EXAMINATION

September 2007

Subject CT5 — Contingencies
Core Technical

EXAMINERS' REPORT

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

M A Stocker
Chairman of the Board of Examiners

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1

$$({}_tV_x + P_x)(1+i) = q_{x+t} + p_{x+t}({}_{t+1}V_x)$$
$$\Rightarrow (0.468+0.017)(1.03) = 0.024 + (0.976)({}_{t+1}V_x)$$
$$\Rightarrow {}_{t+1}V_x = (0.49955 - 0.024)/0.976 = 0.487$$

2

$$p_{[x]} = ({}_{0.5}p_{[x]})({}_{0.5}p_{[x]+0.5}) = (1 - {}_{0.5}q_{[x]})(1 - {}_{0.5}q_{[x]+0.5})$$
$$= (1-0.33q_x)(1-0.5q_x) = (1-0.33(1-p_x))(1-0.5(1-p_x))$$
$$= (0.67+0.33p_x)(0.5+0.5p_x)$$
$$= 0.335 + 0.5p_x + 0.165p_x^{\ 2}$$

3

(+1, -1, +1, +1, +1, -1, 0, -1, +1, -1, +1, +1)
→ (+1, -1, +1, +1, +1, -1, 0, -1, 0, 0, +1, +1)
→ (+1, -1, +1, +1, +1, -1, -1, 0, 0, 0, +1, +1)
→ (+1, -1, +1, +1, +1, -2, 0, 0, 0, 0, +1, +1)
→ (+1, -1, +1, +1, -1, 0, 0, 0, 0, 0, +1, +1)
→ (+1, -1, +1, 0, 0, 0, 0, 0, 0, 0, +1, +1)
→ (0, 0, +1, 0, 0, 0, 0, 0, 0, 0, +1, +1)

(Following not necessary for marks but to help explain)

All positives after last negative remain unchanged.

Consider last negative in year 10. The underlying cash flow that year, per policy in force at start of year 10, is

$$\frac{-1}{{}_9p_x} = (NUCF)_{10}.$$

The reserve needed at t = 9 to counter this is

$$\frac{-(NUCF)_{10}}{1+i} = -(NUCF)_{10} \text{since } i = 0.$$

This is funded from year 9 cash flows at a cost of

$$= (p_{x+8})(-(NUCF)_{10})$$

per policy in force at the start of year 9.

The change in year 9 cash flow is

$$(NUCF)_9 = -(p_{x+8})(-(NUCF)_{10}) = p_{x+8}(NUCF)_{10}$$

and the change in year 9 profit signature becomes

$$(PS)_9^* = {}_8p_x\, p_{x+8} (NUCF)_{10} = {}_9p_x (NUCF)_{10} = -1$$

resulting in a revised year 9 profit signature of +1-1=0.

The other results follow by repeating this step from year 9 towards year 1 wherever there are negative values in the profit signature.

4 $EPV = 1{,}000\left({}_{1/12}p_{65} \frac{1}{1.09^{1/12}} + {}_{2/12}p_{65} \frac{1.0039207}{1.09^{2/12}} + {}_{3/12}p_{65} \frac{1.0039207^2}{1.09^{3/12}} + + \right.$

But $1.0039207^{12} = 1.048076$ and $\frac{1.048076}{1.09} = \frac{1}{1.04}$ leading to

$$EPV = \frac{1{,}000}{1.0039207}\left({}_{1/12}p_{65} \frac{1}{1.04^{1/12}} + {}_{2/12}p_{65} \frac{1}{1.04^{2/12}} + {}_{3/12}p_{65} \frac{1}{1.04^{3/12}} + + \right.$$

$$= \frac{12{,}000}{1.0039207} a_{65}^{(12)} @ 4\%\text{p.a.}$$

EPV = 11,953.14 * (14.871 - 1+(11/24)) = 11,953.14 * 14.329 = 171,276

5 (i) Directly standardised mortality rate = $\dfrac{\sum_x {}^sE_{x,t}^c m_{x,t}}{\sum_x {}^sE_{x,t}^c}$.

Where:

${}^sE_{x,t}^c$: Central exposed to risk in standard population between ages x and $x+t$

$m_{x,t}$: central rate of mortality either observed or from a life table in population being studied for ages x to $x+t$

(ii) The main disadvantage is that it requires age-specific mortality rates, $m_{x,t}$, for the group / population in question, and these are often not available conveniently. To overcome this, indirect standardisation, which relies on easily available data, can be used.

6 $\quad {}_{0.5|}q_{75} = {}_{0.5}p_{75}(q_{75.5}) = {}_{0.5}p_{75}[{}_{0.5}q_{75.5} + ({}_{0.5}p_{75.5})({}_{0.5}q_{76})]$

$= [({}_{0.5}p_{75})({}_{0.5}q_{75.5}) + (p_{75})({}_{0.5}q_{76})]$

(a) $\quad UDD \Rightarrow {}_tq_x = (t)q_x,\ 0 \le t \le 1$

$${}_{0.5|}q_{75} = {}_{0.5}p_{75}(q_{75.5}) = {}_{0.5}p_{75}(1 - p_{75.5}) = {}_{0.5}p_{75}[1 - ({}_{0.5}p_{75.5})({}_{0.5}p_{76})]$$
$$= {}_{0.5}p_{75} - (p_{75})({}_{0.5}p_{76}) = (1 - {}_{0.5}q_{75}) - (1 - q_{75})(1 - {}_{0.5}q_{76})$$
$$= (1-(0.5)(.05) - (1-0.05)(1-(0.5)(.06)) = 0.975 - (0.95)(0.97) = 0.0535$$ or

using

$${}_{0.5|}q_{75} = [({}_{0.5}p_{75})({}_{0.5}q_{75.5}) + (p_{75})({}_{0.5}q_{76})] = [(1 - {}_{0.5}q_{75})({}_{0.5}q_{75.5}) + (1 - q_{75})({}_{0.5}q_{76})]$$
$$= [((1-(0.5)(.05))(\tfrac{(0.5)(.05)}{1-(0.5)(.05)}) + (1-.05)(0.5)(.06)] = 0.025 + 0.0285 = 0.0535$$

(b) $\quad$ Constant force of mortality $\Rightarrow {}_tp_{x+r} = e^{-\mu t} = (e^{-\mu})^t = (p_x)^t,\ 0 \le r+t \le 1$

$${}_{0.5|}q_{75} = {}_{0.5}p_{75}[1 - ({}_{0.5}p_{75.5})({}_{0.5}p_{76})]$$
$$= (0.95)^{0.5}[1 - (0.95)^{0.5}(0.94)^{0.5}]$$
$$= (0.974679)[1 - 0.944987] = 0.05362$$

7 $\quad$ Policy A:

${}_{10}V = 145{,}000A_{50} + 5{,}000(IA)_{50} - (NP)\ddot{a}_{50}^{(12)}$ where NP from

$$95{,}000A_{[40]} + 5{,}000(IA)_{[40]} = (NP)\ddot{a}_{[40]}^{(12)}$$

$\Rightarrow \quad NP \quad = \{(95{,}000)(0.23041) + (5{,}000)(7.95835)\}/(20.009 - 0.458) = 3{,}154.86$

and $\ {}_{10}V \quad = \{(145{,}000)(0.32907) + (5{,}000)(8.55929)\} - (3{,}154.86)(17.444 - 0.458)$
$= 36{,}923.15$

Policy B:

${}_{10}V = 150{,}000A_{50} - (NP)\ddot{a}_{50}^{(12)}$ where NP from

$$100{,}000A_{[40]} = (NP)\ddot{a}_{[40]}^{(12)}$$

$\Rightarrow \quad NP \quad = \quad (100{,}000)(0.23041)/(20.009 - 0.458) = 1{,}178.51$

and $_{10}V$ = (150,000)(0.32907) - (1,178.51)(17.444 - 0.458)
= 29,342.33

8 (a) Class selection: groups with different permanent attributes having different mortality

e.g. sex, male and female rates differ at all ages

(b) Spurious selection: ascribing mortality differences to groups formed by factors which are not the true causes of these differences. The influence of some confounding factor has been ignored.

e.g. Regional mortality differences actually explained by the different composition of occupations in the different regions.

(c) Time selection: within a population, mortality varies over calendar time. The effect is usually noticed at all ages and usually rates become lighter over time

e.g. ELT12 male mortality vs. ELT15male

9 (i)

$$EPV = 20,000a_{\overline{68:65}} = 20,000(a_{68}^{m} + a_{65}^{f} - a_{68:65})$$
$$= 20,000(11.412 + 13.871 - 10.112) = 20,000(15.171) = 303,420$$

(ii) The office loses money if PV of payments > 320,000
i.e. if 20,000 $a_{\overline{n|}}$ >320,000 or $a_{\overline{n|}}$ >16.

At 4% p.a., $a_{\overline{26|}}$ =15.9828 and $a_{\overline{27|}}$ = 16.3296 so if the office makes the 27th payment under this annuity, it incurs a loss. It therefore makes a profit so long as both lives have died before this time, with probability $_{27}q_{\overline{68:65}}$

$$_{27}q_{\overline{68:65}} = ({}_{27}q_{68}^{m})({}_{27}q_{65}^{f}) = (1 - \frac{l_{95}^{m}}{l_{68}^{m}})(1 - \frac{l_{92}^{f}}{l_{65}^{f}})$$
$$= (1 - \tfrac{1,020.409}{9,440.717})(1 - \tfrac{3,300.559}{9,703.708}) = (0.891914)(0.65987) = 0.5885$$

10 (i) $g(T) = \begin{cases} 500,000 v^{T_y} & T_y > T_x \\ 0 & T_y \le T_x \end{cases}$

(ii) $E[g(T)] = 500,000 \int_0^\infty v^t (1 - {}_t p_x)\, {}_t p_y \mu_{y+t} dt$

(iii) Lifetime of (*y*). If (y) dies first, no benefit is possible and if (*y*) dies second, SA becomes payable immediately. (*x*)'s lifetime is irrelevant in this context. Premium could be payable for joint lifetime of (*x*) and (*y*) but this is shorter than (*y*) and therefore we use (*y*)'s lifetime.

11 (i) $X = \begin{cases} v^n & K_x \ge n \\ 0 & K_x < n \end{cases}$ $\qquad Y = \begin{cases} 0 & K_x \ge n \\ v^{K_x+1} & K_x < n \end{cases}$

$\Rightarrow XY = 0$ for all K_x

$COV(X,Y) = E[XY] - E[X]E[Y] = 0 - (A_{x:\overline{n|}}^{\;\;1})(A^1_{x:\overline{n|}})$

(ii) $VAR(X+Y) = VAR(X) + VAR(Y) + 2COV(X,Y)$

$$= {}^2A_{x:\overline{n|}}^{\;\;1} - (A_{x:\overline{n|}}^{\;\;1})^2 + {}^2A^1_{x:\overline{n|}} - (A^1_{x:\overline{n|}})^2 - 2(A_{x:\overline{n|}}^{\;\;1})(A^1_{x:\overline{n|}})$$
$$= \{{}^2A_{x:\overline{n|}}^{\;\;1} + {}^2A^1_{x:\overline{n|}}\} - \{(A_{x:\overline{n|}}^{\;\;1})^2 + (A^1_{x:\overline{n|}})^2 + 2(A_{x:\overline{n|}}^{\;\;1})(A^1_{x:\overline{n|}})\}$$
$$= \{{}^2A_{x:\overline{n|}}^{\;\;1} + {}^2A^1_{x:\overline{n|}}\} - \{(A_{x:\overline{n|}}^{\;\;1}) + (A^1_{x:\overline{n|}})\}^2$$
$$= {}^2A_{x:\overline{n|}} - (A_{x:\overline{n|}})^2$$

12 (i) $P\ddot{a}_{[40]:\overline{20|}} = 75,000 A_{[40]:\overline{20|}}^{\;\;\;\;1} = 75,000 v^{20}\, {}_{20}p_{[40]}$

$\Rightarrow P(13.930) = (75,000)(0.45639)(0.94245)$

$\Rightarrow P = 32,259.45 / 13.93 = 2,315.83$

Mortality profit = Expected Death Strain – Actual Death Strain

$$DSAR = 0 - {}_{15}V = -(75,000 A_{55:\overline{5|}}^{\;\;1} - P\ddot{a}_{55:\overline{5|}})$$
$$= -(75,000 v^5\, {}_5p_{55} - P\ddot{a}_{55:\overline{5|}})$$
$$= -\{(75,000)(0.82193)(0.97169) - (2,315.83)(4.585)\} = -49,281.51$$

EDS = (q_{54})(500)(-49,281.51) = (0.003976)(500)(-49,281.51) = -97,971.64
ADS = (3)(-49,281.51) = -147,844.53

Mortality Profit = -97,971.64 - (-147,844.53) = 49,872.89 profit.

(ii) We expected $500q_{54}$ = 1.988 deaths. Actual deaths were 3. With pure endowments, the death strain is negative because no death claim is paid and there is a release of reserves to the company on death. In this case, more deaths than expected means this release of reserves is greater than required by the equation of equilibrium and the company therefore makes a profit.

13 (i) $$P\ddot{a}_{30:\overline{35}|} = 200,600A_{30:\overline{35}|} - 400A_{\substack{1\\30:\overline{35}|}} + (0.02)P\ddot{a}_{30:\overline{35}|} - 0.02P + 300 + (0.5)(P)$$

Expected present value of premiums:

$$P\ddot{a}_{30:\overline{35}|} = 15.150P$$

EPV of benefit and claim expenses:

$$A_{30:\overline{35}|} = 0.14246$$

$$A_{\substack{\\30:\overline{35}|}}^{\;\;\;\;\;\;\;1} = v^{35}\ {}_{35}p_{30} = (0.13011)(0.88877) = 0.11563$$

$\Rightarrow$ EPV of benefits and claim expenses

= (200,600)(0.14246) - (400)(0.11563)

= 28,577.48 - 46.25 = 28,531.23

EPV of remaining expenses:

[(0.02)(15.150P)] - 0.02P + 0.5P + 300 = 0.783P + 300

Equation of value:

15.150P = 28,531.23 + 300 + 0.783P $\Rightarrow$14.367P = 28,831.23

$\Rightarrow$ P = 2,006.77 per annum = 2,007 p.a.

(ii) $$\text{GFLRV} = \begin{cases} 200,600v^{K_{55}+1} - (0.98)(2,007)(\ddot{a}_{\overline{K_{55}+1}|}) & K_{55} < 10 \\ 200,200v^{10} - (0.98)(2,007)(\ddot{a}_{\overline{10}|}) & K_{55} \geq 10 \end{cases}$$

(iii) $${}_{25}V^{retro} = \frac{1}{v^{25}\,{}_{25}p_{30}}\{0.98P\ddot{a}_{30:\overline{25}|} - 0.48P - 300 - 200{,}600A^{1}_{30:\overline{25}|}\}$$

$$v^{25}\,{}_{25}p_{30} = (0.37512)(0.96298) = 0.36123$$

$$\ddot{a}_{30:\overline{25}|} = \ddot{a}_{30} - v^{25}\,{}_{25}p_{30}\ddot{a}_{55} = 21.834 - (0.36123)(15.873) = 16.100$$

$$A^{1}_{30:\overline{25}|} = A_{30} - v^{25}\,{}_{25}p_{30}A_{55} = 0.16023 - (0.36123)(0.38950) = 0.01953$$

$${}_{25}V^{retro} = \frac{1}{0.36123}\{[2{,}007][(0.98)(16.100) - (0.48)] - 300 - (200{,}600)(0.01953)\}$$

$$= \frac{1}{0.36123}\{30{,}703.09 - 300 - 3{,}917.72\} = 73{,}319.96$$

(iv) It would have been larger. At 6% both would be the same but

$$V^{retro}_{@4\%} < V^{retro}_{@6\%} = V^{pro}_{@6\%} < V^{pro}_{@4\%}$$

since retrospective reserves are accumulating premiums in excess of claims and expenses and lower interest leads to lower reserves but prospective reserves are meeting the excess of future benefits claims over future premiums and lower interest leads to higher reserves.

14 (i), (ii) and (iii)

Transition probabilities not given explicitly are

t	p^{HH}_{63+t}	p^{CC}_{63+t}	p^{DD}_{63+t}
0	0.94	0.75 (not needed)	1.00 (not needed)
1	0.91	0.67	1.00

Outcome	*PV of Cash flow (000's)*	*PV Ben*	*(PV Ben)^2*	*Prob.*	*Prob.*	*E[PVB]*	*E[PVB^2]*
HH	0	0.00	0.00	0.94*0.91	0.8554	0	0
HC	$60v^2$	49.59	2458.85	0.94*0.06	0.0564	2.79669	138.67905
HD	$100v^2$	82.64	6830.13	0.94*0.03	0.0282	2.33058	192.60979
CC	$60v$	54.55	2975.21	0.04*0.67	0.0268	1.46182	79.73554
CD	$60v+40v^2$	87.60	7674.34	0.04*0.33	0.0132	1.15636	101.30128
DD	$100v$	90.91	8264.46	0.02	0.02	1.81818	165.28926
Total				1		9.56364	677.61492

(iv) Mean $= (1{,}000)(9.56364) = 9{,}563.64$

Var. $= (1{,}000)^2\{(677.61492 - (9.56364)^2\}$

$= 586{,}151{,}710 = (24{,}210.57)^2$

(v) Var.(profit) = Var.(SP - EPV(bens)) = Var.(EPV(bens))

EPV(profit) = (10,000)(SP - 9,563.64) = 10,000SP - 95,636,400

Var.(profit) = Var.(SP - EPV(bens)) = Var.(EPV(bens))

For 10,000 independent policies,

Var.(profit) = (10,000)(586,151,710) = $(2{,}421{,}057)^2$

St. Dev.(profit) = 2,421,057

We need SP so Prob.(profit > 0) = 0.95

$$\Rightarrow \text{Pr.}\left(\frac{\text{profit} - (EPV(\text{profit})}{\text{StDev(profit)}} > \frac{0 - (10{,}000SP - 95{,}636{,}400)}{2{,}421{,}057}\right) = 0.95$$

Assuming profit is normally distributed

$$\Rightarrow \text{Pr.}\left(z > \frac{95{,}636{,}400 - 10{,}000SP}{2{,}421{,}057}\right) = 0.95 \Rightarrow \Phi\left(\frac{95{,}636{,}400 - 10{,}000SP}{2{,}421{,}057}\right) = 0.05$$

$$\Rightarrow \left(\frac{95{,}636{,}400 - 10{,}000SP}{2{,}421{,}057}\right) = \Phi^{-1}(0.05) = -1.6449$$

$$\Rightarrow SP = \left(\frac{95{,}636{,}400 + (1.6449)(2{,}421{,}057)}{10{,}000}\right) = 9{,}961.88 = 9{,}962$$

END OF EXAMINERS' REPORT