INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

4 October 2017 (pm)

Subject CT6 – Statistical Methods
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 11 questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Claim amounts on a portfolio of insurance policies follow a Weibull distribution. The median claim amount is £1,000 and 90% of claims are less than £5,000.

Estimate the parameters of the Weibull distribution, using the method of moments. [4]

2 (i) Explain why a sequence of pseudo-random numbers are often preferred to truly random numbers for Monte Carlo simulation. [2]

An actuary is generating pairs of standard Normal variates using the Polar algorithm and pairs of pseudo-random variates from a $U(0,1)$ distribution.

(ii) Determine the pairs of standard Normal variates generated by the following pairs of pseudo-random variates where possible.

(a) 0.062, 0.293

(b) 0.984, 0.794

(c) 0.008, 0.961

[3]
[Total 5]

3 On 1 January 2014 an insurance company writes a policy for a European farmer. At the end of each year, the farmer's crop is assessed, and if it is less than 100 tonnes, it is deemed to have failed. If the crop fails for two years in a row the insurance policy pays out €1m and then is immediately terminated.

Premiums are €25k per month, paid in advance, and there are no expenses. This is the only policy the insurance company writes and it has initial surplus $U > 0$. Denote by $\Psi(U,t)$ the probability of ruin by time t, measured in years; and $\Psi(U)$ as the ultimate probability of ruin.

Explain whether the following statements are TRUE or FALSE:

(a) $\Psi(U,1) < \Psi\left(U,1\frac{1}{2}\right)$

(b) $\Psi\left(U,2\frac{1}{2}\right) = \Psi\left(U,3\frac{1}{3}\right)$

(c) $\Psi(U,3) < \Psi(U,4)$

(d) $\Psi(U,4) = \Psi(U)$

[8]

4 Craig and Shivon are playing a two person zero sum game.

Craig picks an integer i from 1 to n, Shivon picks an integer j from 1 to n, and Shivon receives from Craig:

1 if $|i - j| = 1$,
0 otherwise.

(i) Set out the payoff matrix, A, in the case that $n = 7$. [2]

(ii) Show that A can be reduced to a 4×4 matrix by eliminating dominated strategies. [4]

Craig and Shivon adopt randomised strategies.

(iii) Determine the randomised strategy Shivon should adopt, and the value of the game to her. [2]
[Total 8]

5 The table below shows the cost of claims settled per calendar year for a set of car insurance policies, with figures in €000s.

	Development Year		
Accident Year	*0*	*1*	*2*
2014	5,419	908	239
2015	6,234	1,088	
2016	7,719		

The corresponding number of settled claims is as follows:

	Development Year		
Accident Year	*0*	*1*	*2*
2014	760	98	37
2015	819	93	
2016	881		

(i) Calculate the outstanding claims reserve for this portfolio, using the average cost per claim method with grossing up factors. [7]

(ii) State four key assumptions made in part (i). [2]
[Total 9]

6 (i) State the key characteristics of the individual risk model. [2]

(ii) State all possible values for the number of claims that may arise from a given risk over the period being modelled. [1]

(iii) Suggest an example of an insurance contract where the individual risk model may be suitable, and an example where it is unlikely to be. [2]

(iv) Describe the differences between the individual risk model and the collective risk model. [3]

(v) State the additional assumption required such that the individual risk model can be reduced to the collective risk model. [1]

[Total 9]

7 A random variable X follows a Poisson distribution with parameter λ.

(i) Show that the distribution of X is a member of the Exponential family of distributions. [4]

(ii) Show that the mean of X equals the variance of X, using your answer to part (i). [2]

(iii) Describe the three key components required when fitting a Generalised Linear Model (GLM). [3]

[Total 9]

8 (i) Describe the key difference between excess of loss and proportional reinsurance. [1]

A random variable X follows an Exponential distribution.

(ii) Show that $E(X|X > M) = E(X) + M$. [3]

An insurance company writes travel insurance policies, with a premium loading factor of 15%. There are two types of claims, and a maximum of one claim per policy.

Type I claims are for a delay. Claim amounts follow a Uniform distribution with a minimum of $500 and a maximum of $1,500.

Type II claims are for a cancellation. Claim amounts are Exponentially distributed with parameter $\lambda = 0.001$.

5% of policies result in a claim, 85% of which are Type I and 15% of which are Type II.

(iii) Calculate the premium charged for each policy. [2]

The insurer is choosing between two different reinsurance policies:

Policy A: The reinsurer covers 10% of every claim, and uses a premium loading factor of 35%.

Policy B: The reinsurer covers the maximum of {0, claim amount – $1,500}, and uses a premium loading factor of 45%.

(iv) Determine the reinsurance policy the insurance company should purchase, under:

(a) the Bayes criterion.
(b) the minimax criterion. [3]

The insurance company's actuary, Tom, believes the minimax criterion is more relevant in this case.

(v) Suggest a reason for Tom's belief. [2]
[Total 11]

9 Consider a random variable X, with probability density:

$$h(x) = 2(1-x) \qquad 0 < x < 1$$

(i) Construct an algorithm to sample from $h(x)$ using the inverse transform method. [4]

Now consider the probability density function:

$$f(x) = 6x(1-x) \quad 0 < x < 1$$

(ii) Construct an algorithm to sample from $f(x)$ using the acceptance-rejection method and $h(x)$. [4]

(iii) Explain whether the method used in part (ii) would be more efficient when using samples from a standard Uniform distribution $U(0,1)$ instead of $h(x)$. [3]

[Total 11]

10 Let $X_t = a + bt + Y_t$, where Y_t is a stationary time series, and a and b are fixed non-zero constants.

(i) Show that X_t is not stationary. [2]

Let $\Delta X_t = X_t - X_{t-1}$.

(ii) Show that ΔX_t is stationary. [1]

(iii) Determine the autocovariance values of ΔX_t in terms of those of Y_t. [4]

Now assume that Y_t is an MA(1) process, i.e. $Y_t = \varepsilon_t + \beta\varepsilon_{t-1}$

(iv) Set out an equation for ΔX_t in terms of b, β, ε_t and L, the lag operator. [1]

(v) Show that ΔX_t has a variance larger than that of Y_t. [4]

[Total 12]

11 (i) Explain what is meant by a conjugate prior distribution. [1]

The random variables $X_1, X_2, \ldots, X_n$ are independent and have density function:

$$P(X = x) = p(1-p)^x, 0 < p < 1$$

(ii) Show that the conjugate prior for p is a beta distribution. [3]

Assume that we have an independent sample $X_1, X_2, \ldots, X_n$ from a geometric distribution with parameter p, with the prior density function for p given by:

$$f(p) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\alpha-1}(1-p)^{\beta-1}, 0 < p < 1$$

(iii) Show that $E\left(\frac{1-p}{p}\right) = \frac{\beta}{(\alpha-1)}$. [3]

(iv) Show that the posterior mean of this distribution can be expressed as a weighted average of the prior mean and the sample average, including statement of the credibility factor Z. [2]

Every day Amit and Bonnie catch the bus home from work at a bus stop next to their office. Most buses which arrive at the bus stop do not go to their destination. Denote the average number of buses they have to wait for as N such that the $(N+1)^{\text{th}}$ bus to arrive at the bus stop goes to their destination. Amit's prior belief is that $N = 10$. Bonnie's prior belief is that $N = 5$. They both use a beta prior distribution with $\alpha = 5$ but with different β.

(v) Calculate the number of bus trips home required such that the absolute difference in Amit and Bonnie's posterior estimates for N is less than 0.5. [5]

[Total 14]

END OF PAPER