INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2018

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Luke Hatter
Chair of the Board of Examiners
June 2018

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Contingencies subject is to provide a grounding in the mathematical techniques which can be used to model and value cashflows dependent on death, survival, or other uncertain risks.

2. CT5 introduces the fundamental building blocks of all life insurance and pensions actuarial work.

B. **General comments on *student performance in this diet of the*** examination

In general, students who had prepared properly for this examination did well and passed relatively easily. However, it was clear that a large number of students were sitting CT5 in advance of the transition to Curriculum 19 (there was a virtually double normal intake this session). There was considerable evidence that many such attempts were poorly prepared for, resulting in very low marks.

Most questions were easily answered. The ones which gave most difficulty were Q7, 8(ii), 9(ii), 10 and 12(i) and it is hoped the detailed solutions will assist students in future preparation.

C. Pass Mark

The Pass Mark for this exam was 57.

Solutions

Q1

$$_{2.75}p_{84.5} = {}_{0.5}p_{84.5} \times p_{85} \times p_{86} \times {}_{0.25}p_{87}$$

$$= (p_{84})^{0.5} \times p_{85} \times p_{86} \times (p_{87})^{0.25}$$

$$= (1-q_{84})^{0.5} \times (1-q_{85}) \times (1-q_{86}) \times (1-q_{87})^{0.25}$$

$$= (1-0.101007)^{0.5} \times (1-0.110600) \times (1-0.120929) \times (1-0.132028)^{0.25}$$

$$= 0.94815 \times 0.88940 \times 0.87907 \times 0.96522$$

$$= 0.71553$$

$$_{2.75}q_{84.5} = 1 - {}_{2.75}p_{84.5}$$

$$= 0.28447$$

[2 marks for first 2 lines; 1 for result]
[Total 3]

Straightforward and generally well answered. The most common error was not using the method asked for and using the uniform distribution of deaths method instead. This was given minor credit if done correctly.

Q2 The following are three types of reversionary bonuses. The bonuses are usually allocated annually in arrears, following a valuation.

Simple – the rate of bonus each year is a percentage of the initial (basic) sum assured under the policy. The effect is that the sum assured increases linearly over the term of the policy. [1]

Compound – the rate of bonus each year is a percentage of the initial (basic) sum assured and the bonuses previously added. The effect is that the sum assured increases exponentially over the term of the policy. [1]

Super compound – two compound bonus rates are declared each year. The first rate (usually the lower) is applied to the initial (basic) sum assured. The second rate is applied to bonuses previously added. The effect is that the sum assured increases exponentially over the term of the policy. The sum assured usually increases more slowly than under a compound allocation in the earlier years and faster in the later years. [2]
[Total 4]

(Note: credit to be given if special reversionary bonus mentioned.)

> *Relatively well answered. To score full marks the exponential growth aspect in Super Compound needed to be mentioned.*

Q3 Net retrospective reserve = accumulated value of single premium less accumulated value of benefits provided [½]

$$= \frac{l_{[50]} \times (1+i)}{l_{[50]+1}} \left[7500 a_{[50]:\overline{10}|} - 7500 a_{[50]:\overline{1}|} \right] \qquad [1½]$$

$$= 7500 \times \frac{9706.0977}{9686.9669} \times 1.04 \times 7.0048 = 54,745.03 \qquad [½]$$

$$\text{where } a_{[50]:\overline{10}|} = \ddot{a}_{[50]:\overline{10}|} - 1 + v^{10} \frac{l_{60}}{l_{[50]}} = 8.318 - 1 + 0.67556 \times \frac{9287.2164}{9706.0977} = 7.9644 \qquad [1]$$

$$\text{and } a_{[50]:\overline{1}|} = v \frac{l_{[50]+1}}{l_{[50]}} = 0.96154 \times \frac{9686.9669}{9706.0977} = 0.9596 \qquad [½]$$

[Total 4]

> *Reasonable marks scored but many had errors in the first formula above, particularly using just 7500 without the 1 year annuity factor in accumulated value of benefits provided.*

Q4

- Occupation determines a person's environment for 40 or more hours each week.
    - The environment may be rural or urban.
    - The occupation may involve exposure to harmful substances e.g. chemicals, or to potentially dangerous situations e.g. working at heights.
    - Much of this is moderated by health and safety at work regulations.

- Some occupations are more healthy by their very nature e.g. bus drivers have a sedentary and stressful occupation while bus conductors are more active and less stressed.

- Some work environments e.g. publicans, give exposure to a less healthy lifestyle.

- Some occupations by their very nature attract more healthy or unhealthy workers. This may be accentuated by health checks made on appointment or by the need to pass regular health checks e.g. airline pilots.

- This effect can be produced without formal checks, e.g. former miners who have left the mining industry as a result of ill-health and then chosen to sell newspapers. This will inflate the mortality rates of newspaper sellers.

- A person's occupation largely determines their income, and this permits them to adopt a particular lifestyle e.g. content and pattern of diet, quality of housing. This effect can be positive and negative e.g. over indulgence.

[1 for each bullet; maximum 5]

Most students covered some of the points but generally omitted some as well. Other valid points were credited especially the relationship between level of education and health.

Q5 (i) The death strain at risk (DSAR) per annuity is given by:

$$\left[0-(\text{payment due 31.12.17}+\text{reserve at 31.12.17})\right]$$
$$=[0-30{,}000-30{,}000a_{73}]=-30{,}000\left(1+9.288\right)=-308{,}640$$ [1]

Expected death strain (EDS) =
$-q_{72}\times 5{,}650\times 308{,}640=-(0.01838)\times 5{,}650\times 308{,}640=-32{,}051{,}338$ [1]

Actual death strain (ADS) $=-80\times 308{,}640=-24{,}691{,}200$ [1]

Profit = EDS–ADS = –32,051,338 + 24,691,200 = – 7,360,138 i.e. a mortality loss [1]

(ii) The company expected 103.8 deaths during 2017 and experienced fewer than this (i.e. 80). There is no death benefit for the annuity. However, there is a release of reserves on death, so fewer actual deaths than expected leads to a mortality loss. [2]

[Total 6]

Part (i) was done well by many well-prepared students. The explanation in (ii) was generally not well stated, especially the release of reserves point.

Q6 (i) (a) $E\left[\dfrac{1-v^{\min(K_x+1,n)}}{d}\right]$ [1]

(b) $E\left[v^{\min(T_x,T_y)}\right]$ [1]

where:

x = age first life, y=age second life
n = duration of temporary annuity
K_x = random variable for curtate duration of life
T_x , T_y = random variables for complete duration of life

[½ for each definition]
[Total 4]

(ii) (a) $\bar{Z} = \bar{a}_{\overline{T_y}|} - \bar{a}_{\overline{T_x}|}$ if $T_y > T_x$

$= 0$ otherwise [2]

(b) $E(\bar{Z}) = \int_0^{\infty} (v^t \, {}_t p_{xy} \mu_{x+t} \bar{a}_{y+t}) dt$ [2]

[Total 8]

Generally well answered. There were many different correct ways to express the answers all of which were given credit.

Q7 (i) $\text{EPV} = \dfrac{500 \times n \times (3 \times M_x^r + 2 \times M_x^i) + 500 \times (3 \times \bar{R}_x^r + 2 \times \bar{R}_x^i)}{D_x}$ [3]

(ii) $\text{EPV} = \dfrac{500 \times 15 \times (3 \times M_{45}^r + 2 \times M_{45}^i) + 500 \times (3 \times \bar{R}_{45}^r + 2 \times \bar{R}_{45}^i)}{D_{45}}$

$= \dfrac{500 \times 15 \times (3 \times 782 + 2 \times 334) + 500 \times (3 \times 13773 + 2 \times 3916)}{2329}$

$= \dfrac{22605000 + 24575500}{2329}$

$= 20257.8$

[1 for line 1; 2 for calculation]

(iii) If C is the contribution then:

$C \times \dfrac{\bar{N}_{45}}{D_{45}} = 20257.8$

$$\Rightarrow C \times \frac{26693}{2329} = 20257.8$$

Hence $C = \dfrac{20257.8 \times 2329}{26693} = 1767.52$

[1 for line 2; 1 for calculation]
[Total 8]

> *Poorly answered for a straightforward question. . A very large percentage of students used pension annuity and salary scaled functions which was totally wrong given the question was about a lump sum benefit without scaling.*

Q8 (i) EPV $= 20000\int_0^{20}(0.03 \times e^{-0.08t})dt - 10000\int_0^{10}(0.03 \times e^{-0.08t})dt$

$$= 600 \times \left[-\frac{e^{-0.08t}}{0.08}\right]_0^{20} - 300 \times \left[-\frac{e^{-0.08t}}{0.08}\right]_0^{10}$$

$$= 7500 \times \left(1 - e^{-1.6}\right) - 3750 \times \left[1 - e^{-0.8}\right]$$

$$= 3750 + (3750 \times 0.44933) - (7500 \times 0.20190)$$

$$= 3920.8 \text{ or say } 3921.$$

[1 mark lines 1 and 2, ½ mark line 3 and ½ mark for answer]

(ii) First calculate 2nd moment at interest rate ∂ of 10%

Value $= (10000)^2\int_0^{10} 0.03 \times e^{-0.13t}dt + (20000)^2 \times \int_{10}^{20} 0.03 \times e^{-0.13t}dt$

$$= 0.03 \times (10)^8 \times \left(\frac{1 - e^{-1.3}}{0.13}\right) + 0.03 \times 4 \times (10)^8 \times \left(\frac{e^{-1.3} - e^{-2.6}}{0.13}\right)$$

$$= 16787727.9 + 18300758.3$$

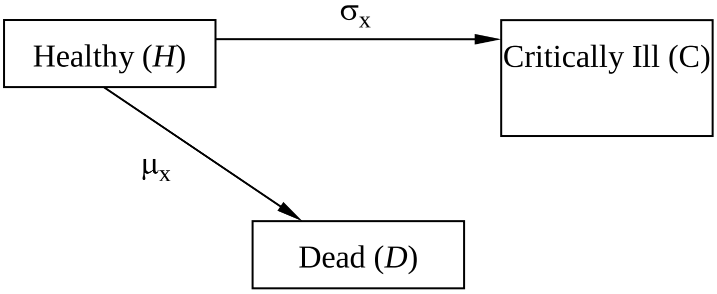
$$= 35088486.2$$

Variance $= 35088486.2 - (3920.8)^2 = 19715813.6 = (4440)^2$

[2 mark lines 1 and 2, 1 mark for answer]
[Total 8]

Part (i) was generally done well. Part (ii) was done less well as the approach was different to (i) with the necessity to return to first principles to ensure the squaring of the benefit was properly allowed for. The answer in (ii) was very susceptible to rounding and other close values were accepted provided the method was correct.

Q9 (i)



[2½ mark plus ½ for not including the irrelevant mortality of C]

(ii) The expected present value of premiums is:

$$(750/12)\times\sum_{1}^{240} e^{-.046(t-1)/12} = (750/12)\times\left(\frac{1-e^{-0.92}}{1-e^{-(.046/12)}}\right)$$

$$= 62.5\times\left(\frac{1-0.398519}{1-0.996174}\right) = 9825.56$$

[3]

The expected present value of benefits is:

$$\text{EPV} = 100000\times\int_0^{20} e^{-0.04t}\, {}_tp_{45}^{HH}\, m_{45+t}\,dt + 50000\times\int_0^{20} e^{-0.04t}\, {}_tp_{45}^{HH}\, s_{45+t}\,dt$$

$$= (100000\times 0.004 + (50000\times 0.002))\times\int_0^{20} e^{-0.046t}\,dt$$

$$= 500\times 13.07567$$

$$= 6537.8$$ [2]

So value to company = 9,825.6 – 6,537.8 = 3,288 rounded [1]
[Total 9]

Part (i) was very straightforward. Part (ii) was more complex and many students struggled to develop the correct formulae.

Q10 Value of benefits:

$$\text{EPV} = v^{4.25} \times \frac{l_{65}}{l_{60.75}} \times \left[12,000 \times \ddot{a}^{(2)}_{\overline{5}|} + 12,000 \times v^5 \times \frac{l_{70}}{l_{65}} \times \ddot{a}^{(2)}_{70:\overline{5}|} + 10,000 \times v^{10} \times \frac{l_{75}}{l_{65}} \times (1 + \ddot{a}^{(2)}_{75}) \right]$$

$$= v^{4.25} \times \frac{9703.708}{9833.230} \times \left[12,000 \times 4.58483 + 12,000 \times 0.82193 \times \frac{9392.621}{9703.708} \times 4.471 + 10,000 \times 0.67556 \times \frac{8784.955}{9703.708}(1+10.683) \right]$$

$$= 141,303$$ [6]

where:

$$l_{60.75} = \left(0.75 l_{61} + 0.25 l_{60}\right) = \left(0.75 \times 9828.163 + 0.25 \times 9848.431\right) = 9833.230$$

$$\ddot{a}^{(2)}_{\overline{5}|} = \frac{\left(1 - v^5\right)}{d^{(2)}} = \frac{(1 - 0.82193)}{0.038839} = 4.58483$$

$$\ddot{a}^{(2)}_{70:\overline{5}|} = (\ddot{a}_{70} - \frac{1}{4}) - v^5 \times \frac{l_{75}}{l_{70}} \times (\ddot{a}_{75} - \frac{1}{4})$$

$$= (12.934 - 0.25) - 0.82193 \times \frac{8784.955}{9392.621} \times (10.933 - 0.25)$$

$$= 4.471$$

$$\ddot{a}^{(2)}_{75} = 10.933 - 0.25 = 10.683$$

Value of premiums:

Let quarterly premium = *P*

Value of premiums:

$$= P \times (1 + v^{1/4} \times \frac{l_{61}}{l_{60.75}} \times 4 \times \ddot{a}^{(4)}_{61:\overline{4}|})$$

$$= P \times (1 + 0.99025 \times \frac{9828.163}{9833.230} \times 4 \times ((16.311 - 0.375)$$

$$- 0.85480 \times \frac{9703.708}{9828.163} \times (14.871 - 0.375)))$$

$$= P \times (1 + 3.95896 \times (15.936 - 12.234))$$

$$= 15.656P$$

Hence:

Quarterly premium = 141,303/ 15.656 = 9026 or 9030 approximately [3]

[Total 9]

> *This higher skills question was generally poorly done. Credit was given for correct formulae and approach even if all the arithmetic was not accurate.*

Q11 (i) Let P be the monthly premium. Then:

EPV of premiums:

$$12P\ddot{a}^{(12)}_{[35]:\overline{30}|} \; @ \; 4\% = 207.5844P$$

where:

$$\ddot{a}^{(12)}_{[35]:\overline{30}|} = \ddot{a}_{[35]:\overline{30}|} - \frac{11}{24}\left(1 - {}_{30}p_{[35]}v^{30}\right)$$

$$= 17.631 - \frac{11}{24}\left(1 - \frac{8821.2612}{9892.9151} \times 0.30832\right) = 17.2987$$ [2]

EPV of benefits and claim expense:

$$= 90,295 \times \bar{A}^{1}_{[35]:\overline{30}|} + 45,000 \times v^{15} \times {}_{15}p_{[35]}\bar{A}^{1}_{50:\overline{15}|}$$

$$= 90,295 \times (1.04)^{0.5}\left[A_{[35]} - v^{30} \times {}_{30}p_{[35]}A_{65}\right]$$

$$+45,000\times(1.04)^{0.5}\times v^{15}\times {}_{15}p_{[35]}\left[A_{50}-v^{15}\times {}_{15}p_{50}A_{65}\right]$$

$$=90,295\times(1.04)^{0.5}\left[0.19207-0.30832\times\frac{8821.2612}{9892.9151}\times 0.52786\right]$$

$$+45,000\times(1.04)^{0.5}\times 0.55526$$
$$\times\frac{9712.0728}{9892.9151}\left[0.32907-0.55526\times\frac{8821.2612}{9712.0728}\times 0.52786\right]$$

$$=90,295\times(1.04)^{0.5}\times 0.04695+45,000\times(1.04)^{0.5}\times 0.54511\times 0.06285$$

$$=4323.306+1572.239=5895.545$$ [3]

EPV of expenses (at 4% unless otherwise stated)

$$=0.5\times 12P+375+0.025\times 12P\ddot{a}^{(12)}_{[35]:\overline{30}|}-0.025\times 12P\ddot{a}^{(12)}_{[35]:\overline{1}|}+72\left[\ddot{a}^{@0\%}_{[35]:\overline{30}|}-1\right]$$

$$=6P+375+0.025\times 12P\times 17.2987-0.025\times 12P\times 0.98212+72\times 28.1751$$

$$=6P+375+5.18961P-0.29464P+2028.607$$

$$=10.89497P+2403.607$$

where:

$$\ddot{a}^{(12)}_{[35]:\overline{1}|}=\ddot{a}_{[35]:\overline{1}|}-\frac{11}{24}\left(1-v\times p_{[35]}\right)$$

$$=1-\frac{11}{24}\left(1-\frac{9887.2069}{9892.9151}\times 0.96154\right)=0.98212$$

$$\ddot{a}^{@0\%}_{[35]:\overline{30}|}-1=\frac{1}{l_{[35]}}\left(l_{[35]+1}+\dots+l_{64}\right)=e_{[35]}-\frac{l_{64}}{l_{[35]}}e_{64}$$

$$=43.909-\frac{8934.8771}{9892.9151}\times 17.421=28.1751$$ [4½]

Equation of value gives:

$$207.584P=5895.545+10.89497P+2403.607$$

$$\Rightarrow P=\frac{8299.152}{196.689}=42.19$$ [½]

[Total 10]

Well done by fully prepared students. Credit was given for developing the correct methods.

Q12 (i)

$$150,000\left[1+0.04\left(K_{[25]}+1\right)\right]v^{T_{[25]}}+315v^{T_{[25]}}+265+0.05P\left(\ddot{a}_{\overline{K_{[25]}+1|}}-1\right)-P\ddot{a}_{\overline{K_{[25]}+1|}}$$

[1] [½] [½] [½] [½]

where P is the annual premium.

(ii) The annual gross premium P is given by:

$$P\ddot{a}_{[25]}=150,315\bar{A}_{[25]}+6,000\left(I\bar{A}\right)_{[25]}+265+0.05Pa_{[25]}$$ [3]

$$\Rightarrow P\times 16.662=150,315\times 1.06^{0.5}\times 0.05686+6,000\times 1.06^{0.5}\times 2.40151$$
$$+265+0.05P\times(16.662-1)$$

$$\Rightarrow 15.8789P=8799.583+14835.035+265$$

$$\Rightarrow P=1505.12$$ [1]

(iii) After 5 years, the total bonus on the policy is given by:

$$150,000\times\left[0.04\times 2+0.0375+0.035+0.03\right]=27,375$$

Therefore, the gross premium prospective reserve is:

$$177,700\bar{A}_{30}+4,500\left(I\bar{A}\right)_{30}-0.95\times 1,505.12\times\ddot{a}_{30}$$ [3]

$$=177,700\times 1.04^{0.5}\times 0.16023+4,500\times 1.04^{0.5}\times 6.91559$$
$$-0.95\times 1,505.12\times 21.834$$

$$=29,036.745+31,736.456-31,219.651$$

$$=29,553.55$$ [1]

[Total 11]

> *Part (i) was poorly done-students traditionally have problems in developing random variable expressions. The other parts were done well by properly prepared students.*

Q13 Multiple decrement table

x	q_x^d	q_x^s
62	0.010112	0.100
63	0.011344	0.050
64	0.012716	0.000

x	$(aq)_x^d$	$(aq)_x^s$	$(ap)_x$	$_{t-1}(ap)_{62}$
62	0.010112	0.09899	0.890899	1.000000
63	0.011344	0.04943	0.939223	0.890899
64	0.012716	0.00000	0.987284	0.836753

[2]

Unit fund

Year	*1*	*2*	*3*
Fund at the start of the year	0	5332.635	10875.909
Premium allocation	5400.000	5400.000	5400.000
B/O spread	270.00	270.00	270.00
Interest	256.500	523.132	800.295
Management charge	53.865	109.858	168.062
Fund at the end of the year	5332.635	10875.909	16638.142

[½ mark for each line]
[Total 3]

Non-unit fund before reserves

Year	*1*	*2*	*3*
Unallocated premium + B/O spread	870.00	870.00	870.00
Expenses	525.00	215.00	215.00
Interest	10.350	19.650	19.650
Extra death cost	67.420	12.752	0
Extra maturity cost	0	0	1642.657
Management charge	53.865	109.858	168.062
End of year cash flow	341.795	771.756	–799.945

[½ mark for each line; ½ extra for death cos]
[Total 4]

Reserve required at the start of year 3 = 799.945 / 1.03 =776.646 [1]

Reduced profit at the end of year 2 = 771.756 – 776.646 × $(ap)_{63}$ = 42.312 [1]

Revised profit vector: $(341.795, 42.312, 0)$ [½]

Net present value = $\dfrac{341.795}{1.07}+\dfrac{42.312\times(ap)_{62}}{1.07^2}=352.359$ [1½]

Present value of premiums = $6000\times\left(1+\dfrac{(ap)_{62}}{1.07}+\dfrac{{}_2(ap)_{62}}{1.07^2}\right)=15,380.812$

[1½]

Profit margin = $\dfrac{352.359}{15380.812}=2.29\%$ [½]

[Total 6]
[Total 15]

This question was generally done reasonably well by prepared students. The main errors other than arithmetical were omitting calculating dependant decrements and the revision of the profit vector. Again the correct approach earned substantial credit even where there were arithmetic errors.

END OF EXAMINERS' REPORT