INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2013 examinations

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

D C Bowie
Chairman of the Board of Examiners

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General comments on Subject CT5

CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions actuarial work.

Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given where appropriate to different valid points made which do not appear in the solutions below.

In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

Comments on the September 2013 paper

The general performance was similar this session to previous ones. Well prepared students generally scored well. Questions that were done less well were 15, 18, 11, 21, 22(iii) and 23(iii). The examiners hope that the detailed solutions given below will assist students with further revision.

As in past examinations most of the short questions were very straightforward and this is where many successful candidates scored particularly well. Students should note that for long questions some credit is given if they can describe the right procedures although to score well reasonably accurate numerical calculation is necessary.

11 (a) $${}_{10}q_{63} = \frac{l_{63} - l_{73}}{l_{63}} = \frac{9775.888 - 9073.650}{9775.888} = 0.07183$$

(b) $$\ddot{a}_{63}^{(2)} = \ddot{a}_{63} - \frac{1}{4} = 15.606 - 0.25 = 15.356$$

(c) $$s_{55:\overline{10}|} = \frac{(1.04)^{10} * a_{55:\overline{10}|}}{{}_{10}p_{55}} = \frac{(1.04)^{10} * ((\ddot{a}_{55} - 1) - (1.04)^{-10} * (l_{65} / l_{55}) * (\ddot{a}_{65} - 1))}{(l_{65} / l_{55})}$$

$$= \frac{(1.04)^{10} * 17.210 - (0.97843 * 13.871)}{0.97843}$$

$$= 12.166$$

This question was generally well done.

12 Temporary Initial Selection describes the modelling of rates by sub-dividing a population by duration since entry to that class. The rates modelled are dependent on duration up to a duration of s (the length of the select period) and after s they are independent of duration, so the effect is "temporary".

An example is a life purchasing a life assurance policy who has been medically selected and thus initially would be expected to enjoy better mortality. This advantage however wears off over time.

This question was generally well done. Credit was given for all relevant comments. To earn full marks it was important to stress in the answer the fact that the effect of selection wears off.

13 (a) For the first policy year

$$[{}_0V + P - \frac{a}{100}P - B] \times (1+i) = (1 + \frac{e}{100}) \times S \times q_x + {}_1V \times p_x$$

(b) For subsequent policy years

$$({}_tV + P - \frac{c}{100}P - D) \times (1+i) = (1 + \frac{e}{100}) \times S \times q_{x+t} + {}_{t+1}V \times p_{x+t}$$

Students had in many cases difficulties in setting out these standard formulae which are fundamental in CT5. In (a) expressing ${}_0V$ as zero was fine so long as this definition was stated. Also using t-1and t instead of t and t+1 respectively was acceptable.

14 $${}_{2.25}p_{90.25} = {}_{0.75}p_{90.25} * p_{91} * {}_{.5}p_{92}$$

$$= (1 - {}_{0.75}q_{90.25})(1 - q_{91})(1 - {}_{.5}q_{92})$$

$$= \left(1 - \frac{.75q_{90}}{(1 - .25q_{90})}\right)(1 - q_{91})(1 - .5q_{92})$$

$$= \left(1 - \frac{.75 * .170247}{(1 - .25 * .170247)}\right) * .815286 * .89996$$

$$= 0.63587$$

Generally well done. An alternative correct method is to use straight line interpolation on l factors. This is fine so long as it produces an accurate answer.

15 (a) If $q^d_{[40]}$ and q^s_{40} represent the independent rates of mortality and surrender respectively in the 1st policy year, then the dependent rate of surrender at the end of the 1st policy year is:

$$(aq)^s_{40} = \left[1 - q^d_{[40]}\right] \times q^s_{40} = (1 - 0.000788) \times 0.15 = 0.14988$$

The cash flows are now modified to include a surrender charge at the end of the 1st policy year

$$= 500 \times (aq)^s_{40} = 500 \times 0.14988 = 74.94$$

The revised profit vector = revised profit signature
= –209.80 + 74.94 = –134.86

(b) Although the profit vector for this policy will remain the same for policy years 2 and 3, the profit signature for each year will reduce as the probability of the policy being in force at the start of each year will reduce.

This question was done poorly overall with few students being able to derive the correct answer.

16

$$PV = 1100\ddot{a}_{75:\overline{10|}} + 100(I\ddot{a})_{75:\overline{10|}}$$

$$= 1100(\ddot{a}_{75} - v^{10} * {}_{10}p_{75}\ddot{a}_{85}) + 100((I\ddot{a})_{75} - v^{10} * {}_{10}p_{75}(10\ddot{a}_{85} + (I\ddot{a})_{85}))$$

$$= 1100\left(7.679 - 0.55839 * \frac{3385.2479}{6879.1673} * 4.998\right) + 100(48.128 - 0.55839 * \frac{3385.2479}{6879.1673} * (49.98 + 21.503))$$

$$= 6936.2 + 2848.6$$

= £9785 rounded

This was a very straightforward question that was generally well done. The most common error was for the first function above to be multiplied by 1000 rather than the correct 1100.

17 $EPV =$

$$\left(.03 * \int_0^{20} te^{-.08t}dt\right) + \left(.04 * e^{-1.6} * e^{1.8}\int_{20}^{\infty} te^{-.09t}dt\right) = \left(.03 * \int_0^{20} te^{-.08t}dt\right) + \left(.04 * e^{0.2}\int_{20}^{\infty} te^{-.09t}dt\right)$$

$$\int_0^{20} te^{-.08t}dt = \left[-\frac{te^{-.08t}}{.08}\right]_0^{20} + \frac{1}{.08}\int_0^{20} e^{-.08t}dt = \left[-\frac{te^{-.08t}}{.08} - \frac{e^{-.08t}}{(.08)^2}\right]_0^{20}$$

$$= -\frac{20e^{-1.6}}{.08} + 0 - \frac{e^{-1.6}}{(.08)^2} + \frac{1}{(.08)^2} = -50.474 - 31.546 + 156.25$$

$$= 74.230$$

$$\int_{20}^{\infty} te^{-.09t}dt = \left[-\frac{te^{-.09t}}{.09}\right]_{20}^{\infty} + \frac{1}{.09}\int_{20}^{\infty} e^{-.09t}dt = \left[-\frac{te^{-.09t}}{.09} - \frac{e^{-.09t}}{(.09)^2}\right]_{20}^{\infty}$$

$$= -0 + \frac{20e^{-1.8}}{.09} - 0 + \frac{e^{-1.8}}{(.09)^2} = 36.733 + 20.407 = 57.140$$

$$EPV = .03 * 74.230 + .04 * 1.2214 * 57.140$$

$$= 5.019$$

A challenging question. Well prepared students coped well but many failed at the basic level in constructing the integral.

18 Define the random variable $\mathbf{K}_x$ for the curtate duration of life aged x.

The expected present value is:

$$\Sigma_{k=0}^{n} 0 \times P[\mathbf{K}_x = k] + \Sigma_{k=n+1}^{\infty} {}_{n|}a_{\overline{k-n}|} \times P[\mathbf{K}_x = k]$$

$$= (\Sigma_{k=0}^{n} a_{\overline{k}|} \times P[\mathbf{K}_x = k] + a_{\overline{n}|} \times P[\mathbf{K}_x > n]) - (\Sigma_{k=0}^{n} a_{\overline{k}|} \times P[\mathbf{K}_x = k] + a_{\overline{n}|} \times P[\mathbf{K}_x > n])$$
$$+\Sigma_{k=n+1}^{\infty} {}_{n|}a_{\overline{k-n}|} \times P[\mathbf{K}_x = k]$$

$$= (\Sigma_{k=0}^{n} a_{\overline{k}|} \times P[\mathbf{K}_x = k] + a_{\overline{n}|} \times P[\mathbf{K}_x > n] + \Sigma_{k=n+1}^{\infty} {}_{n|}a_{\overline{k-n}|} \times P[\mathbf{K}_x = k])$$
$$-(\Sigma_{k=0}^{n} a_{\overline{k}|} \times P[\mathbf{K}_x = k] + a_{\overline{n}|} \times P[\mathbf{K}_x > n])$$

$$= (\Sigma_{k=0}^{\infty} a_{\overline{k}|} \times P[\mathbf{K}_x = k]) - (\Sigma_{k=0}^{n} a_{\overline{k}|} \times P[\mathbf{K}_x = k] + a_{\overline{n}|} \times P[\mathbf{K}_x > n])$$

$$= a_x - a_{x:\overline{n}|}$$

This is a straight bookwork question taken straight from Core Reading. Most students struggled to reproduce it and the primary error was that students did not appreciate the random variable aspect often trying to solve it in a non random variable manner. This gained no credit.

19 (i) (a)

	Region A			*Country*		
Age	*Population exposed*	*Number of Deaths*	*Mortality*	*Population exposed*	*Number of Deaths*	*Mortality*
18–35	25000	25	0.00100	500000	1000	0.00200
36–50	50000	80	0.00160	125000	375	0.00300
51–70	70000	170	0.00243	110000	500	0.00455
	145000	275		735000	1875	

The mortality rates are shown in Columns 4 and 7 above.

(b) Crude Mortality Rate (Region A) = 275/145000 = 0.00190
Crude Mortality Rate (Country) = 1875/735000 = 0.00255

(c) The directly standardised mortality rate for Region A is:

((500000 * .00100) + (125000 * .00160) + (110000 * .00243))/735000
= 0.00132

(d) The standardised mortality ratio for Region A is:

Actual deaths in Region A/Expected Deaths in Region A based on Country mortality rates i.e.

275/((25000 * .00200) + (50000 * .00300) + (70000 * .00455))
= 275/518.5 = 0.53

(ii)

- Crude mortality rate in Region A suggests Region A has only 75% of the mortality rate of Country as a whole.
- However the directly standardised mortality rate for Region A is significantly lighter than the appropriate crude rate.
- This difference is explained by the fact that Region A has a much higher proportion of older lives than the Country as a whole thus inflating the crude rate.
- The standardised mortality ratio shows the true difference i.e. the mortality rates for Region A are on average 53% of those for the Country as a whole.

Generally this was another straightforward question on which students did well. The most common error was that not all points were covered in (ii).

20

$$(aq)_{85}^{d} = \frac{1400}{10000} = 0.14; (aq)_{86}^{d} = \frac{1000}{6300} = 0.15873$$

$$(aq)_{85}^{w} = \frac{2300}{10000} = 0.23; (aq)_{86}^{w} = \frac{1100}{6300} = 0.17460$$

$$q_{85}^{d} = \frac{(aq)_{85}^{d}}{\left(1 - \frac{1}{2}(aq)_{85}^{w}\right)} = \frac{0.14}{0.885} = 0.158192$$

Similarly $q_{86}^{d} = \dfrac{0.15873}{0.9127} = 0.173913$

$$q_{85}^{w} = \frac{(aq)_{85}^{w}}{\left(1 - \frac{1}{2}(aq)_{85}^{d}\right)} = \frac{0.23}{0.93} = 0.247312$$

Similarly $q_{86}^{w} = \dfrac{0.17460}{0.9206} = 0.189659$

But q_{85}^{w} and q_{86}^{w} are now reduced by 50% so their new values are:

$$q_{85}^{w} = 0.123656 \text{ and } q_{86}^{w} = .0948295$$

Hence $(aq)_{85}^{d} = 0.158192 * \left(1 - \frac{1}{2} * 0.123656\right) = 0.14841; (aq)_{86}^{d} = 0.173913 * \left(1 - \frac{1}{2} * .0948295\right) = 0.16567$

Hence $(aq)_{85}^{w} = 0.123656 * \left(1 - \frac{1}{2} * 0.158192\right) = 0.113875; (aq)_{86}^{w} = 0.0948295 * \left(1 - \frac{1}{2} * 0.173913\right) = 0.086583$

Using the above the new table is:

Age x	$(al)_x$	$(ad)_x^d$	$(ad)_x^w$
85	10000	1484	1139
86	7377	1222	638
87	5518		

Note that values are sensitive to rounding-other close values accepted.

This was another relatively straightforward question generally well done by well prepared students. Most marks were awarded on knowing the principles of calculation rather than the precision of the calculations themselves.

21 (i) Assume that decrements on average occur at time x+ ½.

$$3 \times 25{,}000 \times \left\{ \left(\frac{d_{40}}{l_{35}} \frac{s_{39.5}}{s_{34}} \right) \right\}$$

$$= 3 \times 25{,}000 \times \left\{ \left(\frac{14}{18866} \frac{(7.623 + 7.814)/2}{6.389} \right) \right\}$$

$$= 67.24$$

(ii) Expected present value

$$\sum_{t=0}^{t=64-x} 3 \times 25{,}000 \times \frac{s_{34+t+1/2}}{s_{34}} \frac{d_{35+t}}{l_{35}} \frac{v^{35+t+½}}{v^{35}}$$

Define:

$$^{s}D_{35} = s_{34} l_{35} v^{35}$$

$$^{s}C^{d}_{x+t} = s_{34+t+½} d_{35+t} v^{35+t+½}$$

$$^{s}M^{d}_{35} = \sum_{t=0}^{t=64-x} {}^{s}C^{d}_{35+t}$$

Then the expected value is:

$$3 \times 25,000 \times \frac{^{s}M^{d}_{35}}{^{s}D_{35}}$$

This question was very poorly done. Students seem to struggle continually with questions involving pension commutation functions and this was felt to be a reasonably straightforward derivation from 1st principles.

22

(i) Let P be the annual premium for the policy. Then (functions at 4%):

EPV of premiums:

$$P\ddot{a}_{[40]:\overline{20|}} = 13.930P$$

EPV of benefits:

$$75,000A^{1}_{[40]:\overline{20|}} + 150,000v^{20}\ {}_{20}p_{[40]}$$

where:

$$v^{20}\ {}_{20}p_{[40]} = 0.45639 \times \frac{9287.2164}{9854.3036} = 0.43013$$

$$A^{1}_{[40]:\overline{20|}} = A_{[40]:\overline{20|}} - v^{20}\ {}_{20}p_{[40]} = 0.46423 - 0.43013 = 0.0341$$

$$= 75,000 \times 0.0341 + 150,000 \times 0.43013$$

$$= 67,077.0$$

EPV of expenses:

$$0.25P + 400 + 45(\ddot{a}_{[40]:\overline{20|}} - 1) \; = \; 0.25P + 400 + 45 \times 12.93$$

$$= 0.25P + 981.85$$

Equation of value gives:

$$13.93P = 67,077.0 + 0.25P + 981.85$$

$$\Rightarrow P = \frac{68,058.85}{13.68} = 4,975.06$$

(ii) The gross prospective policy reserve at the end of the 8th policy year is given by:

$${}_8V = 75,000 A^1_{48:\overline{12|}} + 150,000 v^{12} {}_{12}p_{48} + (45 - P)\ddot{a}_{48:\overline{12|}}$$

where:

$$v^{12} {}_{12}p_{48} = 0.62460 \times 0.95220 = 0.59474$$

$$\Rightarrow {}_8V = 75,000 \times (0.63025 - 0.59474) + 150,000 \times 0.59474 + (45 - 4975.06) \times 9.613$$

$$= 44,481.58$$

The gross prospective policy reserve at the end of the 9th policy year is given by:

$${}_9V = 75,000 A^1_{49:\overline{11|}} + 150,000 v^{11} {}_{11}p_{49} + (45 - P)\ddot{a}_{49:\overline{11|}}$$

where:

$$v^{11} {}_{11}p_{49} = 0.64958 \times 0.95411 = 0.61977$$

$$\Rightarrow {}_9V = 75,000 \times (0.65477 - 0.61977) + 150,000 \times 0.61977 + (45 - 4975.06) \times 8.976$$

$$= 51338.28$$

Note: students can alternatively calculate these reserves on a retrospective basis i.e.

$${}_8V = \frac{D_{[40]}}{D_{48}} \left[P\ddot{a}_{[40]:\overline{8|}} - 75,000 A^1_{[40]:\overline{8|}} - 400 - 45(\ddot{a}_{[40]:\overline{8|}} - 1) - 0.25P \right]$$

where:

$$A^1_{[40]:\overline{8}|} = A_{[40]} - v^8 \, {}_8p_{[40]} \times A_{48} = 0.23041 - 0.73069 \times 0.98977 \times 0.30695 = 0.008419$$

and:

$$\ddot{a}_{[40]:\overline{8}|} = \ddot{a}_{[40]} - v^8 \, {}_8p_{[40]} \times \ddot{a}_{48} = 20.009 - 0.73069 \times 0.98977 \times 18.019 = 6.9774$$

$$\Rightarrow {}_8V = 1.382713[4975.06 \times 6.9774 - 75,000 \times 0.008419 - 400 - 45 \times 5.9774 - 0.25 \times 4975.06]$$

$${}_9V = \frac{D_{[40]}}{D_{49}}\left[P\ddot{a}_{[40]:\overline{9}|} - 75,000A^1_{[40]:\overline{9}|} - 400 - 45(\ddot{a}_{[40]:\overline{9}|} - 1) - 0.25P\right]$$

where:

$$A^1_{[40]:\overline{9}|} = A_{[40]} - v^9 \, {}_9p_{[40]} \times A_{49} = 0.23041 - 0.70259 \times 0.98778 \times 0.31786 = 0.009814$$

and:

$$\ddot{a}_{[40]:\overline{9}|} = \ddot{a}_{[40]} - v^9 \, {}_9p_{[40]} \times \ddot{a}_{49} = 20.009 - 0.70259 \times 0.98778 \times 17.736 = 7.7001$$

$$\Rightarrow {}_9V = 1.440915\left[4975.06 \times 7.7001 - 75,000 \times 0.009814 - 400 - 45 \times 6.7001 - 0.25 \times 4975.06\right]$$

$= 51,335.68$

(iii) Using the gross prospective policy reserve calculated in b) above then:

Sum at risk per policy in the 9th policy year is:

DSAR = 75,000 – 51,338.28 = 23,661.72

Mortality profit = EDS – ADS

EDS = $625 \times q_{48} \times 23,661.72 = 625 \times 0.002008 \times 23,661.72 = 29,695.46$

ADS = $3 \times 23,661.72 = 70,985.16$

i.e. mortality profit = –41,289.7 (i.e. a loss)

total profit/loss in 2012 =

$$= 625 \times ({}_8V + P - E) \times (1 + i) - S \times \text{actual deaths} - {}_9V \times \text{number of policies inforce}$$

$= 625 \times (44,481.58 + 4,975.06 - 45) \times 1.045 - 75,000 \times 3 - 51,338.28 \times 622$

$= 114,567.22$

i.e. total profit from mortality, interest and expense combined = 114,567.22

As expenses incurred per policy during 2012 were the same as assumed in the premium basis, then expense surplus = 0

$= \ 44,480.23$

Therefore interest surplus = 114,567.22 – (–41,289.7) = 155,856.92

Most well prepared students did parts (i) and (ii) well. Part (iii) was less well done as few students realised expense surplus was zero and many attempted only the mortality surplus.

23

(i) Let P be the monthly premium payable for this policy. Then:

EPV of premiums (at 6% p.a.)

$12P\ddot{a}^{(12)}_{[50]:\overline{15|}} = 117.114P$

where:

$$\ddot{a}^{(12)}_{[50]:\overline{15|}} = \ddot{a}_{[50]:\overline{15|}} - \frac{11}{24}(1 - v^{15}\,{}_{15}p_{[50]}) = 10.044 - \frac{11}{24}(1 - 0.379230) = 9.7595$$

EPV of benefits: (at 6% p.a.)

$$= 50,000\bar{A}^{\ 1}_{[50]:\overline{15|}} + 10,000(I\bar{A})^{\ 1}_{[50]:\overline{15|}}$$

$$= 50,000\{\bar{A}_{[50]} - v^{15}\,{}_{15}p_{[50]}\bar{A}_{65}\} + 10,000\{(I\bar{A})_{[50]} - v^{15}\,{}_{15}p_{[50]}(15\bar{A}_{65} + (I\bar{A})_{65})\}$$

$$= 1.06^{0.5} \times [50,000A_{[50]} + 10,000(IA)_{[50]} - v^{15}\,{}_{15}p_{[50]}(200,000A_{65} + 10,000(IA)_{65})]$$

$$= 1.02956\begin{bmatrix} 50,000 \times 0.20463 + 10,000 \times 4.84789 \\ -0.41727 \times \dfrac{8821.2612}{9706.0977}\left(200,000 \times 0.40177 + 10,000 \times 5.50985\right) \end{bmatrix}$$

$$= 1.02956[10,231.5 + 48,478.9 - 0.37923 \times (80,354.0 + 55,098.5)]$$

$= 7,559.80$

EPV of expenses (functions @6% p.a. unless otherwise stated):

$$225 + 0.3\times 12P + 0.04\times 12P\left(\ddot{a}^{(12)}_{[50]:\overline{15}|} - \frac{1}{12}\right) + 65\left(\ddot{a}^{4\%}_{[50]:\overline{15}|} - 1\right) + 275\bar{A}^{1\ \ 4\%}_{[50]:\overline{15}|}$$

$$= 225 + 3.6P + 0.48P\times 9.6762 + 65\times 10.259 + 275\left(\bar{A}^{@4\%}_{[50]} - v^{15}_{(4\%)}\ {}_{15}p_{[50]}\bar{A}^{4\%}_{65}\right)$$

$$= 225 + 3.6P + 4.6446P + 666.835 + 275\times 1.04^{0.5}[0.32868 - 0.55526\times 0.90884\times 0.52786]$$

$$= 909.307 + 8.2446P$$

Equation of value gives:

$117.114P = 7559.80 + 909.307 + 8.2446P \Rightarrow P = 77.79$

(ii) Gross prospective reserve at the end of the 14th policy year is given by (functions @6% p.a. unless otherwise stated):

$${}_{14}V = 200,000q_{64}v^{0.5} + 275(1.0192308)^{14}q_{64}v^{0.5}_{0.04}$$

$$+65(1.0192308)^{14} - 0.96\times 12P\ddot{a}^{(12)}_{64:\overline{1}|}$$

$$= 200,000\times 0.012716\times 0.97129 + 359.044\times 0.012716\times 0.98058 + 84.865 - 867.967$$

$$= 2,470.185 + 4.4769 + 84.865 - 867.967 = 1691.60$$

where:

$$\ddot{a}^{(12)}_{64:\overline{1}|} = \ddot{a}_{64:\overline{1}|} - \frac{11}{24}(1 - v\ \times p_{64}) = 1 - \frac{11}{24}(1 - 0.9434\times 0.98728) = 0.96856$$

(iii) If $K_{64} \geq 1$

GFLRV = $65(1.0192308)^{14} - 0.96\times 12\times 77.79\times \ddot{a}^{(12)}_{\overline{1}|.06}$

If $K_{64} < 1$

$$\text{GFLRV} = 200,000v^{T_{64}}_{.06} + 275(1.0192308)^{14}v^{T_{64}}_{.04}$$

$$+65(1.0192308)^{14} - 0.96\times 12\times 77.79\times \ddot{a}^{(12)}_{\overline{\frac{1}{12}\left(1+[12T_{64}]\right)}|}\ @6\%$$

where $[12T_{64}]$ represents the integer part of $12T_{64}$

Again well prepared students did part (i) well although part (ii) was done less well. Very few students made a serious attempt at part (iii) which was set to test higher skills.

24

(i)

Let P be the annual premium payable. Then equation of value gives (functions at 6% unless otherwise stated):

$$P\ddot{a}_{[56]:\overline{4}|} = 25,000A^{@i'}_{[56]:\overline{4}|} + 0.25P + 100 + (0.025P + 40)\left[\ddot{a}_{[56]:\overline{4}|} - 1\right]$$

where $i' = \left(\dfrac{1.06}{1.0192308}\right) - 1 = 0.04$

$$\Rightarrow 3.648P = 25,000 \times 0.8558 + 0.25P + 100 + (0.025P + 40) \times 2.648$$

$$\Rightarrow P = \frac{21,600.92}{3.3318} = 6,483.26$$

(ii) Decrement table

x	q_x	$q'_x = 0.8q_x$	p'_x	${}_{t-1}p'_x$
56	0.003742	0.002994	0.997006	1
57	0.005507	0.004406	0.995594	0.997006
58	0.006352	0.005082	0.994918	0.992614
59	0.007140	0.005712	0.994288	0.987570

Accrued bonus at start of policy year t for each in force policy is given by:

t	*Accrued bonus*
1	480.77
2	970.79
3	1470.22
4	1979.27

Reserves required on the policy at 4% interest are:

$${}_1V_{56:\overline{4}|} = 25,480.77A_{57:\overline{3}|} - NP\ddot{a}_{57:\overline{3}|}$$

$$= 25,000\left(1 - \frac{\ddot{a}_{57:\overline{3}|}}{\ddot{a}_{56:\overline{4}|}}\right) + 480.77A_{57:\overline{3}|} = 25,000\left(1 - \frac{2.87}{3.745}\right) + 480.77 \times 0.88963 = 6268.83$$

$$ {}_2V_{56:\overline{4}|} = 25,000\left(1-\frac{\ddot{a}_{58:\overline{2}|}}{\ddot{a}_{56:\overline{4}|}}\right)+970.79A_{58:\overline{2}|} = 25,000\left(1-\frac{1.955}{3.745}\right)+970.79\times 0.92479 = 12847.04 $$

$$ {}_3V_{56:\overline{4}|} = 25,000\left(1-\frac{\ddot{a}_{59:\overline{1}|}}{\ddot{a}_{56:\overline{4}|}}\right)+1470.22A_{59:\overline{1}|} = 25,000\left(1-\frac{1.0}{3.745}\right)+1470.22\times 0.96154 = 19738.11 $$

Cash flows for the policy under the profit test are given by:

Year T	*Opening reserve*	*Premium*	*Expense*	*Interest*	*Death Claim*	*Maturity Claim*	*Closing reserve*
1	0	6483.26	1720.82	357.18	76.28	0	6250.06
2	6268.83	6483.26	202.08	941.25	114.42	0	12790.44
3	12847.04	6483.26	202.08	1434.62	134.51	0	19637.81
4	19738.11	6483.26	202.08	1951.45	154.11	26825.16	0

Year t	*Profit vector*	${}_{t-1}p$	*Profit signature*	*Discount factor*	*NPV of profit signature*
1	–1206.71	1.0	–1206.71	.913242	–1102.02
2	586.40	0.997006	584.64	.834011	487.60
3	790.51	0.992614	784.67	.761654	597.65
4	991.47	0.987570	979.15	.695574	681.07

NPV of profit signature = £664.30

Year t	*Premium*	${}_{t-1}p$	*Discount factor*	*NPV of premium*
1	6483.26	1.0	1	6483.26
2	6483.26	0.997006	.913242	5903.06
3	6483.26	0.992614	.834011	5367.17
4	6483.26	0.987570	.761654	4876.62

NPV of premiums = £22,630.11

Profit margin = $\dfrac{664.30}{22,630.11} = 0.0294$ i.e. 2.94%

A relatively straightforward if detailed question where well prepared students scored well. In these types of question credit is given for understanding of the method and how to approach the calculations even if the calculation part contains numerical errors.

END OF EXAMINERS' REPORT