Subject CT5 — Contingencies
Core Technical

EXAMINERS' REPORT

September 2008

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

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Chairman of the Board of Examiners

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1 Let t equal future lifetime. Lower quartile means that 25% of people have future lifetime less than t.

$$_{t}q_{25} = 0.25 \Rightarrow \frac{l_{25+t}}{l_{25}} = 0.75 \qquad \frac{l_{25+t}}{98,797} = 0.75 \Rightarrow l_{25+t} = 74,098$$

$$l_{73} = 74,287 \quad l_{74} = 72,048,$$

So 73–25 = 48 is lower quartile future lifetime to nearest integer.

2 Profit margin = (EPV profit / EPV premiums)

EPV profit = $-250v + 150v^2 + 200v^3 = 38.72$

EPV premiums = $500(1 + {_1p_{57}}v + {_2p_{57}}v^2) = 500(1 + 0.8878125 + 0.7876546)$ = 1,337.73

Profit margin = 38.72/1,337.37 = 2.89%

3 £75,000 represents earnings from 1/7/2007 to 1/7/2008

i.e. from age 46.25 to 47.25

2008 is the period from age 46.75 to 47.75

2008 expected earnings = $75,000\dfrac{s_{46.75}}{s_{46.25}}$

$(s_{46} + 3s_{47})$ is a satisfactory alternative to the numerator above and $(3s_{46} + s_{47})$ a satisfactory alternative for the denominator.

Alternative: $75,000\{0.5 + 0.5(\dfrac{s_{47.25}}{s_{46.25}})\}$

4 (i) ${}_n q_{\overline{xy}}$

(ii) $A^{1}_{x:\overline{n}|}$

(iii) ${}_{n|m-n} q_x$

(iv) $\mu_{x+t:y+t}$ or $\mu_{x+t} + \mu_{y+t}$

(v) $\ddot{a}_{x:\overline{n}|}$

5 It is the accumulation of an n-year annuity due i.e. the expected fund per survivor after *n* years, from a group of people, initially aged *x*, who each put 1 at the start of each of the *n* years, if they are still alive, into a fund earning interest at rate *i* per annum.

$$\ddot{s}_{50:\overline{20}|} = \frac{\ddot{a}_{50:\overline{20}|}}{v^{20}\ {}_{20}p_{50}}$$

$$= \frac{1}{v^{20}\ {}_{20}p_{50}}(\ddot{a}_{50} - v^{20}\ {}_{20}p_{50}\ddot{a}_{70})$$

$$= \frac{1}{(0.45639)(0.82928)}(17.444) - 10.375 = 35.715$$

6

x	$q_{[x]}$	$q_{[x-1]+1}$	$q_{[x-2]+2}$	$q_{[x-3]+3}$	q_x
62	**0.0045**	0.006	0.009	0.018	0.018
63	0.005	**0.006667**	0.01	0.02	0.02
64	0.0055	0.007333	**0.011**	0.022	0.022

EPV premiums = 30,000$\{1 + v^*p_{[62]} + v^{2*}\ p_{[62]}{}^*p_{[62]+1}\}$

= 30,000$\{1 + v^*(1 - 0.0045) + v^{2*}(1 - 0.0045)^*(1 - 0.006667)\}$

= (30,000)(2.781742) = 83,452.27

EPV benefits = 100,000$\{v^*q_{[62]} + v^{2*}p_{[62]}{}^*q_{[62]+1} + v^{3*}p_{[62]}{}^*p_{[62]+1}{}^*(q_{[62]+2} + p_{[62]+2})\}$ = 80,592.50

EPV profit = 83,452.27 - 80,592.50 = 2,859.77

Cash flow approach:

Year	*Premium*	*Interest*	*Death Cost*	*Maturity Cost*	*Profit Vector*	*Profit Signature*	*NPV*
1	30,000.00	2,250.00	450.00		31,800.00	31,800.00	29,581.40
2	30,000.00	2,250.00	666.67		31,583.33	31,441.21	27,207.10
3	30,000.00	2,250.00	1,100.00	98,900.00	-67,750.00	–66,995.49	–53,928.73
							2,859.77

7 (i) $(aq)_x^{\alpha} = q_x^{\alpha}\{1-\tfrac{1}{2}(q_x^{\beta}+q_x^{\gamma})+\tfrac{1}{3}(q_x^{\beta}q_x^{\gamma})\}$

(ii) $(aq)_x^{\alpha} = \int_0^1 {}_tp_x^{\alpha}\,{}_tp_x^{\beta}\,{}_tp_x^{\gamma}\mu_{x+t}^{\alpha}dt$

$${}_tp_x^{\alpha} = 1-t^2q_x^{\alpha} \Rightarrow {}_tp_x^{\alpha}\mu_{x+t}^{\alpha} = -\frac{d}{dt}\,{}_tp_x^{\alpha} = 2tq_x^{\alpha}$$

With β and γ uniformly distributed, then

$$(aq)_x^{\alpha} = \int_0^1 {}_tp_x^{\alpha}\,{}_tp_x^{\beta}\,{}_tp_x^{\gamma}\mu_{x+t}^{\alpha}dt = \int_0^1 2tq_x^{\alpha}(1-tq_x^{\beta})(1-tq_x^{\gamma})dt$$

$$= q_x^{\alpha}\int_0^1\{2t-2t^2(q_x^{\beta}+q_x^{\gamma})+2t^3(q_x^{\beta}q_x^{\gamma})\}1dt$$

$$= q_x^{\alpha}\{1-\frac{2}{3}(q_x^{\beta}+q_x^{\gamma})+\frac{2}{4}(q_x^{\beta}q_x^{\gamma})\}$$

8 (i) The Death Strain at risk per policy is
[0 – (payment due 31.12 + reserve @ 31.12] = $-25,000\ddot{a}_{66}$

Expected DS =
$-q_{65}*1,000*25,000\ddot{a}_{66} = -(0.004681)(25,000,000)(14.494) = -1,696,160$

Actual DS = $-5*25,000\ddot{a}_{66} = -1,811,750$

Profit = EDS – ADS = -1,696,160 + 1,811,750 = 115,590 profit

(ii) We expected 4.681 deaths and had more than this with 5. There is no death benefit, just a release of reserves on death, so more deaths than expected leads to profit.

9 EPV of benefits: $\dfrac{40,000}{60}\dfrac{{}^{z}\overline{R}^{ra}_{35}+{}^{z}\overline{R}^{ia}_{35}}{{}^{s}D_{35}}=\dfrac{40,000}{60}\dfrac{3,524,390+1,187,407}{31,816}=98,730$

PV of contribution:

$$40,000\frac{(0.04)\,{}^{s}\overline{N}_{35}+(0.01)\,{}^{s}\overline{N}_{50}}{{}^{s}D_{35}}=40,000\frac{(0.04)502,836+(0.01)163,638}{31,816}=27,345$$

Employer's proportion = (98,730 - 27345)/27,345 = 2.61 times employee's contribution.

10 Variance of $\bar{a}_{\overline{T_x}|}=\dfrac{{}^{2}\overline{A}_x-(\overline{A}_x)^2}{\delta^2}$

$$\overline{A}_x=\int_0^\infty e^{-\delta t}\,{}_tp_x\mu_{x+t}dt=\int_0^\infty e^{-\delta t}e^{-\mu t}\mu dt=\mu\int_0^\infty e^{-t(\delta+\mu)}dt=-\frac{\mu}{\mu+\delta}(e^{-t}\Big|_0^\infty)=-\frac{\mu}{\mu+\delta}(0-1)$$
$$=\frac{\mu}{\mu+\delta}=\frac{0.02}{0.07}$$

Similarly, ${}^{2}\overline{A}_x=\displaystyle\int_0^\infty e^{-2\delta t}\,{}_tp_x\mu_{x+t}dt=\int_0^\infty e^{-\delta t}e^{-\mu t}\mu dt=\frac{\mu}{\mu+2\delta}=\frac{0.02}{0.12}$

Variance of $\bar{a}_{\overline{T_x}|}=\dfrac{\dfrac{0.02}{0.12}-\left(\dfrac{0.02}{0.07}\right)^2}{0.05^2}=34.01$

Alternatively

Variance of $X=E[X^2]-\{E[X]\}^2$

Here X= $\quad\bar{a}_{\overline{T_x}|}=\dfrac{1-e^{-\delta T_x}}{\delta}$

$$E[X]=\int_0^\infty\frac{1-e^{-\delta t}}{\delta}\,{}_tp_x\mu_{x+t}dt=\int_0^\infty\frac{1-e^{-\delta t}}{\delta}e^{-\mu t}\mu dt=\frac{1}{\delta}\int_0^\infty(e^{-\mu t}\mu)-(e^{-(\delta+\mu)t}\mu)dt$$
$$=\frac{1}{\delta}\{(-e^{-\mu t}\Big|_0^\infty)-(\frac{\mu}{\delta+\mu})(-e^{-(\delta+\mu)t}\Big|_0^\infty)\}=\frac{1}{\delta}\{(0-(-1))-(\frac{\mu}{\delta+\mu})(0-(-1))\}$$
$$=\frac{1}{\delta}\{1-(\frac{\mu}{\delta+\mu})\}=\frac{1}{\delta}\{\frac{\delta+\mu-\mu}{\delta+\mu}\}=\frac{1}{\delta+\mu}=\frac{1}{0.07}=14.2857$$

$$E[X^2] = \int_0^\infty \left(\frac{1-e^{-\delta t}}{\delta}\right)^2 {}_t p_x \mu_{x+t} dt = \int_0^\infty \left(\frac{1-2e^{-\delta t}+e^{-2\delta t}}{\delta^2}\right) e^{-\mu t}\mu dt$$

$$= \frac{1}{\delta^2}\int_0^\infty (e^{-\mu t}\mu) - (2e^{-(\delta+\mu)t}\mu) + (e^{-(2\delta+\mu)t}\mu) dt$$

$$= \frac{1}{\delta^2}\{(-e^{-\mu t}\Big|_0^\infty) - (2)(\frac{\mu}{\delta+\mu})(-e^{-(\delta+\mu)t}\Big|_0^\infty) + (\frac{\mu}{2\delta+\mu})(-e^{-(2\delta+\mu)t}\Big|_0^\infty)\}$$

$$= \frac{1}{\delta^2}\{(0-(-1)) - 2(\frac{\mu}{\delta+\mu})(0-(-1)) + (\frac{\mu}{2\delta+\mu})(0-(-1))\}$$

$$= \frac{1}{\delta^2}\{1-(\frac{2\mu}{\delta+\mu}) + (\frac{\mu}{2\delta+\mu})\} = \frac{1}{0.05^2}\{1-\frac{0.04}{0.07}+\frac{0.02}{0.12}\} = 238.0952$$

Variance = $238.0952-(14.2857)^2 = 34.01$

11 Class selection

People with same age definition will have different underlying mortality due to particular permanent attributes, e.g. sex. The existence of such classes would be certainly found in these data: e.g. male / female smoker / non-smoker, people having different occupational and/or social backgrounds, etc.

Solution would be to subdivide the data according to the nature of the attribute.

Time selection

Where mortality is changing over calendar time, people of the same age could experience different levels of mortality at different times. This might well be a problem here, as data from as much as ten years apart are being combined.

Solution would be to subdivide the data into shorter time periods.

Temporary initial selection

Mortality changes with policy duration and the combination of subgroups of policyholders with different durations into a single sample will cause heterogeneity. Lives accepted for insurance have passed a medical screening process. The longer that has elapsed since screening (i.e. since entry) the greater the proportion of lives who may have developed impairments since the screening date and hence the higher the mortality. Mortality rates would then be expected to rise with policy duration, and hence result in heterogeneous data.

The solution would be to perform a select mortality investigation, that is one in which the data are subdivided by policy duration as well as by age.

Self selection

By purchasing a particular product type, policyholders are putting themselves in a particular group. People expecting lighter than normal mortality might purchase annuities and experience better mortality rates than, for example, term assurance buyers.

The solution would be to subdivide the data by product type.

12 (i) (a) 100,000{1 + (20)(0.045)) = 190,000

(b) $100{,}000(1.0384615)^{20} = 212{,}720$

(c) $100{,}000\{1 + 0.03\, s^{6\%}_{\overline{20|}}\} = 210{,}357$

(ii) EPV maturity benefits:

$$100{,}000 A_{[45]:\overline{20|}}^{\ \ \ \ \ 1} = 100{,}000 v^{20}\ {}_{20}p_{[45]} \ @\ \frac{1.08}{1.0384615} - 1 = 4\%$$

= 100,000*(0.45639)(8,821.2612/9,798.0837) = 0.41089*100,000 = 41,089

EPV death benefits:

$$\frac{100{,}000}{1.0384615} A^1_{[45]:\overline{20|}} \ @\ 4\% = \frac{100{,}000}{1.0384615}\left(A_{[45]:\overline{20|}} - A_{[45]:\overline{20|}}^{\ \ \ \ \ 1}\right)$$

$$= \frac{100{,}000}{1.0384615}(0.46982 - 0.41089)$$

= (100,000)(0.05893 / 1.0384615) = 0.05675*100,000 = 5,675

EPV total benefits = 41,089 + 5,675 = 46,764

(iii) Making appropriate adjustments to (ii)

(a) 1.0384615*5,675+41,089 = 46,982

(b) $(1.08)^{0.5}$*5,675+41,089 = 46,987

(c) = $(1.04)^{0.5}$*1.0384615*5,675 + 41,089 = 47,099

Alternatively, just starting a fresh for each condition:

(a) $A_{[45]:\overline{20|}}$ at 4%

(c) $\bar{A}_{[45]:\overline{20|}}$ at 4% = $\bar{A}^{1}_{[45]:\overline{20|}} + A_{[45]:\overline{20|}}^{\ \ \ \ 1} = \{(1.04)^{0.5} A^{1}_{[45]:\overline{20|}}\} + A_{[45]:\overline{20|}}^{\ \ \ \ 1}$

13 Let P be the monthly premium

$$12P\ddot{a}^{(12)}_{\overline{65:60}:\overline{10|}} = 350 + 0.025P(12\ddot{a}^{(12)}_{\overline{65:60}:\overline{10|}} - 1) + 100,000 A_{\underbrace{65:60}_{}:\overline{10|}}^{\ \ 1} + 20,000\,{}_{10|}\ddot{a}_{\overline{65:60}}:$$

$$\ddot{a}_{65:60:\overline{10|}} = \ddot{a}_{65:60} - v^{10}\,{}_{10}p_{65}\,{}_{10}p_{60}\ddot{a}_{75:70}$$

$$= 12.682 - (0.67556)(0.87120)(0.95372)(8.357)$$
$$= 12.682 - (0.56131)8.357 = 7.991$$

$$\ddot{a}^{(12)}_{65:60:\overline{10|}} = \ddot{a}_{65:60:\overline{10|}} - \frac{11}{24}(1 - v^{10}\,{}_{10}p_{65}\,{}_{10}p_{60})$$
$$= 7.991 - (0.458)(1 - 0.56131)$$
$$= 7.991 - 0.201 = 7.790$$

$$A_{\underbrace{65:60}_{}:\overline{10|}}^{\ \ 1} = A_{65:60:\overline{10|}} - v^{10}\,{}_{10}p_{65}\,{}_{10}p_{60} = 1 - d\ddot{a}_{65:60:\overline{10|}} - v^{10}\,{}_{10}p_{65}\,{}_{10}p_{60}$$

$$= 1 - \frac{0.04}{1.04}(7.991) - 0.56131 = 0.13134$$

$$A_{\underbrace{65:60}_{}}^{\ \ 1\ \ \overline{10|}} = A_{65:60} - v^{10}\,{}_{10}p_{65}\,{}_{10}p_{60}A_{75:70} = 1 - d\ddot{a}_{65:60} - v^{10}\,{}_{10}p_{65}\,{}_{10}p_{60}(1 - d\ddot{a}_{75:70})$$

or

$$= 1 - \frac{0.04}{1.04}(12.682) - (0.67556)(0.87120)(0.95372)(1 - \frac{0.04}{1.04}8.357$$
$$= 0.51223 - 0.38089 = 0.13134$$

$${}_{10|}\ddot{a}_{\overline{65:60}} = v^{10}\,{}_{10}p_{65}\,{}_{10}p_{60}\ddot{a}_{\overline{75:70}} + v^{10}\,{}_{10}p_{65}(1 - {}_{10}p_{60})\ddot{a}_{75} + v^{10}(1 - {}_{10}p_{65})\,{}_{10}p_{60}\ddot{a}_{70}$$
$$= v^{10}\,{}_{10}p_{65}\,{}_{10}p_{60}(\ddot{a}_{75} + \ddot{a}_{70} - \ddot{a}_{75:70}) + v^{10}\,{}_{10}p_{65}(1 - {}_{10}p_{60})\ddot{a}_{75} + v^{10}(1 - {}_{10}p_{65})\,{}_{10}p_{60}\ddot{a}_{70}$$
$$= (0.67556)(0.87120)(0.95372)(9.456 + 12.934 - 8.357)$$
$$+ (0.67556)(0.87120)(1 - 0.95372)(9.456)$$
$$+ (0.67556)(1 - 0.87120)(0.95372)(12.934)$$
$$= 7.877 + 0.258 + 1.073 = 9.208$$

or

$$ {}_{10|}\ddot{a}_{\overline{65:60}} = \ddot{a}_{\overline{65:60}} - \ddot{a}_{\overline{65:60}:\overline{10|}} = (\ddot{a}_{65} + \ddot{a}_{60} - \ddot{a}_{65:60}) - (\ddot{a}_{65:\overline{10|}} + \ddot{a}_{60:\overline{10|}} - \ddot{a}_{65:60:\overline{10|}}) $$

$$ = (\ddot{a}_{65} + \ddot{a}_{60} - \ddot{a}_{65:60}) - (\ddot{a}_{65} - v^{10}\,{}_{10}p_{65}\ddot{a}_{75} + \ddot{a}_{60} - v^{10}\,{}_{10}p_{60}\ddot{a}_{70} - \ddot{a}_{65:60} - v^{10}\,{}_{10}p_{65:60}\ddot{a}_{75:70} $$

$$ = 13.666 + 16.652 - 12.682 $$
$$ -\{13.666 - (0.67556)(0.87120)(9.456)\} $$
$$ -\{16.652 - (0.67556)(0.95372)(12.934)\} $$
$$ +\{12.682 - (0.67556)(0.87120)(0.95372)(8.357) $$

$$ = 17.636 - 8.101 - 8.319 + 7.991 = 17.636 - 8.429 = 9.207 $$

$$ 12P(7.790) = 350 + 0.025P(12*7.790 - 1) + 100,000(0.13134) + 20,000*9.208 $$
$$ \Rightarrow P(93.480 - 2.312) = 350 + 13,134 + 184,160 $$
$$ \Rightarrow P(91.168) = 197,644 \Rightarrow P = 2,168 \ \textit{per month} $$

14 (i) GFLRV=

$$ 300 + 0.25P + 0.05P * a^{4\%}_{\overline{\min(K_{[55]},9)|}} + 50 * a^{0\%}_{\overline{\min(K_{[55]},9)|}} - P * \ddot{a}^{4\%}_{\overline{\min(K_{[55]}+1,10)|}} $$
$$ +(\textit{if } K_{[55]} < 10 \textit{ only})\ v^{T_{[55]}}(100,000 - 10,000 * K_{[55]}) + 200 $$

(ii)

$$ P * \ddot{a}^{4\%}_{[55]:\overline{10|}} = 250 + 0.20P + 0.05P * \ddot{a}^{4\%}_{[55]:\overline{10|}} + 50\ddot{a}^{0\%}_{[55]:\overline{10|}} $$
$$ +110,000\bar{A}^{1}_{[55]:\overline{10|}} - 10,000(I\bar{A})^{1}_{[55]:\overline{10|}} + 200 * {}_{10}q_{[55]} $$

$$ \ddot{a}^{4\%}_{[55]:\overline{10|}} = 8.228 $$

$$ \ddot{a}^{0\%}_{[55]:\overline{10|}} = (1 + e_{[55]}) - {}_{10}p_{[55]}(1 + e_{65}) $$
$$ = 26.037 - \left(\frac{8,821.2612}{9,545.9929}\right)17.645 $$
$$ = 26.037 - (0.92408)17.645 $$
$$ = 9.732 $$

$$ \bar{A}^{1}_{[55]:\overline{10|}} = (1.04)^{0.5}(A_{[55]\overline{10|}} - v^{10}\,{}_{10}p_{[55]}) $$
$$ = (1.04)^{0.5}(0.68354 - 0.67556 * 0.92408) $$
$$ = 0.06044 $$

$$(I\bar{A})^{1}_{[55]:\overline{10}|} = (1.04)^{0.5}[(IA)_{[55]} - v^{10}\,{}_{10}p_{[55]}(10A_{65} + (IA)_{65}]$$

$$= (1.04)^{0.5}[8.58908 - 0.67556 * 0.92408(10 * 0.52786 + 7.89442)]$$

$$= 0.37278$$

$$P * 8.228$$

$$= 250 + 0.20P + 0.05P * 8.228 + 50 * 9.732 + 110,000 * 0.06044$$

$$-10,000 * 0.37278 + 200 * (1 - 0.92408)$$

$$\Rightarrow P * 7.6166 = 250 + 486.60 + 6,648.40 - 3,727.80 + 15.18$$

$$\Rightarrow P = 3,672.38 / 7.6166 = 482.15$$

(iii) $${}_9V = q_{64}v^{0.5}[10,000 + 200 * (1.04)^{9.5}] - [0.95P - 50 * (1.04)^{9}]$$

$$= (0.012716)(0.980581)[10,000 + 290.30] - [0.95 * 482.15 - 71.17]$$

$$128.31 - 386.87 = -258.56$$

END OF EXAMINERS' REPORT