

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

12 April 2021 (am)

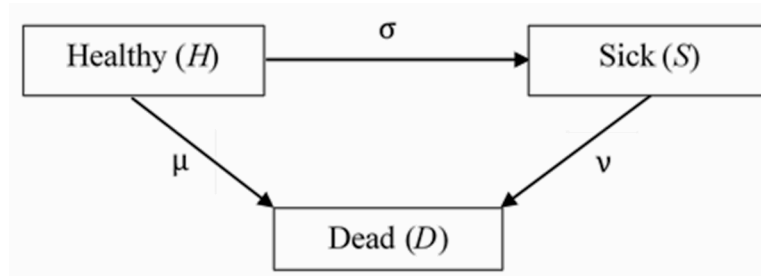
Subject CM1A – Actuarial Mathematics Core Principles

Time allowed: Three hours and fifteen minutes

<p>In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.</p>
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If you encounter any issues during the examination please contact the Assessment Team on
T. 0044 (0) 1865 268 873.

- 1 Calculate ${}_{3|5}q_{45:45}^1$ assuming AM92 mortality for both individuals and that the individuals are independent with regards to mortality. [4]
- 2 A life insurance company uses the following three-state model, with constant forces of transition, to price its stand-alone critical illness policies.



Under these policies, a lump sum benefit is payable when a life becomes critically ill during the policy term. No other benefits are payable.

A 30-year policy with sum assured \$150,000 is issued to a healthy life aged 35 exact.

The expected present value of the benefit at outset is given by the following formula:

$$m \times \int_a^b n \times e^{zt} dt.$$

- (i) State the numerical values of a , b , m , n and z . [3]
- (ii) Calculate the expected present value of the benefit for this policy based on your answer to part (i).

Basis: $\mu = 0.01$
 $\sigma = 0.02$
 $\nu = 0.04$

Interest: 3% p.a. effective

[3]
 [Total 6]

- 3** A fixed interest security of nominal amount \$100,000 was issued on 1 March 2017 and was redeemed at par on 1 March 2020. Coupons were paid at the rate of 4% p.a. annually in arrears.

The value of the inflation index at various dates during the term of the security was as follows:

<i>Date</i>	<i>Inflation index</i>
1 March 2017	240.5
1 March 2018	256.0
1 March 2019	272.8
1 March 2020	286.6

- (i) Demonstrate that the effective annual real rate of return achieved over the term of the security is approximately equal to -1.9% p.a. [5]
- (ii) Comment on the result in part (i). [3]
- [Total 8]

- 4** The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.03 + 0.005t & 0 \leq t \leq 6 \\ 0.1 - 0.01t & t > 6 \end{cases}$$

$A(0, t)$, the accumulation at time t of a unit of money invested at time 0, can be written as:

$$A(0, t) = \begin{cases} e^{a+bt+ct^2} & 0 \leq t \leq 6 \\ e^{f+gt+ht^2} & t > 6 \end{cases}$$

- (i) Calculate the values of a, b, c, f, g and h . [5]

A sum of \$5,000 is invested at $t = 2$ for 5 years.

- (ii) Calculate the annual nominal rate of return convertible monthly on the investment. [3]
- [Total 8]

- 5** The force of mortality, μ_x^* , experienced by a particular population at all ages x (where x is not necessarily an integer) is 20% higher than that under the PMA92C20 table.

Calculate the following, based on μ_x^* , assuming a rate of interest of 7% p.a.

(i) $\ddot{a}_{70:\overline{3}|}$ [5]

(ii) $A_{70:\overline{3}|}$ [2]

[Total 7]

- 6** A special whole life assurance policy is issued to a life aged 45 exact. The policy provides a benefit of \$40,000 on death within 15 years of inception, and \$50,000 on death thereafter. Benefits are payable at the end of the year of death.

(i) Calculate the expected present value of the benefit payments. [3]

(ii) Calculate the variance of the present value of the benefit payments.

Basis:

Mortality: AM92 Ultimate

Interest: 6% p.a.

[6]

[Total 9]

- 7** A life insurance company issues a 20-year with-profit endowment assurance policy to a life aged 45 exact for a sum assured of \$150,000. The sum assured, together with any attaching bonuses, is payable on survival to the end of the term or at the end of the year of death if earlier.

The company assumes that future reversionary bonuses will be declared on the policy annually at a rate of 1.92308% of the sum assured, compounded and vesting at the end of the policy year (i.e. the death benefit does not include any bonus relating to the policy year of death).

Calculate the level premium payable annually in advance throughout the policy term, and ceasing on earlier death.

Basis:

Mortality: AM92 Select

Interest: 6% p.a.

Initial expenses: \$200 plus 75% of the annual premium

Renewal expenses: 2.5% of each annual premium excluding the first

Claim expenses: \$140 on death or maturity

[10]

8 On 1 January 2022, a student plans to take out a 10-year bank loan for \$15,000.

Under the repayment schedule, instalments will be paid monthly in arrears until the end of the term. The first instalment, at the end of January 2022, will be X , and the second instalment, at the end of February 2022, will be $2X$, and so on, until the instalment at the end of December 2026, which will be $60X$. The remaining instalments from the end of January 2027 will also be $60X$.

The bank charges a rate of interest of 12% p.a. effective.

- (i) Write down an equation of value to calculate X . [2]
- (ii) Calculate the value of X using the equation of value in part (i). [5]
- (iii) Write down an equation to calculate the loan outstanding, after the instalment paid at the end of December 2026, using the retrospective method. [2]
- (iv) Calculate the loan outstanding after the instalment at the end of December 2026 has been paid, using the equation in part (iii). [1]
- (v) Comment on your answer to part (iv). [2]
- (vi) Write down an equation to calculate the total interest paid during 2027. [2]
- (vii) Calculate the total interest paid during 2027 using the equation in part (vi). [2]

The bank also offers the 10-year loan with the same interest rate but where the monthly instalments remain level throughout the term.

- (viii) Comment on whether the total interest paid by the student under this revised offer would be greater or less than that paid under the original repayment schedule. You should not perform any further calculations. [2]

[Total 18]

- 9** The Green Investment Company has the opportunity to purchase a factory for \$400,000. The factory is to be leased and two different companies, A and B, are interested in the lease. The two companies have made the following proposals.

Company A

The Green Investment Company will need to spend another \$50,000 refurbishing the factory for Company A.

Company A will pay rent annually in advance for 20 years starting immediately. The rent will increase by 3% p.a. compound each year. At the end of 20 years, Company A will purchase the factory from the Green Investment Company for \$450,000.

Company B

Company B will pay rent at an initial level amount of \$44,600 p.a. payable monthly in advance starting immediately. The rent will increase by 50% at the end of the 10th year and remain at this level for the next 10 years. At the end of 20 years, ownership of the factory will pass to Company B at no further cost.

- (i) Calculate the initial annual rent payable by Company A, to give the Green Investment Company an internal rate of return of 9% p.a. effective on the proposal. [3]
- (ii) Demonstrate that the internal rate of return from Company B's proposal would be greater than 9% p.a. effective. [3]

The Green Investment Company does not have the capital available to purchase the factory but can take out a loan at an interest rate of 9.5% p.a. effective. The loan is to be repaid over 20 years in level instalments payable annually in arrears.

The Green Investment Company decides to accept the proposal from Company B, and takes out a loan in order to purchase the factory.

- (iii) Calculate the accumulated profit of the investment after 20 years using an effective rate of interest of 9.5% p.a. [4]
- [Total 10]

10 On 1 January 2011, a life insurance company planned to issue the following two policies to lives then aged 45 exact:

- a 15-year without-profit endowment assurance with a sum assured of S payable on maturity or immediately on earlier death, and with premiums of P payable annually in advance
- a 15-year temporary life annuity payable annually in advance purchased with a single premium of \$50,000.

The annual annuity payments were calculated to be exactly sufficient to pay the premiums for the endowment assurance as they fell due.

(i) Calculate P and S . [7]

The policies were actually issued with $S = \$90,000$ payable under each endowment assurance policy and an annual premium of $P = \$4,450$.

On 31 December 2020, there were 550 policies still in force. During 2020, there were six deaths with no other decrements taking place.

(ii) Calculate the mortality profit for the calendar year 2020. [10]

(iii) Comment on your numerical result obtained in part (ii).

Basis:

Mortality: AM92 Ultimate

Interest: 4% p.a.

Expenses: Ignore

[3]

[Total 20]

END OF PAPER