INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

17 April 2019 (am)

Subject CP2 – Modelling Practice
Core Practices

Paper One

Time allowed: Three hours and fifteen minutes

INSTRUCTIONS TO THE CANDIDATE

1. *You are given this question paper and the Excel file.*

2. *Mark allocations are shown in brackets.*

3. *Attempt all questions. Questions are to be answered as per "exam requirements".*

If you encounter any issues during the examination, please contact the Examinations Team at
T. +44 (0) 1865 268 255

Exam requirements

1 Read the background document, which describes the scenarios that need to be modelled and documented for this project.

2 Construct a spreadsheet model that produces the following calculations and charts. You should ensure that your spreadsheet contains appropriate self-checks and that you have performed reasonableness checks at each stage of your calculations.

(i) Carry out data checks on the data provided to confirm that the data is complete and fit for use, making any corrections that are needed. [4]

(ii) Calculate the average mark and standard deviation of marks for each school. [2]

(iii) Produce a chart to illustrate whether the data for all schools combined appears to follow a normal distribution. [3]

(iv) For each pupil, recalculate their mark as a percentage and, for each school, calculate:

(a) the percentage of pupils achieving the expected standard. [1]
(b) the percentage of pupils achieving the higher standard. [1]
(c) the rank of each pupil (within their school). [1]
[Total 3]

(v) Produce a chart to show the percentage of pupils for each school achieving the expected and higher standards. [3]

(vi) Calculate the total grant that will be awarded to each school. [3]

(vii) Produce a chart that shows the level of base and additional grant awarded to each school. [3]

Next year, the City Board of Education has a budget of only £75,000 for the total amount of grants to be awarded. The base grant will remain at £50,000.

(viii) Calculate, using this year's results, the amount that should be awarded for each pupil that achieves the higher standard for the Education Board to remain within their budget of £75,000. [2]

It has been suggested by one government official that there is bias in the data caused by the location of each of the schools. Schools A and E are from one location and Schools B, C and D are from another location.

(ix) Calculate, using a *t*-test, whether the mean difference in test scores between schools in the two different locations is significant or not. [5]

An additional two marks are available for auto checks on steps (iii) to (viii). [2]

Note: all scenarios outlined on page 2 should be modelled separately within your spreadsheet. The user should not need to change the parameters to see the results.

Marks available for spreadsheet model and checks:

Accurate completion of above modelling steps and associated auto checks. [30]
Demonstration of good modelling technique and practice. [7]

[Sub-total 37]

3 Produce an audit trail for your spreadsheet model which includes the following aspects:

(a) purpose of the model
(b) data and assumptions used
(c) methodology, i.e. description of how each calculation stage in the model has been produced
(d) explanation of the checks performed.

You should ensure that your audit trail is suitable for both a senior actuary, who has been asked to approve your work, and a fellow student, who has been asked to peer review and correct your model, or may be asked to continue to work on it or to use it again for a similar purpose in the future.

Marks available for audit trail:

Audit approach

- Communication skills *(the audit trail provides enough detail to be read as a self-standing document)* [4]
- Fellow student can review and check methods used in the model [7]
- Senior actuary can scrutinise and understand what has been done [7]
- Written in clear English [4]
- Written in a logical order [3]

Audit content

- All steps clearly explained [8]
- Clear signposting included throughout [4]
- Statement of assumptions made [5]
- Reasonableness checks [5]
- All model steps accurately covered [16]

[Sub-total 63]

[Total 100]

Background

The Board of Education in a particular city monitors the performance of schools by testing all children at the age of 11 in mathematics.

All of the pupils' raw marks in each school are received by the City Board of Education and analysed to assess the performance of each school.

The tests have a maximum mark of 60.

A pupil will be considered to have reached the expected standard in mathematics if they get 50% of the answers correct. If they get 75% of the answers correct, then they are considered to have achieved the higher standard in mathematics.

There are five schools in the city and each year the City Board of Education needs to work out the ranking of each school. The City Board of Education gives an annual payment or "grant" to each school based on their performance. The ranking is based on the percentage of children in each school that achieve the expected standard. First ranked is for the highest percentage of children achieving the expected standard; fifth ranked is for the lowest percentage of children achieving the expected standard.

These results are then used to identify what grant each school will receive from the City Board of Education.

The base grant available is £50,000.

The base grant is distributed as follows:

- The top ranked school (based on the highest percentage achieving the expected standard) will receive 40% of the base grant available.
- The second ranked school will receive 25% of the base grant available.
- The third ranked school will receive 15% of the base grant available.
- The fourth ranked school will receive 10% of the base grant available.
- The fifth ranked school will receive 10% of the base grant available.

In addition, each school will receive £1,000 for each pupil that achieves the higher standard.

The City Board of Education has provided the marks for this year's tests for the five schools and has requested that your company carries out the analysis to work out the total grant that the Board will need to pay out this year.

Next year, the City Board of Education has a budget of only £75,000 for the total amount of grants to be awarded. Using this year's results, they have also requested that you calculate the amount that should be awarded for each pupil that achieves the higher standard for the Education Board to remain within their budget of £75,000. The base grant will remain at £50,000.

It has been suggested by one government official that there is bias in the data which is caused by the location of each of the schools. Schools A and E are from one location and Schools B, C and D are from a different location. Your boss has suggested using a *t*-test, with a significance level of 5%, to calculate whether the mean difference in test scores between schools in the two locations is significant or not and ascertain whether the government official has a valid observation. (See additional guidance.)

This *t*-test should be built within your model. It should NOT be completed using Excel's data analysis tool.

Your boss has asked you to carry out this work and is now out of the office and cannot be contacted for the next three hours. They would like the above calculations finished and documented in the audit trail ready for their return.

Additional Guidance

Testing the value of the difference between two population means

Situation: independent random samples, sizes n_1 and n_2 from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ respectively.

Testing: $\boldsymbol{H_0 : \mu_1 - \mu_2 = \delta}$

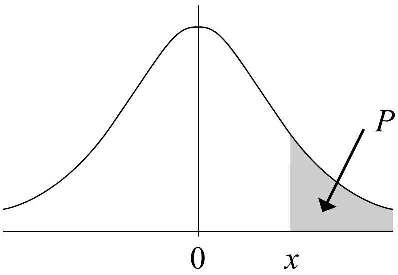
Where σ_1^2 and σ_2^2 are unknown, and under the assumption $\sigma_1^2 = \sigma_2^2$, this common variance is estimated by S_p^2, the test statistic is:

$$t = \frac{\bar{x}_1 - \bar{x}_2 - \delta}{S_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$

distributed as t with $n_1 + n_2 - 2$ degrees of freedom under H_0.

Where S_p^2 is the pooled variance and is equal to $\dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$.

Percentage points for the *t* distribution



P =	**40%**	**30%**	**25%**	**20%**	**15%**	**10%**	**5%**	**2.5%**	**1%**	**0.5%**	**0.1%**	**0.05%**
ν												
1	0.3249	0.7265	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	0.2887	0.6172	0.8165	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.2767	0.5844	0.7649	0.9785	1.250	1.638	2.353	3.182	4.541	5.841	10.21	12.92
4	0.2707	0.5686	0.7407	0.9410	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.2672	0.5594	0.7267	0.9195	1.156	1.476	2.015	2.571	3.365	4.032	5.894	6.869
6	0.2648	0.5534	0.7176	0.9057	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.2632	0.5491	0.7111	0.8960	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.2619	0.5459	0.7064	0.8889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.2610	0.5435	0.7027	0.8834	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.2602	0.5415	0.6998	0.8791	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.2596	0.5399	0.6974	0.8755	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.2590	0.5386	0.6955	0.8726	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.2586	0.5375	0.6938	0.8702	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.2582	0.5366	0.6924	0.8681	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.2579	0.5357	0.6912	0.8662	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.2576	0.5350	0.6901	0.8647	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.2573	0.5344	0.6892	0.8633	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.2571	0.5338	0.6884	0.8620	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.2569	0.5333	0.6876	0.8610	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.2567	0.5329	0.6870	0.8600	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.2566	0.5325	0.6864	0.8591	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.2564	0.5321	0.6858	0.8583	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.2563	0.5317	0.6853	0.8575	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.2562	0.5314	0.6848	0.8569	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.2561	0.5312	0.6844	0.8562	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.2560	0.5309	0.6840	0.8557	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.2559	0.5306	0.6837	0.8551	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	0.2558	0.5304	0.6834	0.8546	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.2557	0.5302	0.6830	0.8542	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.660
30	0.2556	0.5300	0.6828	0.8538	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	0.2555	0.5297	0.6822	0.8530	1.054	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	0.2553	0.5294	0.6818	0.8523	1.052	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	0.2552	0.5291	0.6814	0.8517	1.052	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	0.2551	0.5288	0.6810	0.8512	1.051	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	0.2550	0.5286	0.6807	0.8507	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	0.2547	0.5278	0.6794	0.8489	1.047	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	0.2545	0.5272	0.6786	0.8477	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	0.2539	0.5258	0.6765	0.8446	1.041	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	0.2533	0.5244	0.6745	0.8416	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291

END OF PAPER