Subject CT5 — Contingencies.
Core Technical

September 2009 Examinations

EXAMINERS' REPORT

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

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Chairman of the Board of Examiners

December 2009

Comments for individual questions are given with the solutions that follow.

1

$$
\begin{aligned}
{}_{20}q^{2}_{[45]:[45]} &= 1/2 * {}_{20}q_{[45]:[45]} \\
&= 1/2 * (1 - {}_{20}p_{[45]})^2 \\
&= 1/2 * (1 - l_{65} / l_{[45]})^2 \\
&= 1/2 * (1 - 8821.2612 / 9798.0837)^2 \\
&= .00497
\end{aligned}
$$

In general well this question was well done.

2

Define

${}_{k}(ap)_x$ = the probability that a life aged x is alive and not diagnosed as critically ill at time k

$(aq)^t_{x+k}$ = the probability that a life aged $x + k$ is diagnosed as critically ill in the following year

Then the value is

$$\sum_{k=0}^{n-1} v^{k+1} {}_{k}(ap)_x . (aq)^t_{x+k}$$

Where the benefit is payable at the end of the year of diagnosis

Students often failed to define symbols adequately. The continuous alternative was also fully acceptable

3

For contant force of mortality at age 72:

$$p_{72} = (1 - q_{72}) = .969268 = e^{-\int_0^1 \mu dt} = e^{-\mu}$$

Hence $\mu = -\ln(.969268) = .031214$

$$
\begin{aligned}
{}_{0.5}p_{72.25} &= e^{-\int_{0.25}^{0.75} .031214 dt} = e^{-.015607} \\
&= .984514
\end{aligned}
$$

$$
\begin{aligned}
{}_{0.5}q_{72.25} &= 1 - .984504 \\
&= .015486
\end{aligned}
$$

Generally well done. The alternative very quick answer of $1-(p_{72})^{1/2}$ *was fully acceptable*

4

(i) Crude rate = (104+127+132)/(121376+134292+133277)=0.000933

(ii)

Age	*Population*	q_x	*Expected Number of deaths*
40	121,376	0.000937	114
41	134,292	0.001014	136
42	133,277	0.001104	147

SMR = actual deaths / expected deaths

= (104+127+132) / (114+136+147) = 0.914

Generally well done.

5

$$\text{var}\left[\ddot{a}_{\overline{\min(K_x+1,n)}|}\right] = \text{var}\left[\frac{1-v^{\min(K_x+1,n)}}{d}\right]$$

$$= \frac{1}{d^2}\text{var}\left[v^{\min(K_x+1,n)}\right]$$

$$= \frac{1}{d^2}\left[{}^2A_{x:\overline{n}|} - (A_{x:\overline{n}|})^2\right] \text{ where } {}^2A_{x:\overline{n}|} \text{ is at rate } (1+i)^2-1$$

Straightforward bookwork where considerable information was given in Handbook. The Examiners were looking to see students knew how to derive the relationship. Generally well done.

6

a. $({}_tV' + GP\text{-}e_t)(1+i) = q_{x+t}S + p_{x+t}({}_{t+1}V')$

where $_tV'$ = gross premium reserve at time t

GP = office premium

e_t = renewal expenses incurred at time t

i = interest rate in premium/valuation basis

S = Sum Assured

q_{x+t} = probability life aged $x+t$ dies within one year on premium/valuation basis

p_{x+t} = probability life aged $x+t$ survives one year on premium/valuation basis

b. Income (opening reserve and excess of premium over renewal expenses) plus interest equals outgo (death claims and closing reserve for survivors) if assumptions are borne out.

Generally well done.

7

Direct expenses are those that vary with the amount of business written. Direct expenses are divided into:

Initial expenses

Renewal expenses

Termination expenses

Examples of each:

Initial expenses – those arising when the policy is issued e.g. initial commission

Renewal expenses – those arising regularly during the policy term e.g. renewal commission

Termination expenses – those arising when the policy terminates as a result of an insured contingency (e.g. death claim for a temporary life insurance policy)

Generally well done and other valid comments and examples were credited.

8

(i) Pensioners retiring at normal retirement age

Pensioners retiring before normal retirement age

Pensioners retiring before normal retirement age on the grounds of ill-health

(ii) Class selection – ill-health pensioners will have different mortality to other retirements.

Temporary initial selection – the difference between these classes will diminish with duration since retirement

Anti-Selection and Time Selection were credited provided they were properly justified. Generally well done.

9

(i) To set premium rates to ensure the probability of a profit is set at an acceptable level then the insurer takes advantage of the Central Limit Theorem while pooling risks which are independent and homogeneous.

Independence of risk usually follows naturally.

Homogeneity is ensured by careful underwriting. Risk groups are separated by the use of risk factors, such as age and sex.

The life assurance company uses responses to questions to allocate prospective customers to the appropriate risk group.

Enough questions should be asked to ensure that the variation between categories is smaller than the random variation that remains but in practice there will be limits on the number and type of questions that can be asked.

(ii) Equity – insurance is about pooling of risks and the use of genetic information reduces that pooling.

Ethics – use of genetic information could create an "underclass" of lives who are not able to obtain insurance products at an affordable price, given the results of their genetic tests.

In some countries legislation may prohibit genetic testing or there might be political or social reasons why it is avoided.

Generally part (i) was done poorly with students failing to appreciate the key points. Part (ii) was done better but in this case also most students failed to obtain all the main valid points.

10

Pension at retirement = $3 \times 1000 = 3000$

Annuity at retirement

$$\ddot{a}_{\overline{5}|}^{(12)} + v^5 \cdot {}_5p_{65}\,\ddot{a}_{70}^{(12)}$$

$$= a_{\overline{5}|} \cdot \frac{i}{d^{(12)}} + v^5 \cdot {}_5p_{65}\,\ddot{a}_{70}^{(12)}$$

$$= 4.4518 \times 1.021537 + 0.82193 * \frac{9238.134}{9647.797} \cdot \left(11.562 - \tfrac{11}{24}\right) = 13.28659$$

Multiple decrement table

Use the formula

$$(aq)_x = q_x^{\alpha} + q_x^{\beta} - q_x^{\alpha}.q_x^{\beta}$$

To derive the following

$(aq)_{62}$= 0.021233, $(aq)_{63}$= 0.084295, $(aq)_{64}$= 0.062397

And ${}_3(ap)_{62}$ = (1 – 0.021233).(1–0.084295).(1–0.062397)= 0.840338

So value = $3000\times13.28659\times0.840338\times(1.04)^{-3}$ = 29778

A large proportion of students whilst understanding how to approach this question failed to calculate some or all of it correctly. In some cases certain parts were omitted or calculated wrongly. Credit was given where parts of the solution were correct.

11

Let EDS and ADS denote the expected and actual death strain in 2008. Then

$$EDS = \sum_i q_{60}\left[S_i - \left\{S_i(A_{\overline{61:4}|} + \frac{l_{65}}{l_{61}}v^4) - P_i\ddot{a}_{\overline{61:4}|}\right\}\right]$$

where S_i is the death benefit per policy and the summation is over all policies in force at start of the year i.e. (where figures are in £000's)

$$EDS = q_{60}\left[\left(\sum S_i\right) - \left(\sum S_i\right)(A_{\overline{61:4}|} + \frac{l_{65}}{l_{61}}v^4) + \left(\sum P_i\right)\ddot{a}_{\overline{61:4}|}\right]$$

$$= 0.008022\times\left\{6125 - 6125\left(0.85685 + \frac{8821.2612}{9212.7143}\times0.854804\right) + 440\times3.722\right\}$$

$$= 0.008022\times\ 6125 - 8623.75\ = -20.045$$

The actual death strain is obtained by summation of the death strains at risk over the policies that become claims. Therefore

$$ADS = \sum_{claims}\left[S_i - \left\{S_i(A_{\overline{61:4}|} + \frac{l_{65}}{l_{61}}v^4) - P_i\ddot{a}_{\overline{61:4}|}\right\}\right]$$

$$= \left(\sum_{claims} S_i\right) - \left(\sum_{claims} S_i\right)(A_{\overline{61:4}|} + \frac{l_{65}}{l_{61}}v^4) + \left(\sum_{claims} P_i\right)\ddot{a}_{\overline{61:4}|}$$

$$= 100 -\ 167.5333 - 26.054\ = -41.479$$

Therefore, mortality profit = -20.045 + 41.479 = 21.434 (i.e. a profit of £21,434).

This question was very poorly done. Students failed to properly identify the data and the subtleties of a Pure Endowment contract.

12

(i) The expected present value of a **continuous assurance** for **a sum assured of 1000** calculated at a **force of interest** δ on **2 lives aged** x **and** y whereby the **sum is paid on the death of** x only **if life aged** x **dies after life aged** y**.**

(ii) For both parts (a) and (b):

for the life aged 30

$$ {}_tp_{30} = e^{-\int_0^t \mu_{30+r}dr} = e^{-\int_0^t 0.02dr} = e^{-0.02t} $$

Similarly for the life 40

$$ {}_tp_{40} = e^{-0.03t} $$

$$ \text{(a)}\,1000\,\bar{A}^{\,2}_{30:40} = 1000\int_0^\infty v^t\,{}_tp_{30}(1-{}_tp_{40})\mu_{30+t}\,dt $$

$$ = 1000\int_0^\infty e^{-.05t} * e^{-.02t} * (1-e^{-.03t}) * .02dt $$

$$ = 1000\int_0^\infty .02 * (e^{-.07t} - e^{-.1t})dt $$

$$ = 1000[-.02/.07 * e^{-.07t} + .02/.1 * e^{-.1t}]_0^\infty $$

$$ = 1000(.02/.07 - .02/.1) $$

$$ = 85.714 $$

(b) To calculate premium we need $\bar{a}_{\overline{30:40}}$

$$ \bar{a}_{\overline{30:40}} = \int_0^\infty v^t * (1-(1-{}_tp_{30})(1-{}_tp_{40}))dt $$

$$ = \int_0^\infty e^{-.05t}(e^{-.02t} + e^{-.03t} - e^{-.05t})dt $$

$$ = \int_0^\infty (e^{-.07t} + e^{-.08t} - e^{-1.t})dt $$

$$ = [-e^{-.07t}/.07 - e^{-.08t}/.08 + e^{-.1t}/.1]_0^\infty $$

$$ = (1/.07 + 1/.08 - 1/.1) $$

$$ = 16.786 $$

So the required premium = $85.714/16.786$

$= 5.11$

(iii) If the life age 30 dies first the policy ceases without benefit yet the premium is expected to be maintained by the life aged 40 so long as they survive. There is no incentive to continue.

The sensible option would be to establish the premium paying period as ceasing on the death of the life aged 30.

A single premium is possible as an alternative if affordable.

In general terms this question was reasonably well done although a large number of students failed to obtain all of the required numerical solutions (the main error being failure to calculate the joint life last survivor annuity). In part (iii) a student who suggested a joint life first death approach was given credit although this is an expensive option.

13

(i) Let P be the monthly premium for the contract. Then:

EPV of premiums valued at rate i where i = 0.06 is:

$$12P\ddot{a}^{(12)}_{[30]:\overline{35}|} = 12P(\ddot{a}_{[30]:\overline{35}|} - \frac{11}{24}(1 - v^{35}\frac{l_{65}}{l_{[30]}}))$$

$$where\ v^{35}\frac{l_{65}}{l_{[30]}} = 0.13011 \times \frac{8821.2612}{9923.7497} = 0.11566$$

$$= 12P(15.152 - \frac{11}{24}(1 - .11566)) = 12P \times 14.74668 = 176.9601P$$

EPV of benefits valued at rate i where i = 0.06 is:

$$= 75,000A_{[30]:\overline{35}|} = 75,000 \times 0.14234 = 10,675.5$$

EPV of expenses not subject to inflation and therefore valued at rate i where i = 0.06 is:

$$0.025 \times 12P\ddot{a}^{(12)}_{[30]:\overline{35}|} - 0.025P + 250 + 0.5 \times 12P$$
$$= 250 + 10.399P$$

EPV of expenses subject to inflation and therefore valued at rate j where

$$1 + j = \frac{1.06}{1.0192308} = 1.04 \text{ is:}$$
$$75(\ddot{a}_{[30]:\overline{35}|} - 1) + 300A_{[30]:\overline{35}|} = 75 \times 18.072 + 300 \times 0.26647 = 1435.341$$

Equating EPV of premiums and EPV of benefits and expenses gives:

176.9601P = 10,675.5 + 250 + 10.399P + 1,435.341
=> P = 12,360.841/166.5611 = £74.21

(ii) Gross retrospective policy value is given by:

$V^{retrospective}$

$$= (1+i)^{30}\frac{l_{[30]}}{l_{60}}\left[\begin{array}{r}12P\times 0.975\ddot{a}^{(12)}_{[30]:\overline{30|}}@i\% + 0.025P - 12\times 0.5P - 250 - 75{,}000A^{1}_{[30]:\overline{30|}}@i\% \\ -75(\ddot{a}^{@j\%}_{[30]:\overline{30|}} - 1) - 300A^{1}_{[30]:\overline{30|}}@j\%\end{array}\right]$$

where,

$$\frac{l_{[30]}}{l_{60}} = \frac{9923.7497}{9287.2164} = 1.06854$$

and at rate i = 0.06

$$\ddot{a}^{(12)}_{[30]:\overline{30|}} = \ddot{a}_{[30]:\overline{30|}} - \frac{11}{24}\left(1 - v^{30}\frac{l_{60}}{l_{[30]}}\right) = 14.437 - \frac{11}{24}(1 - 0.16294) = 14.0533$$

$$A^{1}_{[30]:\overline{30|}} = \left(A_{[30]:\overline{30|}} - v^{30}\frac{l_{60}}{l_{[30]}}\right) = (0.18283 - 0.16294) = 0.0198$$

and at rate j = 0.04

$$\ddot{a}_{[30]:\overline{30|}} = 17.759$$

$$A^{1}_{[30]:\overline{30|}} = A_{[30]:\overline{30|}} - v^{30}\frac{l_{60}}{l_{[30]}} = 0.31697 - 0.30832\times 0.93586 = 0.02843$$

$$\Rightarrow V^{retrospective}$$
$$= 6.13715\ 12{,}201.876 + 1.8553 - 445.26 - 250 - 1{,}491.75 - 1{,}256.925 - 8.529$$
$$= £53{,}707.84$$

Generally part (i) was done well. Students did however often struggle to reproduce part (ii) which is often the case with retrospective reserves.

In this case because the reserve basis matched the premium basis the retrospective reserve equalled the prospective reserve. If the student realised this, fully stated the fact and then calculated the prospective reserve full credit was given.

Minimal credit was however given if just a prospective reserve method was attempted without proper explanation.

14

(i) First calculate net premium NP and reserve ${}_tV_{57:\overline{3|}}$ for t = 1 and 2

$$NP\ddot{a}_{57:\overline{3}|} = 10000(A_{57:\overline{3}|} - v^3 \frac{l_{60}}{l_{57}}) + 0.5 \times 3 \times NP \times v^3 \frac{l_{60}}{l_{57}}$$

$$NP \times 2.870 = 8896.3 - 10000 \times 0.889 \times 9287.2164 / 9467.2906$$
$$+1.5NP \times 0.889 \times 9287.2164 / 9467.2906$$
$$= 175.394 + NP \times 1.308$$
$$\Rightarrow NP = 112.29$$

$${}_1V_{57:\overline{3}|} = (112.29 \times (1.04) - 10000 \times q_{57}) / (1 - q_{57})$$
$$= (116.782 - 56.50) / 0.99435$$
$$= 60.62$$

$${}_2V_{57:\overline{3}|} = ((112.29 + 60.62) \times (1.04) - 10000 \times q_{58}) / (1 - q_{58})$$
$$= (179.826 - 63.520) / 0.993648$$
$$= 117.05$$

The end 3rd year reserve needs to be 1.5 times the office premium to be calculated so as to meet the return guarantee.

We can complete the following table (denoting the office premium by *P*). Note as withdrawals are assumed at the end of the year the decrements of mortality and withdrawal are not dependent.

	Year 1 Age 57	*Year 2 Age 58*	*Year 3 Age 59*
80% AM92 q select	0.0033368	.004944	.005712
Withdrawal	.19933264	.0995056	0
In force factor begin year	1	.79733056	.7140497
Premium	*P*	*P*	*P*
Expenses	0.2*P*	0.05*P*	0.05*P*
Death Claims	33.368	49.440	57.120
Opening Reserve	0	60.62	117.05
Closing Reserve	48.334	104.824	1.4914*P*
Interest	.048*P*	.057*P*+3.6372	.057*P*+7.023
Profit vector	.848*P*–81.702	1.007*P*–90.007	–0.4844*P*+66.953
Profit signature	.848*P*–81.702	.8029*P*–71.7653	–0.3459*P*+47.808

Alternatively the Closing Reserve at End Year 3 can be taken as zero and an additional item termed "Maturity Value" can be shown in Year 3 only equal to 1.4914*P*.

To obtain 10% return the equation is:

$P \times [.848/(1.1) + 0.8029/(1.1)^2 - 0.3459/(1.1)^3] - [81.702/(1/1)$

$+ 71.7653/(1.1)^2 - 47.808/(1.1)^3] = 0$

$\Rightarrow 1.1746 \times P - 97.6659 = 0 \quad \Rightarrow P = 83.15$ say £83

(ii) The impact of increasing withdrawal rates depends primarily on the relationship between expenses, reserves and any surrender value. In this case there is no surrender value, a substantial reserve for a maturity benefit and low expenses.

In that scenario, increasing the lapse rates actually improves the return to the company as it retains a substantial premium and reserve with low expected death costs and returns nothing to the policyholder. This return comes earlier also and benefits from the high risk discount rate.

(iii) A revised office premium is now required say P'.

In this case a life who surrenders obtains $0.25P'$ at the end of year1 and $0.5P'$ at the end of year 2.

On the same parameters the present value of these 2 cash flow items are:

$P' \times [0.25 \times 0.19933264/(1.1)+0.5 \times 0.0995056 \times 0.79733056/(1.1)^2]$

$= 0.07809P'$

Hence from above with adjustment:

$1.1746 \times P' - 97.6659 - 0.07809P' = 0 \Rightarrow P' = 89.07$ say £89

Most students found this a very daunting question and overall performance was lower than expected. Certain comments are appropriate:

- *Because of the stated fact that withdrawals happened at the end of the year calculating dependent decrements was not necessary. Many students wasted much time attempting to perform this.*
- *Many students did not know how to calculate a net premium for this contract.*
- *The reserve process was very straightforward if done on a recursive basis (see question 6)*
- *Once these facts were realised the question was then a relatively simple manipulation of cash flows.*

Credit was given to students who gave some reasonable verbal explanation of what needed to be done even if calculations were incomplete.

END OF EXAMINERS' REPORT