EXAMINATION

15 April 2008 (am)

Subject CT1 — Financial Mathematics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 An eleven month forward contract is issued on 1 March 2008 on a stock with a price of £10 per share at that date. Dividends of 50 pence per share are expected to be paid on 1 April and 1 October 2008.

Calculate the forward price at issue, assuming a risk-free rate of interest of 5% per annum effective and no arbitrage. [4]

2 Describe the characteristics of the following investments:

(a) Eurobonds
(b) Certificates of deposit [4]

3 A mortgage company offers the following two deals to customers for twenty-five year mortgages.

Product A

A mortgage of £100,000 is offered with level repayments of £7,095.25 made annually in arrear. There are no arrangement or exit fees.

Product B

A mortgage of £100,000 is offered whereby a monthly payment in advance is calculated such that the customer pays an effective rate of return of 4% per annum ignoring arrangement and exit fees. In addition the customer also has to pay an arrangement fee of £6,000 at the beginning of the mortgage and an exit fee of £5,000 at the end of the twenty-five year term of the mortgage.

Compare the annual effective rates of return paid by customers on the two products. [8]

4 A loan of nominal amount £100,000 is to be issued bearing coupons payable quarterly in arrear at a rate of 7% per annum. Capital is to be redeemed at 108% on a coupon date between 15 and 20 years after the date of issue, inclusive. The date of redemption is at the option of the borrower.

An investor who is liable to income tax at 25% and capital gains tax at 35% wishes to purchase the entire loan at the date of issue.

Calculate the price which the investor should pay to ensure a net effective yield of at least 5% per annum. [8]

5 The n –year spot rate of interest, i_n, is given by:

$$i_n = a - bn$$

for $n = 1, 2$ and 3, and where a and b are constants.

The one-year forward rates applicable at time 0 and at time 1 are 6.1% per annum effective and 6.5% per annum effective respectively. The 4–year par yield is 7% per annum.

Stating any assumptions:

(i) calculate the values of a and b. [4]

(ii) calculate the price per £1 nominal at time 0 of a bond which pays annual coupons of 5% in arrear and is redeemed at 103% after 4 years. [5]

[Total 9]

6 (i) An investor is considering the purchase of an annuity, payable annually in arrear for 20 years. The first payment is £500. Using a rate of interest of 8% per annum effective, calculate the duration of the annuity when:

(a) the payments remain level over the term.
(b) the payments increase at a rate of 8% per annum compound. [6]

(ii) Explain why the answer in (i)(b) is higher than the answer in (i)(a). [2]

[Total 8]

7 The shares of a company currently trade at £2.60 each, and the company has just paid a dividend of 12p per share. An investor assumes that dividends will be paid annually in perpetuity and will grow in line with a constant rate of inflation. The investor estimates the assumed inflation rate from equating the price of the share with the present value of all estimated future gross dividend payments using an effective interest rate of 6% per annum.

(i) Calculate the investor's estimation of the effective inflation rate per annum based on the above assumptions. [4]

(ii) Suppose that the actual inflation rate turns out to be 3% per annum effective over the following twelve years, but that all the investor's other assumptions are correct.

Calculate the investor's real rate of return per annum from purchase to sale, if she sold the shares after twelve years for £5 each immediately after a dividend has been paid. You may assume that the investor pays no tax. [6]

[Total 10]

8 An investor is considering investing in a capital project.

The project requires an outlay of £500,000 at outset and further payments at the end of each of the first 5 years, the first payment being £100,000 and each successive payment increasing by £10,000.

The project is expected to provide a continuous income at a rate of £80,000 in the first year, £83,200 in the second year and so on, with income increasing each year by 4% per annum compound. The income is received for 25 years.

It is assumed that, at the end of 15 years, a further investment of £300,000 will be required and that the project can be sold to another investor for £700,000 at the end of 25 years.

(i) Calculate the net present value of the project at a rate of interest of 11% per annum effective. [9]

(ii) Without doing any further calculations, explain how the net present value would alter if the interest rate had been greater than 11% per annum effective. [3]

[Total 12]

9 The force of interest, $\delta(t)$, is a function of time and at any time *t*, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.06 & 0 \le t \le 4 \\ 0.10 - 0.01t & 4 < t \le 7 \\ 0.01t - 0.04 & 7 < t \end{cases}$$

(i) Calculate the value at time $t = 5$ of £1,000 due for payment at time $t = 10$. [5]

(ii) Calculate the constant rate of interest per annum convertible monthly which leads to the same result as in (i) being obtained. [2]

(iii) Calculate the accumulated amount at time $t = 12$ of a payment stream, paid continuously from time $t = 0$ to $t = 4$, under which the rate of payment at time *t* is $\rho(t) = 100e^{0.02t}$. [6]

[Total 13]

10 An insurance company holds a large amount of capital and wishes to distribute some of it to policyholders by way of two possible options.

Option A

£100 for each policyholder will be put into a fund from which the expected annual effective rate of return from the investments will be 5.5% and the standard deviation of annual returns 7%. The annual effective rates of return will be independent and $(1+i_t)$ is lognormally distributed, where i_t is the rate of return in year t. The policyholder will receive the accumulated investment at the end of ten years.

Option B

£100 will be invested for each policyholder for five years at a rate of return of 6% per annum effective. After five years, the accumulated sum will be invested for a further five years at the prevailing five-year spot rate. This spot rate will be 1% per annum effective with probability 0.2, 3% per annum effective with probability 0.3, 6% per annum effective with probability 0.2, and 8% per annum effective with probability 0.3. The policyholder will receive the accumulated investment at the end of ten years.

Deriving any necessary formulae:

(i) Calculate the expected value and the standard deviation of the sum the policyholders will receive at the end of the ten years for each of options A and B. [17]

(ii) Determine the probability that the sum the policyholders will receive at the end of ten years will be less than £115 for each of options A and B. [5]

(iii) Comment on the relative risk of the two options from the policyholders' perspective. [2]

[Total 24]

END OF PAPER