INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2015

Subject CT4 – Models
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

F Layton
Chairman of the Board of Examiners
December 2015

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Models subject is to provide a grounding in stochastic processes and survival models and their application.

2. Subject CT4 comprises five main sections:

   (1) a study of the properties of models in general, and their uses for actuaries, including advantages and disadvantages (and a comparison of alternative models of the same processes)

   (2) stochastic processes, especially Markov chains and Markov jump processes

   (3) models of a random variable measuring future lifetime

   (4) the calculation of exposed to risk and the application of the principle of correspondence

   (5) the reasons why mortality (or other decremental) rates are graduated, and a range of statistical tests used both to compare a set of rates with a previous experience and to test the adherence of a graduated set of rates to the original data. Throughout the subject the emphasis is on estimation and the practical application of models. Theory is kept to the minimum required in order usefully to apply the models to real problems.

3. Different numerical answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations but candidates are not penalised for this. However, candidates may be penalised where excessive rounding has been used or where insufficient working is shown. Credit is given for valid solutions different from those shown below. Partial credit is also given to candidates submitting incomplete solutions with valid intermediate workings.

B. General comments on *student performance in this diet of the examination*

1. The average performance was similar to that of recent September sessions, with 50 per cent of candidates passing. Well-prepared candidates scored highly across most of the paper, with one in eight candidates scoring 70% or more, and a highest mark of 92%.

2. In general, there was a tendency for candidates to fail to score marks by missing out the more "wordy" sections of questions even when these were straightforward bookwork.

3. The comments that follow the questions concentrate on areas where candidates could have improved their performance. Candidates approaching the subject for the first time are advised to include these areas in their revision.

C. Comparative pass rates for the past 3 years for this diet of examination

Year	*Rate (%)*
September 2015	50
April 2015	52
September 2014	43
April 2014	55
September 2013	52
April 2013	52

Reasons for any significant change in pass rates in current diet to those in the past:

The pass rate in this diet was within the range of pass rates in previous September diets, though slightly higher than the average of the last five September diets. It was felt that, as a group, the candidates in this diet were better prepared than have been candidates in some recent September diets.

Solutions

Q1 Type of policy (which often reflects the reason for insuring)
Smoker/non-smoker status
Level of underwriting
Duration in force
Sales channel
Policy size
Occupation of policyholder
Known impairments
Postcode/geographical region
Marital status

Most candidates scored full marks on this question.

Q2 A stochastic model is one which recognises the random nature of the input components, whereas a deterministic model does not contain any random components.

Running a stochastic model many times will produce a distribution of results for possible scenarios, whereas a deterministic model will produce results for a single scenario.
Thus a deterministic model can be seen as a special case of a stochastic model.

In a stochastic model the output of each run is one value from a distribution.

By contrast, in a deterministic model, the output is determined once the set of fixed inputs and the relationships between them have been defined.

For many stochastic models, it is necessary to use numerical approximations in order to integrate functions or solve differential equations.

The results for a deterministic model can often be obtained by direct calculations.

In a stochastic model, several independent runs are required for each set of inputs so that statistical theory can be used to help study the implications of a set of inputs.

A deterministic model only requires one run.

Full marks could be obtained for rather less than is written in this solution.

Q3 (i) Right censoring refers to a life ceasing to be observed prior to the event of interest occurring.

Type I censoring occurs when the censoring times are known in advance and lives will be considered censored on a pre-determined date regardless of whether the event of interest has occurred.

Random censoring refers to the time of censoring being a random variable such that censoring may occur as a random event prior to the event of interest.

(ii) (a) Right censoring occurs because the censoring means no information is available about whether the policy would subsequently have lapsed.

This is not Type I censoring as it would not be known in advance when the policyholder would die.

Random censoring occurs as the time of death is a random variable.

(b) It is right censoring as it removes information about whether the policies subsequently lapsed.

It is not clear whether this is Type I censoring because it is not known whether the migration was anticipated in the observation plan.

For the same reason it is not clear whether it is random censoring.

(c) It is in theory right censoring, but in practice the event of interest cannot occur after the censoring date.

It is Type I censoring as the maturity date would be known in advance.

The policy reaching its maturity date is not a random variable.

> A common error in part (i) was to state that Type I censoring occurred when the investigation runs for a fixed period. This is true, but is not a definition of Type I censoring. Type I censoring can occur in an investigation of indeterminate duration provided that, for each life in the investigation, the censoring time is known in advance of the study. In part (ii), many candidates failed to read the question and did not describe those types of censoring that were not present in each situation, and explain why they were not there. Such candidates lost marks needlessly. Other candidates simply stated whether each form of censoring was present or not, without offering an explanation of why. In part (ii) (c) credit was given for arguing either that right censoring would occur in theory as a result of the maturity date being reached, or that right censoring would not occur in practice, as a mature policy could not lapse.

Q4 (i) The deaths data carries more information, so the exposed to risk data must be amended to correspond with the deaths data.

The exposed to risk may be calculated using the census formula

$$E_x^c = \int_0^1 P_{x,t}\,dt\,,$$

where P_x is the population aged x last birthday.

For Company A, 53 next corresponds to 52 last.

Assuming that the population varies linearly between census dates, this can be approximated using the trapezium rule.

So $E_x^c = \frac{1}{2}(P^*{}_{x+1,1/1/13} + P^*{}_{x+1,1/1/14})$,

where $P^*{}_x$ is the population aged x next birthday

= (8,016 + 9,026) / 2
= 8,521

For Company B the age definition does not need adjusting.

Again assuming the population varies linearly between census dates we can calculate the population at 1/1/2013 as

5,218 + (3/12) (5,281 − 5,218) = 5,233.75, and the exposed to risk for the first three months of 2013 is

(1/2) (3/12) (5,218 + 5,233.75) = 1,306.4688.

Similarly the population at 1/1/2014 is

5,218 - (9/12)(5,218 – 3,812) = 4,163.5

and the exposed to risk for the last nine months of 2013 is

(1/2)(9/12)(5,218 + 4,163.5) = 3,518.0625.

Assuming that the force of mortality is constant over the year of age,

we have $\hat{\mu}_{52} = \dfrac{28+17}{8,521+1,306.47+3,518.06} = 0.0034$

(ii) The estimate $\hat{\mu}_{52}$ applies to the age at the middle of the rate interval, which is age 52.5 exact.

> The credit for stating the assumptions was only given if they were stated in the right place in the argument. In fact, few candidates stated the correct assumptions and many stated assumptions that were unnecessary (e.g. that deaths should be uniformly distributed across the year of age). Very few candidates realised that it was necessary to assume a constant force of mortality across the year of age.

Q5 (i) A model is an imitation of a real world system or process.

Different future policies or possible actions can be compared to see which best suits the requirements of a user.

We can examine different scenarios without carrying them out in practice, or to avoid potential costs or risks associated with trialling in real life.

Parameters can be sensitivity tested using a model so the effect of changing certain input parameters can be studied before a decision is made to implement the plans in the real world.

A model allows systems with long time-frames to be analysed in compressed time.

Models also allow complex processes involving stochastic elements to be investigated.

Models can be developed for many activities (e.g. the economy of a country, future cash flows of a broker distribution channel).

(ii) Sample path C – discrete time, discrete state process
Sample path A – continuous time, discrete state process
Sample path B – discrete time, continuous state process
Sample path D – continuous time, continuous state process

(iii) The objectives of the study.

The real world process may only be able to change in a discrete fashion.

Outputs may only be required at discrete points.

To simulate a process may need to discretise the process (for example with Monte Carlo simulation)

It may be easier to model certain situations with a probability density function, which is therefore continuous.

Data may only be available at discrete points, for example the position at the end of each day.

There may be other modelling constraints for example it may be that a limited number of states can be used so a discrete model would be preferred.

Answers to part (ii) were disappointing. Few candidates realised that in sample paths A and C the process could only be in one of a finite number of states (hence discrete state), and that in sample paths B and C the observations were equally spaced on the time axis (hence discrete time). In part (iii) other sensible comments were given credit. Unfortunately, many candidates simply gave examples of cases where a discrete or a continuous process would be most appropriate without explaining the reasons why this would be the case. Such candidates were given only limited credit.

Q6 (i) A proportional hazards model is used to estimate the effect of covariates on the hazard of experiencing an event.

In a proportional hazards model the hazard is assumed to factorise into two components, one depending only on duration, and the other depending only on the covariates.

The ratio between the hazards for persons with any two values of a covariate is the same at all durations.

(ii) A cow who started the previous treatment immediately the condition appeared.

(iii) $h_0(t)\exp(\beta_0 + \beta_1 x + \beta_2 x) = h_0(t)\exp(\beta_1 x)$

$\exp(\beta_0) = \exp(-\beta_2 x)$

$\beta_0 = -\beta_2 x$

$0.8 = 0.1x$

So $x = 8$ days.

(iv) The median recovery time is the value of t such that $S(t) = 0.5$.

For the previous treatment, we have

$$S(14) = \exp\left(-e^{0.4(3)}\int_0^{14} h_0(t)dt\right) = \exp\left(-3.320\int_0^{14} h_0(t)dt\right) = 0.5\,.$$

So $\int_0^{14} h_0(t)dt = \dfrac{\log_e(0.5)}{-3.320} = 0.209.$

For the new treatment we have

$$S(14) = \exp(-e^{0.8+1.2-0.3}\int_0^{14} h_0(t)dt)$$

$$= \exp[-5.474(0.209)] = 0.319\ .$$

Answers to part (ii) were disappointing. Some candidates wrote that the cow has not suffered from the condition before, which may or may not have been the case. Others wrote that the cow was not suffering from the condition when the treatment started, which was absurd as the treatment would not have been given were the cow not suffering from the condition. Part (iii) was well answered by most candidates. Part (iv) was more demanding, but it was disappointing that so many candidates framed their answers entirely in terms of the hazard function, whereas the question clearly refers to the median recovery time and hence implies that the survival function is required.

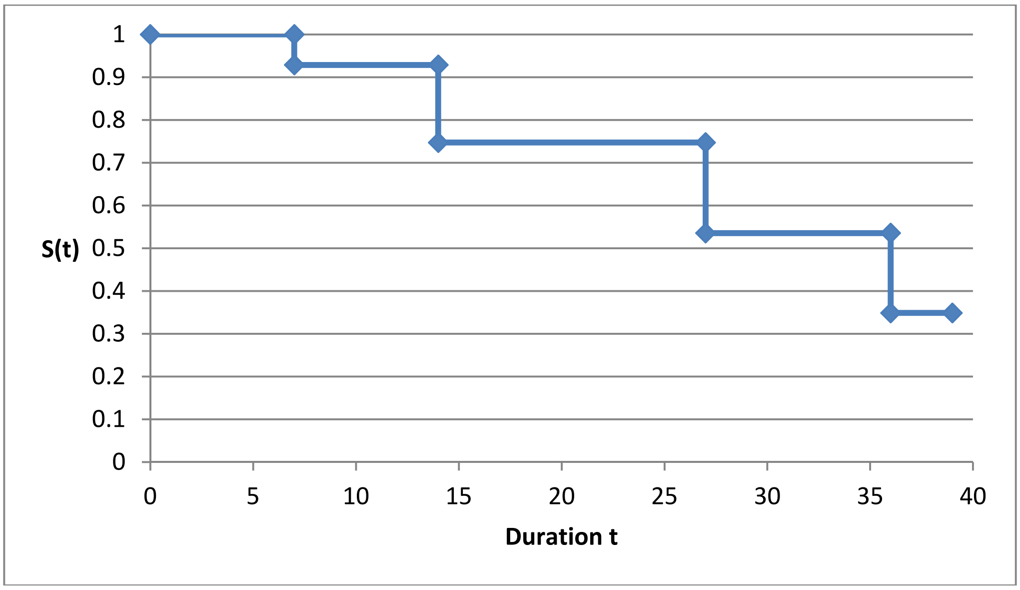
Q7 (i) The Nelson-Aalen estimate for Λ is $\Lambda_t = \sum_{x_j \le x} \frac{d_j}{n_j}$.

t_j	n_j	d_j	c_j	d_j/n_j	Λ_t
0	33				
1	33		5		
6	28		1		
7	27	2		2/27	0.0741
13	25		2		
14	23	5		5/23	0.2915
27	18	6		6/18	0.6248
28	12		4		
30	8		1		
36	7	3	4	3/7	1.0534

Since $S(t) = \exp(-\Lambda_t)$ we have:

t	$S(t)$
$0 \le t < 7$	1
$7 \le t < 14$	0.9286
$14 \le t < 27$	0.7472
$27 \le t < 36$	0.5354
$36 \le t < 39$	0.3488

(ii)



(iii) EITHER

$1 - S(39) = 65.13\%$

OR

Since 16 students passed the test and 33 started the year, the required probability is 16/33 = 48.48%.

(iv) The school has ignored those students who dropped out during the year.

Since they did not pass, their exclusion would clearly increase the proportion who pass.

> In part (i) most candidates managed to compute the correct Nelson-Aalen estimate. A minority of candidates supposed that the decrement was dropping out rather than passing the test. As the wording of the question did not rule out this interpretation, full marks were awarded for the correct survival function for this alternative decrement.

Q8 (i) A process with a continuous time space and discrete state space

satisfying the Markov property that

EITHER

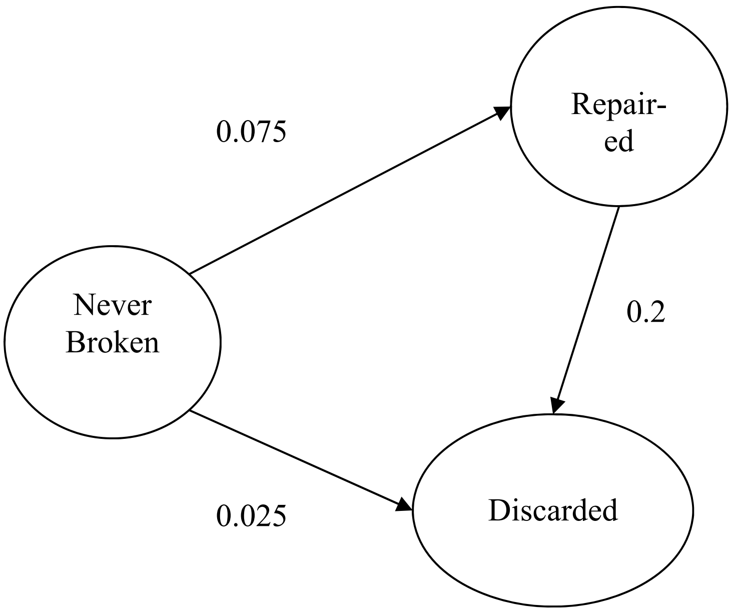
the future progression of the process does not depend on the history of the process prior to arrival in the current state.

OR

$$P[X_t \in A \mid X_{s_1} = x_1, X_{s_2} = x_2, ..., X_{s_n} = x_n, X_s = x] = P[X_t \in A \mid X_s = x]$$

for all times $s_1 < s_2 < ... < s_n < s < t$, all states $x_1, x_2, ..., x_n, x$ in S and all subsets A of S.

(ii)



(iii) $\frac{d}{dt} P_{NB}(t) = -0.1P_{NB}(t)$

$$\frac{d}{dt} P_R(t) = 0.075P_{NB}(t) - 0.2P_R(t)$$

$$\frac{d}{dt} P_D(t) = 0.2P_R(t) + 0.025P_{NB}(t)$$

(iv) $\frac{d}{dt} P_{NB}(t) = -\,0.1P_{NB}(t)$

$$\frac{d}{dt}[\ln P_{NB}(t)] = -0.1$$

$\ln P_{NB}(t) = -0.1t + \text{const}$

$P_{NB}(0) = 1$ so constant = 0

$P_{NB}(t) = \exp(-0.1t)$

$$\frac{d}{dt} P_R(t) = 0.075 \exp(-0.1t) - 0.2P_R(t)$$

$$\left[\frac{d}{dt} P_R(t) + 0.2P_R(t)\right] \exp(0.2t) = 0.075 \exp(-0.1t) \exp(0.2t)$$

$$\frac{d}{dt}[\exp(0.2t)\, P_R(t)] = 0.0750 \exp(0.1t)$$

$\exp(0.2t)\, P_R(t) = 0.750 \exp(0.1t) + \text{const}$

$P_R(0) = 0$ so constant = –0.75

$P_R(t) = 0.750\, (\exp(-0.1t) - \exp(-0.2t))$

(v) This is $P_{NB}(t) + P_R(t)$

$= 1.75 \exp(-0.1t) - 0.75 \exp(-0.2t)$

> Answers to parts (iii)-(v) of this question were very disappointing. Few candidates worked out a constant of integration for P_{NB}(t) (even though it was equal to zero). Few candidates produced the correct transition diagram in part (ii). Those candidates who wrote down incorrect transition diagrams were given credit in parts (iii)-(v) for answers which correctly followed through from the incorrect transition diagrams.

Q9 (i) Let the number of transitions observed between states i and j be d^{ij}.

Let the transition rate between states i and j be μ^{ij}.

Let the observed waiting time in state i be v^i.

Then the likelihood of the data can be written

$$L \propto \exp[(-\mu^{12} - \mu^{13} - \mu^{14})\nu^1]\exp[(-\mu^{21} - \mu^{23} - \mu^{24})\nu^2](\mu^{12})^{d^{12}}(\mu^{13})^{d^{13}} (\mu^{14})^{d^{14}}(\mu^{21})^{d^{21}}(\mu^{23})^{d^{23}}(\mu^{24})^{d^{24}}$$

(ii) Taking logarithms of the likelihood we have:

$\log_e L = -\mu^{13}\nu^1 + d^{13}\log_e(\mu^{13})$ + terms not dependent on μ^{13}.

Differentiating with respect to μ^{13} gives:

$$\frac{d(\log_e L)}{d\mu^{13}} = -\nu^1 + \frac{d^{13}}{\mu^{13}}.$$

Setting the derivative to zero to get the maximum likelihood estimator (MLE):

$$\hat{\mu}^{13} = \frac{d^{13}}{\nu^1}.$$

As the second derivative of the log likelihood

$$\frac{d^2(\log_e L)}{(d\mu^{13})^2} = -\frac{d^{13}}{(\mu^{13})^2}$$

is negative this is a maximum.

(iii) The MLE of heart related death for an Obese person is $\frac{178}{14,392} = 0.012368$

and its associated variance is $\frac{0.012368}{14,392} = 0.859365 \times 10^{-6}$.

The MLE of heart related death for a person who is Not obese is $\frac{190}{18,109} = 0.010492$

and its associated variance is $\frac{0.010492}{18,109} = 0.579302 \times 10^{-6}$.

The null hypothesis is that the death rate for Obese people is no higher than that for people who are Not obese.

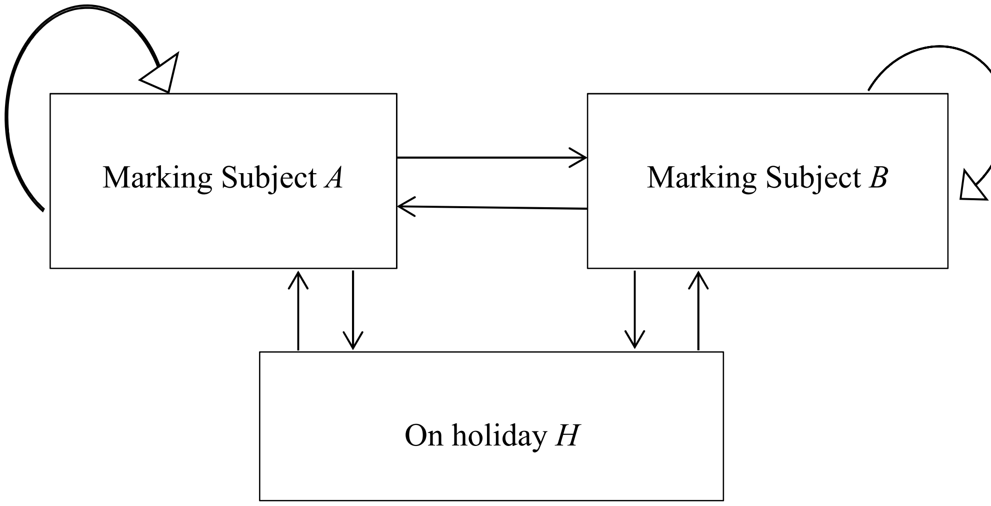
The test statistic is $\dfrac{\hat{\mu}^{23} - \hat{\mu}^{13}}{\sqrt{\dfrac{\hat{\mu}^{23}}{t_2} + \dfrac{\hat{\mu}^{13}}{t_1}}} \sim N(0,1)$

Evaluating this gives $(0.012368 - 0.010492)/\sqrt{1.43875 \times 10^{-6}} = 1.564$
This is a one-tailed test, at the 90% confidence level.

Therefore, as $1.564 > 1.28$, we can reject the null hypothesis and conclude that the death rate for Obese people is statistically larger at the 90% confidence level.

In part (i) many candidates supposed that it was not possible to move from Obese to Not obese and vice versa. The experience of many people demonstrates that this is not the case! Candidates who omitted these transitions without justification were penalised modestly. Part (iii) produced many variant attempts involving confidence intervals. The calculation of confidence intervals around the *difference* between the two rates would be a valid way of conducting a two-sided test, but here a one-sided test seems more appropriate, hence the z-score approach is to be preferred. Some candidates computed confidence intervals around each rate separately and then argued that because they overlapped (or did not overlap) the rates were not (or were) significantly different. This approach gained only limited credit.

Q10 (i)



(ii)

$$\begin{array}{r} \text{Subject A} \\ \text{Subject B} \\ \text{Holiday} \end{array} \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.5 & 0.5 & 0 \end{bmatrix}.$$

(iii) We have $\pi P = \pi$.

The stationary distribution of examiners can be found as the solution of the set of equations

$\pi_A = 0.8\pi_A + 0.2\pi_B + 0.5\pi_H$ (1)
$\pi_B = 0.1\pi_A + 0.6\pi_B + 0.5\pi_H$ (2)
$\pi_H = 0.1\pi_A + 0.2\pi_B$ (3)

(1) gives

$0.2\pi_A = 0.2\pi_B + 0.5\pi_H$
$0.4\pi_A = 0.4\pi_B + \pi_H$
$0.4\pi_B = 0.4\pi_A - \pi_H$

(2) gives

$0.4\pi_B = 0.1\pi_A + 0.5\pi_H$

so

$0.4\pi_A - \pi_H = 0.1\pi_A + 0.5\pi_H$
$0.3\pi_A = 1.5\pi_H$
$\pi_A = 5\pi_H$

In (2) this gives

$\pi_B = 0.5\pi_H + 0.6\pi_B + 0.5\pi_H$
$0.4\pi_B = \pi_H$
$\pi_B = 2.5\pi_H$

So, since $\pi_A + \pi_B + \pi_H = 1$,

the stationary distribution is $\{5\pi_H, 2.5\pi_H, \pi_H\}$

and hence

$$\pi_A = \frac{10}{17}$$
$$\pi_B = \frac{5}{17}$$
$$\pi_H = \frac{2}{17}.$$

So in the long run 58.8% of examiners are marking subject *A* and 29.4% are marking subject *B*.

(iv) Let the new transition probability from *H* to *A* be *x*, and that from *H* to *B* be $1 - x$.

The proportion we require is thus just *x*. The new transition matrix is

$$\begin{array}{r} \text{Subject A} \\ \text{Subject B} \\ \text{Holiday} \end{array} \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ x & 1-x & 0 \end{bmatrix}.$$

The stationary probability distribution is given by the three equations

$$\vartheta_A = 0.8\vartheta_A + 0.2\vartheta_B + x\vartheta_H \qquad (1)$$
$$\vartheta_B = 0.1\vartheta_A + 0.6\vartheta_B + (1-x)\vartheta_H \qquad (2)$$
$$\vartheta_H = 0.1\vartheta_A + 0.2\vartheta_B \qquad (3)$$

We also have $\vartheta_A = \vartheta_B$.

EITHER

From (3) $\vartheta_H = 0.3\vartheta_B = 0.3\vartheta_A$

Therefore the new stationary probability distribution is $\left\{\frac{10}{23}, \frac{10}{23}, \frac{3}{23}\right\}$.

In (1) we have

$10 = 8 + 2 + 3x$

Hence $x = 0$.

OR

If $\vartheta_A = \vartheta_B$ then in (1)

$$\vartheta_A = 0.8\vartheta_A + 0.2\vartheta_B + x\vartheta_H = 0.8\vartheta_A + 0.2\vartheta_A + x\vartheta_H = \vartheta_A + x\vartheta_H$$

Hence $x = 0$.

AND HENCE

All those returning from holiday will have to be allocated to subject B.

> There were a gratifying number of completely correct answers to this question, and many candidates scored full marks on parts (i)–(iii). Part (iv) was more demanding, and required candidates to invert the usual question and establish what transition matrix would give rise to a particular stationary distribution.

Q11 (i) If the previous experience is the recent experience of the policyholders of a life insurance company, the comparison could be important for pricing life insurance contracts.

Mortality rates are expected to change over time due to for example, improved medical processes or change in the mix of the population.

It helps to validate the results of the investigation.

It is can indicate whether the office's experience is out of line with the population as a whole.

Unexpected changes in mortality may have an impact on the underwriting process.

It is important for the company to know whether the investigation's results are consistent with published life tables, especially if the company plans to use published tables for any financial calculations.

(ii) The null hypothesis is that the company's underlying mortality experience is the same as the standard table.

The calculations are shown in the table below.

Age x	*Number of policies*	*Actual deaths*	*Expected deaths*	z_x	z_x^2
70	1,000	13	23.74	–2.204	4.859
71	1,200	28	31.80	–0.674	0.454
72	1,100	31	32.50	–0.263	0.069
73	1,100	34	36.20	–0.366	0.134
74	1,000	39	36.63	0.392	0.153
75	1,000	41	40.73	0.042	0.002
76	950	41	42.99	–0.304	0.092
77	900	40	45.20	–0.773	0.598
78	850	46	47.34	–0.195	0.038
79	800	48	49.35	–0.192	0.037
			Sum		6.436

An overall test of the hypothesis is the chi-squared test.

The test statistic is $\sum_x z_x^2$

where

$$z_x = \frac{E_x^c \mu_x - E_x^c \mu_x^s}{\sqrt{E_x^c \mu_x^s}}$$

and

E_x^c is the central exposed to risk at age x, μ_x is the observed death rate at age x, and μ_x^s is the death rate at age x in the standard table.

The calculated chi-squared statistic is 6.436.

We have 10 ages, so 10 degrees of freedom.

At the 95% level, the critical value is 18.31.

Since 6.436 < 18.31

we do not reject the null hypothesis that the underlying mortality of the company's policyholders is, overall, represented by the standard table.

(iii) Overall, the data fit the standard table pretty well, but there are features which the chi-squared test fails to detect such as small but consistent bias, or the existence of outliers.

EITHER SIGNS TEST

This tests for bias.

We have only 2 positive signs out of 10 ages.

The probability of getting only 2 positive signs under the hypothesis that the underlying mortality of the company's policyholders is, overall, represented by the standard table is equal to

$$\binom{10}{8 or 2} 0.5^{10} = 0.0439$$

which is greater than 0.025.

Therefore at the 95% significance level we can say that there is no bias.

OR CUMULATIVE DEVIATIONS TEST

This tests for bias.

the test statistic $\dfrac{\sum_x (\text{Observed deaths} - \text{Expected deaths})}{\sqrt{\sum_x \text{Expected deaths}}} \sim \text{Normal}(0,1)$.

So, using the results in the table, the value of the test statistic is

$$\frac{361 - 386.48}{\sqrt{386.48}} = -1.30$$

Since –1.96 < test statistic < +1.96

Therefore at the 95% significance level we can say that there is no bias.

GROUPING OF SIGNS TEST

This tests to see if the shape of the mortality is the same, or whether there is "clumping" of the deviations.

We also have just one run of positive signs with 2 positive signs and 8 negative signs.

We could try the Grouping of Signs test. The table on p. 189 of the Golden Book shows no value for these data. So we cannot reject the null hypothesis of no difference between the two sets of mortality rates.

The run of negatives, then positives, then negatives does look odd but we have too few age groups to conclude that this is a problem.

INDIVIDUAL STANDARDISED DEVIATIONS TST

This is a test for outliers.

Under the null hypothesis we would expect the individual z_xs to be distributed Normal (0,1)

and therefore only 1 in 20 $z_x s$ should have absolute magnitude greater than 1.96 (or none should be outside −3 to +3)

OR

table showing split of deviations, actual versus expected as below

Range	−∞,−2	−2,−1	−1,0	0,1	1,2	2,+∞
Expected	0.2	1.4	3.4	3.4	1.4	0.2
Actual	1	0	7	2	0	0

In fact, $z_{70} = -2.20$, which should give cause for concern.

Nevertheless this test is inconclusive (1 deviation out of 10 ages is greater than 1.96).

(iv) The variance of the number of claims at age 70 years will increase in the ratio

$$\frac{\sum_i i^2 \pi_i}{\sum_i i \pi_i},$$

where π_i is the proportion of policyholders who have i policies.

If, at age 70 years, the other 975 policyholders each have one policy, then we have

$\pi_1 = 0.99898, \pi_{25} = 0.00102$ and $\pi_i = 0$ for all other values of i.

OR

$\pi_1 = \dfrac{975}{976}, \pi_2 = \dfrac{1}{976}$ and $\pi_i = 0$ for all other values of *I*.

Thus for age 70 the variance ratio is

$$\frac{(1*1*0.99898)+(25*25*0.00102)}{(1*0.99898)+(25*0.00102)} = \frac{1.639}{1.026} = 1.598$$

OR

$$\frac{(975/976)+25*25*(1/976)}{(975/976)+25(1/976)} = 1.6$$

So the variance of the number of claims will be inflated by a factor of 1.598 (or 1.6).

Most candidates made a fair effort at part (i) and the majority scored highly on part (ii). Part (iii), however, was uncertainly answered by many candidates. Some candidates carried out both the Signs Test and the Cumulative Deviations Test, which were really checking for the same feature of the data. In part (iii), credit was only given for two tests. Only a minority of candidates attempted part (iv).

END OF EXAMINERS' REPORT