EXAMINATION

8 April 2008 (am)

Subject CT6 — Statistical Methods
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Give two examples of the main types of liability insurance, stating for each example a typical insured peril. [4]

2 A claim amount distribution is normal with unknown mean μ and known standard deviation £50. Based on past experience a suitable prior distribution for μ is normal with mean £300 and standard deviation £20.

(i) Calculate the prior probability that μ, the mean of the claim amount distribution, is less than £270. [1]

(ii) A random sample of 10 current claims has a mean of £270.

(a) Determine the posterior distribution of μ.

(b) Calculate the posterior probability that μ is less than £270 and comment on your answer.

[5]
[Total 6]

3 (i) X and Y are independent Poisson random variables with mean λ. Derive the moment generating function of X, and hence show that $X + Y$ also has a Poisson distribution. [4]

(ii) An insurer has a portfolio of 100 policies. Annual premiums of 0.2 units per policy are payable annually in advance. Claims, which are paid at the end of the year, are for a fixed sum of 1 unit per claim. Annual claims numbers on each policy are Poisson distributed with mean 0.18.

Calculate how much initial capital is needed in order to ensure that the probability of ruin at the end of the year is less than 1%. [4]
[Total 8]

4 $Y_1, Y_2, \ldots, Y_n$ are independent observations from a normal distribution with $E[Y_i] = \mu_i$ and $\text{Var}[Y_i] = \sigma^2$.

(i) Write the density of Y_i in the form of an exponential family of distributions. [2]

(ii) Identify the natural parameter and derive the variance function. [3]

(iii) Show that the Pearson residual is the same as the deviance residual. [4]
[Total 9]

5 The following table shows the claim payments for an insurance company in units of £5,000:

Accident year	*Development year* 0	*1*	*2*	*3*
2004	410	814	216	79
2005	575	940	281	
2006	814	1066		
2007	1142			

The inflation for a 12 month period to the middle of each year is given as follows:

2005	*2006*	*2007*
5%	5.5%	5.4%

The future inflation from 2007 is estimated to be 8% per annum.

Claims are fully run-off at the end of the development year 3.

Calculate the amount of outstanding claims arising from accidents in year 2007, using the inflation adjusted chain ladder method.

[9]

6 A portfolio of general insurance policies is made up of two types of policies. The policies are assumed to be independent, and claims are assumed to occur according to a Poisson process. The claim severities are assumed to have exponential distributions.

For the first type of policy, a total of 65 claims are expected each year and the expected size of each claim is £1,200.

For the second type of policy, a total of 20 claims are expected each year and the expected size of each claim is £4,500.

(i) Calculate the mean and variance of the total cost of annual claims, S, arising from this portfolio. [3]

The risk premium loading is denoted by θ, so that the annual premium on each policy is $(1+\theta)\times$expected annual claims on each policy. The initial reserve is denoted by u.

A normal approximation is used for the distribution of S, and the initial reserve is set by ensuring that

$$P(S < u + \text{annual premium income}) = 0.975.$$

(ii) (a) Derive an equation for u in terms of θ.

(b) Determine the annual premium required in order that no initial reserve is necessary. [7]

[Total 10]

7 Consider the following model applied to some quarterly data:

$$Y_t = e_t + \beta_1 e_{t-1} + \beta_4 e_{t-4} + \beta_1\beta_4 e_{t-5}$$

where e_t is a white noise process with mean zero and variance σ^2.

(i) Express in terms of β_1 and β_4 the roots of the characteristic polynomial of the MA part, and give conditions for invertibility of the model. [2]

(ii) Derive the autocorrelation function (ACF) for Y_t. [5]

For our particular data the sample ACF is:

Lag	*ACF*
1	0.73
2	0.14
3	0.37
4	0.59
5	0.24
6	0.12
7	0.07

(iii) Explain whether these results confirm the initial belief that the model could be appropriate for these data. [3]

[Total 10]

8 The NCD scale policy for an insurance company is:

Level 0	0%
Level 1	25%
Level 2	50%

The premium at the Level 0 is £800. The probability that a policyholder has an accident in a year is 0.2, and it is assumed that a policyholder does not have more than one accident each year.

In the event of a claim free year the policyholder moves to the next higher level of discount in the coming year or remains at Level 2.

In the event of a claim the policyholder moves to the next lower level of discount in the coming year or remains at Level 0.

Following an accident the policyholder decides whether or not to make a claim based on the claim size and the amount of premiums over the period of the next 2 policy years, assuming no more claims are made.

(i) For each discount level, find the minimum claim amount for which the policyholder will make a claim. [2]

(ii) Assuming that the cost of repair for each accident has an exponential distribution with mean £600, calculate the probability that a policyholder makes a claim at each level of discount. [5]

(iii) Write down the transition matrix and calculate the average premium payment for a year when the system has reached the equilibrium. [6]

[Total 13]

9 (i) Describe the difference between *strictly* stationary processes and *weakly* stationary processes. [2]

(ii) Explain why weakly stationary multivariate normal processes are also strictly stationary. [1]

(iii) Show that the following bivariate time series process, $(X_n, Y_n)^t$, is weakly stationary:

$$X_n = 0.5X_{n-1} + 0.3Y_{n-1} + e_n^x$$
$$Y_n = 0.1X_{n-1} + 0.8Y_{n-1} + e_n^y$$

where e_n^x and e_n^x are two independent white noise processes. [5]

(iv) Determine the positive values of c for which the process

$$X_n = (0.5 + c)\, X_{n-1} + 0.3Y_{n-1} + e_n^x$$
$$Y_n = 0.1X_{n-1} + (0.8 + c)\, Y_{n-1} + e_n^y$$

is stationary. [6]

[Total 14]

10 A bicycle wheel manufacturer claims that its products are virtually indestructible in accidents and therefore offers a guarantee to purchasers of pairs of its wheels. There are 250 bicycles covered, each of which has a probability p of being involved in an accident (independently). Despite the manufacturer's publicity, if a bicycle is involved in an accident, there is in fact a probability of 0.1 for each wheel (independently) that the wheel will need to be replaced at a cost of £100. Let S denote the total cost of replacement wheels in a year.

(i) Show that the moment generating function of S is given by

$$M_S(t) = \left[\frac{pe^{200t} + 18pe^{100t} + 81p}{100} + 1 - p\right]^{250}.$$ [4]

(ii) Show that $E(S) = 5{,}000p$ and $Var\left(S\right) = 550{,}000p - 100{,}000p^2$ [6]

Suppose instead that the manufacturer models the cost of replacement wheels as a random variable T based on a portfolio of 500 wheels, each of which (independently) has a probability of 0.1p of requiring replacement.

(iii) Derive expressions for $E(T)$ and $\text{Var}(T)$ in terms of p. [2]

(iv) Suppose $p = 0.05$.

(a) Calculate the mean and variance of S and T.
(b) Calculate the probabilities that S and T exceed £500.
(b) Comment on the differences. [5]

[Total 17]

END OF PAPER