EXAMINATION

29 March 2006 (am)

Subject CT4 — Models
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 12 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

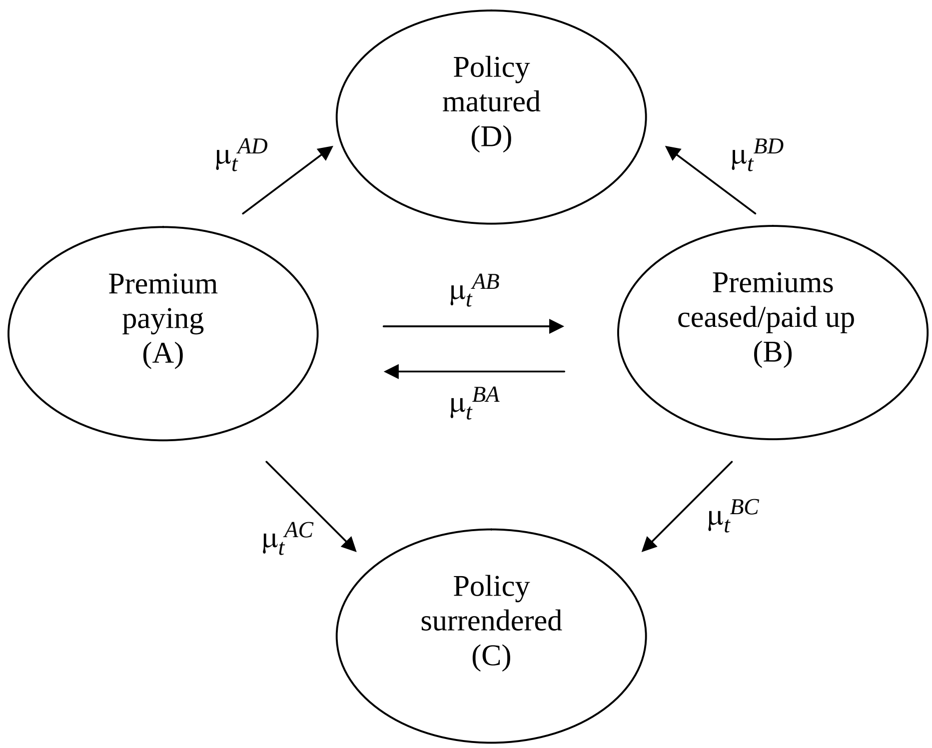
103 Questions

A1 In the context of a stochastic process $\{X_t : t \in J\}$, explain the meaning of the following conditions:

(a) strict stationarity
(b) weak stationarity

[3]

A2 A savings provider offers a regular premium pension contract, under which the customer is able to cease paying in premiums and restart them at a later date. In order to profit test the product, the provider set up the four-state Markov model shown in the following diagram:



Show, from first principles, that under this model:

$$\frac{\partial}{\partial t}\,{}_t p_0^{AB} = {}_t p_0^{AA}.\mu_t^{AB} - {}_t p_0^{AB}.(\mu_t^{BA} + \mu_t^{BC} + \mu_t^{BD})$$ [5]

A3 A motor insurer's No Claims Discount system uses the following levels of discount {0%, 25%, 40%, 50%}. Following a claim free year a policyholder moves up one discount level (or remains on 50% discount). If the policyholder makes one (or more) claims in a year they move down one level (or remain at 0% discount).

The insurer estimates that the probability of making at least one claim in a year is 0.1 if the policyholder made no claims the previous year, and 0.25 if they made a claim the previous year.

New policyholders should be ignored.

(i) Explain why the system with state space {0%, 25%, 40%, 50%} does not form a Markov chain. [2]

(ii) (a) Show how a Markov chain can be constructed by the introduction of additional states.

(b) Write down the transition matrix for this expanded system, or draw its transition diagram.

[4]

(iii) Comment on the appropriateness of the current No Claims Discount system.

[2]

[Total 8]

A4 (i) List the benefits of modelling in actuarial work. [2]

(ii) Describe the difference between a stochastic and a deterministic model. [2]

(iii) Outline the factors you would consider in deciding whether to use a stochastic or deterministic model to study a problem. [3]

(iv) Explain how a deterministic model might be used to validate model outcomes where a stochastic approach has been selected. [2]

[Total 9]

A5 Employees of a company are given a performance appraisal each year. The appraisal results in each employee's performance being rated as High (H), Medium (M) or Low (L). From evidence using previous data it is believed that the performance rating of an employee evolves as a Markov chain with transition matrix:

$$
\begin{array}{cc} & \begin{array}{ccc} \quad H \quad & \quad M \quad & \quad L \quad \end{array} \\ P = \begin{array}{c} H \\ M \\ L \end{array} & \begin{pmatrix} 1-\alpha-\alpha^2 & \alpha & \alpha^2 \\ \alpha & 1-2\alpha & \alpha \\ \alpha^2 & \alpha & 1-\alpha-\alpha^2 \end{pmatrix} \end{array}
$$

for some parameter α.

(i) Draw the transition graph of the chain. [2]

(ii) Determine the range of values for α for which the matrix P is a valid transition matrix. [2]

(iii) Explain whether the chain is irreducible and/or aperiodic. [2]

(iv) For $\alpha = 0.2$, calculate the proportion of employees who, in the long run, are in state L. [3]

(v) Given that $\alpha = 0.2$, calculate the probability that an employee's rating in the third year, X_3, is L:

(a) in the case that the employee's rating in the first year, X_1, is H

(b) in the case $X_1 = M$

(c) in the case $X_1 = L$

[2]

[Total 11]

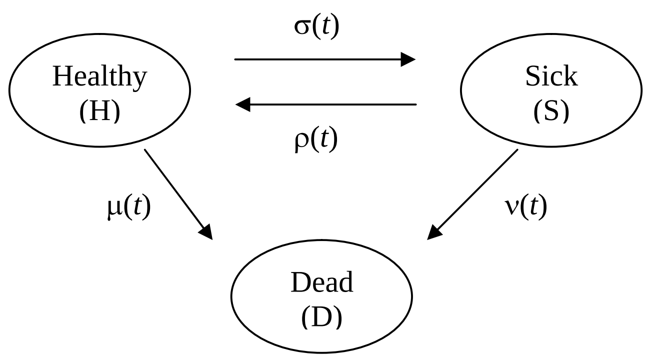
A6 (i) (a) Explain what is meant by a Markov jump process.

(b) Explain the condition needed for such a process to be time-homogeneous.

[2]

(ii) Outline the principal difficulties in fitting a Markov jump process model with time-inhomogeneous rates. [2]

A company provides sick pay for a maximum period of six months to its employees who are unable to work. The following three-state, time-inhomogeneous Markov jump process has been chosen to model future sick pay costs for an individual:



Where "Sick" means unable to work and "Healthy" means fit to work.

The time dependence of the transition rates is to reflect increased mortality and morbidity rates as an employee gets older. Time is expressed in years.

(iii) Write down Kolmorgorov's forward equations for this process, specifying the appropriate transition matrix. [1]

(iv) (a) Given an employee is sick at time $w < T$, write down an expression for the probability that he or she is sick throughout the period $w < t < T$.

(b) Given that a transition out of state H occurred at time w, state the probability that the transition was into state S.

(c) For an employee who is healthy at time τ, give an approximate expression for the probability that there is a transition out of state H in a small time interval $[w, w + dw]$, where $w > \tau$. Your expression should be in terms of the transition rates and $P_{HH}(\tau, w)$ only.

[3]

(v) Using the results of part (iv) or otherwise, derive an expression for the probability that an employee is sick at time T and has been sick for less than 6 months, given that they were healthy at time $\tau < T - ½$. Your expression should be in terms of the transition rates and $P_{HH}(\tau, w)$ only. [3]

(vi) Comment on the suggestions that:

(a) $\rho(t)$ should also depend on the holding time in state S, and
(b) mortality rates can be ignored.

[3]
[Total 14]

104 Questions

B1 A Cox proportional hazards model was estimated to assess the effect on survival of a person's sex and his or her self-esteem (measured on a three-point scale as "low", "medium" or "high"). The baseline category was males with "low" self-esteem.

Write down the equation of the model, using algebraic symbols to represent variables and parameters and defining all the symbols that you use. [4]

B2 (i) (a) Explain why it is important to sub-divide data when carrying out mortality investigations.

(b) Describe the problems that can arise with sub-dividing data.

[4]

(ii) List four factors which are often used to sub-divide life assurance data. [2]

[Total 6]

B3 (i) Assume that the force of mortality between consecutive integer ages, y and $y + 1$, is constant and takes the value μ_y.

Let T_x be the future lifetime after age x ($x \le y$) and $S_x(t)$ be the survival function of T_x.

Show that:

$$\mu_y = \log[S_x(y-x)] - \log[S_x(y+1-x)].$$

[4]

(ii) An investigation was carried out into the mortality of male life office policyholders. Each life was observed from his 50th birthday until the first of three possible events occurred: his 55th birthday, his death, or the lapsing of his policy. For those policyholders who died or allowed their policies to lapse, the exact age at exit was recorded.

Using the result from part (i) or otherwise, describe how the data arising from this investigation could be used to estimate:

(a) μ_{50}

(b) ${}_5q_{50}$

[3]

[Total 7]

B4 A company is interested in estimating policy lapse rates by age. It conducts an investigation into this, which lasts for the whole of the calendar year 2003. The investigation collects the following data for a sample of policies which are funded by annual premiums:

- the age last birthday of the policyholder when the policy was taken out;
- the number of premiums the policyholder paid before the policy lapsed.

In addition, the number of policies in-force on 1 January each year is available, classified by age x last birthday and years t elapsed since 1 January 2003, $(P_{x,t}^{*})$.

(i) State the rate interval in this investigation. [1]

(ii) Derive an expression for the exposed-to-risk in terms of $P_{x,t}^{*}$, stating any assumptions you make. [7]

(iii) Comment on the reasonableness or otherwise of the assumptions you made in your answer to part (ii). [2]

[Total 10]

B5 A life assurance company carried out an investigation of the mortality of male life assurance policyholders. The investigation followed a group of 100 policyholders from their 60th birthday until their 65th birthday, or until they died or cancelled their policy (whichever event occurred first).

The ages at which policyholders died or cancelled their policies were as follows:

Died	**Cancelled policy**
Age in years and months	*Age in years and months*
60y 5m	60y 2m
61y 1m	60y 3m
62y 6m	60y 8m
63y 0m	61y 0m
63y 0m	61y 0m
63y 8m	61y 0m
64y 3m	61y 5m
	62y 2m
	62y 9m
	63y 9m
	64y 5m

(i) Explain which types of censoring are present in the investigation. [2]

(ii) Calculate the Nelson-Aalen estimate of the integrated hazard for these policyholders. [5]

(iii) Sketch the estimated integrated hazard function. [2]

(iv) Estimate the probability that a policyholder will survive to age 65. [2]
[Total 11]

B6 An investigation was undertaken into the mortality of male term assurance policyholders for a large life insurance company. The crude mortality rates were graduated using a formula of the form:

$$\overset{\circ}{q}_x = \alpha + \beta e^{\gamma x}$$

An extract of the results is shown below.

Age	*Exposure (years)*	*Crude mortality rate*	*Graduated mortality rate*	*Standardised deviation*
x	E_x	$\hat{q}_x$	$\overset{\circ}{q}_x$	$z_x = \dfrac{E_x\left(\hat{q}_x - \overset{\circ}{q}_x\right)}{\sqrt{E_x \overset{\circ}{q}_x\left(1 - \overset{\circ}{q}_x\right)}}$
40	11,037	0.0029	0.00348	-1.035
41	12,010	0.00333	0.00358	-0.459
42	11,654	0.003	0.00368	-1.212
43	9,658	0.003	0.00379	-1.264
44	8,457	0.00319	0.00391	-1.061
45	10,541	0.00427	0.00402	0.406
46	7,410	0.00472	0.00415	0.763
47	12,042	0.00399	0.00428	-0.487
48	14,038	0.00406	0.00441	-0.626
49	11,479	0.00375	0.00455	-1.274
50	12,480	0.00409	0.00469	-0.981
51	10,567	0.00407	0.00485	-1.154
52	9,187	0.00512	0.00500	0.163
53	14,027	0.00456	0.00517	-1.007
54	11,581	0.00466	0.00534	-1.004

(i) Test the graduation for goodness of fit using the chi-squared test. [5]

(ii) (a) By inspection of the data, suggest one aspect of the graduated rates where adherence to data seems inadequate.

(b) Explain why this may not be detected by the chi-squared test.

(c) Carry out one other test that may detect this deficiency.

[5]

(iii) Suggest how the graduation could be adjusted to correct the deficiency identified. [2]

[Total 12]

END OF PAPER