INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2010 examinations

Subject CT5 — Contingencies
Core Technical

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

T J Birse
Chairman of the Board of Examiners

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1 (a) $_{20|10}q_{[45]} = (l_{65} - l_{75}) / l_{[45]}$

$= (8,821.2612 - 6,879.1673) / 9,798.0837 = 0.198212$

(b) $_{30}p_{[45]:[50]} = \dfrac{l_{75}}{l_{[45]}} \dfrac{l_{80}}{l_{[50]}} = \dfrac{6,879.1673}{9,798.0837} \dfrac{5,266.4604}{9,706.0977} = 0.380951$

Question generally done well.

2 $_{.5}p_{45.75} = {}_{.25}p_{45.75} * {}_{.25}p_{46}$

$_{.25}q_{45.75} = .25 * q_{45} / (1 - .75 * q_{45}) = .25 * .001465 / (1 - .75 * .001465)$

$= .000367$ by UDD

$_{.25}q_{46} = .25 * q_{46} = .25 * .001622 = .000406$

Hence $_{.5}p_{45.75} = (1 - .000367) * (1 - .000406) = .999227$

In general question done well. However many students did not appreciate the split in line 1 above and attempted to apply formula directly.

3 Value of Single Premium is:

$$12 \times 1,000 \times \left(a^{(12)}_{55:\overline{20|}} - a^{(12)}_{50:55:\overline{20|}} \right)$$

$$= 12,000\left(\left[\left(\ddot{a}_{55} - \tfrac{13}{24}\right) - v^{20}\,{}_{20}p_{55}\left(\ddot{a}_{75} - \tfrac{13}{24}\right)\right] - \left[\left(\ddot{a}_{50:55} - \tfrac{13}{24}\right) - v^{20}\,{}_{20}p_{50:55}\left(\ddot{a}_{70:75} - \tfrac{13}{24}\right)\right]\right)$$

$$= 12,000\left(\left[\left(18.210 - \tfrac{13}{24}\right) - v^{20} \frac{8784.955}{9917.623}\left(10.933 - \tfrac{13}{24}\right)\right]\right.$$

$$\left. - \left[\left(16.909 - \tfrac{13}{24}\right) - v^{20} \frac{8784.955}{9917.623} \frac{9238.134}{9941.923}\left(8.792 - \tfrac{13}{24}\right)\right]\right)$$

$$= 12,000((17.668 - 4.201) - (16.367 - 3.099))$$

$$= 2,388$$

Many students struggled with how to break down the monthly annuity functions into those which could then utilise the Tables. However question generally done well by well prepared students.

4 The value of 1 per annum payable monthly for 1 year is

$$\ddot{a}^{(12)}_{x:\overline{1}|} = \ddot{a}^{(12)}_{x} - v.p_x \ddot{a}^{(12)}_{x+1} = \ddot{a}_{x:\overline{1}|} - 11/24(1 - v.p_x)$$

Where $\ddot{a}_{x:\overline{1}|} = 1$

Therefore

$$\ddot{a}^{(12)}_{x:\overline{1}|} = 1 - 11/24(1 - 0.99/1.06) = 0.96973$$

The probability of reaching the beginning of each year is :

Year $1 = 1$
Year 2 = 0.99*0.8 = 0.792
Year 3 = 0.792 * 0.792 = 0.6273

The value is therefore

$$120 \times 240 \times 0.96973 \times (1 + 0.792/1.06 + 0.6273/(1.06)^2)$$
$$= 64,388$$

This question was overall done very poorly with few students realising that the key element to the calculation involved a one year annuity due payable monthly.

5 The formula is:

$$25000\left[\sum_{t=15}^{19} \frac{12}{80} \frac{z_{45+t+0.5}}{s_{44}} \frac{r_{45+t}}{l_{45}} \frac{(rl)_{66+t}}{(rl)_{45+t+0.5}}\right] + 25000\left[\frac{12}{80} \frac{z_{65}}{s_{44}} \frac{r_{65}}{l_{45}} \frac{(rl)_{66}}{(rl)_{65}}\right]$$

Question done very poorly. Many students attempted to use annuity functions whereas the question sought was a pure cash flow one.

6 (a)

$$\bar{A}_{\overline{30:40}} = \int_0^\infty e^{-.04t}\{e^{-.01t}(1-e^{-.02t})*.01+e^{-.02t}(1-e^{-.01t})*.02\}dt$$
$$= \int_0^\infty \{.01*(e^{-.05t}-e^{-.07t})+.02*(e^{-.06t}-e^{-.07t})\}dt$$
$$= \int_0^\infty (.01*e^{-.05t}+.02*e^{-.06t}-.03*e^{-.07t})dt$$
$$= \left[-\frac{.01}{.05}*e^{-.05t}-\frac{.02}{.06}*e^{-.06t}+\frac{.03}{.07}*e^{-.07t}\right]_0^\infty$$
$$= (1/5+1/3-3/7) = .10476$$

(b)

$$\bar{a}_{30:40:\overline{20}|} = \int_0^{20} e^{-.04t}*e^{-.01t}*e^{-.02t}dt$$
$$= \int_0^{20} e^{-.07t}dt$$
$$= \left[-\frac{1}{.07}e^{-.07t}\right]_0^{20}$$
$$= (1/.07)-e^{-1.4}/.07) = 10.763$$

Question generally done well.

7 Let P be the monthly premium. Then equating expected present value of premiums and benefits gives:

$$12P\ddot{a}^{(12)}_{[55]:\overline{10}|} = 45000\bar{A}^{1}_{[55]:\overline{10}|} + 5000(I\bar{A})^{1}_{[55]:\overline{10}|}$$

where

$$\ddot{a}^{(12)}_{[55]:\overline{10}|} = \ddot{a}_{[55]:\overline{10}|} - \frac{11}{24}\left(1-v^{10}\times{}_{10}p_{[55]}\right) = 8.228-0.458\left(1-.67556\times\frac{8821.2612}{9545.9929}\right) = 8.056$$
$$\bar{A}^{1}_{[55]:\overline{10}|} = 1.04^{0.5}\left(A_{[55]:\overline{10}|}-v^{10}\times{}_{10}p_{[55]}\right) = 1.04^{0.5}\left(0.68354-0.62427\right) = 0.06044$$
$$(I\bar{A})^{1}_{[55]:\overline{10}|} = 1.04^{0.5}\left((IA)_{[55]}-v^{10}\times{}_{10}p_{[55]}\times(IA)_{65}-10v^{10}\times{}_{10}p_{[55]}\times A_{65}\right)$$
$$= 1.04^{0.5}(8.58908-0.62427\times7.89442-10\times0.62427\times0.52786) = 0.3728$$
$$\Rightarrow 12P = \frac{45000\times0.06044+5000\times0.3728}{8.056} = 568.99$$
$$\Rightarrow P = £47.42$$

In general question done well by well prepared students.

8 Occupation – either because of environmental or lifestyle factors mortality may be directly affected. Occupations may also have health barriers to entry, e.g. airline pilots

Nutrition – poor quality nutrition increases morbidity and hence mortality

Housing – standard of housing (reflecting poverty) increases morbidity

Climate – climate can influence morbidity and may also be linked to natural disaster

Education – linked to occupation but better education can reduce morbidity, e.g. by reducing smoking

Genetics – there is genetic evidence of a predisposition to contracting certain illnesses, even if this has no predictive capability

A straightforward bookwork question generally done well although not all students captured the full range. All valid examples not shown above were credited.

Students who misunderstood the question and tried to answer using Class, Time, Temporary Initial Selection were given no credit.

9 Use the formula

$$q_x^\alpha = \frac{(aq)_x^\alpha}{(1-0.5((aq)_x^{-\alpha}))}$$

to derive the independent probabilities:

$$q_x^d = \frac{(aq)_x^d}{(1-0.5((aq)_x^{-d}))} = \frac{(50/6548)}{(1-0.5*((219+516)/6548))} = 0.00809$$

$$q_x^i = \frac{(aq)_x^i}{(1-0.5((aq)_x^{-i}))} = \frac{(219/6548)}{(1-0.5*((50+516)/6548))} = 0.03496$$

$$q_x^r = \frac{(aq)_x^r}{(1-0.5((aq)_x^{-r}))} = \frac{(516/6548)}{(1-0.5*((50+219)/6548))} = 0.080455$$

Then the revised $q_x^d = 80\% * 0.00809 = 0.006472$
then use the formula

$$(aq)_x^\alpha = q_x^\alpha(1-\frac{1}{2}(q_x^\beta+\ldots)+\frac{1}{3}(q_x^\beta.q_x^\gamma+\ldots)-\ldots)$$

to derive dependent probabilities:

$$(aq)_x^d = q_x^d(1-\frac{1}{2}(q_x^i + q_x^r)+\frac{1}{3}(q_x^i.q_x^r)) = 0.0061046$$

$$(aq)_x^i = q_x^i(1-\frac{1}{2}(q_x^d + q_x^r)+\frac{1}{3}(q_x^d.q_x^r)) = 0.0334465$$

$$(aq)_x^r = q_x^r(1-\frac{1}{2}(q_x^d + q_x^i)+\frac{1}{3}(q_x^d.q_x^i)) = 0.0787948$$

The resulting service table is:

l_x	d_x	i_x	r_x
6,548	40	219	516

This question was done poorly. Many students appeared not to remember the derivation process for multiple decrements etc. Some students wrote down the final table without showing intermediate working. This gained only a proportion of the marks.

10 (a) Crude mortality rate = actual deaths / total exposed to risk

$$= \frac{\sum_x E_{x,t}^c m_{x,t}}{\sum_x E_{x,t}^c}$$

where

$E_{x,t}^c$ is central exposed to risk in population between age x and $x+t$
$m_{x,t}$ is central rate of mortality in population between age x and $x+t$

(b) Indirectly standardised mortality rate

$$= \frac{\sum_x {}^sE_{x,t}^c \, {}^sm_{x,t}}{\sum_x {}^sE_{x,t}^c} \Bigg/ \frac{\sum_x E_{x,t}^c \, {}^sm_{x,t}}{\sum_x E_{x,t}^c m_{x,t}}$$

$^{s}E^{c}_{x,t}$ is central exposed to risk in standard population between age x and $x+t$

$^{s}m_{x,t}$ is central rate of mortality in standard population between age x and $x+t$

This question generally done well. Other symbol notation was accepted provided it was consistent and properly defined.

11

Year t	q_x	p_x	$_{t-1}p_x$	$NUCF_t$	*Profit Signature*
1	0.01	0.99	1	–50.2	–50.2
2	0.01	0.99	0.99	–43.1	–42.7
3	0.01	0.99	0.9801	–32.1	–31.5
4	0.01	0.99	0.9703	145.5	141.2

(i) PV of profit @ 6%

$$= -50.2v - 42.7v^2 - 31.5v^3 + 141.2v^4$$
$$= -47.4 - 38.0 - 26.4 + 111.8$$
$$= 0.0 \quad \Rightarrow IRR = 6\%$$

(ii) $${}_2V = \frac{32.1}{1.025} = 31.3$$

$${}_1V \times 1.025 - p_x \times {}_2V = 43.1 \Rightarrow {}_1V = 72.3$$

revised cash flow in year 1 $= -50.2 - p_x \times {}_1V = -50.2 - 71.6 = -121.8$

and NPV of profit = –121.8/1.06 + 111.8 = -3.1

(iii) As expected, the *NPV* after zeroisation is smaller because the emergence of the non- unit cash flow losses have been accelerated and the risk discount rate is greater than the accumulation rate.

Parts (i) and (iii) done well generally. In Part (ii) many students failed to develop the formulae properly although they realised the effect in (iii).

12 (i) The gross future loss random variable is

$$50,000\left[1+b\left(K_{40}+1\right)\right]v^{T_{40}}+(I-e)+e\ddot{a}_{\overline{K_{40}+1}|}+fv^{T_{40}}-P\ddot{a}_{\overline{\min(K_{40}+1,25)}|}$$

Note: select functions also acceptable

where b is the annual rate of bonus
I is the initial expense
e is the annual renewal expense payable in the 2nd and subsequent years
f is the claim expense
P is the gross annual premium
$K_{40}(T_{40})$ is the curtate (complete) random future lifetime of a life currently aged 40

(ii) The annual premium P is given by

$$P\ddot{a}_{[40]:\overline{25}|}=50,250\bar{A}_{[40]}+1,250\left(I\bar{A}\right)_{[40]}+300+25(\ddot{a}_{[40]}-1)$$

$$\Rightarrow P\times 13.29=50,250\times 1.06^{0.5}\times 0.12296+1,250\times 1.06^{0.5}\times 3.85489$$
$$+300+25(15.494-1)$$

$$\Rightarrow 13.29P=6361.402+4961.065+300+362.35$$

$$\Rightarrow P=£901.79$$

(iii) The required reserve is

$$64,000\bar{A}_{50}+1,500\left(I\bar{A}\right)_{50}+35\ddot{a}_{50}-901.79\times\ddot{a}_{50:\overline{15}|}$$

$$=64,000\times 1.04^{0.5}\times 0.32907+1,500\times 1.04^{0.5}\times 8.55929$$
$$+35\times 17.444-901.79\times 11.253$$

$$=21,477.560+13,093.196+610.54-10,147.84$$

$$=£25,033.32$$

In general question done well by well prepared students. In (i) credit also given if the formulae included a limited term on the expense element although in reality this is unlikely.

13 (i) Let P be the annual premium. Then equating expected present value of premiums and benefits gives:

$$P\ddot{a}_{\overline{60^m:55^f}} = 100000\bar{A}_{\overline{60^m:55^f}}$$

where $\ddot{a}_{\overline{60^m:55^f}} = \ddot{a}_{60^m} + \ddot{a}_{55^f} - \ddot{a}_{60^m:55^f} = 15.632 + 18.210 - 14.756 = 19.086$

$$\bar{A}_{\overline{60^m:55^f}} = 1.04^{0.5} \times A_{\overline{60^m:55^f}} = 1.04^{0.5} \times (1 - d \times \ddot{a}_{\overline{60^m:55^f}})$$

$= 1.04^{0.5} \times (1 - 0.038462 \times 19.086) = 0.2711804$

$\therefore P \times 19.086 = 100000 * 0.2711804$

$\Rightarrow P = £1,420.83$.

(ii) Reserves at the end of the first policy year:

- Where both lives are alive:

$$100000 \times 1.04^{0.5} \times \left(1 - \frac{\ddot{a}_{\overline{61^m:56^f}}}{\ddot{a}_{\overline{60^m:55^f}}}\right)$$

$$= 100000 \times 1.04^{0.5} \times \left(1 - \frac{15.254 + 17.917 - 14.356}{15.632 + 18.210 - 14.756}\right) = 1448.01$$

- Where the male life is alive only:

$$100000\bar{A}_{61^m} - P\ddot{a}_{61^m}$$
$$100000 \times 1.04^{0.5} \times \left(1 - \frac{0.04}{1.04} \times 15.254\right) - 1420.83 \times 15.254 = 20475.94$$

- Where the female life is alive only:

$$100000\bar{A}_{56^f} - P\ddot{a}_{56^f}$$
$$100000 \times 1.04^{0.5} \times \left(1 - \frac{0.04}{1.04} \times 17.917\right) - 1420.83 \times 17.917 = 6247.12$$

Mortality Profit = Expected Death Strain – Actual Death Strain

(a) Both lives die during 2009 = 1 actual claim.

Mortality Profit

$$= \left(10{,}000 \times q_{60^m} \times q_{55^f} - 1\right) \times \left(100000 \times 1.04^{0.5} - 1448.01\right)$$
$$= \left(10{,}000 \times 0.002451 \times 0.001046 - 1\right) \times \left(100532.38\right) = -97954.99$$

(b) Males only die during 2009 = 20 actual deaths (and therefore we need to change reserve from joint life to female only surviving).

Mortality Profit

$$= \left(10{,}000 \times p_{55^f} \times q_{60^m} - 20\right) \times \left(6247.12 - 1448.01\right)$$
$$= \left(10{,}000 \times 0.998954 \times 0.002451 - 20\right) \times \left(4799.11\right) = 21520.95$$

(c) Females only die during 2009 = 10 actual deaths (and therefore we need to change reserve from joint life to male only surviving).

Mortality Profit

$$= \left(10{,}000 \times p_{60^m} \times q_{55^f} - 10\right) \times \left(20475.94 - 1448.01\right)$$
$$= \left(10{,}000 \times 0.997549 \times 0.001046 - 10\right) \times \left(19027.93\right) = 8265.02$$

Hence overall total mortality profit

$$= -97954.99 + 21520.95 + 8265.02 = -£68{,}169.01$$

i.e. a mortality loss

Part (i) generally done well. Part (ii) was challenging and few students realised the full implications of "reserve change" on 1st death. Only limited partial credit was given if students used only joint life situations.

14 Reserves required on the policy per unit sum assured are:

$$ {}_0V_{56:\overline{4}|} = 1 - \frac{\ddot{a}_{56:\overline{4}|}}{\ddot{a}_{56:\overline{4}|}} = 0 $$

$$ {}_1V_{56:\overline{4}|} = 1 - \frac{\ddot{a}_{57:\overline{3}|}}{\ddot{a}_{56:\overline{4}|}} = 1 - \frac{2.870}{3.745} = 0.23364 $$

$$ {}_2V_{56:\overline{4}|} = 1 - \frac{\ddot{a}_{58:\overline{2}|}}{\ddot{a}_{56:\overline{4}|}} = 1 - \frac{1.955}{3.745} = 0.47797 $$

$$ {}_3V_{56:\overline{4}|} = 1 - \frac{\ddot{a}_{59:\overline{1}|}}{\ddot{a}_{56:\overline{4}|}} = 1 - \frac{1.0}{3.745} = 0.73298 $$

Multiple decrement table:

T	$q^d_{[56]+t-1}$	$q^s_{[56]+t-1}$	$(aq)^d_{[56]+t-1}$	$(aq)^s_{[56]+t-1}$	$(ap)_{[56]+t-1}$	${}_{t-1}(ap)_{[56]}$
1	0.003742	0.1	0.003742	0.09963	0.896632	1.000000
2	0.005507	0.1	0.005507	0.09945	0.895044	0.896632
3	0.006352	0.1	0.006352	0.09936	0.894283	0.802525
4	0.007140	0.0	0.007140	0.0	0.992860	0.717685

Probability in force $(ap)_{[56]+t-1} = (1 - q^d_{[56]+t-1}) \times (1 - q^s_{[56]+t-1})$

The calculations of the profit vector, profit signature and NPV are set out in the table below:

Policy year	*Premium*	*Expenses*	*Interest*	*Death claim*	*Maturity claim*	*Surrender claim*	*In force cash flow*
1	5000	600.00	176.00	80.45	0.00	350.31	4145.23
2	5000	45.00	198.20	118.40	0.00	715.38	4319.42
3	5000	45.00	198.20	136.57	0.00	1096.1613	3920.475
4	5000	45.00	198.20	153.51	21346.49	0.00	–16346.80

Policy year	*Increase in reserves*	*Interest on reserves*	*Profit vector*	*Cum probability of survival*	*Discount factor*	*NPV profit*
1	4504.02	0.00	–358.78	1.00000	0.943396	–338.47
2	4174.53	200.93	345.82	0.89663	0.890000	275.96
3	3816.72	411.05	514.84	0.80253	0.839619	346.91
4	–15759.07	630.36	42.63	0.71768	0.792094	24.24

Total *NPV* = 308.63

The calculations of the premium signature and profit margin are set out in the table below:

Policy year	*1*	*2*	*3*	*4*
Premium	5000.00	5000.00	5000.00	5000.00
probability in force	1.00000	0.89663	0.80253	0.71768
discount factor	1.00000	0.943396	0.890000	0.839619
p.v. of premium signature	5000.000	4229.40	3571.22	3012.91
=> expected p.v. of premiums	**15813.53**			
profit margin =	**2.0%**			

Many well prepared students were able to outline the process required without being totally accurate on the calculation. Significant credit was awarded in such situation.

Many students failed to appreciate the multiple decrement element.

END OF EXAMINERS' REPORT