# EXAMINATION

28 September 2007 (am)

# Subject CT5 — Contingencies
# Core Technical

*Time allowed: Three hours*

***INSTRUCTIONS TO THE CANDIDATE***

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

***Graph paper is not required for this paper.***

*AT THE END OF THE EXAMINATION*

*Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.*

*In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.*

**1** Calculate ${}_{t+1}V_x$ given the following:

$P_x = 0.017$
${}_tV_x = 0.468$
$i = 0.03$
$q_{x+t} = 0.024$ [2]

**2** In a special mortality table with a select period of one year, the following relationships are true for all ages:

$${}_{0.5}q_{[x]} = (0.33)q_x$$
$${}_{0.5}q_{[x]+0.5} = (0.5)q_x$$

Express $p_{[x]}$ in terms of $p_x$. [3]

**3** A twelve-year life insurance contract has the following profit signature before any non-unit reserves are created:

(+1, -1, +1, +1, +1, -1, 0, -1, +1, -1, +1, +1)

Non-unit reserves are to be set up to zeroise the negative cash flows.

Write down the revised profit signature, ignoring interest. [3]

**4** An annuity makes monthly payments in arrear to a life aged 65 exact where each payment is 1.0039207 times greater than the one immediately preceding. The first monthly amount is £1,000.

Calculate the expected present value of the annuity using the following basis:

Mortality: PFA92C20
Interest: 9% per annum [4]

**5** (i) Write down the formula for a directly standardised mortality rate. [2]

(ii) State the main disadvantage of this rate and outline how is it overcome in practice. [2]

[Total 4]

**6** For a certain group of pensioners, $q_{75} = 0.05$ and $q_{76} = 0.06$.

Calculate the probability that a pensioner aged 75 exact will die between ages 75.5 and 76.5 assuming:

(a) a uniform distribution of deaths between consecutive birthdays
(b) a constant force of mortality between consecutive birthdays. [5]

**7** A life insurance company sells two whole life contracts to lives aged 40 exact at entry. Level monthly premiums are payable in advance until the death of the life assured. Death benefits are paid at the end of the year of death.

Under policy A, the sum assured is £100,000 during the first year and it increases by £5,000 at the end of each year for surviving policyholders.

Policy B is a with profit policy with initial sum assured of £100,000. The company intends to declare simple annual reversionary bonuses of 5% of the original sum assured each year, vesting at the end of each policy year.

After ten years, the total declared bonuses under the with profit policy amount to £50,000.

Calculate the net premium reserve required for each policy after ten years.

Basis:

Mortality: AM92 Select
Interest: 4% per annum [6]

**8** Explain the following terms and give an example of each:

(a) class selection
(b) spurious selection
(c) time selection. [6]

**9** A life office issues an annuity to a woman aged 65 exact and a man aged 68 exact. The annuity of £20,000 per annum is payable annually in arrears for as long as either of the lives is alive.

The office values this benefit using the following basis:

| | | |
|---|---|---|
| Interest: | | 4% per annum |
| Mortality: | Female: | PFA92C20 |
| | Males: | PMA92C20 |

(i) Calculate the expected present value of this benefit. [2]

(ii) Calculate the probability that the life office makes a profit in this case if it charges a single premium of £320,000. [4]

[Total 6]

**10** A policy provides a benefit of £500,000 immediately on the death of ($y$) if she dies after ($x$).

(i) Write down an expression in terms of $T_x$ and $T_y$ (random variables denoting the complete future lifetimes of ($x$) and ($y$) respectively) for the present value of the benefit under this policy. [2]

(ii) Write down an expression for the expected present value of the benefit in terms of an integral. [2]

(iii) Suggest, with a reason, the most appropriate term for regular premiums to be payable under this policy. [2]

[Total 6]

**11** Let $X$ be a random variable representing the present value of the benefits of a pure endowment contract and $Y$ be a random variable representing the present value of the benefits of a term assurance contract which pays the death benefit at the end of the year of death. Both contracts have unit sum assured, a term of $n$ years and were issued to the same life aged $x$.

(i) Derive and simplify as far as possible using standard actuarial notation an expression for the covariance of $X$ and $Y$. [4]

(ii) Hence or otherwise, derive an expression for the variance of $(X+Y)$ and simplify it as far as possible using standard actuarial notation. [4]

[Total 8]

**12** On 1 January 1992 a life insurance company issued a number of 20-year pure endowment policies to a group of lives aged 40 exact. In each case, the sum assured was £75,000 and premiums were payable annually in advance.

On 1 January 2006, 500 policies were still in force. During 2006, 3 policyholders died, and no policy lapsed for any other reason.

The office calculates net premiums and net premium reserves on the following basis:

Interest: 4% per annum
Mortality: AM92 Select

(i) Calculate the profit or loss from mortality for this group for the year ending 31 December 2006. [7]

(ii) Explain why the mortality profit or loss has arisen. [2]

[Total 9]

**13** A life insurance company issues a 35-year endowment assurance contract to a life aged 30 exact. The sum assured of £200,000 is payable at maturity or at the end of the year of death if earlier. Level premiums are payable annually in advance for the duration of the contract.

(i) Show that the annual premium is approximately £2,007, using the following basis:

| | | |
|---|---|---|
| Interest: | | 6% p.a. |
| Mortality: | | AM92 Ultimate |
| Expenses: | Initial: | £300 plus 50% of the annual premium |
| | Renewal: | 2% of the second and subsequent annual premiums |
| | Claim: | £600 on death; £200 on maturity |

[6]

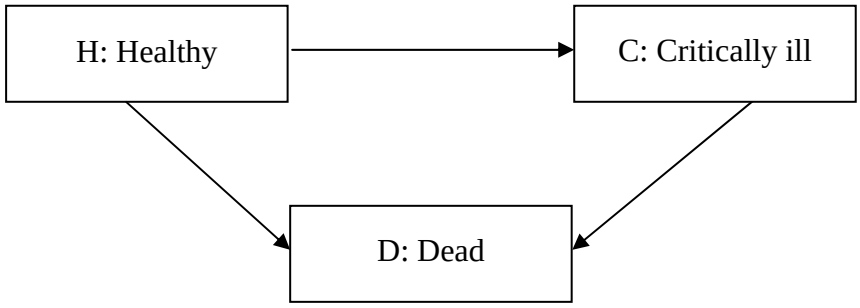
(ii) Write down the gross premium future loss random variable after 25 years, immediately before the premium then due is paid. [3]

(iii) Calculate the retrospective policy reserve after 25 years, using the same basis as in (i), but with 4% p.a. interest. [6]

(iv) Explain whether the reserve in (iii) would have been smaller, the same or greater than in (iii) if the office had used the prospective gross premium reserve, on the same basis. [3]

[Total 18]

**14** A life office uses the following three-state model to calculate premiums for a 2-year accelerated critical illness policy issued to healthy policyholders aged 63 exact at entry.




In return for a single premium payable at entry, the office will pay benefits of:

£100,000 if the policyholder dies from the healthy state;
£60,000 if he is diagnosed as having a critical illness;
£40,000 if he dies from the critically ill state.

All benefits are payable at the end of the relevant policy year.

Let $S_t$ represent the state of the policyholder at age 63 + $t$, so that $S_0 = H$ and for $t = 1$, 2, $S_t = H$, $C$ or $D$. The transition probabilities are defined as follows:

$$p_{63+t}^{ij} = \Pr(S_{t+1} = j \mid S_t = i).$$

Their values are:

| t | $p_{63+t}^{HC}$ | $p_{63+t}^{HD}$ | $p_{63+t}^{CD}$ |
|---|---|---|---|
| 0 | 0.04 | 0.02 | 0.25 |
| 1 | 0.06 | 0.03 | 0.33 |

(i) Identify all 6 possible outcomes under this policy. [3]

(ii) Calculate the net present value at entry of the benefits assuming a rate of interest of 10% per annum for each of the outcomes in (i). [3]

(iii) Calculate the probability that each outcome occurs. [3]

(iv) Calculate the mean and variance of the present value at entry of the total benefits per policy. [5]

(v) The office expects to sell 10,000 of these policies. The single premium is set at a level which will ensure that the probability that the office makes a profit is 0.95. Calculate the amount of the single premium, assuming the profit is normally distributed. [6]

[Total 20]

## END OF PAPER