EXAMINATION

28 April 2009 (am)

Subject CT6 — Statistical Methods
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Describe the essential characteristic of liability insurance. List four distinct examples of types of liability insurance. [4]

2 (i) Express the probability density function of the gamma distribution in the form of a member of the exponential family of distributions. Specify the natural and scale parameters. [3]

(ii) State the corresponding canonical link function for generalised linear modelling if the response variable has a gamma distribution. [1]
[Total 4]

3 An insurer's portfolio consists of three independent policies. Each policy can give rise to at most one claim per month, which occurs with probability θ independently from month to month. The prior distribution of θ is beta with parameters $\alpha = 2$ and $\beta = 4$. A total of 9 claims are observed on this portfolio over a 12 month period.

(i) Derive the posterior distribution of θ. [2]

(ii) Derive the Bayesian estimate of θ under all or nothing loss. [4]
[Total 6]

4 Individual claim amounts on a particular insurance policy can take the values 100, 150 or 200.

There is at most one claim in a year. Annual premiums are 60.

The insurer must choose between three reinsurance arrangements:

A no reinsurance
B individual excess of loss with retention 150 for a premium of 10
C proportional reinsurance of 25% for a premium of 20

(i) Complete the loss table for the insurer. [4]

	Reinsurance		
Claims	*A*	*B*	*C*
0			
100			
150			
200			

(ii) Determine whether any of the reinsurance arrangements is dominated from the viewpoint of the insurer. [2]

(iii) Determine the minimax solution for the insurer. [1]
[Total 7]

5 An insurance portfolio contains policies for three categories of policyholder: A, B and C. The number of claims in a year, N, on an individual policy follows a Poisson distribution with mean λ. Individual claim sizes are assumed to be exponentially distributed with mean 4 and are independent from claim to claim. The distribution of λ, depending on the category of the policyholder, is

Category	*Value of* λ	*Proportion of policyholders*
A	2	20%
B	3	60%
C	4	20%

Denote by S the total amount claimed by a policyholder in one year.

(i) Prove that $E(S) = E[E(S|\lambda)]$. [2]

(ii) Show that $E(S|\lambda) = 4\lambda$ and $\text{Var}(S|\lambda) = 32\lambda$. [2]

(iii) Calculate $E(S)$. [2]

(iv) Calculate $\text{Var}(S)$. [2]
[Total 8]

6 The following information is available for a motor insurance portfolio:

The number of claims settled:

	Development Year		
Accident year	*0*	*1*	*2*
2006	442	151	50
2007	623	111	
2008	681		

The cost of settled claims during each year (in 000's):

	Development Year		
Accident year	*0*	*1*	*2*
2006	6321	1901	701
2007	7012	2237	
2008	7278		

Claims are fully run off after year 2. Calculate the outstanding claims reserve using the average cost per claim method with grossing up factors. Inflation can be ignored.
[10]

7 It is necessary to simulate samples from a distribution with density function $f(x) = 6x(1-x) \quad 0 < x < 1$.

(i) Use the acceptance-rejection technique to construct a complete algorithm for generating samples from f by first generating samples from the distribution with density $h(x) = 2(1-x)$. [5]

(ii) Calculate how many samples from h would on average be needed to generate one realisation from f. [1]

(iii) Explain whether the acceptance-rejection method in (i) would be more efficient if the uniform distribution were to be used instead. [4]

[Total 10]

8 An insurer has an initial surplus of U. Claims up to time t are denoted by $S(t)$. Annual premium income is received continuously at a rate of c per unit time.

(i) Explain what is meant by the insurer's surplus process $U(t)$. [2]

(ii) Define carefully each of the following probabilities:

(a) $\psi(U,t)$

(b) $\psi_h(U,t)$

[2]

(iii) Explain, for each of the following pairs of expressions, whether one of each pair is certainly greater than the other, or whether it is not possible to reach a conclusion.

(a) $\psi(10,2)$ and $\psi(20,1)$

(b) $\psi(10,2)$ and $\psi(5,1)$

(c) $\psi_{0.5}(10,2)$ and $\psi_{0.25}(10,2)$

[6]

[Total 10]

9 Individual claims under a certain type of insurance policy are for either 1 (with probability α) or 2 (with probability $1 - \alpha$).

The insurer is considering entering into an excess of loss reinsurance arrangement with retention $1 + k$ (where $k < 1$). Let X_i denote the amount paid by the insurer (net of reinsurance) on the ith claim.

(i) Calculate and simplify expressions for the mean and variance of X_i. [5]

Now assume that $\alpha = 0.2$. The number of claims in a year follows a Poisson distribution with mean 500. The insurer wishes to set the retention so that the probability that aggregate claims in a year will exceed 700 is less than 1%.

(ii) Show that setting $k = 0.334$ gives the desired result for the insurer. [5]
[Total 10]

10 Let Y_t be a stationary time series with autocovariance function $\gamma_Y(s)$.

(i) Show that the new series $X_t = a + bt + Y_t$ where a and b are fixed non-zero constants, is not stationary. [2]

(ii) Express the autocovariance function of $\Delta X_t = X_t - X_{t-1}$ in terms of $\gamma_Y(s)$ and show that this new series is stationary. [7]

(iii) Show that if Y_t is a moving average process of order 1, then the series ΔX_t is not invertible and has variance larger than that of Y_t. [6]
[Total 15]

11 A motor insurance company operates a No Claims Discount scheme with discount levels 0%, 25% and 50% of the annual premium of 1,000. The probability of having an accident during any year is 0.1 (ignore the possibility of more than one accident in a year). The policyholder moves one level up in the discount scheme (or stays at the 50% level) in the event of a claim free year and moves one level down (or stays at the 0% level) if the claim does not involve a criminal offence. If the claim involves a criminal offence then the policyholder automatically moves to the 0% discount level. One in ten accidents involves a criminal offence. The policyholder makes a claim only if the cost of repairs is higher than the aggregate additional premiums payable in the next two claim-free years.

(i) Calculate, for each level of discount, the cost of a repair below which the policyholder will not claim, distinguishing between claims that involve a criminal offence and claims that do not. [4]

(ii) Calculate the probability of a claim for each level of discount given that an accident has occurred, given that the repair cost following an accident has an exponential distribution with mean 400. [5]

(iii) Calculate the stationary distribution of the proportion of policyholders at each discount level. [7]

[Total 16]

END OF PAPER