EXAMINATION

September 2006

Subject CT5 — Contingencies
Core Technical

EXAMINERS' REPORT

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

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Chairman of the Board of Examiners

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Comments

No comments are given

1 If funds chose at random which annuities to insure and which to self-insure, we would expect approximately the same mortality experience in both groups. The self-insured experience is heavier, meaning their lives are somehow below standard on average.

The most likely explanation is that the pension funds make informed decisions based on the health or reason for retirement of the pensioners. Those retiring early due to ill-health or those known to have poor health are retained for payment directly by the fund. That should be cheaper than paying a premium to the insurer based on normal mortality for these lives. The remainder of the lives, known to be on reasonable health are insured.

This is adverse selection.

Sensible comments regarding other forms of selection are also acceptable.

2 (i)

$$EPV = 200,000\int_0^{20} e^{-\delta t}\, {}_t p_{40}^{HH}\sigma_{40+t}dt = 200,000\int_0^{20} e^{-\delta t}\, {}_t p_{40}^{\overline{HH}}\sigma_{40+t}dt$$

$$= 200,000\int_0^{20} e^{-\delta t} e^{-\int_0^t (\mu_{40+r}+\sigma_{40+r})dr}\sigma_{40+t}dt$$

where:

${}_t p_{40}^{HH}$ is the probability that the healthy life aged 40is healthy at age 40+t

${}_t p_{40}^{\overline{HH}}$ is the probability that the healthy life aged 40 is healthy at all points up to age 40+t (These 2 probabilities are the same for this model)

$\delta = \ln(1.08) = 0.076961$

(ii)

From $EPV = 200,000\int_0^{20} e^{-\delta t} e^{-\int_0^t (\mu_{40+r}+\sigma_{40+r})dr}\sigma_{40+t}dt$

$$EPV = 200,000\int_0^{20} e^{-(0.076961)t} e^{-\int_0^t \{(0.01)+(0.02)\}dr}(0.02)dt$$

$$= 200,000\int_0^{20} e^{-(0.076961)t}e^{-(0.03)t}(0.02)dt = 200,000\int_0^{20} e^{-(0.106961)t}(0.02)dt$$

$$= -\frac{(200,000)(0.02)}{0.106961}\left[e^{-(0.106961)t}\right]_0^{20} = 37,396.79[1-0.11775] = 32,993.32$$

3 $\bar{A}^{\;1}_{70:\overline{1|}} = \int_0^1 e^{-\delta_{.075}t}\, {}_tp_{70}\mu_{70+t}\,dt$ in the general case.

Here, assuming μ is constant for $0 < t < 1$, we get

$\mu = -\ln(p_{70}) = -\ln(1 - .03930) = 0.040093$
${}_tp_{70} = \exp(-\mu t) = \exp(-.040093t)$
$\delta_{.075} = \ln(1.075) = 0.07232$

$$\bar{A}^{\;1}_{70:\overline{1|}} = \int_0^1 e^{-0.07232t}e^{-0.040093t}(0.040093)dt$$

$$= \frac{-(0.040093)}{(0.07232+0.040093)}\left[e^{-(0.07232+0.040093)t}\right]_0^1$$

$$= (-0.35610)(0.89368-1) = 0.0379$$

4 EPV of benefits:

$$20,000a^{(12)}_{65|60} = 20,000(a^{(12)}_{60} - a^{(12)}_{65:60}) = 20,000(a_{60} - a_{65:60}) = 20,000(15.652 - 11.682)$$
$$= 79,400$$

EPV premiums:

(The premium term will be the joint lifetime of the two lives because if his death is first the annuity commences or if her death is first, there will never be any annuity.)

Let P be the monthly premium

$$12P\ddot{a}^{(12)}_{65:60} = 12P(\ddot{a}_{65:60} - \tfrac{11}{24}) = 12P(12.682 - 0.458) = 146.688P$$

Equation of value allowing for expenses:

$1.015(79{,}400) = (1 - 0.05)(146.688P) \Rightarrow 80{,}591 = 139.3536P \Rightarrow P$
$= 578.32$ per month

5 (i) This is the present value of a joint life annuity of amount 1 per annum payable continuously until the first death of 2 lives (x) and (y).

(ii) $E[g(T)] = \int_0^\infty {}_tp_{xy}\mu_{x+t:y+t}\bar{a}_{\overline{t|}}dt$ or $E[g(T)] = \int_0^\infty {}_tp_{xy}e^{-\delta t}dt$

(iii) $Var[g(T)] = \dfrac{{}^2\bar{A}_{xy} - (\bar{A}_{xy})^2}{\delta^2}$ where ${}^2\bar{A}_{xy}$ indicates that the function is to be evaluated at force of interest 2δ.

6 (i) EPV past service benefits:

$$40,000\,\frac{20}{80}\,\frac{({}^zM_{55}^{ia} + {}^zM_{55}^{ra})}{s_{54}D_{55}} = 40,000\,\frac{20}{80}\,\frac{(34,048 + 128,026)}{(9.745)(1,389)} = 119,737$$

EPV future service benefits:

$$\frac{40,000}{80}\,\frac{({}^z\bar{R}_{55}^{ia} + {}^z\bar{R}_{55}^{ra})}{s_{54}D_{55}} = \frac{40,000}{80}\,\frac{(163,063 + 963,869)}{(9.745)(1,389)} = 41,628$$

EPV total pension benefits = 119,737 + 41,628 = £161,365

(ii) $(k)(40,000)\dfrac{{}^s\bar{N}_{55}}{s_{54}D_{55}} = 41,628 \Rightarrow (k)(40,000)\dfrac{88,615}{(9.745)(1,389)} = 41,628 \Rightarrow k = .159$

i.e. 15.9% salary per annum

7 ACF $= \dfrac{\sum_x {}^sE_{x,t}^c\,{}^sm_{x,t}}{\sum_x {}^sE_{x,t}^c} \bigg/ \dfrac{\sum_x E_{x,t}^c\,{}^sm_{x,t}}{\sum_x E_{x,t}^c}$

Here

	${}^sE_{x,t}^c$	${}^sE_{x,t}^c\,{}^sm_{x,t}$	$E_{x,t}^c$
Age-group	Population	Deaths	Population
0–19	2,900,000	580	800,000
20–44	3,500,000	2,450	1,000,000
45–69	2,900,000	20,300	900,000
70 and over	700,000	49,000	300,000
Total	10,000,000	72,330	3,000,000

leading to

${}^sm_{x,t}$	$E_{x,t}^c\,{}^sm_{x,t}$
0.0002	160
0.0007	700
0.007	6,300
0.07	21,000
	28,160

$$\text{ACF} \;=\; \frac{72,330}{10,000,000} \Big/ \frac{28,160}{3,000,000} = \quad (0.007233/\ 0.0093867) = 0.77056$$

Indirectly standardised mortality rate = (ACF)*(Province crude rate)

$$= (0.77056)\left(\frac{25,344}{3,000,000}\right) = (0.77056)(0.008448) = 0.00651$$

8 (i)

$$_t\overline{V} = {}_{n-t}p_{x+t}e^{-\delta(n-t)}$$

$$\frac{\partial}{\partial t}{}_t\overline{V} = \frac{\partial}{\partial t}({}_{n-t}p_{x+t}e^{-\delta(n-t)}) = \{e^{-\delta(n-t)}\frac{\partial}{\partial t}({}_{n-t}p_{x+t})\} + \{{}_{n-t}p_{x+t}\frac{\partial}{\partial t}(e^{-\delta(n-t)})\}$$

$$\frac{1}{{}_{n-t}p_{x+t}}\frac{\partial}{\partial t}({}_{n-t}p_{x+t}) = \frac{\partial}{\partial t}\ln({}_{n-t}p_{x+t}) = \frac{\partial}{\partial t}\ln\left(\frac{l_{x+n}}{l_{x+t}}\right) = \frac{\partial}{\partial t}\{\ln(l_{x+n}) - \ln(l_{x+t})\} = \mu_{x+t}$$

$$\Rightarrow \frac{\partial}{\partial t}({}_{n-t}p_{x+t}) = (\mu_{x+t})({}_{n-t}p_{x+t})$$

$$\frac{\partial}{\partial t}(e^{-\delta(n-t)}) = \delta e^{-\delta(n-t)}$$

$$\Rightarrow \frac{\partial}{\partial t}{}_t\overline{V} = \{e^{-\delta(n-t)}(\mu_{x+t})({}_{n-t}p_{x+t})\} + \{{}_{n-t}p_{x+t}\delta e^{-\delta(n-t)}\} = {}_{n-t}p_{x+t}e^{-\delta(n-t)}(\mu_{x+t} + \delta)$$

$$\Rightarrow \frac{\partial}{\partial t}{}_t\overline{V} = (\mu_{x+t} + \delta){}_t\overline{V}$$

(ii) The change in reserve at time t consists of the interest earned and the release of reserves from the deaths.

(*The release may be more easily seen if the last line of (i) is rewritten as:* $\frac{\partial}{\partial t}{}_t\overline{V} = \delta\, {}_t\overline{V} - \mu_{x+t}(0 - {}_t\overline{V})$ *where the pure endowment has zero death benefit.)*

(iii) ${}_n\overline{V} = 1$.

9 (i) Survival table

x	q_x	p_x	${}_{t-1}p_x$
60	0.008022	0.99198	1
61	0.009009	0.99099	0.991978
62	0.010112	0.98989	0.983041

Unit fund

	Value of units at start of year	*Allocation*	*Bid/offer*	*Interest*	*Management charge*	*Value of units at end of year*
Year 1	0.00	4,250.00	212.50	242.25	32.10	4,247.65
Year 2	4,247.65	5,200.00	260.00	551.26	73.04	9,665.87
Year 3	9,665.87	5,200.00	260.00	876.35	116.12	15,366.10

Non-unit fund

	Unallocated premium	*Bid/offer*	*Expenses*	*Interest*	*Management charge*	*Extra death benefit*	*End of year cashflows*
Year 1	750.00	212.50	600.00	14.50	32.10	126.37	282.73
Year 2	-200.00	260.00	100.00	-1.60	73.04	93.10	-61.66
Year 3	-200.00	260.00	100.00	-1.60	116.12	46.86	27.66

	Non-unit fund cash flow (profit vector)	*Probability in force at start of year*	*Profit signature*	*Discount factor*	*Expected present value of profit*
Year 1	282.73	1	282.73	0.909091	257.03
Year 2	-61.66	0.991978	-61.16	0.826446	-50.55
Year 3	27.66	0.983041	27.19	0.751315	20.43

Total NPV 226.91

Expected NPV = 226.91

(ii) The NPV would decrease. Holding reserves would delay the emergence of some of the Year 1 cash flow, and as the non-unit fund earns 4%, well below the risk discount rate, the NPV would reduce.

10 (i) The 2 deaths were 70 and 69 respectively at 1/1/2005. The reserves for these policies at 31/12/2005 were

$${}_{26}V = 12,000\left(1 - \frac{\ddot{a}_{71}}{\ddot{a}_{45}}\right) = 12,000\left(1 - \frac{9.998}{18.823}\right) = 5,626.10 \text{ and}$$

$${}_{24}V = 10,000\left(1 - \frac{\ddot{a}_{70}}{\ddot{a}_{46}}\right) = 10,000\left(1 - \frac{10.375}{18.563}\right) = 4,410.92$$

Total death strain at risk, sorted by age at 1/1/2005:

Age 69: 500,000 - (175,000 + 4,410.92) = 320,589.08
Age 70: 400,000 - (150,000 + 5,626.10) = 244,373.90

Expected death strain:

$(q_{69})(320{,}589.08) + (q_{70})(244{,}373.90)$
= (0.022226)(320,589.08) + (0.024783)(244,373.90)
= 7,125.41 + 6,056.32
= 13,181.73

Actual death strain:

(12,000 - 5,626.10)+(10,000 - 4,410.92) = 6,373.90+5,589.08 = 11,962.98

Mortality profit = EDS – ADS = 13,181.73-11,962.98 = 1,218.75 profit

(ii) (a) Expected claims:

$(q_{69})(500{,}000)+(q_{70})(400{,}000)$
= (0.022226)(500,000) + (0.024783)(400,000)
= 11,113 + 9,913.2 = 21,026.20

Actual claims:

12,000 + 10,000 = 22,000

(b) Actual claims were higher than expected claims but the company still made a mortality profit. This can only have occurred because the deaths were disproportionately concentrated on lower DSAR lives (policies more mature on average). (This can be seen by comparing the ratio of reserves to sum assured for the death claim policies with the corresponding ratio for the full portfolio.)

11 (i)

$$g(T) = \begin{cases} 5{,}000 v^2 \bar{a}_{\overline{T_{63}-2}|} & \text{if } T_{63} \geq 2 \quad (\text{or } 5{,}000(\bar{a}_{\overline{T_{63}}|} - \bar{a}_{\overline{2}|})) \\ 0 & \text{if } T_{63} < 2 \end{cases}$$

(ii)

$$E[g(T)] = (100)(5{,}000)v^2 \, {}_2p_{63}\bar{a}_{65} = (500{,}000)(0.92456)(0.992617)(14.871 - 0.5)$$
$$= (500{,}000)(13.1887) = 6{,}594{,}350$$

(iii) $Var[g(T)] = E[g(T)^2] - E[g(T)]^2$

For £1 of annuity:

$$E[g(T)^2] = \int_2^\infty {}_tp_{63}\mu_{63+t}[v^2\bar{a}_{\overline{t-2}|}]^2 dt$$

Let $t = r + 2 \Rightarrow$

$$E[g(T)^2] = \int_0^\infty {}_{r+2}p_{63}\mu_{63+r+2}[v^2\bar{a}_{\overline{r}|}]^2 dr$$
$$= \int_0^\infty {}_rp_{65}\,{}_2p_{63}\mu_{65+r}v^4\left[\frac{1-v^r}{\delta}\right]^2 dr$$
$$= \frac{{}_2p_{63}v^4}{\delta^2}\int_0^\infty {}_rp_{65}\mu_{65+r}[1 - 2v^r + v^{2r}]dr$$
$$= \frac{{}_2p_{63}v^4}{\delta^2}[1 - 2\bar{A}_{65} + {}^2\bar{A}_{65}]$$

where

$$\bar{A}_{65} = (1.04)^{0.5}(1 - d\ddot{a}_{65}) = 1.019804\{1 - \left(\frac{0.04}{1.04}\right)(14.871)\} = 0.436515$$

and ${}^2\bar{A}_{65} = (1.04)({}^2A_{65}) = (1.04)(0.20847) = 0.21681$

$$\therefore E[g(T)^2] = \frac{(0.992617)(0.85480)}{(0.039221)^2}[1 - (2)(0.436515) + (0.21681)] = 189.622$$

$$Var[g(T)] = 189.622 - (13.1887)^2 = 15.680$$

For annuity of 5,000 we need to increase by $5{,}000^2$ and for 100 (independent) lives we need to multiply by 100.

Total variance = $(15.680)(5{,}000^2)(100) = 39{,}200{,}000{,}000 = (197{,}999)^2$

12 EPV benefits:

$110{,}000 A^{\ 1}_{[50]:\overline{10|}} - 10{,}000(IA)^{\ 1}_{[50]:\overline{10|}}$ (functions @ 6% p.a.)

$$= 110{,}000\{A_{[50]} - v^{10}\ {}_{10}p_{[50]}A_{60}\} - 10{,}000\{(IA)_{[50]} - v^{10}\ {}_{10}p_{[50]}(10A_{60} + (IA)_{60})\}$$
$$= 110{,}000A_{[50]} - 10{,}000(IA)_{[50]} + v^{10}\ {}_{10}p_{[50]}\{10{,}000((IA)_{60} - A_{60})\}$$
$$= (110{,}000)(0.20463) - (10{,}000)(4.84789) + (0.55839)(0.95684)\{10{,}000(5.46572 - 0.32692)\}$$
$$= 22{,}509.30 - 48{,}478.90 + 27{,}456.09 = 1{,}486.49$$

EPV gross premiums

Let P be annual premium

$$P\ddot{a}^{6\%}_{[50]:\overline{10|}} = 7.698P$$

EPV expenses

$$200 + 0.25P + 0.02Pa^{6\%}_{[50]:\overline{9|}} + 50a^{4\%}_{[50]:\overline{9|}} + 200A^{\ 1\ \ 4\%}_{[50]:\overline{10|}}$$
$$= 150 + 0.23P + 0.02P\ddot{a}^{6\%}_{[50]:\overline{10|}} + 50\ddot{a}^{4\%}_{[50]:\overline{10|}} + 200(A_{[50]} - v^{10}_{(4\%)}\ {}_{10}p_{[50]}A^{4\%}_{60})$$
$$= 150 + 0.23P + 0.02P(7.698) + (50)(8.318) + 200(0.32868 - (0.67556)(0.95684)(0.45640))$$
$$= 150 + 415.90 + 6.73 + P(.23 + 0.15396) = 572.63 + 0.38396P$$

Equation of value:

$7.698P = 1{,}486.49 + 572.63 + 0.38396P \Rightarrow 7.31404P = 2{,}059.12$ so $P = 281.53$ p.a.

(ii) If $K_{59} \geq 1$ $GFLRV = 50(1.01923)^9 - 0.98(281.53)$ else (i.e. $K_{59} = 0$)

$$GFLRV = 10{,}000v_{.06} + 200(1.01923)^9 v_{.04}$$
$$+50(1.01923)^9 - 0.98(281.53)$$

or

$$GFLRV = 10{,}000v_{.06} + 200(1.01923)^{10} v_{.06}$$
$$+50(1.01923)^9 - 0.98(281.53)$$

(iii)

$$
\begin{aligned}
{}_9V &= 10,000 q_{59} v_{.06} + 200(1.01923)^9 q_{59} v_{.04} \\
&\quad + 50(1.01923)^9 - 0.98(281.53) \\
&= (10,000)(0.007140)(0.94340) + (237.40)((0.007140)(0.96154) + 59.35 - 275.90 \\
&= 67.36 + 1.63 + 59.35 - 275.90 = -147.56
\end{aligned}
$$

(iv) The reserve is negative. The expected future income exceeds expected future outgo, because past outgo exceeded past income, meaning the office needs a net inflow in the last year to recoup previous losses. However, it is at risk of the policy lapsing, and never getting this net inflow.

END OF EXAMINERS' REPORT