INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

25 September 2014 (am)

Subject CT6 – Statistical Methods
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all nine questions, beginning your answer to each question on a new page.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 An insurance company has a portfolio of 240 insurance policies. The probability of a claim on the i^{th} policy in a year is p_i independently from policy to policy and there is no possibility of more than one claim. Claim amounts on the i^{th} policy follow an exponential distribution with mean $100/p_i$.

Let X denote the aggregate annual claims on the portfolio.

Determine the mean and variance of X. [6]

2 (i) List the three main components of a generalised linear model. [3]

(ii) Explain what is meant by a saturated model and discuss whether such a model is useful in practice. [3]
[Total 6]

3 Sara is a car mechanic for a racing team. She knows that there is a problem with the car, but is unsure whether the fault is with the gearbox or the engine. Sara is able to observe one practice race.

If the underlying problem is with the gearbox there is a 40% chance the car will not complete the practice race. If the underlying problem is with the engine there is a 90% chance the car will not complete the practice race.

At the end of the practice race Sara must decide, on the basis of whether the car completes the practice race, whether the fault lies with the gearbox or the engine.

(i) Write down the four decision functions Sara could adopt. [2]

If Sara correctly identifies the fault there is no cost. The cost of incorrectly deciding the fault is with the gearbox is £1m. The cost of incorrectly deciding the fault is with the engine is £5m.

(ii) Show that one of the decision functions is dominated. [3]

The probability that the fault lies with the gearbox is p.

(iii) Determine the range of values of p for which Sara will, under the Bayes criterion, choose a decision function whose outcome is affected by whether or not the car completes the practice race. [4]
[Total 9]

4 As part of a simulation study an actuary is asked to design an algorithm for simulating claims from a particular type of insurance policy. The probability distribution of the annual number of claims on a policy is given by:

	No claims	*One claim*	*Two claims*
Probability	0.4	0.4	0.2

The claim size distributions of the first and second claims are different. The size of the first claim follows an exponential distribution with mean 10. The size of the second claim follows a Weibull distribution with parameters $c = 1$ and $\gamma = 4$.

(i) Construct an algorithm to simulate the first claim on a given policy. [3]

(ii) Construct an algorithm to simulate the second claim on a given policy. [3]

(iii) Construct an algorithm to simulate the total annual claims on a given policy. [4]

[Total 10]

5 The table below shows the incremental claims incurred for a certain portfolio of insurance policies.

Accident year	*Development year*		
	0	*1*	*2*
2011	2,233	1,389	600
2012	3,380	1,808	
2013	4,996		

Cumulative numbers of claims are shown in the following table:

Accident year	*Development year*		
	0	*1*	*2*
2011	140	203	224
2012	180	230	
2013	256		

(i) Calculate the outstanding claim reserve for this portfolio using the average cost per claim method with grossing up factors. [7]

(ii) State the assumptions underlying the calculations in part (i). [3]

[Total 10]

6 For three years an insurance company has insured buildings in three different towns against the risk of fire damage. Aggregate claims in the j^{th} year from the i^{th} town are denoted by X_{ij} for i = 1, 2, 3 and j = 1, 2, 3. The data is given in the table below.

Town i	*Year j* 1	2	3
1	8,130	9,210	8,870
2	7,420	6,980	8,130
3	9,070	8,550	7,730

Calculate the expected claims from each town for the next year using the assumptions of Empirical Bayes Credibility Theory model 1. [10]

7 The random variable X follows a Pareto distribution with parameters α and λ.

(i) Show that for L, $d > 0$

$$\int_d^{L+d} xf(x)dx = \frac{\lambda^\alpha}{\alpha - 1}\left[\frac{d\alpha + \lambda}{(\lambda + d)^\alpha} - \frac{\alpha(L+d)+\lambda}{(\lambda + L + d)^\alpha}\right]$$ [5]

Claims on a certain type of motor insurance policy follow a Pareto distribution with mean 16,000 and standard deviation 20,000. The insurance company has an excess of loss reinsurance policy with a retention level of 40,000 and a maximum amount paid by the reinsurer of 25,000.

(ii) Determine the mean claim amount paid by the reinsurer on claims that involve the reinsurer. [8]

Claim amounts increase by 5%.

(iii) State the new distribution of claim amounts. [1]
[Total 14]

8 Claims on a portfolio of insurance policies follow a Poisson process with rate λ. Individual claim amounts follow a distribution X with mean μ and variance σ^2. The insurance company charges premiums of c per policy per year.

(i) Write down the equation satisfied by the adjustment coefficient R. [1]

(ii) Show that R can be approximated by

$$\hat{R} = \frac{2(c-\mu)}{\sigma^2+\mu^2}$$ [4]

Now suppose that individual claims follow a distribution given by

Value	10	20	50	100
Probability	0.3	0.5	0.15	0.05

The insurance company uses a premium loading of 30%. It is considering the following reinsurance arrangements:

A No reinsurance.

B Proportional reinsurance where the insurer retains 70% of all claims with a reinsurer using a 20% premium loading.

C Excess of loss reinsurance with retention 70 with a reinsurer using a 40% premium loading.

(iii) Determine which arrangement gives the insurance company the lowest probability of ultimate ruin, using the approximation in part (ii) [10]

(iv) Comment on your result in part (iii). [2]
[Total 17]

9 (i) List the main steps in the Box-Jenkins approach to fitting an ARIMA time series to observed data. [3]

Observations $x_1, x_2, \ldots, x_{200}$ are made from a stationary time series and the following summary statistics are calculated:

$$\sum_{i=1}^{200} x_i = 83.7 \qquad \sum_{i=1}^{200} (x_i - \bar{x})^2 = 35.4 \qquad \sum_{i=2}^{200} (x_i - \bar{x})(x_{i-1} - \bar{x}) = 28.4$$

$$\sum_{i=3}^{200} (x_i - \bar{x})(x_{i-2} - \bar{x}) = 17.1$$

(ii) Calculate the values of the sample auto-covariances $\hat{\gamma}_0, \hat{\gamma}_1$ and $\hat{\gamma}_2$. [3]

(iii) Calculate the first two values of the partial correlation function $\hat{\phi}_1$ and $\hat{\phi}_2$. [3]

The following model is proposed to fit the observed data:

$$X_t - \mu = a_1 (X_{t-1} - \mu) + e_t$$

where e_t is a white noise process with variance σ^2.

(iv) Estimate the parameters μ, a_1 and σ^2 in the proposed model. [5]

After fitting the model in part (iv) the 200 observed residual values $\hat{e}_t$ were calculated. The number of turning points in the residual series was 110.

(v) Carry out a statistical test at the 95% significance level to test the hypothesis that $\hat{e}_t$ is generated from a white noise process. [4]

[Total 18]

END OF PAPER