INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

22 April 2021 (am)

Subject CM2A - Financial Engineering and Loss Reserving Core Principles

Time allowed: Three hours and fifteen minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

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1 An investor has \$800 to invest, for a period of 1 year, and has identified two investment opportunities in which to invest. The first is a direct investment in a stock index for a period of 1 year. The annual return, X, on the index follows a Normal distribution with mean $\mu = 7\%$ p.a. and standard deviation $\sigma = 5.5\%$ p.a.

(i) Calculate the following in respect of the investment at the end of 1 year:

(a) The shortfall probability below a value of \$720

(b) The 99.5% value at risk.

[5]

The second opportunity is a derivative that offers the following payoff in 1 year's time based on the performance of the index during the year.

Payoff (\$)	*Scenario*
730	when $X \leq -7.1\%$
750	when $-7.1\% < X \leq 7\%$
962	when $X > 7\%$

(ii) Calculate the expected payoff from the derivative at the end of the year. [4]

(iii) Calculate the following in respect of the payoff from the derivative:

(a) The shortfall probability below a value of \$720

(b) The 99.5% value at risk.

[2]

(iv) Comment on how the investor may choose between the two investments. [3]

[Total 14]

2 A dividend-paying share is currently priced at \$120. Every 3 months, the share price will either increase by 25% or decrease by 20%.

An insurer is considering purchasing a 6-month European call option on the share with a strike price of \$120. Each share will pay a dividend of \$4 just before the option expires.

The risk-free force of interest is 0% per annum.

(i) State the difference between American and European options. [1]

(ii) Calculate the risk neutral price of the option, using a binomial tree. [5]

(iii) Calculate the volatility of the share implied by the option price. [2]

(iv) State the benefits of using a risk-neutral derivative pricing approach. [2]

(v) Explain why it may be difficult to price this option using the Black–Scholes option pricing model. [2]

Now consider an American option on the same underlying share with all other parameters unchanged.

(vi) Explain why you may expect the price of the American option to be different to your answer in part (ii). [2]

(vii) Calculate the price of the American put option. [2]

[Total 16]

3 Two assets are available for investment. Asset A returns $2X\%$, where X is a Binomial random variable with parameters $n = 6$ and $p = 0.4$. Asset B returns $1.5Y\%$, where Y is a Normal random variable with parameters $\mu = 3.2$ and $\sigma = 2$.

(i) Calculate the following separately for each of assets A and B:

(a) The variance of the return

(b) The shortfall probability below a return of 3%.

[4]

An investor with a quadratic utility function wishes to invest in either Asset A, or Asset B.

(ii) Explain which asset the investor should choose. [3]

The investor is now considering whether to split their investment between both assets.

(iii) Explain whether the investor may decide to split the investment in each of the following circumstances:

(a) If the assets are independent

(b) If the assets exhibit some positive correlation.

[4]
[Total 11]

4 (i) Explain the similarities and differences between the basic chain ladder method and the inflation-adjusted chain ladder method for calculating run-off triangles. [3]

(ii) Discuss when it may be more appropriate to use the inflation-adjusted chain ladder method than the basic chain ladder method. [2]

(iii) Discuss possible reasons why neither method may be appropriate for calculating run-off triangles. [2]
[Total 7]

5 A risk-averse, non-satiated investor is trying to determine their utility of wealth function, $U(w)$. They have decided to use the utility function $U(w) = w + dw^2$, where $d < 0$ is a constant that the investor has chosen.

(i) Derive an upper bound in terms of d for the range of values of w over which $U(w)$ can be used. [2]

(ii) Explain why $d < 0$ is a necessary condition for $U(w)$ to be a valid utility function. [2]

The investor lives on a tropical island. On this island, root vegetables can be bought once a week for \$10 per box. The investor knows that they will be able to sell any vegetables they buy for \$30, \$12, \$10 or \$0.5 per box with equal probability. All boxes of vegetables sold at a given time will be sold for the same price.

The investor's current wealth is \$100. If the investor were to buy seven boxes of vegetables, their expected utility of wealth after selling them would be 50.

(iii) Calculate the value of d. [5]

(iv) Calculate the expected utility of wealth of the investor, if they do not buy any vegetables. [1]

The investor has decided they want to buy seven boxes of vegetables.

(v) Discuss whether $U(w)$ is appropriate for the investor. [2]

[Total 12]

6 An insurer assumes that the number of claims it incurs follows a Poisson process with parameter μ. The adjustment coefficient, *R*, for this insurer is a risk measurement parameter associated with its surplus process. It is the unique positive root of the following equation:

$$\mu M_X(R) = \mu + cR$$

where $M_X(R)$ is the moment generating function of the individual claim amount distribution, *X*.

(i) State what quantity is represented by *c*. [1]

The insurer assumes that claim amounts follow the gamma distribution with parameters $\alpha = 2$ and $\lambda = 1$.

(ii) Write down the moment generating function of this gamma distribution. [1]

(iii) Show that *R* satisfies the following equation:

$$cR^2 + R(\mu - 2c) + (c - 2\mu) = 0. \quad [4]$$

The insurer sets $\mu = 3.5$ and $c = 1$.

(iv) Solve the equation in part (iii) to determine the adjustment coefficient. [2]
[Total 8]

7 Explain, for each of the following scenarios, which behavioural biases are exhibited.

(i) A portfolio manager says that 50% of their portfolio performed well because of their careful stock selection process, while 50% underperformed because of a global economic crash. [1]

(ii) An individual investor claims that their stock-picking skills are very strong because 75% of their portfolio produced a 3% return. The other 25% of their portfolio produced a 50% loss. [2]

(iii) A risk modelling team at an insurance company decides to continue to use a modelling technique for a risk because it is used by modelling teams at other similar companies. The team makes this decision despite academic research suggesting that a new approach is substantially better. [2]

(iv) An actuary, with no knowledge of equity risk modelling, strongly opposes a new equity risk model that has suggested that the 1-in-50 value at risk of an equity is a 25% loss. The actuary's rationale for their opposition is that the previous model suggested a 10% loss. [3]
[Total 8]

8 A general insurer specialising in catastrophes is trying to understand its future probability of ruin. It models future claims payments in millions of dollars over each yearly interval as Exponentially distributed with parameter $\lambda = 1$. The insurer assumes that the claims payments are independent each year.

The insurer holds an initial surplus of \$3m and assumes no future premium income.

(i) Calculate the probability that the insurer is solvent at the following times measured in years:

(a) $t = 0$

(b) $t = 1$.

[2]

(ii) State, without performing any further calculations, how your answers to part (i) would change if the insurer changed its net claims distribution to be Exponentially distributed with parameter $\lambda = 2$. [1]

(iii) Discuss the appropriateness of the insurer's approach to assessing its future probability of ruin. [4]

[Total 7]

9 In a particular market, there are two assets, A and B, for which the annualised rates of return have the following characteristics:

Asset	*Expected value (%)*	*Standard deviation (%)*
A	10	20
B	5	0

An investor is considering investing a proportion of their wealth x_A in asset A and x_B in asset B.

(i) State the formula for the market price of risk in this market. [1]

(ii) Show that the efficient frontier for this investor is a straight line passing through the points (0, 0.05) and (0.1, 0.075) in (Standard Deviation, Expected Return) space. [5]

A third asset, C, becomes available in the market. It has an annualised expected return of 6% and an annualised standard deviation of 10%. It is uncorrelated with assets A and B.

(iii) Show that the new efficient frontier using A, B and C, passes through the point (0.1, 0.0769). [6]

[**Hint:** the market price of risk for a portfolio involving only assets A and C is maximised when $x_A = {}^5/_9$ and $x_C = {}^4/_9$.]

[Total 12]

10 Consider two investment portfolios, A and B, with cumulative probability distribution functions of returns F_A and F_B respectively.

Assume that portfolio A exhibits first-order stochastic dominance over portfolio B.

(i) Explain in words:

(a) what this tells us about the returns on portfolio A and portfolio B.

(b) why a non-satiated investor will prefer portfolio A to portfolio B.

[2]

Assume now that portfolio A exhibits second-order stochastic dominance over portfolio B.

(ii) State the requirements on an investor for them to prefer portfolio A over portfolio B. [1]

Assume now that portfolio B provides a return of either –5%, or +10%, with equal probability.

(iii) Set out how you may determine a distribution that the return on portfolio A could take, such that A is second-order stochastic dominant over B. [2]

[Total 5]

END OF PAPER