INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

27 September 2013 (pm)

Subject CT5 – Contingencies
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional booklets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

11 Calculate:

(a) ${}_{10}q_{63}$

(b) $\ddot{a}_{63}^{(2)}$

(c) $s_{55:\overline{10}|}$

Basis:

Mortality PFA92C20
Interest 4% per annum

[2]

12 Define temporary initial selection, giving a distinct example of the process. [2]

13 A whole life assurance policy was issued to a life aged x exact for a sum assured of S payable at the end of year of death. A premium of P is payable annually in advance until death. The following expense assumptions were used to derive the gross premium payable on the policy:

Initial commission	a% of the annual premium
Initial expenses	B
Renewal commission	c% of each annual premium excluding the first
Renewal expenses	D per annum at the start of the second and subsequent policy years
Claim expenses	e% of the sum assured

Let ${}_tV_x$ represent the gross premium reserve on this policy at duration t.

Write down an equation linking the gross premium reserve at the beginning and the end of:

(a) the first policy year

(b) the t^{th} policy year where $t > 1$

[3]

14 Calculate ${}_{2.25}p_{90.25}$ using the method of Uniform Distribution of Deaths.

Basis:

Mortality AM92 [3]

15 A three-year unit-linked endowment assurance policy is sold to a male life aged 40 exact. The profit signature for this policy, calculated using AM92 Select mortality and making no allowance for surrenders, is:

(–209.80, 253.55, 109.85)

It is now assumed for the cash flows for this policy that 15% of all policies in force at the end of the first policy year are surrendered at that time. The surrender value payable at that time is the bid value of units at the end of the policy year less a surrender penalty of £500. There are no other changes to the policy.

(a) Calculate the revised profit signature in the first policy year.

(b) Comment on the impact on the profit signature in the second and third policy years.

[4]

16 A life aged 75 exact purchases a ten-year temporary annuity of an initial amount of £1,200 per annum. This annuity increases on each policy anniversary by £100 per annum, the last increase being at the beginning of the tenth policy year. All annuity payments are annual in advance.

Calculate the expected present value of the annuity benefits.

Basis:

Mortality	AM92 Ultimate
Interest	6% per annum
Expenses	Nil

[4]

17 Calculate $(\overline{IA})_{20}$ (the present value of a whole life assurance issued to a life aged 20 exact payable immediately on death where the benefit paid on death at time t is t) using the following basis:

Basis:

Mortality	$\mu_x = 0.03$ for $x<40$ inclusive and 0.04 for $x \geq 40$
Force of interest	5% per annum

[6]

18 Show, using the random variable approach, that the expected present value of an annuity of 1 per annum payable annually in arrears to a life now aged x, deferred for n years is equal to $a_x - a_{x:\overline{n}|}$. [7]

19 The following statistics have been provided in relation to a particular country and one of its regions:

	Region A		*Country*	
Age band	*Population exposed*	*Number of Deaths*	*Population exposed*	*Number of Deaths*
18–35	25,000	25	500,000	1,000
36–50	50,000	80	125,000	375
51–70	70,000	170	110,000	500

(i) Calculate:

(a) the mortality rates for each age band both for Region A and Country

(b) the crude mortality rate for each of Region A and Country

(c) the directly standardised mortality rate for Region A by reference to the Country

(d) the standardised mortality ratio for Region A by reference to the Country [5]

(ii) Comment on the results. [2]

[Total 7]

20 The following is an extract of a decrement table assumed for a funeral plan, showing deaths (d) and withdrawals (w):

Age x	$(al)_x$	$(ad)_x^d$	$(ad)_x^w$
85	10,000	1,400	2,300
86	6,300	1,000	1,100
87	4,200		

It has been established that the independent rates of decrement of withdrawal are now only 50% of those assumed in the table above for the ages of 85 and 86. The underlying independent mortality rates are unchanged.

Construct a revised decrement table to reflect this change. [7]

21 A pension scheme provides a lump sum benefit on death in service of three times salary in the 12 months before death. Normal retirement age is 65 exact.

(i) Calculate the expected cashflow between ages 40 and 41 in respect of the death benefit for a life now aged 35 exact with salary in the previous 12 months of 25,000.

Basis: Pension scheme tables in the Formulae and Tables for Examinations [3]

(ii) (a) Give a formula for the expected present value of the death benefit.

(b) Hence express this value using commutation factors.

[5]
[Total 8]

22 At the beginning of 2004, a life insurance company issued a number of 20-year "special" endowment assurance policies to male lives then aged 40 exact. Each policy provides a death benefit of £75,000 payable at the end of year of death and a maturity benefit of £150,000.

Premiums on each policy are payable annually in advance for the term of the policy, ceasing on earlier death.

(i) Calculate the annual gross premium for each policy using the following premium basis: [4]

Mortality	AM92 Select
Interest	4% per annum
Initial commission	25% of the first annual premium
Initial expenses	£400
Renewal expenses	£45 per annum at the start of the second and subsequent policy years

(ii) Determine the gross premium reserve for each policy in force at the end of the eighth policy year **and** for each policy in force at the end of the ninth policy year, using the same basis as above. [6]

At the beginning of 2012, there were 625 policies in force. Actual experience for this portfolio of business during 2012 was as follows:

Number of deaths	3
Interest earned	4.5%
Expense incurred per policy in force at beginning of policy year	£45

(iii) Derive, using the recursive relationship between the opening and closing reserves, the profit/loss from this portfolio of business in 2012 separately from:

- mortality
- interest
- expenses [4]

[Total 14]

23 A life insurance company issues a 15-year increasing term assurance policy to a life aged 50 exact.

The death benefit on the policy, payable immediately on death, is given by the formula:

$$£10{,}000 \times [6+t] \quad t = 0, 1, 2, ..., 14$$

where t denotes the curtate duration in years since the inception of the policy.

Level premiums on the policy are payable monthly in advance for the term of the policy, ceasing on death if earlier.

(i) Calculate the monthly premium for the policy using the following premium basis:

Mortality	AM92 Select
Interest	6% per annum
Expenses	
Initial	£225
Renewal	£65 per annum inflating at 1.92308% per annum, at the start of the second and subsequent policy years
Commission	
Initial	30% of the total premium payable in the first policy year
Renewal	4% of the second and subsequent monthly premiums
Claim	£275 on termination, inflating at 1.92308% per annum
Inflation	For renewal and claim expenses, the amounts quoted are at outset, and the increases due to inflation start immediately.

[8]

(ii) Calculate the gross prospective reserve for the policy at the end of the 14th policy year using the elements of the premium basis that are relevant. [3]

(iii) Write down an expression for the gross future loss random variable at the end of the 14th policy year, again using the elements of the premium basis that are relevant. [4]

[Total 15]

24 A life insurance company issues a four-year with profits endowment assurance policy for a basic sum assured of £25,000 to a life aged 56 exact. Level premiums are payable annually in advance throughout the term of the policy.

Compound reversionary bonuses are added to the policy at the start of each year, including the first. The basic sum assured (together with any bonuses attaching) is payable at maturity or at the end of year of death, if earlier.

(i) Show that the annual premium is approximately £6,483 using the following premium basis:

Mortality	AM92 Select
Interest	6% per annum
Initial expenses	£100 plus 25% of the first premium (all incurred on policy commencement)
Renewal expenses	2.5% of the second and subsequent premiums plus £40 at the start of the second and subsequent policy years
Bonus rates	A compound reversionary bonus will be declared each year at a rate of 1.92308% per annum

[5]

(ii) The insurance company holds net premium reserves using a rate of interest of 4% per annum and AM92 Ultimate mortality.

Calculate the expected profit margin on this policy using the following profit test basis:

Mortality	80% AM92 Select
Interest earned on funds	7.5% per annum
Initial expenses	as per premium basis
Renewal expenses	as per premium basis
Bonus rates	as per premium basis
Risk discount rate	9.5% per annum

[13]
[Total 18]

END OF PAPER