INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

29 April 2015 (pm)

Subject CT6 – Statistical Methods
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a new page.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 The matrix below shows the losses to Player A in a two player zero sum game. The strategies for Player A are denoted I, II, III and IV.

		Player A			
		I	*II*	*III*	*IV*
	1	−10	4	−6	−3
Player B	2	−8	−6	*X*	*Y*
	3	−3	−7	−9	4

(i) Determine the values of X and Y for which there are dominated strategies for Player A. [4]

(ii) Determine whether there exist values of X and Y which give rise to a saddle point. [3]
[Total 7]

2 The table below shows cumulative claim amounts incurred on a portfolio of insurance policies.

Accident Year	*Development Year*			
	0	*1*	*2*	*3*
2011	1,509	1,969	2,106	2,207
2012	1,542	2,186	2,985	
2013	1,734	1,924		
2014	1,773			

Annual premiums written in 2014 were 4,013 and the ultimate loss ratio has been estimated as 93.5%. Claims can be assumed to be fully run off by the end of development year 3.

Estimate the total claims arising from policies written in 2014 only, using the Bornhuetter-Ferguson method. [7]

3 (i) (a) Explain why an insurance company might purchase reinsurance.
(b) Describe two types of reinsurance.
[3]

The claim amounts on a particular type of insurance policy follow a Pareto distribution with mean 270 and standard deviation 340.

(ii) Determine the lowest retention amount such that under excess of loss reinsurance the probability of a claim involving the reinsurer is 5%. [4]
[Total 7]

4 Let X be a random variable with density $f(x) = e^{-x}$ for $x > 0$.

(i) Construct an algorithm for generating random samples from X. [2]

A sequence of simulated observations is required from the density function
$h(x) = 2xe^{-x^2} \quad x > 0$.

(ii) Construct a procedure using the Acceptance-Rejection method to obtain the required observations. [5]

(iii) Calculate the expected number of pseudo-random numbers required to generate 10 observations from h using the algorithm in part (ii). [2]
[Total 9]

5 An insurance company has for five years insured three different types of risk. The number of policies in the j^{th} year for the i^{th} type of risk is denoted by P_{ij} for $i = 1, 2, 3$ and $j = 1, 2, 3, 4, 5$. The average claim size per policy over all five years for the i^{th} type of risk is denoted by $\bar{X}_i$. The values of P_{ij} and $\bar{X}_i$ are tabulated below.

	Number of policies					*Mean claim size*
Risk type i	*Year 1*	*Year 2*	*Year 3*	*Year 4*	*Year 5*	$\bar{X}_i$
1	17	23	21	29	35	850
2	42	51	60	55	37	720
3	43	31	62	98	107	900

The insurance company will be insuring 30 policies of type 1 next year and has calculated the aggregate expected claims to be 25,200 using the assumptions of Empirical Bayes Credibility Theory Model 2.

Calculate the expected annual claims next year for risks 2 and 3 assuming the number of policies will be 40 and 110 respectively. [9]

6 Annual numbers of claims on three different types of insurance policy follow a Poisson distribution with parameter μ_i for $i = 1, 2, 3$. Data for the last four years is given in the table below.

			Year		
Type	*1*	*2*	*3*	*4*	*Total*
1	5	5	0	1	11
2	2	5	4	5	16
3	5	6	4	5	20

(i) Derive the maximum likelihood estimate of μ_1 and calculate the corresponding estimates of μ_2 and μ_3. [5]

(ii) Test the hypothesis that μ_1, μ_2 and μ_3 are equal using the scaled deviance. [5]
[Total 10]

7 The following time series model is being used to model monthly data:

$$Y_t = Y_{t-1} + Y_{t-12} - Y_{t-13} + e_t + \beta_1 e_{t-1} + \beta_{12} e_{t-12} + \beta_1 \beta_{12} e_{t-13}$$

where e_t is a white noise process with variance σ^2.

(i) Perform two differencing transformations and show that the result is a moving average process which you may assume to be stationary. [3]

(ii) Explain why this transformation is called seasonal differencing. [1]

(iii) Derive the auto-correlation function of the model generated in part (i). [8]
[Total 12]

8 The number of claims, N, in a given year on a particular type of insurance policy is given by:

$$P(N = n) = 0.8 \times 0.2^n \quad n = 0, 1, 2, \ldots$$

Individual claim amounts are independent from claim to claim and follow a Pareto distribution with parameters $\alpha = 5$ and $\lambda = 1{,}000$.

(i) Calculate the mean and variance of the aggregate annual claims per policy. [4]

(ii) Calculate the probability that aggregate annual claims exceed 400 using:

(a) a Normal approximation.
(b) a Lognormal approximation.

[6]

(iii) Explain which approximation in part (ii) you believe is more reliable. [2]
[Total 12]

9 Let p be an unknown parameter and let $f(p|\underline{x})$ be the probability density of the posterior distribution of p given information $\underline{x}$.

(i) Show that under all-or-nothing loss the Bayes estimate of p is the mode of $f(p|\underline{x})$. [2]

John is setting up an insurance company to insure luxury yachts. In year 1 he will insure 100 yachts and in year 2 he will insure 100 + g yachts where g is an integer.

If there is a claim the insurance company pays a fixed sum of \$1m per claim.

The probability of a claim on a policy in a given year is p. You may assume that the probability of more than one claim on a policy in any given year is zero. Prior beliefs about p are described by a Beta distribution with parameters $\alpha = 2$ and $\beta = 8$.

In year 1 total claims are \$13m and in year 2 they are \$20m.

(ii) Derive the posterior distribution of p in terms of g. [4]

(iii) Show that it is not possible in this case for the Bayes estimate of p to be the same under quadratic loss and all-or-nothing loss. [6]

[Total 12]

10 Claims on a certain portfolio of insurance policies arise as a Poisson process with annual rate λ. Individual claim amounts are independent from claim to claim and follow an exponential distribution with mean μ. The insurance company has purchased excess of loss reinsurance with retention M from a reinsurer who calculates premiums using a premium loading of θ. Denote by X_i the amount paid by the reinsurer on the i^{th} claim (so that $X_i = 0$ if the i^{th} claim amount is below M).

(i) Explain why the claims arrival process for the reinsurer is also a Poisson process and specify its parameter. [3]

(ii) Show that

$$M_{X_i}(t) = 1 + e^{-M/\mu} \times \frac{\mu t}{1-\mu t}.$$

[4]

(iii) (a) Determine $E(X_i)$.

(b) Write down and simplify the equation for the reinsurer's adjustment coefficient. [6]

(iv) Comment on your results to part (iii). [2]

[Total 15]

END OF PAPER