INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2011 examinations

Subject CT5 — Contingencies
Core Technical

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

T J Birse
Chairman of the Board of Examiners

July 2011

1 (a) Time selection – because it is based on a period of three calendar years
(b) Class selection – applies only to male pensioners
(c) Temporary initial selection – as there are select rates

Other valid answers acceptable

This question was generally done well. However some students did not supply <u>different</u> selection types for each part and this was penalised.

2 (a) $${}_{23}p_{65} = \frac{l_{88}}{l_{65}} = \frac{3534.054}{9647.797} = 0.366307$$

(b) $${}_{10|5}q_{60} = \frac{(l_{70} - l_{75})}{l_{60}} = \frac{(9238.134 - 8405.160)}{9826.131} = 0.084771$$

(c)

$$\ddot{s}_{65:\overline{10}|} = \frac{(1+i)^{10}\ddot{a}_{65:\overline{10}|}}{{}_{10}p_{65}} = \frac{(1+i)^{10}(\ddot{a}_{65} - v^{10}\,{}_{10}p_{65}\ddot{a}_{75})}{{}_{10}p_{65}}$$

$$= \frac{(1.04)^{10}(13.666 - (1.04^{-10}) \times \left(8,405.160 \big/ 9,647.797\right) \times 9.456)}{\left(8,405.160 \big/ 9,647.797\right)}$$

$$= 1.48024 \times (13.666 - 0.67556 \times 0.87120 \times 9.456) / 0.87120$$

$$= 13.764$$

This question was generally done well for parts (a) and (b) but students struggled more with part (c).

3

$$(I\bar{a})_x = \int_0^1 v^t\,{}_tp_x dt + 2\int_1^2 v^t\,{}_tp_x dt + 3\int_2^3 v^t\,{}_tp_x dt + ...$$

Now $vp_x = e^{-.04} * e^{-.02} = e^{-.06}$ throughout.

Hence

$(I\bar{a})_x = (1 + 2e^{-0.06} + 3(e^{-.06})^2 + 4(e^{-.06})^3 + ...)\bar{a}_{\overline{1}|}$ at force of interest 6%

$= (1/(1 - e^{-.06}))^2 \times ((1 - e^{-.06}) / .06)$

$= 294.8662 \times 0.970591$

$= 286.19$

This question was not done well. The majority of students failed to realise that the increasing function I was not continuous, although the payment ā is continuous. Instead most attempted to compute $(\overline{Ia})_x = \int_0^\infty tv^t\,{}_tp_x dt$. Only minimal credit was given for this.

4 Schemes usually allow members to retire on grounds of ill-health and receive a pension benefit after a minimum length of scheme service.

Benefits are usually related to salary at the date of ill-health retirement in similar ways to age retirement benefits.

However, pensionable service is usually more generous than under age retirement with years beyond those served in the scheme being credited to the member e.g. actual pensionable service subject to a minimum of 20 years, or pensionable service that would have been completed by normal retirement age.

A lump sum may be payable on retirement and a spouse pension on death after retirement.

Other valid points were credited. Generally this bookwork question was done well.

5 The Kolmogorov equations in this case are:

$$\frac{\delta}{\delta t}\,{}_t(aq)_x^r = \rho e^{-(\mu+\rho)t}$$

$$\frac{\delta}{\delta t}\,{}_t(aq)_x^d = \mu e^{-(\mu+\rho)t}$$

For the case where $t = 1$ the solution for the dependent probability of retirement is:

$$(aq)_x^r = \frac{\rho}{\rho+\mu}(1-e^{-(\mu+\rho)})$$

Hence the dependent probability of retirement is

$$(aq)_x^r = \frac{0.08}{0.08+0.05}(1-e^{-(0.05+0.08)})$$
$$= 0.07502$$

The formula for the independent probability of death is

$$q_x^d = 1-e^{-\mu}$$

Hence the independent probability of death is:

$$q_x^d = 1-e^{-0.05} = 0.04877$$

Generally this question was completed satisfactorily by well prepared students.

6 (i) The definition of the uniform distribution of deaths (UDD) is ${}_s q_x = s.q_x$ (alternatively ${}_t p_x \mu_{x+t}$ is constant).

(ii) We have

$${}_{1.25}p_{65.5} = {}_{0.5}p_{65.5} \times {}_{0.75}p_{66}$$

$${}_{0.5}p_{65.5} = (1 - {}_{0.5}q_{65.5}) = (1-(0.5q_{65}/(1-0.5q_{65}))) \text{ by UDD}$$
$$= (1-((0.5\times 0.02447)/(1-0.5\times 0.02447)))$$
$$= 0.98761$$

$${}_{0.75}p_{66} = 1 - {}_{0.75}q_{66} = 1-0.75\times q_{66} = 1-0.75\times 0.02711$$
$$= 0.97967$$

Hence

$${}_{1.25}p_{65.5} = 0.98761\times 0.97967 = 0.96753$$
$$\Rightarrow {}_{1.25}q_{65.5} = 1 - {}_{1.25}p_{65.5} = 1-0.96753$$
$$= 0.03247$$

A straightforward question that was generally done well.

7 Education influences the awareness of a healthy lifestyle, which reduces morbidity.

Education includes formal and informal processes, such as public health awareness campaigns.

Shows in:

- Increased income
- Better diet
- Increased exercise
- Better health care
- Reduced alcohol and tobacco consumption
- Lower levels of illicit drug use
- Safer sexual practices

Some effects are direct (e.g. drug use); some are indirect (e.g. exercise)

Students generally scored on a range of points but in most cases did not write enough of them to gain all the marks.

Students who mentioned over indulgence risks for the better educated were given credit.

8 Let b be the simple bonus rate (expressed as a percentage of the sum assured). Then the equation of value at 4% p.a. interest is (where P = 3,212):

$$P(.975\ddot{a}_{[40]:\overline{25}|}+0.025)=(100,000+350)\bar{A}_{[40]}+1,000b(I\bar{A})_{[40]}+300+0.5P$$

$$P(.975\times 15.887+0.025)=$$

$$(100,000+350)\times(1.04)^{0.5}\times 0.23041+1,000b\times(1.04)^{0.5}\times 7.95835+300+0.5P$$

$$\Rightarrow 49,833.6179=23,579.5423+8,115.9564b+1,906$$

$$\Rightarrow b=\frac{24,348.0756}{8,115.9564}=3.00$$

i.e. a simple bonus rate of 3% per annum

Generally done well although some students treated b as not vesting in the first year.

9 Value of benefits using premium conversion

$$100,000\bar{A}_{52:50}=100,000\times(1.04)^{1/2}\times A_{52:50}$$

$$=100,000\times(1.04)^{1/2}\times(1-(0.04/1.04)\times\ddot{a}_{52:50})$$

$$=101,980.4\times(1-0.0384615\times 17.295)$$

$$=34,143.89$$

Value of monthly premium of P

$$12P\ddot{a}^{(12)}_{52:50:\overline{5}|}=12P\left(\ddot{a}^{(12)}_{52:50}-\left(v^5\times {}^{l_{57:55}}\!\!/_{l_{52:50}}\right)\times\ddot{a}^{(12)}_{57:55}\right)$$

$$\ddot{a}^{(12)}_{52:50}=\ddot{a}_{52:50}-11/24=17.295-0.458=16.837$$

$$\ddot{a}^{(12)}_{57:55}=\ddot{a}_{57:55}-11/24=15.558-0.458=15.100$$

$$\left(v^5\times {}^{l_{57:55}}\!\!/_{l_{52:50}}\right)=(0.82193\times 9,880.196\times 9,917.623)/(9,930.244\times 9,952.697)$$

$$=0.81491$$

Hence $12P\ddot{a}^{(12)}_{52:50:\overline{5}|}=12P(16.837-0.81491\times 15.100)=54.3823P$

Therefore:

$$P=34,143.89/54.3823=627.85$$

There was an anomaly in this question in that it was not fully clear that the premium paying period ceased on 1st death within the 5 year period. Even though the vast majority of students who completed this question used the above solution a small minority used $12P\ddot{a}^{(12)}_{\overline{5}|}$ *i.e. ignoring the joint life contingency. This was credited.*

None the less many students struggled with this question

10 Expected present value is $A_{[40]:\overline{15|}}$ where

$$A_{[40]:\overline{15|}} = A^{1}_{[40]:\overline{15|}} + A_{[40]:\overset{\,1}{\overline{15|}}}$$

$$= A_{[40]} - v^{15}\,{}_{15}p_{[40]}A_{55} + v^{15}\,{}_{15}p_{[40]}$$

$$= 0.23041 - \left(0.55526 \times \frac{9,557.8179}{9,854.3036} \times 0.38950\right) + 0.55526 \times \frac{9,557.8179}{9,854.3036}$$

$$= 0.23041 - 0.20977 + 0.53855$$

$$= 0.55919$$

Variance

$$= {}^{2}A_{[40]:\overline{15|}} - (A_{[40]:\overline{15|}})^2$$

$${}^{2}A_{[40]:\overline{15|}} = {}^{2}A^{1}_{[40]:\overline{15|}} + {}^{2}A_{[40]:\overset{\,1}{\overline{15|}}}$$

$$= {}^{2}A_{[40]} - (v^2)^{15}\,{}_{15}p_{[40]}\,{}^{2}A_{55} + (v^2)^{15}\,{}_{15}p_{[40]}$$

$$= 0.06775 - \left(0.30832 \times \frac{9,557.8179}{9,854.3036} \times 0.17785\right) + 0.30832 \times \frac{9,557.8179}{9,854.3036}$$

$$= 0.06775 - 0.05318 + 0.29904$$

$$= 0.31361$$

So variance $= 0.31361 - 0.55919^2 = 0.000917$

Note answers are sensitive to number of decimal places used.

Question done well by well prepared students. Many students failed to realise that the endowment function needed to be split into the term and pure endowment portions.

11 Summary of assumptions:

Annual premium	2,500.00	Allocation % (1st yr)	40%
Risk discount rate	5.5%	Allocation % (2nd yr +)	110%
Interest on investments	4.25%	Man charge	0.5%
Interest on sterling provisions	3.5%	B/O spread	5.0%
Minimum death benefit	10,000.00	Maturity benefit	105%

	£	% prm	Total
Initial expense	325	10.0%	575
Renewal expense	74	2.5%	136.5

(i) Multiple decrement table:

x	q_x^d	q_x^s
61	0.006433	0.06
62	0.009696	0.06
63	0.011344	0.06
64	0.012716	0.06

x	$(aq)_x^d$	$(aq)_x^s$	(ap)	$_{t-1}(ap)$
61	0.006240	0.05981	0.933953	1.000000
62	0.009405	0.05971	0.930886	0.933953
63	0.011004	0.05966	0.929337	0.869404
64	0.012335	0.05962	0.928047	0.807969

(ii) Unit fund (per policy at start of year)

	yr 1	*yr 2*	*yr 3*	*yr 4*
value of units at start of year	0.00	985.42	3,732.08	6,581.15
alloc	1,000.00	2,750.00	2,750.00	2,750.00
B/O	50.00	137.50	137.50	137.50
interest	40.37	152.91	269.65	390.73
management charge	4.95	18.75	33.07	47.92
value of units at year end	985.42	3,732.08	6,581.15	9,536.46

Non-unit fund (per policy at start of year)

	yr 1	*yr 2*	*yr 3*	*yr 4*
unallocated premium	1,500.00	–250.00	–250.00	–250.00
B/O spread	50.00	137.50	137.50	137.50
expenses/commission	575.00	136.50	136.50	136.50
interest	34.12	–8.72	–8.72	–8.72
man charge	4.95	18.75	33.07	47.92
extra death benefit	56.25	58.95	37.62	5.72
extra surrender benefit	–58.94	–148.20	–168.91	–121.41
extra maturity benefit	0.00	0.00	0.00	442.51
end of year cashflow	1,016.76	–149.71	–93.36	–536.62

(iii)

probability in force	1	0.933953	0.869404	0.807969
discount factor	0.947867	0.898452	0.851614	0.807217
expected p.v. of profit	**419.03**			
premium signature	2,500.00	2,213.16	1,952.79	1,720.19
expected p.v. of premiums	**8,386.15**			
profit margin	**5.00%**			

Credit was given to students who showed good understanding of the processes involved even if the calculations were not correct. Generally well prepared students did this question quite well.

12 (i) Let P be the monthly premium. Then:

EPV of premiums:

$$12P\ddot{a}^{(12)}_{[40]:\overline{25}|} @ 4\% = 186.996P$$

where

$$\ddot{a}^{(12)}_{[40]:\overline{25}|} = \ddot{a}_{[40]:\overline{25}|} - \frac{11}{24}\left(1 - {}_{25}p_{[40]}v^{25}\right)$$

$$= 15.887 - \frac{11}{24}\left(1 - \frac{8821.2612}{9854.3036} \times 0.37512\right)$$

$$= 15.583$$

EPV of benefits:

$$75,000(q_{[40]}v^{0.5} + {}_{1}|q_{[40]}(1+b)v^{1.5} + \ldots + {}_{24}|q_{[40]}(1+b)^{24}v^{24.5})$$

where b = 0.04

$$= \frac{75,000 \times (1+i)^{0.5}}{(1+b)} \times A^{1}_{[40]:\overline{25|}} @ i' = \frac{75,000 \times (1+i)^{0.5}}{(1+b)} \left[A_{[40]} - {}_{25}p_{[40]} v^{25} A_{65} \right] @ i'$$

$$= \frac{75,000}{(1.04)^{0.5}} \times \left(1 - \frac{8821.2612}{9854.3036} \times 1 \times 1 \right)$$

$$= 7709.6880$$

where

$$i' = \frac{1.04}{1+b} - 1 = 0.00 \text{ i.e. } i' = 0\%$$

EPV of expenses (at 4% unless otherwise stated

$$= 0.5 \times 12P + 400 + 0.025 \times 12P\ddot{a}^{(12)}_{[40]:\overline{25|}} - 0.025 \times 12P\ddot{a}^{(12)}_{[40]:\overline{1|}} + 75\left[\ddot{a}^{@0\%}_{[40]:\overline{25|}} - 1 \right]$$
$$+300\bar{A}^{1}_{[40]:\overline{25|}}$$
$$= 6P + 400 + 0.025 \times 12P \times 15.583 - 0.025 \times 12P \times 0.982025 + 75 \times 23.27542$$
$$+300 \times 0.05422$$
$$= 6P + 400 + 4.6749P - 0.2946P + 1745.6558 + 16.266$$
$$= 10.3803P + 2161.9218$$

where

$$\ddot{a}^{(12)}_{[40]:\overline{1|}} = \ddot{a}_{[40]:\overline{1|}} - \frac{11}{24}\left(1 - p_{[40]}v\right)$$
$$= 1 - \frac{11}{24}\left(1 - \frac{9846.5384}{9854.3036} \times 0.96154\right) = 0.982025$$

$$\ddot{a}^{@0\%}_{[40]:\overline{25|}} - 1 = \frac{1}{l_{[40]}}\left(l_{[40]+1} + \ldots. + l_{64}\right) = e_{[40]} - \frac{l_{64}}{l_{[40]}} e_{64}$$
$$= 39.071 - \frac{8934.8771}{9854.3036} \times 17.421 = 23.27541$$

$$\bar{A}^{1}_{[40]:\overline{25|}} = 1.04^{0.5} A^{1}_{[40]:\overline{25|}} = 1.04^{0.5}\left[A_{[40]:\overline{25|}} - v^{25}\frac{l_{65}}{l_{[40]}}\right]$$

$$= 1.04^{0.5}\left[0.38896 - 0.37512 \times \frac{8821.2612}{9854.3036}\right] = 0.05422$$

Equation of value gives:

$$186.996P = 7709.6880 + 10.3803P + 2161.9218$$

$$\Rightarrow P = \frac{9871.6098}{176.6157} = £55.89$$

(ii) Gross prospective policy value at $t = 23$ (calculated at 4%) is given by:

$$V^{\text{prospective}} = 75{,}000 \times (1.04)^{23} \times v^{0.5}\left[q_{63} + (1.04) \times p_{63}q_{64} \times v\right] + 300v^{0.5}\left[q_{63} + p_{63}q_{64}v\right] + 0.025 \times 12P\ddot{a}^{(12)}_{63:\overline{2|}}$$
$$+75 \times (1.04)^{23}\left[1 + (1.04)p_{63}v\right] - 12P\ddot{a}^{(12)}_{63:\overline{2|}}$$

$$= 184{,}853.66 \times 0.98058 \times \left[0.011344 + (1.04) \times 0.988656 \times 0.012716 \times 0.96154\right]$$
$$+300 \times 0.98058 \times \left[0.011344 + 0.988656 \times 0.012716 \times 0.96154\right] + 0.025 \times 12 \times 55.89 \times 1.90629$$
$$+184.854\left[1 + (1.04) \times 0.988656 \times 0.96154\right] - 12 \times 55.89 \times 1.90629$$

where $\ddot{a}^{(12)}_{63:\overline{2|}} = \ddot{a}_{63:\overline{2|}} - \frac{11}{24}\left(1 - v^2 \times \frac{l_{65}}{l_{63}}\right) = 1.951 - \frac{11}{24}\left(1 - 0.92456 \times \frac{8821.2612}{9037.3973}\right) = 1.90629$

$$= 4{,}335.0628 + 6.8932 + 31.9628 + 367.6104 - 1{,}278.5106$$
$$= £3{,}463.02$$

This question was generally not done well especially part (ii). In part (i) although it was commonly recognised that a resultant rate of interest of 0% emerged students did not often seem to know how to progress from there.

13 (i) The death strain at risk for a policy for year $t + 1$ ($t = 0, 1, 2\ldots$) is the excess of the sum assured (i.e. the present value at time $t + 1$ of all benefits payable on death during the year $t + 1$) over the end of year provision.

i.e. DSAR for year $t + 1 = S - {}_{t+1}V$

The "expected death strain" for year $t + 1$ ($t = 0, 1, 2\ldots$) is the amount that the life insurance company expects to pay extra to the end of year provision for the policy.

i.e. EDS for year $t + 1 = q(S - {}_{t+1}V)$

The "actual death strain" for year t + 1 (t = 0, 1, 2…) is the observed value at t+1 of the death strain random variable

i.e. ADS for year t + 1 $= (S - {}_{t+1}V)$ if the life died in the year t to t+1
= 0 if the life survived to t + 1

Note: Full credit given if definition of death strain is given for a block of policies rather than for a single policy as per above.

(ii) (a) Annual premium for endowment assurance with £100,000 sum assured given by:

$$P^{EA} = \frac{100,000}{\ddot{a}_{35:\overline{25}|}} \times A_{35:\overline{25}|} = \frac{100,000}{16.027} \times 0.38359 = 2,393.40$$

Annual premium for term assurance with £200,000 sum assured given by:

$$P^{TA} = \frac{200,000 A^{1}_{40:\overline{25}|}}{\ddot{a}_{40:\overline{25}|}}$$

where $A^{1}_{40:\overline{25}|} = A_{40:\overline{25}|} - v^{25}\,{}_{25}p_{40}$

$$= 0.38907 - 0.37512 \times \frac{8,821.2612}{9,856.2863} = 0.38907 - 0.33573 = 0.05334$$

$$P^{TA} = \frac{200,000 \times 0.05334}{15.884} = 671.62$$

Reserves at the end of the 11th year:

– for endowment assurance with £100,000 sum assured given by:

$${}_{11}V^{EA} = 100,000 \times A_{46:\overline{14}|} - P^{EA}\ddot{a}_{46:\overline{14}|}$$

$$= 100,000 \times 0.58393 - 2,393.40 \times 10.818$$

$$= 58,393.0 - 25,891.8 = 32,501.2$$

– for term assurance with £200,000 sum assured given by:

$$ {}_{11}V^{TA} = 200,000 A^{1}_{51:\overline{14|}} - P^{TA} \ddot{a}_{51:\overline{14|}} $$

where $A^{1}_{51:\overline{14|}} = A_{51:\overline{14|}} - v^{14}\,{}_{14}p_{51}$

$$ = 0.58884 - 0.57748 \times \frac{8,821.2612}{9,687.7149} = 0.58884 - 0.52583 = 0.06301 $$

$$ {}_{11}V^{TA} = 200,000 \times 0.06301 - 671.62 \times 10.69 $$

$$ = 12,602.0 - 7,179.6 = 5,422.4 $$

Therefore, sums at risk are:

Endowment assurance: DSAR = 100,000 – 32,501.2 = 67,498.8

Term assurance: DSAR = 200,000 – 5,422.4 = 194,577.6

(b) Mortality profit = EDS – ADS

For endowment assurance

$$ EDS = 19768 \times q_{45} \times 67,498.8 = 19768 \times 0.001465 \times 67,498.8 = 1,954,773.3 $$

$$ ADS = 36 \times 67,498.8 = 2,429,956.8 $$

mortality profit = –475,183.5 (i.e. a loss)

For term assurance

$$ EDS = 9,855 \times q_{50} \times 194,577.6 = 9,855 \times .002508 \times 194,577.6 = 4,809,246.1 $$

$$ ADS = 22 \times 194,577.6 = 4,280,707.2 $$

mortality profit = 528,538.9
Hence, total mortality profit = 528,538.9 – 475,183.5 = £53,355.4

(c) Although there is an overall mortality profit in 2010, the actual number of deaths for the endowment assurances is approximately 25% higher than expected, which is a concern. Further investigation would be required to determine reasons for poor mortality experience for the endowment assurances, e.g. there may have been limited underwriting requirements applied to this type of contract when they were written.

Generally (a) was done well. The most common error in (b) was to assume reserves at 10 years rather than 11. On the whole well prepared students coped with (b) well. Many students did not attempt (c) or at best gave a somewhat sketchy answer.

END OF EXAMINERS' REPORT