

EXAMINATION

14 April 2008 (am)

Subject CT5 — Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 13 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

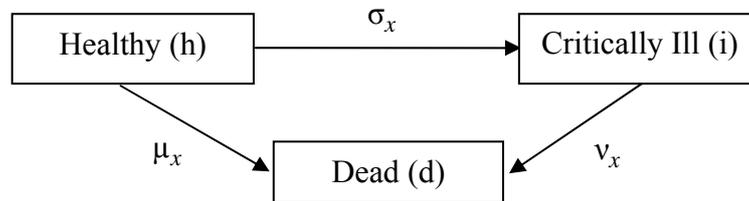
Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1 (a) Express ${}_{5|10}q_{40}$ in words.
 (b) Calculate its value using AM92 mortality. [2]
- 2 Describe three types of reversionary bonus that may be given to a with-profits contract. [4]
- 3 Explain why a life insurance company will need to set up reserves for the endowment assurance contracts it has sold. [4]
- 4 A life insurance company sells a term assurance and critical illness policy with a 20 year term to a life aged 40 exact. The policy provides a benefit of £50,000 payable immediately on death or earlier diagnosis of critical illness. No further benefit is paid in the event of death within the term after a prior critical illness claim has been paid. The company prices the policy using the following multiple state model:



Calculate the expected present value of the benefits under the policy.

- Basis: $i = 5\%$ per annum
 $\mu_x = 0.005$ at all ages
 $\nu_x = 0.006$ at all ages
 $\sigma_x = 0.003$ at all ages [5]

- 5 A reversionary annuity is payable continuously beginning on the death of a life aged x to an annuitant aged y .
- (a) Derive an expression for the present value of the reversionary annuity using random variables for the future lifetimes.
- (b) Derive an expression for the expected present value of the reversionary annuity in terms of assurance functions. [5]

- 6** A parent who has just died left a bond in their will that provides a single payment of £15,000 in 10 years' time. The payment of £15,000 will be shared equally between the local cats' home and such of the parent's two sons (currently aged 25 and 30 exact) who are then still alive. Calculate the expected present value of the share due to the cats' home.

Basis: Mortality AM92 Ultimate
Interest 3% per annum

[5]

- 7** A defined benefit pension scheme provides a pension on retirement for any reason of one-sixtieth of final pensionable salary for each year of service (with proportion for part years of service). Final pensionable salary is average salary over the three years immediately preceding retirement. Calculate the cost of providing future service benefits for a new member aged 40 exact as a percentage of salary.

Basis: Example Pension Scheme Table in the Formulae and Tables for Examinations Handbook

[6]

- 8** (i) Show that

$${}_{t-s}q_{x+s} = \frac{(t-s)q_x}{(1-sq_x)}, \quad (0 \leq s < t \leq 1)$$

using an assumption of a uniform distribution of deaths. [4]

- (ii) Calculate the value of ${}_{0.5}q_{62.25}$ using assumptions of:

- (a) a uniform distribution of deaths
- (b) a constant force of mortality

Basis: Mortality PMA92C20

[3]

[Total 7]

9 A life insurance company prices annuities using a basis which incorporates the location of the proposing annuitants as an additional rating factor.

- (i) Identify three factors that influence mortality and would cause the insurance company to adopt location as a rating factor. State which form of selection is demonstrated by the use of location as a rating factor. [4]
- (ii) The company has produced the following data in respect of two locations. Calculate the standardised mortality ratio for each location based on the standard mortality table ELT15(Males).

Age	Location A		Location B	
	Initial exposed to risk	Number of deaths	Initial exposed to risk	Number of deaths
60	100	1	200	3
61	175	3	150	3
62	190	2	170	3
63	210	3	100	2

[4]
[Total 8]

10 A male life aged 60 exact wants to buy the following benefits within one policy:

- (a) an annuity of £5,000 per annum payable monthly in arrear to his wife currently aged 55 exact commencing on his death and for the rest of her life, and
- (b) an annuity of £2,000 per annum payable monthly in arrear to his grandson currently aged 13 exact commencing on the death of either grandparent and ceasing when the grandson reaches age 21

Calculate the overall single premium.

Basis:

Mortality Male life – PMA92C20
Wife – PFA92C20
Grandson – ignore

Interest 4% per annum

[10]

11 A life insurance company issues a 10-year with-profits endowment policy to a life then aged 50 exact. Under the policy, the basic sum assured of £75,000 and attaching bonuses are payable at maturity or immediately on death, if earlier. The company declares compound reversionary bonuses vesting at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year of death). Level premiums are payable annually in advance under the policy.

- (i) Show that the annual premium, using the equivalence principle, is approximately £7,487.

Basis:		
Mortality		AM92 Select
Interest		6% per annum
Bonus loading		1.92308% of the sum assured, compounded and vesting at the end of each policy year
Expenses	Initial	£350 plus 50% of the annual premium
	Renewal	5% of each premium payable in the second and subsequent years

[7]

At aged 55 exact, immediately before the premium then due and just after the declared bonus relating to the 5th policy year has been added to the policy, the policy is still in force.

- (ii) Calculate the reserve for the policy at this point in time using a gross premium prospective basis assuming the same basis as in (i) above. You should also assume that the life insurance company has declared a compound bonus throughout the duration of the policy consistent with the bonus loading assumption used to derive the premium in (i) above.

[5]

[Total 12]

12 A life assurance company issues the following policies:

- 10-year term assurances with a sum assured of £50,000 where the death benefit is payable at the end of the policy year of death
- 10-year pure endowment assurances with a sum assured of £50,000 payable on maturity

For the term assurance and pure endowment policies, premiums are paid annually in advance.

The company sold 5,000 policies of each type to lives then aged 50 exact. During the first policy year, there were five actual deaths from each of the two types of policies written.

- (i) Assuming each type of policy was sold to a distinct set of lives (i.e. no life buys more than one type of policy).
- (a) Calculate the death strain at risk for each type of policy at the end of the second policy year of the policies.
- (b) During the second policy year, there were ten deaths from each of the two types of policy written. Calculate the total mortality profit or loss to the company during the second policy year.

Basis:

Interest 4% per annum

Mortality AM92 Ultimate for term assurance and pure endowment

Expenses Nil

[11]

- (ii) The company now discovers that 5,000 lives had bought one of each type of policy.
- (a) State whether the mortality profit or loss calculated would now be higher, lower or unchanged to that calculated in (i)(b).
- (b) State whether the variance of the benefits paid out by the company in future years would be higher, lower or unchanged to that in (i). Explain your answer by general reasoning. [3]

[Total 14]

- 13** A life insurance company issues a 4-year unit-linked endowment policy to a life aged 50 exact under which level premiums of £750 are payable yearly in advance throughout the term of the policy or until earlier death. In the first policy year, 25% of the premium is allocated to units and 102.5% in the second and subsequent years. The units are subject to a bid-offer spread of 5% and an annual management charge of 1% of the bid value of units is deducted at the end of each policy year.

Management charges are deducted from the unit fund before death, surrender and maturity benefits are paid.

If the policyholder dies during the term of the policy, the death benefit of £3,000 or the bid value of the units, whichever is higher, is payable at the end of the policy year of death. The policyholder may surrender the policy only at the end of each policy year. On surrender, the bid value of the units is payable at the end of the policy year of exit. On maturity, 110% of the bid value of the units is payable.

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund	6.5% per annum
Rate of interest on non-unit fund cash flows	5.5% per annum
Mortality	AM92 Select
Initial expenses	£150
Renewal expenses	£65 per annum on the second and subsequent premium dates
Initial commission	10% of first premium
Renewal commission	2.5% of the second and subsequent years' premiums
Risk discount rate	8.5% per annum

In addition assume that at the end of each of the first 3 years, 10% of all policies still in force then surrender.

- (i) Calculate the profit margin for the policy on the assumption that the company does not zeroise future expected negative cash flows. [13]
- (ii) Suppose the company sets up reserves in order to zeroise future expected negative cash flows.
- (a) Calculate the expected reserve that must be set up at the end of each policy year, per policy in force at the start of each policy year.
- (b) Calculate the profit margin allowing for the cost of setting up these reserves.

[5]

[Total 18]

END OF PAPER