INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORTS

September 2020

Subject CM1A - Actuarial Mathematics

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision

Mike Hammer
Chair of the Board of Examiners
September 2020

A. General comments on the *aims of this subject and how it is marked*

1. CM1 provides a grounding in the principles of modelling as applied to actuarial work - focusing particularly on deterministic models which can be used to model and value known cashflows as well as those which are dependent on death, survival, or other uncertain risks.

2. Please note that different answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations but candidates are not penalised for this. However, candidates may lose marks where excessive rounding has been used or where insufficient working is shown.

3. These solutions use full actuarial notation although candidates who used notation based on standard keystrokes were given full credit.

B. Comments on *candidates' performance in this diet of the examination.*

1. The comments that follow the questions concentrate on areas where candidates could have improved their performance. Where no comment is made, the question was generally answered well by most candidates. The examiners look most closely at the performance of the candidates close to the pass mark and the comments therefore often relate to those candidates.
2. The move to an online exam meant that there were less knowledge based questions on the paper. The examiners included new types of questions (Qs 3 and 7) which reduced the need for typing of long formulae although these questions proved to be among the more challenging on the paper. In addition, the longest question (Q10) was designed so that it should not take longer to answer in a typed format rather than a handwritten format. Other questions offered more marks than would have been offered in previous years to allow for the greater length of time to type the answers. Despite this, there was some limited evidence that the typing requirement led to greater time pressure for the marginal candidate and the pass mark was slightly lowered to allow for this.
3. Where candidates made numerical errors, examiners would award marks for the correct method used and also for the parts of the calculation that were correct. However, many candidates often did not show enough of their working to fully benefit from this.
4. The Examiners felt that the "open book" nature of the online exam led some candidates to rely on their notes much more than if the exam had been "closed book". The Examiners strongly recommend that candidates prepare for online exams just as thoroughly as they would do if the exam were of the traditional "closed book" format. Candidates should treat it as a bonus that they can refer to their notes but they should not be relying on being able to do so.

C. Pass Mark

The Pass Mark for this exam in combination with CM1B was 58.
1717 presented themselves and 797 passed.

Q1

The investor has an initial negative cashflow. [½]
In return the investor receives…
… a series of regular interest payments… [½]
…which are linked to an 'index'… [½]
…which reflects the effects of inflation… [½]
…and a final capital repayment… [½]
…that is also linked to the index. [½]
The indexation might be subject to a time lag [½]

[Marks available 3 ½, maximum 3]

This Questions was generally well-answered although some candidates chose to include non-cashflow related characteristics in their answer.

Q2

(a)

$$ {}_{10|4}q_{[36]} = \frac{l_{46} - l_{50}}{l_{[36]}} $$ [1 formula]

$$ = \frac{9{,}786.9534 - 9{,}712.0728}{9{,}886.0395} $$

$$ = 0.007574 $$ [1]

(b)

$$ \bar{A}^{\,1}_{46:\overline{25|}} = (1+i)^{0.5}\left(A_{46} - v^{25}\frac{l_{71}}{l_{46}}A_{71} \right) $$ [1.5 formula]

$$ = \sqrt{1.04}\left(0.28605 - 0.375117\frac{7{,}854.4508}{9{,}786.9534}0.61548 \right) $$

$$ = \sqrt{1.04}(0.10076) = 0.10276 $$ [1.5 calculation]

[Total 5]

This question was well-answered by the candidates.

Q3

(i) $a = 0, b = 6, c = 0, d = t, p = 0.03, q = 0.04, X = 0.03 \times 0.04 = 0.0012$
[or $a = 0, b = 6, c = t, d = 6, p = 0.04, q = 0.03, X = 0.03 \times 0.04 = 0.0012$ – see end of part (ii) for solution with these numbers]
[3, deduct 0.5 mark for each error, min(0)]

(ii) Answer = $\displaystyle\int_{t=0}^{6} 0.03\times 0.04 e^{-0.04t}\left(\int_{s=0}^{t} e^{-0.03s}ds\right)dt$

$$= 0.04\int_{t=0}^{6} e^{-0.04t}\left(1-e^{-0.03t}\right)dt = 0.04\int_{t=0}^{6}\left(e^{-0.04t}-e^{-0.07t}\right)dt \qquad [1]$$

$$= 0.04\left[\frac{e^{-0.04t}}{-0.04}\right]_{t=0}^{6} - 0.04\left[\frac{e^{-0.07t}}{-0.07}\right]_{t=0}^{6} = \left(1-0.786628\right)-\frac{0.04}{0.07}\left(1-0.657047\right) \qquad [2]$$

$$= 0.017399 \qquad [1]$$

[or Answer = $\displaystyle\int_{t=0}^{6} 0.03\times 0.04 e^{-0.03t}\left(\int_{s=t}^{6} e^{-0.04s}ds\right)dt$

$$= 0.03\int_{t=0}^{6} e^{-0.03t}\left(e^{-0.04t}-e^{-0.24}\right)dt = 0.03\int_{t=0}^{6} e^{-0.07t}-e^{-0.03t-0.24}dt$$

$$= \frac{0.03}{0.07}\left[1-e^{-0.42}\right]-\left(e^{-0.24}-e^{-0.42}\right) = 0.017399]$$

[Total 7]

Part (i) was a new style of question which aimed to reduce the amount of typing required from candidates. Most candidates scored some marks on part (i) and a significant number of candidates were then able to use the answers from part (i) to make an attempt at part (ii).

Q4 Let P denote the the monthly premium, then the present value of future premiums is given by

$$12P\ddot{a}^{(12)}_{55:53:\overline{20|}} = 12P\left(\ddot{a}_{55:53:\overline{20|}}-\left(\frac{11}{24}\right)\times\left(1-\frac{v^{20}\times l_{75:73}}{l_{55:53}}\right)\right) \qquad [2]$$

$$= 12P\times\left(13.463-\frac{11}{24}\times\left(1-0.353725\right)\right)$$

$$= 12\times P\times 13.16683$$

$$= 158.002P \qquad [1]$$

where $\ddot{a}_{55:53:\overline{20|}} = \ddot{a}_{55:53} - \left(\frac{v^{20} \times l_{75:73}}{l_{55:53}}\right) \times \ddot{a}_{75:73} = 16.284 - 0.353725 \times 7.975 = 13.463$

and

$$\left(\frac{v^{20} \times l_{75:73}}{l_{55:53}}\right) = v^{20} \times \left(\frac{l_{75}^{m} \times l_{73}^{f}}{l_{55}^{m} \times l_{53}^{f}}\right) = 0.456387 \times \left(\frac{8,405.160 \times 9,073.650}{9,904.805 \times 9,934.574}\right) = 0.353725$$

[2]

The present value of death benefits is given by

$$150000 \times \left(1 - d \times \ddot{a}_{55:53:\overline{20|}} - \left(\frac{v^{20} \times l_{75:73}}{l_{55:53}}\right)\right)$$

[2]

$$= 150,000 \times \left(1 - \left(\frac{0.04}{1.04}\right) \times 13.463 - 0.353725\right)$$

$$= 150,000 \times (0.482191 - 0.353725) = 150,000 \times 0.128465 = 19269.82$$ [1]

<u>Alternativly present value of death benefit is given by :</u>

$$150000 \times \left(A_{55:53} - \left(\frac{v^{20} \times l_{75:73}}{l_{55:53}}\right) \times A_{75:73}\right)$$

$$= 150000 \times \left(\left(1 - d \times \ddot{a}_{55:53}\right) - \left(\frac{v^{20} \times l_{75:73}}{l_{55:53}}\right) \times \left(1 - d \times \ddot{a}_{75:73}\right)\right)$$

$$= 150000 \times \left(\left(1 - \left(\frac{0.04}{1.04}\right) \times 16.284\right) - 0.353725 \times \left(1 - \left(\frac{0.04}{1.04}\right) \times 7.975\right)\right)$$

Using the principle of equivalence the monthly premium is found by equating the present values of premiums and benefits.

$$\Rightarrow 158.002P = 19269.82$$

$$\Rightarrow P = \$121.96$$

[1]

[Total 9]

Common errors here were

- *to incorrectly apply the monthly premium adjustment (or to ignore the adjustment completely)*
- *to calculate the benefit as an endowment assurance*

Q5

Equation of value is given by:

$$50,000 = 4,000a_{\overline{4}|}^{(4)} + Xa_{\overline{4}|}^{(2)} \times v^4 + 12,000\bar{a}_{\overline{4}|} \times v^8$$ [2]

where the effective rate of interest per annum is i such that $i^{(12)} = 9\%$.

Thus, we have: $1+i = \left(1+\dfrac{i^{(12)}}{12}\right)^{12} = \left(1+\dfrac{0.09}{12}\right)^{12} \Rightarrow i = 9.38069\%$ per annum [1]

Thus, we have:

$$a_{\overline{4}|}^{(4)} = \frac{1-v^4}{i^{(4)}} = \frac{1-0.6986141}{0.0906767} = 3.323742\text{, where}$$

$$v^4 = \left(\frac{1}{1+i}\right)^4 = \left(\frac{1}{1.0938069}\right)^4 = 0.6986141$$

$$\left(1+\frac{i^{(4)}}{4}\right)^4 = 1+i = 1.0938069 \Rightarrow i^{(4)} = 0.0906767$$

[2½]

$$a_{\overline{4}|}^{(2)} = \frac{1-v^4}{i^{(2)}} = \frac{1-0.6986141}{0.0917045} = 3.286491\text{, where}$$

$$\left(1+\frac{i^{(2)}}{2}\right)^2 = 1+i = 1.0938069 \Rightarrow i^{(2)} = 0.0917045$$ [2]

$$\bar{a}_{\overline{4}|} = \frac{1-v^4}{\delta} = \frac{1-0.6986141}{0.0896642} = 3.361274\text{, where}$$

$$e^{\delta} = 1+i = 1.0938069 \Rightarrow \delta = 0.0896642$$

[1½]

Thus, we have:

$$50,000 = 4,000 \times 3.323742 + 0.6986141 \times 3.286491X$$

$$+12,000 \times \left(\frac{1}{1.0938069}\right)^8 \times 3.361274$$

$$= 13,294.968 + 2.2959890X + 19,686.109$$

$$\Rightarrow X = \$7,412.46$$

[2]

[Total 11]

Candidates generally answered this question well Common errors were

- *to assume that the effective interest rate was 9% per annum which simplified the calculations significantly and so was penalised accordingly*
- *to omit the deferral factors for the second and third elements of the annuity*

Q6

(i) Let DPP be t. Working is \$000's, we want

$39{,}500 = 5{,}000\ a_{\overline{t|}}^{(4)}$ at 8% pa [1]

$$= 5{,}000 \frac{i}{i^{(4)}} a_{\overline{t|}}$$

$$\Rightarrow a_{\overline{t|}} = \frac{39{,}500}{5000 \times 1.029519} = 7.67349 \quad [1]$$

$$\Rightarrow v^t = 1 - 7.67349 \times 0.08$$

$$\Rightarrow t = \frac{\ln 0.38612}{\ln (1/1.08)} = 12.365 \quad [1]$$

So DPP is 12 years 6 months [1]

[Total 4]

(ii) Profit at the end of 15 years is:

$$-39500 \times (1.08)^{12.5} \times (1.06)^{2.5} + 5000 s_{\overline{12.5|}@8\%}^{(4)} (1.06)^{2.5} + 5000 s_{\overline{2.5|}@6\%}^{(4)} \quad [2]$$

where:

$$s_{\overline{12.5|}8\%}^{(4)} = \frac{(1.08)^{12.5} - 1}{i^{(4)}} = \frac{(1.08)^{12.5} - 1}{0.077706} = 20.80868 \quad [1½]$$

$$s_{\overline{2.5|}@6\%}^{(4)} = \frac{(1.06)^{2.5} - 1}{i^{(4)}} = \frac{(1.06)^{2.5} - 1}{0.058695} = 2.67173 \quad [1½]$$

Profit = -119,580 + 120,359 + 13,359 =14,138 (=\$14,138,000) [1]

[Total 6]

Part (i) was answered well although many candidates did not recognise that the end of the DPP would be measured in a whole number of quarters i.e. the investment would only become profitable at the time of an income payment.
Part (ii) was answered less well with many candidates incorrectly assuming that the accumulated profit would be zero at the end of the DPP.

Q7

(i) Since the policy will terminate on a critical illness claim, we do not need to model ρ_x and ν_x and these can be assumed to be zero. [1]

(ii)

$a = 100{,}000$ [½]
$b = 0.04$ [½]
$c = 0$ [½]
$d = 20$ [½]
$z = -ln(1.03) - 0.04 = -0.069559$ [1]

(iii) $\frac{4{,}000}{-0.069559}\left[e^{-0.069559t}\right]_0^{20}$ [1]

$= \$43{,}199$ [1]

(iv) $f = 0$
$g = 19$ [1 for *f* and *g*]

$h = -ln(1.03) - 0.04 = -0.069559$ [1]

(v) $P\left(\frac{1-e^{-0.069559\times 20}}{1-e^{-0.069559}}\right) = 11.1797P$ [½]

43,199 = 11.1797P
P = \$3,864 [½]

<u>Alternatively:</u>

$\ddot{a}_{\overline{20}|}$ with $\delta = 0.069559$

[Total 9]

Some of the points highlighted in Q3 also apply here with many candidates struggling to use the equations developed in parts (ii) and (iv) to obtain answers to parts (iii) and (v). In general this question was not answered well with many candidates making errors without showing much intermediate working, especially in part (iii).

Q8

(i) Loan = 185 $a_{\overline{16}|} + 15(Ia)_{\overline{16}|}$ at 5% [1½]

$= 185\times 10.8378 + 15\times 80.9975$
$= \$3{,}219.96$ [1½]

(ii) Capital o/s after 4 payments comes from:

5^{th} payment =\$260 [½]

$\Rightarrow$ Capital o/s $= 245a_{\overline{12}|} + 15(Ia)_{\overline{12}|}$ at 5%

$= 245 \times 8.8633 + 15 \times 52.4873 = 2958.82$ [2]

Year	Capital o/s at start	Instalment	Interest	Capital repaid	Capital o/s at end
5	2958.82	260.00	147.94	112.06	2846.76
6	2846.76	275.00	142.34	132.66	2714.10
7	2714.10	290.00			

[2½]

(iii) Final instalment =\$425

$\Rightarrow$ Loan o/s at start of final year = 425v = \$404.76 [1]

= capital paid in final payment

$\Rightarrow$ Interest = 425 – 404.76 = \$20.24 [1]

[Total 10]

This question was well answered by candidates. Many candidates seemed to use Excel to help with their calculations although their final answers needed to be typed into their script in order to receive credit.

Q9

(i)

$$i^{(4)} = 4\left(1.049^{1/4} - 1\right) = 4.812\%$$

$$\frac{D}{R}(1 - t_1) = \frac{6}{1.05} \times 0.8 = 4.571\%$$ [3]

$$i^{(4)} > \frac{D}{R}(1 - t_1) \Rightarrow \text{Capital gain}$$

(ii) Since there is a capital gain, the security is least valuable to the investor if the repayment is made by the borrower at the latest possible date. [1]

Since that <u>decision is beyond the control of the investor</u>, we must assume that the redemption occurs after 25 years to find the minimum yield obtained. [1]

(iii) If P is the price per \$100 of the security:

$$P = 100 \times 0.06 \times 0.8a^{(4)}_{\overline{25}|} + \left(105 - 0.25(105 - P)\right)v^{25} \text{ at } 4.9\%$$ [2]

where:

$$a_{\overline{25}|}^{(4)} = \frac{1-v^{25}}{0.04812} = 14.4966$$ [1]

$$\Rightarrow P = 4.8 \times 14.4966 + \left(105 - 0.25\left(105 - P\right)\right) \times 0.30242$$

$$\Rightarrow P = \frac{69.5837 + 23.8156}{1 - 0.25 \times 0.30242} = \$101.03157$$ [2]

(iv) If the coupons were paid less frequently (i.e. half-yearly not quarterly) then the investor would have to wait longer, on average, for the coupon payments to be made. This will make the investment less valuable, and therefore the price would be lower than in (iii). [2]

[Total 12]

The calculation elements of the question were answered well but the explanations given as part of answers to parts (ii) and (iv) were often unclear.

Q10

(i) Mortality Table

	[½]	[1]	[½]	[1]
x	q_x^{base}	120% q_x^{base}	p_x	$_{t-1}(ap)$
62	0.010112	0.012134	0.987866	1
63	0.011344	0.013613	0.986387	0.987866
64	0.012716	0.015259	0.984741	0.974418

Year	1	2	3	
Death Benefit	15,000	15,300	15,606	[1½]
Maturity Benefit			17,509.93	[1½]

Profit Test

Year	1	2	3	
Premium	P	P	P	[½]
Commission	0.15P	0.015P	0.015P	[½]
Expenses	200	30	30	
Interest	0.017P-4	0.0197P-0.6	0.0197P-0.6	[1]
Death Outgo	182.02	208.28	238.14	[1]
Maturity Outgo			17,242.74	[½]
Claim Expense	0.61	0.68	50	[½]

Profit Vector	0.867P-386.62	1.0047P-239.56	1.0047P-17,561.48

[1]

Probability in force	1	0.987866	0.974418
Profit Signature	0.867P-386.62	0.993P-236.65	0.979P-17,112.22
Discount Factor	0.934579	0.873439	0.816298
Present Value of Future Profits	0.810P-361.33	0.867P-206.70	0.799P-13,968.67

[2]

Total Present Value of Future Profits is 2.47633P-14,536.70

[½]

Thus the annual premium is found by:

$$P = \frac{14,536.70}{2.47633} = 5,870.26$$

i.e. the annual premium is approximately $5,871 [½]

(ii) Taking the profit vector from above and allowing for the transfer to reserves.

Year	1	2	3	
Profit Vector	0.867P-386.62	1.0047P-239.56	1.0047P-17,561.48	[½]
Reserve at start	0	5,000	10,000	[½]
Interest on reserve	0	100	200	[½]
Reserve at end	4,939.33	9,863.87	0	[1]
Transfer to Reserves	-4,939.33	-4,763.87	10,200	[1]
Revised Profit Vector	0.867P-5,325.95	1.0047P-5,003.43	1.0047P-7,361.48	[½]
Probability in force	1	0.987866	0.974418	
Revised Profit Signature	0.867P-5,325.95	0.993P-4,4942.71	0.979P-7,173.16	[½]
Discount Factor	0.934579	0.873439	0.816298	
Present Value of Future Profits	0.810P-4,977.52	0.867P-4,317.16	0.799P-5,855.43	[½]

Total Present Value of Future Profits is 2.47633P - 15,150.11

$$P = \frac{15,150.11}{2.47633} = \$6,117.87$$ [1]

(iii) With the premium calculated in (ii) we can calculate the expected cashflows arising in each policy year.

Year	1	2	3
Profit Vector	-$21	$1,144	-$1,214

[1]

Here we see negative cashflows arising in policy years 1 and 3 and a positive cashflow for policy year 2. [1]

Despite having set up reserves the life insurance company is facing a negative cashflow in policy year 3. The company may find itself with insufficient funds to meet all the claims that fall due in policy year 3. [1]
This implies that the reserves set up are not sufficient. It would be prudent to hold larger reserves. [1]

[Total 24]

This question seemed to discriminate well between the stronger and weaker candidates although some candidates seemed to be under time pressure. Despite the instruction to use a discounted cash-flow approach, many candidates used an equation of value approach in part (i) and were given partial credit for this.
In parts (i) and (ii), a zero profit- criterion was implicitly assumed although candidates who used other criteria were given full credit.
Common errors in part (i) were

- *to incorrectly calculate the maturity benefit (although full credit was given for candidates who did not include the final reversionary bonus)*
- *to ignore claims expenses*

There was a significant amount of calculation work and it was not surprising that even the strongest candidates made the occasional numerical error.
Many candidates failed to score any marks at all on parts (ii) and (iii) although it was not clear whether this was due to time pressure.

END OF MARKING SCHEDULE

