INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2012 examinations

Subject CT6 – Statistical Methods
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and who are using past papers as a revision aid, and also those who have previously failed the subject. The Examiners are charged by Council with examining the published syllabus. Although Examiners have access to the Core Reading, which is designed to interpret the syllabus, the Examiners are not required to examine the content of Core Reading. Notwithstanding that, the questions set, and the following comments, will generally be based on Core Reading.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report. Other valid approaches are always given appropriate credit; where there is a commonly used alternative approach, this is also noted in the report. For essay-style questions, and particularly the open-ended questions in the later subjects, this report contains all the points for which the Examiners awarded marks. This is much more than a model solution – it would be impossible to write down all the points in the report in the time allowed for the question.

T J Birse
Chairman of the Board of Examiners

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General comments on Subject CT6

The examiners for CT6 expect candidates to be familiar with basic statistical concepts from CT3 and so to be comfortable computing probabilities, means, variances etc for the standard statistical distributions. Candidates are also expected to be familiar with Bayes' Theorem, and be able to apply it to given situations. Many of the weaker candidates are not familiar with this material.

The examiners will accept valid approaches that are different from those shown in this report. In general, slightly different numerical answers can be obtained depending on the rounding of intermediate results, and these will still receive full credit. Numerically incorrect answers will usually still score some marks for method providing candidates set their working out clearly.

Comments on the April 2012 paper

Candidates found this paper to be slightly harder than the typical CT6 paper. Nevertheless, well prepared candidates were able to score well. Once again, the question on simulation techniques was poorly answered (or not attempted in many cases). Both this question and the decision theory question required no difficult mathematics, but did require a good understanding of the underlying ideas. Many candidates also struggled on questions using material from CT3.

The questions on ruin theory and time series were again well answered.

1 (i) A random variable *Y* belongs to an exponential family if the pdf of *Y* can be written in the form

$$f(y;\theta,\phi) = \exp\left[\frac{y\theta - b(\theta)}{a(\phi)} - c(y,\phi)\right]$$

Where *a*, *b* and *c* are functions.

(ii) Suppose that the parameter of the exponential distribution *Y* is λ. Then

$$f(y) = \lambda \exp(-\lambda y)$$

$$= \exp[\log\lambda - \lambda y]$$

$$= \exp\left[\frac{\lambda y - \log\lambda}{-1}\right]$$

Which is of the required form with

$$\theta = \lambda$$

$$a(\varphi) = -1$$

$$b(\theta) = \log\theta$$

$$c(y,\varphi) = 0$$

Alternative solution: $\theta = -\lambda$; $a(\varphi) = 1$; $b(\theta) = -\log(-\theta)$; $c(y,\varphi) = 0$

This question was answered well.

2 (i) The decision function must nominate a choice of die for each potential outcome from the observation.

There are 6 possible outcomes from the die roll and hence 2 × 2 × 2 × 2 × 2 × 2 = 64 possible decision functions.

(ii) The most natural candidate is to nominate the conventional die on rolls of 1,3,5,6 and the special die on rolls of 2 or 4.

The expected payoff from this approach is:

$$0.5\times\left(\frac{4}{6}\times 1+\frac{2}{6}\times 0\right)+0.5\times 1=0.83333$$

Candidates who understood what a decision function is scored well. However, many candidates struggled to make any headway with this question. For part (i) some candidates observed that if the dice roll is 1,3,5 or 6 it is obvious that you must chose the conventional dice. Therefore a choice is only needed on a roll of a 2 or a 4 giving a total of 2 × 2 = 4 functions. This was given full credit if carefully explained.

3 The likelihood function is given by:

$$L=D\times\prod_{i=1}^{6}3cx_i^{\,2}e^{-cx_i^3}\times e^{-4\times 50^3 c}$$

where D is a constant.

Where the x_i are the claims below the retention.

$$l=\log L=\log D+\sum_{i=1}^{6}log\,3cx_i^{\,2}-c\sum_{i=1}^{6}x_i^{\,3}-4\times 50^3 c$$

$$=\log D+6\log 3+6\log c+\sum_{i=1}^{6}\log x_i^{\,2}-c\sum_{i=1}^{6}x_i^{\,3}-4\times 50^3 c$$

Differentiating we get

$$\frac{dl}{dc}=\frac{6}{c}-\sum_{i=1}^{6}x_i^{\,3}-500000$$

So our estimate is given by

$$\hat{c} = \frac{6}{\sum_{i=1}^{6} x_1^3 + 500000} = \frac{6}{175868 + 500000} = 8.8775 \times 10^{-6}$$

This question was answered well.

4 Let the individual total claim costs be denoted by *X*. Then *X*=*Y*+*Z* where *Y* is the cost of the claim and *Z* is the claim handling expense.

Then

$$E(X) = E(Y) + E(Z) = 100 + 0.3 \times 30 = 109$$

And

$$E\left(X^2\right) = E\left(Y^2 + 2YZ + Z^2\right) = E\left(Y^2\right) + 2E(Y)E(Z) + E(Z^2)$$

Using the independence of *Y* and *Z*. Now

$$E\left(Y^2\right) = 2E(Y)^2 = 2 \times 100^2 = 20000$$

and

$$E\left(Z^2\right) = 0.3 \times 30^2 = 270$$

So that

$$E\left(X^2\right) = 20000 + 2 \times 100 \times 9 + 270 = 22070 = 148.56^2$$

Now if there are *n* policies in the portfolio, total claim amounts *S* will have an approximately Normal distribution with mean $0.2 \times n \times 109 = 21.8n$ and variance $0.2 \times n \times 148.56^2$.

The premium income will be 35*n*.

We need to solve for *n* in the following equation:

$$P\left(N\left(21.8n, 66.44^2 n\right) > 35n\right) < 0.05$$

i.e. $$P\left(N(0,1) > \frac{13.2n}{66.44\sqrt{n}}\right) < 0.05$$

So

$$0.198675496\sqrt{n} > 1.6449$$
$$n > 68.55$$

i.e. at least 69 policies must be sold.

Most candidates struggled with this question. Many did not calculate the variance correctly and a lot did not correctly use the number of policies, n, as a multiplier for the mean and variance of the claims. Others used n and the claim rate when calculating the additional claim handling expense.

5 (i) The posterior distribution of θ is Normal with variance given by

$$\sigma_*^2 = \frac{1}{\left(\frac{n}{200^2} + \frac{1}{50^2}\right)}$$

And mean given by

$$\mu_* = \sigma_*^2\left(\frac{n\bar{x}}{200^2} + \frac{600}{50^2}\right)$$

(ii) Set

$$Z = \sigma_*^2 \frac{n}{200^2}$$

Then

$$Z = \frac{\frac{n}{200^2}}{\left(\frac{n}{200^2} + \frac{1}{50^2}\right)} = \frac{n}{(n+16)}$$

And

$$1 - Z = \frac{\frac{1}{50^2}}{\left(\frac{n}{200^2} + \frac{1}{50^2}\right)} = \sigma_*^2 \frac{1}{50^2}$$

And so

$$\mu_* = Z\bar{x} + (1 - Z)600$$

Which is in the form of a credibility estimate with 600 being the prior mean, $\overline{x}$ being the observed sample mean and Z being the credibility factor.

(iii) In this case we have

$$\sigma_*^2 = \frac{1}{\left(\frac{n}{200^2}+\frac{1}{50^2}\right)} = \frac{1}{\left(\frac{5}{200^2}+\frac{1}{50^2}\right)} = 43.64^2$$

and

$$\mu_* = \sigma_*^2\left(\frac{n\overline{x}}{200^2}+\frac{600}{50^2}\right) = 43.64^2\left(\frac{3400}{200^2}+\frac{600}{50^2}\right) = 619.0476$$

So

$$P(\theta > 600) = P\left(N\left(619.0476, 43.64^2\right) > 600\right)$$

$$= P\left(N(0,1) > \frac{600-619.0476}{43.64}\right) = P(N(0,1) > -0.436)$$

$$= 0.6\times 0.67003 + 0.4\times 0.66640$$

$$= 0.669$$

This question was well answered. Some candidates attempted to derive the answer to part (i) from first principles which was not required. Parts (ii) and (iii) were generally answered well.

6 (i) The posterior distribution has a likelihood given by

$$f(p|n_1) \propto f\left(n_1|p\right) f(p)$$

$$\propto (1-p)^{n_1-1} p \times 1$$

Which is the pdf of a Beta distribution with parameters $\alpha = 2$ and $\beta = n_1$.

(ii) Now the posterior distribution has likelihood given by

$$f(p|n_1, n_2, \ldots, n_5) \propto f\left(n_1, n_2, \ldots, n_5|p\right) f(p)$$

$$\propto (1-p)^{n_1-1} p \times (1-p)^{n_2-1} p \times \cdots \times (1-p)^{n_5-1} \times p$$

$$\propto (1-p)^{n_1+n_2+\cdots+n_5-5} \times p^5$$

Which is the pdf of a Beta distribution with parameters $\alpha = 6$ and $\beta = n_1 + n_2 + \cdots + n_5 - 4$.

(iii) Under squared error loss the Bayes estimate is given by the mean of the posterior distribution which in this case is

$$\hat{p} = \frac{\alpha}{\alpha+\beta} = \frac{6}{n_1 + n_2 + \cdots + n_5 + 2}$$

The maximum likelihood estimate is given by maximising the likelihood which is

$$L \propto (1-p)^{n_1+n_2+\cdots+n_5-5} \times p^5$$

The log-likelihood is given by

$$l = \log L = \log C + \left(n_1 + \cdots + n_5 - 5\right)\log(1-p) + 5\log p$$

And so $\dfrac{dl}{dp} = -\left(n_1 + \cdots + n_5 - 5\right) \times \dfrac{1}{1-p} + \dfrac{5}{p}$

And setting this expression to zero gives

$$\left(n_1 + \cdots + n_5 - 5\right)\hat{p} = 5(1-\hat{p})$$

And so $\left(n_1 + \cdots + n_5\right)\hat{p} = 5$

i.e. $\hat{p} = \dfrac{5}{n_1 + \cdots + n_5}$

So the two estimates are not the same. This is perhaps a little surprising given that we started with an uninformative prior, but arises because the estimates are calculated in two different ways – i.e. one maximises the likelihood and the other minimises the expected squared error. If we wanted the two to be the same we should use an "all-or-nothing" loss function.

A reasonably well answered question. Weaker candidates failed to identify the geometric distribution in part (i). Stronger candidates demonstrated a good understanding of loss functions in part (iii).

7 (i) Suppose that the Poisson rate for risk i is λ_i for =1,2,3.

For the first risk, the likelihood is given by:

$$L = e^{-4\lambda_1} \frac{(4\lambda_1)^{10}}{10!}$$

And so the log-likelihood is given by

$$l = \log L = -4\lambda_1 + 10 \log 4\lambda_1 + \text{Constants}$$

Differentiating gives

$$\frac{dl}{d\lambda_1} = -4 + \frac{10}{\lambda_1}$$

And setting this equal to zero gives a maximum likelihood estimate of

$$\hat{\lambda}_1 = \frac{10}{4} = 2.5$$

Since $\dfrac{d^2 l}{d\lambda^2} = -\dfrac{10}{\lambda_i^2} < 0$ we do have a maximum.

In the same way $\hat{\lambda}_2 = \dfrac{17}{4} = 4.25$ and $\hat{\lambda}_3 = \dfrac{24}{4} = 6$.

(ii) Under the assumption that these risks share the same rate i.e. $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$ then the mle for this is simply

$$\hat{\lambda} = \frac{51}{12} = 4.25$$

In order to compare these models we can use the scaled deviances to compare these models and use the chi-squared test.

The difference in the scaled deviance here should have a chi-square distribution with 3−1=2 degrees of freedom.

$$2\left(\log L_1 + \log L_2 + \log L_3 - \log L\right) = 10\log\hat{\lambda}_1 - 4\hat{\lambda}_1 + 17\log\hat{\lambda}_2 - 4\hat{\lambda}_2 + 24\log\hat{\lambda}_3 - 4\hat{\lambda}_3 - 51\log\hat{\lambda} + 12\hat{\lambda}$$

With the $\sum_{i=1}^{4}\log y_{1i}! + \sum_{i=1}^{4}\log y_{2i}! + \sum_{i=1}^{4}\log y_{3i}!$ cancelling out in the difference.

Hence

$$2\left(\log L_1 + \log L_2 + \log L_3 - \log L\right)$$

$$= 2\left(10\log 2.5 + 17\log 4.25 + 24\log 6 - 51\log\frac{51}{12} - \frac{4(10+17+24)}{4} + 12\frac{51}{12}\right)$$

$$= 2\left(10\log 2.5 + 17\log 4.25 + 24\log 6 - 51\log\frac{51}{12}\right) = 5.939778$$

This value is below 5.991 which is the critical value at the upper 5% level and therefore there is not a significant improvement by considering different rates for each risk.

Part (i) was answered very well. Most candidates struggled with part (ii).

8 The claims uplifted to 2011 prices are as follows:

Underwriting Year	*Development Year*			
	0	*1*	*2*	*3*
2008	520.93	343.98	122.85	41
2009	554.56	408.45	162	
2010	641.55	438		
2011	555			

Accumulating gives:

Underwriting Year	*Development Year*			
	0	*1*	*2*	*3*
2008	520.93	864.91	987.76	1028.76
2009	554.56	963.01	1125.01	
2010	641.55	1079.55		
2011	555			

Hence the development factors are given by:

$$DF_{0,1} = \frac{864.91 + 963.01 + 1079.55}{520.93 + 554.56 + 641.55} = 1.693304$$

$$DF_{1,2} = \frac{987.76 + 1125.01}{864.91 + 963.01} = 1.155833$$

$$DF_{2,3} = \frac{1028.76}{987.76} = 1.041508$$

The completed triangle of cumulative claims is:

Underwriting year	*Development Year*			
	0	*1*	*2*	*3*
2008	520.93	864.91	987.76	1028.76
2009	554.56	963.01	1125.01	1171.70
2010	641.55	1079.55	1247.78	1299.57
2011	555.00	939.78	1086.23	1131.32

Dis-accumlating gives (in 2011 prices):

Underwriting year	*Development Year*			
	0	*1*	*2*	*3*
2008				
2009				46.70
2010			168.23	51.79
2011		384.78	146.45	45.09

Inflating for future claims growth gives:

Underwriting year	*Development Year*			
	0	*1*	*2*	*3*
2008				
2009				51.37
2010			185.05	62.67
2011		423.26	177.20	60.01

And the outstanding claims are:

51.37+62.67+60.01+185.05+177.20+423.26 = 959.56

This question was tackled very well by most candidates

9 (i) The characteristic polynomial is $(1-\alpha Y)^3 = 0$.

This has a triple root at $1/\alpha$ and so the process is stationary when $\left|\frac{1}{\alpha}\right| > 1$ i.e. $|\alpha| < 1$.

(ii) Expanding the cubic equation and rearranging gives:

$$X_t - 3\alpha X_{t-1} + 3\alpha^2 X_{t-2} - \alpha^3 X_{t-3} = e_t$$

So the Yule-Walker equations give:

$$\rho_0 - 3\alpha\rho_1 + 3\alpha^2\,\rho_2 - \alpha^3\rho_3 = \sigma^2$$
$$\rho_1 - 3\alpha + 3\alpha^2\rho_1 - \alpha^3\rho_2 = 0 \quad \text{(A)}$$
$$\rho_2 - 3\alpha\rho_1 + 3\alpha^2 - \alpha^3\rho_1 = 0 \quad \text{(B)}$$
$$\rho_3 - 3\alpha\rho_2 + 3\alpha^2\,\rho_1 - \alpha^3\rho_0 = 0$$

So re-writing we have from (A) $\rho_1\left(1+3\alpha^2\right) - 3\alpha = \alpha^3\rho_2$

And substituting into (B) gives

$$\frac{\rho_1\left(1+3\alpha^2\right)-3\alpha}{\alpha^3} - 3\alpha\rho_1 + 3\alpha^2 - \alpha^3\rho_1 = 0$$

i.e. $$\frac{\rho_1(1+3\alpha^2-3\alpha^4-\alpha^6)}{\alpha^3} = \frac{3\alpha-3\alpha^5}{\alpha^3}$$

i.e. $$\rho_1 = \frac{3\alpha(1-\alpha^4)}{(1+3\alpha^2-3\alpha^4-\alpha^6)} = 0.83573487$$

And so

$$\rho_2 = \frac{3\alpha\left(1-\alpha^4\right)(1+3\alpha^2)}{(1+3\alpha^2-3\alpha^4-\alpha^6)\alpha^3} - \frac{3}{\alpha^2} = 0.576368876$$

Alternative solution:

Express the Yule-Walker equations in terms of the covariances:

$$X_t = 1.2X_{t-1} - 0.48X_{t-2} + 0.064X_{t-3} + e_t$$

$$\begin{aligned}
\gamma_0 &= 1.2\gamma_1 - 0.48\gamma_2 + 0.064\gamma_3 + \sigma^2 \\
\gamma_1 &= 1.2\gamma_0 - 0.48\gamma_1 + 0.064\gamma_2 \\
\gamma_2 &= 1.2\gamma_1 - 0.48\gamma_0 + 0.064\gamma_1 \\
\gamma_3 &= 1.2\gamma_2 - 0.48\gamma_1 + 0.064\gamma_0
\end{aligned}$$

Or in general:

$$\begin{aligned}
\gamma_0 &= 1.2\gamma_1 - 0.48\gamma_2 + 0.064\gamma_3 + \sigma^2 \\
\gamma_k &= 1.2\gamma_{k-1} - 0.48\gamma_{k-2} + 0.064\gamma_{k-3} \quad k \geq 1
\end{aligned}$$

Simplifying the second and third equations:

$$\begin{aligned}
148\gamma_1 &= 1.2\gamma_0 + 0.064\gamma_2 \Rightarrow \gamma_1 = \tfrac{30}{37}\gamma_0 + \tfrac{8}{185}\gamma_2 \\
\gamma_2 &= 1.264\gamma_1 + 0.064\gamma_1
\end{aligned}$$

To obtain:

$$\gamma_2 = \tfrac{200}{347}\gamma_0 \quad \lambda_1 = \tfrac{290}{347}\gamma_0$$

Dividing both by γ_0 gives the same solutions as above.

(iii) The series is an AR(3) series. The asymptotic behaviour is therefore that ρ_k decays exponentially to zero
whilst ϕ_k is zero for k>3.

The latter parts of this question were not particularly well answered. Candidates generally showed an understanding of how to solve the problem, but made a number of arithmetic and algebraic slips.

10 (i) $M_Y(t) = E\left(e^{tY}\right) = pE\left(e^{tX_1}\right) + (1-p)E\left(e^{tX_2}\right)$
$= pM_{X_1}(t) + (1-p)M_{X_2}(t)$

(ii) Let S_1, S_2 denote aggregate claims on the type 1 and type 2 policies respectively, and let N_1, N_2 denote the number of claims from type 1 and type 2 policies respectively. Let $S = S_1 + S_2$ denote the aggregate claims on the combined portfolio. We know that S_1, S_2 follow compound Poisson processes and so

$$M_{S_i}(t) = M_{N_i}(\log M_{X_i}(t)) = \exp(\lambda_i(M_{X_i}(t) - 1))$$

Now

$$M_S(t) = M_{S_1+S_2}(t) = M_{S_1}(t) M_{S_2}(t)$$

$$= \exp\left(\lambda_1\left(M_{X_1}(t) - 1\right)\right)\exp(\lambda_2(M_{X_2}(t) - 1))$$

$$= \exp\left[(\lambda_1 + \lambda_2)\left(\frac{\lambda_1}{\lambda_1+\lambda_2} M_{X_1(t)} + \frac{\lambda_2}{\lambda_1+\lambda_2} M_{X_2(t)} - 1\right)\right]$$

$$= \exp((\lambda_1 + \lambda_2)(pM_{X_1}(t) + (1-p)M_{X_2}(t) - 1) \text{ where } p = \frac{\lambda_1}{\lambda_1+\lambda_2}$$

$$= \exp((\lambda_1 + \lambda_2)(M_Y(t) - 1))$$

Where *Y* is defined as in part (i). This is of the form $M_N(logM_Y(t))$ where *N* is a Poisson distribution with parameter $\lambda_1 + \lambda_2$. Hence S has a compound Poisson distribution with rate $\lambda_1 + \lambda_2$ and where individual claim amounts are taken from distribution X_1 with probability $p = \frac{\lambda_1}{\lambda_1+\lambda_2}$ and from distribution X_2 with probability $1 - p = \frac{\lambda_2}{\lambda_1+\lambda_2}$.

(iii) **Step 1**

We first begin by generating a random sample from $N \sim P(25)$ as follows:

Let u be a random sample from a Uniform distribution on (0,1).

Find the positive integer i such that $P(N \le i-1) < u \le P(N \le i)$ (using the cumulative Poisson tables)

Then i is the simulated number of claims.

Step 2

Now we simulate the individual claim amount

Generate v a sample from a Uniform distribution on (0,1).

If $v \le \frac{10}{10+15} = \frac{10}{25} = 0.4$ then we have a type 1 claim otherwise we have a type 2 claim. Let the claim type be j.

Put $\mu_1 = 50$ and $\mu_2 = 70$. Generate w a sample from a uniform distribution on (0,1).

The simulated claim Z is given by setting

$$F_{X_j}(Z) = w$$

So $1 - \exp\left(\frac{1}{\mu_j} Z\right) = w$

So $Z = -\mu_j \ln(1-w)$

Step 3

Repeat Step 2 for a total of i samples and add the results.

Alternative algorithm: simulate the two results separately and add together at the end.

This question was not answered well. In particular, many candidates did not attempt part (iii). Of those that did, most had a good attempt at step 2, but very few got step 1 (to deduce the simulated number of claims).

11 (i) Let $S(t)$ denote cumulative claims to time *t*. Let the annual rate of premium income be *c* and let the insurer's initial surplus be *U*=100.

Then the surplus at time *t* is given by:

$$U(t) = U + ct - S(t)$$

And the relevant probabilities are defined by:

$$\psi(100) = P(U(t) < 0 \text{ for some } t > 0)$$
$$\psi(100,1) = P(U(t) < 0 \text{ for some } t \text{ with } 0 < t \le 1)$$
$$\psi_1(100,1) = P(U(1) < 0)$$

(ii) The adjustment coefficient is the unique positive root of the equation

$$\lambda M_X(R) = \lambda + cR$$

Where λ is the rate of the Poisson process (i.e. 100) and *X* is the normal distribution with mean 30 and standard deviation 5.

(iii) In this case we have:

$$c = 100 \times 30 \times 1.2 = 3600$$

And

$$M_X(R) = \exp(30R + 12.5R^2)$$

So *R* is the root of

$$100\exp\left(30R + 12.5R^2\right) - 100 - 3600R = 0$$

Denote the left hand side of this equation by *f*(*R*).

When *R* = 0.0115 we have

$$f(0.0115) = 100\exp(0.346653125) - 100 - 41.4 = 0.032604592 > 0$$

And when *R* = 0.0105 we have

$$f(0.0105) = 100\exp(0.316378125) - 100 - 37.8 = -0.585099862 < 0$$

Since the function changes sign between 0.0105 and 0.0115 the unique positive root must lie between these values and hence the root is 0.011 correct to 3 decimal places.

(iv) By Lundberg's inequality $\psi(100) < \exp(-100 \times 0.011) = 0.33287$

Claims in the first year are approximately Normal, with mean $100 \times 30 = 3000$

And variance given by $100 \times \left(25 + 30^2\right) = 92500$

So approximately

$$\psi_1(100,1) = P(100 + 3600 - N(3000, 92500) < 0)$$

$$= P\left(N(3000, 92500) > 3700\right) = P\left(N(0,1) > \frac{3700 - 3000}{\sqrt{92500}}\right)$$

$$= P(N(0,1) > 2.302)$$

$$= 1 - \left(0.98928 \times 0.8 + 0.98956 \times 0.2\right)$$

$$= 0.0107.$$

(v) The probability of ruin is much smaller in the first year than the long-term bound provided by Lundberg's inequality. This suggests that either the bound in Lundberg's inequality may not be that tight or that there is significant probability of ruin at times greater than 1 year.

In part (i) many candidates lost straightforward marks by failing to give sufficiently precise definitions. In particular, many candidates gave solutions along the lines of P(U(t)<0, t>0). It isn't clear whether this refers to all positive values of t or just some positive value.

Most candidates got part (ii).

For part (iii), many candidates were able to show that when R=0.011 the two sides of the equation are approximately equal. Very few were able to give a precise demonstration that the root is at R=0.011 by considering where the curve cross the axis. Candidates for future exams should note this technique carefully.

For part (iv) most candidates got the upper bound for R.

Part (v) was well answered by stronger candidates.

END OF EXAMINERS' REPORT