INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

28 April 2011 (am)

Subject CT6 — Statistical Methods
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Give two examples of exercises where Monte-Carlo simulation should be performed using the same choice of random numbers, explaining your reasoning in each case. [4]

2 An insurance company has collected data for the number of claims arising from certain risks over the last 10 years. The number of claims in the *j*th year from the *i*th risk is denoted by X_{ij} for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, 10$. The distribution of X_{ij} for $j = 1\ 2, \ldots, 10$ depends on an unknown parameter θ_i and given θ_i the X_{ij} are independent identically distributed random variables.

(i) Give a brief interpretation of $E[s^2(\theta)]$ and $V[m(\theta))]$ under the assumptions of Empirical Bayes Credibility Theory Model 1. [2]

(ii) Explain how the value of the credibility factor Z depends on $E[s^2(\theta)]$ and $V[m(\theta)]$. [3]

[Total 5]

3 Let $y_1, \ldots, y_n$ be samples from a uniform distribution on the interval $[0, \theta]$ where $\theta > 0$ is an unknown constant. Prior beliefs about θ are given by a distribution with density

$$f(\theta) = \begin{cases} \alpha\beta^{\alpha}\theta^{-(1+\alpha)} & \theta > \beta \\ 0 & \text{otherwise} \end{cases}$$

where α and β are positive constants.

(i) Show that the posterior distribution of θ given y_1 is of the same form as the prior distribution, specifying the parameters involved. [4]

(ii) Write down the posterior distribution of θ given $y_1, \ldots, y_n$. [2]

[Total 6]

4 The annual number of claims on an insurance policy within a certain portfolio follows a Poisson distribution with mean μ. The parameter μ varies from policy to policy and can be considered as a random variable that follows an exponential distribution with mean $1/\lambda$.

Find the unconditional distribution of the annual number of claims on a randomly chosen policy from the portfolio. [6]

5 The number of claims under an insurance policy in a year is either 0 (with probability 40%) or 1 (with probability 20%) or 2 (with probability 40%). Individual claim amounts are equally likely to be 50 or 20. The insurance company calculates premiums using a premium loading of 50% and is considering operating one of the following arrangements:

(A) Making no changes.

(B) Introducing a policy excess of 10 (per claim) in return for a reduction of 5 in premiums.

(C) Effecting an individual excess of loss reinsurance arrangement with retention 30 for a premium of 10.

Construct a table of the insurance company's profits under all the possible outcomes for each of (A) (B) and (C) and hence determine the optimal arrangement using the Bayes criteria. [8]

6 The double exponential distribution with parameter $\lambda > 0$ has density given by

$$g(x) = \tfrac{1}{2}\lambda e^{-\lambda|x|} \quad x \in \mathbb{R}.$$

(i) Construct an algorithm for generating samples from this distribution. [3]

(ii) Construct an algorithm for producing samples from a $N(0,1)$ distribution using samples from the double exponential distribution and the acceptance-rejection method. [6]

[Total 9]

7 Consider the time series

$$Y_t = 0.7 + 0.4Y_{t-1} + 0.12Y_{t-2} + e_t$$

where e_t is a white noise process with variance σ^2.

(i) Identify the model as an ARIMA(p,d,q) process. [1]

(ii) Determine whether Y_t is a stationary process. [2]

(iii) Calculate $E(Y_t)$. [2]

(iv) Calculate the auto-correlations ρ_1, ρ_2, ρ_3 and ρ_4. [4]

[Total 9]

8 Suppose that Y is a random variable belonging to a special subset of the exponential family where the density function of Y has the form

$$f(y,\theta,\varphi)=\exp\left[\frac{y\theta-b(\theta)}{\varphi}+c(y,\varphi)\right]$$

For some constants θ and φ and functions b and c.

(i) Show that the moment generating function of Y is given by

$$M_Y(t)=\exp\left[\frac{b(\theta+t\varphi)-b(\theta)}{\varphi}\right]$$ [3]

Hint: Note that the function $f(y, \theta + \varphi t, \varphi)$ is the density of another random variable of the same family and hence $\int_{-\infty}^{\infty} f(y,\theta+\varphi t,\varphi)dy=1$.

(ii) Show that $E(Y) = b'(\theta)$ and $\text{Var}(Y) = \varphi b''(\theta)$ using the result in (i). [4]

(iii) Verify that the result in (i) holds if Y has a Poisson distribution. [4]

[Total 11]

9 Claims on a portfolio of insurance policies arise as a Poisson process with parameter λ. Individual claim amounts are taken from a distribution X and we define $m_i = E(X^i)$ for i = 1, 2, …. The insurance company calculates premiums using a premium loading of θ.

(i) Define the adjustment coefficient R. [1]

(ii) (a) Show that R can be approximated by $\dfrac{2\theta m_1}{m_2}$ by truncating the series expansion of $M_X(t)$.

(b) Show that there is another approximation to R which is a solution of the equation $m_3y^2 + 3m_2y - 6\theta m_1 = 0$. [6]

Now suppose that X has an exponential distribution with mean 10 and that θ = 0.3.

(iii) Calculate the approximations to R in (ii) and (iii) and compare them to the true value of R. [6]

[Total 13]

10 The number of claims on a portfolio of insurance policies has a Poisson distribution with mean 200. Individual claim amounts are exponentially distributed with mean 40. The insurance company calculates premiums using a premium loading of 40% and is considering entering into one of the following re-insurance arrangements:

(A) No reinsurance.

(B) Individual excess of loss insurance with retention 60 with a reinsurance company that calculates premiums using a premium loading of 55%.

(C) Proportional reinsurance with retention 75% with a reinsurance company that calculates premiums using a premium loading of 45%.

(i) Find the insurance company's expected profit under each arrangement. [6]

(ii) Find the probability that the insurer makes a profit of less than 2000 under each of the arrangements using a normal approximation. [8]
[Total 14]

11 The table below shows cumulative claims paid on a portfolio of insurance policies.

	Development Year			
Accident Year	*0*	*1*	*2*	*3*
2007	240	281.4	302	305
2008	260	320	322	
2009	270	312.9		
2010	276			

All claims are fully run off by the end of development year 3.

(i) Calculate the total reserve for outstanding claims using the basic chain ladder technique. [7]

An actuary is considering modelling the future claims assuming that individual development factors are lognormally distributed with the following parameters:

	Development Year		
Parameter	*0 to 1*	*1 to 2*	*2 to 3*
μ	0.171251	0.035850	0.008787
σ	0.032148	0.045606	0.046853

(ii) Show that under these assumptions the cumulative development factor to ultimate is also lognormally distributed. [3]

(iii) Calculate a 99% upper confidence limit for the outstanding claims relating to the 2010 accident year. [5]
[Total 15]

END OF PAPER