INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2012 examinations

Subject CT5 – Contingencies Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

D C Bowie
Chairman of the Board of Examiners

December 2012

General comments on Subject CT5

CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions actuarial work.

Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given to alternative valid points which do not appear in the solutions below.

In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

Comments on the September 2012 paper

The general performance was lower than average this session. Questions that were done less well were 6, 11, 12, and 15(ii) and (iii); more commentary on these questions is given in this report to assist candidates with further revision.

However most of the short questions were very straightforward and this is where many successful candidates scored particularly well. Students should note that for long questions a reasonable level of credit is given if they can describe the right procedures; however, to score well reasonable, accurate numerical calculation is necessary.

1 (a) $$ {}_{12}p_{43} = \frac{l_{55}}{l_{43}} = \frac{9557.8179}{9826.2060} = 0.97269 $$

(b) $$ {}_{10|5}q_{55} = \frac{l_{65} - l_{70}}{l_{55}} = \frac{8821.2612 - 8054.0544}{9557.8179} = 0.08027 $$

(c) $$ \ddot{a}_{45:\overline{10|}} = \ddot{a}_{45} - \left(v^{10} \times \left(\frac{l_{55}}{l_{45}} \right) \times \ddot{a}_{55} \right) \text{ at } 6\% $$

$$ = 14.850 - \left(0.55839 \times \frac{9557.8179}{9801.3123} \times 13.057 \right) $$

$$ = 7.740 $$

Generally well done

2

- Class selection – e.g. males and females have different mortality/longevity characteristics.
- Time Selection – mortality rates vary over time (in annuities generally improves).
- Adverse Selection – lives in better health than average may be more likely to purchase an annuity

Generally well done-other plausible types and examples were credited.

3 Nutrition has an important influence on morbidity and in the longer term on mortality.

Poor quality nutrition can increase the risk of contracting many diseases and hinder recovery from sickness.

Excessive or inappropriate (e.g. too much fat) eating can lead to obesity and an increased risk of associated diseases (e.g. heart disease, hypertension) leading to increased morbidity and mortality.

Inappropriate nutrition may be the result of economic factors e.g. lack of income to buy appropriate foods or the result of a lack of health and personal education resulting in poor nutritional choices.

Also, social and cultural factors encourage or discourage the eating of certain foods e.g. alcohol consumption.

Many candidates gave a reasonable answer but there was a tendency to overlook the obesity risk in the second paragraph.

4 $_3p_{55.75} = {}_{0.25}p_{55.75} \times p_{56} \times p_{57} \times {}_{0.75}\,p_{58}$

$$= (1 - {}_{0.25}\,q_{55.75}) \times (1 - q_{56}) \times (1 - q_{57}) \times (1 - {}_{0.75}\,q_{58})$$

$$= \left(1 - \left(\frac{0.25 \times 0.00475}{1 - 0.75 \times 0.00475}\right)\right) \times 0.99469 \times 0.99408 \times (1 - 0.75 \times 0.00660)$$

$$= 0.99881 \times 0.99469 \times 0.99408 \times 0.99505$$

$$= 0.98274$$

Generally well done.

5 (a) Definitions of three terms are:

$E^c_{x,t}$ Central exposed to risk in population being studied between ages x and $x + t$.

$m_{x,t}$ Central rate of mortality either observed or from a life table in population being studied for ages x to $x + t$.

${}^sE^c_{x,t}$ Central exposed to risk for a standard population between x and $x + t$.

${}^sm_{x,t}$ Central rate of mortality either observed or from a life table in standard population for ages x to $x + t$.

(b) The area comparability factor (F) is the ratio of the mortality rates in the standard population weighted by the age structure distribution of the standard population to the mortality rates in the standard population weighted by the age structure distribution of the observed population.

F is therefore is a measure of variation between population age structures.

Many candidates gave formulae that were not required. Also many did not give a complete answer.

6 The net future loss random variable is given by:

$$S(1+b)^{K_{40}+1} v^{T_{40}} - P\ddot{a}_{\overline{\min(K_{40}+1,45)}|}$$

b = annual rate of future bonus
P = annual net premium
K_{40} = curtate future lifetime of a life aged 40 exact
T_{40} = complete future lifetime of a life aged 40 exact

Generally not done well. It is often the case that candidates have difficulties in setting out the random variable expressions.

7 Reserve at the end of the 5th policy year is given by:

$$ {}_5V = 125{,}000v^{25}\,{}_{25}p_{40} - P\ddot{a}_{40:\overline{25}|} = 125{,}000 \times 0.37512 \times \frac{8821.2612}{9856.2863} - P \times 15.884 $$

where annual net premium for the policy is given by

$$ P = \frac{125{,}000v^{30}\,{}_{30}p_{[35]}}{\ddot{a}_{[35]:\overline{30}|}} = \frac{125{,}000 \times 0.30832 \times \dfrac{8821.2612}{9892.9151}}{17.631} = 1949.13 $$

$$ \Rightarrow {}_5V = 41{,}966.00 - 30{,}959.98 = 11{,}006.02 $$

$DSAR = 0 - {}_5V = -11{,}006.02$

$EDS = 3521q_{39} \times DSAR = 3521 \times .00087 \times DSAR = -33{,}714.41$

$ADS = -8 \times 11{,}006.02 = -88{,}048.16$

Profit = $EDS - ADS$=54,333.75

Generally done fine by well-prepared candidates

8

- Mortality just after birth ("infant mortality") is very high.
- Mortality then falls dramatically during the first few years of life and is at lowest around ages 8–10.
- There is a distinct "hump" in the deaths at ages around 18–25. This is often attributed to a rise in accidental deaths during young adulthood, and is called the "accident hump".
- From middle age onwards there is a steep increase in mortality, reaching a peak at about age 80.
- The number of deaths at higher ages falls again (even though the mortality rate q_x continues to increase) since the probabilities of surviving to these ages are small.

Generally well done but many candidates did not score all available marks.

9 (i) (a) The gross premium prospective policy reserve is the expected present value of future benefits (including declared bonus and an allowance for future bonus if applicable) and future expenses less the expected present value of future gross premiums.

(b) The gross premium retrospective policy reserve is the expected accumulation of past gross premiums received, less expected expenses and benefits including bonuses included in past claims.

(ii) Gross premium retrospective and prospective reserves will be equal if:

- the mortality and interest rate basis used is the same as used to determine the gross premium at the date of issue of the policy; and
- the expenses valued are the same as those used to determine the original gross premium; and
- the gross premium is that determined on the original basis (mortality, interest, expenses) using the equivalence principle

Generally done well but many answers were incomplete on a standard bookwork question.

10 Value of benefit:

$$\frac{r_{65}}{l_{35}} v^{(65-35)} 20{,}000\left(\frac{1}{12}\right)(65-35)\frac{\frac{s_{63}+s_{64}}{2}}{s_{35}}$$

$$= \frac{3757}{18866} v^{(65-35)} 20{,}000\left(\frac{1}{12}\right)(65-35)\frac{\frac{11.151+11.328}{2}}{6.655}$$

$= 5185$

Assume value of contributions is *K%* of salary

Value of contributions of *K%* of salary

$$20{,}000.K\%.\frac{{}^{s}\bar{N}_{35}}{{}^{s}D_{35}} = 20{,}000.K\%.\frac{502{,}836}{31{,}816} = 316{,}090.K\%$$

Therefore $K = 1.64$

This question was generally poorly answered despite being a relatively straightforward question. The main issue was in understanding how the benefit value arose.

11 Because the annuity is payable weekly this can reasonably be represented by continuous annuity functions.

Working initially for a unit annualised payment:

$$\text{PV} = \bar{a}_{\overline{5}|} + v^5 \frac{l^m_{70}}{l^m_{65}} \times \frac{l^f_{67}}{l^f_{62}} \times \left(\frac{2}{3} \times \bar{a}_{\overline{70(m):67(f)}} + \frac{1}{3} \times \bar{a}_{70(m):67(f)} \right)$$

$$+v^5 \frac{l^m_{70}}{l^m_{65}} \times \left(1 - \frac{l^f_{67}}{l^f_{62}} \right) \times \left(\frac{2}{3} \bar{a}_{70(m)} \right) + v^5 \left(1 - \frac{l^m_{70}}{l^m_{65}} \right) \times \frac{l^f_{67}}{l^f_{62}} \times \left(\frac{2}{3} \bar{a}_{67(f)} \right) \text{ at 4\%}$$

$$\bar{a}_{\overline{5}|} = (i/\delta) \times a_{\overline{5}|} = 1.019869 \times 4.4518 = 4.5403$$

$$\frac{l^m_{70}}{l^m_{65}} = \frac{9238.134}{9647.797} = 0.95754, \quad \frac{l^f_{67}}{l^f_{62}} = \frac{9605.483}{9804.173} = 0.97973$$

$$\bar{a}_{70(m)} = \ddot{a}_{70(m)} - 0.5 = 11.062, \ \bar{a}_{67(f)} = \ddot{a}_{67(f)} - 0.5 = 13.611,$$

$$\bar{a}_{70(m):67(f)} = \ddot{a}_{70(m):67(f)} - 0.5 = 9.733$$

$$\bar{a}_{\overline{70(m):67(f)}} = 11.062 + 13.611 - 9.733 = 14.940$$

$$PV = 4.5403 + \left(0.82193 \times 0.95754 \times 0.97973 \times \left(\frac{2}{3} \times 14.940 + \frac{1}{3} \times 9.733 \right) \right)$$

$$+0.82193 \times 0.95754 \times 0.02027 \times \frac{2}{3} \times 11.062$$

$$+0.82193 \times 0.04246 \times 0.97973 \times \frac{2}{3} \times 13.611$$

$$= 4.5403 + 10.1816 + 0.1176 + 0.3103$$

$$= 15.1498$$

The annualised benefit is 500 × 52.18 = 26090 p.a. (NB 52 acceptable)

So PV = 26090 ×15.1498 = £395,258

The key to this question is to break down carefully the component parts of the annuity. Once this is done the question is then a relatively simple calculation of annuity functions. The question was generally done poorly and many candidates failed to realise that a weekly annuity could be closely approximated by a continuous one.

12 The value of the death benefit is:

$$100000\int_0^{25} e^{-.05t}\{e^{-.02t}(1-e^{-.03t})\times .02+e^{-.03t}(1-e^{-.02t})\times .03\}dt$$

$$=100000\int_0^{25}(.02e^{-.07t}+.03e^{-.08t}-.05e^{-.1t})dt$$

$$=100000\left\{\left[-\frac{0.02e^{-.07t}}{.07}\right]_0^{25}+\left[-\frac{0.03e^{-.08t}}{.08}\right]_0^{25}-\left[-\frac{.05e^{-.1t}}{.1}\right]_0^{25}\right\}$$

$$=100000\left\{\left(\frac{2}{7}+\frac{3}{8}-\frac{1}{2}\right)-\left(\frac{2e^{-1.75}}{7}+\frac{3e^{-2}}{8}-\frac{e^{-2.5}}{2}\right)\right\}$$

$$=100000\{(0.28571+0.375-0.5)-(.04965+.05075-.04104)\}$$

$$=10135$$

The value of the survival benefits are:

$$e^{-1.25}\times(100000e^{-.5}\times e^{-.75}+50000e^{-.5}(1-e^{-.75})+50000e^{-.75}(1-e^{-.5}))$$

$$=50000e^{-1.75}+50000e^{-2}$$

$$=8688.7+6766.8=15456 \text{ say}$$

The value of annualised premium P is:

$$P\int_0^{25} e^{-.05t}\{e^{-.02t}(1-e^{-.03t})+e^{-.03t}(1-e^{-.02t})+e^{-.05t}\}dt$$

$$=P\int_0^{25}(e^{-.07t}+e^{-.08t}-e^{-.1t})dt$$

$$=P\left\{\left[\frac{-e^{-.07t}}{0.07}\right]_0^{25}+\left[\frac{-e^{-.08t}}{0.08}\right]_0^{25}-\left[\frac{-e^{-.1t}}{0.1}\right]_0^{25}\right\}$$

$$=P\left\{\left(\frac{1}{.07}+\frac{1}{.08}-\frac{1}{.1}\right)-\left(\frac{e^{-1.75}}{.07}+\frac{e^{-2}}{.08}-\frac{e^{-2.5}}{.1}\right)\right\}$$

$$=P\{(14.2857+12.5-10)-(2.4825+1.6917-0.8208)\}$$

$$=13.432P$$

So $P = \dfrac{10135 + 15456}{13.432} = £1905.23$

Many well prepared candidates made a very good attempt at this difficult question but in general terms it was done quite poorly. As in Question 11 the key is to organise the component parts logically.

13 (i) If the monthly premium and sum assured are denoted by P and S respectively then:

$$0.975 \times 12 P \ddot{a}^{(12)}_{[55]:\overline{30|}} + 0.025P$$

$$= (0.98S + 200)\bar{A}_{[55]} + 0.02S(I\bar{A})_{[55]} + 275 + 65(\ddot{a}_{[55]} - 1) + 0.75 \times 12P$$

$$\Rightarrow 0.975 \times 12 P \ddot{a}^{(12)}_{[55]:\overline{30|}} + 0.025P$$

$$= (1.04)^{0.5}\left[(0.98 \times 75{,}000 + 200)A_{[55]} + 0.02 \times 75{,}000(IA)_{[55]}\right] + 275 + 65(\ddot{a}_{[55]} - 1) + 9P$$

where

$$\ddot{a}^{(12)}_{[55]:\overline{30|}} = \ddot{a}^{(12)}_{[55]} - v^{30}\,{}_{30}p_{[55]}\ddot{a}^{(12)}_{85}$$

$$= \left(\ddot{a}_{[55]} - \frac{11}{24}\right) - v^{30}\,{}_{30}p_{[55]}\left(\ddot{a}_{85} - \frac{11}{24}\right)$$

$$= \left(15.891 - \frac{11}{24}\right) - .30832 \times \frac{3385.2479}{9545.9929}\left(5.333 - \frac{11}{24}\right)$$

$$= 15.433 - 0.533 = 14.900$$

$$\Rightarrow (0.975 \times 12 \times 14.9 + 0.025)P$$

$$= (1.04)^{0.5}\left[73{,}700 \times 0.38879 + 1{,}500 \times 8.58908\right] + 275 + 65 \times 14.891 + 9P$$

$$\Rightarrow 174.355P = (1.04)^{0.5}[28{,}653.823 + 12{,}883.62] + 275 + 967.915 + 9P$$

$$\Rightarrow 165.355P = 42{,}360.046 + 275 + 967.915 \Rightarrow P = £263.69$$

(ii) Gross prospective policy value (calculated at 4%) is given by:

$$V^{\text{prospective}} = \left(0.975S + B + 250\right)\bar{A}_{85} + 0.025S(I\bar{A})_{85} + 80\ddot{a}_{85}$$

where $B = 30 \times 0.02 \times 75{,}000 = 45{,}000$

$\Rightarrow$

$$V^{\text{prospective}} = (1.04)^{0.5}\left\{\left(0.975 \times 75{,}000 + 45{,}000 + 250\right)A_{85} + 0.025 \times 75{,}000(IA)_{85}\right\} + 80\ddot{a}_{85}$$

$$= (1.04)^{0.5}(118{,}375 \times 0.7949 + 1{,}875 \times 4.40856) + 80 \times 5.333$$

$$= 104{,}389.51 + 426.64 = £104{,}816.15$$

Generally part (i) was done well. Very few candidates successfully completed part (ii) as is often the case with prospective reserve calculations.

14

We have the following multiple decrement table:

Year t	$(aq)^d_{[30]+t-1}$	$(aq)^s_{30+t-1}$	$(aq)^r_{30+t-1}$	$(ap)_{[x]+t-1}$	${}_{t-1}(ap)_{[30]}$
1	.000447	.098727	.023744	.877082	1.000000
2	.000548	.049361	.024368	.925723	.877082
3	.000602	.024680	.024680	.950038	.811935
4	.000636	0	0	.999364	.771370

Cash flows:

Year t	*Premium P*	*Expenses E*	*Interest on P-E*	*Death Claim*	*Surrender Claim*	*Redundancy Claim*	*Maturity Claim*	*Profit Vector*
1	14000.00	700.00	399.00	27.22	701.38	337.37	0	12633.04
2	14000.00	700.00	399.00	33.37	701.34	692.46	0	12271.82
3	14000.00	700.00	399.00	36.66	526.00	1051.99	0	12084.35
4	14000.00	700.00	399.00	38.73	0	0	59961.84	-46301.57

Note: allowance for ½ year interest roll up is included in death, surrender and redundancy costs

Year t	*Profit Vector*	*Cum probability of survival*	*Profit signature*	*Discount factor*	*NPV of Profit signature*
1	12633.04	1.0	12633.04	.952381	12031.46
2	12271.82	.877082	10763.40	.907029	9762.72
3	12084.35	.811935	9811.71	.863838	8475.72
4	-46301.57	.771370	-35715.60	.822702	-29383.31

=> Total NPV of profit = **886.59**

NPV of premium = 14,000 × (1 + .877082 × .952381 + .811935 × .907029 + .771370 × .863838)
= 45,333.44

Therefore, profit margin = 886.61/45,333.44 = 1.96%

Credit was given for correct data items and many well prepared candidates scored a reasonable proportion of the marks available. Very few got to the final answer however.

15

Annual premium	£3000.00	Allocation % (1st yr)	75.0%
Risk discount rate	6.5%	Allocation % (2nd yr)	100.0%
Interest on investments (1st yr)	5.0%	Allocation % (3rd yr)	105.0%
Interest on investments (2nd yr)	4.5%	B/O spread	5.0%
Interest on investments (3rd yr)	4.0%	Management charge	1.5%
Interest on non-unit funds	3.0%	Policy Fee	£35
Death benefit (% of bid value of units)	150%		

	£	*% premium*
Initial expense	275	20.0%
Renewal expense	80	2.5%
Expense inflation	2.0%	

Mortality table:

X	$q_{[x]+t-1}$	$p_{[x]+t-1}$	${}_{t-1}p_{[x]}$
45	0.001201	0.998799	1.000000
46	0.001557	0.998443	0.998799
47	0.001802	0.998198	0.997244

Unit fund (per policy at start of year)

	yr 1	*yr 2*	*yr 3*
value of units at start of year	0.000	2174.511	5135.828
Alloc	2250.000	3000.000	3150.000
B/O	112.50	150.000	157.500
policy fee	35.000	35.000	35.000
interest	105.125	224.528	323.733
management charge	33.114	78.211	126.256
value of units at year end	2174.511	5135.828	8290.805

Cash flows (per policy at start of year)

	yr 1	*yr 2*	*yr 3*
unallocated premium + pol fee	785.000	35.000	–115.000
b/o spread	112.500	150.000	157.500
expenses	875.000	156.600	158.232
interest	0.675	0.852	–3.472
man charge	33.114	78.211	126.256
extra death benefit	1.306	3.998	7.470
profit vector	54.984	103.464	–0.418

(i) If policyholder dies in the 3rd year of contract, non unit cash flows at end of each year are:

$$yr\ 1 = (785 + 112.5 - 875 + 0.675 + 33.114) = 56.289$$
$$yr\ 2 = (35 + 150 - 156.6 + 0.852 + 78.211) = 107.463$$
$$yr\ 3 = (-115 + 157.5 - 158.232 - 3.472 + 126.256 - 0.5 \times 8290.805) = -4138.351$$

=> expected present value of these cash flows is given by:

$$\left[56.289 \times v + 107.463 \times v^2 - 4138.351 \times v^3\right] \times p_{[45]} \times p_{[45]+1} \times q_{47}$$
$$= [52.854 + 94.746 - 3425.930] \times 0.998799 \times 0.998443 \times 0.001802 = -5.891$$

(ii) (a) If policyholder dies in the 1st year of contract, non unit cash flow at end of 1st year is:

$$yr\ 1 = (785 + 112.5 - 875 + 0.675 + 33.114 - 0.5 \times 2174.511) = -1030.967$$

=> expected present value of this cash flow is given by:

$$-1030.967 \times v \times q_{[45]} = -968.967 \times 0.001201 = -1.163$$

(b) If policyholder dies in the 2nd year of contract, non unit cash flows at end of each year are:

$$yr\ 1 == 56.289 \text{ (derived above)}$$
$$yr\ 2 = (35 + 150 - 156.6 + 0.852 + 78.211 - 0.5 \times 5135.828) = -2460.451$$

=> expected present value of these cash flows is given by:

$$\left[56.289 \times v - 2460.451 \times v^2\right] \times p_{[45]} \times q_{[45]+1}$$
$$= [52.854 - 2169.279] \times 0.998799 \times 0.001557 = -3.291$$

(iii) If policyholder survives until end of contract, non unit cash flows at end of each year are:

$yr\ 1 == 56.289$ (derived above)
$yr\ 2 = 107.463$ (derived above)
$yr\ 3 = (-115 + 157.5 - 158.232 - 3.472 + 126.256) = 7.052$

=> expected present value of these cash flows is given by:

$$\left[56.289 \times v + 107.463 \times v^2 + 7.052 \times v^3\right] \times {}_3p_{[45]}$$
$$= [52.854 + 94.746 + 5.838] \times 0.998799 \times 0.998443 \times 0.998198 = 152.737$$

Expected present value of policy is therefore = −1.163 − 3.291 − 5.891 + 152.737
= **142.39**

Candidates generally found this question difficult particularly parts (ii) and (ii). Part credit was given in (i) for correctly calculating the data items and well prepared candidates scored a fair proportion of the marks here.

END OF EXAMINERS' REPORT