INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

21 September 2018 (am)

Subject CT6 – Statistical Methods
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all nine questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

 © Institute and Faculty of Actuaries

1 An actuary is simulating claim sizes, X, for a particular insurance policy. X follows an exponential distribution with varying parameter λ, where λ can take one of three possible values.

The table shows the distribution function of this external factor, and its impact on λ.

Probability	0.3	0.3	0.4
λ	1	2	3

Set out an algorithm to generate samples from X, using the inverse transform method. [5]

2 (i) State the three main components of a generalised linear model. [3]

Consider the discrete random variable Y, with the following probability density function

$$f(y,\mu) = \binom{n}{ny}\mu^{ny}(1-\mu)^{n-ny} \qquad y = 0, \frac{1}{n}, \frac{2}{n}, \ldots, 1$$

(ii) Show that Y belongs to the exponential family of distributions, specifying each component. [4]

(iii) State the canonical link function in this case. [1]
[Total 8]

3 (i) State the fundamental difference between Bayesian estimation and Classical estimation. [2]

(ii) State three different loss functions which may be used under Bayesian estimation, indicating for each its link to the posterior distribution. [3]

The proportion, θ, of the population of a particular country who use online banking is being estimated. Of a sample of 500 people, 326 do use online banking.

An actuary is estimating θ using a suitable uniform distribution as a prior.

(iii) (a) Determine the posterior distribution of θ.

(b) Calculate an estimate of θ using the loss function that minimises the mean of the posterior distribution. [4]
[Total 9]

4 An insurance company has a portfolio of policies, where claim amounts follow a Pareto distribution with parameters $\alpha = 3$ and $\lambda = 100$. The insurance company has entered into an excess of loss reinsurance agreement with a retention of M, such that 90% of claims are still paid in full by the insurer.

(i) Calculate M. [4]

(ii) Calculate the average claim amount paid by the reinsurer, on claims which involve the reinsurer. [6]
[Total 10]

5 The cumulative claim amounts incurred on a portfolio of motor insurance policies are as follows:

Accident Year	*Development Year*			
	0	1	2	3
2014	3,907	5,606	6,061	6,145
2015	4,831	7,319	7,470	
2016	6,042	8,282		
2017	7,061			

The cumulative number of reported claims are as follows

Accident Year	*Development Year*			
	0	1	2	3
2014	435	469	528	534
2015	485	525	541	
2016	509	558		
2017	544			

(i) Estimate the ultimate number of claims, for each accident year, using the chain-ladder technique. [4]

(ii) Estimate the ultimate average incurred cost per claim, for each accident year, using the grossing-up method. [5]

(iii) Calculate the total reserve required, using the results from (i) and (ii), assuming that claims paid to date are 19,544. [2]
[Total 11]

6 In a two-player zero sum game, the matrix below shows the value to Player 1. Player 1's strategies are labelled I to VI, where Player 2's strategies are labelled A to F.

	A	B	C	D	E	F
I	13	29	8	12	16	23
II	18	22	21	22	29	31
III	18	22	31	31	27	37
IV	11	22	12	21	21	26
V	18	16	19	14	19	28
VI	23	22	19	23	30	34

(i) Show, by eliminating dominated strategies, that the game can be reduced to the following 3 x 3 matrix. [3]

	a	b	c
α	13	29	8
β	18	22	31
γ	23	22	19

(ii) Explain whether or not this new 3 x 3 matrix has any saddle points. [2]

Now consider a randomised strategy for Player 2, denoted X, whereby strategy 'a' is chosen with probability p and strategy 'c' is chosen with probability $1 - p$, $0 < p < 1$.

(iii) Find the range of values for p such that X dominates strategy 'b'. [3]

(iv) Solve the game and determine the value to Player 1 given that 'b' is dominated. [3]

[Total 11]

7 Claims on a portfolio of insurance policies arise as a Poisson process with parameter λ. Individual claim amounts are taken from a distribution X and we define $m_i = E(X^i)$ for $i = 1, 2, \ldots$. The insurance company calculates premiums using a premium loading of θ.

(i) Define the adjustment coefficient R. [1]

(ii) Show that R can be approximated as $\dfrac{2\theta m_1}{m_2}$, by truncating the series expansion of $M_X(t)$. [3]

Now suppose that X follows an exponential distribution with parameter γ.

(iii) Show that $R = \dfrac{\theta\gamma}{(1+\theta)}$. [3]

The insurance company uses a premium loading of 12%, and the mean claim amount is 200.

(iv) Calculate R, commenting on the difference with the approximation to R shown in part (ii). [3]

The initial surplus is 5,000.

(v) Calculate an upper bound for the ultimate probability of ruin. [1]

(vi) Suggest two methods by which the insurance company can reduce the probability of ruin. [2]

[Total 13]

8 For a portfolio of insurance policies, claims X_i are independent and follow a gamma distribution, with parameters $\alpha = 6$ and β, which is unknown.

A random sample of n claims, $X_1,\ldots, X_n$ is selected, with mean $\overline{X}$.

(i) Derive an expression for the estimator of β using the method of moments. [2]

(ii) Explain what the Maximum Likelihood Estimator (MLE) of β represents. [2]

(iii) Derive an expression for the MLE of β, commenting on the result. [5]

(iv) State the Moment Generating Function (MGF) of X. [1]

Let $Y = 2n\beta\overline{X}$.

(v) Derive the MGF of Y, and hence its distribution, including statement of parameters. [5]

[Total 15]

9 An actuary is modelling a set of data which consists of 100 consecutive observations, $y_1, y_2, \ldots, y_{100}$. The data has the following statistics:

$$\bar{y} = \frac{1}{100}\sum_{i=1}^{100} y_i = A = 10.5$$

$$\sum_{i=1}^{100} \left(y_i - \bar{y}\right)^2 = B = 290$$

$$\sum_{i=2}^{100} \left(y_{i-1} - \bar{y}\right)\left(y_i - \bar{y}\right) = C = 60$$

$$\sum_{i=3}^{100} \left(y_{i-2} - \bar{y}\right)\left(y_i - \bar{y}\right) = D = -240$$

(i) Calculate the values of the sample auto-correlations r_1 and r_2. [3]

(ii) Calculate the first two sample partial auto-correlation values $\hat{\phi}_1$ and $\hat{\phi}_2$. [2]

The actuary is considering two different models for this data:

Model X: $y_t = a_0 + a_1 y_{t-1} + \varepsilon_t$

Model Y: $y_t = b_0 + b_1 y_{t-1} + b_2 y_{t-2} + \varepsilon_t$

where ε_t is a standard white-noise process, with variance σ^2.

(iii) Estimate the parameters (including σ^2) for both Models X and Y, using the method of moments. [10]

(iv) Explain whether each of Models X and Y satisfy the Markov property. [3]

[Total 18]

END OF PAPER