INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

18 September 2018 (am)

Subject CT4 – Models
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 12 questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Explain why each of the following matrices is, or is not, a valid transition matrix for a Markov chain.

(a) $\begin{pmatrix} 0 & 0 \\ 0.5 & 0.5 \end{pmatrix}$

(b) $\begin{pmatrix} 0.5 & 0.3 & 0.2 \\ 0.3 & 0.1 & 0.6 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.2 & 0 & 0.8 \\ 0.4 & 0.6 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 0.5 & 0.3 & 0.2 \\ 0.3 & 0.4 & 0.3 \\ 0.4 & -0.1 & 0.7 \end{pmatrix}$

[4]

2 Describe how you would determine the number of degrees of freedom to use in a chi-square test when graduating a set of crude mortality rates.

[4]

3 For each of the following processes:

- simple random walk
- Markov jump process
- compound Poisson process
- Markov chain
- counting process

(a) State whether the state space is discrete, continuous, or can be either.
(b) State whether the time set is discrete, continuous, or can be either.

[5]

4 (i) Suggest three types of information source which could be used in recommending parameters to use in an actuarial model. [3]

(ii) Comment on a practical difficulty which could arise with using each type of information source. [3]

[Total 6]

5 The following data are available for a sample of lives that were alive for at least some time between 1 January 2017 and 31 December 2017:

- date of birth;
- date of entry into observation (if entering after 1 January 2017);
- date of death (if died between 1 January 2017 and 31 December 2017);
- date of exit from observation while still alive (if leaving before 31 December 2017).

(i) Derive a maximum likelihood estimator of the hazard of death which could be used with these data and which uses all the information available on the timing of death. [4]

(ii) Explain how these data can be used to estimate a life table. [3]

[Total 7]

6 A Markov jump process has the following generator matrix:

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{pmatrix} -0.3 & 0.2 & 0.1 \\ 0.1 & -0.5 & 0.4 \\ 0.3 & 0.1 & -0.4 \end{pmatrix}$$

(i) Draw a transition graph for this process. [2]

The process is in state A at time zero.

(ii) Give Kolmogorov's forward equations for $\frac{d}{dt}P_{AA}(t)$, $\frac{d}{dt}P_{AB}(t)$ and $\frac{d}{dt}P_{AC}(t)$. [2]

(iii) Calculate the probability that the process remains in state A throughout the period $t = 0$ to $t = 2$. [2]

(iv) Determine the probability that the third jump of the process is into state C. [3]

[Total 9]

7 (i) Define the following types of censoring in the context of a mortality investigation:

- random censoring;
- right censoring;
- informative censoring.

[3]

Anwen received a bunch of 17 fresh red roses on the evening of her birthday from her boyfriend. She arranged them in a vase and placed them on the table in the garden for all to admire. She needed to do a project for school so decided to use them to conduct an experiment as to how long roses live before they start to wilt. She checked them very often, and noted down the date when any was showing signs of wilting, and immediately removed the wilting rose from the vase. The following shows what she discovered.

Day 2. Very disappointing, already two roses wilting.
Day 3. A neighbour passed with his goat which took a nibble at the bunch, so three damaged, but otherwise fresh, roses had to be removed.
Day 5. One more wilting.
Day 7. Three more wilting.
Day 8. The boy down the road stole a fresh rose to give to his sweetheart.
Day 9. Another one wilting and it is hard to make the remaining ones look good in the vase, so the project is terminated.

(ii) For each of the three types of censoring listed in part (i):

(a) State which roses (if any) experience that censoring.
(b) Explain why those roses (if any) experience that censoring.

[6]
[Total 9]

8 A drug named Nimble is often prescribed to the elderly for periods greater than a year to reduce the pain of arthritis. It has been proven that taking Nimble increases the chance of death by heart disease. One local health authority is looking into whether the increased risk continues once a patient stops taking the drug. It proposes to use a model with five states: (1) Never taken Nimble, (2) Taking Nimble, (3) No longer taking Nimble, (4) Dead through heart disease, (5) Dead through other causes.

(i) Draw a diagram showing the possible transitions between the five states. [2]

Let the transition intensity between state i and state j at time $x + t$ be μ^{ij}_{x+t}. Let the probability that a person in state i at time x will be in state j at time $x + t$ be ${}_tp_x^{ij}$.

(ii) Show from first principles that

$$\frac{d}{dt}\left({}_tp_x^{34}\right) = {}_tp_x^{32}\mu_{x+t}^{24} + {}_tp_x^{33}\mu_{x+t}^{34}$$

[5]

The health authority wishes to calculate death rates aged 50 last birthday from each cause for individuals in states 1, 2 and 3.

(iii) State what data would need to be extracted from the health authority's medical records in order to do this. [3]

[Total 10]

9 (i) Describe why a mortality experience would need to be graduated. [3]

(ii) Describe how smoothness is achieved when using the following graduation methods:

(a) parametric formula;
(b) graphical;
(c) with reference to a standard table.

[3]

An insurance company conducts an investigation into the mortality rates of policyholders who choose to retire at a relatively young age.

The following table shows data from the investigation, together with graduated rates $\overset{o}{q}_x$ which were fitted with reference to standard table rates, $\overset{s}{q}_x$ using a link function $\overset{o}{q}_x = \overset{s}{q}_x$ + constant.

Age x	*Exposed to risk*	*Deaths*	$\overset{o}{q}_x$
55	1,550	15	0.00673
56	2,100	18	0.00689
57	2,300	15	0.00709
58	2,450	21	0.00736
59	2,700	18	0.00770
60	3,250	29	0.00820
61	3,100	25	0.00891
62	3,450	30	0.00978
63	3,600	45	0.01084
64	3,750	41	0.01210

(iii) Test the goodness-of-fit of the graduated rates using a chi-square test. [5]

[Total 11]

10 A religious organisation maintains two lists of members:

- a list of sick members, so that members may pray for them (the Sick List);
- a list of recently deceased members (the Dead List).

Each list is published in a bulletin given to those attending the regular weekly meetings of religious worship. The lists are updated each week half way between the religious worship meetings.

A study was made of the mortality of sick members. A sample of members joining the Sick List in the first quarter of 2016 was followed until they left the list. Those who left the list but who did not move to the Dead List were assumed to have recovered. The study terminated on 31 March 2017.

Below are given some data from the study. 'Week first appeared on Sick List' and 'Week last appeared on Sick List' are measured in weeks from the first week of 2016.

Member number	*Week first appeared on Sick List*	*Week last appeared on Sick List*	*Outcome*
1	1	1	Assumed recovered
2	1	3	Moved to Dead List
3	3	4	Moved to Dead List
4	3	65	Still on Sick List 31 March 2017
5	6	17	Moved to Dead List
6	7	14	Assumed recovered
7	9	11	Assumed recovered
8	10	60	Moved to Dead List
9	11	11	Moved to Dead List
10	12	65	Still on Sick List 31 March 2017

(i) Calculate the central death rate per week of these members using the exact exposed to risk. [4]

(ii) Determine the probability of survival for 52 weeks using your result from part (i). [1]

One member of the organisation has been studying statistics and recommends using the Nelson-Aalen estimator to calculate the probability of survival for 52 weeks.

(iii) Calculate the Nelson-Aalen estimate of $S(52)$. [5]

(iv) Comment on your results in parts (ii) and (iii). [2]

[Total 12]

11 Man Life is an insurance company which only sells life insurance to males. It has recently bought another smaller company called Mixed Life which sells business to both males and females. The company is reviewing the premium rates it charges for life insurance.

Man Life has records of the number of policies in force at their year end, which is 30 September, recorded by age last birthday. Mixed Life has records of the number of policies in force on 31 December each year recorded by age last birthday for males and age nearest birthday for females.

These are the data for the most recent years.

Man Life

Age last birthday	*Number of policies 30 Sept. 2015*	*Number of policies 30 Sept. 2016*	*Number of policies 30 Sept. 2017*
49	4,789	4,296	4,367
50	4,953	5,009	4,809
51	5,300	5,186	5,902

Mixed Life Males

Age last birthday	*Number of policies 31 Dec. 2015*	*Number of policies 31 Dec. 2016*	*Number of policies 31 Dec. 2017*
49	1,832	1,650	1,698
50	1,800	1,750	1,550
51	1,966	1,756	1,569

Mixed Life Females

Age nearest birthday	*Number of policies 31 Dec. 2015*	*Number of policies 31 Dec. 2016*	*Number of policies 31 Dec. 2017*
49	1,602	1,568	1,639
50	1,506	1,497	1,508
51	1,610	1,587	1,411

(i) Calculate the central exposed to risk of the combined portfolio for males aged 50 last birthday for the calendar year 2016, stating each assumption you make at the point where you make it. [5]

(ii) Calculate the central exposed to risk of the combined male and female portfolio for persons aged 50 last birthday for the calendar year 2016, stating each assumption you make at the point where you make it. [3]

Legislation has been brought in which means that males and females must be charged the same premium rates for life insurance. The company is considering basing its future premium rates on the number of deaths across the whole male and female portfolio of the two companies at each age divided by the exposed to risk across the combined male and female portfolio at each age.

(iii) Discuss the appropriateness of the company's approach to determining its future premium rates. [3]

[Total 11]

12 A small town is served by a single funeral director. The funeral director collects corpses immediately following death and stores them in a refrigerator pending embalming. The number of deaths per day in this town has the following probability distribution:

Number of deaths per day	*Probability*
0	0.497
1	0.348
2	0.122
3	0.028
4	0.005

The embalmer can embalm exactly one corpse per day. He works on a corpse from the refrigerator if there is one, but if the refrigerator is empty he works on the first corpse to arrive that day. Corpses are removed from the refrigerator immediately before being embalmed and are not returned there after embalming.

The refrigerator has room for four corpses. If more space is needed, the funeral director has to ask the local hospital if there is spare capacity in the hospital's refrigerator.

(i) Determine the transition matrix for the number of corpses in the funeral director's refrigerator. [3]

(ii) Calculate the long-run probability of there being 0, 1, 2, 3 and 4 corpses in the refrigerator. [5]

(iii) Calculate the probability that the funeral director has to contact the hospital on any given day. [2]

The embalmer has not had a day off for years. The funeral director says that from now on the embalmer must not work on Christmas Day.

(iv) Calculate the probability that the funeral director will need to contact the hospital on Christmas Day when the embalmer is not working. [2]

[Total 12]

END OF PAPER