

# EXAMINATION

7 September 2005 (am)

## Subject CT1 — Financial Mathematics Core Technical

*Time allowed: Three hours*

### ***INSTRUCTIONS TO THE CANDIDATE***

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

***Graph paper is not required for this paper.***

### ***AT THE END OF THE EXAMINATION***

*Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.*

*In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.*

- 1** Describe how cashflows are exchanged in an “interest rate swap”. [2]
- 2** An investor has earned a money rate of return from a portfolio of bonds in a particular country of 1% per annum effective over a period of ten years. The country has experienced deflation (negative inflation) of 2% per annum effective during the period.
- Calculate the real rate of return per annum over the ten years. [2]
- 3** Calculate the time in days for £1,500 to accumulate to £1,550 at:
- (a) a simple rate of interest of 5% per annum
  - (b) a force of interest of 5% per annum
- [4]
- 4** The force of interest  $\delta(t)$  at time  $t$  is  $a + bt^2$  where  $a$  and  $b$  are constants. An amount of £200 invested at time  $t = 0$  accumulates to £210 at time  $t = 5$  and £230 at time  $t = 10$ .
- Determine  $a$  and  $b$ . [5]
- 5** (i) Calculate the present value of £100 over ten years at the following rates of interest/discount:
- (a) a rate of interest of 5% per annum convertible monthly
  - (b) a rate of discount of 5% per annum convertible monthly
  - (c) a force of interest of 5% per annum
- [4]
- (ii) A 91-day treasury bill is bought for \$98.91 and is redeemed at \$100. Calculate the annual effective rate of interest obtained from the bill. [3]
- [Total 7]
- 6** (i) State the features of a eurobond. [3]
- (ii) An investor purchases a eurobond on the date of issue at a price of £97 per £100 nominal. Coupons are paid annually in arrear. The bond will be redeemed at par twenty years from the issue date. The rate of return from the bond is 5% per annum effective.
- (a) Calculate the annual rate of coupon paid by the bond.
  - (b) Calculate the duration of the bond.
- [6]
- [Total 9]

**7** A bank makes a loan to be repaid in instalments annually in arrear. The first instalment is 50, the second 48 and so on with the payments reducing by 2 per annum until the end of the 15th year after which there are no further payments. The rate of interest charged by the lender is 6% per annum effective.

(i) Calculate the amount of the loan. [6]

(ii) Calculate the interest and capital components of the second payment. [3]

(iii) Calculate the amount of capital repaid in the instalment at the end of the fourteenth year. [3]

[Total 12]

**8** An insurance company has just written contracts that require it to make payments to policyholders of £1,000,000 in five years' time. The total premiums paid by policyholders amounted to £850,000. The insurance company is to invest half the premium income in fixed interest securities that provide a return of 3% per annum effective. The other half of the premium income is to be invested in assets that have an uncertain return. The return from these assets in year  $t$ ,  $i_t$ , has a mean value of 3.5% per annum effective and a standard deviation of 3% per annum effective.  $(1 + i_t)$  is independently and lognormally distributed.

(i) Deriving all necessary formulae, calculate the mean and standard deviation of the accumulation of the premiums over the five-year period. [9]

(ii) A director of the company suggests that investing all the premiums in the assets with an uncertain return would be preferable because the expected accumulation of the premiums would be greater than the payments due to the policyholders.

Explain why this still may be a more risky investment policy. [2]

[Total 11]

- 9** (i) Explain what is meant by the “expectations theory” for the shape of the yield curve. [2]
- (ii) Short-term, one-year annual effective interest rates are currently 8%; they are expected to be 7% in one years time, 6% in two years time and 5% in three years time.
- (a) Calculate the gross redemption yields (spot rates of interest) from 1-year, 2-year, 3-year and 4-year zero coupon bonds assuming the expectations theory explanation of the yield curve holds.
- (b) The price of a coupon paying bond is calculated by discounting individual payments from the bond at the zero-coupon bond yields in (a).
- Calculate the gross redemption yield of a bond that is redeemed at par in exactly four years and pays a coupon of 5 per annum annually in arrear.
- (c) A two-year forward contract has just been issued on a share with a price of 400p. A dividend of 4p is expected in exactly one year.
- Calculate the forward price using the above spot rates of interest, assuming no arbitrage. [12]
- [Total 14]

**10** An investor purchased a bond with exactly 15 years to redemption. The bond, redeemable at par, has a gross redemption yield of 5% per annum effective. It pays coupons of 4% per annum, half yearly in arrear. The investor pays tax at 25% on the coupons only.

- (i) Calculate the price paid for the bond. [3]
- (ii) After exactly eight years, immediately after the payment of the coupon then due, this investor sells the bond to another investor who pays income tax at a rate of 25% and capital gains tax at a rate of 40%. The bond is purchased by the second investor to provide a net return of 6% per annum effective.
- (a) Calculate the price paid by the second investor.
- (b) Calculate, to one decimal place, the annual effective rate of return earned by the first investor during the period for which the bond was held. [10]
- [Total 13]

**11** (i) Explain what is meant by the following terms:

- (a) equation of value
- (b) discounted payback period from an investment project

[4]

- (ii) An insurance company is considering setting up a branch in a country in which it has previously not operated. The company is aware that access to capital may become difficult in twelve years time. It therefore has two decision criteria. The cashflows from the project must provide an internal rate of return greater than 9% per annum effective and the discounted payback period at a rate of interest of 7% per annum effective must be less than twelve years.

The following cashflows are generated in the development and operation of the branch.

### **Cash Outflows**

Between the present time and the opening of the branch in three years time the insurance company will spend £1.5m per annum on research, development and the marketing of products. This outlay is assumed to be a constant continuous payment stream. The rent on the branch building will be £0.3m per annum paid quarterly in advance for twelve years starting in three years time. Staff costs are assumed to be £1m in the first year, £1.05m in the second year, rising by 5% per annum each year thereafter. Staff costs are assumed to be incurred at the beginning of each year starting in three years time and assumed to be incurred for 12 years.

### **Cash Inflows**

The company expects the sale of products to produce a net income at a rate of £1m per annum for the first three years after the branch opens rising to £1.9m per annum in the next three years and to £2.5m for the following six years. This net income is assumed to be received continuously throughout each year. The company expects to be able to sell the branch operation 15 years from the present time for £8m.

Determine which, if any, of the decision criteria the project fulfils.

[17]

[Total 21]

**END OF PAPER**