EXAMINATION

9 April 2008 (am)

Subject CT4 — Models
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 List four factors in respect of which life insurance mortality statistics are often subdivided. [2]

2 Describe how smoothness is ensured when mortality rates are graduated using each of the following methods:

(a) fitting a parametric formula
(b) graphical graduation

[3]

3 (i) Define the following stochastic processes:

(a) Poisson process
(b) compound Poisson process

[4]

(ii) Identify the circumstances in which a compound Poisson process is also a Poisson process. [1]

[Total 5]

4 Describe the benefits and limitations of modelling in actuarial work. [6]

5 A survey of first marriage patterns among women in a remote population in central Asia collected the following data for a sample of women:

- calendar year of birth
- calendar year of first marriage

Data are also available about the population of never-married women on 1 January each year, classified by age last birthday.

You have been asked to estimate the intensity, λ_x, of first marriage for women aged x.

(i) State the rate interval implied by the first marriages data. [1]

(ii) Derive an appropriate exposed to risk which corresponds to the first marriages data. State any assumptions that you make. [4]

(iii) Explain to what age x your estimate of λ_x applies. State any assumptions that you make. [2]

[Total 7]

6 An investigation was carried out into mortality rates among a certain class of female pensioners. Crude mortality rates were estimated by single years of age from ages 65–89 years last birthday inclusive. The investigators decided to ask an actuary to compare the crude rates with a standard table. They calculated the relevant standardised deviations, printed them out and sent them to the actuary.

Unfortunately, because of a printing error, the right-hand edge of the document containing the standardised deviations failed to print properly. The actuary was unable to read the magnitude of the standardised deviations. However, the sign of each deviation was clear. This revealed that the crude mortality rates were higher than the standard table rates at ages 65–72 years and 75–84 years inclusive, but that the crude mortality rates were lower than the standard table rates at ages 73–74 years and 85–89 years inclusive.

The null hypothesis to be tested is that the crude mortality rates come from a population with underlying mortality consistent with that in the standard table.

(i) List two statistical tests of the null hypothesis which the actuary could carry out on the basis of the information received. [1]

(ii) Carry out both tests. For each test, state what feature of the experience it is specifically testing, and give your conclusion. [10]

[Total 11]

7 In a certain small country all listed companies are required to have their accounts audited on an annual basis by one of the three authorised audit firms (A, B and C). The terms of engagement of each of the audit firms require that a minimum of two annual audits must be conducted by the newly appointed firm. Whenever a company is able to choose to change auditors, the likelihood that it will retain its auditors for a further year is (80%, 70%, 90%) where the current auditor is (A,B,C) respectively. If changing auditors a company is equally likely to choose either of the alternative firms.

(i) A company has just changed auditors to firm A. Calculate the expected number of audits which will be undertaken before the company changes auditors again. [2]

(ii) Formulate a Markov chain which can be used to model the audit firm used by a company, specifying:

(a) the state space
(b) the transition matrix

[4]

(iii) Calculate the expected proportion of companies using each audit firm in the long term. [5]

[Total 11]

8 An education authority provides children with musical instrument tuition. The authority is concerned about the number of children giving up playing their instrument and is testing a new tuition method with a proportion of the children which it hopes will improve persistency rates. Data have been collected and a Cox proportional hazards model has been fitted for the hazard of giving up playing the instrument. Symmetric 95% confidence intervals (based upon standard errors) for the regression parameters are shown below.

Covariate	*Confidence Interval*
Instrument	
Piano	0
Violin	[-0.05,0.19]
Trumpet	[0.07,0.21]
Tuition method	
Traditional	0
New	[-0.15,0.05]
Sex	
Male	[-0.08,0.12]
Female	0

(i) Write down a general expression for the Cox proportional hazards model, defining all terms that you use. [3]

(ii) State the regression parameters for the fitted model. [2]

(iii) Describe the class of children to which the baseline hazard applies. [1]

(iv) Discuss the suggestion that the new tuition method has improved the chances of children continuing to play their instrument. [3]

(v) Calculate, using the results from the model, the probability that a boy will still be playing the piano after 4 years if provided with the new tuition method, given that the probability that a girl will still be playing the trumpet after 4 years following the traditional method is 0.7. [3]

[Total 12]

9 An investigation into the mortality of patients following a specific type of major operation was undertaken. A sample of 10 patients was followed from the date of the operation until either they died, or they left the hospital where the operation was carried out, or a period of 30 days had elapsed (whichever of these events occurred first). The data on the 10 patients are given in the table below.

Patient number	*Duration of observation (days)*	*Reason for observation ceasing*
1	2	Died
2	6	Died
3	12	Died
4	20	Left hospital
5	24	Left hospital
6	27	Died
7	30	Study ended
8	30	Study ended
9	30	Study ended
10	30	Study ended

(i) State whether the following types of censoring are present in this investigation. In each case give a reason for your answer.

(a) Type I
(b) Type II
(c) Random [3]

(ii) State, with a reason, whether the censoring in this investigation is likely to be informative. [1]

(iii) Calculate the value of the Kaplan-Meier estimate of the survival function at duration 28 days. [5]

(iv) Write down the Kaplan-Meier estimate of the hazard of death at duration 8 days. [1]

(v) Sketch the Kaplan-Meier estimate of the survival function. [2]

[Total 12]

10 An internet service provider (ISP) is modelling the capacity requirements for its network. It assumes that if a customer is not currently connected to the internet ("offline") the probability of connecting in the short time interval $[t,dt]$ is $0.2dt + o(dt)$. If the customer is connected to the internet ("online") then it assumes the probability of disconnecting in the time interval is given by $0.8dt + o(dt)$.

The probabilities that the customer is online and offline at time t are $P_{ON}(t)$ and $P_{OFF}(t)$ respectively.

(i) Explain why the status of an individual customer can be considered as a Markov Jump Process. [2]

(ii) Write down Kolmogorov's forward equation for $P'_{OFF}(t)$. [2]

(iii) Solve the equation in part (ii) to obtain a formula for the probability that a customer is offline at time t, given that they were offline at time 0. [3]

(iv) Calculate the expected proportion of time spent online over the period $[0,t]$. [HINT: Consider the expected value of an indicator function which takes the value 1 if offline and 0 otherwise.] [4]

(v) (a) Sketch a graph of your answer to (iv) above.
(b) Explain its shape. [3]

[Total 14]

11 An investigation was carried out into the relationship between sickness and mortality in an historical population of working class men. The investigation used a three-state model with the states:

1 Healthy
2 Sick
3 Dead

Let the probability that a person in state i at time x will be in state j at time $x+t$ be ${}_tp_x^{ij}$. Let the transition intensity at time $x+t$ between any two states i and j be μ_{x+t}^{ij}.

(i) Draw a diagram showing the three states and the possible transitions between them. [2]

(ii) Show from first principles that

$$\frac{\partial}{\partial t}\,{}_tp_x^{23} = {}_tp_x^{21}\mu_{x+t}^{13} + {}_tp_x^{22}\mu_{x+t}^{23}\,. \qquad [5]$$

(iii) Write down the likelihood of the data in the investigation in terms of the transition rates and the waiting times in the Healthy and Sick states, under the assumption that the transition rates are constant. [3]

The investigation collected the following data:

- man-years in Healthy state 265
- man-years in Sick state 140
- number of transitions from Healthy to Sick 20
- number of transitions from Sick to Dead 40

(iv) Derive the maximum likelihood estimator of the transition rate from Sick to Dead. [3]

(v) Hence estimate:

(a) the value of the constant transition rate from Sick to Dead
(b) 95 per cent confidence intervals around this transition rate

[4]

[Total 17]

END OF PAPER