EXAMINATION

30 September 2010 (am)

Subject CT6 — Statistical Methods
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 An actuary is using a simulation technique to estimate the probability θ that a claim on an insurance policy exceeds a given amount. The actuary has carried out 50 simulations and has produced an estimate that $\hat{\theta} = 0.47$. The variance of the simulated values is 0.15.

Calculate the minimum number of simulations that the actuary will have to perform in order to estimate θ to within 0.01 with 95% confidence. [4]

2 Claims on a portfolio of insurance policies follow a compound Poisson process with annual claim rate λ. Individual claim amounts are independent and follow an exponential distribution with mean μ. Premiums are received continuously and are set using a premium loading of θ. The insurer's initial surplus is U.

Derive an expression for the adjustment coefficient, R, for this portfolio in terms of μ and θ. [4]

3 An underwriter has suggested that losses on a certain class of policies follow a Weibull distribution. She estimates that the 10th percentile loss is 20 and the 90th percentile loss is 95.

(i) Calculate the parameters of the Weibull distribution that fit these percentiles. [3]

(ii) Calculate the 99.5th percentile loss. [2]
[Total 5]

4 An office worker receives a random number of e-mails each day. The number of emails per day follows a Poisson distribution with unknown mean μ. Prior beliefs about μ are specified by a gamma distribution with mean 50 and standard deviation 15. The worker receives a total of 630 e-mails over a period of ten days.

Calculate the Bayesian estimate of μ under all or nothing loss. [7]

5 The table below shows aggregate annual claim statistics for three risks over a period of seven years. Annual aggregate claims for risk i in year j are denoted by X_{ij}.

Risk, i	$\bar{X}_i = \frac{1}{7}\sum_{j=1}^{7} X_{ij}$	$S_i^2 = \frac{1}{6}\sum_{j=1}^{7}\left(X_{ij} - \bar{X}_i\right)^2$
i = 1	127.9	335.1
i = 2	88.9	65.1
i = 3	149.7	33.9

(i) Calculate the credibility premium of each risk under the assumptions of EBCT Model 1. [6]

(ii) Explain why the credibility factor is relatively high in this case. [2]
[Total 8]

6 The probability density function of a gamma distribution is given in the following parameterised form:

$$f(x) = \frac{\alpha^{\alpha}}{\mu^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x\alpha}{\mu}} \quad \text{for } x > 0.$$

(i) Express this density in the form of a member of the exponential family, specifying all the parameters. [6]

(ii) Hence show that the mean and variance of the distribution are given by μ and $\frac{\mu^2}{\alpha}$ respectively. [3]
[Total 9]

7 An insurance company has a portfolio of policies under which individual loss amounts follow an exponential distribution with mean $1/\lambda$. There is an individual excess of loss reinsurance arrangement in place with retention level 100. In one year, the insurer observes:

- 85 claims for amounts below 100 with mean claim amount 42; and
- 39 claims for amounts above the retention level.

(i) Calculate the maximum likelihood estimate of λ. [5]

(ii) Show that the estimate of λ produced by applying the method of moments to the distribution of amounts paid by the insurer is 0.011164. [5]
[Total 10]

8 Claims on a portfolio of insurance policies arrive as a Poisson process with rate λ. The claim sizes are independent identically distributed random variables $X_1, X_2, \ldots$with:

$$P(X_i = k) = p_k \text{ for } k = 1, 2, \ldots, M \text{ and } \sum_{k=1}^{M} p_k = 1.$$

The premium loading factor is θ.

(i) Show that the adjustment coefficient R satisfies:

$$\frac{1}{M}\log(1+\theta) < R < \frac{2\theta m_1}{m_2}$$

where $m_i = E(X_1^i)$ for $i = 1, 2$. [7]

[The inequality $e^{Rx} \le \frac{x}{M}e^{RM} + 1 - \frac{x}{M}$ for $0 \le x \le M$ may be used without proof.]

(ii) (a) Determine upper and lower bounds for R if $\theta = 0.3$ and X_i is equally likely to be 2 or 3 (and cannot take any other values).

(b) Hence derive an upper bound on the probability of ruin when the initial surplus is U. [3]

[Total 10]

9 An actuarial student has been working on some claims projections but some of her workings have been lost. The cumulative claim amounts and projected ultimate claims are given by the following table:

Accident Year	*Development Year* 0	1	2	3	*Ultimate*
1	1001	1485	1762	W	X
2	1250	Y	1820		1862.3
3	1302	1805			2122.5
4	Z				2278.8

All claims are paid by the end of development year 3.

It is known that ultimate claims for accident years 2 and 3 have been estimated using the Basic Chain Ladder method.

(i) Calculate the values of W, X and Y. [5]

For accident year 4 the student has used the Bornhuetter-Ferguson method using an earned premium of 2,500 and an expected loss ratio of 90%.

(ii) Calculate the value of Z. [4]

(iii) Calculate the outstanding claims reserve for all accident years implied by the completed table. [1]

[Total 10]

10 An insurance company has a portfolio of 10,000 policies covering buildings against the risk of flood damage.

(i) State the conditions under which the annual number of claims on the portfolio can be modelled by a binomial distribution $B(n, p)$ with $n = 10{,}000$. [3]

These conditions are satisfied and $p = 0.03$. Individual claim amounts follow a normal distribution with mean 400 and standard deviation 50. The insurer wishes to take out proportional reinsurance with the retention α set such that the probability of aggregate payments on the portfolio after reinsurance exceeding 120,000 is 1%.

(ii) Calculate α assuming that aggregate annual claims can be approximated by a normal distribution. [7]

This reinsurance arrangement is set up with a reinsurer who uses a premium loading of 15%.

(iii) Calculate the annual premium charged by the reinsurer. [2]

As an alternative, the reinsurer has offered an individual excess of loss reinsurance arrangement with a retention of M for the same annual premium. The reinsurer uses the same 15% loading to calculate premiums for this arrangement.

(iv) Show that the retention M is approximately 358.50. [4]

[You may wish to use the following formula which is given on page 18 of the Tables:

If $f(x)$ is the PDF of the $N(\mu, \sigma^2)$ distribution then

$$\int_L^U xf(x)dx = \mu\left[\Phi(U') - \Phi(L')\right] - \sigma\left[\varphi(U') - \varphi(L')\right]$$

where $L' = \dfrac{L-\mu}{\sigma}$ and $U' = \dfrac{U-\mu}{\sigma}$.

Here $\Phi(z)$ is the cumulative density function of the $N(0, 1)$ distribution and

$\varphi(z) = \dfrac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}$.] [Total 16]

11 A time series model is specified by

$$Y_t = 2\alpha Y_{t-1} - \alpha^2 Y_{t-2} + e_t$$

where e_t is a white noise process with variance σ^2.

(i) Determine the values of α for which the process is stationary. [2]

(ii) Derive the auto-covariances γ_0 and γ_1 for this process and find a general recursive expression for γ_k for $k \geq 2$. [10]

(iii) Show that the auto-covariance function can be written in the form:

$$\gamma_k = A\alpha^k + kB\alpha^k$$

for some values of A, B which you should specify in terms of the constants α and σ^2. [5]

[Total 17]

END OF PAPER