INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

6 October 2016 (am)

Subject CT4 – Models
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 12 questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

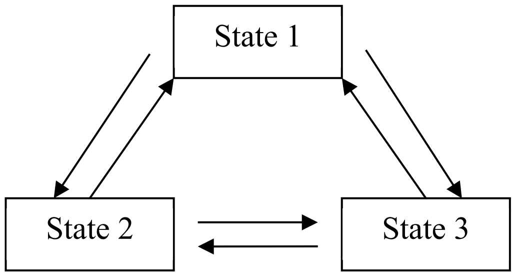
Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

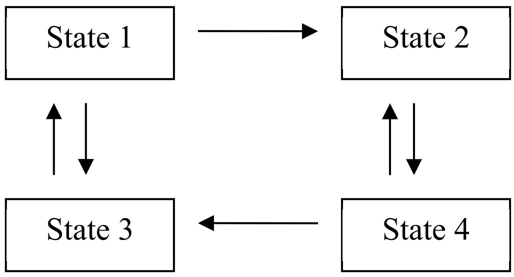
1 State THREE advantages of the two-state model over the Binomial model for estimating transition intensities where exact dates of entry into and exit from observation are known. [3]

2 The diagrams below show three Markov chains, where arrows indicate a non-zero transition probability.

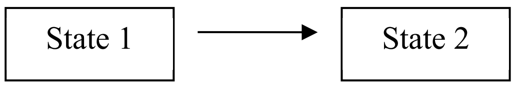
A Markov Chain 1



B Markov Chain 2



C Markov Chain 3



State whether each of the chains is:

- irreducible.
- periodic, giving the period.

[3]

3 Describe the similarities and differences between the following processes:

- Markov Chain
- Markov Jump Chain
- Markov Jump Process

[4]

4 An insurance company's business consists only of policies covering a specified event and which pay a sum assured of £Z immediately on occurrence of this event. Claims on the portfolio of policies are considered to occur in accordance with a Poisson process with annual rate λ.

The insurance company currently has assets of £S (£Z > £S). It charges a premium which is to be 50% more than the expected outgo. Premiums can be assumed to be received continuously. The insurance company's expenses are small and can be ignored.

(i) Derive the total annual premium charged by the insurance company on the portfolio. [1]

(ii) Show that the probability that the insurance company has insufficient assets to pay the next claim made is given by:

$$1-\exp\left[-\frac{1}{1.5}\left(1-\frac{S}{Z}\right)\right]$$ [3]

[Total 4]

5 (i) Describe why the raw data gathered from a mortality investigation need to be graduated. [3]

(ii) Explain which method of graduation would be most suitable for each of the following mortality investigations:

(a) the female population of a large European country

(b) a study of the mortality of rhinoceroses in the safari parks of South Africa

(c) the pension scheme of a large company [3]

[Total 6]

6 Brian worked in a large open-plan office with a communal kitchen in which the workers made coffee. Each worker supplied his or her own coffee cup. For several years Brian was annoyed by his coffee cups being taken away by colleagues and never returned to the kitchen, so he decided to do an experiment. He brought into the kitchen 20 cups which were distinguishable from the other cups in the kitchen. At the end of each day for 15 days he counted the number of his 20 cups which remained. The results were as follows:

Day	*Number of cups*	*Day*	*Number of cups*
1	20	9	15
2	19	10	15
3	18	11	15
4	18	12	15
5	17	13	13
6	17	14	12
7	17	15	10
8	16		

Brian noted that:

- the cup that "disappeared" during day 2 was taken home by Brian to be used by his mother.
- the two cups that "disappeared" during day 13 were accidentally broken by Brian when doing his daily check.

Let $h(x)$ be the hazard that each of Brian's cups is taken by colleagues during day x and not returned, and let $S(x)$ be the corresponding survival function.

(i) Determine an estimate of $S(x)$ for Brian's cups using the Nelson-Aalen estimator. [6]

(ii) Sketch a chart for your estimated $S(x)$. [2]
[Total 8]

7 (i) List EIGHT factors which should be considered when deciding whether a model is suitable for a particular purpose. [4]

A colleague has been asked to present a model which might be used to determine the number of new schools required throughout a country over the next 40 years. He forgot all about it until the last minute when he was reading an article in a newspaper about immigration and education which provided some figures to back up the article. Your colleague has the following suggestion for a model:

- Start with the number of children in the education system over the last twenty years (as provided by the country's central statistical office). Project these forward using a straight line approach.

- Use the number of immigrants predicted to arrive in each of the next five years as given in the newspaper article. Apply to this an estimate of "number and age of children for each immigrant" also provided by the newspaper. Project this forward also using a straight line approach.

- Add the two together to get the total number of children in the education system for the next 40 years.

(ii) Assess whether this model is suitable with regards to SIX of the factors which you listed in your answer to part (i). [6]

[Total 10]

8 An analysis of the number of term assurance policies in force for three companies has revealed the following information:

	Year	*Company A*	*Company B*	*Company C*
	2013	6,728	2,643	4,132
Age 50	2014	6,189	2,548	
	2015	5,962	2,496	4,630
	2013	5,987	2,333	4,012
Age 51	2014	6,002	2,417	
	2015	5,056	2,213	4,500
	2013	5,359	2,155	3,895
Age 52	2014	5,600	1,992	
	2015	4,906	2,006	4,367

- Company *A* has reported the number of policies in force on 1 January each year using age nearest birthday.
- Company *B* has reported the number of policies in force on 1 November each year using age last birthday.
- Company *C* has reported the number of policies in force on 31 December each year using age next birthday, but failed to provide data for 2014.

(i) Calculate the contribution to the central exposed to risk for lives age 51 last birthday for the calendar year 2014 for each company individually. [5]

(ii) (a) State the assumptions you have made in order to perform your calculations.

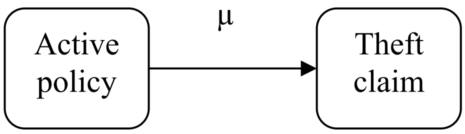
(b) Explain why these assumptions were required. [6]

[Total 11]

9 (i) Describe how transition rates can be estimated under multiple state models with constant transition rates, including a statement of the data required. [3]

A specialist insurance policy provides cover only for the theft of valuable items (such as jewellery) stored in safety deposit boxes in a bank vault. The premium for cover for an item worth £C is paid in advance. If a claim is made, the cover ceases.

Claims are modelled using a two state model as follows, where μ is a constant transition rate:



(ii) Give Kolmogorov's forward equations for this process. [1]

(iii) Determine the expected cost of claims incurred by time T.
[2]

If the item is no longer stored in the safety deposit box (for example, if the item is sold) then the insurance cover lapses. The transition rate for lapses of such policies is a constant λ.

(iv) Draw a transition diagram for a revised process allowing for lapses. [2]

(v) Derive the revised expected cost of claims incurred by time T. [3]
[Total 11]

10 A researcher is investigating the contributing factors to the speed at which patients recover from a common minor surgical procedure undertaken in hospitals across the country. He has the questionnaires which each patient completed before the surgery and the length of time the patient remained in hospital after surgery and is attempting to fit a Cox proportional hazards model to the data.

He has fitted a model with what he assumes are the most common contributing factors and has calculated the parameters as shown in the table below:

Covariate	*Category*	*Parameter*
Gender	Male	0
	Female	0.065
Smoker	Non Smoker	–0.035
	Smoker	0
Drinker	Non Drinker	–0.06
	Moderate Drinker	0
	Heavy Drinker	0.085

(i) Give the hazard function for this Cox proportional hazards model, defining all the terms and covariates. [4]

A male moderate drinker who does not smoke has a hazard of leaving hospital after three days of 0.6.

(ii) Calculate the probability that a female heavy drinker who smokes and who is still in hospital after three days is NOT discharged at that point. [3]

A colleague suggests that, in his experience, gender has no material impact on the length of time in hospital after surgery.

(iii) Explain how the researcher could test this suggestion statistically. [2]

Another colleague suggests that the original model is good, but could be improved by including an additional factor as to whether a patient is married or not.

(iv) Set out how the researcher could establish whether an additional factor representing marital status would improve the model. [4]

[Total 13]

11 An individual's marginal tax rate depends upon his or her total income during a calendar year and may be 0% (that is, he or she is a non-taxpayer), 20% or 40%.

The movement in the individual's marginal tax rate from year to year is believed to follow a Markov Chain with a transition matrix as follows:

$$\begin{array}{r} 0\% \\ 20\% \\ 40\% \end{array}\begin{pmatrix} 1-\beta-\beta^2 & \beta & \beta^2 \\ \beta & 1-3\beta & 2\beta \\ \beta^2 & \beta & 1-\beta-\beta^2 \end{pmatrix}$$

(i) Draw the transition diagram of the process, including the transition rates. [2]

(ii) Determine the range of values of β for which this is a valid transition matrix. [3]

(iii) Explain whether the chain is:

(a) irreducible.
(b) periodic.

including whether this depends on the value of β. [2]

The value of β has been estimated as 0.1.

(iv) Calculate the long term proportion of taxpayers at each marginal rate. [4]

Lucy pays tax at a marginal rate of 20% in 2011.

(v) Calculate the probabilities that Lucy's marginal tax rate in 2013 is:

(a) 0%.
(b) 20%.
(c) 40%.

[2]
[Total 13]

12 A large life insurance company is conducting an investigation into the mortality of its policyholders to see if this has changed since the previous investigation ten years ago. Below is a sample of the results:

	Current investigation		*Previous investigation*
Age	*Exposed to risk*	*Observed deaths*	*mortality rates*
55	5,842	150	0.0267
56	5,630	132	0.0278
57	4,281	126	0.0301
58	3,955	98	0.0325
59	3,879	142	0.0356
60	3,550	149	0.0387
61	4,006	162	0.0396
62	4,150	173	0.0410
63	3,520	158	0.0433
64	3,057	150	0.0458
65	3,666	200	0.0490

(i) Explain how many degrees of freedom would be used to conduct a chi-squared test for goodness of fit on these data. [2]

(ii) Carry out a chi-squared test on these data. [5]

(iii) Perform a test to determine whether the shape of the mortality rates has changed over the age range. [3]

(iv) Comment on your results to parts (ii) and (iii). [2]

(v) Suggest reasons why the mortality experience may have changed over the past ten years. [2]

[Total 14]

END OF PAPER