Subject CT5 — Contingencies
Core Technical

EXAMINERS' REPORT

April 2008

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

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Chairman of the Board of Examiners

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1 The probability that an ultimate life age 40 dies between 45 and 55 (all exact)

$$ {}_{5|10}q_{40} = \frac{(l_{45} - l_{55})}{l_{40}} = \frac{(9801.3123 - 9557.8179)}{9856.2863} = 0.024704 $$

2 The following are three types of guaranteed reversionary bonuses. The bonuses are usually allocated annually in arrears, following a valuation.

<u>Simple</u> – the rate of bonus each year is a percentage of the initial (basic) sum assured under the policy. The effect is that the sum assured increases linearly over the term of the policy.

<u>Compound</u> – the rate of bonus each year is a percentage of the initial (basic) sum assured and the bonuses previously added. The effect is that the sum assured increases exponentially over the term of the policy.

<u>Super compound</u> – two compound bonus rates are declared each year. The first rate (usually the lower) is applied to the initial (basic) sum assured. The second rate is applied to bonuses previously added. The effect is that the sum assured increases exponentially over the term of the policy. The sum assured usually increases more slowly than under a compound allocation in the earlier years and faster in the later years.

(Note: credit given if special reversionary bonus mentioned)

3 The expected cost of paying benefits usually increases as the life ages and the probability of a claim by death increases. In the final year the probability of payment is large, since the payment will be made if the life survives the term, and for most contracts the probability of survival is large.

Level premiums received in the early years of a contract are more than enough to pay the benefits that fall due in those early years, but in the later years, and in particular in the last year of an endowment assurance policy, the premiums are too small to pay for the benefits. It is therefore prudent for the premiums that are not required in the early years of the contract to be set aside, or reserved, to fund the shortfall in the later years of the contract.

If premiums received that were not required to pay benefits were spent by the company, perhaps by distributing to shareholders, then later in the contract the company may not be able to find the money to pay for the excess of the cost of benefits over the premiums received.

(Credit given for other valid points)

4

$Value =$

$50,000\int_0^{20} v^t \cdot {}_t p_{40}^{hh} (\mu_{40+t} + \sigma_{40+t}) dt$

$= 50,000\int_0^{20} e^{-\ln(1.05)t} \cdot {}_t p_{40}^{hh} 0.008 dt$

${}_t p_{40}^{hh} = {}_t p_{40}^{\overline{hh}} = \exp(-\int_{40}^{40+t} (\mu_s + \sigma_s) ds)$

$= \exp(-\int_{40}^{40+t} 0.008 ds)$

$= e^{-0.008t}$

$Therefore, value =$

$50,000 * 0.008\int_0^{20} e^{-\ln(1.05)t} . e^{-0.008t} dt$

$= 400\int_0^{20} e^{-0.05679t} dt$

$= 400 * [-e^{-.05679t} / .05679]_0^{20}$

$= 400 * (-5.65531 + 17.60873)$

$= 4781.4$

5 (a) Define random variables T_x and T_y for the complete duration of life for the lives aged x and y.

Define a random variable $\overline{Z}$ for the value of the reversionary annuity, which has the following definition:

$\overline{Z} = \bar{a}_{\overline{T_y}|} - \bar{a}_{\overline{T_x}|}$ if $T_y > T_x = 0$ otherwise

$\overline{Z} = \bar{a}_{\overline{T_y}|} - \bar{a}_{\overline{T_{xy}}|}$ where T_{xy} is a random variable for the duration to the first death

$$\overline{Z} = \frac{(1 - v^{T_y})}{\delta} - \frac{(1 - v^{T_{xy}})}{\delta} = \frac{(v^{T_{xy}} - v^{T_y})}{\delta}$$

(b) $$E[\overline{Z}] = \frac{(E[v^{T_{xy}}] - E[v^{T_y}])}{\delta} = \frac{(\overline{A}_{xy} - \overline{A}_y)}{\delta}$$

6 The probability that the life age 25 survives 10 years
= 9894.4299 / 9953.6144 = 0.994054
The probability that the life age 30 survives 10 years
= 9856.2863 / 9925.2094 = 0.993056

There are four possible outcomes:

Outcome	*Expression for value*	*value*
Both survive	$V^{10}\times 0.994054\times 0.993056\times 15000/3$	3672.67
Only 25 survives	$V^{10}\times 0.994054\times(1-0.993056)\times 15000/2$	38.52
Only 30 survives	$V^{10}\times(1-0.994054)\times 0.993056\times 15000/2$	32.95
Neither survive	$V^{10}\times(1-0.994054)\times(1-0.993056)\times 15000$	0.46

Total value is 3744.60

7 Value of future service benefits

$$\frac{1}{60}.S.\frac{({}^{z}\overline{R}^{ra}_{40}+{}^{z}\overline{R}^{ia}_{40})}{{}^{s}D_{40}}=\frac{1}{60}.S.\frac{(2884260+887117)}{25059}=2.5S$$

Value of contributions of k% of future salary

$$\frac{k}{100}.S.\frac{{}^{s}\overline{N}_{40}}{{}^{s}D_{40}}=\frac{k}{100}.S.\frac{(363573)}{25059}=k/100*14.5S$$

Equating these values give $k = 17.3$

8 (i) The uniform distribution of deaths is consistent with an assumption that

$$_{s}q_x = s.q_x$$

$$\begin{aligned}
{}_{t-s}q_{x+s} &= (1 - {}_{t-s}p_{x+s}) \\
&= (1 - \frac{{}_t p_x}{{}_s p_x}) \\
&= (1 - \frac{(1 - {}_t q_x)}{(1 - {}_s q_x)}) \\
&= (1 - \frac{(1 - tq_x)}{(1 - sq_x)}) \\
&= \frac{(t-s).q_x}{(1 - s.q_x)}
\end{aligned}$$

(ii) (a) $_{0.5}q_{62.25} = 0.5q_{62}/(1 - 0.25q_{62}) = 0.5 \times 0.00355/(1 - 0.25 * 0.00355)$
$= 0.001777$

(b) The assumption of a constant force of mortality requires μ to be derived from the expression $p = e^{-\mu}$. p_{62}=0.99645 => μ_{62}=0.003556

$$_{t-s}p_{x+s} = e^{-(t-s)\mu} = e^{-0.5x0.003556} = 0.998224$$
$$_{t-s}q_{x+s} = 0.001776$$

9 (i) Occupation – as some occupations have a regional distribution

Housing – as quality of housing will be impacted by occupationally influenced income levels

Climate – different locations having different climates

Using location is a spurious form of class selection as it disguises the underlying causes

(ii) Actual deaths (location A) = 9
Actual deaths (location B) = 11

The calculation of the expected deaths is

		Location A		*Location B*	
Age	*Standard Mortality Rate*	*Initial Exposed to risk*	*Number of deaths*	*Initial Exposed to risk*	*Number of deaths*
60	0.01392	100	1.4	200	2.8
61	0.01560	175	2.7	150	2.3
62	0.01749	190	3.3	170	3.0
63	0.01965	210	4.1	100	2.0
Total			11.5		10.1

The SMRs are therefore

Location A = 9/11.5 = 0.78
Location B = 11/10.1 = 1.09

10 (a) wife

value = $5000(a_{55}^{(12)} - a_{60:55}^{(12)}) = 5000(18.210 - 14.756) = 17,270$

Note no effect of monthly payments

(b) grandson

value =

$2000(a_{\overline{8}|}^{(12)} - a_{60:55:\overline{8}|}^{(12)})$

$$a_{\overline{8}|}^{(12)} = \frac{(1-v^8)}{i^{(12)}} = 6.7327x\, {}^{0.04}\!\big/_{0.039285} = 6.855$$

$$a_{60:55:\overline{8}|}^{(12)} = a_{60:55}^{(12)} - v^8\, {}_8p_{60} \cdot {}_8p_{55}\, a_{68:63}^{(12)}$$

$$= a_{60:55} + 11/24 - v^8\, {}_8p_{60} \cdot {}_8p_{55}(a_{68:63} + 11/24)$$

$$= (14.756 - 1) + 11/24 - v^8 \frac{9440.717}{9826.131}\frac{9775.888}{9917.623}(11.372 - 1 + 11/24)$$

$= 6.721$

Therefore value = 2000(6.855-6.721) = 268

Total value = 17270 + 268 = 17,538

11 (i) Let P be the annual premium. Then:

EPV of premiums:

$$P\ddot{a}_{[50]:\overline{10|}} = 7.698P$$

EPV of benefits:

$$\frac{75,000}{(1+b)} \times (1.06)^{1/2} \{q_{[50]}(1+b)v \ \ +_1|q_{[50]}(1+b)^2 v^2$$

$$+....+_9|q_{[50]}(1+b)^{10} v^{10}\} + 75,000\,{}_{10}p_{[50]}(1+b)^{10} v^{10}$$

where b = 0.0192308

$$= \frac{75,000}{(1+b)} \times (1.06)^{1/2} A^1_{[50]:\overline{10|}} @ i' + 75,000 \times {}_{10}p_{[50]} \times \frac{1}{(1+i')^{10}}$$

$$= \frac{75,000}{1.0192308} \times (1.06)^{1/2} \times (.68007 - .64641) + 75,000 \times .64641 = 2,550.091 + 48,480.75$$

$$= 51,030.84$$

where $i' = \dfrac{1.06}{1+b} - 1 = 0.04$

EPV of other expenses:

$$.5 \times P + 350 + 0.05 \times P(\ddot{a}_{[50]:\overline{10|}} - 1) = 0.8349P + 350$$

Equation of value gives $7.698P = 51{,}030.84 + 0.8349P + 350$

and P = £7,486.54

(ii) Gross premium prospective reserve (calculated at 6%) is given by:

EPV of benefits and expenses less EPV of premiums

EPV of benefits and expenses:

$$= \frac{82,494.3}{(1+b)} \times (1.06)^{1/2} A^1_{55:\overline{5|}} @ i' + 82,494.3 \times {}_5p_{55} \times \frac{1}{(1+i')^5} + 0.05P\ddot{a}_{55:\overline{5|}}$$

$$= \frac{82,494.3}{1.0192308} \times (1.06)^{1/2} \times (.82365 - .79866) @ i' + 82,494.3 \times 0.79866 + 0.05 \times 7486.54 \times 4.423$$

$$= 2,082.43 + 65,884.90 + 1,655.65 = 69,622.98$$

EPV of premiums:

$P\ddot{a}_{55:\overline{5}|} = 4.423P = 33{,}112.97$

=> Gross premium prospective reserve = £36,510.00

12 (i) (a) Annual premium for pure endowment with £50,000 sum assured given by:

$$P^{PE} = \frac{50{,}000}{\ddot{a}_{50:\overline{10}|}} \times {}_{10}p_{50} \times v^{10} = \frac{50{,}000}{8.314} \times 0.64601 = 3885.10$$

Annual premium for term assurance with £50,000 sum assured given by:

$$P^{TA} = P^{EA} - P^{PE} = \frac{50{,}000 A_{50:\overline{10}|}}{\ddot{a}_{50:\overline{10}|}} - P^{PE}$$

$$= \frac{50{,}000 \times 0.68024}{8.314} - 3885.10 = 205.83$$

Reserves at the end of the second year:

for pure endowment with £50,000 sum assured given by:

$${}_2V^{PE} = 50{,}000 \times {}_8p_{52} \times v^8 - P^{PE}\ddot{a}_{52:\overline{8}|}$$

$$= 50{,}000 \times 0.70246 - 3885.10 \times 6.910 = 35123.0 - 26846.04 = 8276.96$$

for term assurance with £50,000 sum assured given by:

$${}_2V^{TA} = {}_2V^{EA} - {}_2V^{PE}$$

$$= 50{,}000 A_{52:\overline{8}|} - (3885.1 + 205.83)\ddot{a}_{52:\overline{8}|} - 8276.96$$

$$= 50{,}000 \times 0.73424 - 4090.93 \times 6.91 - 8276.96$$

$$= 166.71$$

Sums at risk:

Pure endowment: DSAR = 0 − 8,276.96 = −8,276.96
Term assurance: DSAR = 50,000 – 166.71 = 49,833.29

(b) Mortality profit = EDS – ADS

For term assurance

$$EDS = 4995 \times q_{51} \times 49{,}833.29 = 4995 \times .002809 \times 49{,}833.29 = 699{,}208.65$$

$$ADS = 10 \times 49{,}833.29 = 498{,}332.90$$

mortality profit = 200,875.75

For pure endowment

$$EDS = 4995 \times q_{51} \times -8{,}276.96 = 4995 \times .002809 \times -8{,}276.96 = -116{,}133.65$$

$$ADS = 10 \times -8{,}276.96 = -82{,}769.60$$

mortality profit = −33,364.05

Hence, total mortality profit = £167,511.70

(ii) (a) The actual mortality profit would remain as that calculated in (i) (b).

(b) The variance of the benefits would be lower than that calculated in (i).

In this case, the company would not pay out benefits under both the PE and the TA but will definitely pay out one of the benefits. Under the scenario in (i), the company could pay out all the benefits (if all the TA policyholders die and the PE policyholders survive). Alternatively, they could pay out no benefits at all (if all the TA policyholders survive and the PE policyholders immediately die).

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Annual premium	750.00	Allocation % (1st yr)	25.0%
Risk discount rate	8.5%	Allocation % (2nd yr +)	102.50%
Interest on investments	6.5%	Man charge	1.0%
Interest on sterling provisions	5.5%	B/O spread	5.0%
Minimum death benefit	3000.00		

	£	% prm	Total
Initial expense	150	10.0%	225
Renewal expense	65	2.5%	83.75

(i) Multiple decrement table

x	q_x^d	q_x^s
50	0.001971	0.1
51	0.002732	0.1
52	0.003152	0.1
53	0.003539	0.0

x	$(aq)_x^d$	$(aq)_x^s$	(ap)	${}_{t-1}(ap)$
50	0.001971	0.09980	0.898226	1.000000
51	0.002732	0.09973	0.897541	0.898226
52	0.003152	0.09968	0.897163	0.806195
53	0.003539	0.00000	0.996461	0.723288

Unit fund (per policy at start of year)

	yr 1	*yr 2*	*yr 3*	*yr 4*
value of units at start of year	0.000	187.806	968.018	1790.635
alloc	187.500	768.750	768.750	768.750
B/O	9.375	38.4375	38.4375	38.4375
interest	11.578	59.678	110.392	163.862
management charge	1.897	9.778	18.087	26.848
value of units at year end	187.806	968.018	1790.635	2657.961

Cash flows (per policy at start of year)

	yr 1	*yr 2*	*yr 3*	*yr 4*
unallocated premium	562.500	−18.750	−18.750	−18.750
B/O spread	9.375	38.4375	38.4375	38.4375
expenses	225.000	83.750	83.750	83.750
interest	19.078	−3.523	−3.523	−3.523
man charge	1.897	9.778	18.087	26.848
extra death benefit	5.543	5.551	3.812	1.210
Extra maturity benefit	0.000	0.000	0.000	264.855
end of year cashflow	362.307	−63.359	−53.311	−306.804

probability in force	1	0.898226	0.806195	0.723288
discount factor	0.921659	0.849455	0.782908	0.721574
expected p.v. of profit	91.809			
premium signature	750.000	620.894	513.620	424.701
expected p.v. of premiums	2309.215			
profit margin	3.98%			

(ii) (a) To calculate the expected provisions at the end of each year we have (utilising the end of year cashflow figures and decrement tables in (i) above):

$$_3V = \frac{306.804}{1.055} = 290.809$$
$$_2V \times 1.055 - (ap)_{52} \times {}_3V = -53.311 \Rightarrow {}_2V = 297.833$$
$$_1V \times 1.055 - (ap)_{51} \times {}_2V = -63.359 \Rightarrow {}_1V = 313.437$$

These need to be adjusted as the question asks for the values in respect of the beginning of the year. Thus we have:

Year 3 $290.809(ap)_{52} = 260.903$
Year 2 $297.833(ap)_{51} = 267.318$
Year 1 $313.437(ap)_{50} = 281.538$

(b) Based on the expected provisions calculated in (a) above, the cash flow for years 2, 3 and 4 will be zeroised whilst year 1 will become:

$362.307 - 281.538 = 80.769$

Hence the table below can now be completed for the revised profit margin.

revised end of year cash flow	80.769	0	0	0
probability in force	1	0.898226	0.806195	0.723288
discount factor	0.921659	0.849455	0.782908	0.721574
expected p.v. of profit	74.442			
profit margin	3.22%			

END OF EXAMINERS' REPORT