INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

29 September 2017 (pm)

Subject CT4 – Models
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 10 questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

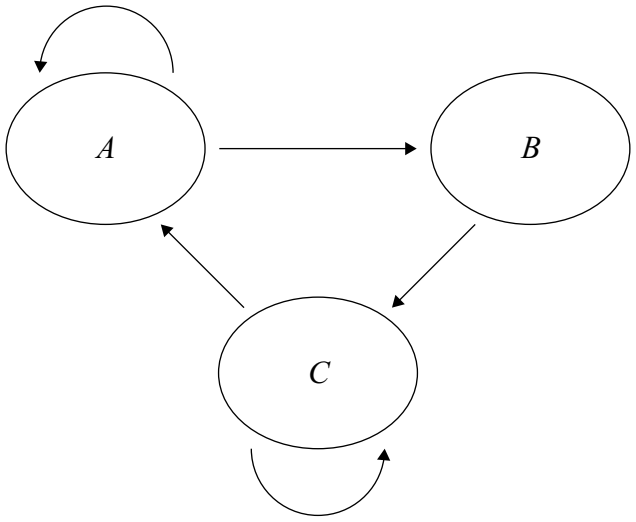
Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 A Markov Chain has the following transition graph:



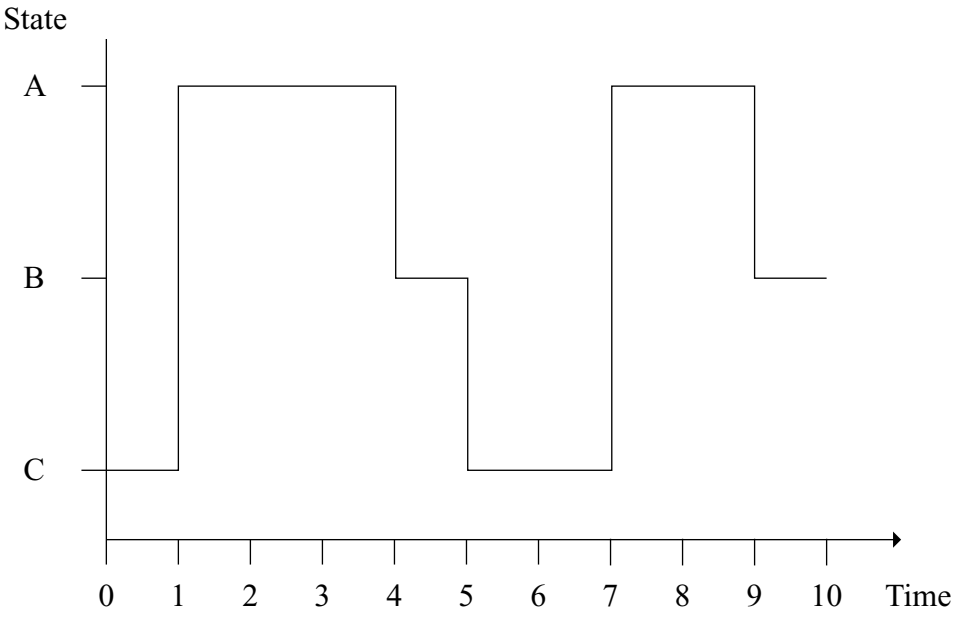
The following is a partially completed transition matrix for this Markov Chain:

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} \quad & \quad & \quad \end{array} \\ \begin{pmatrix} 0.2 & – & – \\ – & – & 1.0 \\ – & – & 0.4 \end{pmatrix} \end{array}$$

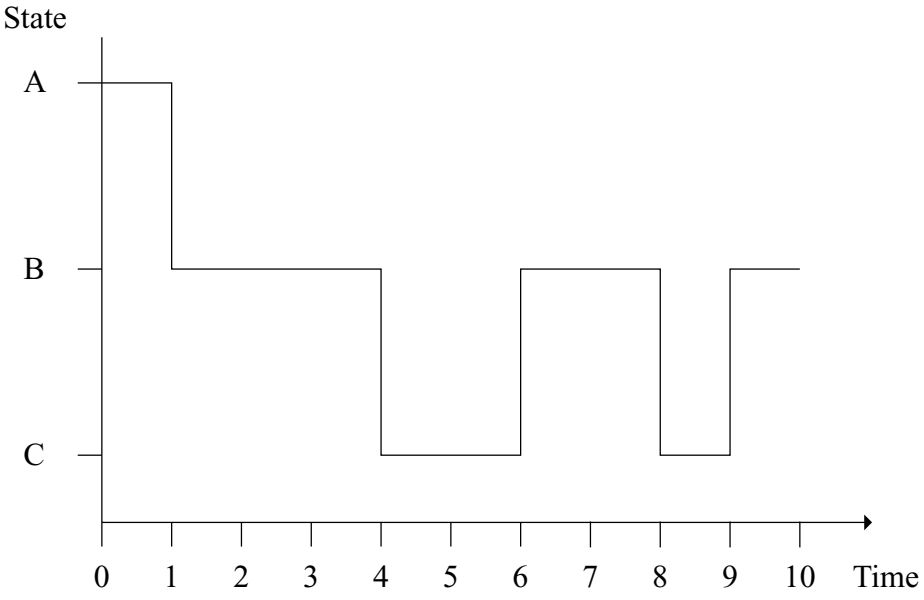
(i) Determine the remaining entries in the transition matrix. [2]

(ii) Explain whether each of the following is a valid sample path for this process.

(a) Path 1:



(b) Path 2:



[2]

[Total 4]

2 For each of the following processes:

- General Random Walk
- Markov Jump Process
- Compound Poisson Process
- Markov Chain

(a) State whether the state space is discrete, continuous or can be either.

(b) State whether the time set is discrete, continuous, or can be either.

[4]

3 Calls arrive on Fred's desk phone according to a Poisson Process with parameter 3, with time measured in hours.

(i) Write down the expected number of phone calls Fred receives each hour. [1]

Fred has not received a phone call for 15 minutes.

(ii) Give the expected time until Fred next receives a phone call. [1]

Fred goes into a meeting for half an hour.

(iii) Determine the probability that Fred has NOT missed a call when he returns to his desk. [1]

The average length of a call to Fred is 7 minutes.

(iv) Determine the probability that if a caller phones Fred the line will be engaged, assuming that Fred is at his desk to receive calls. [2]

[Total 5]

4 A study was conducted into the mortality of persons aged between exact ages 85 and 86 years. The study took place from 1 April 2015 to 31 March 2016. The following table shows information on 10 lives observed in the study.

Life number	*Date of 85th birthday*	*Date of death*
1	1 August 2014	–
2	1 November 2014	–
3	1 January 2015	1 February 2016
4	1 February 2015	–
5	1 March 2015	–
6	1 April 2015	1 January 2016
7	1 June 2015	1 November 2015
8	1 July 2015	–
9	1 September 2015	1 March 2016
10	1 January 2016	–

(i) Calculate a central exposed to risk for the 10 lives in the sample, working in months. [3]

(ii) Give the maximum likelihood estimate of the mortality hazard at age 85 last birthday. [1]

(iii) Estimate q_{85}. [1]

[Total 5]

5 (i) List the key steps involved in developing an actuarial model. [4]

(ii) Comment on considerations which would apply if you were developing a model of the spread of a newly discovered disease. [3]

[Total 7]

6 A pharmaceutical company is undertaking trials on a new drug which, it claims, cures a particularly uncomfortable but not life threatening condition. It has conducted extensive testing of the drug on a large group of people suffering from the condition and has noticed that the drug is much more effective in some groups of patients than others. It has fitted a Cox regression for the hazard of symptoms disappearing $h(t)$ with three parameters

$$h(t) = h_0(t)\exp\left(S\beta_S + A\beta_A + G\beta_G\right)$$

where β_S, β_A, and β_G are parameters and

- S represents the sex of the patient and takes a value of 1 if the patient is female, 0 if male.
- A represents the age, in years minus 20, of the patient when the drug was administered.
- G takes the value 1 if the patient attended a gym, 0 otherwise.

The company has discovered the following, where the age given is the age when the drug was administered:

- a 25 year old female who attended a gym had a hazard of symptoms disappearing equal to twice that of a male of the same age who did not attend a gym;
- a 45 year old male who did not attend a gym had a hazard of symptoms disappearing half that of a 43 year old male who attended a gym; and
- a 32 year old female who attended a gym had a hazard of symptoms disappearing 60% greater than that of a 45 year old female who did not attend a gym.

(i) Calculate the values of the parameters β_S, β_A, and β_G. [5]

(ii) Determine for which group of people the drug is most effective. [3]

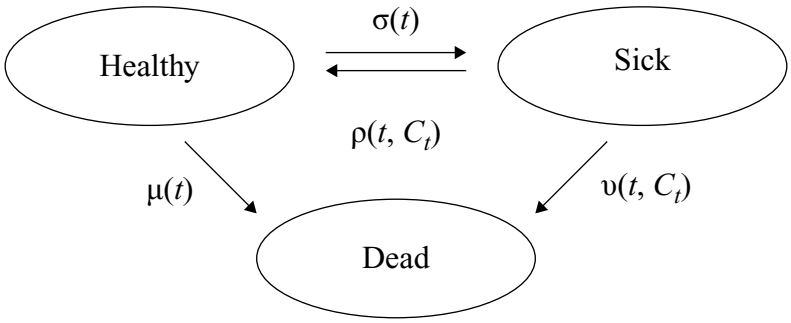
The probability that a woman who attended a gym and was aged 38 years when she was given the drug still had symptoms of the condition after 28 days was found to be 0.75.

(iii) Calculate the probability of still having symptoms after 28 days for a male aged 26 years when given the drug who did not attend a gym. [4]

[Total 12]

7 The following diagram shows the transitions under a Healthy-Sick-Dead multiple state model under which:

- transition rates are dependent on time, t.
- transitions out of the Sick state are dependent on the duration, C_t, a person has been in the Sick state as well as on time.



(i) Show from first principles, that if $p_{ij}(x,t)$ is the probability of being in state j at time t conditional on being in state i at time x, that

$$\frac{\partial}{\partial t} p_{HH}(x,t) = p_{HH}(x,t)\left(-\sigma(t) - \mu(t)\right) + p_{HS}(x,t)\rho\left(t, C_t\right) \qquad [5]$$

(ii) Determine the probability that a life is in the Healthy state throughout the period 0 to t if the life is in the Healthy state at time 0. [2]

(iii) Describe how integrated Kolmogorov equations can be constructed by conditioning on the first or the last jump, illustrating your answer with a diagram. [3]

(iv) Explain the difference in approach between deriving forward and backward integrated Kolmogorov equations. [1]

An actuarial student suggests the following integrated Kolmogorov equation for this model:

$$\Pr[X_t = H | X_s = S,\, C_s = w] = \int_0^t e^{-\int_0^y (\rho(u, w-s+u) + \upsilon(u, w-s+u))\,du} \upsilon\left(y, w - s + y\right) p_{HH}\left(y,t\right) dy$$

(v) Identify TWO errors in this equation. [2]

[Total 13]

8 A company has for many years offered a car insurance policy with four levels of No Claims Discount (NCD): 0%, 15%, 30% and 40%. A policyholder who does not claim in a year moves up one level of discount, or remains at the highest level. A policyholder who claims one or more times in a year moves down a level of discount or remains at the lowest level. The company pays a maximum of three claims in any year on any one policy.

The company has established that:

- the arrival of claims follows a Poisson process with a rate of 0.35 per year.
- the average cost per claim is £2,500.
- the proportion of policyholders at each level of discount is as follows:

Discount level	*Proportion of policyholders*
0%	4.4%
15%	10.5%
30%	25.1%
40%	60.0%

(i) Calculate the premium paid by a policyholder at the 40% discount level ignoring expenses and profit. [4]

The company has decided to introduce a protected NCD feature whereby policyholders can make one claim on their policy in a year and, rather than move down a level of discount, remain at the level they are at. All other features of the policy remain the same.

(ii) Draw the transition graph for this process. [2]

(iii) Calculate the premium paid, in the long term, by a policyholder at the 40% discount level of the policy with protected NCD, ignoring expenses and profit. [6]

(iv) Discuss THREE issues with the policy with protected NCD which may each be either a disadvantage or an advantage to the company. [3]

[Total 15]

9 (i) (a) List TWO different methods of graduating crude mortality data.

(b) State, for each method, TWO advantages and ONE disadvantage.

[3]

A large pension scheme is examining its most recent experience and has graduated its data over a range of ages using $\mu_x = 0.0005 + 0.00005(1.1^x)$. The table below gives some of the data.

Age	*Exposed to Risk*	*Observed Deaths*	*Graduated Rates*
60	7,966	127	0.015724
61	7,728	139	0.017246
62	7,870	162	0.018921
63	7,622	167	0.020763
64	7,097	205	0.022790
65	7,208	179	0.025019
66	6,833	185	0.027470
67	6,474	212	0.030167
68	6,208	209	0.033134
69	5,914	195	0.036398

(ii) Perform an overall goodness of fit test on the data. [5]

(iii) (a) State THREE possible defects of the graduation which the test you performed in (ii) would fail to detect.

(b) Suggest, for each defect in part (a), an alternative test which would detect each defect.

[3]

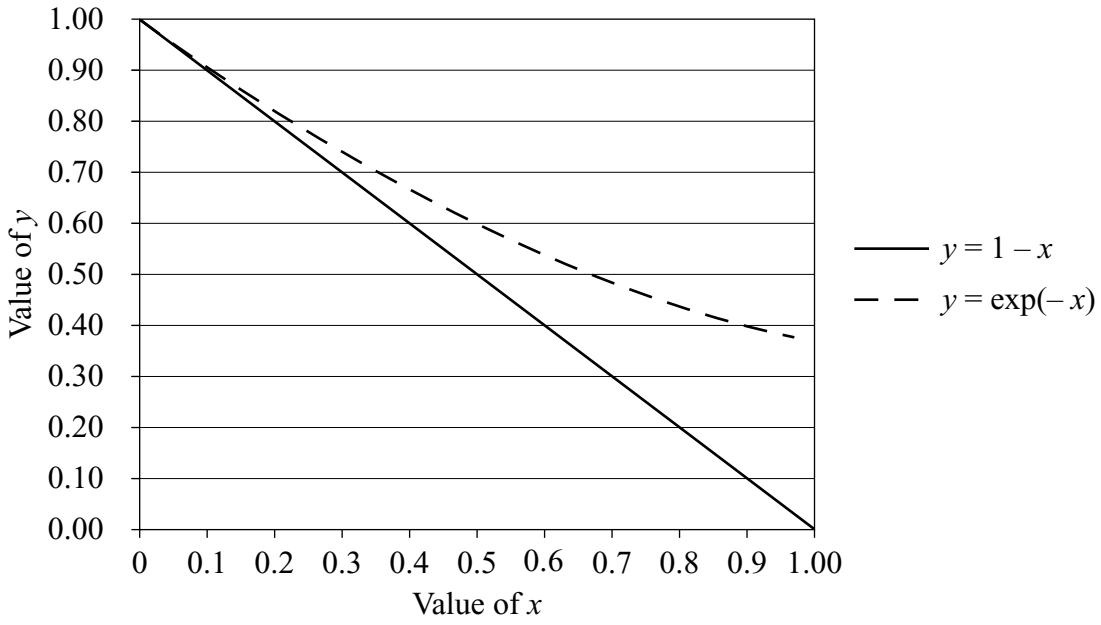
(iv) Carry out TWO of the tests you mentioned in part (iii), clearly stating your conclusions in relation to the relevant defects. [6]

[Total 17]

10 (i) Write down the formulae for the Kaplan-Meier estimator $\hat{S}(t)$ and Nelson-Aalen estimator $\tilde{S}(t)$ of survival in the presence of a stated hazard, defining all terms used. [2]

The following graph shows the functions:

$y = 1 - x$ and
$y = e^{-x}$ over the range $0 \leq x \leq 1$.



(ii) Demonstrate that the Nelson-Aalen estimator is never lower than the Kaplan-Meier estimator. [2]

A trial is conducted amongst 20 patients who have suffered from eczema but are in remission (that is, they are clear of the condition). The trial is to assess whether continuing with periodic doses of a certain steroid cream in remission reduces the rate at which eczema recurs. Patients are invited to tests every 3 months for a period of up to 5 years from when first declared to be in remission.

(iii) Describe THREE types of censoring present in the investigation. [3]

The data for the trial are subdivided into a group who continued to receive the steroid cream, and a control group who did not receive the steroid cream. The data for the patients in the trial showing the quarterly test at which eczema recurred, or censoring occurred, are as follows (an * indicates a patient who was censored):

For group receiving steroid cream: 3, 5, 6*, 7*, 10, 10, 12*, 14*, 18, 19*
For control group: 6, 8, 8, 10*, 11*, 12*, 14, 15*, 18, 18

(iv) Calculate the Kaplan-Meier estimates of the survival function for remaining clear of eczema for:

(a) the group who continued to receive the steroid cream; and

(b) the control group.

[8]

(v) (a) Recommend, without performing any calculations, a method of establishing whether the hazard of eczema returning is statistically lower for those continuing to receive the steroid cream.

(b) Comment on the chance of being able to conclude from the trial data that continuing to receive the steroid cream reduces the risk of recurrence of eczema.

[3]
[Total 18]

END OF PAPER