INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

1 October 2015 (am)

Subject CT4 – Models
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 11 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 List four factors, other than age and sex, by which mortality statistics are often subdivided. [2]

2 Describe the differences between a stochastic and a deterministic model. [4]

3 (i) Define how the following forms of censoring arise in a survival investigation:

- right censoring
- type I censoring
- random censoring [3]

An experience analysis is conducted where the event of interest is the lapse of a term assurance policy.

(ii) Explain whether each form of censoring listed in part (i) occurs in each of the following situations. If it is not possible to state whether a form of censoring occurs, explain why this is the case.

(a) A policyholder dies.

(b) A subset of the policies is migrated to a new administration system and no data are provided from the new system to the experience analysis team.

(c) A policy reaches its maturity date. [4]

[Total 7]

4 Company *A* and Company *B* are two small insurance companies which have recently merged to form Company *C*. Company *C* is reviewing its premium rates for a whole of life product and so is conducting an analysis of mortality rates experienced.

Company *A* recorded the number of policies in force every 1 January using a definition of age next birthday whereas Company *B* recorded the number of policies in force every 1 April using an age definition of age last birthday. Both companies recorded deaths as they happened using an age definition of age last birthday.

These are the data for the most recent years.

Company A

Age next birthday	*Number of policies 1 Jan. 2012*	*Number of policies 1 Jan. 2013*	*Number of policies 1 Jan. 2014*
51	8,192	6,421	8,118
52	7,684	8,298	7,187
53	9,421	8,016	9,026

Company B

Age last birthday	*Number of policies 1 April 2012*	*Number of policies 1 April 2013*	*Number of policies 1 April 2014*
51	4,496	3,817	4,872
52	5,281	5,218	3,812
53	4,992	5,076	5,076

In the calendar year 2013 Company *A* recorded 28 deaths of those aged 52 last birthday and Company *B* recorded 17 deaths of those aged 52 last birthday.

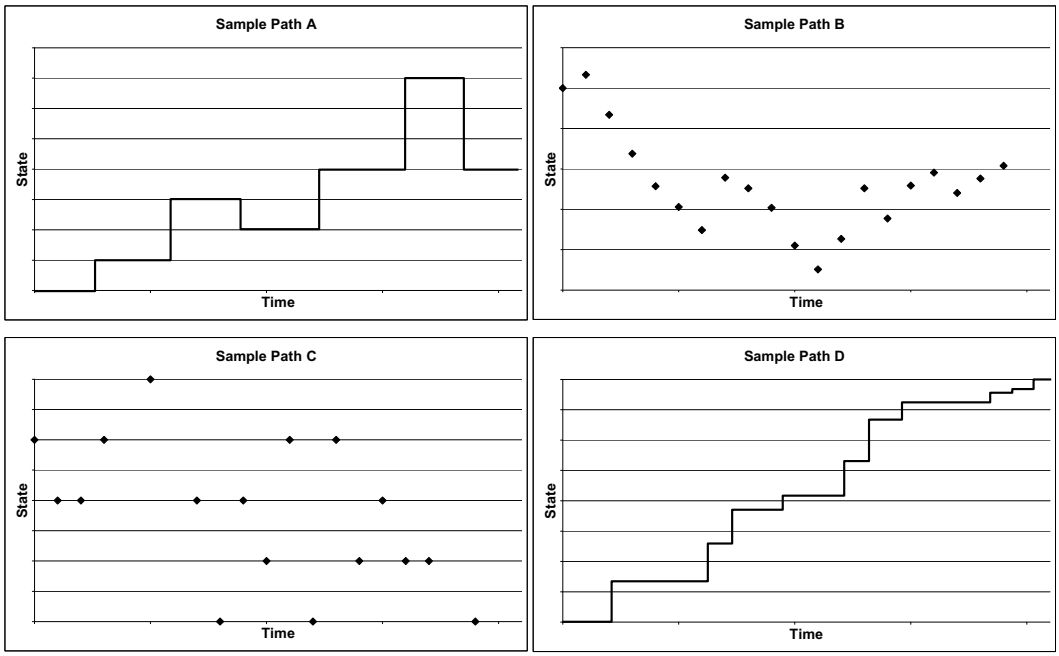
(i) Estimate the force of mortality for the combined company for age 52 last birthday, stating all assumptions that you make. [6]

(ii) Explain the exact age to which your estimate applies. [1]

[Total 7]

5 (i) Describe why models are used in actuarial work. [4]

The following diagrams illustrate sample paths for four stochastic processes.



(ii) Identify which sample path is most likely to correspond to a:

- discrete time, discrete state process.
- continuous time, discrete state process.
- discrete time, continuous state process.
- continuous time, continuous state process.

[2]

(iii) Discuss the reasons why a discrete or continuous process may be selected for a modelling exercise. [3]

[Total 9]

6 (i) Describe what is meant by a proportional hazards model. [3]

A pharmaceutical company is interested in testing a new treatment for a debilitating but non-fatal condition in cows. A randomised trial was carried out in which a sample of cows with the condition was assigned to either the new treatment or the previous treatment. The event of interest was the recovery of a cow from the condition. The results were analysed using a Cox regression model.

The final model estimated the hazard, $h(t, x)$ as:

$$h(t, x) = h_0(t) \exp(\beta_0 z + \beta_1 x + \beta_2 xz),$$

where:

$h_0(t)$ is the baseline hazard;

z is a covariate taking the value 1 if the cow was assigned the new treatment and 0 if the cow was assigned the previous treatment;

x is a covariate denoting the length of time (in days) for which the cow had been suffering from the condition when treatment was started;

and t is the number of days since treatment started.

β_0 , β_1 and β_2 are parameters. Their estimated values were $\beta_0 = 0.8$, $\beta_1 = 0.4$ and $\beta_2 = -0.1$.

(ii) Determine the characteristics of the baseline cow. [1]

For a particular cow, the new treatment and the previous treatment have exactly the same hazard.

(iii) Calculate the number of days for which that cow had the condition before the initiation of treatment. [2]

Under the previous treatment, cows whose treatment began after they had been suffering from the condition for three days had a median recovery time of 14 days once treatment had started.

(iv) Calculate the proportion of these cows which would still have had the condition after 14 days if they had been given the new treatment. [4]

[Total 10]

7 A school offers a one year course in a foreign language as an evening class. This is divided into three terms of 13 weeks each with one lesson per week. At the end of each lesson all the students sit a test and any that pass are awarded a qualification, and no longer attend the course.

Last year 33 students started the course. Of these 13 dropped out before completing the year, and 16 passed the test before the end of the year. The last lesson attended by the students who did not stay for the whole 39 lessons is shown in the table below along with their reason for leaving.

Number of students	*Last lesson attended*	*Reason for leaving*
5	1	Dropped out
1	6	Dropped out
2	7	Passed test
2	13	Dropped out
5	14	Passed test
6	27	Passed test
4	28	Dropped out
1	30	Dropped out
3	36	Passed test

(i) Calculate the Nelson-Aalen estimate of the survival function. [5]

(ii) Sketch a graph of the Nelson-Aalen estimate of the survival function, labelling the axes. [2]

(iii) Determine the probability that a student who starts the course passes by the end of the year. [1]

Since only four students had not passed by the end of the year and a total of 16 had passed, the school claims in its publicity that 80% of students are awarded the qualification by the end of the year.

(iv) Comment on the school's claim in light of your answer to part (iii). [2]

[Total 10]

8 (i) Define a Markov Jump Process. [1]

A company provides phones on contracts under which it is responsible for repairing or replacing any phones which break down.

When a customer reports a fault with a phone, it is immediately taken to the company's repair shop and it is assessed whether it can be fixed (meaning fixable at reasonable cost). Based on previous experience, it is estimated that the probability of a phone being fixable is 0.75. If a phone is not fixable it is discarded and the customer is provided with a new phone.

If a repaired phone breaks again the company, in line with its customer charter, will not attempt to repair it again, and so discards the phone and replaces it with a new one.

The status of a phone is to be modelled as a Markov Jump Process with state space {Never Broken (*NB*), Repaired (*R*), Discarded (*D*)}.

The company considers the rate at which phones break down to vary according to whether a phone has previously been repaired as follows:

Status	*Probability of break down in small interval of time, dt*
Never Broken	$0.1dt + o(dt)$
Repaired	$0.2dt + o(dt)$

(ii) Draw a transition diagram for the possible transitions between the states, including the associated transition rates. [2]

Let $P_{NB}(t)$, $P_R(t)$ and $P_D(t)$ be the probabilities that a phone is in each state after time t since it was provided as a new phone.

(iii) Determine Kolmogorov's forward equations in component form for $P_{NB}(t)$, $P_R(t)$ and $P_D(t)$. [2]

(iv) Solve the equations in part (iii) to obtain $P_{NB}(t)$ and $P_R(t)$. [4]

(v) Calculate the probability that a phone has not been discarded by time t. [1]

[Total 10]

9 Doctors at a health centre are carrying out an investigation to see if obesity affects the likelihood of dying from heart disease. They propose to use a model with four states:

1. Obese
2. Not obese
3. Dead due to heart disease, and
4. Dead due to other causes

(i) Write down, defining all the terms you use, the likelihood for the transition intensities. [3]

(ii) Derive the maximum likelihood estimator of the force of mortality from heart disease for Obese people. [3]

The investigation has followed several thousand people aged 50–59 years for five years and has the following data:

Waiting time in state Obese (in person-years)	14,392
Waiting time in state Not obese (in person-years)	18,109
Number of deaths due to heart disease for those persons who are Obese	178
Number of deaths due to heart disease for those persons who are Not obese	190
Number of deaths due to other causes for those persons who are Obese	89
Number of deaths due to other causes for those persons who are Not obese	53

The doctors want to promote healthy living and therefore wish to claim that Obese people have a much higher chance, statistically, of dying from heart disease than do people who are Not obese.

(iii) Test whether this claim is true at the 90% confidence level. [5]

[Total 11]

10 A profession has examination papers in two subjects, *A* and *B*, each of which is marked by a team of examiners. After each examination session, examiners are given the choice of remaining on the same team, switching to the other team, or taking a session's holiday.

In recent sessions, 10% of subject *A*'s examiners have elected to switch to subject *B* and 10% to take a holiday. Subject *B* is more onerous to mark than subject *A*, and in recent sessions, 20% of subject *B*'s examiners have elected to take a holiday in the next session, with 20% moving to subject *A*.

After a session's holiday, the profession allocates examiners equally between subjects *A* and *B*. No examiner is permitted to take holiday for two consecutive sessions.

(i) Sketch the transition graph for the process. [2]

(ii) Determine the transition matrix for this process. [2]

(iii) Calculate the proportion of the profession's examiners marking for subjects *A* and *B* in the long run. [4]

The profession considers that in future, an equal number of examiners is likely to be required for each subject. It proposes to try to ensure this by adjusting the proportion of those examiners on holiday who, when they return to marking, are allocated to subjects *A* and *B*.

(iv) Calculate the proportion of examiners who, on returning from holiday, should be allocated to subject *B* in order to have an equal number of examiners on each subject in the long run. [4]

[Total 12]

11 (i) Describe why an insurance company might want to compare the results of a mortality investigation with previous experience. [2]

A large life insurance company has undertaken an investigation of the mortality of its policyholders. Currently it assumes that mortality at age x, μ_x, is equal to a standard table. The company wishes to use the results from the investigation to see whether the standard table is still appropriate. Below are shown some data from the investigation.

Age x	*Number of policies in force*	*Actual death claims*	*Expected death claims from standard table*
70	1,000	13	23.74
71	1,200	28	31.80
72	1,100	31	32.50
73	1,100	34	36.20
74	1,000	39	36.63
75	1,000	41	40.73
76	950	41	42.99
77	900	40	45.20
78	850	46	47.34
79	800	48	49.35

(ii) Perform an overall test of the hypothesis that the underlying mortality of the company's policyholders is, over this range of ages, represented by the standard table. [6]

(iii) Evaluate the suitability of the standard table for use in the company's financial modelling by performing two additional tests for different possible inconsistencies between the actual death rates and those represented by the standard table. [6]

The company discovers that at age 70 years, one individual owns 25 of the policies in the investigation, the remaining policies each being owned by different individuals.

(iv) Assess the impact of this on the variance of the number of claims at age 70 years. [4]

[Total 18]

END OF PAPER