INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

6 October 2011 (am)

Subject CT6 — Statistical Methods
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

 © Institute and Faculty of Actuaries

1 The loss function for a decision problem is given below.

	θ_1	θ_2	θ_3
D1	30	20	15
D2	20	25	10
D3	17	19	24
D4	35	20	17

(i) Explain which strategies, if any, are dominated. [3]

(ii) Find the minimax solution to this decision problem. [1]
[Total 4]

2 An accountant is using a psychic octopus to predict the outcome of tosses of a fair coin. He claims that the octopus has a probability $p > 0.5$ of successfully predicting the outcome of any given coin toss. His actuarial colleague is extremely sceptical and summarises his prior beliefs about p as follows: there is an 80% chance that $p = 0.5$ and a 20% chance that p is uniformly distributed on the interval [0.5,1]. The octopus successfully predicts the results of 7 out of 8 coin tosses.

Calculate the posterior probability that $p = 0.5$. [4]

3 Loss amounts under a class of insurance policies follow an exponential distribution with mean 100. The insurance company wishes to enter into an individual excess of loss reinsurance arrangement with retention level M set such that 8 out of 10 claims will not involve the reinsurer.

(i) Find the retention M. [2]

For a given claim, let X_I denote the amount paid by the insurer and X_R the amount paid by the reinsurer.

(ii) Calculate $E(X_I)$ and $E(X_R)$. [3]
[Total 5]

4 Claims on a portfolio of insurance policies follow a Poisson process with parameter λ. The insurance company calculates premiums using a premium loading of θ and has an initial surplus of U.

(i) Define the surplus process $U(t)$. [1]

(ii) Define the probabilities $\psi(U, t)$ and $\psi(U)$. [2]

(iii) Explain how $\psi(U, t)$ and $\psi(U)$ depend on λ. [2]
[Total 5]

5 An insurance company covers pedigree cats against the costs of medical treatment. The cost of claims from a policy in a year is assumed to have a normal distribution with mean μ (which varies from policy to policy) and known variance 25^2. It is assumed that $\mu = \alpha + \beta x$ where α and β are fixed constants and x is the age of the cat. You are given the following data for the pairs (y_i, x_i) for $i = 1, 2, \ldots, 50$ where y_i is the cost of claims last year for the ith policy and x_i is the age of the corresponding cat.

$$\sum_{i=1}^{50} x_i = 637 \quad \sum_{i=1}^{50} y_i = 5,492 \quad \sum_{i=1}^{50} y_i x_i = 74,532 \quad \sum_{i=1}^{50} x_i^2 = 8,312$$

Calculate the maximum likelihood estimates of α and β. [6]

6 Let X_1 and X_2 be two independent exponentially distributed random variables with parameters λ_1 and λ_2 respectively. The random variable X is related to X_1 and X_2 such that a single observation from X is chosen from X_1 with probability p and from X_2 with probability $1 - p$.

(i) Show that the density function of X is

$$pf_1(x) + (1 - p)\, f_2(x).$$

where $f_i(x)$ is the density function of X_i. [2]

(ii) Construct an algorithm for generating samples from X. [4]

(iii) Describe how the algorithm in (ii) could be generalised for k independent components $p_1 f_1(x) + \ldots + p_k f_k(x)$ where $p_1 + \ldots + p_k = 1$, each $p_i \geq 0$ and $f_i(x)$ is the density of an exponential distribution with parameter λ_i. [2]

[Total 8]

7 A portfolio of insurance policies contains two types of risk. Type I risks make up 80% of claims and give rise to loss amounts which follow a normal distribution with mean 100 and variance 400. Type II risks give rise to loss amounts which are normally distributed with mean 115 and variance 900.

(i) Calculate the mean and variance of the loss amount for a randomly chosen claim. [3]

(ii) Explain whether the loss amount for a randomly chosen claim follows a normal distribution. [2]

The insurance company has in place an excess of loss reinsurance arrangement with retention 130.

(iii) Calculate the probability that a randomly chosen claim from the portfolio results in a payment by the reinsurer. [3]

(iv) Calculate the proportion of claims involving the reinsurer that arise from Type II risks. [2]
[Total 10]

8 Consider the time series

$$Y_t = 0.1 + 0.4Y_{t-1} + 0.9e_{t-1} + e_t$$

where e_t is a white noise process with variance σ^2.

(i) Identify the model as an ARIMA(p,d,q) process. [1]

(ii) Determine whether Y_t is:

(a) a stationary process
(b) an invertible process

[2]

(iii) Calculate $E(Y_t)$ and find the auto-covariance function for Y_t. [6]

(iv) Determine the MA(∞) representation for Y_t. [4]
[Total 13]

9 Claim events on a portfolio of insurance policies follow a Poisson process with parameter λ. Individual claim amounts follow a distribution X with density

$$f(x) = 0.01^2 xe^{-0.01x} \qquad x > 0.$$

The insurance company calculates premiums using a premium loading of 45%.

(i) Derive the moment generating function $M_X(t)$. [3]

(ii) Determine the adjustment coefficient and hence derive an upper bound on the probability of ruin if the insurance company has initial surplus U. [5]

(iii) Find the surplus required to ensure the probability of ruin is less than 1% using the upper bound in (ii). [2]

Suppose instead that individual claims are for a fixed amount of 200.

(iv) Determine whether the adjustment coefficient is higher or lower than in (ii) and comment on your conclusion. [4]
[Total 14]

10 The table below shows cumulative claims paid on a portfolio of motor insurance policies.

	Development Year			
Accident Year	*0*	*1*	*2*	*3*
2007	120	134	146	148
2008	140	180	185	
2009	135	149		
2010	138			

All claims are fully run off by the end of development year 3.

(i) Calculate the total reserve for outstanding claims using the basic chain ladder technique. [7]

An actuarial student suggests an alternative approach to projecting the claims as follows:

- For each of development years 1 to 3 calculate the observed development factor separately for each accident year.

- Then project claims assuming the development factor for a given year is the maximum of the observed development factors for the relevant accident year.

- For example for the development factor from development year 1 to development year 2 we can observe actual factors for accident years 2007 and 2008. To project claims, we assume that the development factor for development year 1 to development year 2 is the maximum of the two observed factors.

(ii) Calculate the increase in the reserve for outstanding claims if claims are projected in this way. [5]

(iii) Discuss why the method in (ii) may not be appropriate. [2]
[Total 14]

11 Five years ago, an insurance company began to issue insurance policies covering medical expenses for dogs. The insurance company classifies dogs into three risk categories: large pedigree (category 1), small pedigree (category 2) and non-pedigree (category 3). The number of claims n_{ij} in the ith category in the jth year is assumed to have a Poisson distribution with unknown parameter θ_i. Data on the number of claims in each category over the last 5 years is set out as follows:

	Year 1	*2*	*3*	*4*	*5*	$\sum_{j=1}^{5} n_{ij}$	$\sum_{j=1}^{5} n_{ij}^2$
Category 1	30	43	49	58	60	240	12,144
Category 2	37	49	58	52	64	260	13,934
Category 3	26	31	18	37	32	144	4,354

Prior beliefs about θ_1 are given by a gamma distribution with mean 50 and variance 25.

(i) Find the Bayes estimate of θ_1 under quadratic loss. [5]

(ii) Calculate the expected claims for year 6 of each category under the assumptions of Empirical Bayes Credibility Theory Model 1 [6]

(iii) Explain the main differences between the approach in (i) and that in (ii). [3]

(iv) Explain why the assumption of a Poisson distribution with a constant parameter may not be appropriate and describe how each approach might be generalised. [3]

[Total 17]

END OF PAPER