INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

29 September 2016 (am)

Subject CT6 – Statistical Methods
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 10 questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 An actuary is considering a portfolio of insurance policies. This portfolio only contains five policies. Each policy has been taken out by a different couple who are getting married on the same day in the same city. Each policy would pay out in the event of rain on that day in that city.

(i) Explain why these policies may not meet the ideal criteria for an insurable risk. [3]

(ii) Suggest two ways in which either the portfolio or the policy terms could be changed in order to better meet the criteria for insurable risks. [2]
[Total 5]

2 Andy is playing a game, which involves rolling four-sided fair dice. Each time a dice is rolled, it is equally likely to show one of the numbers: 1, 2, 3 or 4.

Before each roll, he has three strategies:

a_1: Receive 1.5 times the number showing.

a_2: Receive half the number showing if it is odd, and twice the number if it is even.

a_3: Receive the number showing if it is even, and twice the number if it is odd.

(i) Construct Andy's payoff matrix. [2]
(ii) State which, if any, of the decision functions are dominated. [1]
(iii) Determine Andy's optimal strategy under the Bayes criterion. [3]
[Total 6]

3 The table below shows aggregate annual claim statistics for four risks over a period of six years. Annual aggregate claims for risk i in year j are denoted by X_{ij}.

$Risk, i$	$\overline{X_i} = \frac{1}{6}\sum_{j=1}^{6} X_{ij}$	$S_i^2 = \frac{1}{5}\sum_{j=1}^{6}\left(X_{ij} - \overline{X_i}\right)^2$
$i=1$	46.8	1227.4
$i=2$	30.2	1161.4
$i=3$	74.5	1340.3
$i=4$	60.7	1414.7

(i) Calculate the credibility premium of each risk under the assumptions of Empirical Bayes Credibility Theory (EBCT) Model 1. [7]

(ii) Comment on why the credibility factor is relatively low in this case. [2]
[Total 9]

4 An insurance company has three types of policyholders: Standard, Premium and Elite. If a Standard policyholder does not make a claim in a given year, they move to being a Premium policyholder the following year. If a Standard policyholder makes a claim in a given year they stay as a Standard policyholder for the following year.

If a Premium policyholder makes a claim in a given year, they move to being a Standard policyholder for the following year. If a Premium policyholder does not make a claim in a given year, they move to being an Elite policyholder the following year.

Similarly if an Elite policyholder makes a claim in a given year, they move to being a Premium policyholder for the following year. If an Elite policyholder does not make a claim in a given year, they stay as an Elite policyholder for the following year.

You may assume the probability of more than one claim in a given year is negligible.

The transition probability matrix for the change in the policyholder type for each year is:

		Type next year	
	Standard	*Premium*	*Elite*
Current type			
Standard	80%	20%	0%
Premium	50%	0%	50%
Elite	0%	90%	10%

(i) Set out three algorithms (one for each possible initial policyholder type) which simulate the policyholder type for each of the next three years, using the inverse transform method. [5]

Assume the starting policyholder type is Standard and that the random numbers drawn from a Uniform [0,1] distribution for the first simulation are 0.89, 0.64 and 0.12.

(ii) Determine how the policyholder type evolves over a three year period in this simulation, including statement of how many claims are simulated to have occurred. [4]
[Total 9]

5 (i) (a) Explain what is meant by a sequence of independent, identically distributed (I.I.D.) random variables.

(b) Give one example of a sequence of I.I.D. random variables. [3]

Claim amounts X_i from a portfolio of insurance policies are assumed to be I.I.D. and exponentially distributed, with parameter λ. In a given year there are n claims.

(ii) Show that the total claim amounts follow a gamma distribution, specifying its parameters. [2]

In practice the individual claim amounts are not I.I.D. but instead the exponential parameter λ_i varies between each claim. λ_i follows a gamma distribution with parameters α and β.

(iii) Show that the marginal distribution of claim amounts follows a Pareto distribution with parameters α and β. [5]
[Total 10]

6 Assume that the numbers of accidents for three different risks in five years are as follows:

	Year 1	*Year 2*	*Year 3*	*Year 4*	*Year 5*	***Total***
Risk A	1	4	5	0	2	**12**
Risk B	1	6	4	6	5	**22**
Risk C	5	6	4	9	4	**28**

An actuary is modelling each risk according to a Poisson distribution.

(i) Determine the Poisson parameter for each risk using the method of maximum likelihood estimation. [5]

(ii) Test the hypothesis that the three risks have the same claim rate, using the scaled deviances. [5]
[Total 10]

7 Claim amounts, X, arising from a portfolio of insurance policies follow a Pareto distribution, with parameters α and λ. The insurance company has bought excess of loss reinsurance cover, with retention $M > 0$.

The reinsurer only has a record of claims greater than M. Consider the truncated distribution of claim amounts, $Z = X - M \mid X > M$.

(i) Show that Z also follows a Pareto distribution, but with parameters α and $\lambda + M$. [4]

Claim amounts, X', have now increased by a factor k, such that a claim incurred is k times an equivalent claim previously incurred, and k is greater than 1. The retention level M is unchanged.

(ii) Show that the distribution of X' still follows a Pareto distribution, and determine its parameters. [4]

The truncated distribution of claim amounts is now Z', where $Z' = X' - M \mid X' > M$.

(iii) State the distribution of Z', using the results from parts (i) and (ii), including statement of parameters. [1]

(iv) Comment on whether or not the average claim amount retained by the insurance company has increased by a factor of k. [2]

[Total 11]

8 The table below shows incremental claim amounts paid on a portfolio of general insurance policies, where claims are assumed to fully run off after three years.

Underwriting Year	*Development Year* 0	1	2	3
2012	504	286	110	35
2013	621	302	120	
2014	685	340		
2015	801			

Past and projected future inflation is given by the following index (measured to the mid-point of the relevant year).

Year	*Index*
2012	100
2013	103
2014	105
2015	106
2016	105
2017	107
2018	110

Estimate the outstanding claims reserve using the inflation-adjusted chain ladder technique. [12]

9 In order to model the seasonality of a particular data set an actuary is asked to consider the following model:

$$(1-B^{12})(1-(\alpha+\beta)B+\alpha\beta B^2)X_t = \varepsilon_t$$

where B is the backshift operator and ε_t is a white noise process with variance σ^2.

The actuary intends to apply a seasonal difference $\nabla_s X_t = Y_t$.

(i) Explain why s should be 12 in this case (i.e. $Y_t = X_t - X_{t-12}$). [1]

(ii) Determine the range of values for α and β for which the process will be stationary after applying this seasonal difference. [3]

Assume that after the appropriate seasonal differencing the following sample autocorrelation values for observations of Y_t are $\hat{\rho}_1 = 0$ and $\hat{\rho}_2 = 0.09$.

(iii) Estimate the parameters α and β. [5]

The actuary observes a sequence of observations $x_1, x_2, \ldots, x_T$ of X_t, with $T > 12$.

(iv) Derive the next two forecasted values for next two observations $\hat{x}_{T+1}$ and $\hat{x}_{T+2}$, as a function of the existing observations. [4]

[Total 13]

10 Claims on portfolio of insurance policies arise as a Poisson process with parameter $\lambda = 125$. Individual claim amounts, X_i follow a gamma distribution with parameters $\alpha = 20$ and $\beta = 0.5$.

The insurance company calculates premiums using a premium loading factor of 15% and has an initial surplus of 300.

(i) Define the adjustment coefficient R. [1]

(ii) Show that for this portfolio the value of R is 0.00648 correct to three significant figures. [5]

(iii) (a) Calculate an upper bound for $\Psi(300)$.
(b) Calculate an estimate of $\Psi_1(300,1)$, using a Normal approximation. [5]

The parameter β now reduces to 0.4.

(iv) Explain what would happen to the estimate of $\Psi_1(300,1)$, without carrying out any further calculations. [2]

(v) Propose two ways in which the insurance company could reduce $\Psi_1(300,1)$. [2]

[Total 15]

END OF PAPER