EXAMINATION

6 April 2005 (am)

Subject CT1 — Financial Mathematics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

1 A bond is priced at £95 per £100 nominal, has a coupon rate of 5% per annum payable half-yearly, and has an outstanding term of five years.

An investor holds a short position in a forward contract on £1 million nominal of this bond, with a delivery price of £98 per £100 nominal and maturity in exactly one year, immediately following the coupon payment then due.

The continuously compounded risk-free rates of interest for terms of six months and one year are 4.6% per annum and 5.2% per annum, respectively.

Calculate the value of this forward contract to the investor assuming no arbitrage. [5]

2 An investment fund had a market value of £2.2 million on 31 December 2001 and £4.2 million on 31 December 2004. It had received a net cashflow of £1.44 million on 31 December 2003.

The money weighted rate of return and the time weighted rate of return for the period from 31 December 2001 to 31 December 2004 are equal (to two decimal places).

Calculate the market value of the fund immediately before the net cashflow on 31 December 2003. [7]

3 A computer manufacturer is to develop a new chip to be produced from 1 January 2008 until 31 December 2020. Development begins on 1 January 2006. The cost of development comprises £9 million payable on 1 January 2006 and £12 million payable continuously during 2007.

From 1 January 2008 the chip will be ready for production and it is assumed that income will be received half yearly in arrear at a rate of £5 million per annum.

(i) Calculate the discounted payback period at an effective rate of interest of 9% per annum. [6]

(ii) Without doing any further calculations, explain whether the discounted payback period would be greater than, less than or equal to that given in part (i) if the effective interest rate were substantially greater than 9% per annum. [2]

[Total 8]

4 The force of interest, $\delta(t)$, is a function of time and at any time t (measured in years) is given by

$$\delta(t) = \begin{cases} 0.07 - 0.005t & \text{for } t \le 8 \\ 0.06 & \text{for } t > 8 \end{cases}$$

(i) Calculate the accumulation at time $t = 10$ of £500 invested at time $t = 0$. [3]

(ii) Calculate the present value at time $t = 0$ of a continuous payment stream at the rate of $£200e^{0.1t}$ paid from $t = 10$ to $t = 18$. [5]

[Total 8]

5 A university student receives a 3-year sponsorship grant. The payments under the grant are as follows:

Year 1 £5,000 per annum paid continuously.
Year 2 £5,000 per annum paid monthly in advance.
Year 3 £5,000 per annum paid half yearly in advance.

Calculate the total present value of these payments at the beginning of the first year using a rate of interest of 8% per annum convertible quarterly. [8]

6 At time $t = 0$ an investor purchased an annuity-certain which paid her £10,000 per annum annually in arrear for three years. The purchase price paid by the investor was £25,000.

The value of the retail price index at various times was as shown in the table below:

Time t (years):	$t = 0$	$t = 1$	$t = 2$	$t = 3$
Retail price index:	170.7	183.3	191.0	200.9

(i) Calculate, to the nearest 0.1%, the following effective rates of return per annum achieved by the investor from her investment in the annuity:

(a) the real rate of return; and
(b) the money rate of return

[7]

(ii) By considering the average rate of inflation over the three-year period, explain the relationship between your answers in (a) and (b) of (i). [2]

[Total 9]

7 A loan of nominal amount £100,000 is to be issued bearing coupons payable quarterly in arrear at a rate of 5% per annum. Capital is to be redeemed at 103 on a single coupon date between 15 and 20 years after the date of issue, inclusive. The date of redemption is at the option of the borrower.

An investor who is liable to income tax at 20% and capital gains tax of 25% wishes to purchase the entire loan at the date of issue. Calculate the price which the investor should pay to ensure a net effective yield of at least 4% per annum. [9]

8 A small insurance fund has liabilities of £4 million due in 19 years time and £6 million in 21 years time. The manager of the fund has sold the assets previously held and is creating a new portfolio by investing in the zero-coupon bond market. The manager is able to buy zero-coupon bonds for whatever term he requires and has adequate monies at his disposal.

(i) Explain whether it is possible for the manager to immunise the fund against small changes in the rate of interest by purchasing a single zero-coupon bond. [2]

(ii) In fact, the manager purchases two zero-coupon bonds, one paying £3.43 million in 15 years time and the other paying £7.12 million in 25 years time. The current interest rate is 7% per annum effective.

Investigate whether the insurance fund satisfies the necessary conditions to be immunised against small changes in the rate of interest. [8]

[Total 10]

9 The one-year forward rate of interest at time $t = 1$ year is 5% per annum effective.

The gross redemption yield of a two-year fixed interest stock issued at time $t = 0$ which pays coupons of 3% per annum annually in arrear and is redeemed at 102 is 5.5% per annum effective.

The issue price at time $t = 0$ of a three-year fixed interest stock bearing coupons of 10% per annum payable annually in arrear and redeemed at par is £108.9 per £100 nominal.

(i) Calculate the one-year spot rate per annum effective at time $t = 0$. [4]

(ii) Calculate the one-year forward rate per annum effective at time $t = 2$ years. [3]

(iii) Calculate the two-year par yield at time $t = 0$. [3]

[Total 10]

10 (i) In any year, the interest rate per annum effective on monies invested with a given bank has mean value j and standard deviation s and is independent of the interest rates in all previous years.

Let S_n be the accumulated amount after n years of a single investment of 1 at time $t = 0$.

(a) Show that $E[S_n] = (1+j)^n$.

(b) Show that $\text{Var}[S_n] = (1+2j+j^2+s^2)^n - (1+j)^{2n}$.

[5]

(ii) The interest rate per annum effective in (i), in any year, is equally likely to be i_1 or $i_2 (i_1 > i_2)$. No other values are possible.

(a) Derive expressions for j and s^2 in terms of i_1 and i_2.

(b) The accumulated value at time $t = 25$ years of £1 million invested with the bank at time $t = 0$ has expected value £5.5 million and standard deviation £0.5 million.

Calculate the values of i_1 and i_2.

[8]
[Total 13]

11 (i) A loan is repayable over 20 years by level instalments of £1,000 per annum made annually in arrear. Interest is charged at the rate of 5% per annum effective for the first 10 years, increasing to 7% per annum effective for the remaining term.

Show that the amount of the original loan is £12,033.56. (Minor discrepancies due to rounding will not be penalised). [2]

(ii) The following are the details from the loan schedule for year x, i.e. the year running from exact duration $x - 1$ years to exact duration x years.

		Instalment paid at the end of the year	
	Loan outstanding at the beginning of the year	*Interest*	*Capital*
Year x	£8,790.48	£439.52	£560.48

Determine the value of x. [4]

(iii) At the beginning of year 11, it is agreed that the increase in the rate of interest will not take place, so that the rate remains at 5% per annum effective for the remainder of the loan. The annual instalment will continue to be payable at the same level so that there may be a reduced term and a reduced final instalment.

(a) Calculate by how many years, if any, the repayment schedule is shortened.

(b) Calculate the amount of the reduced final instalment.

(c) Calculate the reduction in the total interest paid during the existence of the loan as a result of the interest rate not increasing.

[7]

[Total 13]

END OF PAPER