INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

15 April 2016 (am)

Subject CT6 – Statistical Methods
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 (i) Derive the median of a Pareto distribution with parameters α and λ. [3]

Let $\alpha = 2$ and $\lambda = 3$.

(ii) Comment on the skewness of this Pareto distribution. [3]
[Total 6]

2 A portfolio of insurance policies has two types of claims:

- Loss amounts for Type I claims are exponentially distributed with mean 120.
- Loss amounts for Type II claims are exponentially distributed with mean 110.

25% of claims are Type I, and 75% are Type II.

(i) Calculate the mean and variance of the loss amount for a randomly chosen claim. [3]

An actuary wants to model randomly chosen claims using an exponential distribution as an approximation.

(ii) Explain whether this is a good approximation. [2]
[Total 5]

3 A child playing a game believes that a six sided die is unfair, and that he has a probability $p > 1/6$ of predicting the outcome of any given throw. His mother is less sure, and her prior beliefs about p are as follows:

- a 1/3 chance that $p = 2/6$ and
- a 2/3 chance that $p = 1/6$

The child accurately predicts the results of 4 out of 10 dice throws.

Calculate the posterior probability that $p = 1/6$. [6]

4 Let us consider that we need to sample from a discrete random variable Y with distribution function:

Y	1	2	3
P	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

(i) Set out a direct method of sampling from Y. [2]

Consider now another random variable X with distribution:

X	1	2	3
P	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

(ii) Set out a direct method of sampling from X. [2]

(iii) (a) Explain how you can apply the acceptance-rejection method to sample X by rejecting/accepting samples from Y.

(b) Calculate how many samples from Y on average are needed to generate one sample from X. [6]

[Total 10]

5 (i) Explain why insurance companies make use of run-off triangles. [2]

(ii) The run-off triangle below shows incremental claims incurred on a portfolio of general insurance policies.

	Development Year			
Policy Year	*0*	*1*	*2*	*3*
2011	4,657	3,440	931	572
2012	6,089	5,275	1,381	
2013	5,623	4,799		
2014	7,224			

Calculate the outstanding claims reserve for this portfolio using the basic chain ladder method. [7]

[Total 9]

6 Felicity is a fund manager who is considering investing €1m in a specialist investment contract where the return depends on the performance of a particular company. She has a choice between two contracts as follows:

- Long contract: if the company is deemed a "success", the investment will return +100% and if it is deemed a "failure" it will return –75%.
- Short contract: if the company is deemed a "success", the investment will return –50% and if it is deemed a "failure" it will return +50%.

Before she decides which contract to invest in, Felicity will be able to observe the investment performance of the company's shares relative to the stock market. Companies that are "successes" have a 60% probability of outperforming the stock market. Companies that are "failures" have a 40% probability of outperforming the market.

(i) List Felicity's four decision functions. [2]

(ii) Calculate the values of the risk function for each decision function and type of company. [6]

Two thirds of such companies under consideration are known to be failures.

(iii) Determine Felicity's optimal decision function. [3]
[Total 11]

7 Claims on a portfolio of insurance policies arrive as a Poisson process with parameter λ, claim amounts having a Normal distribution with mean μ and variance σ^2, and there is a loading θ on premiums. The insurance company has an initial surplus of U.

(i) Explain carefully the meaning of $\Psi(U)$, $\Psi(U,t)$ and $\Psi(U,1)$. [3]

(ii) State four factors that affect the size of $\Psi(U,t)$, for a given t. [2]

(iii) Explain, for each factor, what happens to $\Psi(U,t)$ when the factor increases. [4]

Sarah, the insurance company's actuary, prefers to consider the probability of ruin in discrete rather than continuous time.

(iv) Explain an advantage and disadvantage of Sarah's approach. [2]
[Total 11]

8 (i) Show that:

$$\int_a^b \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(\ln x-\mu)^2} dx = e^{\mu+½\sigma^2}\left(\Phi\left(\frac{\ln b-\mu-\sigma^2}{\sigma}\right)-\Phi\left(\frac{\ln a-\mu-\sigma^2}{\sigma}\right)\right).$$ [4]

A general insurance company writes claims, whose amounts have a lognormal distribution, with mean 300 and standard deviation 400. The insurance company purchases excess of loss reinsurance with retention 500 per claim.

(ii) Calculate the average expected claim size payable by the insurance company. [6]

Next year, claim inflation is 10%, but the retention amount remains the same.

(iii) Explain whether the average expected claim size payable by the insurance company next year would increase by 10%. [2]
[Total 12]

9 Consider the following time series model:

$$Y_t = 1 + 0.6Y_{t-1} + 0.16Y_{t-2} + \varepsilon_t$$

where ε_t is a white noise process with variance σ^2.

(i) Determine whether Y_t is stationary and identify it as an ARMA(p,q) process. [3]

(ii) Calculate $E(Y_t)$. [2]

(iii) Calculate for the first four lags:

- the autocorrelation values ρ_1, ρ_2, ρ_3, ρ_4 and
- the partial autocorrelation values ψ_1, ψ_2, ψ_3, ψ_4. [7]

[Total 12]

10 (i) State the general expression of the exponential families of distributions and use this to derive the relevant expressions for the mean and the variance of these distributions. [6]

(ii) Extend the result in (i) to obtain an expression for the third central moment. [4]

(iii) Show that the following density function belongs to the exponential family of distributions: [4]

$$f(x) = \frac{\alpha^{\alpha}}{\mu^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x\frac{\alpha}{\mu}}.$$

(iv) Using the results in (i) and (ii) obtain the second and third central moments for this distribution. [4]

[Total 18]

END OF PAPER