INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

25 September 2019 (am)

Subject CM2B – Financial Engineering and Loss Reserving Core Principles

Time allowed: One hour and forty-five minutes

INSTRUCTIONS TO THE CANDIDATE

1. *You are given this question paper and the Excel file.*

2. *Mark allocations are shown in brackets.*

3. *Attempt all questions. Each question is to be answered in the allocated tab.*

If you encounter any issues during the examination, please contact the Examinations Team at T. +44 (0) 1865 268 255

1 An equity index currently has a value of £7,000. A European call option on the index is currently priced at £480. The option has a strike price of £7,200 and matures two years from now.

Assume that no dividends are paid on the shares underlying the index. The risk-free force of interest is 1.5% per annum.

(i) Calculate the implied volatility of the index to the nearest 1%. [4]

(ii) Calculate the price today of a two-year forward contract on the index. [2]

An investor has purchased one call option and taken one short position in a two-year forward contract on the equity index.

(iii) Plot a chart showing the total payoff of the investor's portfolio at the end of two years for index values from £5,000 to £9,000. [10]

(iv) Suggest how the investor could achieve the same payoff as in part (iii) using the put option and a cash account. [4]

[Total 20]

2 An insurance company has a portfolio of policies. Consider an aggregate claims process, $S(t)$, where:

- $S(t) = \sum_{i=1}^{N(t)} X_i$
- $N(t)$ = the number of claims generated by the portfolio in the time interval $[0, t]$
- X_i = the amount of the i^{th} claim.

You have been given a single realisation of $N(5)$ and the values of the first three claim amounts, which follow an exponential distribution with mean 1.

(i) Calculate the value of the aggregate claims process at time $t = 5$. [2]

Assume that c, the rate of premium income per unit time, is 1.1 and U, the initial surplus, is 0.5.

(ii) Calculate the value of the surplus process, $U(t)$, at time $t = 5$. [4]

$N(t)$ is a Poisson process with $\lambda = 1$, therefore the interval of time between claims is exponentially distributed with a mean of 1. You have been given realisations of the time intervals t_i between the first three claims, where t_1 is the time of the first claim and t_i (for $i > 1$) is the time between claim i and claim $i - 1$.

(iii) Plot a chart showing the surplus process from time $t = 0$ to time $t = 5$. [10]

(iv) Calculate the minimum value of U, to three decimal places, that would avoid ruin before time $t = 5$ in the process in part (iii). [3]

The formula for the probability of ultimate ruin is $\Psi(U)$. Individual claim amounts are exponentially distributed with mean 1 and the premium loading factor is θ. The formula for $\Psi(U)$ is:

$$\Psi(U) = \frac{1}{1+\theta}\exp\left\{-\frac{\theta U}{1+\theta}\right\}$$

(v) Plot a chart showing how the probability of ruin varies for values of θ ranging from 0 to 1, when $U = 0.5$. [7]

(vi) Comment on what can be concluded from your chart in part (v). [6]

[Total 32]

3 An insurer writes one-year motor insurance policies. Each policy pays an amount of £10,000 if the car insured is damaged or stolen, and this has a likelihood of 1% in any policy year. The loss to an individual without insurance whose car is damaged or stolen will also be £10,000.

To determine an appropriate premium, the insurer has decided to test a range of annual premium rates from £90 to £150 in steps of £1.

When considering the gain or loss from an individual policy, the insurer uses a utility function where the utility $U(w)$ of wealth w is $U(w) = 0.001w$. The utility function is linear so we can assume the insurer's initial wealth to be zero and a negative utility value is allowable.

Customers use a log utility function where the utility $U(w)$ of wealth w is $U(w) = \ln(w)$. Each customer has initial wealth of £20,000 including the value of their car.

(i) Calculate the expected utility at the end of the year for an individual without insurance. [4]

(ii) Calculate, for the range of premiums above, the expected utility at the end of the year for a customer with insurance. [6]

(iii) Calculate the insurer's expected utility at the end of the year under the same circumstances as part (ii). [6]

(iv) Determine the range of premiums where both the customer's and the insurer's expected utility is higher with insurance than without. [4]

(v) Explain why the customer is willing to pay a premium which exceeds the expected cost of a claim. [4]

(vi) Discuss how the range of premiums calculated in part (iv) will change with varying levels of customer initial wealth. [6]

[Total 30]

4 An investment product allows a customer to save an amount of €100 at the start of each year. The total savings in the product earn interest at a random rate i_t for the period $[t - 1, t]$. The rate i_t is independently normally distributed with mean 7% and standard deviation 20%.

You have been given 100 simulations of i_t for $t = 1, 2, \ldots, 10$.

Let the total savings at time n be A_n, so that $A_n = (100 + A_{n-1}) \times (1 + i_n)$ and $A_0 = 0$.

(i) Calculate at time $t = 10$, using the probability distribution of i_t and **not** the simulated values in the spreadsheet:

(a) The expected value of the savings product.
(b) The standard deviation of the value of the savings product.

[8]

(ii) Calculate at time $t = 10$ using the simulated values of i_t in the spreadsheet:

(a) The mean value of the savings product.
(b) The standard deviation of the value of the savings product.

[6]

(iii) Comment on any differences between your answers to (i) and (ii). [4]

[Total 18]

END OF PAPER