

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

16 April 2020 (am)

Subject CP2 – Modelling Practice Core Practices

Paper Two

Time allowed: Three hours and fifteen minutes

INSTRUCTIONS TO THE CANDIDATE

1. You are given this question paper and Excel file.
2. Mark allocations are shown in brackets.
3. Attempt all questions. Questions are to be answered as per “exam requirements”.

If you encounter any issues during the examination please contact the Examination team at
T. 0044 (0) 1865 268 255

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Exam requirements

Read the background document that describes the scenarios that have been modelled and documented for this project, and the work that remains outstanding

Read the audit trail that has been written by your colleague, another actuarial student, for the calculations that they performed. This will assist you in following and understanding the calculations performed in the Excel model provided.

You are not required to add to or amend the audit trail.

You should assume that your colleague's calculations have been checked and are correct.

1 Spreadsheet model

Expand the spreadsheet model to produce the following additional calculations. You should ensure that the additional work you undertake on the spreadsheet contains appropriate self-checks and you should not overwrite the existing calculations.

- (i) Repeat the calculations for the 'overall guarantee' option. [5]
- (ii) Calculate, using Goal Seek or otherwise, an adjusted guarantee **charge** (as a percentage of the fund) that would give the same expected average fund value under the annual guarantee option after 10 years as the no guarantee option. [2]
- (iii) Calculate, using Goal Seek or otherwise, an adjusted guarantee **level** (as a percentage of the fund) that would give the same expected average fund value under the overall guarantee option after 10 years as the no guarantee option. [2]

2 Charts

Construct, for each of the following, a suitable chart to illustrate:

- the mean, minimum and maximum expected fund values, after 10 years, under the three options.
- the annual guarantee **charge** under the annual guarantee option and with the new charge that gives the same expected average fund value (found in part 1(ii)).
- the guarantee **level** for the overall guarantee option and with the new level that gives the same expected average fund value (found in part 1(iii)).

[6]

[Sub-total 15]

3 Summary document

Prepare a summary document of around five to seven pages, capturing the main features and results of the work done by you and your colleague. You can assume that the summary is being prepared for your boss, a senior actuary, who will present the work to the client.

Your summary should include the following:

- purpose of the project, data, method and assumptions used by you and your colleague
- results, including charts
- commentary on the results
- key conclusions
- suggested next steps.

Commentary on the results should cover, but not be limited to:

- analytical comments on each stage of the results, including explaining patterns in the results and any unusual features.
- an explanation of the differences between the results under the various options modelled.

Next steps need to be specific to the project, with some mention of why each is a valid next step. The summary should cover the full scope of the project, including the current approach that was modelled in the spreadsheet provided.

You are not required to add to or amend the audit trail.

Marks available for the summary:

(i)	Methodology (including purpose, data, method and assumptions)	[27]
(ii)	Results, including charts	[5]
(iii)	Commentary on results and conclusions	[23]
(iv)	Next steps	[20]
(v)	Drafting	[10]
		[Sub-total 85]
		[Total 100]

Background

You are an actuarial student working for a consultancy in a developed country.

One of the clients of the consultancy, Mr White, has approached your boss, a qualified actuary, and asked them to perform some modelling in order to help him assess a potential major investment.

Mr White is planning on investing \$100,000 into a mixed asset fund that invests in a well-diversified portfolio of assets (equities, bonds, etc.). He is considering holding the investment for the next 10 years. The investment fund has no initial charge but does levy an annual management charge of 1% p.a., which is applied before any charges for guarantees. Based on an historic analysis of the performance of the fund, the average annual return before charges was 5% p.a. and the standard deviation in return was 15% p.a. The mix of assets in the fund has been stable over time and is not expected to change in the future.

Under the provisions of the fund, Mr White has the option, but not the obligation, to choose one of two guarantees as detailed below:

‘Annual guarantee’ option

- On each anniversary of the initial investment, the value of the fund, net of the management charge, is guaranteed to be no less than 80% of the highest of all the fund values at each previous anniversary, including the initial investment (i.e. the guarantee level is 80%). The asset management company will make up any shortfall to meet this guarantee.
- There is an additional charge for the guarantee of 1.5% p.a. of the fund value taken annually in arrears. There is no one-off initial charge.

‘Overall guarantee’ option

- At the end of the 10-year period, the asset management company will guarantee to pay Mr White the highest of the initial investment (after allowing for the one-off charge), and the fund value at each of the subsequent 10 year-end anniversaries (i.e. the guarantee level is 100%).
- There is a one-off charge of 15% of the total initial investment, taken prior to investment in the fund.
- There is no additional annual charge for the guarantee.

Mr White would like to have a better understanding of the range of investment values that are expected to be achieved at the end of 10 years under three scenarios, i.e. with no guarantee and also under each of the two guarantee options set out above. Mr White is interested in the expected final fund value after 10 years as well as the volatility in the expected value.

To illustrate the range of investment outcomes Mr White could expect, your colleague, Sam, has begun some analysis. He has used 100 sets of 10 random numbers to perform 100 simulations of the fund’s performance. He sourced the random numbers from the Statistics and Analysis Department of your consultancy.

The annual investment returns are assumed to follow a Normal distribution with a mean of 5% p.a. and a standard deviation of 15% p.a. Sam used the random numbers to perform 100 simulations of investment returns for each year. Using these simulated returns he completed

100 projections of the fund value and has produced summary statistics of the projected investment fund after 10 years, under the first two scenarios described above (i.e. with no guarantee and with the annual guarantee option). He produced an audit trail for the work completed and a copy is below.

Sam is currently at a client meeting and cannot be contacted. In Sam's absence your boss has asked you to enhance the model to include the overall guarantee option.

To help Mr White understand the value of the guarantees, your boss would also like you to calculate the following and compare these to the original values in appropriate charts:

1. Under the 'annual guarantee' option: an adjusted guarantee **charge** (as a percentage of the fund) that would give the same expected average fund value under the annual guarantee option after 10 years as the no guarantee option.
2. Under the 'overall guarantee' option: an adjusted guarantee **level** (as a percentage of the fund) that would give the same expected average fund value under the overall guarantee option after 10 years as the no guarantee option.

Finally, your boss needs you to prepare a summary document covering all elements of the work (both the original work your colleague completed and the additional modelling you are undertaking).

Your summary should include the following:

- purpose of the project, data, method and assumptions used by you and your colleague
- results, including charts
- commentary on the results and key conclusions
- suggested next steps.

You are not expected to include the additional modelling you undertake in the audit trail, but your results should be included in the summary.

Additional guidance

Function NORMINV

The Excel NORMINV function calculates the inverse of the Cumulative Normal Distribution Function for a supplied value of x , and a supplied distribution mean and standard deviation.

The syntax of the function is NORMINV(probability, mean, standard deviation)

where the arguments are:

probability – the value at which you want to evaluate the inverse function

mean – the arithmetic mean of the distribution

standard deviation – the standard deviation of the distribution.

Function STDEV

The Excel STDEV function calculates the standard deviation of a supplied set of values.

The syntax of the function is STDEV([range of numbers])

Audit trail

Objective

Mr White is planning on making an investment of \$100,000 in a mixed asset fund that offers two guarantee options, in return for additional charges. The investment term being considered is 10 years. The mixed asset fund levies an annual management charge of 1% p.a.

The first guarantee option is:

- ‘Annual guarantee’: in return for an additional annual management charge, at each policy year anniversary the fund value is guaranteed to be no less than 80% of the highest fund value at all the previous anniversaries (including the initial investment). The cost of the guarantee biting is funded by the investment company.

The purpose of the model is to perform the following:

- Calculation of the average of the projected values of the investment fund after 10 years under 100 random scenarios, under the no guarantee option and the two guarantee options (Note – the ‘Overall guarantee’ option has not been modelled at this stage.)
- Calculation of summary statistics on the outcomes under the scenarios.

Data

This sheet contains the random data from a $U(0,1)$ distribution from the Statistics and Analysis Department.

Data checks

This sheet does some checks on the random data to ensure that it is from a uniform distribution.

The first test compares the mean and standard deviation of the data with the expected values of a $U(0,1)$ distribution. The expected mean is 0.5 and the expected standard deviation is the square root of 1/12. The tests are passed as the actual values are within 1% of the expected values.

The second test is a Chi-square test. For bands of 0.1 from 0 to 1, the actual frequency observed is compared to the expected frequency. This shows that there is a very low probability (less than 5%) that the differences seen are errors and not just expected random variances.

Parameters

This sheet summarises the parameters used in the calculations.

Assumptions

The following assumptions were made in the model:

- There are no other charges associated with the investment or guarantees.
- The past performance data is suitable for projecting over the next 10 years.
- Future investment returns follow a Normal distribution.
- All charges are applied annually in arrears, unless stated otherwise.
- The original investment of \$100,000 is also within the scope of the annual guarantee, i.e. a fall of more than 20% in year 1 would trigger the guarantee.

Annual guarantee

This is the main calculation sheet that does the projections and summarises the results.

In columns B to K and for 100 rows, the formula NORMINV() is used to transform each random number from the 'Data' sheet into a random number from a Normal distribution with the mean and standard deviation parameters set out in the 'Parameters' sheet. This gives the projection of the performance of the investment before charges for 10 years and under 100 different random scenarios.

In columns M to W the projection of the investment value is calculated based on the annual performance figures as calculated above. At the start of the investment period (time 0) the value of the investment is fixed at the initial investment amount of \$100,000.

In each subsequent year the value is projected as:

the value at the previous point in time
multiplied by 1 plus the relevant annual performance for that year and that simulation
multiplied by 1 minus the annual management charge set out in the 'Parameters' sheet.

This gives a range of 100 values after the 10-year investment period in column W.

In columns Y to AI the projected value of the investment under the annual guarantee option is calculated as follows:

The maximum of the value of the investment at the previous point in time
multiplied by 1 plus the relevant annual performance
multiplied by 1 minus the annual management charge
multiplied by 1 minus the annual guarantee charge
and the maximum previous value of the fund at either the initial investment
or any year end anniversary, multiplied by the guarantee level of 80%

This gives a range of 100 values after the 10-year investment period in column AI, including any benefit received from the annual guarantee.

Finally, in columns AK to AM the key statistics for the projections with and without the annual guarantee are given, by analysing the projected results after 10 years in each case. These use the functions MEAN() for the average, MIN() for the minimum, MAX() for the maximum and STDEV() for the standard deviations.

END OF PAPER