INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

17 April 2018 (am)

Subject CT1 – Financial Mathematics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 11 questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 State the characteristics of a Eurobond. [4]

2 (i) Describe what is meant by the term "ex-dividend". [1]

An individual purchased 10,000 shares on 1 December 2017. Dividends are payable on 1 January and 1 July each year, and are assumed to be payable in perpetuity. The next dividend, due on 1 January 2018, is \$0.07 per share.

The two dividend payments in any calendar year are expected to be the same, but the dividend payment is expected to increase at the end of each year at a rate of 2% per annum compound.

Assume that the share is ex-dividend on 1 December 2017 and use an effective rate of interest of 7% per annum.

(ii) Calculate the present value of the investment at the date of purchase. [5]

[Total 6]

3 An investor pays £80 at the start of each month into a 25-year savings plan.

The contributions accumulate at an effective rate of interest of 3% per half-year for the first 10 years, and at a force of interest of 6% per annum for the final 15 years.

Calculate the accumulated amount in the savings plan at the end of 25 years. [6]

4 The annual investment return achieved by an insurance company in year t is i_t.

Returns in successive years are assumed to be independent and:

$$\ln(1+i_t) \sim N(\mu, \sigma^2), \text{ where } \mu = 0.08 \text{ and } \sigma = 0.15.$$

The insurance company has a liability of €800,000 payable at the end of year 10.

The company wishes to invest an amount now so that there is a 95% probability that the accumulated amount at the end of year 10 will be sufficient to meet this liability.

(i) Calculate the amount of money that the insurance company should invest. [5]

(ii) Explain, without doing any further calculations, how your answer to part (i) would change if each of the following occurs separately, with all other parameters as in part (i):

(a) The value of μ is increased to 0.1.
(b) The value of σ is increased to 0.2.
(c) The desired probability of meeting the liability is increased to 99%.

[3]

[Total 8]

5 (i) Describe what is meant by the "no arbitrage" assumption in financial mathematics. [2]

An investor entered into a long forward contract for a security three years ago and the contract is due to mature in six years' time. The price of the security was £7.10 three years ago and is now £10.20. The risk-free rate of interest can be assumed to be 2% per annum effective throughout the nine-year period.

(ii) Calculate the current value of the contract with the following dividend payments, assuming no arbitrage:

(a) The security will pay dividends of £1.10 annually in arrear from now until the maturity of the contract.

(b) The security has paid and will continue to pay annually in arrear a dividend equal to 2.5% of the market price of the security at the time of payment.

[6]
[Total 8]

6 On 1 April 2018 a government issued a 10-year bond redeemable at £105 per £100 nominal and paying coupons at the rate of 3% per annum half-yearly in arrear. The price of the bond was £102 per £100 nominal.

An investor subject to income tax of 25% and capital gains tax of 35% purchased £10,000 nominal of the bond at issue.

The investor assumes that inflation will be constant over the term of the bond at a rate of 2% per annum.

(i) Calculate the net effective real redemption yield which the investor expects to earn on the investment. [6]

(ii) Explain how your answer to part (i) would change if inflation were less than 2% per annum throughout the term. [2]
[Total 8]

7 A retailer is considering opening a new store as a business venture. The purchase price of the store will be £2 million and there will be a further investment required of £0.5 million 6 months after purchase.

The store will open 12 months after purchase. Revenues less running costs are expected to occur continuously and will be £0.2 million in the first year of operation, £0.25 million in the second year of operation and thereafter increasing at yearly intervals by 4% per annum compound.

Eight years after purchase, a major refit costing £0.8 million will be required. Fifteen years after purchase, it is assumed that the store will be closed and sold for £6.4million.

The retailer requires a rate of return on its investment of 10% per annum effective.

(i) Calculate the net present value of the venture. [8]

It is now assumed that the revenue less running costs will be received mid-way through each year, rather than continuously.

(ii) Explain how your answer to part (i) would change. [2]

[Total 10]

8 An investment fund has liabilities of £20 million due in 8 years' time and £15 million due in 12 years' time.

The manager wishes to immunise the fund against small changes in the rate of interest and seeks to achieve this by purchasing two zero-coupon bonds. One bond is for a term of exactly 7 years and the other bond is for a term of exactly 14 years. The current interest rate is 4.5% per annum effective.

(i) Calculate the amount that should be invested in each bond, demonstrating that all three Redington conditions are met. [9]

(ii) Explain, without performing any further calculations, how the relative values of the assets and the liabilities will change if the interest rate changes immediately to 4.7% per annum effective. [2]

[Total 11]

9 Two bonds paying annual coupons of 6% in arrear and redeemable at par have terms to maturity of exactly one year and two years.

The gross redemption yield from the 1-year bond is 5.2% per annum effective. The gross redemption yield from the 2-year bond is 6.1% per annum effective. The 3-year par yield is 6.6% per annum.

Calculate the following as a percentage to three decimal places:

(i) the annual effective spot yields for each of the three years [8]

(ii) the annual effective one-year forward rates for each of the three years [4]

[Total 12]

10 The force of interest $\delta(t)$ is a function of time, and at any time t, measured in years is given by the formula:

$$\delta(t) = \begin{cases} 0.24 - 0.02t & 0 < t \le 6 \\ 0.12 & 6 < t \end{cases}$$

(i) Derive, and simplify as far as possible, expressions in terms of t for the present value of a unit investment made at any time, t. You should derive separate expressions for each time interval $0 < t \le 6$ and $6 < t$. [5]

(ii) Determine the discounted value at time $t = 4$ of an investment of £1,000 due at time $t = 10$. [2]

(iii) Calculate the constant nominal annual interest rate convertible monthly implied by the transaction in part (ii). [2]

(iv) Calculate the present value of a continuous payment stream invested from time $t = 6$ to $t = 10$ at a rate of $\rho(t) = 20e^{0.36+0.32t}$ per annum. [4]

[Total 13]

11 An n-year decreasing annuity is payable annually in arrear where the payment at the end of the first year is n, the payment at the end of the second year is $(n - 1)$, and so on until the final payment at the end of year n is 1.

(i) Show that the present value of this annuity is $\frac{n - a_{\overline{n}|}}{i}$ [3]

A loan is to be repaid over 25 years by means of annual instalments payable in arrear.

The amount of the first instalment is £8,000 and each subsequent instalment reduces by £200.

The effective rate of interest charged by the lender is 5.5% per annum.

(ii) Calculate the initial amount of the loan. [3]

(iii) Determine the interest and capital components of the 10^{th} instalment. [6]

(iv) Calculate the total amount of interest payable over the term of the loan. [2]

[Total 14]

END OF PAPER