

# **INSTITUTE AND FACULTY OF ACTUARIES**

## **EXAMINATION**

28 September 2021 (am)

### **Subject CM2 - Financial Engineering and Loss Reserving Core Principles**

#### **Paper B**

Time allowed: One hour and fifty minutes

<p>In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.</p>
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If you encounter any issues during the examination please contact the Assessment Team on  
T. 0044 (0) 1865 268 873.

- 1 In a particular country, it is possible to invest in two assets: A and B. You are given the following information about the expected returns, variances and correlation of these two assets:

	<i>Asset A</i>	<i>Asset B</i>
Expected return, $E$	$E_A = 4.5\%$	$E_B = 6\%$
Variance of return, $V$	$V_A = 1\%$	$V_B = 9\%$
Correlation of return	$\rho_{AB} = 65\%$	

An investor holds a proportion  $x_A$  of Asset A and  $x_B = 1 - x_A$  of Asset B in their portfolio. The expected return of the portfolio is  $E_P$  and its variance is  $V_P$ . Assume that short selling is permitted and that assets can be held in any amount.

Consider  $x_A$  ranging from  $-0.5$  to  $1.5$  (inclusive) in intervals of  $0.1$ .

- (i) Calculate, for each value of  $x_A$ :
  - (a)  $x_B$ .
  - (b)  $E_P$ .
  - (c)  $V_P$ .

[7]
- (ii) Plot a chart of  $E_P$  and  $V_P$  against  $x_A$ . [4]
- (iii) Comment on how  $E_P$  and  $V_P$  vary with  $x_A$ . In particular, comment on the approximate value of  $x_A$  for which  $V_P$  is minimised (you do not need to calculate this accurately). [5]

Now consider a situation in which  $\rho_{AB}$  varies between  $-100\%$  and  $100\%$  (inclusive) in intervals of  $10\%$ .

- (iv) Calculate, for each value of  $\rho_{AB}$ :
  - (a) the value of  $x_A$  that minimises  $V_P$ .
  - (b) the value of  $V_P$  at this value of  $x_A$ .

[7]

The government passes a law that bans short selling. Define the new portfolio variance as  $V_{Law}$ .

- (v) Calculate, for each value of  $\rho_{AB}$ :
    - (a) the value of  $x_A$  that minimises  $V_{Law}$ .
    - (b) the value of  $V_{Law}$  at this  $x_A$ .

[4]
  - (vi) Plot a chart of  $V_P$  and  $V_{Law}$  against  $\rho_{AB}$ . [4]
  - (vii) Discuss the extent to which this law will affect the investor's portfolio. [4]
- [Total 35]

- 2 Consider two consultants with respective utility functions  $U_1(w)$  and  $U_2(w)$ :

$$U_1(w) = \ln(w)$$

$$U_2(w) = (w - 20^{-4}w^2)$$

Assume the consultants have zero initial wealth, and that they are non-satiated and risk averse.

The consultants have been offered a 1-year contract with profit-sharing arrangements by Company A. The fee paid to each consultant will be  $\$(30,000 + Y)$  where  $Y$  depends on the company's success.  $Y \sim 10,000X$  where  $X \sim \text{Bin}(4, 0.25)$ . You have been provided with the probability distribution of  $X$  in the 'Q2 data' worksheet.

- (i) Calculate the mean and variance of the total fee. [5]

- (ii) Calculate, for each consultant, the expected utility of the fee. [6]

Suppose the second consultant, with utility function  $U_2(w)$ , is also offered a 1-year contract with a fixed fee by Company B.

- (iii) Calculate the minimum fixed fee that would ensure that Company B's offer is more attractive to the second consultant than Company A's offer. [6]

The owners of Company A, who are risk-averse, wish to ensure that their offer is more attractive to the second consultant than Company B's offer.

- (iv) Explain whether Company A should agree to pay a fixed fee and, if so, how much. [7]

[Total 24]

- 3** The incremental claims paid to date on a motor insurance policy were as follows (figures in \$000s):

<i>Accident year</i>	<i>Development year</i>			
	<i>0</i>	<i>1</i>	<i>2</i>	<i>3</i>
2017	1,476	1,031	724	506
2018	1,537	1,076	892	
2019	1,754	1,234		
2020	1,632			

Inflation for the 12 months to the middle of each calendar year was as follows:

2018	3.20%
2019	1.70%
2020	−0.90%

Future inflation from mid-2020 is estimated to be 1.5% per annum.

Claims are assumed to be fully run-off at the end of development year 3.

Estimate the total outstanding claims arising from accidents in 2020 using the inflation adjusted chain ladder method. [19]

- 4** Consider a non-dividend paying share currently valued at £10. In each future 6-month period its value can either rise by 25% or fall by 20%. The continuously compounded risk-free rate of interest is 5% p.a.

Consider a European call option written on this share with a strike price of £11 and a maturity of 5 years.

- (i) Calculate the value of the option, using state price deflators. You have been provided with the Binomial combinations  ${}_nC^r$  in the ‘Q4 data’ worksheet if you wish to use them. [13]
  - (ii) Estimate the implied volatility of the underlying share. [2]
  - (iii) Calculate the value of the option using the Black–Scholes model and your answer from part (ii). [3]
  - (iv) Comment on the differences between your answers to parts (i) and (iii). [4]
- [Total 22]

**END OF PAPER**