EXAMINATION

15 September 2005 (am)

Subject CT6 — Statistical Methods
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

1 Claims occur on a portfolio of insurance policies according to a Poisson process at a rate λ. All claims are for a fixed amount d, and premiums are received continuously. The insurer's initial surplus is U ($< d$) and the annual premium income is $1.2\lambda d$. Show that the probability that ruin occurs at the first claim is:

$$1 - e^{-\frac{1}{1.2}\left(1-\frac{U}{d}\right)}.$$ [4]

2 $Y_1, Y_2, \ldots, Y_n$ are independent random variables, and

$Y_i \sim$ Poisson, mean μ_i.

The fitted values for a particular model are denoted by $\hat{\mu}_i$. Derive the form of the scaled deviance. [5]

3 A manufacturer of specialist products for the retail market must decide which product to make in the coming year. There are three possible choices — basic, deluxe or supreme — each with different tooling up costs. The manufacturer has fixed overheads of £1,300,000.

The revenue and tooling up costs for each product are as follows:

	Tooling up costs £	*Revenue per item sold* £
Basic	100,000	1.00
Deluxe	400,000	1.20
Supreme	1,000,000	1.50

Last year the manufacturer sold 2,100,000 items and is preparing forecasts of profitability for the coming year based on three scenarios: Low sales (70% of last year's level); Medium sales (same as last year) and High sales (15% higher than last year).

(i) Determine the annual profits in £ under each possible combination. [3]

(ii) Determine the minimax solution to this problem. [2]

(iii) Determine the Bayes criterion solution based on the annual profit given the probability distribution: P(Low) = 0.25; P(Medium) = 0.6 and P(High) = 0.15. [2]

[Total 7]

4 XYZ bank are about to offer a new mortgage product to consumers with a poor creditrating. They currently offer a similar product to customers with normal credit ratings. The normal product charges all customers a Standard Variable Rate (SVR) of 6% which moves up and down in line with short term interest rates. In addition there is a maximum "loan to value" of 90% — in other words the customer cannot borrow more than 90% of the value of the property. For loans above this level an additional Mortgage Indemnity Guarantee insurance premium must be paid — this protects the bank in the event that the borrower defaults and the value of the property has fallen. There are no other charges on the normal product.

The bank intends to use its experience from the normal business as a basis for setting terms on the new product.

(i) List the factors the bank should take into account when setting terms on the new product compared with the "normal" business. [3]

(ii) Suggest ways in which the bank may mitigate the additional risks associated with this product. [4]

[Total 7]

5 An insurer believes that claims from a particular type of policy follow a Pareto distribution with parameters $\alpha = 2.5$ and $\lambda = 300$. The insurer wishes to introduce a deductible such that 25% of losses result in no claim on the insurer.

(i) Calculate the size of the deductible. [4]

(ii) Calculate the average claim amount net of the deductible. [6]

[Total 10]

6 The number of claims on a portfolio of washing machine insurance policies follows a Poisson distribution with parameter 50. Individual claim amounts for repairs are a random variable $100X$ where X has a distribution with probability density function

$$f(x) = \begin{cases} \frac{3}{32}(6x - x^2 - 5) & 1 < x < 5 \\ 0 & \text{otherwise} \end{cases}$$

In addition, for each claim (independently of the cost of the repair) there is a 30% chance that an additional fixed amount of £200 will be payable in respect of water damage.

(i) Calculate the mean and variance of the total individual claim amounts. [7]

(ii) Calculate the mean and variance of the aggregate claims on the portfolio. [3]

[Total 10]

7 An insurer operates a simple no claims discount system with 5 levels: 0%, 20%, 40%, 50% and 60%.

The rules for moving between levels are:

- An introductory discount of 20% is available to new customers.
- If no claims are made during a year the policyholder moves up to the next discount level or remains at the maximum level.
- If one or more claims are made during the year, a policyholder at the 50% or 60% discount level moves to the 20% level and a policyholder at 0%, 20% or 40% moves to or remains at the 0% level.

The full annual premium is £600.

When an accident occurs, the distribution of loss is exponential with mean £1,750. A policyholder will only claim if the loss is greater than the extra premiums over the next four years, assuming no further accidents occur.

(i) For each discount level, calculate the smallest cost for which a policyholder will make a claim. [3]

(ii) For each discount level, calculate the probability of a claim being made in the event of an accident occurring. [3]

(iii) Comment on the results of (ii). [2]

(iv) Currently, equal numbers of customers are in each discount level and the probability of a policyholder not having an accident each year is 0.9. Calculate the expected proportions in each discount level next year. [4]

[Total 12]

8 The following time series model is used for the monthly inflation rate (Y_t) in a particular country:

$$Y_t = 0.4Y_{t-1} + 0.2Y_{t-2} + Z_t + 0.025$$

where $\{Z_t\}$ is a sequence of uncorrelated identically distributed random variables whose distributions are normal with mean zero.

(i) Derive the values of p, d and q, when this model is considered as an ARIMA(p, d, q) model. [3]

(ii) Determine whether $\{Y_t\}$ is a stationary process. [2]

(iii) Assuming an infinite history, calculate the expected value of the rate of inflation over this. [1]

(iv) Calculate the autocorrelation function of $\{Y_t\}$. [5]

(v) Explain how the equivalent infinite-order moving average representation of $\{Y_t\}$ may be derived. [2]

[Total 13]

9 The claims paid to date on a motor insurance policy are as follows (Figures in £000s):

	Development year			
Policy Year	*0*	*1*	*2*	*3*
2001	1,256	945	631	378
2002	1,439	1,072	723	
2003	1,543	1,133		
2004	1,480			

Inflation for the 12 months to the middle of each year was as follows:

2002	2.10%
2003	1.20%
2004	–0.80%

Annual premiums written in 2004 were £5,250,000.

Future inflation from mid-2004 is estimated to be 2.5% per annum.

The ultimate loss ratio (based on mid-2004 prices) has been estimated at 75%.

Claims are assumed to be fully run-off at the end of development year 3.

Estimate the outstanding claims arising from policies written in 2004 only (taking explicit account of the inflation statistics in **both** cases), using:

(i) The chain ladder method. [9]

(ii) The Bornhuetter-Ferguson method. [7]

[Total 16]

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10 The total amounts claimed each year from a portfolio of insurance policies over n years were $x_1, x_2, \ldots, x_n$. The insurer believes that annual claims have a normal distribution with mean θ and variance σ_1^2, where θ is unknown. The prior distribution of θ is assumed to be normal with mean μ and variance σ_2^2.

(i) Derive the posterior distribution of θ. [4]

(ii) Using the answer in (a), write down the Bayesian point estimate of θ under quadratic loss. [2]

(iii) Show that the answer in (b) can be expressed in the form of a credibility estimate and derive the credibility factor. [2]

The claims experience over five years for two companies was as follows:

	Year	*1*	*2*	*3*	*4*	*5*
Company A	Amount	421	417	438	456	463
Company B	Amount	343	335	356	366	380

(iv) Determine the Bayes credibility estimate of the premiums the insurer should charge for each company based on the modelling assumptions of part (i), a profit loading of 25% and the following parameters:

	Company A	*Company B*
μ	400	300
σ_1^2	500	350
σ_2^2	800	600

[6]

(v) Comment on the effect on the result of increasing σ_1^2 and σ_2^2. [2]

[Total 16]

END OF PAPER