INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

25 September 2012 (am)

Subject CT6 – Statistical Methods
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 The potential losses from a decision problem are given in the table below

	D1	*D2*	*D3*	*D4*
θ_1	5	8	12	3
θ_2	10	15	7	8
θ_3	7	12	16	9
θ_4	17	4	10	12

(i) Find the optimal decision using the minimax criteria. [2]

Now suppose that $p(\theta_1) = p(\theta_2) = p(\theta_3) = 0.3$ and $p(\theta_4) = 0.1$.

(ii) Find the optimal decision using the Bayes criteria. [2]
[Total 4]

2 Claim amounts on a certain type of insurance policy depend on a parameter α which varies from policy to policy. The mean and variance of the claim amount X given α are specified by

$$E\left[X\,|\alpha\right] = 200 + \alpha$$
$$V[X\,|\alpha] = 10 + 2\alpha$$

The parameter α follows a normal distribution with mean 20 and variance 4.

Find the unconditional mean and variance of X. [6]

3 An actuary needs to generate samples from the standard normal distribution for use in a simulation model he is constructing.

(i) Describe the polar algorithm for generating pairs of samples from the standard normal distribution given pairs of samples from a uniform distribution on [0,1]. [3]

(ii) Calculate the probability that a pair of samples from a uniform distribution on [0,1] results in an acceptable pair of samples from the standard normal distribution under the algorithm in (i). [3]
[Total 6]

4 Claims arising on a particular type of insurance policy are believed to follow a Pareto distribution. Data for the last several years shows the mean claim size is 170 and the standard deviation is 400.

(i) Fit a Pareto distribution to this data using the method of moments. [4]

(ii) Calculate the median claim using the fitted parameters and comment on the result. [3]
[Total 7]

5 A discrete probability distribution is defined by

$$f(y,\mu) = \binom{n}{ny}\mu^{ny}(1-\mu)^{n-ny} \qquad y = 0, \frac{1}{n}, \frac{2}{n}, \ldots, 1$$

where μ is a parameter between 0 and 1.

(i) Explain why this distribution belongs to an exponential family. [4]

(ii) State the three main components that need to be taken into account when constructing a generalised linear model. [3]

(iii) Suggest a natural choice of link function if the response variable followed the distribution defined above. [1]

(iv) Suggest a natural choice of link function if instead the response variable followed a lognormal distribution. [2]
[Total 10]

6 Individual claim amounts from a particular type of insurance policy follow a normal distribution with mean 150 and standard deviation 30. Claim numbers on an individual policy follow a Poisson distribution with parameter 0.25. The insurance company uses a premium loading of 70% to calculate premiums.

(i) Calculate the annual premium charged by the insurance company. [1]

The insurance company has an individual excess of loss reinsurance arrangement with a retention of 200 with a reinsurer who uses a premium loading of 120%.

(ii) Calculate the probability that an individual claim does not exceed the retention. [2]

(iii) Calculate the probability for a particular policy that in a given year there are no claims which exceed the retention. [2]

(iv) Calculate the premium charged by the reinsurer. [4]

(v) Calculate the insurance company's expected profit. [2]
[Total 11]

7 The table below shows claims paid on a portfolio of general insurance policies. Claims from this portfolio are fully run off after 3 years.

Underwriting year	*Development Year*			
	0	*1*	*2*	*3*
2008	85	42	30	7
2009	103	65	25	
2010	93	47		
2011	111			

(i) Estimate the outstanding claims using the basic chain ladder approach. [7]

You are asked to investigate the fit of the model by applying the development factors from part (i) to the claims paid in development year 0 and then comparing the fitted claim payments to the actual payments.

(ii) Construct a table showing the difference between the fitted payments and the actual payments in the table above. [3]

(iii) Comment on the results of the analysis in part (ii). [2]
[Total 12]

8 An insurer classifies the buildings it insures into one of three types. For Type 1 buildings, the number of claims per building per year follows a Poisson distribution with parameter λ. Data are available for the last five years as follows:

Year	*1*	*2*	*3*	*4*	*5*
Number of type 1 buildings covered	89	112	153	178	165
Number of claims	15	23	29	41	50

(i) Determine the maximum likelihood estimate of λ based on the data above. [5]

The insurer also has data for the other two types of building for all five years. Define

P_{ij} = the number of buildings insured in the jth year from type i and

Y_{ij} = the corresponding number of claims.

The five years of data can be summarised as follows:

$Type(i)$	$\bar{P}_i = \sum_{j=1}^{5} P_{ij}$	$\bar{X}_i = \sum_{j=1}^{5} \frac{Y_{ij}}{\bar{P}_i}$	$\sum_{j=1}^{5} P_{ij}\left(\frac{Y_{ij}}{P_{ij}} - \bar{X}_i\right)^2$	$\sum_{j=1}^{5} P_{ij}\left(\frac{Y_{ij}}{P_{ij}} - \bar{X}\right)^2$
Type 1	697	0.226686	1.527016	2.502737
Type 2	295	0.237288	0.96605	1.178133
Type 3	515	0.330097	4.53253	6.775614

$$\bar{X} = \sum_{i=1}^{3}\sum_{j=1}^{5} \frac{Y_{ij}}{\bar{P}} = 0.264101 \text{ where } \bar{P} = \sum_{i=1}^{3} \bar{P}_i$$

There are 191 buildings of Type 1 to be insured in year six.

(ii) Estimate the number of claims from Type 1 buildings in year six using Empirical Bayes Credibility Theory model 2. [6]

(iii) Explain the main differences between the approaches in parts (i) and (ii). [2]
[Total 13]

9 In order to model a particular seasonal data set an actuary is considering using a model of the form

$$(1 - B^3)\left(1 - (\alpha + \beta)B + \alpha\beta B^2\right)X_t = e_t$$

where B is the backward shift operator and e_t is a white noise process with variance σ^2.

(i) Show that for a suitable choice of s the seasonal difference series $Y_t = X_t - X_{t-s}$ is stationary for a range of values of α and β which you should specify. [3]

After appropriate seasonal differencing the following sample autocorrelation values for the series Y_t are observed: $\hat{\rho}_1 = 0.2$ and $\hat{\rho}_2 = 0.7$.

(ii) Estimate the parameters α and β based on this information. [7]

[HINT: let $X = \alpha + \beta$, $Y = \alpha\beta$ and find a quadratic equation with roots α and β.]

(iii) Forecast the next two observations $\hat{x}_{101}$ and $\hat{x}_{102}$ based on the parameters estimated in part (ii) and the observed values $x_1, x_2, \ldots, x_{100}$ of X_t. [4]
[Total 14]

10 Claims occur on a portfolio of insurance policies according to a Poisson process. Individual claim amounts are either 1 (with probability 0.7) or 8 (with probability 0.3). The insurance company uses a premium loading of 60% to calculate premiums and buys excess of loss reinsurance with a retention of M ($1<M<8$) from a reinsurer. The reinsurer uses a premium loading of 120%.

(i) Calculate the smallest value of M that the insurance company should consider if it wishes to expect to make a profit on this portfolio. [3]

(ii) Derive the adjustment coefficient equation for the insurance company. [2]

(iii) Calculate the adjustment coefficient (correct to 2 decimal places) if M=4. [4]

The same reinsurer also offers proportional reinsurance with the same premium loading such that the reinsurer pays a proportion α of each claim.

(iv) Show that the insurance company may either purchase excess of loss reinsurance with retention M or proportional reinsurance with $\alpha = \dfrac{3(8-M)}{31}$ for the same premium. [2]

(v) Determine whether the adjustment coefficient with proportional reinsurance is higher or lower than that with excess of loss reinsurance when M=4. [4]

(vi) Comment on the implications of part (v). [2]

[Total 17]

END OF PAPER