

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

4 October 2011 (am)

Subject CT5 — Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

<p><i>In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.</i></p>
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1 Calculate:

- (a) ${}_{10|}q_{[50]}$
- (b) ${}_{10}P_{[60]+1}$
- (c) $\ddot{a}_{[40]:20}^{(12)}$

Basis:

Mortality AM92

Rate of interest 6% per annum

[3]

2 Calculate ${}_{0.5}q_{75.25}$ using the assumption of a constant force of mortality.

Basis:

Mortality AM92

[3]

3 In a special mortality table with a select period of one year, the following relationships are true for all ages:

$${}_{0.5}q_{[x]} = 0.25q_x$$

$${}_{0.5}q_{[x]+0.5} = 0.45q_x$$

Express $p_{[x]}$ in terms of p_x .

[3]

4 A term assurance contract with a term of 20 years pays a sum assured of 1 immediately on death to a life now aged 30 exact.

Calculate the expected value and variance of this contract.

Basis:

Mortality AM92 Ultimate

Rate of interest 4% per annum

[4]

5 (a) Write down the random variable form of $\bar{A}_{x:y}^1$.

(b) Calculate $\bar{A}_{x:y}^1$ on the following assumptions:

$$\mu_x = 0.02 \text{ for all } x$$

$$\mu_y = 0.03 \text{ for all } y$$

$$\delta = 4\% \text{ per annum}$$

[5]

- 6 Explain why it is necessary to have different mortality tables for different classes of lives. [6]

- 7 A special joint life last survivor annuity of £10,000 per annum is payable continuously in respect of a male and female life each aged 60 exact. Payments commence on the first death and continue for 5 years after the second death.

Calculate the expected present value of this annuity.

Basis:

Mortality PMA92C20 (male life), PFA92C20 (female life)

Rate of interest 4% per annum

Expenses Nil

[6]

- 8 The following data is extracted from a population census:

Age	<i>All Professions</i>		<i>Profession A</i>	
	<i>Population</i>	<i>Deaths</i>	<i>Population</i>	<i>Deaths</i>
20–29	120,000	256	12,500	30
30–39	178,000	458	15,000	40
40–49	156,000	502	16,000	50
50–64	123,000	600	14,000	60

- (a) Calculate the area comparability factor for Profession A using the data for All Professions as the standard population.
- (b) Hence or otherwise derive the standardised mortality ratio and the indirectly standardised mortality rate.

[6]

- 9 Members of a pension scheme are subject to three decrements:

- (a) Deaths - with independent decrement rates that are assumed to follow ELT15(Males)
- (b) Ill-health retirement - with an independent decrement rate of 0.01 at age 50 exact increasing by 0.005 for each additional year of age (so the ill-health independent decrement at age 53 exact is 0.025)
- (c) Age retirement - with an independent decrement rate of 0.2 at each age from 60 to 64 all exact.

Age retirements are assumed to take place on the attainment of the exact age, whilst other decrements act uniformly across the year of age.

Calculate the probability that a member currently aged 59 exact will retire at age 62 exact.

[6]

- 10** (i) Five years ago a with profits whole life assurance policy was sold to a life then aged 30 exact.

The sum assured is £150,000 payable at the end of year of death and premiums are payable annually in advance throughout life. The super compound method of adding bonuses to the policy is used as follows:

- each year there is a simple bonus of 2.5% on the sum assured
- and an additional bonus of 5% on all existing bonuses (excluding the simple bonus relating to that policy year)

Assume that bonuses vest at the start of each policy year and that the actual past bonus additions have followed the assumptions stated above.

Calculate the net premium policy value just before payment of the 6th premium.

Basis:

Mortality	AM92 Select	
Rate of interest	4% per annum	[5]

- (b) Suggest two reasons why a life insurance company might use the super compound method of adding bonuses to with profits policies, as opposed to the compound method. [2]
- [Total 7]

- 11** A pension scheme provides a pension on retirement of 1% of final pensionable salary for each completed year of pensionable service. On retirement due to ill-health, pensionable service is calculated as service that would have been completed by the normal retirement age of 65. Final pensionable salary is defined as the average salary in the last three years before retirement.

Derive an expression, without using commutation functions, for the present value of the benefits for a new member age 30 with salary of £20,000 in the year after entry to the scheme. Define all symbols used. [8]

- 12** (i) List the main features of a unit-linked policy. [4]

A four-year unit-linked policy issued by a life insurance company to a life aged 56 exact has the following profit vector:

(1525.89, −334.08, −292.05, −933.82)

- (ii) Determine the net present value of the profits of this policy, assuming that the company sets up reserves in order to zeroise future negative expected cash flows on the policy.

Basis:

Mortality	AM92 Ultimate	
Rate of interest on non-unit fund cash flows	4.5% per annum	
Risk discount rate	7.5% per annum	[5]
		[Total 9]

- 13** A life insurance company issues a 3-year without profits endowment assurance policy to a male life aged 57 exact for a sum assured of £15,000 payable on maturity or at the end of the year of death if earlier. Premiums of £4,700 are payable annually in advance throughout the term of the policy.

The office holds net premium reserves for these policies, calculated using AM92 Ultimate mortality and interest of 4% per annum.

Surrenders occur only at the end of a year immediately before a premium is paid. The surrender value payable is 75% of total premiums paid on the contract at the time the surrender value is payable. Assume that at the end of the first and second policy years, 10% and 5% respectively of all policies still in force at that time then surrender.

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of interest on cash flows and Reserves	5% per annum
Mortality	AM92 Select
Initial expenses	10% of the annual premium
Renewal expenses	£65 per annum on the second and subsequent premium dates
Risk discount rate	7% per annum

- (i) Calculate the net present value of profits for this contract. [10]
- (ii) Calculate the internal rate of return for this contract. [2]

The company weakens the reserving basis by assuming that net premium reserves for these policies are now calculated using AM92 Ultimate mortality and interest of 6% per annum.

- (iii) Calculate the revised net present value of profits and comment on your answer. [4]
- [Total 16]

- 14** On 1 January 2001, a life insurance company issued a number of 30-year endowment assurance policies that pay £100,000 at maturity, or £50,000 at the end of the year of earlier death to lives then aged 35 exact. Premiums are payable annually in advance.

The company uses the following basis for calculating premiums and reserves:

Mortality	AM92 Select
Interest	4% per annum
Initial commission	50% of the premium payable in the first policy year
Initial expenses	£300 paid at policy commencement date
Renewal expenses	2.5% of each premium from the start of the second policy year

- (i) Write down the recursive relationship between the gross premium reserves at successive durations of these policies, defining all symbols used. [4]
- (ii) Show that the annual premium for each policy is approximately £1,803. [4]

There were 385 policies in force on 1 January 2010. During 2010, there were three actual deaths, actual interest earned by the company was 5% and expenses were as expected.

- (iii) Calculate the profit or loss made by the company from both mortality and interest in respect of these policies for the year 2010 based on the formula stated in (i) above. [10]
- [Total 18]

END OF PAPER