INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

13 September 2021 (am)

Subject CM1 – Actuarial Mathematics

Core Principles

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

1 A 10-year unit linked contract has the following profit signature before any non-unit reserves are created:

Year	1	2	3	4	5	6	7	8	9	10
	+1	–1	+1	+1	+1	–1	0	–1	+1	+1

Non-unit reserves are set up to zeroise the negative cashflows.

Determine the revised profit signature, ignoring interest and mortality. [3]

2 Calculate, showing all working

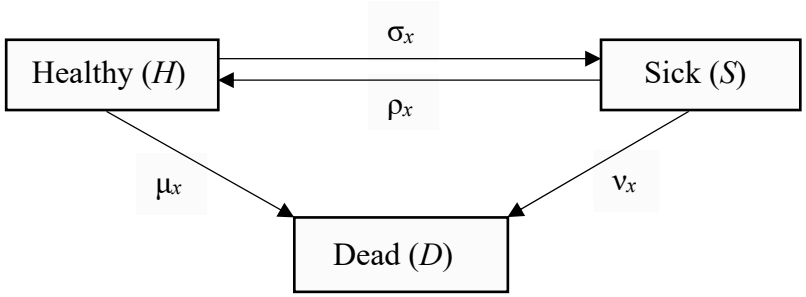
(a) $\mu_{55:60}$

(b) ${}_5p_{55:60}$

(c) ${}_2q^{1}_{60:60}$

Assume lives are independent with regards to mortality and that both lives are subject to the PFA92C20 mortality table. [4]

3 A life insurance company uses the following multiple state model to price its sickness policies.



Describe, in words, what each of the following integral expressions represents. You may assume the time periods are measured in years.

(a) $2{,}000 \times \int_0^{20} e^{-\delta t} \times {}_tp_{40}^{HS}\,dt$

(b) $1{,}000 \times \int_0^{20} e^{-\delta t} \times {}_tp_{40}^{\overline{HH}}\,dt$

(c) $20{,}000 \times \int_0^{20} e^{-\delta t} \times {}_tp_{40}^{HS} \times v_{40+t}\,dt$

[6]

4 A company has agreed to build and operate a ferry service for a regional government.

The company will invest \$10 million at the outset, and a further \$8 million after 1 year.

The ferry will then come into operation and the company will receive payments at the end of each year, the first payment occurring at the end of the second year of the project.

The amount of payment at the end of the second year will be \$4 million, increasing by \$0.5 million in each of the subsequent years until the annual payment is \$7 million, after which the payments will reduce by \$1 million each year. When the payments have reduced to zero, the company's involvement in the project will end.

Calculate the net present value of the project at a rate of interest of 6% p.a. effective.
Note: You should show your working and determine the present value of income using annuity functions.

[6]

5 An equity is expected to pay its first dividend in exactly 2 years' time. It is assumed that this dividend will be \$0.20 per share.

Subsequent annual dividends are assumed to grow at 6% p.a. compound for the following 10 years, and at 3% p.a. compound in perpetuity thereafter.

Calculate, showing all working, the price of the share to the nearest \$0.01, that would give an effective rate of return of 7% p.a. [7]

6 A bond is issued at time $t = 0$ at a price of \$107.60 per \$100 nominal. The bond pays coupons of 6% p.a., annually in arrears, and will be redeemed at par in 3 years' time.

The 2-year par yield at time $t = 0$ is 6.5% p.a. The 1-year forward rate of interest at time $t = 1$ year is 4.5% p.a. effective.

[7]

Calculate, showing all working and assuming no arbitrage, the implied 1-year, 2-year and 3-year annual effective spot rates.

7 A life insurance company issues a reversionary annuity policy to a male and female, both aged exactly 65.

The annuity of \$30,000 p.a., payable monthly in arrears, commences on the first death, and payments cease on the death of the second life, or on the 15th anniversary of the policy inception if earlier.

Calculate, showing all working, the single premium for the policy.

Basis:

Mortality:	PMA92C20 for the male life and PFA92C20 for the female life The lives are independent with respect to mortality
Interest:	4% p.a.
Expenses:	Initial: \$250 incurred at the outset Renewal: 3% of each annuity payment

[10]

8 The force of interest, $\delta(t)$, is a function of time, and at any time, t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.06 + 0.02t & 0 \le t \le 4 \\ 0.08 - 0.01t & t > 4 \end{cases}$$

$A(0,t)$ the accumulation at time t of a unit of money invested at time 0, can be written as:

$$A(0,t) = \begin{cases} e^{\boldsymbol{a}+\boldsymbol{b}t+\boldsymbol{c}t^2} & 0 \le t \le 4 \\ e^{\boldsymbol{f}+\boldsymbol{g}t+\boldsymbol{h}t^2} & t > 4 \end{cases}$$

(i) Determine the values of $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, $\boldsymbol{f}$, $\boldsymbol{g}$ and $\boldsymbol{h}$. [6]

A sum of \$600 is invested at $t = 3$ and a further sum of \$900 is invested at $t = 9$.

(ii) Calculate, showing all working, the accumulated amount at $t = 13$. [4]

(iii) Calculate, showing all working, the yield of the investment described in part (ii) expressed as an effective rate of interest per month to the nearest 0.1% [3]

(iv) Comment on your answer to part (iii). [1]

[Total 14]

9 A life insurance company issues 30-year pure endowment assurance policies to a group of lives aged exactly 30. Each policy provides a sum assured of \$50,000 payable on survival to the end of the term. Premiums on the policy are payable annually in advance for 30 years or until earlier death.

There were two deaths during the 25th policy year and the number of policies in force at the end of that year was 315. There were no exits other than death during the year.

(i) Calculate, showing all working, the mortality profit or loss arising in the 25th policy year. [5]

(ii) Comment on your result obtained in part (i).

Basis:

Mortality: AM92 Ultimate
Interest: 4% p.a. effective
Expenses: None [3]

[Total 8]

10 The table below is an extract from a multiple decrement table that is currently used to model the deaths and withdrawals of employees working for a large company in the hospitality industry. No decrements occur other than by death or withdrawal.

Age (x)	*Number of employees* $(al)_x$	*Number of deaths* $(ad)_x^d$	*Number of withdrawals* $(ad)_x^w$
47	50,000	390	1,500

Recent experience has resulted in an estimate that, at all ages:

- the annual independent force of mortality for employees is now 60% of that implied by the q_x rates in the ELT15 (Females) table.
- the annual independent probability of withdrawal for employees is now 250% of that used to construct the above table.

(i) Calculate, showing all working, the revised independent forces of mortality and withdrawal, each to six significant figures, for age 47. You should state any assumptions that you make. [7]

(ii) Construct the revised multiple decrement table, showing your results to two decimal places. [5]

(iii) Identify any concerns with the use of this revised multiple decrement table to model the future deaths and withdrawals of employees of the company. [3]

[Total 15]

11 A life insurance company issues a 15-year with profit endowment assurance policy to a life aged 50 exact. Premiums are payable monthly in advance for 15 years or until earlier death. The sum assured is payable at the end of the year of death or at the end of the term if earlier.

(i) Demonstrate that the basic sum assured a policyholder can purchase for a premium of \$500 per month is approximately \$93,000 (to the nearest \$1,000). [8]

Pricing basis:

Mortality:	AM92 Ultimate
Interest:	6% p.a. effective
Reversionary bonus:	1.9231% p.a. compound, vesting at the end of each year (i.e. The death benefit does not include the bonus relating to the policy year of death).
Initial expenses:	60% of the annual premium, incurred at policy commencement
Renewal expenses:	4% of the annual premium, incurred annually from the start of the second year onwards

Assume that the policyholder purchased a basic sum assured of \$93,000 with the premium of \$500 per month.

(ii) Demonstrate that the annual effective rate of return that a policyholder will earn on this contract, if they survive to the end of the 15 years, is at least 0.434% p.a. [3]

(iii) Explain why the contract may be attractive to policyholders in spite of the low level of minimum rate of return given in part (ii). [3]

During each of the first 5 years of the contract, the office declared compound reversionary bonuses of 5% p.a.

(iv) Calculate, showing all working, the prospective gross premium reserve at the end of the fifth year of the contract, using the basis given in part (i). [6]

[Total 20]

END OF PAPER