# EXAMINATION

April 2006

## Subject CT5 — Contingencies
## Core Technical

**Introduction**

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

M Flaherty
Chairman of the Board of Examiners

June 2006

**Comments**

No comments are given.

**1** $$10,000\int_0^\infty e^{-\delta t}\left({}_t p_x^{aa}\mu_{x+t} + {}_t p_x^{ar}\nu_{x+t}\right)dt$$

**2** (i) The super compound bonus method is a method of allocating bonuses (mostly these days on an annual basis) under which two bonus rates are declared each year. The first rate, usually the lower, is applied to the basic sum assured and the second rate is applied to the bonuses already declared.

(ii) The sum assured and bonuses increase more slowly than under other methods for the same ultimate benefit, enabling the office to retain surplus for longer and thereby providing greater investment freedom.

**3** (a) $\mu_{65:60} = \mu_{65} + \mu_{60} = 0.005543 + 0.002266 = 0.007809$

(b) $${}_5 p_{65:60} = \frac{l_{70}}{l_{65}}.\frac{l_{65}}{l_{60}} = \frac{9238.134}{9647.797}.\frac{9647.797}{9826.131} = 0.940160$$

(c) $${}_2 q^{1}_{65:65} = ½.{}_2 q_{65:65} = ½.(1 - {}_2 p_{65:65})$$

$$= ½(1 - {}_2 p_{65}\cdot{}_2 p_{65}) = ½\left(1 - \frac{9521.065}{9647.797}.\frac{9521.065}{9647.797}\right) = 0.013050$$

**4** Overhead expenses are those that in the short term do not vary with the amount of business.

An example of an overhead expense is the cost of the company's premises (as the sale of an extra policy now will have no impact on these costs).

Direct expenses are those that do vary with the amount of business.
An example of a direct expense is commission payment to a direct salesman (as the sale of an extra policy now will have an impact on these costs).

**5** The expected share of the fund is

$$\frac{10,000(1.04)^{25}.A^{1}_{[30]:\overline{25}|}}{\cdot_{25}p_{[30]}}$$

$$=\frac{10,000(1.04^{25}(A_{[30]}-v^{25}\cdot_{25}p_{[30]}.A_{55}))}{_{25}p_{[30]}}$$

$$=10,000\left[\frac{2.66584(0.16011-0.37512x\,{}^{9557.8179}\!/\!_{9923.7497}\,x0.38950)}{\frac{9557.8179}{9923.7497}}\right]$$

$$=536.65$$

**6** The insurer should expect to find:

Time selection — the experience would be different in different time periods; in developed economies mortality has tended to improve with time.

Class selection — The insurer may price policies differently depending on fixed factors such as age/sex. Also different groups of recipients may have different mortality based on factors such as occupation.

Temporary Initial Selection — if there is no evidence of health required then there is an expectation that poor lives would be likely to take out the insurance and in the short term the experience would be adverse. This effect should reduce with duration. Conversely, if there are medical questions on the application form then we would expect to see some form of self selection and mortality experience would be better in the short term.

This is also evidence of adverse selection as highlighted above.

Spurious selection — If there is no evidence of health required then the duration effect would be confounded by the differential mortality experience of withdrawals, as those lives withdrawing would be expected to have lighter mortality.

**7** (i) $$\frac{1}{{}_s p_{x+t}} \times \frac{\partial}{\partial t}\, {}_s p_{x+t} = \frac{\partial}{\partial t} \ln({}_s p_{x+t}) = \frac{\partial}{\partial t}(\ln l_{x+t+s} - \ln l_{x+t})$$

$$= -\mu_{x+t+s} + \mu_{x+t}$$

Multiplying through by ${}_s p_{x+t}$ gives the required result.

(ii) Now

$${}_t\bar{V}_x = \bar{A}_{x+t} - \bar{P}_x \times \bar{a}_{x+t} = 1 - \frac{\bar{a}_{x+t}}{\bar{a}_x}$$

$$\frac{\partial}{\partial t}\bar{a}_{x+t} = \frac{\partial}{\partial t}\int_0^\infty e^{-\delta s}\, {}_s p_{x+t} ds = \int_0^\infty e^{-\delta s} \frac{\partial}{\partial t}\, {}_s p_{x+t} ds$$

$$= \int_0^\infty e^{-\delta s}\, {}_s p_{x+t}(\mu_{x+t} - \mu_{x+t+s}) ds = \mu_{x+t}\bar{a}_{x+t} - \bar{A}_{x+t}$$

$$\Rightarrow \frac{\partial}{\partial t}\, {}_t\bar{V}_x = \frac{-(\mu_{x+t}\bar{a}_{x+t} - \bar{A}_{x+t})}{\bar{a}_x} = -\mu_{x+t}(1 - {}_t\bar{V}_x) + \frac{(1 - \delta\bar{a}_{x+t})}{\bar{a}_x}$$

$$= -\mu_{x+t}(1 - {}_t\bar{V}_x) + \delta\left(1 - \frac{\bar{a}_{x+t}}{\bar{a}_x}\right) - \delta + \frac{1}{\bar{a}_x}$$

$$= -(1 - {}_t\bar{V}_x)\mu_{x+t} + \delta\, {}_t\bar{V}_x + \left(\frac{1 - \delta\bar{a}_x}{\bar{a}_x}\right)$$

$$= -(1 - {}_t\bar{V}_x)\mu_{x+t} + \delta\, {}_t\bar{V}_x + \bar{P}_x$$

**8** (i) Expected present value:

$$\ddot{a}_{\overline{70:67}} = \ddot{a}_{70} + \ddot{a}_{67} - \ddot{a}_{70:67}$$

$$= 11.562 + 14.111 - 10.233$$

$$= 15.440$$

(ii) Variance:

$$\frac{1}{d^2}\left\{{}^2A_{\overline{xy}} - (A_{\overline{xy}})^2\right\} = \frac{1}{d^2}\left\{(1-(1-v^2).{}^2\ddot{a}_{\overline{xy}}) - (1-d.\ddot{a}_{\overline{xy}})^2\right\}$$

where normal functions are at a rate of interest $i$ and functions with a left superscript are at a rate of interest $i^2+2i$.

The expression $(1-v^2)$ in the right hand side of the above equation can also be expressed as ${}^2d$.

**9** (i) The expected present value
of 1 per annum
payable continuously
until the second death
of 2 lives
currently age $x$ and $y$
for a maximum $n$ years

(ii) $\bar{a}_{\overline{xy}:\overline{n|}} = E(\bar{a}_{\overline{\min(\max(T_x,T_y),n)|}})$

$T_x$ and $T_y$ are random variables which measures the complete lifetime of two lives aged $x$ and $y$

$$E(\bar{a}_{\overline{\min(\max(T_x,T_y),n)|}}) = E\left(\frac{1-v^{\min(\max(T_x,T_y),n)}}{\delta}\right)$$

$$= \frac{1-E(v^{\min(\max(T_x,T_y),n)})}{\delta}$$

$$= \frac{1-\bar{A}_{\overline{xy}:\overline{n|}}}{\delta}$$

**10** (i) Let $P$ be the net premium for the policy payable annually in advance. Then, equation of value becomes:

$$P\ddot{a}_{45:\overline{15}|} = 10,000(A_{45:\overline{20}|} + v^{20}\,{}_{20}p_{45})$$

$$11.386P = 10,000(0.46998 + 0.41075)$$

$$\Rightarrow P = £773.52$$

Net premium reserve at the end of the 13th policy year is

$${}_{13}V = 10,000(A_{58:\overline{7}|} + v^{7}\,{}_{7}p_{58}) - P\ddot{a}_{58:\overline{2}|}$$

$$= 10,000(0.76516 + 0.71209) - 773.52 \times 1.955$$

$$= 14,772.48 - 1,512.23 = 13,260.25$$

Death strain at risk per policy = 10,000 – 13,260.25 = –3,260.25

$$EDS = 199q_{57} \times -3,260.25 = 199 \times 0.00565 \times -3,260.25 = -3,665.66$$
$$ADS = 4 \times -3,260.25 = -13,041.00$$

mortality profit = –3,665.66 + 13,041.00 = £9,375.34

(ii) The death strain at risk is negative. Hence, the life insurance company makes money on early deaths.

More people die than expected during the year considered so the company makes a mortality profit.

**11** (i) $$1,000.n.\frac{M_x^r}{D_x} + 1,000.\frac{\overline{R}_x^r}{D_x}$$

Where $D_x = v^x l_x$

$$C_x^r = v^{x+½} r_x \text{ for } x < 65$$
$$C_{65}^r = v^{65} r_{65}$$

$$M_x^r = \sum_{t=0}^{65-x} C_{x+t}^r$$

$$\overline{M}_x^r = M_x^r - ½C_x^r \text{ for } x < 65$$

$$\overline{R}^{r}_{x} = \sum_{t=0}^{64-x} \overline{M}^{r}_{x+t}$$

(ii) $$1{,}000.10.\frac{782}{7{,}874} + 1000.\frac{25{,}502}{7{,}874} = 4{,}231.902$$

(iii) Given contribution of $C$ then

$$C.\frac{\overline{N}_x}{D_x} = 4{,}231.902$$
$$\overline{N}_{30} = 90684,\ D_{30} = 7874$$

Therefore $C = £367.45$

**12** Definitions:

(i) (a) Crude Mortality Rate — the ratio of the total number of deaths in a category to the total exposed to risk in the same category.

(b) Directly Standardised Mortality Rate — the mortality rate of a category weighted according to a standard population.

(c) Indirectly Standardised Mortality Rate — an approximation to the directly standardised mortality rate being the crude rate for the standard population multiplied by the ratio of actual to expected deaths for the region.

This is the same as the crude rate for the local population multiplied by the Area Comparability Factor.

(ii) (a) Calculations.

| | England and Wales | | Tyne and Wear | |
|---|---|---|---|---|
| | *Population* | *Number of births* | *Population* | *Number of births* |
| Total | 10,845,000 | 595,000 | 227,000 | 11,000 |

Crude birth rate: England and Wales 595,000/10,845,000 = 5.49%

Tyne and Wear: 11,000/227,000 = 4.85%

| | England and Wales | | Tyne and Wear | |
|---|---|---|---|---|
| | *Population* | | *Fertility rate* | *Expected number of births* |
| Under 25 | 3,149,000 | | 0.0563 | 177,408 |
| 25–35 | 3,769,000 | | 0.0811 | 305,595 |
| 35+ | 3,927,000 | | 0.0122 | 47,890 |
| Total | 10,845,000 | | | 530,893 |

Directly standardised rate: 530,893/10,845,000 = 4.90%

| | England and Wales | | Tyne and Wear | |
|---|---|---|---|---|
| | *Fertility rate* | | *Population* | *Expected Births* |
| Under 25 | 0.0486 | | 71,000 | 3,450 |
| 25–35 | 0.0899 | | 74,000 | 6,656 |
| 35+ | 0.0262 | | 82,000 | 2,151 |
| Total | | | 227,000 | 12,256 |

Indirectly standardised rate: 5.49%/(12,256/11,000) = 4.93%

(b) The indirectly standardised rate does not require local records of births to be analysed by age grouping.

The standardised rates are similar so the approximation is acceptable.

Both standardised rates are higher than the crude rate, showing that the reason for the low cruder rate compared to the national population is due to population distribution by age.

Both standardised rates are below the crude rate for England and Wales so the birth rate of Tyne and Wear is lower, even allowing for the age distribution.

**13** (i) Let $P$ denote the monthly premium for the contract. Then

EPV of premiums =

$$12P\ddot{a}^{(12)}_{[35]:\overline{30|}} = 12\left(\ddot{a}_{[35]:\overline{30|}} - \frac{11}{24}(1 - v^{30}\ {}_{30}p_{[35]})\right)$$

$$= 12P\left(17.631 - \frac{11}{24}\left(1 - \frac{689.23}{2507.02}\right)\right) = 207.5841P$$

EPV of benefits and expenses =

$$(245,000 + 300)A_{[35]:\overline{30|}} + 5000\left(IA\right)^{1}_{[35]:\overline{30|}} + (155,000 - 150)v^{30}\ {}_{30}p_{[35]}$$

$$+0.03\times 12P\ddot{a}^{(12)}_{[35]:\overline{30|}} - 0.03P + 250 + 0.5\times 12P$$

where

$$\left(IA\right)^{1}_{[35]:\overline{30|}} = \left(IA\right)_{[35]} - v^{30}\ {}_{30}p_{[35]}\left((IA)_{65} + 30A_{65}\right) =$$

$$7.47005 - \frac{689.23}{2507.02}\left(7.89442 + 30\times 0.52786\right) = 0.946137$$

EPV of benefits and expenses =

$$245,300\times 0.32187 + 5,000\times 0.946137 + 154,850\times\frac{689.23}{2507.02}$$

$$0.03\times 12P\times 17.298675 - 0.03P + 250 + 0.5\times 12P$$

Equating EPV of premiums with EPV of benefits and expenses gives

$$207.5841P = 78,954.711 + 4,730.685 + 42,571.366 + 6.227523P - 0.03P + 250 + 6P$$

$$\Rightarrow P = \frac{126,506.762}{195.3866} = £647.47$$

(ii) (a)

$$ {}_{25}V^{\text{retrospective}} = \frac{(1+i)^{25}}{{}_{25}p_{[35]}}\left(0.97\times 12P\ddot{a}^{(12)}_{[35]:\overline{25}|} - 245,300A^{1}_{[35]:\overline{25}|} - 5,000\left(IA\right)^{1}_{[35]:\overline{25}|} + 0.03P - 250 - 0.5\times 12P\right) $$

where

$$ \left(IA\right)^{1}_{[35]:\overline{25}|} = \left(IA\right)_{[35]} - v^{25}\,{}_{25}p_{[35]}\left(\left(IA\right)_{60} + 25A_{60}\right) $$

$$ = 7.47005 - \frac{882.85}{2507.02}\left(8.36234 + 25\times 0.4564\right) = 0.507198 $$

$$ A^{1}_{[35]:\overline{25}|} = A_{[35]:\overline{25}|} - v^{25}\,{}_{25}p_{[35]} = 0.3835 - 0.35215 = 0.03135 $$

$$ \ddot{a}^{(12)}_{[35]:\overline{25}|} = \ddot{a}_{[35]:\overline{25}|} - \frac{11}{24}(1 - v^{25}\,{}_{25}p_{[35]}) = 16.029 - 0.29693 = 15.73207 $$

$$ {}_{25}V^{\text{retrospective}} = 2.83969(11.64P\times 15.73207 - 245,300\times 0.03135 - 5000\times 0.507198 + 0.03P - 250 - 6P) $$

$$ = 2.83969(177.151295P - 10476.145) = £295,963.86 $$

(b) Surrender value would be the same i.e. ${}_{25}V^{\text{retrospective}} = {}_{25}V^{\text{prospective}}$ at 4% per annum rate of interest as the equality of bases ensures that the prospective and retrospective reserves of any policy at any given time $t$ should be equal.

**14** (i) Let $P$ be the annual premium required to meet the company's profit criteria. Then:

(a) Multiple decrement table

Here not all decrements are uniform as whilst deaths can be assumed to be uniformly distributed over the year, surrenders occur only at the year end.

Hence:

$$(aq)_x^d = q_x^d \text{ and } (aq)_x^w = q_x^w(1-q_w^d)$$

| $x$ | $q_x^d$ | $q_x^w$ | $(aq)_x^d$ | $(aq)_x^w$ | $(ap)_x$ | ${}_{t-1}(ap)_x$ |
|---|---|---|---|---|---|---|
| 45 | 0.001465 | 0.05 | 0.001465 | 0.049927 | 0.948608 | 1 |
| 46 | 0.001622 | 0.05 | 0.001622 | 0.049919 | 0.948459 | 0.948608 |
| 47 | 0.001802 | 0.05 | 0.001802 | 0.049910 | 0.948288 | 0.899716 |

(b) Unit fund cashflows (per policy at start of year)

| | *Year 1* | *Year 2* | *Year 3* |
|---|---|---|---|
| Value of units at start of year | 0 | 0.347379$P$ | 1.405063$P$ |
| Allocation | 0.35$P$ | 1.05$P$ | 1.05$P$ |
| Bid/offer | 0.0175$P$ | 0.0525$P$ | 0.0525$P$ |
| Interest | 0.016625$P$ | 0.067244$P$ | 0.120128$P$ |
| Management charge | 0.001746$P$ | 0.007061$P$ | 0.012613$P$ |
| Value of units at start of year | 0.347379$P$ | 1.405063$P$ | 2.510077$P$ |

(c) Non-unit fund cashflows

| | *Year 1* | *Year 2* | *Year 3* |
|---|---|---|---|
| Unallocated premium | $0.65P$ | $-0.05P$ | $-0.05P$ |
| Bid/offer | $0.0175P$ | $0.0525P$ | $0.0525P$ |
| Expenses | $0.2P+250$ | $0.025P+50$ | $0.025P+50$ |
| Interest | $0.0187P-10$ | $-0.0009P-2$ | $-0.0009P-2$ |
| Management charge | $0.001746P$ | $0.007061P$ | $0.012613P$ |
| End of year cashflows | $0.487946P-260$ | $-0.016339P-52$ | $-0.010787P-52$ |

| | | | |
|---|---|---|---|
| Probability in force | 1 | 0.948608 | 0.899716 |
| Discount factor | 0.925926 | 0.857339 | 0.793832 |
| Expected present value of profit | $0.430809P-320.170863$ | | |

NPV = $.15P = 0.430809P - 320.170863 \Rightarrow P = £1140.17$

(ii) Later expected future negative cashflows should be reduced to zero by establishing reserves in the non-unit fund at earlier durations so that the company does not expect to have to input further money in the future. The expected non-unit fund cashflows derived in (i) are negative in years 2and 3 so need to be eliminated.

**END OF EXAMINERS' REPORT**