

# EXAMINATION

6 April 2005 (pm)

## Subject CT5 — Contingencies Core Technical

*Time allowed: Three hours*

### ***INSTRUCTIONS TO THE CANDIDATE***

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

***Graph paper is not required for this paper.***

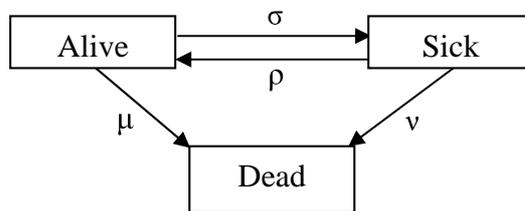
### ***AT THE END OF THE EXAMINATION***

*Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.*

*In addition to this paper you should have available the 2002 edition of the  
Formulae and Tables and your own electronic calculator.*

- 1** Explain the difference between a profit vector and a profit signature. [2]
- 2** A 20-year temporary annuity-due of 1 per annum is issued to a life aged 50 exact.
- (a) Express the expected present value of the annuity in terms of an assurance function.
- (b) Hence calculate the value using the mortality table AM92 Ultimate with 4% interest. [3]
- 3** A life insurance company sells an annual premium whole life assurance policy where the sum assured is payable at the end of the year of death. Expenses are incurred at the start of each policy year, and claim expenses are nil.
- (a) Write down a recursive relationship between the gross premium provisions at successive durations, with provisions calculated on the premium basis. Define all the symbols that you use.
- (b) Explain in words the meaning of the relationship. [4]
- 4** A life insurance company issues an annuity to a life aged 60 exact. The purchase price is £200,000. The annuity is payable monthly in advance and is guaranteed to be paid for a period of 10 years and for the whole of life thereafter.
- Calculate the annual annuity payment.
- Basis:
- Mortality AM92 Ultimate
- Interest 6% per annum [4]

- 5 A three-state transition model is shown in the following diagram:



Assume that the transition probabilities are constant at all ages with  $\mu = 2\%$ ,  $\nu = 4\%$ ,  $\rho = 1\%$  and  $\sigma = 5\%$ .

Calculate the present value of a sickness benefit of £2,000 p.a. paid continuously to a life now aged 40 exact and sick, during this period of sickness, discounted at 4% p.a. and payable to a maximum age of 60 exact. [4]

- 6 Calculate the probability of survival to age 60 exact using ELT15 (Males) for a life aged  $45\frac{1}{2}$  exact using two approximate methods. State any assumptions you make. [5]

- 7 A joint life annuity of 1 per annum is payable continuously to lives currently aged  $x$  and  $y$  while both lives are alive. The present value of the annuity payments is expressed as a random variable, in terms of the joint future lifetime of  $x$  and  $y$ .

Derive and simplify as far as possible expressions for the expected present value and the variance of the present value of the annuity. [5]

- 8 A pension scheme provides a pension on ill-health retirement of  $\frac{1}{80}$ <sup>th</sup> of Final Pensionable Salary for each year of pensionable service subject to a minimum pension of  $\frac{20}{80}$ <sup>ths</sup> of Final Pensionable Salary. Final Pensionable Salary is defined as the average salary earned in the three years before retirement. Normal retirement age is 65 exact.

Derive a formula for the present value of the ill-health retirement benefit for a member currently aged 35 exact with exactly 10 years past service and salary for the year before the calculation date of £20,000. [5]

- 9 Explain how an insurance company uses risk classification to control the profitability of its life insurance business. [5]

**10** You are given the following statistics in respect of the population of Urbania:

<i>Age band</i>	<i>Males</i>		<i>Females</i>	
	<i>Exposed to risk</i>	<i>Observed Mortality rate</i>	<i>Exposed to risk</i>	<i>Observed Mortality rate</i>
20–29	125,000	0.00356	100,000	0.00125
30–39	200,000	0.00689	250,000	0.00265
40–49	100,000	0.00989	200,000	0.00465
50–59	90,000	0.01233	150,000	0.00685

Calculate the directly and indirectly standardised mortality rates for the female lives, using the combined population as the standard population. [6]

**11** A life insurance company issues a 25-year with profits endowment assurance policy to a male life aged 40 exact. The sum assured of £100,000 plus declared reversionary bonuses are payable on survival to the end of the term or immediately on death, if earlier.

Calculate the monthly premium payable in advance throughout the term of the policy if the company assumes that future reversionary bonuses will be declared at a rate of 1.92308% of the sum assured, compounded and vesting at the end of each policy year.

Basis:

Interest	6% per annum
Mortality	AM92 Select
Initial commission	87.5% of the total annual premium
Initial expenses	£175 paid at policy commencement date
Renewal commission	2.5% of each monthly premium from the start of the second policy year
Renewal expenses	£65 at the start of the second and subsequent policy years
Claim expense	2.5% of the claim amount

[10]

- 12** (i) By considering a term assurance policy as a series of one year deferred term assurance policies, show that:

$$\bar{A}_{x:n}^1 = \frac{i}{\delta} A_{x:n}^1 \quad [5]$$

- (ii) Calculate the expected present value and variance of the present value of a term assurance of 1 payable immediately on death for a life aged 40 exact, if death occurs within 30 years.

Basis:

Interest 4% per annum

Mortality AM92 Select

Expenses: None

[6]  
[Total 11]

- 13** A life insurance company issues a 4-year unit-linked endowment assurance contract to a male life aged 40 exact under which level premiums of £1,000 per annum are payable in advance. In the first year, 50% of the premium is allocated to units and 102.5% in the second and subsequent years. The units are subject to a bid-offer spread of 5% and an annual management charge of 0.5% of the bid value of the units is deducted at the end of each year.

If the policyholder dies during the term of the policy, the death benefit of £4,000 or the bid value of the units after the deduction of the management charge, whichever is higher, is payable at the end of the year of death. On surrender or on survival to the end of the term, the bid value of the units is payable at the end of the year of exit.

The company uses the following assumptions in its profit test of this contract:

Rate of growth on assets in the unit fund	6% per annum
Rate of interest on non-unit fund cashflows	4% per annum
Independent rates of mortality	AM92 Select
Independent rate of withdrawal	10% per annum in the first policy year; 5% per annum in the second and subsequent policy years.
Initial expenses	£150 plus 100% of the amount of initial commission
Renewal expenses	£50 per annum on the second and subsequent premium dates
Initial commission	10% of first premium
Renewal commission	2.5% of the second and subsequent years' premiums
Risk discount rate	8% per annum

- (i) Calculate the profit margin on the assumption that the office does not zeroise future negative cashflows and that decrements are uniformly distributed over the year. [13]
- (ii) Suppose the office does zeroise future negative cashflows.
- (a) Calculate the expected provisions that must be set up at the end of each year, per policy in force at the start of each year. [4]
- (b) Calculate the profit margin allowing for the cost of setting up these provisions. [4]

[Total 17]

- 14** (i) Write down in the form of symbols, and also explain in words, the expressions “death strain at risk”, “expected death strain” and “actual death strain”. [6]
- (ii) A life insurance company issues the following policies:
- 15-year term assurances with a sum assured of £150,000 where the death benefit is payable at the end of the year of death
  - 15-year pure endowment assurances with a sum assured of £75,000
  - 5-year single premium temporary immediate annuities with an annual benefit payable in arrear of £25,000

On 1 January 2002, the company sold 5,000 term assurance policies and 2,000 pure endowment policies to male lives aged 45 exact and 1,000 temporary immediate annuity policies to male lives aged 55 exact. For the term assurance and pure endowment policies, premiums are payable annually in advance. During the first two years, there were fifteen actual deaths from the term assurance policies written and five actual deaths from each of the other two types of policy written.

- (a) Calculate the death strain at risk for each type of policy during 2004.
- (b) During 2004, there were eight actual deaths from the term assurance policies written and one actual death from each of the other two types of policy written. Calculate the total mortality profit or loss to the office in the year 2004.

Basis:

Interest 4% per annum

Mortality AM92 Ultimate for term assurances and pure endowments  
PMA92C20 for annuities

[13]  
[Total 19]

**END OF PAPER**