INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

24 April 2017 (pm)

Subject CT4 – Models
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 11 questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 In a certain country, the force of mortality, μ_x, in the age range 90-105 years exact is given by:

Age range (years)	μ_x
$90 \le x < 95$	0.10
$95 \le x < 100$	0.15
$100 \le x < 105$	0.20

The head of state sends a congratulatory card on a citizen's 100th birthday and again on reaching age 105.

Derive the probability that a person aged exactly 93 WILL receive a congratulatory card for reaching age 100 but NOT receive a second congratulatory card for reaching age 105. [3]

2 (i) Define an increment of a process. [1]

The rate of mortality in a certain population at ages over exact age 30 years, $h(30+u)$, is described by the process:

$$h(30+u) = B(1+\gamma)^u \quad u \ge 0$$

where B and γ are constants.

(ii) Show that the increments of the process $\log[h(30+u)]$ are stationary. [3]

[Total 4]

3 (i) Define a Markov Chain. [2]

(ii) Describe the difference between a time-homogeneous and a time-inhomogeneous Markov Chain, giving an example of each. [2]

[Total 4]

4 (i) Describe how a classification based on the nature of the state and time spaces of stochastic processes leads to a four-way categorisation. [2]

(ii) List FOUR stochastic processes, one for each of the four categories in your answer to part (i). [2]

[Total 4]

5 A city operates a bicycle rental scheme. Bicycles are stored in racks at locations around the city and may be rented for a fee and ridden from one location and deposited at another, provided there is space in the rack. The rack outside the actuarial profession's headquarters in that city has spaces for four bicycles.

The profession would like the city to increase the size of the rack. The city has said it will do so if the profession can demonstrate that, in the long run, the rack is full or empty for more than 35 per cent of the working day. The profession commissions a study to monitor the rack every 10 minutes during the working day.

The study shows that, on average:

- there is a probability of 0.3 that the number, m, of bicycles in the rack will increase by 1 over a 10-minute interval (where $0 \leq m < 4$).
- there is a probability of 0.2 that the number of bicycles in the rack will decrease by 1 over a 10-minute interval (where $0 < m \leq 4$).
- the probability of more than one increase or decrease per 10-minute interval can be regarded as 0.

(i) Give the transition matrix for the number of bicycles in the rack. [2]

(ii) Determine whether the city will increase the size of the rack. [6]

(iii) Comment on whether an increase in the size of the rack will reduce the proportion of time for which the rack is empty or full. [2]
[Total 10]

6 (i) Define a Poisson process. [2]

An insurance company observed that 200 claims arrived in the 52 weeks of the calendar year 2014. The company has for some years modelled the number of claims per week, D, using a time-homogeneous Poisson process model with parameter λ.

The company proposes to use a value $\lambda = 3.846$ for 2014.

(ii) Explain why this is a sensible value of λ to use. [1]

The company's records for 2014 show that the number of claims per week was distributed as follows:

Claims per week	*Number of weeks*
0	1
1	5
2	8
3	10
4	12
5	4
6	6
7	4
8	1
9 or more	1

(iii) Test the fit of the Poisson distribution with parameter $\lambda = 3.846$ to the data. [7]

(iv) State why the company might perform a serial correlations test on the experience. [1]

[Total 11]

7 (i) Describe the essential feature of a proportional hazards model. [2]

A study was made of the impact of drinking beer on men aged 60 years and over. A sample of men was followed from their 60th birthdays until they died, or left the study for other reasons. The baseline hazard of death, μ, was assumed to be constant, and a proportional hazards model was estimated with a single covariate: the average daily beer intake in standard-sized glasses consumed, x. The equation of the model is:

$$h(t) = \mu \exp(\beta x)$$

where $h(t)$ is the hazard of death at age $60 + t$.

The estimated value of μ is 0.03, and the estimated value of β is 0.2.

(ii) Explain how μ and β should be interpreted, in the context of this model. [2]

(iii) Calculate the estimated hazard of death of a man aged exactly 62 years who drinks two glasses of beer a day. [1]

A man is aged exactly 60 years and drinks three glasses of beer a day.

(iv) (a) Calculate the estimated probability that this man will still be alive in 10 years' time.

(b) Calculate the expectation of life at age 60 years for this man.

[2]

Another man is aged exactly 60 years. He drinks beer only in his local bar. He drinks all the beer he buys and is expected to continue drinking the same amount of beer every day until he dies. The owner of the bar is interested in selling as much beer as possible.

(v) Determine the average number of glasses of beer a day the owner must sell the man in order to maximise the total amount of beer the man buys over his remaining lifetime. [4]

[Total 11]

8 A careful shopkeeper takes delivery of a batch of 20 packets of cheese. Every morning at 8 a.m. precisely she checks to see if any of the cheese has gone mouldy and throws away any mouldy packets.

As she runs a high quality establishment, she has lots of customers and some of the cheese is sold. After ten days she decides the cheese will be too old to sell and throws out the remaining packets.

A curious customer observes that the shopkeeper has created an observational plan for calculating the hazard of cheese going mouldy.

(i) State, with reasons, THREE types of censoring present in this situation. [3]

(ii) Assess, for EACH type of censoring listed in your answer to part (i), whether a change to the observational plan could be made which would remove that type of censoring. [3]

The shopkeeper made notes at 8 a.m. each day as follows:

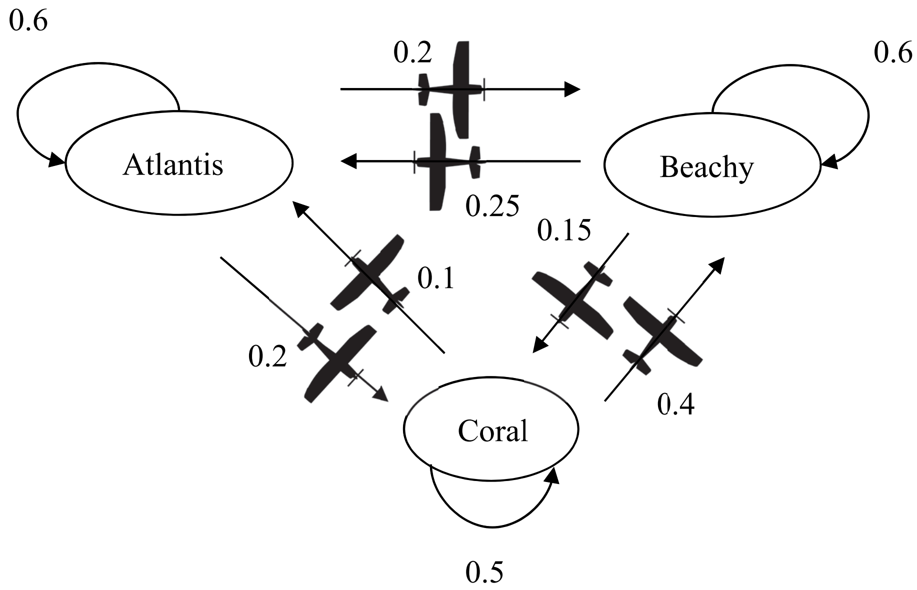
Day	*Shopkeeper's notes*
1	Sold three packets already
2	Sold one more packet
3	One went mouldy
4	Two more mouldy ones, I hope my fridge is cold enough
5	Seems OK, nothing to report
6	Sold four more – all to one customer!
7	Nothing to report
8	Another two mouldy ones this morning
9	Sold two more
10	Three more mouldy ones – I'll throw the rest out

(iii) Calculate the Kaplan-Meier estimate of the survival function for cheese staying free from mould. [6]

[Total 12]

9 A journalist has just been appointed as a foreign correspondent covering news stories in three island countries Atlantis, Beachy and Coral. He will spend each day in the country likely to have the most exciting news, taking the flights available between each country which go once per day at the end of the day.

The previous foreign correspondent tells him "If you want to know how many flights you are likely to take, I estimate my movements have been like this" and she drew this diagram showing the transition probabilities:



(i) Give the transition matrix for this process. [1]

On his first day in the job the new foreign correspondent will be in Atlantis.

(ii) Calculate the probability that the foreign correspondent will be in each country in his third day in the job. [2]

The previous correspondent also reported that Beachy must be the most interesting of the islands in terms of news because she spent 41.9% of her time there compared with 32.6% on Atlantis and 25.6% on Coral.

(iii) Sketch a graph showing the probability that the journalist is in each country over time, using the information above. [3]

(iv) Calculate the proportion of days on which the foreign correspondent will take a flight. [1]

The first time the foreign correspondent visits each of the countries he takes a photograph to mark the occasion.

(v) Identify a suitable state space for modelling as a Markov chain which countries he has visited so far. [2]

(vi) Draw a transition diagram for the possible transitions between these states. [3]

[Total 12]

10 The government statistical service in a country with a population of 10 million has estimated mortality rates among males in that country aged 20 to 99 years inclusive. It wishes to create a new standard mortality table.

(i) Describe why the crude mortality rates should be graduated during the production of this standard mortality table. [3]

(ii) Describe a suitable method of graduation for these mortality rates. [1]

(iii) Explain the limitations of the method described in your answer to part (ii) in this situation. [2]

The government performs the graduation and compares the crude and graduated rates. Below are some of the results of the comparison:

Value of individual standardised deviation at age x, z_x	*Number of ages*
$z_x < -3$	0
$-2 > z_x \geq -3$	7
$-1 > z_x \geq -2$	16
$0 > z_x \geq -1$	26
$1 > z_x \geq 0$	16
$2 > z_x \geq 1$	10
$3 > z_x \geq 2$	2
$z_x \geq 3$	3

(iv) Assess the quality of the graduated rates for use as a new standard mortality table by applying TWO statistical tests to the above information. The two tests should each examine a different aspect of the graduation. [6]

(v) Comment on the implications of your results in part (iv) for the government using the new standard mortality table for economic and financial planning purposes. [2]

[Total 14]

11 A large company operates a health benefits scheme which pays a sickness benefit to any employee who is unable to work through ill-health and a death benefit to any employee who dies.

(i) Draw a transition diagram with three states which could be used to analyse data from this scheme. [2]

(ii) Give the likelihood of the data, defining all the terms you use. [4]

(iii) Derive the maximum likelihood estimator of the rate of falling sick. [3]

The company records data on 1 January each year, classified by age last birthday on:

- the total number of employees (including those in receipt of sickness benefit).
- the number of employees in receipt of a sickness benefit.

Some recent data are given in the table below:

Age last birthday	*Total number of employees on 1 January in year*			*Number of employees in receipt of sickness benefit on 1 January in year*		
	2014	2015	2016	2014	2015	2016
51	148	162	180	12	20	8
52	146	148	160	10	18	7
53	140	144	146	8	20	6

The company wishes to estimate the rates of falling sick and recovery at age 52 years nearest birthday over the two-year period consisting of the calendar years 2014 and 2015.

(iv) Determine suitable exposed-to-risks for calculating the rates of falling sick and recovery. [6]

[Total 15]

END OF PAPER