

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

21 September 2020 (am)

Subject CM1A – Actuarial Mathematics Core Principles

Time allowed: Three hours and fifteen minutes

<p>In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.</p>

If you encounter any issues during the examination please contact the Examination Team on T. 0044 (0) 1865 268 873.

1 Describe the cashflows for an investor who purchases an index-linked bond. [3]

2 Calculate, using standard approximations where necessary:

(a) ${}_{10|4}q_{[36]}$

(b) $\bar{A}_{46:25}^1$

Basis

Mortality: AM92

Interest rate: 4% per annum

[5]

[Note: You should show your working, but intermediate steps can be shown using numerical values – no additional notation is required.]

3 Two independent lives aged x and y have constant annual forces of mortality of 0.04 and 0.03 respectively.

The probability ${}_6q_{xy}^2$ can be expressed as

$$\int_{t=a}^b \int_{s=c}^d X e^{-ps} \times e^{-qt} ds dt$$

where time periods are measured in years.

(i) State the values of a, b, c, d, p, q and X . [3]

(ii) Calculate the probability ${}_6q_{xy}^2$ based on your answers to part (i). [4]

[Total 7]

- 4** An insurance company issues 20-year joint life term assurances. The sum assured of \$150,000 is payable at the end of the year of the first death, if it occurs within the policy term. The premium is payable monthly in advance throughout the term of the policy or until the first death, if earlier.

Calculate the monthly premium for a policy issued to a male life aged 55 exact and a female life aged 53 exact.

Basis

Mortality of male life: PMA92C20

Mortality of female life: PFA92C20

Interest rate: 4% per annum

Expenses: Ignore

Assume that lives are independent with respect to mortality. [9]

- 5** A company invests \$50,000 now and receives the following income over the next 12 years:

During the first 4-year period: \$4,000 per annum paid quarterly in arrears.

During the second 4-year period: \$X per annum paid half-yearly in arrears.

During the final 4-year period: \$12,000 per annum paid continuously.

There are no other payments under the investment.

Calculate X assuming the company achieves a nominal rate of return of 9% per annum convertible monthly. [11]

- 6** An investment bank borrows \$39.5 million at an effective rate of interest of 8% per annum. The bank uses the money to invest in a capital project that provides an income of \$5 million per annum payable quarterly in arrears for a term of 15 years. This income is used to repay the loan. Once the loan has been repaid, the bank can earn interest on the income at an effective rate of interest of 6% per annum.

(i) Calculate the discounted payback period for this project. [4]

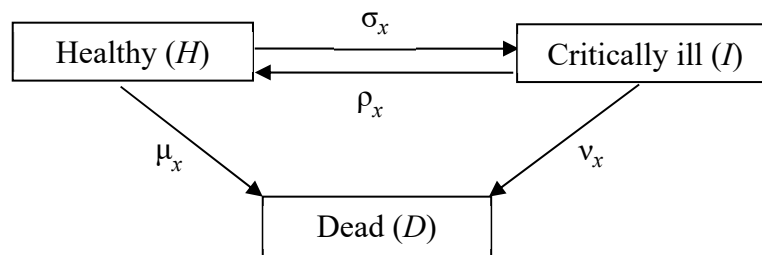
(ii) Calculate the accumulated profit the bank will have made at the end of the term. [6]

[Total 10]

- 7 A life insurance company sells a policy with a 20-year term that provides a benefit of \$100,000 payable immediately on death or on earlier diagnosis of a critical illness. No further benefit is paid in the event of death after a critical illness claim has been paid.

Premiums of P are paid annually in advance throughout the term or until a claim, if earlier.

The company prices the policy using the following multiple state model using the forces of transition σ_x , ρ_x , μ_x and ν_x for a life aged x .



Basis:

$\mu_x = 0.025$ for all ages

$\sigma_x = 0.015$ for all ages

Interest rate is 3% per annum effective

- (i) Determine, with a reason, the values for ρ_x and ν_x that should be used in the pricing model for this policy. [1]

The present value of the benefits on this policy is given by the following formula

$$a \times \int_{t=c}^d b \times e^{zt} dt.$$

- (ii) State the values of a , b , c , d and z . [3]

- (iii) Calculate the present value of the benefits for this policy based on your answers to part (ii). [2]

The present value of the annual premiums for this policy is given by the following formula

$$P \sum_{t=f}^g e^{th}.$$

- (iv) State the values of f , g and h . [2]

- (v) Calculate the annual premium for this policy based on your answers to parts (iii) and (iv). [1]

[Total 9]

- 8** A loan is to be repaid by a series of instalments made annually in arrears. The first instalment is \$200 per annum and thereafter instalments increase by \$15 each year. The instalments are paid for 16 years and are calculated using an effective rate of interest of 5% per annum.
- (i) Calculate the amount of the loan. [3]
 - (ii) Construct a loan schedule that shows the capital and interest elements included in, and the amount of loan outstanding after, each of the 5th and 6th instalments. [5]
 - (iii) Calculate the capital and interest elements of the final instalment. [2]
- [Total 10]
- 9** A fixed interest security of nominal amount \$1,000,000 is to be issued paying coupons quarterly in arrears at a rate of 6% per annum. The security is to be redeemed with a capital payment of \$105 per \$100 nominal on a coupon date between 20 and 25 years after the date of issue, inclusive. The date of redemption is at the option of the borrower.
- An investor, who is liable to income tax at 20% and capital gains tax of 25%, wishes to purchase the entire security at the date of issue, at a price that ensures she achieves a net effective yield of at least 4.9% per annum.
- (i) Determine whether the investor would make a capital gain if she holds the security until redemption. [3]
 - (ii) Explain how your answer to part (i) influences the assumptions made in calculating the price the investor should pay. [2]
 - (iii) Calculate the maximum price that the investor should pay per \$100 nominal. [5]
 - (iv) Explain, without carrying out any further calculations, how your answer to part (iii) would change if the coupons had been payable half-yearly in arrears. [2]
- [Total 12]

- 10** A life insurance company is proposing to launch a 3-year with-profits endowment assurance policy. Compound reversionary bonuses are declared at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year of death). If the policyholder dies during the term, the basic sum assured plus any attaching reversionary bonus is payable at the end of the year of death. On survival to maturity, the basic sum assured plus any attaching reversionary bonus plus a terminal bonus is payable.

Premiums are paid yearly in advance throughout the term of the policy or until earlier death.

Profit test assumptions

Rate of interest:	2% per annum
Mortality:	120% AM92 Ultimate
Initial expenses:	\$200
Renewal expenses:	\$30 per annum on the second and third premium dates
Initial commission:	15% of the first premium
Renewal commission:	1.5% of the second and third years' premiums
Claim expense:	\$50 (payable on death and maturity)
Reversionary bonus:	2% per annum compound
Terminal bonus:	10% of the sum of the basic sum assured and the attaching reversionary bonuses. This is payable on maturity only
Risk discount rate:	7% per annum
Reserves:	Ignore
Surrenders:	Ignore

- (i) Calculate the premium, using a discounted cash flow projection, for a policy with a basic sum assured of \$15,000 issued to a life aged 62 exact. [14]

The life insurance company intends to set up the following reserves for the policy at duration t .

Duration t	${}_tV$
0	0
1	\$5,000
2	\$10,000

- (ii) Calculate the premium for the policy in part (i) after allowing for the reserves. [6]

Assume that the life insurance company charges the premium calculated in part (ii) and sets up reserves as detailed above.

- (iii) Discuss briefly whether the reserves set up by the life insurance company would be sufficient. [4]
[Total 24]

END OF PAPER