INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2014 examinations

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context at the date the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

F Layton
Chairman of the Board of Examiners

November 2014

General comments on Subject CT5

CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions' actuarial work.

Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given where appropriate to different valid points made which do not appear in the solutions below.

In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

Comments on the September 2014 paper

The general performance was similar this session to previous ones although it was felt that this paper was possibly a little harder than some previous ones. Questions that were done less well were 4, 5(ii), 7 (variance), 11, 12(ii) and 14(iii). The examiners hope that the detailed solutions given below will assist students with further revision.

However most of the short questions were very straightforward where an answer could be produced quickly and this is where many successful candidates scored particularly well. Students should note that for long questions reasonable credit is given if they can describe the right procedures although to score high marks reasonable accurate numerical calculation is necessary.

1 Within a population mortality (or morbidity) varies with calendar time. This effect is usually observed at all ages. The usual pattern is for mortality rates to become lighter (improve) over time, although there can be exceptions, due, for example, to the increasing effect of AIDS in some countries.

For example a separate model or table will be produced for different calendar periods e.g. English Life Table No 14 1980–82 and English Life Table No 15 1990–92. The difference between the tables is termed time selection. [2]

This question was generally well done. Other valid examples were credited.

2

- Withdrawal often acts as a selective decrement in respect of mortality. Those withdrawing tend to have lighter mortality than those who keep their policies in force.

- This selective effect results in mortality rates which increase markedly with policy duration and resembles temporary initial selection.

[3]

Generally well done. The main omission was mentioning the worsening mortality of those who did not lapse.

3

$$ {}_{2.5}q_{75.75} = (1 - {}_{2.5}p_{75.75}) = 1 - ({}_{0.25}p_{75.75}) \times p_{76} \times p_{77} \times {}_{0.25}p_{78} $$

$$ p_{76} = 1 - q_{76} = 0.967821 $$

$$ p_{77} = 1 - q_{77} = 0.963304 $$

$$ {}_{0.25}p_{78} = 1 - 0.25 \times q_{78} = 0.989575 $$

$$ {}_{0.25}p_{75.75} = 1 - \frac{0.25q_{75}}{1 - 0.75q_{75}} = 0.992818 $$

$$ {}_{2.5}q_{75.75} = 1 - (0.992818 \times 0.967821 \times 0.963304 \times 0.989575) $$

$$ = 0.08404 $$ [4]

Generally well done.

4 $${}_{5|3}q^{1}_{40:40} = \frac{l_{45}}{l_{40}} \times \frac{l_{45}}{l_{40}} \times \frac{1}{2}\,{}_{3}q_{45:45}$$

$$= 0.5 \times \left(\frac{l_{45}}{l_{40}}\right)^2 \times (1 - {}_{3}p_{45:45})$$

$$= 0.5 \times \left(\frac{l_{45}}{l_{40}}\right)^2 \times \left(1 - \frac{l_{48}}{l_{45}}\frac{l_{48}}{l_{45}}\right)$$

$$= 0.5 \times \left(\frac{9801.3123}{9856.2863}\right)^2 \times \left(1 - \frac{9753.4714}{9801.3123} \times \frac{9753.4714}{9801.3123}\right)$$

$$= 0.00482$$ [4]

This question caused many students problems. The main issue missed was the relationship between the first of 2 equal ages to die and the joint mortality function.

5 (i) $$F = \frac{\sum_x {}^{s}E^{c}_{x,t}\ {}^{s}m_{x,t}}{\sum_x {}^{s}E^{c}_{x,t}} \Bigg/ \frac{\sum_x E^{c}_{x,t}\ {}^{s}m_{x,t}}{\sum_x E^{c}_{x,t}}$$

F is called the area comparability factor and is a measure of the crude mortality rate for the standard population divided by what the crude mortality rate is for the region being studied, assuming the mortality rates are the same as for the standard population. [2]

(ii) If its age/sex profile is such that if it experienced the same age/sex specific mortality rates as the country, then its crude death rate would be 2/3 of that of the country, i.e. the region has either a younger age structure or a higher female proportion (or both) than the country. [2]

[Total 4]

The first part was straight bookwork. Part (ii) was generally poorly explained and the two-thirds relationship was not appreciated.

6 Past Service

$$30000\times\frac{10}{80}\times\frac{{}^{z}M_{30}^{ia}}{{}^{s}D_{30}}=30000\times\frac{10}{80}\times\frac{64061}{41558}=£5781$$

For future service note that maximum ill-health pension will accrue by age 60

$$30000\times\frac{1}{80}\times\frac{{}^{z}\overline{R}_{30}^{ia}-{}^{z}\overline{R}_{60}^{ia}}{{}^{s}D_{30}}=30000\times\frac{1}{80}\times\frac{1502811-35570}{41558}=£13240$$ [4]

Well prepared students did this question well. Many others did not allow for the age limitation properly just setting out the standard formula which was not credited.

7 EPV $=10000\left(A_{[40]:\overline{20|}}+v^{20}\frac{l_{60}}{l_{[40]}}\right)$

$$=10000\left(0.46423+\left(0.45639\times\frac{9287.2164}{9854.3036}\right)\right)$$

$$=8943.6$$

For the variance we need the second moment which can be found as:

$$(10000)^2\left({}^{2}A_{[40]}-(v^{20})^2\frac{l_{60}}{l_{[40]}}\,{}^{2}A_{60}\right)+(20000)^2(v^{20})^2\frac{l_{60}}{l_{[40]}}$$

$$=\ (10000)^2\left(0.06775-0.20829\times\frac{9287.2164}{9854.3036}\times0.23723+4\times0.20829\times\frac{9287.2164}{9854.3036}\right)$$

$$=\ (10000)^2(0.06775-0.04657+0.78521)$$

$$=\ (10000)^2\times0.80639$$

Hence Variance is:

$$(10000)^2\times0.80639-(10000)^2\times(0.89436)^2$$

$$=\ (10000)^2\times0.00651$$

$$=\ (807)^2$$ [6]

The mean was generally easily calculated but many students struggled with the variance not coping properly with the double payment on survival.

8 (i) T_x is the total future lifetime of an ultimate life aged x [2]

K_x is the curtate future lifetime of an ultimate life aged x

(ii) (a) v^{T_x}

(b) $a_{\overline{K_x}|}$

(c) $v^{\min[K_{[x]}+1,n]}$

(d) $v^5\ddot{a}_{\overline{K_x-4}|}$ if $K_x \geq 5$

0 otherwise

[5]

[Total 7]

Very straightforward quick question which well prepared students did well. Main omission was inaccuracies in (ii)(d).

9 The annual premium is found from

$$P\ddot{a}_{60:\overline{20}|} = P(IA)^1_{60:\overline{5}|} + 100,000 \times v^5 \times \frac{l_{65}}{l_{60}} \times A^1_{65:\overline{15}|}$$

$$\ddot{a}_{60:\overline{20}|} = \ddot{a}_{60} - v^{20} \times \frac{l_{80}}{l_{60}} \times \ddot{a}_{80} = 14.134 - 0.45639 \times \frac{5266.4604}{9287.2164} \times 6.818 = 12.369$$

$$(IA)^1_{60:\overline{5}|} = (IA)_{60} - v^5 \times \frac{l_{65}}{l_{60}} \times ((IA)_{65} + 5A_{65}) = 8.36234 - 0.82193 \times \frac{8821.2612}{9287.2164} \times (7.89442 + 5 \times 0.52786)$$

$$= 0.13874$$

$$A^1_{65:\overline{15}|} = A_{65} - v^{15} \times \frac{l_{80}}{l_{65}} \times A_{80} = 0.52786 - 0.55526 \times \frac{5266.4604}{8821.2612} \times 0.73775 = 0.28330$$

Hence:

$$12.369P = 0.13874P + 100000 \times 0.82193 \times \frac{8821.2612}{9287.2164} \times 0.28330$$

$$12.230P = 22117.02$$

$P = £1808$ to nearer £ [7]

Generally done well by well prepared students. Main error related to the treatment of the return of premiums in the first 5 years.

10 (i) The accumulated net cash flow at end of t^{th} policy year per policy in force at the start of that year is given by:

$$(CF)_t = (P - E_t)\times(1+i_t) - (aq)^d_{x+t-1}\times D_t - (aq)^w_{x+t-1}\times B_t$$
$$-\left(1-(aq)^d_{x+t-1}-(aq)^w_{x+t-1}\right)\times S_t$$

[2]

(ii) We need to set the expected present value of the profit signature of the policy equal to zero using a risk discount rate of j% per annum. Hence, if

$$(ap)_{x+k} = \left(1-(aq)^d_{x+k}-(aq)^w_{x+k}\right) \Rightarrow {}_t(ap)_x = \prod_{k=0}^{t-1}(ap)_{x+k}$$

then the level annual premium P is derived from the following equation:

$$\sum_{t=1}^{n} (CF)_t \times {}_{t-1}(ap)_x \times v^t_{j\%} = 0$$ [3]

(iii) Expected profit at the end of the t^{th} policy year for each policy in force at the start of that year

$$= {}_{t-1}V\times(1+i_t)+(CF)_t - (ap)_{x+t-1}\times {}_tV$$ [2]

[Total 7]

Generally done well although students struggled to explain part (ii). Credit was given for reasonable alternative explanations.

11 (i) The reserve for the death claim at 31 December 2013 was

$${}_{14}V = 15,000\left(1-\frac{\ddot{a}_{70}}{\ddot{a}_{56}}\right) = 15,000\left(1-\frac{10.375}{15.537}\right) = 4,983.59$$

Total death strain at risk (*DSAR*) at 31 December 2013:

$$DSAR = 740,000-(371,000+4,983.59) = 364,016.41$$

Expected death strain (*EDS*) =
$q_{69}\times DSAR = 0.022226\times 364,016.41 = 8,090.63$

Actual death strain (*ADS*) = $(15,000-4,983.59) = 10,016.41$
Mortality profit = *EDS* – *ADS* = 8,090.63 – 10,016.41 = –1,925.78 i.e. a loss

[5]

(ii) Expected claims = $q_{69} \times 740,000 = 16,447.24$ [1]

(iii) Actual claims = 15,000

Actual claims were lower than expected although the company made a mortality loss. This was due to the DSAR (expressed as a % of the sum assured) on the one death claim policy being significantly higher than for the group of policies on average. [2]

[Total 8]

This question was not done well overall. Many students failed to understand how to derive the reserve using premium conversion techniques and basically ignored it. This led to totally the wrong conclusions.

12 (i) The probability is:

$$ {}_{25}p_{30} - {}_{35}p_{30} = \frac{l_{55}}{l_{30}} - \frac{l_{65}}{l_{30}} $$

$$ = \frac{91217 - 79293}{97645} = 0.12212 $$ [2]

(ii)

$$ {}_{25}p_{30} = \exp\left(-\int_0^{25} \mu_{30+t}\,dt\right) $$

$$ = \exp\left(-\int_0^{25} 0.005e^{0.09(30+t-20)}\,dt\right) $$

$$ = \exp\left(-0.005 \times e^{0.9} \int_0^{25} e^{0.09t}\,dt\right) $$

$$ = \exp\left(-0.005 \times e^{0.9} \times \left[\frac{e^{0.09t}}{.09}\right]_0^{25}\right) $$

$$ = \exp\left(-0.005 \times e^{0.9} \times \left[\frac{e^{2.25} - 1}{0.09}\right]\right) $$

$$ = \exp(-1.159803) $$

$$ = 0.313548 $$

Similarly:

$$
\begin{aligned}
{}_{35}p_{30} &= \exp\left(-\int_0^{35} \mu_{30+t}\,dt\right) \\
&= \exp\left(-\int_0^{35} 0.005e^{0.09(30+t-20)}\,dt\right) \\
&= \exp\left(-0.005\times e^{0.9}\int_0^{35} e^{0.09t}\,dt\right) \\
&= \exp\left(-0.005\times e^{0.9}\times\left[\frac{e^{0.09t}}{.09}\right]_0^{35}\right) \\
&= \exp\left(-0.005\times e^{0.9}\times\left[\frac{e^{3.15}-1}{0.09}\right]\right) \\
&= \exp(-3.052103) \\
&= 0.047259
\end{aligned}
$$

Hence required probability

$${}_{25}p_{30} - {}_{35}p_{30} = 0.313548 - 0.047259 = 0.266289$$ [7]

[Total 9]

Part (i) was straightforward and well done. Part (ii)was generally poorly done although in essence it was a simple subtraction of 2 similar integrals.

13 (i) Multiple decrement table

	q_x^d	q_x^w	q_x^i	$(aq)_x^d$	$(aq)_x^w$	$(aq)_x^i$	$(ap)_x$	${}_{t-1}(ap)_x$
55	0.004916	0.100	0.040	0.00458	0.09776	0.03791	0.85975	1.00000
56	0.005528	0.080	0.050	0.00518	0.07779	0.04787	0.86917	0.85975
57	0.006215	0.060	0.060	0.00585	0.05802	0.05802	0.87811	0.74727

[3]

(ii) Cash flows for the policy

Let P be the level annual premium for the policy, then

Yr	*Prm*	*Exp*	*Interest*	*Death claim*	*Surrender claim*	*Ill-health claim*	*Mat claim*	*Profit vector*
1	P	150.00	0.05P–7.50	916.00	0.09972P	3791.00	0.00	0.95028P–4864.50
2	P	25.00	0.05P–1.25	1036.00	0.16028P	4787.00	0.00	0.88972P–5849.25
3	P	25.00	0.05P–1.25	1170.00	0.18112P	5802.00	8781.10	0.86888P–15779.25

Yr	*Profit vector*	$_{t-1}(ap_x)$	*Profit signature*	*Discount factor*	*PVFNP*
1	0.95028P–4864.50	1.00000	0.95028P–4864.50	0.952381	0.90503P–4632.86
2	0.88972P–5849.25	0.85975	0.76494P–5028.89	0.907029	0.69382P–4561.35
3	0.86888P–15779.25	0.74727	0.64929P–11791.36	0.863838	0.56088P–10185.82

Total PVFNP = 2.15973P – 19380.03 = 0.05P

$$\Rightarrow P = \frac{19380.03}{2.10973} = 9186.02$$ [9]

(iii) The cash flows show that for this policy, the expected profit vector is positive for policy years 1 and 2 but negative (significantly) for the last policy year (which is expected due to the survival amount being paid at the end of the term of the policy). Unless the company builds up reserves over the period of the policy, it may not have sufficient funds available to pay claims in policy year 3. Therefore, it would be prudent for the company to hold reserves at the beginning and end of each policy year. Indeed, regulations may force the company to do so. [2]

(iv) As the discount rate and the interest rate earned on cash flows items (including reserves) is the same at 5% per annum, holding reserves will not change the premium required for this policy. [2]

[Total 16]

This question was generally well done by students who had prepared well. Other approaches were credited especially where a non tabular approach was adopted.

Credit was also given if the student could demonstrate how the problem might be approached without getting all the arithmetic entirely accurate.

14 (i) Multiple decrement table:

x	q_x^d	q_x^s
61	0.009009	0.075
62	0.010112	0.025
63	0.011344	0.000

x	$(aq)_x^d$	$(aq)_x^s$	(ap)	${}_{t-1}(ap)$
61	0.008108	0.07439	0.917500	1.000000
62	0.009101	0.02477	0.966127	0.917500
63	0.010210	0.00000	0.989790	0.886421

Unit fund (per policy at start of year)

	yr 1	*yr 2*	*yr 3*
value of units at start of year	0.00	729.36	3052.03
allocation	750.00	2362.50	3450.00
B/O spread	45.00	141.75	207.00
interest	31.73	132.75	283.28
management charge	7.37	30.83	65.78
value of units at year end	729.36	3052.03	6512.53

Cash flows (per policy at start of year)

	yr 1	*yr 2*	*yr 3*
unallocated premium	750.00	–112.50	–450.00
B/O spread	45.00	141.75	207.00
expenses	275.00	111.25	130.00
interest	13.00	–2.05	–9.32
man charge	7.37	30.83	65.78
extra death benefit	48.82	33.65	2.42
claim expense	6.19	2.54	0.77
profit vector	485.36	–89.41	–319.73
probability in force	1	0.917500	0.886421
profit signature	485.36	–82.03	–283.42
discount factor	0.938967	0.881659	0.827849
PVFNP	455.74	–72.33	–234.63

Total PVFNP = 148.78

	yr 1	*yr 2*	*yr 3*
premium signature	1500.000	1938.38	2344.57

Total PV of premiums = 7197.448

Total PV of premiums = 5782.95

Profit margin = $\dfrac{148.78}{5782.95} = 2.57\%$ [13]

(ii) Reserves might be required to eliminate/zeroise expected negative cash flows in the future so that the company does not expect to have to input further capital in the future. [2]

(iii) The profit vector for the policy is (485.36, –89.41, –319.73)

In order to set up reserves to zeroise future expected negative cash flows, we require:

$$_2V = \frac{319.73}{1.025} = 311.93$$

$$_1V \times 1.025 - (ap)_{62} \times {}_2V = 89.41 \Rightarrow {}_1V = 381.24$$

revised cash flow in year 1 $= 485.36 - (ap)_{61} \times {}_1V = 135.57$

and PVFNP = 135.57/1.065 = 127.30

$\Rightarrow$Profit margin = $\dfrac{127.30}{5782.95} = 2.20\%$ [4]

[Total 19]

Again well prepared students scored good marks on this question and credit was given if a good understanding of the process was demonstrated even if the result was not entirely arithmetically accurate.

The main difficulty here was the interpretation of zeroising the result in part (iii).

END OF EXAMINERS' REPORT