

# **INSTITUTE AND FACULTY OF ACTUARIES**

## **EXAMINERS' REPORT**

April 2018

### **Subject CT1 – Financial Mathematics Core Technical**

#### **Introduction**

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Luke Hatter  
Chair of the Board of Examiners  
June 2018

**A. General comments on the *aims of this subject and how it is marked***

1. CT1 provides a grounding in financial mathematics and its simple applications. It introduces compound interest, the time value of money and discounted cashflow techniques which are fundamental building blocks for most actuarial work.
2. Please note that different answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations but candidates are not penalised for this. However, candidates may lose marks where excessive rounding has been used or where insufficient working is shown.

**B. General comments on *student performance in this diet of the examination***

1. The comments that follow the questions concentrate on areas where candidates could have improved their performance. Where no comment is made, the question was generally answered well by most candidates. The examiners look most closely at the performance of the candidates close to the pass mark and the comments therefore often relate to those candidates.
2. Student performance was similar to that in recent diets with the average mark being very close to the average of the previous six diets although lower than that in September 2017. Students seemed to have difficulty with the early part of the paper with the four worst answered questions all in the first five questions.

**C. Pass Mark**

The Pass Mark for this exam was 60.

## Solutions

**Q1** The characteristics of a Eurobond are:

- Medium- or long-term borrowing
- Usually unsecured
- Regular interest payments
- Redeemed at par
- Issued and traded internationally/not in the jurisdiction of any one country
- Can be denominated in any currency (e.g. not the currency of issuer)
- Tend to be issued by large companies, governments or supra-national organisations
- Yields depend on issue size and issuer (or marketability and risk)...
- ... (although typically yields will be higher than those on gilts and lower than those on equities)
- Issue characteristics may vary – market free to allow innovation

[½ mark for each point, max 4]

*This was a bookwork question similar to the type asked in most diets. This was generally answered poorly particularly by marginal candidates.*

**Q2** (i) An equity which is offered for sale without the next dividend is called ex-dividend [1]

(ii) Value of dividends to investor =

$$0.07 \times 10,000 \times \left( v^{7/12} + 1.02 \left( v^{13/12} + v^{19/12} \right) + 1.02^2 \left( v^{25/12} + v^{31/12} \right) + \dots \right) \quad [1]$$

$$= 700v^{1/12} \left( v^{6/12} + 1.02 \left( v + v^{1/2} \right) + 1.02^2 \left( v^2 + v^{2/2} \right) + \dots \right)$$

$$= 700v^{7/12} + 700v^{1/12} \cdot 1.02 \left( v + v^{1/2} \right) \times \left[ 1 + 1.02v + 1.02^2 v^2 + \dots \right] @ 7\%$$

$$= 700v^{7/12} + 700v^{1/12} \cdot 1.02 \left( v + v^{1/2} \right) \times \left( \frac{1}{1 - (1.02/1.07)} \right) \quad [2]$$

$$= 672.91 + 709.99 \times 1.83807 \times \left( \frac{1}{1 - (1.02/1.07)} \right)$$

$$= \$28,600 \quad [2]$$

*Candidates who scored well on this question tended to score very well overall but this was poorly answered by marginal candidates. Very few got the timing right, with many failing to include the extra one month offset. Many also struggled with simplifying the long equation into a format which could be more easily calculated.*

**Q3** Effective rate of interest per month for first 10 years,  $i_1$ , comes from:

$$1 + i_1 = (1.03)^{1/6} \Rightarrow i_1 = 0.49386\% \text{ per month} \quad [1]$$

and effective rate of interest per month for last 15 years,  $i_2$ , comes from:

$$1 + i_2 = e^{0.06/12} \Rightarrow i_2 = 0.50125\% \text{ per month} \quad [1]$$

$$\Rightarrow \text{Accumulation after 25 years} = 80 \ddot{s}_{120|}^{0.49386\%} \times (1.0050125)^{180} + 80 \ddot{s}_{180|}^{0.50125\%}$$

$$\begin{aligned} \text{where } \ddot{s}_{120|}^{0.49386\%} &= 1.0049386 \times \frac{(1.0049386^{120} - 1)}{0.0049386} \\ &= 164.0318 \end{aligned} \quad [1\frac{1}{2}]$$

$$\text{and } \ddot{s}_{180|}^{0.50125\%} = 1.0050125 \times \frac{(1.0050125^{180} - 1)}{0.0050125} = 292.6504 \quad [1\frac{1}{2}]$$

$$\begin{aligned} \Rightarrow \text{Accumulation} &= 80 \times 164.0318 \times 1.0050125^{180} + 80 \times 292.6504 \\ &= 32276.13 + 23412.03 = \text{£}55,688.16 \text{ (exact answer is £}55,688.38\text{)} \end{aligned} \quad [1]$$

[or working in years:

$$1 + i_1 = (1.03)^2 \Rightarrow i_1 = 6.09\% \text{ per year}$$

$$1 + i_2 = e^{0.06} \Rightarrow i_2 = 6.1837\% \text{ per year}$$

$$\Rightarrow \text{Accumulation after 25 years} = 960 \ddot{s}_{10|}^{(12)@6.09\%} \times (1.061837)^{15} + 960 \ddot{s}_{15|}^{(12)@6.1837\%}$$

$$\text{where } \ddot{s}_{10|}^{(12)@6.09\%} = \frac{(1.0609^{10} - 1)}{12 \times \left( 1 - \left( 1 - \frac{0.0609}{1.0609} \right)^{1/12} \right)} = 13.6693$$

$$\text{and } \ddot{s}_{\overline{15}|}^{(12)@6.1837\%} = \frac{(1.061837^{15} - 1)}{12 \times \left( 1 - \left( 1 - \frac{0.061837}{1.061837} \right)^{1/12} \right)} = 24.3877$$

$$\begin{aligned} \Rightarrow \text{Accumulation} &= 960 \times 13.6693 \times 1.061837^{15} + 960 \times 24.3877 \\ &= 32276.42 + 23412.17 = \text{£}55,688.59 \end{aligned}$$

*Many of the comments on Q2 also apply here although the performance on this question was better. Common errors included those in the calculation of the appropriate interest rate and in the calculation of the accumulation factors.*

- Q4** (i) Let  $S_n$  denote the accumulation at time  $n$  of an initial investment of 1 at time 0.

Then, the accumulation at time 10 is:

$$S_{10} = \prod_{t=1}^{10} (1 + i_t) \Rightarrow \ln(S_{10}) = \sum_{t=1}^{10} \ln(1 + i_t) \sim N(10\mu, 10\sigma^2) \quad [1]$$

Also, an initial investment of  $X$  at time 0 will accumulate to  $XS_{10}$  at time 10.

Then, we require to find the value of  $X$  such that:

$$P(XS_{10} \geq 800,000) = 0.95 \quad [1]$$

$$\Rightarrow P\left(S_{10} \geq \frac{800,000}{X}\right) = 0.95$$

$$\Rightarrow P\left[\ln(S_{10}) \geq \ln\left(\frac{800,000}{X}\right)\right] = 0.95$$

$$\Rightarrow P\left[Z \geq \frac{\ln\left(\frac{800,000}{X}\right) - 10\mu}{\sqrt{10\sigma^2}}\right] = 0.95$$

$$\Rightarrow \frac{\ln\left(\frac{800,000}{X}\right) - 10\mu}{\sqrt{10\sigma^2}} = -1.645$$

$$\Rightarrow \ln\left(\frac{800,000}{X}\right) = 0.019708$$

$$\Rightarrow X = 784,388 \approx \text{€}784,000$$

$$(\text{exact answer is £}784,333) \quad [3]$$

- (ii) (a) Increasing the value of  $\mu$  will increase the expected annual investment return and so the amount required at time 0 (to meet the liability with probability 95%) will decrease. [1]
- (b) Increasing the value of  $\sigma$  will increase the volatility of the annual investment return  $\Rightarrow$  the amount required at time 0 (to meet the liability with probability 95%) will increase. [1]
- (c) If the probability of meeting the liability is increased from 95% to 99%, then the risk of not meeting the liability has been reduced and so the amount required to be invested now must be increased so that, with a greater initial investment, there is more certainty that the target figure of £800.000 after 10 years will be reached. [1]

*This was the worst answered question on the paper. Some candidates tried to calculate the parameters of the distribution for the 10-year accumulation from first principles and others made method/calculation errors when manipulating the Normal distribution.*

*In part (ii), many candidates stated conclusions with no supporting reasoning. No credit was awarded in such cases.*

- Q5** (i) The “no arbitrage” assumption means that neither of the following applies:
- (a) an investor can make a deal that would give her or him an immediate profit, with no risk of future loss; nor
- (b) an investor can make a deal that has zero initial cost, no risk of future loss, and a non-zero probability of a future profit. [2]
- (ii) (a) The current value of the forward price of the old contract is:

$$7.10 \times (1.02)^3 - 1.1a_{\overline{6}|}^{2\%}$$

whereas the current value of the forward price of a new contract is:

$$10.20 - 1.1a_{\overline{6}|}^{2\%}$$

Hence, current value of old forward contract is:

$$10.20 - 7.10 \times (1.02)^3 = \text{£}2.6654 \quad [3]$$

Alternative solution

$$K_0 = (7.1 - 1.1v^3 a_{\overline{6}|}) \times 1.02^9 = 1.5462$$

$$K_3 = (10.2 - 1.1a_{\overline{6}|}) \times 1.02^6 = 4.5479$$

$$V_4 = (4.5479 - 1.5462) \times 1.02^{-6} = £2.6654$$

Solution if it is assumed that dividends are paid from the start:

$$K_0 = (7.1 - 1.1a_{\overline{9}|}) \times 1.02^9 = -2.22449$$

$$K_3 = (10.2 - 1.1a_{\overline{6}|}) \times 1.02^6 = 4.5479$$

$$V_3 = (4.5479 + 2.2449) \times 1.02^{-6} = £6.0319$$

- (ii) (b) The current value of the forward price of the old contract is:

$$7.10(1.02)^3 (1.025)^{-9} = 6.0331$$

Whereas the current value of the forward price of a new contract is

$$10.20(1.025)^{-6} = 8.7954$$

$\Rightarrow$  current value of old forward contract is

$$8.7954 - 6.0331 = £2.7623$$

[3]

Alternative Solution

$$K_0 = 7.1 \times 1.025^{-9} \times 1.02^9 = 6.7943$$

$$K_3 = 10.2 \times 1.025^{-6} \times 1.02^6 = 9.9051$$

$$V_3 = (9.9051 - 6.7943) \times 1.02^{-6} = £2.7623$$

*In part (i), many candidates seemed confused between 'arbitrage' and 'no arbitrage'. In part (ii)(a), candidates who assumed the dividends were payable from outset also received full credit.*

- Q6** (i) Let  $i$  = money yield per annum.

Consider £100 nominal stock purchased on 1/4/2018.

$$102 = 0.75 \times 3 \times a_{\overline{10}|}^{(2)} + (105 - 0.35 \times (105 - 102))v^{10}$$

$$\Rightarrow 102 = 2.25 a_{\overline{10}|}^{(2)} + 103.95v^{10} \quad [2]$$

$$\text{Try } 2\%, \text{ RHS} = 2.25 \times 1.004975 \times 8.9826 + 85.2752$$

$$= 105.59$$

$$\text{Try } 3\%, \text{ RHS} = 2.25 \times 1.007445 \times 8.5302 + 77.3486$$

$$= 96.68$$

$$i = 0.02 + \frac{105.59 - 102}{105.59 - 96.68} \times 0.01$$

$$= 0.0240 \text{ (exact answer is 0.0239)}$$

[3]

$$\text{and } (1 + i) = (1 + i')(1 + e)$$

$$\Rightarrow i' = \frac{1.0239}{1.02} - 1 = 0.00382 \text{ i.e. Real yield} = 0.4\% \text{ per annum} \quad [1]$$

- (ii) If inflation had been less than 2% per annum throughout the term then the real rate of return would have been higher. This is because one would be stripping out a lower rate of inflation from the money yield to obtain the real yield. [2]

*Generally well-attempted although in part (b) some candidates, as in Q4, gave a conclusion without supporting reasoning.*

**Q7** (i) Present value of initial outlay  $= 2 + 0.5 v^{1/2} = 2.4767 \quad [1]$

$$\text{PV of 1}^{\text{st}} \text{ year's net revenue} = 0.2 v \bar{a}_{\overline{1}|} = 0.2 v^2 \frac{i}{\delta}$$

$$= 0.2 \times 0.82645 \times 1.049206$$

$$= 0.1734 \quad [2]$$

$$\text{PV of 2}^{\text{nd}} \text{ to 14}^{\text{th}} \text{ year of net revenue}$$

$$= 0.25 v^2 \bar{a}_{\overline{1}|} + 0.25 \times 1.04 v^3 \bar{a}_{\overline{1}|} + \dots + 0.25 \times 1.04^{12} v^{14} \bar{a}_{\overline{1}|}$$

$$= 0.25 v^2 \bar{a}_{\overline{1}|} (1 + 1.04 v + \dots + 1.04^{12} v^{12})$$

$$= 0.25 v^3 \frac{i}{\delta} \left[ \frac{1 - \left( \frac{1.04}{1.10} \right)^{13}}{1 - \frac{1.04}{1.10}} \right]$$



$$= 0.25 \times 0.75131 \times 1.049206 \times 9.49094$$

$$= 1.8704 \quad [3]$$

$$\text{PV of refit} = 0.8 v^8 = 0.3732 \quad [1/2]$$

$$\text{PV of sale proceeds} = 6.4 v^{15}$$

$$= 1.5321 \quad [1/2]$$

$$\Rightarrow \text{NPV} = 0.1734 + 1.8704 + 1.5321 - 2.4767 - 0.3732$$

$$= \text{£}0.726\text{m} \quad [1]$$

- (ii) If the net revenue had been received mid-year rather than continuously then we would be replacing  $\bar{a}_{\overline{1}|}$  with  $v^{1/2}$  in the formulae for the PV of the net revenue.

Since we can observe that  $\bar{a}_{\overline{1}|} = \frac{i}{\delta} v > v^{1/2}$  we can see that the PV of the net revenue would decrease. Therefore, the NPV of the profit would decrease.

[2]

*Generally well-attempted. In questions like part (i), candidates are advised to show their working for each element separately as this provides a clear 'audit trail' for markers to follow and appropriate partial credit can be awarded for correct elements.*

- Q8** (i) Let  $X$  and  $Y$  be the maturity proceeds from the amounts invested in the 7-year and 14-year zero-coupon bonds respectively.

Redington's first condition states that the PV of the assets should equal the PV of the liabilities (using  $\frac{1}{1.045} = 0.95694$  and working in £million):

$$V_L = 20v^8 + 15v^{12} = 22.9087$$

$$V_A = Xv^7 + Yv^{14} = 22.9087 \quad (1) \quad [2]$$

Redington's second condition states that the discounted mean term (DMT) of the assets should be equal to the DMT of the liabilities. The denominators for the DMTs will be the respective PVs, which are assumed to be equal from the first condition above, so we can just consider the numerators:

$$\text{For the liabilities: } = 20 \times 8v^8 + 15 \times 12v^{12} = 218.6491$$

$$\text{For the assets: } = 7Xv^7 + 14Yv^{14} = 218.6491 \quad (2) \quad [2]$$

Taking (2) – 7 × (1):

$$7Yv^{14} = 218.6491 - 7 \times 22.9087 = 58.2882 \quad [1]$$

$$Y = \frac{58.2882}{7 \times 1.045^{-14}} = £15.421m$$

$$\text{with an amount invested of } Yv^{14} = £8.327m \quad [1]$$

Sub back in (1):

$$X = \frac{22.9087 - 15.421 \times 1.045^{-14}}{1.045^{-7}} = £19.844m$$

$$\text{with an amount invested of } Xv^7 = £14.582m \quad [1]$$

Since the spread of asset proceeds exceeds the spread of liability outgo (as asset proceeds are received at times 7 and 14, whereas liability outgo is paid at times 8 and 12), the convexity of the assets is greater than the convexity of the liabilities. Thus, the third condition is also satisfied and the company is immunised against small changes in the rate of interest. [2]

- (ii) The small increase in interest rates will mean that the present value of both assets and liabilities will fall. The greater convexity of the assets mean that the assets will fall by a smaller amount. There is a greater positive contribution from the convexity term in the present value of the assets than that of the present value of the liabilities. [2]

*Part (i) was answered well although, for full credit, the amounts invested needed to be given rather than the maturity values. Part (ii) was less well answered with many marginal candidates not appreciating how the greater asset convexity would influence the change in relative values.*

- Q9** (i) Let the 1-year and 2-year zero-coupon yields (spot rates) be  $i_1$  and  $i_2$  respectively.

$$\frac{106}{1+i_1} = 106v \text{ @ } 5.2\%$$

$$\therefore i_1 = 0.052 \text{ (=5.200\% to 3 dp)} \quad [1]$$

For the 2-year spot rate:

$$\frac{6}{1+i_1} + \frac{106}{(1+i_2)^2} = 6a_{\overline{2}|6.1\%} + 100v_{6.1\%}^2 \quad [1]$$

$$\frac{6}{1.052} + \frac{106}{(1+i_2)^2} = 6 \frac{\left(1 - \frac{1}{1.061^2}\right)}{0.061} + \frac{100}{1.061^2}$$

$$= 10.984960 + 88.831957$$

$$= 99.816917$$

$$\frac{106}{(1+i_2)^2} = 99.816917 - \frac{6}{1.052}$$

$$\Rightarrow (1+i_2)^2 = \frac{106}{94.113495}$$

$$\Rightarrow i_2 = 6.1273\% \text{ p.a. (= 6.127\% to 3 dp)} \quad [3]$$

For the 3- year spot rate:

The 3-year par yield is 6.6% p.a.

$$\Rightarrow 1 = 0.066 \left( \frac{1}{1+i_1} + \frac{1}{(1+i_2)^2} + \frac{1}{(1+i_3)^3} \right) + \frac{1}{(1+i_3)^3} \quad [1]$$

$$\Rightarrow \frac{1.066}{(1+i_3)^3} = 1 - \frac{0.066}{1.052} - \frac{0.066}{(1.061273)^2}$$

$$\Rightarrow (1+i_3)^3 = \frac{1.066}{0.878663}$$

$$\Rightarrow i_3 = 6.6543\% \text{ p.a. (= 6.654\% to 3 dp)} \quad [2]$$

(ii) 1-year forward rates:

$$f_0 = 1_i = 5.2\% \text{ p.a.} \quad [1]$$

$$(1+i_1)(1+f_1) = (1+i_2)^2$$

$$\Rightarrow 1+f_1 = \frac{1.061273^2}{1.052}$$

$$\Rightarrow f_1 = 7.0628\% \text{ p.a. } (=7.063\% \text{ to 3dp}). \quad [1\frac{1}{2}]$$

(answer is 7.062% if rounded spot rates used)

$$(1+i_2)^2(1+f_2) = (1+i_3)^3$$

$$\Rightarrow 1+f_2 = \frac{(1.066543)^3}{(1.061273)^2}$$

$$\Rightarrow f_2 = 7.7162\% \text{ p.a. } (=7.716\% \text{ to 3 dp}) \quad [1\frac{1}{2}]$$

(answer unchanged if rounded spot rates used)

*Candidates who made errors in part (i) often scored full marks in part (ii) after allowance was made for the effects of the earlier errors.*

**Q10** (i) We make use of:  $v(t) = \exp\left(-\int_0^t \delta(s) ds\right).$

$$\begin{aligned} \text{For } 0 < t \leq 6 \quad v(t) &= \exp\left(-\int_0^t (0.24 - 0.02s) ds\right) \\ &= \exp\left(-0.24s + 0.01s^2 \Big|_0^t\right) = \exp(-0.24t + 0.01t^2) \end{aligned} \quad [2]$$

$$\begin{aligned} \text{For } t > 6 \quad v(t) &= v(6) \times \exp\left(-\int_6^t 0.12 ds\right) \\ &= \exp(-0.24 \times 6 + 0.01 \times 36) \times \exp(-0.12s \Big|_6^t) \\ &= \exp(-0.36 - 0.12t) \end{aligned} \quad [3]$$

(ii) Discounted value

$$\begin{aligned} &= 1,000 \times v(4,10) = 1,000 \frac{v(10)}{v(4)} = 1,000 \frac{\exp(-0.36 - 0.12 \times 10)}{\exp(-0.24 \times 4 + 0.01 \times 4^2)} \\ &= 1,000e^{-1.56 - (-0.8)} = 1,000e^{-0.76} = 467.67 \end{aligned} \quad [2]$$

$$\begin{aligned} \text{(iii)} \quad \left(1 + \frac{i^{(12)}}{12}\right)^{12 \times (10-4)} &= \frac{1,000}{1,000e^{-0.76}} = e^{0.76} \\ \Rightarrow i^{(12)} &= \left(\sqrt[12]{e^{0.76}} - 1\right) \times 12 = 0.12734 \end{aligned} \quad [2]$$

$$\begin{aligned} \text{(iv)} \quad PV &= \int_6^{10} \rho(t)v(t)dt = \int_6^{10} 20\exp(0.36 + 0.32t) \times \exp(-0.36 - 0.12t)dt \\ &= \int_6^{10} 20e^{0.2t}dt = \frac{20}{0.2}e^{0.2t} \Big|_6^{10} = 100 \times (e^2 - e^{1.2}) = 406.89 \end{aligned} \quad [4]$$

*This calculation question was the best-answered on the paper.*

**Q11** (i) Denote PV of annuity by:

$$(Da)_{\overline{n}|} = nv + (n-1)v^2 + (n-2)v^3 + \dots + 2v^{n-1} + v^n \quad [1]$$

$$\Rightarrow (1+i) \times (Da)_{\overline{n}|} = n + (n-1)v + (n-2)v^2 + \dots + 2v^{n-2} + v^{n-1}$$

$$\Rightarrow i \times (Da)_{\overline{n}|} = n - (v + v^2 + \dots + v^n)$$

$$\Rightarrow (Da)_{\overline{n}|} = \frac{n - a_{\overline{n}|}}{i} \quad [2]$$

(ii) Initial amount of loan,  $L$ , is given by:

$$\begin{aligned} L &= 8,000v_{5.5\%} + 7,800v_{5.5\%}^2 + 7,600v_{5.5\%}^3 + \dots + 3,200v_{5.5\%}^{25} \\ &= 3,000 \times a_{\overline{25}|}^{5.5\%} + 200 \times (Da)_{\overline{25}|}^{5.5\%} \end{aligned} \quad [1]$$

where  $a_{\overline{25}|}^{5.5\%} = \frac{1 - v_{5.5\%}^{25}}{0.055} = 13.4139$ , and

$$(Da)_{\overline{25}|}^{5.5\%} = \frac{25 - a_{\overline{25}|}^{5.5\%}}{0.055} = 210.6558 \quad [1\frac{1}{2}]$$

Thus, initial amount of loan is:

$$L = 3,000 \times 13.4139 + 200 \times 210.6558 = \text{£}82,372.95 \quad [\frac{1}{2}]$$

[or can use

$$\begin{aligned} L &= 8,200 \times a_{\overline{25}|}^{5.5\%} - 200 \times (Ia)_{\overline{25}|}^{5.5\%} \\ &= 8,200 \times 13.4139 - 200 \times 138.1065 \\ &= £82,372.95 \end{aligned}$$

$$\text{where } (Ia)_{\overline{25}|}^{5.5\%} = \frac{\ddot{a}_{\overline{25}|}^{5.5\%} - 25v_{5.5\%}^{25}}{0.055} = \frac{1.055 \times 13.4139 - 25 \times 1.055^{-25}}{0.055} = 138.1065]$$

- (iii) Need loan outstanding immediately after 9<sup>th</sup> instalment (i.e. PV of future repayments).

Amount of tenth instalment is £6,200.

Loan outstanding is PV of future repayments, given by:

$$\begin{aligned} L^* &= 6,200v_{5.5\%} + 6,000v_{5.5\%}^2 + 5,800v_{5.5\%}^3 + \dots + 3,200v_{5.5\%}^{16} \\ &= 3,000 \times a_{\overline{16}|}^{5.5\%} + 200 \times (Da)_{\overline{16}|}^{5.5\%} \end{aligned}$$

where

$$a_{\overline{16}|}^{5.5\%} = \frac{1 - v_{5.5\%}^{16}}{0.055} = 10.4622 \text{ and } (Da)_{\overline{16}|}^{5.5\%} = \frac{16 - a_{\overline{16}|}^{5.5\%}}{0.055} = 100.6880$$

Thus, amount of loan outstanding is:

$$L^* = 3,000 \times 10.4622 + 200 \times 100.6880 = £51,524.08 \quad [4]$$

[or can use

$$\begin{aligned} L^* &= 6,400 \times a_{\overline{16}|}^{5.5\%} - 200 \times (Ia)_{\overline{16}|}^{5.5\%} \\ &= 6,400 \times 10.4622 - 200 \times 77.1688 \\ &= £51,524.08 \end{aligned}$$

$$\text{where } (Ia)_{\overline{16}|}^{5.5\%} = \frac{\ddot{a}_{\overline{16}|}^{5.5\%} - 16v_{5.5\%}^{16}}{0.055} = \frac{1.055 \times 10.4622 - 16 \times 1.055^{-16}}{0.055} = 77.1688]$$

Then, we have:

- interest component of 10<sup>th</sup> instalment is  $0.055 \times 51,524.08 = £2,833.83$ ,  
and
- capital component of 10<sup>th</sup> instalment is  $6,200 - 2,833.83 = £3,366.17$

[2]

- (iv) Total amount repaid is:

$$3,200 + 3,400 + 3,600 + \dots + 7,800 + 8,000$$

$$= (3,000 + 3,000 + \dots + 3,000) + (200 + 400 + \dots + 5,000)$$

$$= 3,000 \times 25 + 200 \times 0.5 \times 25 \times 26$$

$$= 140,000 \quad [1\frac{1}{2}]$$

Thus, total interest paid is:

$$140,000 - 82,372.95 = \text{£}57,627.05 \quad [\frac{1}{2}]$$

*Many attempts at proofs in part (i) were unclear. Part (ii) was generally answered well although a common error was to miscalculate the amount of the level annuity component of the loan outstanding. Some candidates also deducted the decreasing annuity component (or equivalently added the increasing component).*

## END OF EXAMINERS' REPORT