

EXAMINATION

17 April 2007 (am)

Subject CT5 — Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

1 Calculate

- (i) ${}_{5|10}q_{[52]}$
- (ii) $p_{[50]:[60]}$ for two independent lives

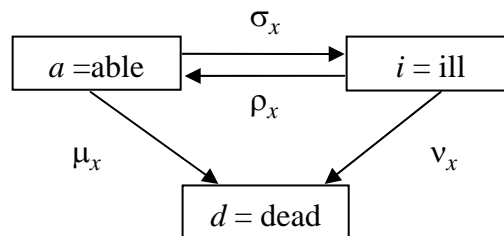
Basis:

Mortality: AM92 Select

[3]

2 State, with examples, three distinct types of selection in the membership of a pension scheme. [3]

3 A three-state transition model is shown in the following diagram:



Assume that the transition probabilities are constant at all ages with $\sigma = 2\%$, $\nu = 6\%$, $\rho = 1\%$ and $\mu = 3\%$.

An able life age 55 exact takes out a 10-year sickness contract that provides a “no-claim” bonus of £100 if the insured remains able for the full duration of the contract. Calculate the expected present value of the bonus at the beginning of the contract with a force of interest of 0.04. [4]

4 (i) In the context of net premiums and reserves, state the conditions necessary for equality of prospective and retrospective reserves. [2]

- (ii) Give two reasons why, in practice, these conditions may not hold. [2]

[Total 4]

- 5** An assurance contract provides a death benefit of £1,000 payable immediately on death, with a savings benefit of £500 payable on every fifth anniversary of the inception of the policy.

The following basis is used:

Force of mortality: $\mu_x = 0.05$ for all x

Force of interest: $\delta = 0.04$

Expenses: None

Calculate the level premium payable annually in advance for life. [5]

- 6** A pension scheme provides a benefit on death in service of 4 times the member's salary at the date of death. Normal Pension Age is 65. State a formula, without using commutation functions, for the present value of this benefit to a life aged 35 exact with salary of £25,000 who has just received a salary increase. Define all symbols used. [5]

- 7** A term assurance contract for a life aged 50 exact for a term of 10 years provides a benefit of £10,000 payable at the end of the year of death. Calculate the expected present value and variance of benefits payable under this contract.

Basis:

Mortality: AM92 Select

Interest: 4% per annum

[6]

- 8** You are given the following statistics in relation to the mortality experience of Actuarial and its province Giro:

| Age | Actuarial | | Giro | |
|-------|-----------------|------------------|-----------------|------------------|
| | Exposed to risk | Number of deaths | Exposed to risk | Number of deaths |
| 0–19 | 300,000 | 25 | 12,000 | 2 |
| 20–39 | 275,000 | 35 | 10,000 | 3 |
| 40–59 | 200,000 | 100 | 9,000 | 6 |
| 60–79 | 175,000 | 500 | 8,000 | 50 |

- (i) Explain, giving a formula, the term Standardised Mortality Ratio (SMR). Define all the symbols that you use. [2]

- (ii) Comment on the relative mortality of the province, by calculating the SMR for Giro. [4]

[Total 6]

- 9** A life insurance company issues an annuity to a life aged 60 exact to provide an annual income of £15,000. The annuity is payable monthly in advance and is guaranteed to be paid for a period of 5 years and for the whole of life thereafter. On the annuitant's death a survivor's pension is paid at the rate £7,500 per annum for the remainder of life for the spouse of the annuitant who is currently aged 55 exact under the following circumstances:

- (a) If the life dies within the guarantee period then the survivor's pension commences with the first payment immediately after the end of the guarantee period.
- (b) If the life dies after the guarantee period has expired then the survivor's pension commences with the first payment immediately after the death of the first life.

Calculate the single premium:

Basis:

Annuitant mortality: PMA92C20
 Spouse mortality: PFA92C20
 Interest: 4% per annum

[6]

- 10** Let X be a random variable representing the present value of the benefits of a whole of life assurance, and Y be a random variable representing the present value of the benefits of a temporary assurance with a term of n years. Both assurances have a sum assured of 1 payable at the end of the year of death and were issued to the same life aged x .

- (i) Describe the benefits provided by the contract which has a present value represented by the random variable $X - Y$. [1]
- (ii) Show that

$$\text{Cov}[X, Y] = {}^2A_{x:n}^1 - A_x * A_{x:n}^1$$

and hence or otherwise that

$$\text{Var}(X - Y) = {}^2A_x - ({}_n|A_x)^2 - {}^2A_{x:n}^1$$

where the functions A are determined using an interest rate of i , and functions 2A are determined using an interest rate of $i^2 + 2i$.

[7]

[Total 8]

- 11** A five-year unit-linked policy issued to a life aged 50 exact has the following pattern of end of year cashflows per policy in force at the start of each year:

(-95.21, -30.18, -20.15, 77.15, 120.29)

- (i) Explain why a life office might need to set up non-unit reserves in respect of a unit-linked life assurance policy. [2]
- (ii) Calculate the non-unit reserves required for the policy in order to zeroise negative cashflows assuming AM92 Ultimate mortality and that reserves earn interest at the rate of 5% per annum. [2]
- (iii) Determine the net present value of the profits before and after zeroisation assuming the risk discount rate used is 8% per annum and state with reasons which of these figures you would expect to be higher. [6]

[Total 10]

- 12** A life office issued 750 identical 25-year temporary assurance policies to lives aged 30 exact each with a sum assured of £75,000 payable at the end of year of death. Premiums are payable annually in advance for 20 years or until earlier death.

- (i) Show that the annual net premium for each policy is approximately equal to £104 using the basis given below. [2]
- (ii) Calculate the net premium reserve per policy at the start and at the end of the 20th year of the policy. [4]
- (iii) Calculate the mortality profit or loss to the life office during the 20th year if twelve policyholders die during the first nineteen years of the policies and two policyholders die during the 20th year. [4]

Basis:

Mortality: AM92 Ultimate

Interest: 4% per annum

[Total 10]

- 13** A life office issues with-profit whole of life contracts, with the sum assured payable immediately on death of the life assured. Level premiums are payable monthly in advance to age 65 or until earlier death.

The life office markets two versions of this policy, one assumed to provide simple bonuses of 4% per annum of the sum assured vesting at the end of each policy year and the other assumed to provide compound bonuses of 4% of the sum assured, again vesting at the end of each policy year. The death benefit under each version does not include any bonus relating to the policy year of death.

The following basis is assumed to price these contracts:

| | |
|--------------------|--|
| Mortality | AM92 Select |
| Interest | 4% per annum |
| Initial expenses | £300 |
| Renewal expenses | 2.5% of the second and subsequent monthly premiums |
| Initial commission | 50% of the gross annual premium |
| Renewal commission | 2.5% of the second and subsequent monthly premiums |
| Claims expenses | £250 at termination of the contract |

Calculate the level monthly premium required for each version of this policy issued to a life aged 30 exact at outset for an initial sum assured of £50,000. [12]

- 14** A life office issues a 4-year non profit endowment assurance policy to a male life aged 61 exact for a sum assured of £100,000 payable on survival to the end of the term or at the end of the year of death if earlier. Premiums are payable annually in advance throughout the term of the policy.

There is a surrender benefit payable equal to a return of premiums paid, with no interest. This benefit is payable at the end of the year of surrender.

The life office uses the following assumptions to price this contract:

| | |
|---|--|
| Mortality | AM92 Select |
| Surrenders | None |
| Interest | 4% per annum |
| Initial expenses | £500 |
| Renewal expenses (on the second and subsequent premium dates) | £50 per annum plus 2.5% of the premium |

In addition, the company holds net premium reserves, calculated using AM92 Ultimate mortality and interest of 4% per annum.

In order to profit test this contract, the life office assumes the same mortality and expense assumptions as per the pricing basis above. In addition, it assumes it earns 5% per annum on funds and that 5% of all policies still in force at the end of 1, 2, and 3 years then surrender.

Calculate, using a risk discount rate of 8% per annum, the expected profit margin on this contract. [18]

END OF PAPER