INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

21 April 2016 (am)

Subject CT4 – Models
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 11 questions, beginning your answer to each question on a new page.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Write down the information required to compute the exact exposed to risk in an investigation of mortality. [3]

2 List the advantages and disadvantages of using models in actuarial work. [4]

3 (i) State the principle of correspondence in the context of a mortality investigation. [1]

A mortality investigation collects the following data:

$n_x(t)$ = total number of policies under which death claims are made when the policyholder is aged x last birthday for each calendar year t.

$P_x(t)$ = number of in-force policies where the policyholder was aged x nearest birthday on 1 January in year t.

(ii) (a) Derive an expression, in terms of $P_x(t)$, for the central exposed to risk, E_x^c, corresponding to the claims data which may be used to estimate the force of mortality in year t at each age x, μ_x.

(b) State any assumptions that you make, indicating at which point in your derivation each assumption is relevant. [5]

[Total 6]

4 The manager of a life insurance company wishes to revise the premiums for term assurance policies. He has asked a trainee to compare the latest mortality estimates from the Continuous Mortality Investigation (CMI) for ages 40–64 years inclusive with the estimates the company has been using in its premium calculations, using a 95% significance level.

The trainee says: "I've done the Signs Test and we just pass – one more positive sign and we would have failed!".

(i) Calculate the number of ages for which the company's mortality estimate was higher than that produced by the CMI. [3]

Ten minutes later the trainee says: " I tried the Grouping of Signs test and we just fail. We needed one more positive run!".

(ii) Determine the number of runs of positive signs in the company's data. [3]

[Total 6]

5 (i) Define the following types of stochastic process:

(a) a Poisson process
(b) a compound Poisson process

[3]

Consider the modelling of the following situations:

A the number of claims for motorcycle accidents received by an insurer's telephone claim line

B the number of breakfast bagels sold by a New York bagel bar

C the number of breakdowns of freezers in a large supermarket

D the cost of wasted food caused by breakdowns of freezers in a large supermarket.

(ii) Comment on which of the following stochastic processes will be most suitable for modelling each of the four situations above:

- time-homogeneous Poisson process
- time-inhomogeneous Poisson process
- time-homogeneous compound Poisson process
- time-inhomogeneous compound Poisson process

[6]
[Total 9]

6 An investigation was undertaken into the time spent waiting in check-out queues at a supermarket. A random sample of customers was surveyed, and the times at which they joined the check-out queue and completed their purchases were recorded. If they left the check-out queue without completing a purchase, the time at which they left was also recorded. Below are the data for 12 customers.

Customer number	*Time joined*	*Time purchase completed*	*Time left without making purchase*
1	10.00 a.m.	10.08 a.m.	
2	10.07 a.m.	10.09 a.m.	
3	10.10 a.m.	10.16 a.m.	
4	10.25 a.m.	10.31 a.m.	
5	10.30 a.m.	10.32 a.m.	
6	10.45 a.m.	10.49 a.m.	
7	11.10 a.m.		11.20 a.m.
8	11.15 a.m.	11.21 a.m.	
9	11.35 a.m.		11.40 a.m.
10	11.58 a.m.	12.09 p.m.	
11	12.10 p.m.	12.14 p.m.	
12	12.15 p.m.		12.22 p.m.

(i) Calculate the Kaplan-Meier estimate of the survival function of the duration between joining the queue and completing a purchase. [6]

The supermarket decides to introduce a scheme under which any customer who has to wait at a check-out for more than 10 minutes receives a $2 refund on the cost of their shopping. The supermarket has 20,000 customers per day.

(ii) Give an estimate of the daily cost of the new scheme. [1]

(iii) Comment on the assumptions that you have made in obtaining the estimate in (ii). [2]

[Total 9]

7 (i) State the condition needed for a Markov Jump Process to be time inhomogeneous. [1]

(ii) Describe the principal difficulties in modelling using a Markov Jump Process with time inhomogeneous rates. [2]

A multi-tasking worker at a children's nursery observes whether children are being "Good" or "Naughty" at all times. Her observations suggest that the probability of a child moving between the two states varies with the time, t, since the child arrived at the nursery in the morning. She estimates that the transition rates are:

From Good to Naughty: $0.2 + 0.04t$
From Naughty to Good: $0.4 - 0.04t$

where t is measured in hours from the time the child arrived in the morning, $0 \leq t \leq 8$.

A child is in the "Good" state when he arrives at the nursery at 9 a.m.

(iii) Calculate the probability that the child is Good for all the time up until time t. [3]

(iv) Calculate the time by which there is at least a 50% chance of the child having been Naughty at some point. [2]

Let $P_G(t)$ be the probability that the child is Good at time t.

(v) Derive a differential equation just involving $P_G(t)$ which could be used to determine the probability that the child is Good on leaving the nursery at 5 p.m. [2]

[Total 10]

8 (i) List THREE different methods of graduating crude mortality rates. [2]

(ii) State the advantages of each method listed in part (i). [4]

A life insurance company has graduated its own mortality experience for term assurance business over the past 15 years against a standard table using the following equation:

$$q_x = 0.94q_x^s - 0.0001$$

where q_x^s is the mortality rate from the standard table.

The following is an extract from the data.

Age x	*Exposed to Risk*	*Deaths*	*Graduated Rate*
40	24,584	14	0.000625
41	32,587	32	0.000683
42	15,784	16	0.000748
43	21,336	22	0.000823
44	25,874	24	0.000908
45	21,544	22	0.001005
46	23,967	25	0.001114
47	25,811	30	0.001239
48	26,911	28	0.001378
49	28,445	38	0.001536
50	30,205	45	0.001713

(iii) Carry out a test for overall goodness of fit of the data, using a 95% significance level. [6]

[Total 12]

9 Orange trees are susceptible to the disease Citrus Greening. There is no known cure for this disease and, although trees often survive for some time with the disease, it can ultimately be fatal.

A researcher decides to model the progression of the disease using a time-homogeneous continuous-time Markov model with the following state space:

{Healthy (i.e. not infected with Citrus Greening);
Infected with Citrus Greening;
Dead (caused by Citrus Greening);
Dead (other causes)}.

The researcher chooses to label the transition rate parameters as follows:

- a mortality rate from the Healthy state, μ
- a rate of infection with Citrus Greening, σ
- a total mortality rate from the Infected state, ρ
- a mortality rate caused by Citrus Greening, τ

(i) Draw a transition diagram for the chosen model, including the transition rates. [2]

(ii) Determine Kolmogorov's forward equations governing the transitions, specifying the generator matrix. [3]

Infected trees display clear symptoms of the disease. This has enabled the researcher to record the following data on trees in the area of his study:

Tree-months in Healthy State	1,200
Tree-months in Infected State	600
Total number of deaths of trees	40
Number of deaths of Healthy trees	10
Number of deaths from Citrus Greening	30

(iii) Give the likelihood of these data. [3]

(iv) Derive the maximum likelihood estimator of the mortality rate caused by Citrus Greening, τ. [3]

(v) Estimate τ. [1]

[Total 12]

10 In a small country there are only four authorised insurance companies, *A*, *B*, *C* and *D*. The law in this country requires homeowners to take out buildings insurance from an authorised insurance company. All policies provide cover for a period of one year.

Based on analysis of the compliance database used to check that every home is insured, the probabilities of a homeowner transferring between the four companies at the end of each year are considered to be described by the following transition matrix:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{pmatrix} A & B & C & D \\ 0.5 & 0.2 & 0.2 & 0.1 \\ 0.2 & 0.6 & 0.1 & 0.1 \\ 0.3 & 0.2 & 0.4 & 0.1 \\ 0 & 0.2 & 0.2 & 0.6 \end{pmatrix}$$

Yolanda has just bought her policy from Company *C* for the first time.

(i) Calculate the probability that Yolanda will be covered by Company *C* for at least five years before she changes provider. [2]

Zachary took out a policy with Company *A* in January 2013. Unfortunately, Zachary's house burnt down on 12 March 2015.

(ii) Calculate the probability that Company *A* does NOT cover Zachary's home at the time of the fire. [2]

(iii) Calculate the expected proportions of homeowners who are covered by each insurance company in the long run. [4]

Company *A* makes an offer to buy Company *D*. It bases its purchase price on the assumption that homeowners who would previously have purchased policies from Company *A* or Company *D* would now buy from the combined company, to be called Addda.

(iv) Determine the transition matrix which will apply after the takeover if Company *A*'s assumption about homeowners' behaviour is correct. [2]

(v) Comment on the appropriateness of Company *A*'s assumption. [2]

[Total 12]

11 An energy provider is worried about the number of its customers who transfer to other companies within the first two years of their contract and is trying to direct its advertising towards the most loyal section of the population.

The company has looked at its records over recent years and has fitted a Cox proportional hazards model to those who have transferred within the first two years using the factors which appear to have the most impact on early transfer rates.

The following figures have been derived from the data:

	Factor	***Parameter Estimate***	***Variance***
Gender	Male	–0.25	0.015
	Female	0	0
Volume of energy consumed	High	0.32	0.008
	Low	0	0
Area of Residence	City Centre	0.19	0.012
	City (not centre)	0	0
	Rural	–0.35	0.005

(i) Give the hazard function for this Cox proportional hazard model defining all the terms and covariates. [3]

(ii) State the features of the person to whom the baseline hazard applies. [1]

(iii) Calculate symmetric 95% confidence intervals for the parameters based on the standard errors. [2]

(iv) Test the suggestion that women change energy providers more frequently than men. [3]

There is a 70% probability that a male customer who is a low consumer of energy and lives in a rural area has transferred providers before the end of two years.

(v) Calculate the probability that a male customer who is a high consumer of energy and lives in a city centre remains with the company for at least two years. [3]

(vi) Set out how you would determine whether the effect of any of the factors depends upon any of the other factors. [5]

[Total 17]

END OF PAPER