share in perpetuity. [6]

The investor purchased a holding of shares on 1 February 2017 at a price of £7.00 per share and sold the holding at a price of £7.50 per share on 1 February 2019, immediately after receiving the dividend payment then due.

(ii) Calculate the effective annual real rate of return achieved by the investor between 1 February 2017 and 1 February 2019 using the following information:

<i>Date</i>	<i>Inflation index</i>	<i>Dividend per share</i>
1 February 2017	211.0	£0.400
1 February 2018	215.7	£0.428
1 February 2019	221.2	£0.449

[5]
[Total 11]

12 A loan of £80,000 was taken out on 1 January 2016. The loan was to be repaid over 10 years in level instalments payable monthly in arrears.

(i) Calculate the level monthly instalment using an effective rate of interest of 8% per annum. [2]

(ii) Calculate the amount of the loan outstanding on 1 November 2018 immediately after payment of the instalment then due. [3]

On 1 November 2018, immediately after payment of the instalment, the borrower asked that the monthly instalment be reduced to £900 and the remaining term extended as required to clear the outstanding loan amount. The final payment would be equal to the outstanding loan at the time, if less than £900.

The lender agreed to this change, subject to the following conditions:

- the interest rate applied in future is increased to 9% per annum convertible monthly; and
- an administration fee of £250 is added to the loan outstanding at 1 November 2018.

(iii) (a) Determine the new date on which the loan will be repaid.
(b) Calculate the final instalment paid.

[6]

[Total 11]

13 On 1 January 2002 a life insurance company issued the following policies:

- Identical 25-year without profit endowment assurances each with a sum assured of £200,000 payable at the end of the policy term or at the end of year of death if earlier. Premiums are payable annually in advance throughout the term or until earlier death. The policies were issued to lives aged 35 exact.
- Identical level whole life annuities, each payable annually in advance at a rate of £10,000 per annum, issued to lives aged 65 exact.

An extract from the company's records gives the following information for 2018 in respect of these policies:

<i>Policy type</i>	<i>Endowment assurance</i>	<i>Annuity</i>
Number of policies in force on 1 January 2018	15,203	12,352
Total annual premium for in-force policies as at 1 January 2018	82,774,000	—
Number of policyholder deaths during 2018	46	746

There were no other exits in 2018.

- (i) Calculate the mortality profit for the year ended 31 December 2018 in respect of:

- (a) endowment assurances
(b) annuities.

[8]

Basis:

Mortality	Endowment assurances	AM92
	Annuities	PMA92C20
Interest rate	4% per annum	
Expenses	Ignore	

- (ii) Discuss your answers in part (i).

[5]

[Total 13]

END OF PAPER