INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2011 examinations

Subject CT5 — Contingencies
Core Technical

Purpose of Examiners' Reports

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and who are using past papers as a revision aid, and also those who have previously failed the subject. The Examiners are charged by Council with examining the published syllabus. Although Examiners have access to the Core Reading, which is designed to interpret the syllabus, the Examiners are not required to examine the content of Core Reading. Notwithstanding that, the questions set, and the following comments, will generally be based on Core Reading.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report. Other valid approaches are always given appropriate credit; where there is a commonly used alternative approach, this is also noted in the report. For essay-style questions, and particularly the open-ended questions in the later subjects, this report contains all the points for which the Examiners awarded marks. This is much more than a model solution – it would be impossible to write down all the points in the report in the time allowed for the question.

T J Birse
Chairman of the Board of Examiners

December 2011

General comments on Subject CT5

CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions actuarial work.

Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given where appropriate to different valid points made which do not appear in the solutions below.

In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

Comments on the September 2011 paper

The general performance was slightly worse than in April 2011 but well-prepared candidates scored well across the whole paper. Questions that were done less well were 7, 9, 10, 11 and 14(iii) and here more commentary is given to students to assist with further revision.

Most of the short questions were very straightforward where an answer could be produced quickly and this is where many successful candidates scored particularly well. Students should note that for long questions a reasonable level of credit is given if they can describe the right procedures although to score well reasonable accurate numerical calculation is necessary.

1 (a) $_{10|1}q_{[50]} = d_{60}/l_{[50]} = 74.5020/9{,}706.0977 = 0.00768$

(b) $_{10}p_{[60]+1} = l_{71}/l_{[60]+1} = 7{,}854.4508/9{,}209.6568 = 0.85285$

(c) $\ddot{a}^{(12)}_{[40];\overline{20}|} = (\ddot{a}_{[40]:\overline{20}|} - 11/24 \times (1 - \dfrac{v^{20} l_{60}}{l_{[40]}}))$

$= 12.000 - 11/24 \times (1 - 0.3118 \times 9{,}287.2164/9{,}854.3036)$

$= 11.676$

Straightforward question generally done well.

2 We have:

$_1p_{75} = 6{,}589.9258/6{,}879.1673 = 0.95795$

$= e^{-\mu}$ where μ is the constant force

Hence $\mu = -\ln(0.95795) = 0.04296$

Hence $_{0.5}p_{75.25} = e^{-\int_{75.25}^{75.75} 0.04296\,dt}$

$= e^{-0.02148} = 0.97875$

Hence $_{0.5}q_{75.25} = 1 - {_{0.5}p_{75.25}} = 0.02125$

Again done well. Credit was given to those students who jumped straight to the solution ***of*** $(1-({_1p_{75}})^{0.5})$.

3 $p_{[x]} = {_{0.5}p_{[x]}} * {_{0.5}p_{[x]+0.5}} = (1 - {_{0.5}q_{[x]}}) * (1 - {_{0.5}q_{[x]+0.5}})$

$= (1 - 0.25q_x) * (1 - 0.45q_x) = (1 - 0.25(1 - p_x)) * (1 - 0.45(1 - p_x))$

$= (0.75 + .25p_x) * (.55 + .45p_x) = 0.4125 + 0.475p_x + 0.1125p_x^2$

This question was done reasonably well but many students failed to make the connection in line 1.

4 The expected value is $\bar{A}^{1}_{30:\overline{20}|}$

This equals

$$(A_{30} - (v^{20} \times (l_{50} / l_{30}) \times A_{50})) \times (1.04)^{1/2}$$
$$= (0.16023 - (0.45639 \times (9,712.0728 / 9,925.2094) \times 0.32907)) \times 1.019804$$
$$= 0.01353$$

The variance equals

$${}^{2}\bar{A}^{1}_{30:\overline{20}|} - (\bar{A}^{1}_{30:\overline{20}|})^{2}$$

$${}^{2}\bar{A}^{1}_{30:\overline{20}|} = (({}^{2}A_{30} - (v^{20})^{2} \times (l_{50} / l_{30}) \times {}^{2}A_{50})) \times (1.04)$$
$$= (0.03528 - (0.20829 \times (9,712.0728 / 9,925.2094) \times 0.13065)) \times 1.04$$
$$= 0.008997$$

$$\text{Variance } = 0.008997 - 0.01353^{2} = 0.008814$$

This question was done reasonably well. The most common error was to forget to use continuous functions which was penalised as one of the key attributes being tested was to see if students could work out the 1.04 factor for the variance.

5 (a)

$$\bar{Z} = \begin{cases} v_i^{T_x} & \text{if} \quad T_x \le T_y \\ 0 & \text{if} \quad T_x > T_y \end{cases}$$

where i is the valuation rate of interest.

(b)

$$\bar{A}^{\;1}_{x:y} = \int_0^\infty e^{-.04t} \cdot {}_tp_{xy} \cdot \mu_{x+t}\, dt$$

$$= \int_0^\infty e^{-.04t} \cdot e^{-.02t} \cdot e^{-.03t} \cdot (0.02) dt = 0.02 \int_0^\infty e^{-.09t} dt$$

$$= 0.02 / 0.09$$

$$= 0.22222$$

In part(a)many students did not appreciate what a random variable form was. Part (b) was generally well done.

Part (a) comes directly from Core Reading but there is some debate about the situation where $T_x = T_y$ *i.e .a simultaneous death where it could be argued either that* $\bar{Z}$ *=0 or is undefined. The examiners decided to accept all these alternative situations.*

6

- When a life table is constructed it is assumed to reflect the mortality experience of a homogeneous group of lives i.e. all the lives to whom the table applies follow the same stochastic model of mortality represented by the rates in the table. This means that the table can be used to model the mortality experience of a homogeneous group of lives which is suspected to have a similar experience.

- If a life table is constructed for a heterogeneous group then the mortality experience will depend on the exact mixture of lives with different experiences that has been used to construct the table. Such a table could only be used to model mortality in a group with the same mixture. It would have very restricted uses.

- For this reason separate mortality tables are usually constructed for groups which are expected to be heterogeneous. This can manifest itself as class selection e.g. separate tables for males and females, whole life and term assurance policyholders, annuitants and pensioners, or as time selection e.g. separate tables for males in England and Wales in 1980–82 (ELT14) and 1990–92 (ELT15).

- Sometimes only parts of the mortality experience are heterogeneous e.g. the experience during the initial select period for life assurance policyholders, and the remainder are homogeneous e.g. the experience after the end of the select period for life assurance policyholders. In such cases the tables are separate (different) during the select period, but combined after the end of the select period. In fact there are separate (homogeneous) mortality tables for each age at selection, but they are tabulated in an efficient (space saving) way.

Well prepared students answered this question well. However many did not get to the heart of the homogeneity discussion and went off on tangents regarding various forms of selection.

7 EPV is

$$10,000(\bar{a}_{\overline{60:60}} - \bar{a}_{60:60}) + 10,000 \times \bar{a}_{\overline{5}|} \times \bar{A}_{\overline{60:60}}$$

$$\bar{a}_{\overline{60:60}} = \bar{a}^m_{60} + \bar{a}^f_{60} - \bar{a}_{60:60} = (15.632 - 0.5) + (16.652 - 0.5) - (14.090 - 0.5) = 17.694$$

$$\bar{A}_{\overline{x:y}} = (1 - \delta \bar{a}_{\overline{x:y}}) = 1 - \ln(1.04) \times 17.694 = 0.30603$$

Therefore

$$\begin{aligned} \text{EPV} &= 10,000 \times (17.694 - (14.090 - 0.5)) + 10,000 \times \frac{(1 - v^5)}{\delta} \times 0.30603 \\ &= 41,040 + 13,894 \\ &= 54,934 \end{aligned}$$

Many students struggled here with the second term in the equation in the 2nd line and did not appreciate how to mix a continuous assurance factor with an annuity.

8

	All professions		*Profession A*		
Age	*Population*	*Deaths*	*Population*	*Deaths*	*Expected deaths*
20–29	120,000	256	12,500	30	26.667
30–39	178,000	458	15,000	40	38.595
40–49	156,000	502	16,000	50	51.487
50–64	123,000	600	14,000	60	68.293
Total	577,000	1,816	57,500	180	185.042

(a) Total Expected deaths 185.042

Area comparability factor = $\frac{1,816}{577,000} \Big/ \frac{185.042}{57,500} = 0.978$

(b) Standardised mortality ratio = 180/185.042 = 0.973

Indirectly standardised mortality rate = $\frac{1,816}{577,000} \Big/ \frac{185.042}{180} = 0.003062$

Straightforward with no issues and generally well done.

9 Age retirement can be ignored in constructing the dependent decrements.

The following rates are required:

Age	q_x^d	q_x^i
59	0.01243	0.055
60	0.01392	0.06
61	0.01560	0.065

The dependent decrements are calculated as:

$$(aq)_x^{\alpha} = q_x^{\alpha}(1 - 0.5 q_x^{\beta})$$

Age	$(aq)_x^d$	$(aq)_x^i$
59	0.012088	0.054658
60	0.013502	0.059582
61	0.015093	0.064493

Probability of reaching 60 = (1 – 0.012088 – 0.054658) = 0.933254
Probability of retiring at age 60 = 0.2 * 0.933254 = 0.186651
Probability of reaching 61 = 0.8 * 0.933254 * (1 – 0.013502-0.059582) = 0.692038
Probability of retiring at age 61 = 0.2 * 0.692038 = 0.138408
Probability of reaching 62 = 0.8 * 0.692038 * (1-0.015093 – 0.064493) = 0.509569
Probability of retiring at age 62 = 0.2 * 0.509569 = 0.101914
Overall required probability thus 10.19%

This question was not done well overall. Students struggled to follow through the logical sequences. In fact this question can be solved with the same answer without using multiple decrements and the few students who realised this were given credit.

10 (a) At the end of the 5th policy year, we have:

Year	*SA*	b_1	b_2	$\sum b$
0	150,000			
1	150,000	3,750	–	3,750.00
2	150,000	3,750	187.50	7,687.50
3	150,000	3,750	384.38	11,821.88
4	150,000	3,750	591.09	16,162.97
5	150,000	3,750	808.15	20,721.12

If net premium denoted by P then

$$P = \frac{150,000 A_{[30]}}{\ddot{a}_{[30]}} = \frac{150,000 \times 0.16011}{21.837} = 1099.81$$

Therefore, net premium reserve at end of 5th policy year is given by:

$$\begin{aligned} {}_5V &= (150,000 + 20,721.12) A_{35} - P\ddot{a}_{35} \\ &= 170,721.12 \times 0.19219 - 1,099.81 \times 21.003 \\ &= 32,810.89 - 23,099.31 = £9,711.58 \end{aligned}$$

(b) The sum assured and bonuses increase more slowly than under other methods for the same ultimate benefit, enabling the office to retain surplus for longer.

This method rewards longer standing policyholders and discourages surrenders, relative to other methods.

This question was also done poorly overall. A very large number of students attempted to construct a complex "net premium" from the existing bonus flow where the question was only seeking the normal net premium method. Part (b) was done better.

11 Retirement other than ill-health:

$$0.01 \times 20,000 \times \left(\sum_{t=0}^{65-30-1} t z_{30+t+0.5} r_{30+t} v^{t+0.5} \bar{a}^{*}_{30+t+0.5} + (65-30) z_{65} r_{65} v^{35} \bar{a}^{*}_{65} \right) / s_{30} l_{30}$$

Retirement due to ill-health:

$$0.01 \times 20,000 \times (65-30) \times \sum_{t=0}^{65-30-1} z_{30+t+0.5} i_{30+t} v^{t+0.5} \bar{a}^{*}_{30+t+0.5} / s_{30} l_{30}$$

Where

a_x^* is the annuity value at age x including any contingent spouse pensions

i_x, r_x, l_x are values from a multiple decrement table at age x

s_x is the salary index for age x where s_{x+1} / s_x is the ratio of salary in the year beginning age $x+1$ to salary in the year beginning age x

z_x $(s_{x-3} + s_{x-2} + s_{x-1})/3$

Other schemes were accepted but overall very few students managed to derive a full answer in this question.

12 (i)

- Allocated premiums are invested in a fund(s) chosen by the policyholder which purchases a number of units within that fund(s)
- Each investment fund is divided into units, which are priced regularly (usually daily)
- Policyholder receives the value of the units allocated to their own policy
- Benefits are directly linked to the value of the underlying investments
- Unallocated premiums are directed to the company's non-unit fund
- Bid/offer spread is used to help cover expenses and contribute towards profit
- Charges are made from the unit account periodically to cover expenses and benefits (i.e. fund management charge) and may be varied after notice of change given.
- Unit-linked contracts may offer guaranteed benefits (e.g. minimum death benefit)
- Unit-linked contracts are generally endowment assurance and whole of life contracts

(ii) To calculate the expected reserves at the end of each year we have (utilising the end of year cashflow figures):

$$p_{58} = 0.99365 \quad p_{57} = 0.99435 \quad p_{56} = 0.99497$$

$${}_3V = \frac{933.82}{1.045} = 893.61$$

$${}_2V \times 1.045 - p_{58} \times {}_3V = 292.05 \Rightarrow {}_2V = 1{,}129.17$$

$${}_1V \times 1.045 - p_{57} \times {}_2V = 334.08 \Rightarrow {}_1V = 1{,}394.13$$

The revised cash flow for year 1 will become:

$$1{,}525.89 - p_{56} \times 1{,}394.13 = 138.77$$

Revised profit vector becomes (138.77, 0, 0, 0) and
Net present value of profits = 138.77/(1.075) = 129.09

This question was generally done well.

13 (i) Reserves required on the policy per unit sum assured are:

$${}_0V_{57:\overline{3|}} = 1 - \frac{\ddot{a}_{57:\overline{3|}}}{\ddot{a}_{57:\overline{3|}}} = 0$$

$${}_1V_{57:\overline{3|}} = 1 - \frac{\ddot{a}_{58:\overline{2|}}}{\ddot{a}_{57:\overline{3|}}} = 1 - \frac{1.955}{2.870} = 0.318815$$

$${}_2V_{57:\overline{3|}} = 1 - \frac{\ddot{a}_{59:\overline{1|}}}{\ddot{a}_{57:\overline{3|}}} = 1 - \frac{1.0}{2.870} = 0.651568$$

Multiple decrement table:

T	$q^d_{[57]+t-1}$	$q^s_{[57]+t-1}$	$(aq)^d_{[57]+t-1}$	$(aq)^s_{[57]+t-1}$	$(ap)_{[57]+t-1}$	${}_{t-1}(ap)_{[57]}$
1	0.004171	0.10	0.004171	0.099583	0.896246	1.000000
2	0.006180	0.05	0.006180	0.049691	0.944129	0.896246
3	0.007140	0.00	0.007140	0.000000	0.992860	0.846172

Probability in force $(ap)_{[56]+t-1} = (1 - q^d_{[56]+t-1}) \times (1 - q^s_{[56]+t-1})$

The calculations of the profit vector, profit signature and NPV are set out in the table below:

Policy Year	*Premium*	*Expenses*	*Interest*	*Death claim*	*Maturity claim*	*Surrender claim*	*In force cash flow*
1	4700	470.00	211.50	62.57	0.00	351.03	4027.91
2	4700	65.00	231.75	92.70	0.00	350.02	4423.73
3	4700	65.00	231.75	107.10	14892.90	0.00	−10133.25

Policy year	*Increase in reserves*	*Interest on reserves*	*Profit vector*	*Cum probability of survival*	*Discount factor*	*NPV Profit*
1	4286.05	0.00	−258.15	1.00000	0.93458	−241.26
2	4445.24	239.11	217.60	0.89625	0.87344	170.34
3	−9773.52	488.68	128.95	0.84617	0.81630	89.07

Total NPV profit = 18.15

(ii) IRR is determined by solving the following equation for i:

$$-258.15(1+i)^{-1}+195.02(1+i)^{-2}+109.11(1+i)^{-3}=0$$

If i = 0.12 then LHS of equation = 2.64
If i = 0.13 then LHS of equation = −0.10
If i = 0.14 then LHS of equation = −2.74

Therefore IRR is 13%

(iii) The revised reserves required on the policy per unit sum assured are:

$${}_0V_{57:\overline{3}|}=1-\frac{\ddot{a}_{57:\overline{3}|}}{\ddot{a}_{57:\overline{3}|}}=0$$

$${}_1V_{57:\overline{3}|}=1-\frac{\ddot{a}_{58:\overline{2}|}}{\ddot{a}_{57:\overline{3}|}}=1-\frac{1.937}{2.817}=0.312389$$

$${}_2V_{57:\overline{3}|}=1-\frac{\ddot{a}_{59:\overline{1}|}}{\ddot{a}_{57:\overline{3}|}}=1-\frac{1.0}{2.817}=0.645012$$

And the revised cashflows become:

Policy year	*Increase in reserves*	*Interest on reserves*	*Revised Profit vector*	*Cum probability of survival*	*Discount factor*	*NPV profit*
1	4199.66	0.00	–171.76	1.00000	0.93458	–160.52
2	4448.78	234.29	209.24	0.89625	0.87344	163.79
3	–9675.18	483.76	25.69	0.84617	0.81630	17.74

Total NPV profit = 21.02

The NPV of profit increases slightly if the reserving basis is weakened, as a result of the surplus emerging being brought forward and the fact that the risk discount rate is greater than the interest rate being earned on reserves.

In general well prepared students made a reasonable attempt with this question. Credit was given to students who showed they understood the processes even if not all the arithmetical calculations were correct.

Note that it is possible to solve (ii) using a quadratic equation process.

14 (i)

Formula is

$$({}_tV + P - e) \times (1+i) = q_{x+t} \times S + p_{x+t} \times {}_{t+1}V$$

Definitions:

${}_tV$ = gross premium reserve at time t

q_{x+t} / p_{x+t} = probability that a life aged $x+t$ dies within /survives one year on premium/valuation basis

P = office premium

e = initial/renewal expense incurred at start of policy year

i = rate of interest in premium/valuation basis

S = sum assured payable at end of year of death

(ii)

Let P be the annual premium. Then equation of value is:

$$P\ddot{a}_{[35]:\overline{30}|} = 50,000A^{1}_{[35]:\overline{30}|} + 100,000v^{30}\,{}_{30}p_{[35]} + 300 + 0.5P + 0.025P\left(\ddot{a}_{[35]:\overline{30}|} - 1\right)$$

where

$$A^{1}_{[35]:\overline{30}|} = A_{[35]:\overline{30}|} - v^{30}\,{}_{30}p_{[35]} = 0.32187 - 0.30832 \times \frac{8,821.2612}{9,892.9151} = 0.32187 - 0.27492 = 0.04695$$

$$\ddot{a}_{[35]:\overline{30}|} = 17.6313$$

$$\Rightarrow \; 17.6313P = 50,000 \times 0.04695 + 100,000 \times 0.27492 + 300 + 0.5P + 0.025P \times 16.6313$$

$$P = \frac{30,139.5}{16.71552} = 1,803.08$$

(iii)

The gross premium prospective reserve per policy at the end of 2009 is given by:

$${}_{9}V^{PRO} = 50,000A^{1}_{44:\overline{21}|} + 100,000v^{21}\,{}_{21}p_{44} - 0.975P\ddot{a}_{44:\overline{21}|}$$

where

$$A^{1}_{44:\overline{21}|} = A_{44:\overline{21}|} - v^{21}\,{}_{21}p_{44} = 0.45258 - 0.43883 \times \frac{8,821.2612}{9,814.3359} = 0.45258 - 0.39443 = 0.05815$$

$$\ddot{a}_{44:\overline{21}|} = 14.2329$$

$$\Rightarrow {}_{9}V^{PRO} = 50,000 \times 0.05815 + 100,000 \times 0.39443 - 0.975P \times 14.2329$$

$$= 2,907.50 + 39,443.00 - 25,021.48 = 17,329.02$$

The gross premium prospective reserve per policy at the end of 2010 is given by:

$${}_{10}V^{PRO} = 50,000A^{1}_{45:\overline{20}|} + 100,000v^{20}\,{}_{20}p_{45} - 0.975P\ddot{a}_{45:\overline{20}|}$$

where

$$A^{1}_{45:\overline{20}|} = A_{45:\overline{20}|} - v^{20}\,{}_{20}p_{45} = 0.46998 - 0.45639 \times \frac{8,821.2612}{9,801.3123} = 0.46998 - 0.41075 = 0.05923$$

$\ddot{a}_{45:\overline{20|}} = 13.7805$

$\Rightarrow {}_{10}V^{PRO} = 50{,}000 \times 0.05923 + 100{,}000 \times 0.41075 - 0.975P \times 13.7805$

$= 2{,}961.50 + 41{,}075.00 - 24{,}226.16 = 19{,}810.34$

Combined mortality and interest profit =

$$385 \times \left(17{,}329.02 + 0.975 \times 1{,}803.08\right) \times 1.05 - 3 \times 50{,}000 - (385-3) \times 19{,}810.34$$
$$= 7{,}715{,}929.05 - 150{,}000 - 7{,}567{,}549.88$$
$$= -1{,}620.83$$

i.e. a combined mortality and interest loss of £1,620.83 which can be split between mortality profit and interest profit separately as follows:

$$DSAR = 50{,}000 - 19{,}810.34 = 30{,}189.66$$
$$EDS = 385 \times q_{44} \times DSAR = 385 \times 0.001327 \times 30{,}189.66 = 15{,}423.75$$
$$ADS = 3 \times DSAR = 3 \times 30{,}189.66 = 90{,}568.98$$

Therefore

Mortality profit = EDS – ADS = 15,423.75 – 90,568.98 = –75,145.23 (i.e. a mortality loss)

Interest profit = 385 × (17,329.02 + 0.975 × 1,803.08) × (0.05 – 0.04) = 73,489.04

Alternatively: Interest profit = 75,145.23 – 1,620.83 = 73,524.40 (the small discrepancy with the figure for interest profit above is due to figures being used from the Actuarial Tables with only a limited number of decimal places)

Part (i) and (ii) were done well. In part (iii) most well prepared students were able to derive the mortality profit but most struggled with the interest portion.

If a student got the combined total correct but then did not split up the content it was decided to give full credit.

END OF EXAMINERS' REPORT