INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

1 October 2013 (pm)

Subject CT6 – Statistical Methods
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 An insurance company has a portfolio of n policies. The probability of a claim in a given year on each policy is p independently from policy to policy, and the possibility of more than one claim can be ignored. Prior beliefs about p are specified by a Beta distribution with parameters α and β. In one year the insurance company has a total of k claims on the portfolio.

Calculate the posterior estimate of p under all or nothing loss and show that it can be written in the form of a credibility estimate. [5]

[You may use without proof the fact that the mode of a Beta distribution with parameters α and β is $\dfrac{\alpha-1}{\alpha+\beta-2}$.]

2 Claim amounts on a certain type of insurance policy follow an exponential distribution with mean 100. The insurance company purchases a special type of reinsurance policy so that for a given claim X the reinsurance company pays

0	if $0 < X < 80$;
$0.5X - 40$	if $80 \le X < 160$;
$X - 120$	if $X \ge 160$.

Calculate the expected amount paid by the reinsurance company on a randomly chosen claim. [6]

3 Andy is a famous weight lifter who will be competing at the Olympic Games. He has taken out special insurance which pays out if he is injured. If the injury is so serious that his career is ended the policy pays \$1m and is terminated. If he is injured but recovers the insurance payment is \$0.1m and the policy continues.

The insurance company's underwriters believe that the probability of an injury in any year is 0.2, and that the probability of more than one injury in a year can be ignored. If Andy is injured, there is a 75% chance that he will recover.

Annual premiums are paid in advance, and the insurance company pays claims at the end of the year. Assume that this is the only policy that the insurance company writes, and that it has an initial surplus of \$0.1m.

(i) Define what is meant by $\psi(\$0.1\text{m},1)$ and $\psi(\$0.1\text{m})$. [2]

(ii) Calculate the annual premium charged assuming the insurance company uses a premium loading of 30%. [2]

(iii) Determine $\psi(\$0.1\text{m},2)$. [4]

[Total 8]

4 The table below shows the probability distribution of a discrete random variable X.

Value	1	2	3
Probability	0.3	0.3	0.4

(i) Construct an algorithm to generate random samples from X. [2]

The random variable Z takes values from X with probability 0.2 and values from an exponential distribution Y with probability 0.8. The upper quartile point of the distribution of Y is 2.5.

(ii) Calculate the expected value of Y. [3]

(iii) Extend the algorithm in part (i) to generate random samples from Z. [4]
[Total 9]

5 An insurance company has a portfolio of life insurance policies for 2,000 workers at a factory. The policies pay out £5,000 if a worker dies in an industrial accident and £2,000 if a worker dies for any other reason. For each worker, the probability of death in any year is 0.02 and 25% of deaths are the result of industrial accidents. The insurance company charges an annual premium of £74.25 per worker.

(i) Calculate the premium loading used by the insurance company. [2]

The insurance company is considering adopting one of the following three approaches to reinsurance:

A None.

B 30% proportional reinsurance at a cost of £27 per worker.

C Individual excess of loss reinsurance with retention £3,000 and a premium of £15 per worker.

(ii) Find the optimal decision under the Bayes criterion. [4]

(iii) Find the optimal decision under the minimax criterion. [2]

(iv) Comment on your answer to part (iii). [2]
[Total 10]

6 The tables below show cumulative data for the number of claims and the total claim amounts arising from a portfolio of insurance policies.

Claim Numbers

	Development Year		
	0	*1*	*2*
2010	87	132	151
2011	117	156	
2012	99		

Total Claim Amounts

	Development Year		
	0	*1*	*2*
2010	43,290	87,430	126,310
2011	68,900	125,290	
2012	74,250		

Claims are fully run off after two development years.

Estimate the outstanding claims using the average cost per claim method with grossing up factors. [10]

7 An insurance company offers dental insurance to the employees of a small firm. The annual number of claims follows a Poisson process with rate 20. Individual loss amounts follow an exponential distribution with mean 100. In order to increase the take-up rate, the insurance company has guaranteed to pay a minimum amount of £50 per qualifying claim. Let S be the total claim amount on the portfolio for a given year.

(i) Show that the mean and variance of S are 2,213.06 and 413,918.40 respectively. [7]

[You may use without proof the result that if $I_n = \int_M^\infty y^n \lambda e^{-\lambda y} dy$

then $I_n = M^n e^{-\lambda M} + \frac{n}{\lambda} I_{n-1}$]

(ii) (a) Fit a log-normal distribution for S using the method of moments.

(b) Estimate the probability that S is greater than 4,000.

[3]

Sarah, the insurance company's actuary, has instead approximated S by a Normal distribution.

(iii) Explain, without performing any further calculations, whether the probability that she calculates that S exceeds 4,000 will be greater or smaller than the calculation in part (ii). [2]

[Total 12]

8 The number of claims per month Y arising on a certain portfolio of insurance policies is to be modelled using a modified geometric distribution with probability density given by

$$p(y|\alpha) = \frac{\alpha^{y-1}}{(1+\alpha)^{y}} \quad y = 1, 2, 3, \ldots$$

where α is an unknown positive parameter. The most recent four months have resulted in claim numbers of 8, 6, 10 and 9.

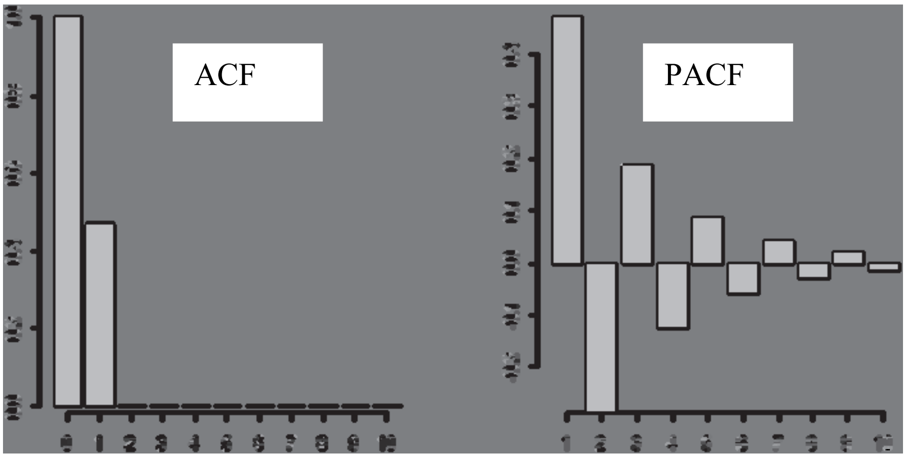
(i) Derive the maximum likelihood estimate of α. [5]

(ii) Show that Y belongs to an exponential family of distributions and suggest its natural parameter. [5]

[Total 10]

9 (i) State the three main stages in the Box-Jenkins approach to fitting an ARIMA time series model. [3]

(ii) Explain, with reasons, which ARIMA time series would fit the observed data in the charts below. [2]



Now consider the time series model given by

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \beta_1 e_{t-1} + e_t$$

where e_t is a white noise process with variance σ^2.

(iii) Derive the Yule-Walker equations for this model. [6]

(iv) Explain whether the partial auto-correlation function for this model can ever give a zero value. [2]

[Total 13]

10 The number of service requests received by an IT engineer on any given day follows a Poisson distribution with mean μ. Prior beliefs about μ follow a gamma distribution with parameters α and λ. Over a period of n days the actual numbers of service requests received are $x_1, x_2, \ldots, x_n$.

(i) Derive the posterior distribution of μ. [3]

(ii) Show that the Bayes estimate of μ under quadratic loss can be written as a credibility estimate and state the credibility factor. [2]

Now suppose that $\alpha = 10$, $\lambda = 2$ and that the IT worker receives 42 requests in 6 days.

(iii) Calculate the Bayes estimate of μ under quadratic loss. [1]

Three quarters of requests can be resolved by telling users to restart their machine, and the time taken to do so follows a Pareto distribution with density

$$f(x) = \frac{3 \times 20^3}{(20+x)^4} \quad \text{for } x>0.$$

One quarter of requests are much harder to resolve, and the time taken to resolve these follows a Weibull distribution with density

$$f(x) = 0.4 \times 0.5 x^{-0.5} e^{-0.4 x^{0.5}} \quad \text{for } x>0.$$

(iv) (a) Calculate the probability that a randomly chosen request takes more than 30 minutes to resolve.

(b) Calculate the average time spent on each request.

(c) Calculate the expected total amount of time the IT worker spends dealing with service requests each day, using the estimate of μ from part (iii).

[5]

The IT worker's line manager is carefully considering his staffing requirements. He decides to model the time taken on each request approximately using an exponential distribution.

(v) (a) Fit an exponential distribution to the time taken per request using the method of moments.

(b) Calculate the probability that a randomly chosen request takes more than 30 minutes to resolve using this approximation.

(c) Comment briefly on your answer to part (v)(b). [2]

The IT engineer needs to devote more of his time to a separate project, so his firm have hired an assistant to help him. The assistant is just as fast at dealing with the straightforward requests, and the time taken to resolve these still follows the Pareto distribution given above. He is significantly slower at dealing with the difficult requests, and the time taken to resolve these now follows a Weibull distribution with density:

$$f(x) = c \times 0.5x^{-0.5}e^{-cx^{0.5}} \quad \text{for } x>0$$

where c is a positive parameter. The line manager again fits an exponential distribution as an approximation to the time taken to service each request using the method of moments. His approximation results in an estimate that the probability that a random service request takes longer than 30 minutes to resolve is 10%.

(vi) Determine the value of c. [4]

[Total 17]

END OF PAPER