INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

17 April 2012 (am)

Subject CT6 – Statistical Methods
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 (i) Define what it means for a random variable to belong to an exponential family. [1]

(ii) Show that if a random variable has the exponential distribution it belongs to an exponential family. [3]

[Total 4]

2 A statistician is told that one of two dice has been chosen and rolled, and he is told the result of the roll. One dice is a conventional dice, but the other has three sides numbered 2 and three sides numbered 4. If the statistician correctly identifies the dice he wins a prize of 1.

(i) Determine the total number of decision functions available to the statistician. [2]

(ii) (a) Identify the most natural candidate for the decision function.

(b) Calculate the expected payoff for this function assuming that each of the two dice are equally likely to be chosen. [3]

[Total 5]

3 Claim amounts on a certain type of insurance policy follow a distribution with density

$$f(x) = 3cx^2 e^{-cx^3} \text{ for } x > 0$$

where c is an unknown positive constant. The insurer has in place individual excess of loss reinsurance with an excess of 50. The following ten payments are made by the insurer:

Losses below the retention: 23, 37, 41, 11, 19, 33
Losses above the retention: 50, 50, 50, 50

Calculate the maximum likelihood estimate of c. [6]

4 Claims on a particular type of insurance policy follow a compound Poisson process with annual claim rate per policy 0.2. Individual claim amounts are exponentially distributed with mean 100. In addition, for a given claim there is a probability of 30% that an extra claim handling expense of 30 is incurred (independently of the claim size). The insurer charges an annual premium of 35 per policy.

Use a normal approximation to estimate how many policies the insurer must sell so that the insurer has a 95% probability of making a profit on the portfolio in the year. [6]

5 The total claim amount per annum on a particular insurance policy follows a normal distribution with unknown mean θ and variance 200^2. Prior beliefs about θ are described by a normal distribution with mean 600 and variance 50^2. Claim amounts $x_1, x_2, \ldots, x_n$ are observed over n years.

(i) State the posterior distribution of θ. [2]

(ii) Show that the mean of the posterior distribution of θ can be written in the form of a credibility estimate. [3]

Now suppose that n=5 and that total claims over the five years were 3,400.

(iii) Calculate the posterior probability that θ is greater than 600. [2]
[Total 7]

6 A proportion p of packets of a rather dull breakfast cereal contain an exciting toy (independently from packet to packet). An actuary has been persuaded by his children to begin buying packets of this cereal. His prior beliefs about p before opening any packets are given by a uniform distribution on the interval [0,1]. It turns out the first toy is found in the $n_1 th$ packet of cereal.

(i) Specify the posterior distribution of p after the first toy is found. [3]

A further toy was found after opening another n_2 packets, another toy after opening another n_3 packets and so on until the fifth toy was found after opening a grand total of $n_1 + n_2 + n_3 + n_4 + n_5$ packets.

(ii) Specify the posterior distribution of p after the fifth toy is found. [2]

(iii) Show the Bayes' estimate of p under quadratic loss is not the same as the maximum likelihood estimate and comment on this result. [5]
[Total 10]

7 The numbers of claims on three different classes of insurance policies over the last four years are given in the table below.

	Year 1	*Year 2*	*Year 3*	*Year 4*	*Total*
Class 1	1	4	5	0	10
Class 2	1	6	4	6	17
Class 3	5	6	4	9	24

The number of claims in a given year from a particular class is assumed to follow a Poisson distribution.

(i) Determine the maximum likelihood estimate of the Poisson parameter for each class of policy based on the data above. [5]

(ii) Perform a test on the scaled deviance to check whether there is evidence that the classes of policy have different mean claim rates and state your conclusion. [5]

[Total 10]

8 The table below shows claims paid on a portfolio of general insurance policies. You may assume that claims are fully run off after three years.

Underwriting year	*Development Year*			
	0	*1*	*2*	*3*
2008	450	312	117	41
2009	503	389	162	
2010	611	438		
2011	555			

Past claims inflation has been 5% p.a. However, it is expected that future claims inflation will be 10% p.a.

Use the inflation adjusted chain ladder method to calculate the outstanding claims on the portfolio. [10]

9 Consider the time series model

$$(1-\alpha B)^3 X_t = e_t$$

where B is the backwards shift operator and e_t is a white noise process with variance σ^2.

(i) Determine for which values of α the process is stationary. [2]

Now assume that $\alpha = 0.4$.

(ii) (a) Write down the Yule-Walker equations.

(b) Calculate the first two values of the auto-correlation function ρ_1 and ρ_2. [7]

(iii) Describe the behaviour of ρ_k and the partial autocorrelation function ϕ_k as $k \to \infty$. [3]

[Total 12]

10 Let X_1 and X_2 be random variables with moment generating functions $M_{X_1}(t)$ and $M_{X_2}(t)$ respectively. A new random variable Y is formed by choosing a sample from X_1 with probability p or a sample from X_2 with probability $1-p$.

(i) Show that the moment generating function of Y is given by

$$M_Y(t) = pM_{X_1}(t) + (1-p)M_{X_2}(t)$$ [2]

A portfolio of insurance policies consists of two different types of policy. Claims on type 1 policies arrive according to a Poisson process with parameter λ_1 and claim amounts have a distribution X_1. Claims on type 2 policies arrive according to a Poisson process with parameter λ_2 and claim amounts have a distribution X_2.

(ii) Show that aggregate claims on the whole portfolio follow a compound Poisson distribution, specifying the claim rate and the claim size distribution. [6]

Now suppose that $\lambda_1 = 10$ and $\lambda_2 = 15$ and that the claim sizes are exponentially distributed with mean 50 for type 1 policies and mean 70 for type 2 policies.

(iii) Construct an algorithm for simulating total claims on the whole portfolio. [6]

[Total 14]

11 Claims on a portfolio of insurance policies arrive as a Poisson process with parameter 100. Individual claim amounts follow a normal distribution with mean 30 and variance 5^2. The insurer calculates premiums using a premium loading of 20% and has initial surplus of 100.

(i) Define carefully the ruin probabilities $\psi(100)$, $\psi(100,1)$ and $\psi_1(100,1)$. [3]

(ii) Define the adjustment coefficient *R*. [1]

(iii) Show that for this portfolio the value of *R* is 0.011 correct to 3 decimal places. [5]

(iv) Calculate an upper bound for $\psi\left(100\right)$ and an estimate of $\psi_1(100,1)$. [5]

(v) Comment on the results in (iv). [2]
[Total 16]

END OF PAPER