EXAMINATION

April 2005

Subject CT5 — Contingencies
Core Technical

EXAMINERS' REPORT

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

M Flaherty
Chairman of the Board of Examiners

15 June 2005

1 The profit vector is the vector of expected end-year profits for policies which are still in force at the start of each year.

The profit signature is the vector of expected end-year profits allowing for survivorship from the start of the contract.

2 (a) $\ddot{a}_{50:\overline{20|}} = \dfrac{1 - A_{50:\overline{20|}}}{d}$

(b) $A_{50:\overline{20|}} = A_{50} + v^{20}\, {}_{20}p_{50}(1 - A_{70})$

$$= 0.32907 + 0.45639 \times \frac{8054.0544}{9712.0728}(1 - 0.60097)$$

$$= 0.480093$$

$$\ddot{a}_{50:\overline{20|}} = \frac{1 - 0.480093}{d} = 13.5176$$

3 (a) $({}_tV' + OP - e_t)(1+i) = q_{x+t}(S) + p_{x+t}({}_{t+1}V')$

where

${}_tV'$ = gross premium provision at time t

OP = office premium

e_t = expenses incurred at time t

i = interest rate in premium/valuation basis

S = sum assured

p_{x+t} is the probability that a life aged $x + t$ survives one year on the premium/valuation mortality basis

q_{x+t} is the probability that a life aged $x + t$ dies within one year on the premium/valuation mortality basis

(b) Income (opening provision plus interest on excess of premium over expense, and provision) equals outgo (death claims and closing provision for survivors) if assumptions are borne out.

4 The value of a pension of £1 p.a. is

$\ddot{a}^{(12)}_{\overline{10}|} + {}_{10|}\ddot{a}^{(12)}_{60}$ where first term is an annuity certain

$$\ddot{a}^{(12)}_{\overline{10}|} = \frac{1-v^{10}}{d^{(12)}} @ 6\% = \frac{1-0.55839}{0.058128} = 7.59720$$

$${}_{10|}\ddot{a}^{(12)}_{60} = \ddot{a}^{(12)}_{60} - \ddot{a}^{(12)}_{60:\overline{10}|} = v^{10}\,{}_{10}p_{60}\ddot{a}^{(12)}_{70}$$

$${}_{10}p_{60} = \frac{8054.0544}{9287.2164} = 0.867219$$

$$\ddot{a}^{(12)}_{70} = \ddot{a}_{70} - 11/24 = 9.140 - 11/24 = 8.682$$

So value of a pension of £1 p.a. is

$7.59720 + v^{10} \times 0.867219 \times 8.682 = 11.801$

So annuity purchased by £200,000 is 200000/11.801 = £16,948

5 The present value is $\int_0^{20} 2000.e^{-\delta t}\,{}_t p_{40}^{\overline{ii}}\,dt$ where $\delta = \ln(1.04)$

$${}_t p_{40}^{\overline{ii}} = \exp\left(-\int_0^t (\rho+\nu)ds\right)$$

$$= \exp(-.05t)$$

So value is

$$2000\int_0^{20} e^{-\delta t} e^{-5\% t} dt \text{ where } \delta = \ln(1.04)$$

$$= 2000\left[\frac{e^{-t(.05+\ln(1.04))}}{-(.05+\ln(1.04))}\right]_0^{20}$$

$$= 18,653$$

6 Require to calculate ${}_{14½} p_{45½} = {}_{½} p_{45½} \cdot {}_{14} p_{46}$

$${}_{14} p_{46} = \frac{l_{60}}{l_{46}} = \frac{86714}{95266} = 0.91023$$

(a) Assume deaths uniformly distributed so ${}_t p_x . \mu_{x+t}$ constant

Then $${}_{½} q_{45½} = \frac{(1-½)q_{45}}{(1-½q_{45})} = \frac{½0.00266}{(1-½.00266)} = .001332$$

So $${}_{14½} p_{45½} = (1-.001332)\times 0.91023 = 0.909018$$

(b) Assume that force of mortality is constant across year of age 45 to 46

$${}_{½} p_{45½} = e^{-½\mu_{45}}$$

$$\mu_{45} = -\ln(1-q_{45}) = -\ln(1-0.00266) = 0.002664$$

$${}_{½} p_{45½} = e^{-½0.002664} = 0.998669$$

So $${}_{14½} p_{45½} = 0.998669\times 0.91023 = 0.909018$$

7 Define a random variable T_{xy}, the lifetime of the joint life status

The expected value at a rate of interest i is

$$\bar{a}_{xy} = E(\bar{a}_{\overline{T_{xy}}|})$$

$$= E\left(\frac{1-v^{T_{xy}}}{\delta}\right)$$

$$= \frac{1-E(v^{T_{xy}})}{\delta}$$

$$= \frac{1-\bar{A}_{xy}}{\delta}$$

The variance is

$$\text{var}\left(\frac{1-v^{T_{xy}}}{\delta}\right)$$

$$= \frac{1}{\delta^2}\text{var}(v^{T_{xy}})$$

$$= \frac{1}{\delta^2}({}^2\bar{A}_{xy} - (\bar{A}_{xy})^2)$$

where ${}^2\bar{A}_{xy}$ is at $(1+i)^2 - 1$

8 Past Service

$$\frac{10}{80}20000\sum_{t=0}^{29}\frac{i_{35+t}}{l_{35}}\frac{v^{35+t+½}}{v^{35}}\frac{z_{35+t+½}}{s_{34}}\bar{a}_{35+t+½}$$

or

$$\frac{10}{80}20000\frac{{}^zM_{35}^{ia}}{{}^sD_{35}}$$

Future Service

$$\frac{10}{80}20000\frac{{}^{z}M_{35}^{ia}}{{}^{s}D_{35}}+\frac{1}{80}20000\frac{{}^{z}\overline{R}_{45}^{ia}}{{}^{s}D_{35}}$$

9

- Insurance works on the basis of pooling independent homogeneous risks
- The central limit theorem then implies that profit can be defined as a random variable having a normal distribution.
- Life insurance risks are usually independent
- Risk classification ensures that the risks are homogeneous
- Lives are divided by risk factors
- More factors implies better homogeneity
- But the collection of more factors is restricted by
  - The cost of obtaining data
  - Problems with accuracy of information
  - The significance of the factors
  - The desires of the marketing department

10

	Males		***Females***		***Male***	***Female***	***Total***	***Total***	***Female***	***Total***
Age band	*Exposed to risk*	*Observed Mortality rate*	*Exposed to risk*	*Observed Mortality rate*	*Actual deaths*	*Actual deaths*	*Actual deaths*	*Exposed to risk*	*Expected deaths using total mortality rates*	*Expected deaths using female rates*
20–29	125000	0.00356	100000	0.00125	445	125	570	225000	253.333333	281.25
30–39	200000	0.00689	250000	0.00265	1378	662.5	2040.5	450000	1133.61111	1192.5
40–49	100000	0.00989	200000	0.00465	989	930	1919	300000	1279.33333	1395
50–59	90000	0.01233	150000	0.00685	1109.7	1027.5	2137.2	240000	1335.75	1644
					3921.7	2745	6666.7	1215000	4002.02778	4512.75
									Direct	0.003714
									Indirect	0.003764

11 Let P be the monthly premium. Then:

EPV of premiums:

$$12P\ddot{a}^{(12)}_{[40]:\overline{25}|} = 155.124P$$

$$\ddot{a}^{(12)}_{[40]:\overline{25}|} = \ddot{a}_{[40]:\overline{25}|} - \frac{11}{24}(1 - {}_{25}p_{[40]}v^{25})$$

$$= 13.290 - \frac{11}{24}\left(1 - (1.06)^{-25} \times \frac{8821.2612}{9854.3036}\right) = 12.927$$

EPV of benefits:

$$\frac{100,000}{(1+b)} \times (1.06)^{1/2}\{q_{[40]}(1+b)v \; + {}_1|q_{[40]}(1+b)^2 v^2$$

$$+ \ldots. + {}_{24}|q_{[40]}(1+b)^{25}v^{25}\} + 100,000\,{}_{25}p_{[40]}(1+b)^{25}v^{25}$$

where b = 0.0192308

$$= \frac{100,000}{(1+b)} \times (1.06)^{1/2} A^{1}_{[40]:\overline{25}|} @ i' + 100,000 \times \frac{D_{65}}{D_{[40]}} @ i'$$

$$= \frac{100,000}{1.0192308} \times (1.06)^{1/2} \times (.38896 - .33579) + 100,000 \times .33579 = 38949.90$$

where $i' = \frac{1.06}{1+b} - 1 = 0.04$

EPV of expenses:

$$.875 \times 12P + 175 + 0.025 \times 12 \times P(\ddot{a}^{(12)}_{[40]:\overline{25}|} - \ddot{a}^{(12)}_{[40]:\overline{1}|}) + 65[\ddot{a}_{[40]:\overline{25}|} - 1] = 14.086P + 973.85$$

$$\ddot{a}^{(12)}_{[40]:\overline{1}|} = \ddot{a}_{[40]:\overline{1}|} - \frac{11}{24}(1 - \; p_{[40]}v \;) = 1 - \frac{11}{24}\left(1 - (1.06)^{-1} \times \frac{9846.5384}{9854.3036}\right) = 0.974$$

EPV of claim expense:

.025 × 38949.9 =973.748

Equation of value gives $155.124P = 38949.9 + 14.086P + 973.85 + 973.75$

and $P = £289.98$

12 (i) $$\bar{A}^{1}_{x:\overline{n}|} = \sum_{t=0}^{n-1} {}_{t|}\bar{A}^{1}_{x:\overline{1}|}$$

$$= \sum_{t=0}^{n-1} v^t \, {}_t p_x \bar{A}^{1}_{x+t:\overline{1}|}$$

$$\bar{A}^{1}_{x+t:\overline{1}|} = \int_0^1 v^s \, {}_s p_{x+t} \mu_{x+t+s} ds$$

Assuming a uniform distribution of deaths, then ${}_s p_{x+t} \mu_{x+t+s} = q_{x+t}$

$$\bar{A}^{1}_{x+t:\overline{1}|} = \int_0^1 v^s q_{x+t} ds = q_{x+t} \int_0^1 v^s ds$$

$$= q_{x+t} \frac{iv}{\delta}$$

$$\bar{A}^{1}_{x:\overline{n}|} = \sum_{t=0}^{n-1} v^t \cdot {}_t p_x \cdot q_{x+t} \frac{iv}{\delta}$$

$$= \frac{i}{\delta} \sum_{t=0}^{n-1} v^{t+1} \cdot {}_t p_x \cdot q_{x+t}$$

$$= \frac{i}{\delta} A^{1}_{x:\overline{n}|}$$

(ii) $$\text{var}(\bar{A}^1_{x:\overline{n}|}) = \text{var}(\frac{i}{\delta} A^1_{x:\overline{n}|}) = \left(\frac{i}{\delta}\right)^2 \text{var}(A^1_{x:\overline{n}|})$$

$$= \left(\frac{i}{\delta}\right)^2 ({}^2A^1_{x:\overline{n}|} - (A^1_{x:\overline{n}|})^2)$$

$$A^1_{[40]:\overline{30}|} = A_{[40]} - v^{30} \cdot {}_{30}p_{[40]} \cdot A_{70}$$

$$= 0.23041 - v^{30} \frac{8054.0544}{9854.3036} 0.60097 = 0.078970$$

$${}^2A^1_{[40]:\overline{30}|} = {}^2A_{[40]} - v^{30} \cdot {}_{30}p_{[40]} \cdot {}^2A_{70}$$

$$= 0.06775 - v^{30} \frac{8054.0544}{9854.3036} 0.38975 = 0.037469$$

where v = 1/1.0816

$$\text{var}(\bar{A}^1_{x:\overline{n}|}) = \left(\frac{0.04}{\ln(1.04)}\right)^2 (0.037469 - (0.078970)^2) = 0.032486$$

$$\text{Expected value} = \frac{i}{\delta} A^1_{[40]:\overline{30}|} = \frac{0.04}{\ln(1.04)} 0.078970 = 0.080539$$

13

Annual premium	1000.00	Allocation % (1st yr)	50.0%
Risk discount rate	8.0%	Allocation % (2nd yr +)	102.50%
Interest on investments	6.0%	Man charge	0.50%
Interest on sterling provisions	4.0%	B/O spread	5.0%
Minimum death benefit	4000.00		

	£	% prm	Total
Initial expense	150	20.0%	350
Renewal expense	50	2.5%	75

(i) Multiple decrement table

x	q_x^d	q_x^s
40	0.000788	0.10
41	0.000962	0.05
42	0.001104	0.05
43	0.001208	0.05

x	$(aq)_x^d$	$(aq)_x^s$	(ap)	${}_{t-1}(ap)$
40	0.000749	0.09996	0.899291	1.000000
41	0.000938	0.04998	0.949086	0.899291
42	0.001076	0.04997	0.948951	0.853504
43	0.001178	0.04997	0.948852	0.809934

Unit fund (per policy at start of year)

	yr 1	*yr 2*	*yr 3*	*yr 4*
value of units at start of year	0.000	500.983	1555.400	2667.495
alloc	500.000	1025.000	1025.000	1025.000
B/O	25	51.25	51.25	51.25
interest	28.500	88.484	151.749	218.475
management charge	2.518	7.816	13.404	19.299
value of units at year end	500.983	1555.400	2667.495	3840.421

Cash flows (per policy at start of year)

	yr 1	*yr 2*	*yr 3*	*yr 4*
unallocated premium	500.000	–25.000	–25.000	–25.000
B/O spread	25.000	51.250	51.250	51.250
expenses	350.000	75.000	75.000	75.000
interest	7.000	–1.950	–1.950	–1.950
man charge	2.518	7.816	13.404	19.299
extra death benefit	2.619	2.293	1.434	0.188
end of year cashflow	181.898	–45.177	–38.730	–31.589
probability in force	1	0.899291	0.853504	0.809934
discount factor	0.925925926	0.85733882	0.793832241	0.735029853
expected p.v. of profit	88.54607934			
premium signature	1000	832.67667	731.74245	642.95174
expected p.v. of premiums	3207.370861			
profit margin	2.76%			

(ii)

(a)

To calculate the expected provisions at the end of each year we have (utilising the end of year cashflow figures and decrement tables in (i) above):

$$ {}_3V = \frac{-31.589}{1.04} = 30.374 $$

$$ {}_2V \times (1.04) - (ap)_{42} \times {}_3V = -38.73 \Rightarrow {}_2V = 64.9552 $$

$$ {}_1V \times (1.04) - (ap)_{41} \times {}_2V = -45.177 \Rightarrow {}_1V = 102.7164 $$

These need to be adjusted as the question asks for the values in respect of the beginning of the year. Thus we have:

Year 3 $30.374(ap)_{42} = 28.823$
Year 2 $64.9552(ap)_{41} = 61.648$
Year 1 $102.7164(ap)_{40} = 92.372$

(b)

Based on the expected provisions calculated in (a) above, the cash flow for years 2, 3 and 4 will be zeroised whilst year 1 will become:

181.898 – 92.372 = 89.526

Hence the table blow can now be completed for the revised profit margin.

revised end of year cash flow	89.526	0	0	0
probability in force	1	0.899291	0.853504	0.809934
discount factor	0.925925926	0.85733882	0.793832241	0.735029853
expected p.v. of profit	82.89461768			
profit margin	2.58%			

14 (i) The death strain at risk for a policy for year $t + 1$ ($t = 0, 1, 2\ldots$) is the excess of the sum assured (i.e. the present value at time $t + 1$ of all benefits payable on death during the year $t + 1$) over the end of year provision.

i.e. DSAR for year $t + 1 = S - {}_{t+1}V$

The "expected death strain" for year $t + 1$ ($t = 0, 1, 2\ldots$) is the amount that the life insurance company expects to pay extra to the end of year provision for the policy.

i.e. EDS for year $t + 1 = q(S - {}_{t+1}V)$

The "actual death strain" for year $t + 1$ ($t = 0, 1, 2\ldots$) is the observed value at $t+1$ of the death strain random variable

i.e. ADS for year $t + 1 = (S - {}_{t+1}V)$ if the life died in the year t to $t + 1$ = 0 if the life survived to $t + 1$

(ii) Annual premium for pure endowment with £75,000 sum assured given by:

$$P^{PE} = \frac{75,000}{\ddot{a}_{45:\overline{15}|}} \times \frac{D_{60}}{D_{45}} = \frac{75,000}{11.386} \times \frac{882.85}{1677.97} = 3465.71$$

Annual premium for term assurance with £150,000 sum assured given by:

$$P^{TA} = P^{EA} - 2P^{PE} = \frac{150,000 A_{45:\overline{15}|}}{\ddot{a}_{45:\overline{15}|}} - 2P^{PE}$$

$$= \frac{150,000 \times 0.56206}{11.386} - 2 \times 3465.71 = 473.20$$

Provisions at the end of the third year:

for pure endowment with £75,000 sum assured given by:

$${}_3V^{PE} = 75,000 \times \frac{D_{60}}{D_{48}} - P^{PE} \ddot{a}_{48:\overline{12}|}$$

$$= 75,000 \times \frac{882.85}{1484.43} - 3465.71 \times 9.613 = 11289.63$$

for term assurance with £150,000 sum assured given by:

$${}_3V^{TA} = {}_3V^{EA} - {}_3V^{PE}$$

$$= 150,000 A_{48:\overline{12}|} - (2 \times 3465.71 + 473.20)\ddot{a}_{48:\overline{12}|} - 2 \times 11289.63$$

$$= 150,000 \times 0.63025 - 7,404.62 \times 9.613 - 22,579.26$$

$$= 777.63$$

for temporary immediate annuity paying an annual benefit of £25,000 given by:

$${}_3V^{IA} = 25,000 a_{58:\overline{2}|}$$

$$= 25,000(\ddot{a}_{58:\overline{3}|} - 1)$$

$$= 25,000(\ddot{a}_{58} - v^3 \, {}_3p_{58} \ddot{a}_{61} - 1)$$

$$= 25,000\left(16.356 - (1.04)^{-3} \frac{9802.048}{9864.803} \times 15.254 - 1\right) = 47,037.91$$

Sums at risk:

Pure endowment: DSAR = 0 – 11,289.63 = –11,289.63

Term assurance: DSAR = 150,000 – 777.63 = 149,222.37

Immediate annuity: DSAR = –(47,037.91 + 25,000) = –72,037.91

Mortality profit = EDS – ADS

For term assurance

$$EDS = 4985 \times q_{47} \times 149,222.37 = 4985 \times .001802 \times 149,222.37 = 1,340,460.07$$

$$ADS = 8 \times 149,222.37 = 1,193,778.96$$

mortality profit = 146,681.11

For pure endowment

$$EDS = 1995 \times q_{47} \times -11,289.63 = 1995 \times .001802 \times -11,289.63 = -40,586.11$$

$$ADS = 1 \times -11,289.63 = -11,289.63$$

mortality profit = –29,296.48

For immediate annuity

$$EDS = 995 \times q_{57} \times -72,037.91 = 995 \times .001558 \times -72,037.91 = -111,673.89$$

$$ADS = 1 \times -72,037.91 = -72,037.91$$

mortality profit = –39,635.98

Hence, total mortality profit = £77,748.65

END OF EXAMINERS' REPORT