# INSTITUTE AND FACULTY OF ACTUARIES

# EXAMINATION

24 September 2019 (pm)

## Subject CM1A – Actuarial Mathematics
## Core Principles

*Time allowed: Three hours and fifteen minutes*

***INSTRUCTIONS TO THE CANDIDATE***

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all questions, begin your answer to each question on a new page.*

5. *Candidates should show calculations where this is appropriate.*

***Graph paper is NOT required for this paper.***

*AT THE END OF THE EXAMINATION*

*Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.*

*In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.*

**1** Describe the main features of an income protection health insurance contract. [4]

**2** List five key preparatory steps in a data analysis process prior to performing an exploratory analysis of the data. [5]

**3** (i) Explain what is meant by the expression ${}_{5|17}q^{1}_{40:40}$ [2]

Two lives, each aged exactly 40, are independent with respect to mortality and are each subject to a constant force of mortality of 0.01 per annum.

(ii) Calculate the value of the expression in part (i). [4]

[Total 6]

**4** Describe the properties that can lead to data being classified as "big data". [5]

**5** A life office issued a whole of life assurance contract to a life aged $x$ exact with a sum assured of 1 payable at the end of the year of death.

Level premiums of $P$ are payable annually in advance, ceasing on death. Ignore expenses.

(i) Using standard actuarial notation, write down:

(a) The equation of value at time 0.
(b) An expression for the prospective reserve at duration $t$ years, denoted by ${}_tV^P_x$.
(c) An expression for the retrospective reserve at duration $t$ years, denoted by ${}_tV^R_x$.

[3]

Assume that the basis used to calculate both the prospective and retrospective reserves is the same as that used to calculate the premium, $P$.

(ii) Show that the prospective and retrospective reserves are equal at time $t$. [4]

[Total 7]

**6** The annual effective forward rate applicable over the period from $t$ to $t + r$ is defined as $f_{t,r}$ where $t$ and $r$ are measured in years.

You are informed that $f_{0,1} = 4\%$, $f_{1,1} = 5\%$, $f_{2,1} = 6\%$ and $f_{3,1} = 7\%$.

(i) Determine the gross redemption yield at issue for a four-year bond, redeemable at par, with a 4% coupon payable annually in arrears. [7]

(ii) Explain why the gross redemption yield in part (ii) is lower than $f_{3,1}$. [3]

[Total 10]

**7** On 1 January 2002, a life insurance company issued whole life increasing assurances to lives then aged 45 exact.

The initial sum assured was £20,000, which increased by £2,000 on each policy anniversary.

Benefits are payable at the end of the year of death. Premiums are payable annually in advance for a maximum of 20 years, ceasing on earlier death.

On 1 January 2018, there were 378 policies in force and, during 2018, 4 of these policyholders died.

(i) Calculate the mortality profit during 2018, assuming the insurance company uses the following basis for both premiums and reserves.

| | |
|---|---|
| Mortality | AM92 ultimate |
| Interest | 4% per annum |
| Expenses | none |

[9]

(ii) Explain why the result in part (i) has arisen. [3]

[Total 12]

**8** A loan of £1,000,000 nominal is issued with coupons payable half-yearly in arrears at a rate of 9% per annum. The loan is to be redeemed at £110 per £100 nominal on a single coupon date between 20 and 25 years after the date of issue, inclusive. The date of redemption is at the option of the borrower.

An investor who is liable to income tax at 15% but not liable to capital gains tax wishes to purchase the loan at the date of issue.

(i) Calculate the price the investor should pay to ensure a net effective yield of at least 8% per annum. [5]

The investor purchases the loan for the price calculated in part (i). Exactly ten years later, immediately after the payment of the coupon then due, a second investor, who is liable to income tax at 25% and capital gains tax of 35%, purchases the loan for a price such that the first investor obtained a net effective yield of 8% per annum. The second investor holds the loan to maturity.

(ii) Calculate:

(a) the price paid by the second investor
(b) the minimum net redemption yield earned by the second investor, to the nearest 0.1% per annum.

[6]
[Total 11]

**9** A man aged 60 exact purchases a whole life level annuity of £20,000 per annum payable monthly in arrears with payment guaranteed for the first five years.

In addition, a reversionary annuity of £10,000 per annum is payable to the man's wife, who is two years younger. This reversionary annuity commences on the monthly payment date following the man's death or on completion of the five-year guaranteed period, if later. The annuity is payable monthly in arrears until the wife's death.

Calculate the single premium payable using the following basis:

| | |
|---|---|
| Interest: | 4% per annum |
| Mortality: | PMA92C20 for the policyholder |
| | PFA92C20 for the spouse |
| Expenses: | Initial expenses of £250 plus £10 on each annuity payment date |

[11]

**10** The force of interest, $\delta(t)$, is a function of time and at any time $t$, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.03 + 0.01t & 0 \le t < 4 \\ 0.07 & 4 \le t < 6 \\ 0.09 & t \ge 6 \end{cases}$$

(i) Calculate the accumulated amount at time $t = 6$ of a lump sum of 10 units invested at time $t = 0$. [3]

(ii) Calculate the present value at time $t = 0$ of a deferred annuity certain of 5 units per year payable continuously from time $t = 4$ to $t = 10$. [6]

(iii) Determine, to the nearest 0.1%, the constant annual effective rate of interest earned by an investor who invests the present value calculated in part (ii) at time $t = 0$ to obtain the payment stream described in part (ii). [3]

[Total 12]

**11** A life insurance company issues a with profit whole life policy. The benefit is payable at the end of the year of death and is equal to the basic sum assured plus any attaching bonus.

Level premiums are paid monthly in advance ceasing after 25 years or on the death of the policyholder if earlier.

Simple reversionary bonuses are added at the start of each policy year.

The company uses the following basis to calculate premiums:

| | |
|---|---|
| Mortality | AM92 Select |
| Interest Rate | 6% per annum |
| | |
| Commission | |
| Initial | 25% of the total premium payable in the first policy year |
| Renewal | 2.5% of the second and subsequent monthly premiums |
| | |
| Expenses | |
| Initial | £300 |
| Renewal | £75 at the start of each year from the first year increasing at a rate of 1.92308% per annum. The first increase will take place at the start of the second year. |
| Claim | £50 inflating at 1.92308% per annum. The first increase will take place at the end of the first year. |
| | |
| Bonus | Simple bonus rate of 1.5% of basic sum assured |

The company issues the policy to lives aged 50 exact. The basic sum assured is £150,000.

(i) Show that the monthly premium is approximately £303. [9]

For the first 24 years of the policy, the actual simple bonuses declared have been at an annual rate of 1% per annum.

(ii) Calculate the gross premium prospective reserve at the end of the 24th policy year.

Prospective Reserving Basis

| | |
|---|---|
| Mortality | AM92 Ultimate |
| Interest Rate | 4% per annum |

Commission

| | |
|---|---|
| Renewal | 2.5% of the monthly premiums |

Expenses

| | |
|---|---|
| Renewal | £125 per annum |
| Claim | £75 |

| | |
|---|---|
| Bonus | Simple bonus rate of 0.75% of basic sum assured |

[8]

[Total 17]

**END OF PAPER**