EXAMINATION

September 2005

Subject CT5 — Contingencies
Core Technical

EXAMINERS' REPORT

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

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Chairman of the Board of Examiners

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In general, this examination was done well by students who were well prepared. Several questions gave difficulties particularly Question 7 and 12(ii) the latter one being very challenging. To help students comments are attached to those questions where particular points are of relevance. Absence of comments can be indicate that the particular question was generally done well.

1 Adverse selection is the manner in which lives form part of a group, which acts against a controlled process of selecting the lives with respect to some characteristic that affects mortality or morbidity.

An example is where a life insurance company does not distinguish between smokers and non-smokers in proposals for term assurance cover. A greater proportion of smokers are likely to select this company in preference to a company that charges different rates to smokers and non-smokers. This would be adverse to the company's selection process, if the company had assumed that its proportion of smokers was similar to that in the general population.

Other examples were credited.

2 Occupation can have several direct effects on mortality and morbidity. Occupation determines a person's environment for 40 or more hours each week. The environment may be rural or urban, the occupation may involve exposure to harmful substances e.g. chemicals, or to potentially dangerous situations e.g. working at heights. Much of this is moderated by health and safety at work regulations.

Some occupations are more healthy by their very nature e.g. bus drivers have a sedentary and stressful occupation while bus conductors are more active and less stressed. Some work environments e.g. pubs, give exposure to a less healthy lifestyle.

Some occupations by their very nature attract more healthy workers. This may be accentuated by health checks made on appointment or by the need to pass regular health checks e.g. airline pilots. Some occupations can attract less healthy workers, for example, former miners who have left the mining industry as a result of ill health and then chosen to sell newspapers. This will inflate the mortality rates of newspaper sellers.

A person's occupation largely determines their income, which permits them to adopt a particular lifestyle e.g. content and pattern of diet, quality of housing. This effect can be positive or negative e.g. over-indulgence.

Other appropriate examples were credited.

3 As t increases, μ_t increases, but ${}_t p_0$ decreases. At t = 80 approximately, the decrease in ${}_t p_0$ is greater than the increase in μ_t, hence $f_0(t) = {}_t p_0 \mu_t$ decreases.

A deceptively straightforward answer which many students struggled to find. The key point is to compare the 2 parameters as shown.

4 $_{1.75}p_{45.5} = {}_{0.5}p_{45.5} * p_{46} * {}_{0.25}p_{47}$

$$= \frac{1-q_{45}}{1-0.5q_{45}} * (1-q_{46}) * (1-0.25q_{47})$$

$$= \frac{1-0.001465}{1-0.5*0.001465} * (1-0.001622) * (1-0.25*0.001802)$$

$$= 0.999267 * 0.998378 * 0.99955 = 0.997197$$

5 (a) The required probability is

$$1-e^{-\int_0^{1.25} 0.015dt} = 1-e^{-0.01875} = 0.018575$$

(b) The curtate expectation is

$$\sum_{k=1}^{\infty} {}_k p_{20} = \sum_{k=1}^{\infty} e^{-\int_0^k 0.015dt} = \sum_{k=1}^{\infty} e^{-0.015k} = \frac{e^{-0.015}}{1-e^{-0.015}} = 66.168.$$

6 $\ddot{a}^{(12)}_{60:50:\overline{20}|}$ is the present value of 1 p.a. payable monthly in advance while two lives aged 60 and 50 are both still alive, for a maximum period of 20 years.

$$\ddot{a}^{(12)}_{60:50:\overline{20}|} = \ddot{a}^{(12)}_{60:50} - v^{20}\,{}_{20}p_{60:50}\ddot{a}^{(12)}_{80:70}$$

$$= (\ddot{a}_{60:50} - 11/24) - v^{20}\,{}_{20}p_{60:50}(\ddot{a}_{80:70} - 11/24)$$

$$= (15.161 - 0.458) - v^{20}\frac{6953.536}{9826.131}\frac{9392.621}{9952.697}(6.876 - 0.458) = 12.747$$

7 $$\text{EPV} = £10,000 \int_0^{45} {}_t p_{20}^{hh} * \sigma_{20+t} * {}_1 p_{20+t}^{\overline{ss}} * \left(\int_0^{44-t} e^{-\delta(t+u+1)} {}_u p_{21+t}^{\overline{ss}} du \right) dt$$

where δ is the force of interest

${}_t p_{20}^{hh}$ is the probability of a healthy life aged 20 being healthy at age $20+t$

${}_1 p_{20+t}^{\overline{ss}}$ is the probability that a life who is sick at age $20+t$ is sick continuously for one year thereafter

${}_u p_{21+t}^{\overline{ss}}$ is the probability that a life who is sick at age $21+t$ is still sick at age $21+t+u$

This question was not done well and few students obtained the whole result. Partial credits were given for correct portions. There were other potentially correct approaches which were credited provided proper definitions of symbols given.

8 Premium = $20,000\left(\ddot{a}_{\overline{5|}}^{(12)} + \frac{D_{70}}{D_{65}}\ddot{a}_{70}^{(12)}\right) + 10,000\left[\left(1-\frac{l_{70}}{l_{65}}\right)\frac{D_{67}}{D_{62}}\ddot{a}_{67}^{(12)} + \frac{l_{70}}{l_{65}}\frac{D_{67}}{D_{62}}\ddot{a}_{70|67}^{(12)}\right]$

$\ddot{a}_{\overline{5|}}^{(12)} = 4.5477$

$\frac{D_{70}}{D_{65}} = v^5 \frac{9238.134}{9647.797} = 0.787027$

$\ddot{a}_{70}^{(12)} = 11.562 - 0.458 = 11.104$

$\frac{l_{70}}{l_{65}} = 0.957538$

$\ddot{a}_{67}^{(12)} = 14.111 - 0.458 = 13.653$

$\frac{D_{67}}{D_{62}} = v^5 \frac{9605.483}{9804.173} = 0.80527$

$\ddot{a}_{70|67}^{(12)} = \ddot{a}_{67}^{(12)} - \ddot{a}_{70:67}^{(12))} = (14.111 - 0.458) - (10.233 - 0.458) = 3.878$

Premium = 265,736.96 + 34,570.77 = £300,308 to nearer £

9 (i) $10{,}000\ddot{a}_{\overline{\max\left(K^A_{60}+1,K^B_{60}+1\right)}|} - P$, where A and B refer to the first and second lives and P is the single premium.

(ii) $P = 10{,}000\left(\ddot{a}^A_{60} + \ddot{a}^B_{60} - \ddot{a}^A_{60} * \ddot{a}^B_{60}\right)$

$$= 10{,}000*\left(15.632 + 16.652 - 14.09\right) = £181{,}940$$

$$\text{Variance } = \frac{10^8}{d^2}\left({}^2A_{\overline{60^A:60^B}} - \left(A_{\overline{60^A:60^B}}\right)^2\right)$$

$$= \frac{10^8}{0.038462^2}*\left(1 - 0.075444*11.957 - \left(1 - 0.038462*18.194\right)^2\right)$$

standard deviation = 22,936.7

10 (i) The gross future loss random variable is

$$100{,}000\left(1 + bK_{20+1}\right)v^{T_{20}} + I + e\ddot{a}_{\overline{K_{20}+1}|} + fv^{T_{20}} - Ga_{\overline{K_{20}+1}|}$$

where b is the annual rate of bonus
I is the initial expense
e is the annual renewal expense and
f is the claim expense
G is the gross annual premium

(ii) The premium is given by

$$G\ddot{a}_{[20]} = 100{,}000\bar{A}_{[20]} + 3{,}000\left(I\bar{A}\right)_{[20]} + 200 + 0.05Ga_{[20]}$$

$$\Rightarrow G*16.877 = 100{,}000*1.06^{0.5}*0.04472 + 3{,}000*1.06^{0.5}*2.00874$$
$$+200 + 0.05G*(16.877-1)\,1] \Rightarrow 16.083G = 4604.206 + 6204.373 + 200$$

$$\Rightarrow G = £684.49$$

(iii) The required provision is

$$110{,}000\bar{A}_{23} + 4{,}000\left(I\bar{A}\right)_{23} - 0.95 * 684.48 * \ddot{a}_{23}$$

$$= 110{,}000 * 1.04^{0.5} * 0.12469 + 4{,}000 * 1.04^{0.5} * 6.09644$$
$$- 0.95 * 684.49 * 22.758$$

$$= 13{,}987.528 + 24{,}868.693 - 14{,}798.742$$

$$= £24{,}057.48$$

11 Unit fund

Year	*1*	*2*	*3*
Fund at the start of the year	0	8611.988	17796.67
Premium	10000	10000	10000
Allocation to units	8075	8075	8075
Interest	646	1334.959	2069.734
Management charge	109.0123	225.274	349.268
Fund at the end of the year	8611.988	17796.67	27592.14

Non-unit fund before provisions

Year	*1*	*2*	*3*
Premium margin	1925	1925	1925
Expenses	600	100	100
Interest	53	73	73
Death cost	115.156	24.995	0
Maturity cost	0	0	4138.821
Management charge	109.013	225.274	349.268
Profit	1371.857	2098.28	–1891.55

Provision required at the start of year 3 = (1891.55 − 4138.821 (1 − p_{64})) / 1.04
= 1768.192

Reduced profit at the end of year 2 = 2098.28 − 1768.192*p_{63} = 350.146

Revised profit vector: $(1371.857, 350.146, 0)$

$p_{62} = 0.989888$

${}_2 p_{62} = 0.978659$

Net Present Value = $\dfrac{1371.857}{1.15} + \dfrac{350.146 * p_{62}}{1.15^2} = 1455.003$

Present value of premiums = $10000 * \left(1 + \dfrac{p_{62}}{1.15} + \dfrac{{}_2 p_{62}}{1.15^2}\right) = 26007.788$

Profit margin = $\dfrac{1455.003}{26007.788} = 5.59\%$

Most students completed the tables satisfactorily in this question but struggled to get the revised profit vectors. Very few produced a complete result.

12 (i) Let P be the annual premium.

$$0.95 * P * \ddot{a}_{\overline{50^m:50^f}} = 1000 + 200000 * \bar{A}_{\overline{50^m:50^f}}$$

$$\ddot{a}_{\overline{50^m:50^f}} = \ddot{a}_{50^m} + \ddot{a}_{50^f} - \ddot{a}_{50^m:50^f} = 18.843 + 19.539 - 17.688 = 20.694$$

$$\bar{A}_{\overline{50^m:50^f}} = 1.04^{0.5} * A_{\overline{50^m:50^f}} = 1.04^{0.5} * (1 - d * \ddot{a}_{\overline{50^m:50^f}})$$

$$= 1.04^{0.5} * (1 - 0.038462 * 20.694) = 0.208109$$

$$\therefore 0.95 * P * 20.694 = 1000 + 200000 * 0.208109$$

$$\Rightarrow P = £2,168.02 .$$

(ii) From part (i) the net premium is:

$$200000 * (1.04)^{0.5} * \left(\frac{1}{\ddot{a}_{\overline{50^m:50^f}}} - d\right) \text{ at } 4\%$$

$$= 200000 * (1.04)^{0.5} * \left(\frac{1}{20.694} - \frac{.04}{1.04}\right)$$

$$= 2011.39$$

We require 3 provisions at end of 5th policy year

Both lives alive

$$200000*(1.04)^{0.5}*\left(1-\frac{\ddot{a}_{\overline{55^m:55^f}}}{\ddot{a}_{\overline{50^m:50^f}}}\right)$$

$$= \ 200000*(1.04)^{0.5}\left(1-\frac{17.364+18.210-16.016}{18.843+19.539-17.688}\right)$$

= 11196.46

Male only alive

$$200000\,\bar{A}_{55^m} - 2011.39\,\ddot{a}_{55^m}$$

$$= \ 200000*(1.04)^{0.5}*\left(1-\frac{.04}{1.04}*17.364\right)-2011.39*17.364$$

= 32820.60

Female only alive

$$200000\,\bar{A}_{55^f} - 2011.39\,\ddot{a}_{55^f}$$

$$= \ 200000*(1.04)^{0.5}*\left(1-\frac{.04}{1.04}*18.210\right)-2011.39*18.210$$

= 24482.39

Mortality Profit Loss

= Expected Death Strain – Actual Death Strain

In this case there are 4 components:

(a) Both lives die during 2004 – no actual claims

Result

$$= \ (4900*q_{54^m}*q_{54^f}-0)(200000*1.04^{0.5}-11196.46)$$

= (4900 * 0.000986 * 0.000912) (192764.32)

= 849.37

(b) Female alive at begin 2004, death during 2004 – 1 actual claim

Result

$$= \ (100*q_{54^f}-1)(200000*1.04^{0.5}-24482.39)$$

= (100 * 0.000912 – 1) (179478.39)

= – 163109.96

(c) Both lives alive beginning 2004, males only die during 2004 -1 actual claim. Here the "claim cost" is the change in provision from joint lives to female only surviving i.e.

Result = $(4900 * q_{54^m} * q_{54^f} - 1)(24482.39 - 11196.46)$

= (4900 * 0.000986 * 0.999088 – 1) (13285.93)

= 50845.17

(d) Both lives alive beginning 2004, females only die during 2004 – no actual claims. Claim cost change in provision from joint lives to male only surviving

Result = $(4900 * p_{54^m} * q_{54^f} - 0)(32820.611 - 11196.46)$

= (4900 * 0.999014 * 0.000912) (21624.14)

= 96538.66

Hence overall total = 849.37 – 163109.16 + 50845.17 + 96538.66

= – 14876.77

i.e. a mortality loss of 14877 when rounded.

For part (i) assuming renewal expenses did not include the first premium (answer £2162.62) was also fully acceptable.

Part (ii) was very challenging and very few students realised the extension of mortality profit/loss extended to joint life contracts involved reserve change costs on first death. Most just considered the first 2 components of the answer and in many cases failed to correctly cost this part. A few exceptional students did manage to reach the final result.

13 (i) Define a service table:

l_{26+t} = no. of members aged 26 + t last birthday

r_{26+t} = no. of members who retire age 26 + t last birthday

s_{x+t} / s_x = ratio of earnings in the year of age $x + t$ to $x + t + 1$ to the earnings in the year of age x to $x + 1$

Define $z_{26+t} = \frac{1}{3}\left(s_{26+t-3} + s_{26+t-2} + s_{26+t-1}\right); \bar{a}^r_{26+t}$ = value of annuity of 1 p.a. to a retiree aged exactly 26 + t.

Past service:

Assume that retirements take place uniformly over the year of age between 60 and 65. Retirement for those who attain age 65 takes place at exact age 65.

Consider retirement between ages 26 + t and 26 + t + 1, $34 \le t \le 38$.

The present value of the retirement benefits related to past service:

$$\frac{50000*5}{60}\frac{z_{26+t+\frac{1}{2}}}{s_{25.25}}\frac{v^{26+t+\frac{1}{2}}}{v^{26}}\frac{r_{26+t}}{l_{26}}\bar{a}^r_{26+t+\frac{1}{2}} = \frac{50000*5}{60}\frac{{}^zC^{ra}_{26+t}}{{}^sD_{26}}$$

where ${}^zC^{ra}_{26+t} = z_{26+t+\frac{1}{2}}v^{26+t+\frac{1}{2}}r_{26+t}\bar{a}^r_{26+t+\frac{1}{2}}$

and ${}^sD_{26} = s_{25.25}v^{26}l_{26}$

For retirement at age 65, the present value of the benefits is:

$$\frac{50000*5}{60}\frac{z_{65}}{s_{25.25}}\frac{v^{65}}{v^{26}}\frac{r_{65}}{l_{26}}\bar{a}^r_{65} = \frac{50000*5}{60}\frac{{}^zC^{ra}_{65}}{{}^sD_{26}}$$

where ${}^zC^{ra}_{65} = z_{65}v^{65}r_{65}\bar{a}^r_{65}$

Summing over all ages, the value is:

$$\frac{50000*5}{60}\frac{{}^zM^{ra}_{60}}{{}^sD_{26}}$$

where ${}^zM^{ra}_{60} = \sum_{t=34}^{39} {}^zC^{ra}_{26+t}$

Future service:

Assume that retirements take place uniformly over the year of age, between ages 60 and 65. Retirement at 65 takes place at exactly age 65.

If retirement takes place between ages 60 and 61, the number of future years service to count is 34. If retirement takes place at age 61 or after, the number of future years service to count is 35.

For retirement between ages 60 and 61, the present value of the retirement benefits is:

$$\frac{34*50000}{60}\frac{z_{60+\frac{1}{2}}}{s_{25.25}}\frac{v^{60+\frac{1}{2}}}{v^{26}}\frac{r_{60}}{l_{26}}\bar{a}^{i}_{60+\frac{1}{2}}=\frac{34*50000}{60}\frac{{}^{z}C^{ra}_{60}}{{}^{s}D_{26}}$$

For retirement at later years, the formula is similar to the above, with 35 in place of 34.

Adding all these together gives:

$$\frac{50000}{60\,{}^{s}D_{26}}\left[34\,{}^{z}C^{ra}_{60}+35({}^{z}C^{ra}_{61}+\ldots+{}^{z}C^{ra}_{65})\right]$$

$$=\frac{50000}{60\,{}^{s}D_{26}}\,{}^{z}\bar{M}^{ra}_{60}$$

where $\displaystyle {}^{z}\bar{M}^{ra}_{60}=\sum_{t=0}^{5}\left(35*{}^{z}C^{ra}_{60+t}-{}^{z}C^{ra}_{60}\right)$

(ii) Define a service table, with l_{26+t} and s_{x+t}/s_{x} defined as in part (i). In addition, define d_{26+t} as the number of members dying age $26+t$ last birthday.

Assume that deaths take place on average in the middle of the year of age.

The present value of the death benefit, for death between ages $26+t$ and $26+t+1$, is

$$50000*4*\frac{s_{26.25+t}}{s_{25.25}}\frac{v^{26+t+\frac{1}{2}}}{v^{26}}\frac{d_{26+t}}{l_{26}}=50000*4*\frac{{}^{s}C^{d}_{26+t}}{{}^{s}D_{26}}$$

where ${}^{s}C^{d}_{26+t}=s_{26.25+t}v^{26+t+\frac{1}{2}}d_{26+t}$

Adding the present value of benefits for all possible years of death gives

$$50000*4*\sum_{t=0}^{38}\frac{{}^{s}C^{d}_{26+t}}{{}^{s}D_{26}}=200000*\frac{{}^{s}M^{d}_{26}}{{}^{s}D_{26}}$$

where $\displaystyle {}^{s}M^{d}_{26}=\sum_{t=0}^{38}{}^{s}C^{d}_{26+t}$

Examiners felt that this question was quite simple provided students constructed proper definitions and followed them through logically allowing of course for the adjusted salary scale. The above answer is one of a number possible and full credit was given for credible alternatives.

Many students, however struggled with this question despite these remarks.

END OF EXAMINERS' REPORT