# INSTITUTE AND FACULTY OF ACTUARIES

# EXAMINATION

13 October 2015 (pm)

## Subject CT6 – Statistical Methods
## Core Technical

*Time allowed: Three hours*

***INSTRUCTIONS TO THE CANDIDATE***

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 12 questions, beginning your answer to each question on a new page.*

5. *Candidates should show calculations where this is appropriate.*

***Graph paper is NOT required for this paper.***

*AT THE END OF THE EXAMINATION*

*Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.*

*In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.*

**1** An actuary is simulating claims $X_i$ on a portfolio of insurance policies.

For each $i$, let $Y_i$ be 1 if $X_i$ exceeds a given amount $M$ and 0 if not. The variance of $Y_i$ is 0.12.

The actuary wishes to estimate the proportion of claims that exceed $M$.

Calculate the number of simulations that the actuary will have to perform in order to estimate the true proportion to within 0.01 with 99% confidence. [3]

**2** (i) State the simplifications usually made in the basic model for short term insurance contracts. [3]

(ii) Give two examples of forms of insurance that can be regarded as short term insurance contracts. [1]
[Total 4]

**3** A particular industry always generates total profits of $1bn in each year, shared between two companies: Raspberry Inc. and Robots Ltd. Every year the companies each need to choose between two distinct business approaches: cautious and aggressive.

If both companies adopt the same approach in a given year, Raspberry Inc. captures 70% of the total profits. If they adopt different approaches, Robots Ltd. captures 80% of the total profits if Raspberry Inc. is cautious, and 60% of the total profits if Raspberry Inc. is aggressive. Neither company knows what the other company's approach will be before adopting its own approach.

(i) Explain why the above can be thought of as a zero-sum two person game. [1]

Raspberry Inc. decides to adopt a randomised strategy to setting its approach each year.

(ii) Explain what is meant by a randomised strategy. [1]

(iii) Determine Raspberry Inc.'s optimal randomised strategy. [4]
[Total 6]

**4** A small island is holding a vote on independence. Two recent survey results are shown below:

| *Poll* | *Sample size* | *Support for independence* |
|---|---|---|
| A | 10 | 5 |
| B | 20 | 11 |

You should assume that the samples are independent.

A politician is using a suitable uniform distribution as the prior distribution in order to estimate the proportion $\theta$ in favour of independence.

(i) Calculate an estimate of $\theta$ under the quadratic loss function. [3]

A rival politician decides to use instead a beta distribution as the prior, with parameters $\alpha$ and $\beta$, where $\alpha = \beta$.

(ii) Determine the new estimate of $\theta$ under the "all-or-nothing" loss function in terms of $\alpha$. [4]

[Total 7]

**5** Claims X each year from a portfolio of insurance policies are normally distributed with mean $\theta$ and variance $\tau^2$. Prior information is that $\theta$ is normally distributed with known mean $\mu$ and known variance $\sigma^2$.

Aggregate claims over the last $n$ years have been $x_i$ for $i = 1$ to $n$, and you should assume that these are independent.

(i) Derive the posterior distribution of $\theta$. [5]

(ii) Write down the Bayesian estimate of $\theta$ under quadratic loss. [1]

(iii) Show that the estimate in your answer to part (ii) can be expressed in the form of a credibility estimate, including statement of the credibility factor Z. [2]

[Total 8]

**6** (i) Explain what is meant by a saturated model. [2]

(ii) State the definition of the scaled deviance in a fitting under generalised linear modelling. [1]

(iii) (a) Define both Pearson and deviance residuals.
(b) Explain how these two types of residuals are generally different.
(c) State in which case they are the same. [5]

[Total 8]

**7** A shipping insurance company has insured ships for six years, and classifies the ships it insures into three types.

Let:

$P_{ij}$ be the number of ships insured in the $j$th year from type $i$,
$Y_{ij}$ be the corresponding number of claims.

The six years of data are summarised as follows:

| *Type (i)* | $\bar{P}_i = \sum_{j=1}^{6} P_{ij}$ | $\bar{X}_i = \sum_{j=1}^{6} \frac{Y_{ij}}{\bar{P}_i}$ | $\sum_{j=1}^{6} P_{ij}\left(\frac{Y_{ij}}{P_{ij}} - \bar{X}_i\right)^2$ | $\sum_{j=1}^{6} P_{ij}\left(\frac{Y_{ij}}{P_{ij}} - \bar{X}\right)^2$ |
|---|---|---|---|---|
| Type 1 | 648 | 0.524 691 | 30.966 692 | 64.392683 |
| Type 2 | 981 | 0.145062 | 4.689 264 | 42.240804 |
| Type 3 | 636 | 0.370 370 | 62.449 512 | 66.467 182 |

$$\bar{X} = \sum_{i=1}^{3} \sum_{j=1}^{6} \frac{Y_{ij}}{\bar{P}} = 0.297572, \text{ where } \bar{P} = \sum_{i=1}^{3} \bar{P}_i$$

There are 100 ships of Type 3 to be insured in year seven.

(i) Estimate the number of claims from Type 3 ships in year seven using empirical Bayes credibility theory (EBCT) Model 2. [6]

The insurance company's actuary is considering using EBCT Model 1 instead.

(ii) Explain an advantage and a disadvantage of using EBCT Model 1 rather than EBCT Model 2. [2]
[Total 8]

**8** The run-off triangle below shows cumulative claims incurred on a portfolio of general insurance policies

| *Policy Year* | *Development Year* 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 2011 | 1,528 | 2,034 | 2,212 | 2,310 |
| 2012 | 1,812 | 2,251 | 2,951 | |
| 2013 | 1,693 | 1,851 | | |
| 2014 | 2,125 | | | |

Annual premiums written in 2014 were 4,023 and the ultimate loss ratio has been estimated as 91%. Claims paid to date for policy year 2014 are 572.

Estimate the outstanding claims to be paid arising from policies written in 2014 only, using the Bornheutter-Ferguson technique, stating any assumptions that you make. [9]

**9** A random variable X follows a gamma distribution with parameters $\alpha$ and $\lambda$.

(i) Derive the moment generating function (MGF) of X. [3]

(ii) Derive the coefficient of skewness of X. [8]
[Total 11]

**10** Claims on a portfolio of insurance policies arrive as a Poisson process with annual rate $\lambda$. Individual claims are uniformly distributed between 0 and 50, and the insurance company uses a premium loading of 12%.

(i) Show that the insurance company's adjustment coefficient is 0.0066 to four decimal places. [3]

The insurance company has entered into an excess of loss insurance agreement with a retention amount of $M$ and with a reinsurer who uses a premium loading of 15%.

(ii) (a) Determine the mean amount per claim paid by the reinsurer as a function of $M$.

(b) Determine the minimum retention level $M_{\min}$ for the insurance company, assuming that expected net premium income needs to be greater than expected net claims.

[6]

The insurance company manages to negotiate a lower reinsurance premium loading.

(iii) Explain what happens to the minimum retention level $M_{\min}$, without doing any further calculations. [2]
[Total 11]

**11** Consider the following pair of equations:

$$X_t = 0.5X_{t-1} + \beta Y_t + \varepsilon_t^1$$

$$Y_t = 0.5Y_{t-1} + \beta X_t + \varepsilon_t^2$$

where $\varepsilon_t^1$ and $\varepsilon_t^2$ are independent white noise processes.

(i) (a) Show that these equations can be represented as

$$M\begin{pmatrix} X_t \\ Y_t \end{pmatrix} = N\begin{pmatrix} X_{t-1} \\ Y_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \end{pmatrix}$$

where $M$ and $N$ are matrices to be determined.

(b) Determine the values of $\beta$ for which these equations represent a stationary bivariate time series model. [9]

(ii) Show that the following set of equations represents a VAR($p$) (vector auto regressive) process, by specifying the order and the relevant parameters:

$$X_t = \alpha X_{t-1} + \alpha Y_{t-1} + \varepsilon_t^1$$

$$Y_t = \beta X_{t-1} - \beta X_{t-2} + \varepsilon_t^2$$

[3]
[Total 12]

**12** An actuary needs to sample from a particular claim size distribution with the following density function:

$$f(x) = \begin{cases} ke^{-(x/2)}, & 0 < x < 2 \\ ke^{-x}, & x \geq 2 \end{cases}$$

(i) Calculate the value of $k$. [2]

(ii) Set out an algorithm for sampling from $f(x)$ using the inverse transform method. [5]

(iii) Set out an algorithm for sampling from $f(x)$ using the acceptance – rejection method, by using samples from the exponential distribution with mean 1. [6]
[Total 13]

## END OF PAPER