

EXAMINATION

12 April 2007 (am)

Subject CT1 — Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

<p><i>In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.</i></p>

- 1** An investor pays £400 every half-year in advance into a 25-year savings plan.

Calculate the accumulated fund at the end of the term if the interest rate is 6% per annum convertible monthly for the first 15 years and 6% per annum convertible half-yearly for the final 10 years. [5]

- 2** The force of interest $\delta(t)$ is a function of time and at any time, measured in years, is given by the formula:

$$\delta(t) = 0.04 + 0.01t \quad 0 \leq t \leq 4$$

$$\delta(t) = 0.12 - 0.01t \quad 4 < t \leq 8$$

$$\delta(t) = 0.06 \quad 8 < t$$

Calculate the present value at time $t = 0$ of a payment stream, paid continuously from time $t = 9$ to $t = 12$, under which the rate of payment at time t is $50e^{0.01t}$.

[6]

- 3** An ordinary share pays annual dividends. The next dividend is due in exactly eight months' time. This dividend is expected to be £1.10 per share. Dividends are expected to grow at a rate of 5% per annum compound from this level and are expected to continue in perpetuity. Inflation is expected to be 3% per annum. The price of the share is £21.50.

Calculate the expected effective annual real rate of return for an investor who purchases the share.

[7]

- 4** An investor entered into a long forward contract for a security five years ago and the contract is due to mature in seven years' time. The price of the security was £95 five years ago and is now £145. The risk-free rate of interest can be assumed to be 3% per annum throughout the 12-year period.

Assuming no arbitrage, calculate the value of the contract now if:

- (i) The security will pay dividends of £5 in two years' time and £6 in four years' time. [3]

- (ii) The security has paid and will continue to pay annually in arrear a dividend of 2% per annum of the market price of the security at the time of payment. [3]

[Total 6]

- 5** In a particular bond market, n -year spot rates per annum can be approximated by the function $0.08 - 0.04e^{-0.1n}$.

Calculate:

- (i) The price per unit nominal of a zero coupon bond with term nine years. [2]
 - (ii) The four-year forward rate at time 7 years. [3]
 - (iii) The three-year par yield. [3]
- [Total 8]

- 6** A fund had a value of £21,000 on 1 July 2003. A net cash flow of £5,000 was received on 1 July 2004 and a further net cash flow of £8,000 was received on 1 July 2005. Immediately before receipt of the first net cash flow, the fund had a value of £24,000, and immediately before receipt of the second net cash flow the fund had a value of £32,000. The value of the fund on 1 July 2006 was £38,000.

- (i) Calculate the annual effective money weighted rate of return earned on the fund over the period 1 July 2003 to 1 July 2006. [3]
 - (ii) Calculate the annual effective time weighted rate of return earned on the fund over the period 1 July 2003 to 1 July 2006. [3]
 - (iii) Explain why the values in (i) and (ii) differ. [2]
- [Total 8]

- 7** An insurance company has liabilities of £87,500 due in 8 years' time and £157,500 due in 19 years' time. Its assets consist of two zero coupon bonds, one paying £66,850 in four years' time and the other paying £ X in n years' time. The current interest rate is 7% per annum effective.

- (i) Calculate the discounted mean term and convexity of the liabilities. [5]
 - (ii) Determine whether values of £ X and n can be found which ensure that the company is immunised against small changes in the interest rate. [5]
- [Total 10]

8 A company has borrowed £800,000 from a bank. The loan is to be repaid by level instalments, payable annually in arrear for 10 years from the date the loan is made. The annual repayments are calculated at an effective rate of interest of 8% per annum.

(i) Calculate the amount of the level annual payment and the total amount of interest which will be paid over the 10 year term. [3]

(ii) At the beginning of the eighth year, immediately after the seventh payment has been made, the company asks for the term of the loan to be extended by two years. The bank agrees to do this on condition that the rate of interest is increased to an effective rate of 12% per annum for the remainder of the term and that payments are made quarterly in arrear.

(a) Calculate the amount of the new quarterly payment.

(b) Calculate the capital and interest components of the first quarterly instalment of the revised loan repayments.

[6]

[Total 9]

9 A property developer is constructing a block of offices. It is anticipated that the offices will take six months to build. The developer incurs costs of £40 million at the beginning of the project followed by £3 million at the end of each month for the following six months during the building period. It is expected that rental income from the offices will be £1 million per month, which will be received at the start of each month beginning with the seventh month. Maintenance and management costs paid by the developer are expected to be £2 million per annum payable monthly in arrear with the first payment at the end of the seventh month. The block of offices is expected to be sold 25 years after the start of the project for £60 million.

(i) Calculate the discounted payback period using an effective rate of interest of 10% per annum. [7]

(ii) Without doing any further calculations, explain whether your answer to (i) would change if the effective rate of interest were less than 10% per annum.

[3]

[Total 10]

10 A loan is issued bearing interest at a rate of 9% per annum and payable half-yearly in arrear. The loan is to be redeemed at £110 per £100 nominal in 13 years' time.

(i) The loan is issued at a price such that an investor, subject to income tax at 25%, and capital gains tax at 30%, would obtain a net redemption yield of 6% per annum effective. Calculate the issue price per £100 nominal of the stock. [5]

(ii) Two years after the date of issue, immediately after a coupon payment has been made, the investor decides to sell the stock and finds a potential buyer, who is subject to income tax at 10% and capital gains tax at 35%. The potential buyer is prepared to buy the stock provided she will obtain a net redemption yield of at least 8% per annum effective.

(a) Calculate the maximum price (per £100 nominal) which the original investor can expect to obtain from the potential buyer.

(b) Calculate the net effective annual redemption yield (to the nearest 1% per annum effective) that will be obtained by the original investor if the loan is sold to the buyer at the price determined in (ii) (a).

[10]

[Total 15]

11 £80,000 is invested in a bank account which pays interest at the end of each year. Interest is always reinvested in the account. The rate of interest is determined at the beginning of each year and remains unchanged until the beginning of the next year. The rate of interest applicable in any one year is independent of the rate applicable in any other year.

During the first year, the annual effective rate of interest will be one of 4%, 6% or 8% with equal probability.

During the second year, the annual effective rate of interest will be either 7% with probability 0.75 or 5% with probability 0.25.

During the third year, the annual effective rate of interest will be either 6% with probability 0.7 or 4% with probability 0.3.

(i) Derive the expected accumulated amount in the bank account at the end of three years. [5]

(ii) Derive the variance of the accumulated amount in the bank account at the end of three years. [8]

(iii) Calculate the probability that the accumulated amount in the bank account is more than £97,000 at the end of three years. [3]

[Total 16]

END OF PAPER