EXAMINATION

April 2007

Subject CT5 — Contingencies
Core Technical

EXAMINERS' REPORT

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

M A Stocker
Chairman of the Board of Examiners

June 2007

Comments

Comments, where applicable, are given in the solutions that follow.

1 (i) $_{5|10}q_{[52]} = (l_{57} - l_{67})/l_{[52]} = (9467.2906 - 8557.0118)/9652.6965 = 0.094303$

(ii) $p_{[50]:[60]} = p_{[50]} \times p_{[60]} = (1 - 0.001971) \times (1 - 0.005774) = 0.992266$

2 Class selection — different classes of members experience different mortality rates. e.g. works versus staff. Alternatively ill-health retirements, other early retirement and normal retirements experience different mortality

Temporary Initial selection — employee turnover rates vary with duration of employment, recent joiners are most likely to leave.

Time selection — turnover rates vary with economic conditions.

Other answers given credit if properly defined with pension fund specific examples.

3
$$\begin{aligned} EPV &= 100e^{-\int_{55}^{65}(0.02+0.03+0.04)dx} \\ &= 100e^{-0.09*[65-55]} \\ &= 100e^{-0.9} \\ &= 40.66 \end{aligned}$$

4 (i) The conditions are:

- The retrospective and prospective reserves are calculated on the same basis.
- The basis is the same as the basis used to calculate the premiums used in the reserve calculation.

(ii) Two reasons are:

- The assumptions used for the retrospective calculation (for which the experienced conditions over the duration of the contract up to the valuation date are used) are not generally appropriate for the prospective calculation (for which the assumptions considered suitable for the remainder of the policy term are used).
- The assumptions considered appropriate at the time the premium was calculated may not be appropriate for the retrospective or prospective reserve some years later.

5 Value of death benefit:

$$1000 * \int_0^\infty 0.05 * \exp(-\int_0^t 0.09 ds) dt =$$

$$1000 * \int_0^\infty 0.05 * e^{-0.09t} dt = 1000 * 0.05 / 0.09$$

$$= 555.56$$

Value of survival benefit every 5th year:

$500*(e^{-0.45} + e^{-0.9} + e^{-1.35} + \ldots\ldots\ldots)$

$= 500*e^{-0.45}/(1 - e^{-0.45}) = 500*0.63763/0.36237$

$= 879.81$

Value of premiums:

$P*(1 + e^{-.09} + e^{-.18} + e^{-.27} + \ldots\ldots)$

$= P*(1/1 - e^{-.09}) = 11.619*P$

Hence

$11.619*P = 555.56+879.81$

$P = 123.54$

6 $$4 \times 25,000 \times \frac{\sum_{t=0}^{29} s_{35+t+1} d_{35+t} v^{35+t+0.5}}{s_{36} l_{35} v^{35}}$$

definitions:

s_x — salary in year to age x
d_x — number of deaths in year of age x to $x+1$
l_x — number of lives alive at age x exact

Other schemes given credit if properly defined.

7 Present value

$$10000A^{1}_{[50]:\overline{10}|} = 10000(A_{[50]} - v^{10}\, {}^{l_{60}}\!/_{l_{[50]}}\, A_{60})$$

$$= 10000(0.32868 - v^{10}\, {}^{9287.2164}\!/_{9706.0977}\, 0.45640) = 336.60$$

Variance

$$= 10000^{2}\left({}^{2}A^{1}_{[50]:\overline{10}|} - (A^{1}_{[50]:\overline{10}|})^{2}\right)$$

$$= 10000^{2}\left(({}^{2}A_{[50]} - v^{20}\, {}^{l_{60}}\!/_{l_{[50]}}\, {}^{2}A_{60}) - (336.66/10000)^{2}\right)$$

$$= 10000^{2}\left((0.13017 - v^{20}\, {}^{9287.2164}\!/_{9706.0977}\, 0.23723) - 0.033666^{2}\right) = 2543992$$

The function with the 2 suffix is calculated at rate i^2+2i i.e 8.16% in this case.

8 (i) The standardised mortality ratio is the ratio of the indirectly standardised mortality rate to the crude mortality rate in the standard population.

$$\text{SMR} = \frac{\sum_{x} \mathrm{E}^{c}_{x,t} m_{x,t}}{\sum_{x} \mathrm{E}^{c}_{x,t}\, {}^{s}m_{x,t}}$$

$E^{c}_{x,t}$ = central exposed to risk in population being studied between age x and age $x+t$

$m_{x,t}$ = central mortality rate in population being studied for ages x to $x+t$

${}^{s}m_{x,t}$ = central mortality rate in standard population for ages x to $x+t$

(ii) SMR = (2 + 3 + 6 + 50) / (25 × 12/300 + 35 × 10/275 + 100 × 9 / 200 + 500 × 8 / 175)
= 2.058

As the SMR is greater than 1, Giro experiences heavier mortality than Actuaria

9 Equation of present value:

$$\text{Purchase price} = 15{,}000\ddot{a}_{\overline{5|}}^{(12)} + 7500\ {}_{5|}\ddot{a}_{60}^{(12)} + 7500\ {}_{5|}\ddot{a}_{\overline{60:55}}^{(12)}$$

$$+7500v^5(1-{}_5p_{60})\,{}_5p_{55}\,\ddot{a}_{60}^{(12)} + 7500v^5(1-{}_5p_{55})\,{}_5p_{60}\,\ddot{a}_{65}^{(12)}$$

$$=15{,}000(1-v^5)/d^{(12)} + 7500\,v^5\,{}^{l_{65}}\!/_{l_{60}}\,(\ddot{a}_{65} - 11/24) + 7500v^5$$

$$\,{}^{l_{65}l_{60}}\!/_{l_{60}l_{55}}\,(\ddot{a}_{65} + \ddot{a}_{60} - \ddot{a}_{65:60} - 11/24)$$

$$+7500v^5[(1-l_{65}/l_{60})l_{60}/l_{55})(\ddot{a}_{60} - 11/24) + (1-l_{60}/l_{55})l_{65}/l_{60})(\ddot{a}_{65} - 11/24)]$$

$$=15000\times(1-0.82193)/0.039157 + 7500\times 0.82193\times {}^{9647.797}\!/_{9826.131}\times(13.666 - 11/24)$$

$$+7500\times 0.82193\times {}^{9647.797}\!/_{9826.131}\times {}^{9848.431}\!/_{9917.623}\times(13.666+16.652-12.682-11/24)$$

$$+7500\times 0.82193\times(1-9647.797/9826.131)\times 9848.431/9917.623\times(16.652-11/24)$$
$$+7500\times 0.82193\times(1-9848.431/9917.623)\times 9647.797/9826.131\times(13.666-11/24)$$
$$= 68213.86 + 79940.67 + 103244.12 + 1799.09 + 557.72$$

= £253755 to nearest £

The following is an alternative derivation of the formula for the purchase price above.

$$15{,}000\ddot{a}_{\overline{5|}}^{(12)} + 15{,}000v^5\,{}_5p_{60}(1-{}_5p_{55})\ddot{a}_{65}^{(12)} + 7{,}500v^5\,{}_5p_{55}(1-{}_5p_{60})\ddot{a}_{60}^{(12)}$$
$$+v^5\,{}_5p_{60}\,{}_5p_{55}(15{,}000\ddot{a}_{65}^{(12)} + 7{,}500[\ddot{a}_{60}^{(12)} - \ddot{a}_{65:60}^{(12)}])$$

10 (i) $X - Y$ is the present value of a deferred whole of life assurance with a sum assured of 1 payable at the end of the year of death of a life now aged x provided the life dies after age $x + n$.

(ii) $X = v^{k+1}$ all k $\qquad Y = \begin{cases} v^{k+1} & 0 \le k < n \\ 0 & k \ge n \end{cases}$

$\text{Cov}(X, Y) = E[XY] - E[X]\,E[Y]$

$$\text{Now } E[XY] = \sum_{k=0}^{k=n-1} (v^{k+1})^2 \; P[K_x = k] + \sum_{k=n}^{k=\infty} v^{k+1} \times 0 \times P[K_x = k]$$

$$= \sum_{k=0}^{k=n-1} (v^2)^{k+1} \; P[K_x = k]$$

$$= {}^2A^1_{x:\overline{n}|}$$

Where ${}^2A$ is determined using a discount function v^2 , i.e. using an interest rate

$$i^* = (1 + i)^2 - 1 = 2i + i^2$$

$$\text{Then: Cov}(X, Y) = {}^2A^1_{x:\overline{n}|} - A_x \, . \, A^1_{x:\overline{n}|}$$

$$\text{Now: Var}(X - Y) = \text{Var}(X) + \text{Var}(Y) - 2\,\text{Cov}(X, Y)$$

$$= ({}^2A_x - (A_x)^2) + ({}^2A^1_{x:\overline{n}|} - (A^1_{x:\overline{n}|})^2) - 2({}^2A^1_{x:\overline{n}|} - A_x \, . A^1_{x:\overline{n}|})$$

$$= ({}^2A_x + {}^2A^1_{x:\overline{n}|} - 2\;{}^2A^1_{x:\overline{n}|}) - ((A_x)^2 + (A^1_{x:\overline{n}|})^2) - 2A^1_{x:\overline{n}|}\;A_x)$$

$$= {}^2A_x - {}^2A^1_{x:\overline{n}|} - (A_x - A^1_{x:\overline{n}|})^2$$

$$= {}^2A_x - {}^2A^1_{x:\overline{n}|} - ({}_{n|}A_x)^2$$

The Examiners regret that two typographical errors occurred in the question wording set in the Examination:

- *In line 2 of 10(ii) the symbol shown as* ${}^2A^1_x$ *should have been* ${}^2A^1_{x:\overline{n}|}$.
- *In the same line the function on the left hand side of the equation should have read Cov(X,Y) and not have included in the brackets 2 assurance functions (which as erroneously stated would have equated to zero).*

In the event this question was done well despite the errors. The majority of students attempting the question noticed the first error as obvious and adjusted accordingly. The second error was rarely noticed by students who often went on to produce an otherwise good proof.

The question has been corrected for publication. The Examiners wish to sincerely apologise for these errors and wish to assure students that the marking system was sympathetically adjusted to meet the circumstances.

11 (i) It is a principle of prudent financial management that once sold and funded at outset a product should be self-supporting. Many products produce profit signatures that usually have a single financing phase. However, some products, particularly those with substantial expected outgo at later policy durations, can give profit signatures which have more than one financing phase. In such cases these later negative cashflows should be reduced to zero by establishing reserves in the non-unit fund at earlier durations. These reserves are funded by reducing earlier positive cashflows.

(ii) The reserves required at the end of year 2 and year 1 are:

$$_2V = \frac{20.15}{1.05} = 19.190$$

$$_1V = \frac{1}{1.05}(30.18 + p_{51} \times 19.190) = \frac{1}{1.05}(30.18 + 0.99719 \times 19.190) = 46.968$$

(iii) Before zeroisation, the net present value (based on a risk discount rate of 8%) is:

$$NPV = -\frac{95.21}{1.08} - \frac{p_{50} \times 30.18}{1.08^2} - \frac{{}_2p_{50} \times 20.15}{1.08^3} + \frac{{}_3p_{50} \times 77.15}{1.08^4} + \frac{{}_4p_{50} \times 120.29}{1.08^5}$$

$$= -\frac{95.21}{1.08} - \frac{0.99749 \times 30.18}{1.08^2} - \frac{0.99469 \times 20.15}{1.08^3} + \frac{0.99155 \times 77.15}{1.08^4} + \frac{0.98804 \times 120.29}{1.08^5}$$

$$= -88.157 - 25.810 - 15.911 + 56.228 + 80.888 = 7.238$$

After zeroisation, the profit in year 1 becomes:

Profit in year 1 $= -95.21 - p_{50} \times {}_1V = -95.21 - 0.99749 \times 46.968 = -142.06$

So profit vector will become (-142.06, 0, 0, 77.15, 120.29)

And *NPV* after zeroisation will be:

$$NPV = -\frac{142.06}{1.08} + 0 + 0 + \frac{{}_3p_{50} \times 77.15}{1.08^4} + \frac{{}_4p_{50} \times 120.29}{1.08^5}$$

$$= -\frac{142.06}{1.08} + 0 + 0 + \frac{0.99155 \times 77.15}{1.08^4} + \frac{0.98804 \times 120.29}{1.08^5}$$

$$= -131.537 + 56.228 + 80.888 = 5.579$$

As expected, the NPV after zeroisation is smaller because the emergence of the profits has been deferred and the risk discount rate is greater than the accumulation rate.

12 (i) Net premium per policy is P where $P\ddot{a}_{30:\overline{20}|} = 75,000A^{1}_{30:\overline{25}|}$

$$P = \frac{75,000\left(A_{30} - v^{25}\,{}_{25}p_{30}A_{55}\right)}{\ddot{a}_{30} - v^{20}\,{}_{20}p_{30}\ddot{a}_{50}}$$

$$= \frac{75,000\left(0.16023 - 1.04^{-25}\dfrac{9557.8179}{9925.2094}0.38950\right)}{21.834 - 1.04^{-20}\dfrac{9712.0728}{9925.2094}17.444}$$

$$= 75,000\frac{(0.16023 - 0.14070)}{(21.834 - 7.7903)} = £104.30$$

(ii) Net premium reserve per policy at the end of the 20th year

$$= 75,000A^{1}_{50:\overline{5}|} - 0 = 75,000\left(A_{50} - v^{5}\,{}_{5}p_{50}A_{55}\right)$$

$$= 75,000\left(0.32907 - 1.04^{-5}\frac{9557.8179}{9712.0728}0.38950\right) = 75,000 \times 0.014014 = £1051.06$$

Net premium reserve per policy at the start of the 20th year

$$= \frac{Sq_{49} + {}_{20}Vp_{49}}{1+i} - P$$

$$= \frac{75,000q_{49} + 1051.06p_{49}}{1.04} - 104.30$$

$$= \frac{75,000 \times 0.002241 + 1051.06 \times 0.997759}{1.04} - 104.30$$

$$= 1065.68$$

(iii) Death strain at risk = 75,000 – 1051 = 73,949

EDS = $738q_{49} \times 73{,}949 = 122,301$

ADS = $2 \times 73{,}949 = 147,898$

Mortality profit = 122,301 – 147,898 = -£25,597 (i.e. a loss)

13 (i) Let P be the monthly premium for the contract with simple bonus. Then equation of value (at 4% p.a. interest) is:

$$12P(.95\ddot{a}^{(12)}_{[30]:\overline{35}|}) - 5.95P = (48,000+250)\bar{A}_{[30]} + 2,000(I\bar{A})_{[30]} + 300$$

where $\ddot{a}^{(12)}_{[30]:\overline{35}|} = \ddot{a}_{[30]:\overline{35}|} - \frac{11}{24}\left(1 - v^{35}\ {}_{35}p_{[30]}\right) = 19.072 - \frac{11}{24}\left(1 - 1.04^{-35}\frac{8821.2612}{9923.7497}\right)$

$$= 18.7169$$

Therefore:

$$12P(.95\times18.7169) - 5.95P = (48,000+250)\times1.04^{0.5}\times0.16011 + 2,000\times1.04^{0.5}\times6.91644 + 300$$

i.e.

$$207.42266P = 7,878.299 + 14,106.825 + 300$$

$$\Rightarrow P = \frac{22,285.124}{207.42266} = £107.44$$

(ii) Let P' be the monthly premium for the contract with compound bonus. Then equation of value (at 4% p.a. interest) is:

$$12P'(.95\ddot{a}^{(12)}_{[30]:\overline{35}|}) - 5.95P' = 50,000\left[v^{0.5}q_{[x]} + v^{1.5}p_{[x]}q_{[x]+1}(1.04) + \ldots\right] + 250\bar{A}^{@4\%}_{[30]} + 300$$

$$= \frac{50,000}{1.04}\left[v^{0.5}\times1.04q_{[x]} + v^{1.5}\times1.04^2\ p_{[x]}q_{[x]+1} + \ldots\right] + 250\bar{A}^{@4\%}_{[30]} + 300$$

$$= \frac{50,000}{(1.04)^{0.5}}\left[v\ \times1.04q_{[x]} + v^2\times1.04^2\ p_{[x]}q_{[x]+1} + \ldots\right] + 250\bar{A}^{@4\%}_{[30]} + 300$$

$$= \frac{50,000}{(1.04)^{0.5}}A^{@0\%}_{[30]} + 250\bar{A}^{@4\%}_{[30]} + 300$$

where $A^{@0\%}_{[30]} = 1$

$$\Rightarrow 12P'(.95\times18.7169) - 5.95P' = \frac{50,000}{(1.04)^{0.5}} + 250\times1.04^{0.5}\times0.16011 + 300$$

$$207.42266P' = 49,029.034 + 40.820 + 300$$

$$\Rightarrow P' = \frac{49,369.854}{207.42266} = £238.02$$

14 Multiple decrement table:

X	$q^d_{[x]} = (aq)^d_{[x]}$	$q^s_{[x]}$	$(aq)^s_{[x]} = q^s_{[x]}\left(1-(aq)^d_{[x]}\right)$
61	0.006433	0.05	0.04968
62	0.009696	0.05	0.04952
63	0.011344	0.05	0.04943
64	0.012716	—	—

T	$(ap)_{[61]+t-1}$	${}_{t-1}(ap)_{[61]}$
1	0.943887	1
2	0.940784	0.94389
3	0.939226	0.88799
4	0.987284	0.83403

Let P be the annual premium payable. Then equation value is:

$$P\ddot{a}_{[61]:\overline{4}|} = 100,000A_{[61]:\overline{4}|} + (50 + 0.025P)(\ddot{a}_{[61]:\overline{4}|} - 1) + 500$$

$$\Rightarrow P(0.975\ddot{a}_{[61]:\overline{4}|} + 0.025) = 100,000A_{[61]:\overline{4}|} + 50(\ddot{a}_{[61]:\overline{4}|} - 1) + 500$$

$$\Rightarrow P(0.975 \times 3.730 + 0.025) = 100,000 \times 0.85654 + 50 \times 2.730 + 500$$

$$P = \frac{85,654 + 636.5}{3.66175} = 23,565.37$$

Reserves required on the policy per unit sum assured are:

$${}_1V_{61:\overline{4}|} = 1 - \frac{\ddot{a}_{62:\overline{3}|}}{\ddot{a}_{61:\overline{4}|}} = 1 - \frac{2.857}{3.722} = 0.23240$$

$${}_2V_{61:\overline{4}|} = 1 - \frac{\ddot{a}_{63:\overline{2}|}}{\ddot{a}_{61:\overline{4}|}} = 1 - \frac{1.951}{3.722} = 0.47582$$

$${}_3V_{61:\overline{4}|} = 1 - \frac{\ddot{a}_{64:\overline{1}|}}{\ddot{a}_{61:\overline{4}|}} = 1 - \frac{1.000}{3.722} = 0.73133$$

Year t	*Prem*	*Expense*	*Opening reserve*	*Interest*	*Death Claim*	*Surr Claim*	*Mat Claim*	*Closing reserve*	*Profit vector*
1	23565.4	500	0	1153.3	643.3	1170.7	0	21935.9	468.8
2	23565.4	639.1	23240.0	2308.3	969.6	2333.9	0	44764.4	406.7
3	23565.4	639.1	47582.0	3525.4	1134.4	3494.5	0	68688.4	716.4
4	23565.4	639.1	73133.0	4803.0	1271.6	0	98728.4	0	862.3

Year t	*Profit signature*	*Discount factor*	*NPV of profit signature*
1	468.8	.92593	434.1
2	383.9	.85734	329.1
3	636.2	.79383	505.0
4	719.2	.73503	528.6

NPV of profit signature = £1,796.8

Year t	*Premium*	${}_{t-1}p_{[61]}$	*Discount factor*	*NPV of premium*
1	23565.4	1	1	23565.4
2	23565.4	0.94389	.92593	20595.6
3	23565.4	0.88799	.85734	17940.6
4	23565.4	0.83403	.79383	15602.1

NPV of premiums = £77,703.7

Profit margin = $\dfrac{1,796.8}{77,703.7} = 0.0231$ i.e. 2.31%

END OF EXAMINERS' REPORT