INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2014 examinations

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

D C Bowie
Chairman of the Board of Examiners

June 2014

General comments on Subject CT5

CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions actuarial work.

Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given where appropriate to different valid points made which do not appear in the solutions below.

In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

Comments on the April 2014 paper

The general performance was similar this session to previous ones. Questions that were done less well were Q3, Q5, Q8, Q11 part (iii) and Q13 part (ii). The examiners hope that the detailed solutions given below will assist students with further revision.

However most of the short questions were very straightforward where an answer could be produced quickly and this is where many successful candidates scored particularly well. Students should note that for long questions some credit is given if they can describe the right procedures although to score well reasonably accurate numerical calculation is necessary.

1 Each group is specified by a category or class of a particular characteristic of the population. The stochastic models (life tables) are different for each class. There are no common features to the models, they are different for all ages. This is termed class selection.

Examples are:

- Gender differences
- Distinction of Smoker and Non-Smoker status
- Occupation

Other examples credited.

Generally well done with no significant issues.

2 (a)

$$\ddot{a}^{(4)}_{25:\overline{20}|} = \ddot{a}^{(4)}_{25} - \frac{l_{45}}{l_{25}} \times v^{20} \times \ddot{a}^{(4)}_{45}$$

$$= \left(\ddot{a}_{25} - \frac{3}{8}\right) - \left(\frac{l_{45}}{l_{25}} \times v^{20} \times \left(\ddot{a}_{45} - \frac{3}{8}\right)\right)$$

$$= (22.520 - 0.375) - \left(\frac{9801.3123 \times 0.45639}{9953.6144} \times (18.823 - 0.375)\right)$$

$$= 22.145 - (0.4494 \times 18.448)$$

$$= 13.854$$

(b)

$$(I\bar{A})^{1}_{25:\overline{20}|} = (1.04)^{1/2} \times \left((IA)_{25} - \frac{l_{45}}{l_{25}} \times v^{20} \times ((IA)_{45} + 20 \times A_{45})\right)$$

$$= 1.0198 \times (6.33195 - 0.4494 \times (8.33628 + 20 \times 0.27605))$$

$$= 0.10656$$

Generally well done with no significant issues. The main error involved the accuracy of the formula in line 1 of part (b) above.

3

$${}_t p_x = \frac{l_0 e^{-0.15(x+t)}}{l_0 e^{-0.15x}} = e^{-0.15t}$$

Therefore:

$$\ddot{a}_{3:\overline{5}|} = \sum_{0}^{4} (1.05)^{-t} \times e^{-0.15t} = \frac{1 - ((1.05)^{-1} e^{-0.15})^5}{1 - ((1.05)^{-1} e^{-0.15})} = \frac{1 - 0.37011}{1 - 0.81972}$$

$$= 3.4940$$

Hence:

$$A_{3:\overline{5}|} = 1 - \left(\frac{.05}{1.05} \times 3.4940 \right)$$
$$= 0.83362$$

This question was poorly done. From the solution above it will be seen that the answer is very straightforward if premium conversion is used. Most students failed to realise this and attempted the question the longer direct way which involves a much more arduous calculation (full credit was given if this produced the correct answer).

4 Cash flow techniques promote understanding and clarity of thought

Cash flow techniques are more easily presented to non-actuaries

Cash flow techniques can be helpful when an office wishes to design an appropriate investment strategy to cope with expected future cash flows.

Cash flow techniques allow much more flexibility e.g.

- Premium basis with varying or stochastic interest rates
- Complex policy designs e.g. varying benefits or options
- Sensitivity analysis can be easy to do on a computer once the model has been set up
- Multiple state models (e.g. PHI) can be dealt with, which is not possible using commutation functions
- Allowance can be made for negative values.

Generally done reasonably well. Each distinct point mentioned above gained a mark up to the maximum for the question. Other valid points not contained above were also credited.

5 At 65 the member would have completed 20 + 16 = 36 years' service so the maximum of 2/3 applies in this case.

$$\text{Value } = \frac{2}{3} \times 40,000 \times \frac{{}^{z}C_{65}^{ra}}{s_{44} D_{45}}$$
$$= \frac{2}{3} \times 40,000 \times \left(\frac{45,467}{8.375 \times 2329} \right)$$
$$= 62,160$$

This very simple question was very poorly done. The question makes it clear that age retirement takes place at age 65 <u>only</u>. A large proportion of students tried to apply Past and Future Service values directly from the Tables which includes all other age retirement possibilities. This is not only arduous given the service limit but is invalid in this case.

6 (a) The method of approximation based on the assumption of a constant force of mortality assumes that for integer x and $0 \le t < 1$, we have:

$\mu_{x+t} = \mu$ = constant.

Then the appropriate relationship is:

$${}_t p_x = \exp\left\{-\int_0^t \mu_{x+s}\, ds\right\} = e^{-t\mu}$$

From this μ can be derived.

(b) From the relationship in (a) we can derive:

$${}_t p_x = ({}_1 p_x)^t$$

Therefore:

$$\begin{aligned}
{}_{2.75} p_{85.5} &= {}_{0.5} p_{85.5} \times p_{86} \times p_{87} \times {}_{0.25} p_{88} \\
&= (p_{85})^{0.5} \times p_{86} \times p_{87} \times p_{88}{}^{0.25} \\
&= (1-0.14372)^{0.5} \times (1-0.15585) \times (1-0.16848) \times (1-0.18061)^{0.25} \\
&= 0.92534 \times 0.84415 \times 0.83152 \times 0.95142 \\
&= 0.61797 \text{ so } {}_{2.75} q_{85.5} = (1-0.61797) \\
&= 0.38203
\end{aligned}$$

Generally well done. Part (a) above is a detailed explanation and lesser detail gained full credit.

7 The annuity is equivalent to:

£6,000 p.a. whilst at least one life survives

An additional £1,500 p.a. if the female is surviving

A further amount of £2,500 p.a. if both lives survive.

The expected present value is:

$$\begin{aligned}
&= 2500a^{(4)}_{65:62} + 1500a^{(4)}_{62} + 6000a^{(4)}_{\overline{65:62}} \\
&= 2500(\ddot{a}_{65:62} - 0.625) + 1500(\ddot{a}_{62} - 0.625) + 6000(\ddot{a}_{65} + \ddot{a}_{62} - \ddot{a}_{65:62} - 0.625) \\
&= 6000\ddot{a}_{65} + 7500\ddot{a}_{62} - 3500\ddot{a}_{65:62} - 6250 \\
&= (6000 \times 13.666) + (7500 \times 15.963) - (3500 \times 12.427) - 6250 \\
&= £151974
\end{aligned}$$

Other approaches were acceptable. Well prepared students coped well with this question but others found difficulties in analysing the contingencies.

8 (i) $l^{\beta}_{x+t} = l^{\beta}_{x} - t^5 d^{\beta}_{x} \Rightarrow {}_t p^{\beta}_{x} = 1 - t^5 q^{\beta}_{x}$ for $0 \le t \le 1$

Thus $\dfrac{\partial({}_t p^{\beta}_{x})}{\partial t} = -5t^4 q^{\beta}_{x}$

Also $\dfrac{\partial({}_t p^{\beta}_{x})}{\partial t} = -{}_t p^{\beta}_{x}\mu^{\beta}_{x+t}$ for $0 \le t \le 1$ (from ${}_t p^{\beta}_{x} = e^{-\int_0^t \mu^{\beta}_{x+r}dr}$)

Therefore ${}_t p^{\beta}_{x}\mu^{\beta}_{x+t} = 5t^4 q^{\beta}_{x}$ as required

(ii)

$$\begin{aligned}
(aq)^{\beta}_{x} &= \int_0^1 {}_t p^{\alpha}_{x}\, {}_t p^{\beta}_{x}\mu^{\beta}_{x+t}\,dt \\
&= \int_0^1 {}_t p^{\alpha}_{x}(5t^4 q^{\beta}_{x})\,dt \text{ from (i) above} \\
&= \int_0^1 (1 - t^3 q^{\alpha}_{x})(5t^4 q^{\beta}_{x})\,dt \\
&= 5q^{\beta}_{x}\int_0^1 t^4(1 - t^3 q^{\alpha}_{x})\,dt \\
&= 5q^{\beta}_{x}\left[\frac{t^5}{5} - \frac{t^8 q^{\alpha}_{x}}{8}\right]_0^1 \\
&= q^{\beta}_{x}\left(1 - \frac{5}{8}q^{\alpha}_{x}\right) \text{ as required}
\end{aligned}$$

This question was poorly done. Part (i) was essentially just the combining together of two bookwork formulae. Part (ii) could have been easily attempted just using the result of part (i) but a majority of students did not seem to really understand how to start this question.

9 $EPV = \bar{A}^1_{x:\overline{20}|} = 0.03\int_0^{20} e^{-.05t} \times e^{-.03t} dt$

$$= \frac{0.03}{0.08}\left[-e^{-.08t}\right]_0^{20}$$
$$= 0.375 \times (1 - e^{-1.6})$$
$$= 0.375 \times 0.79810$$
$$= 0.29929$$

For the Variance:

$${}^2\bar{A}^1_{x:\overline{20}|} = 0.03\int_0^{20} e^{-.1t} \times e^{-.03t} dt$$
$$= \frac{0.03}{0.13}\left[-e^{-.13t}\right]_0^{20}$$
$$= \frac{0.03}{0.13}(1 - e^{-2.6})$$
$$= 0.23077 \times 0.92573$$
$$= 0.21363$$

Hence

$$\text{Variance } = {}^2\bar{A}^1_{x:\overline{20}|} - (\bar{A}^1_{x:\overline{20}|})^2$$
$$= 0.21363 - (0.29929)^2$$
$$= 0.12406$$
$$= (0.35221)^2$$

Generally well done. The question in essence should have been technically posed in random variable form as the $\bar{A}^1_{x:\overline{20}|}$ *function is already the expected value and strictly in those circumstances the variance could be argued as zero. However virtually all students produced the solution above and were not concerned with this point so no difficulties emerged. Anybody pointing out the anomaly gained full credit.*

10 (i) (a) Crude mortality rate is the ratio of the total number of deaths in a category to the total exposed to risk in the same category.

(b) Directly standardised mortality rate is the mortality rate of a category weighted according to a standard population.

(c) Indirectly standardised mortality rate is an approximation to the directly standardised mortality rate being the crude rate for the standard population multiplied by the ratio of actual to expected deaths for the region.

(d) The Area comparability factor is a measure of the crude mortality rate for the standard population divided by what the crude mortality rate is for the region being studied, assuming the mortality rates are the same as for the standard population..

(ii) Crude death rate: Occupational Group = 284/46000 = 0.006174

The Directly Standardised Mortality Rate is:

$$\left(\frac{(1000000\times\frac{67}{20000})+(1500000\times\frac{92}{15000})+(700000\times\frac{125}{11000})}{3200000}\right)$$

$$=\left(\frac{3350+9200+7954.55}{3200000}\right)$$

$$=0.00641$$

The Indirectly Standardised Mortality Rate can be calculated as follow:

Expected Deaths for Occupation:

$$\left(\frac{20000\times3500}{1000000}+\frac{15000\times7800}{1500000}+\frac{11000\times8000}{700000}\right)$$

$$=70+78+125.71=273.71$$

So the Indirectly Standardised Mortality Rate is:

$$\frac{0.006031\times284}{273.71}=0.00626$$

Generally well done. In part (i) students who put the formulae into words were given full credit.

11 (i) Let P be the annual premium for the policy.

Then (functions at 4%) equation of value gives:

$$P\ddot{a}_{[40]:\overline{20}|}=1,000A_{[40]:\overline{20}|}+0.54P+0.06P\ddot{a}_{[40]:\overline{20}|}$$

$$\Rightarrow P=\frac{1,000\times0.46423}{0.94\times13.930-0.54}=36.98$$

(ii) On 31 December 2012, the gross premium prospective reserve per £1,000 sum assured is given by:

$$_5V = 1{,}000 A_{45:\overline{15}|} - 0.94 \times 36.98 \times \ddot{a}_{45:\overline{15}|}$$
$$\Rightarrow {}_5V = 1{,}000 \times 0.56206 - 0.94 \times 36.98 \times 11.386$$
$$= 562.06 - 395.79 = 166.27$$

(iii) If we consider the total portfolio of non-profit endowment policies during 2013, we have:

Reserve on 31 December 2012 = 15,500 ×166.27 = 2,577,185

Premiums (***P***) paid on 1 January 2013 = 15,500 × 36.98 = 573,190

Expenses (***E***) incurred on 1 January 2013 = 76,500

Interest (***I***) earned during 2013 = 0.035 × (2,577,185 + 573,190 – 76,500)
= 107,585.6

Death claims (***D***) during 2013 = 295,000

On 31 December 2013, the gross premium prospective reserve per £1,000 sum assured is given by:

$$_6V = 1{,}000 A_{46:\overline{14}|} - 0.94 \times 36.98 \times \ddot{a}_{46:\overline{14}|}$$
$$\Rightarrow {}_6V = 1{,}000 \times 0.58393 - 0.94 \times 36.98 \times 10.818$$
$$= 583.93 - 376.05 = 207.88$$

Total surrender values paid (***S***) during 2013
= 625 × 0.85 × 207.88 = 110,436.3

Total sum assured in force at 31 December 2013
= 15,500,000 – 295,000 – 625,000 = 14,580,000

Reserve on policies in force at 31 December 2013
= 14,580 × 207.88 = 3,030,890.4
Total Profit for 2013 =

$$= \sum {}_5V + P - E + I - D - S - \sum {}_6V$$
$$= 2{,}577{,}185 + 573{,}190 - 76{,}500 + 107{,}585.6 - 295{,}000 - 110{,}436.3 - 3{,}030{,}890.4$$
$$= -254{,}866.1$$

i.e. an experience loss of £254,866

Parts (i) and (ii) were generally well done. Part (iii) was less well done. Many students successfully obtained the mortality profit but were unable to quantify others as shown above. In particular identifying surrenders caused difficulties.

12 (i) Decrement table

age	q_x	p_x	$_{t-1}p_x$
65	0.006032	0.993968	1.000000
66	0.007147	0.992853	0.993968
67	0.008439	0.991561	0.986864

(ii) Cash flows for policy:

(a) With reserves

Year	*Opening reserve*	*Premium*	*Initial expense*	*Interest*	*Annuity claim*	*Annuity expense*	*Closing reserve*	*Profit vector*
1	0.00	42000.00	770.00	2061.50	14909.52	56.31	29819.04	–1493.37
2	30000.00	0.00	0.00	1500.00	14892.80	57.93	14892.80	1656.47
3	15000.00	0.00	0.00	750.00	14873.42	59.59	0.00	816.99

Year	*Profit vector*	*Profit signature*	*Discount factor*	*PVFNP*
1	–1493.37	–1493.37	0.934579	–1395.67
2	1656.47	1646.48	0.873439	1438.10
3	816.99	806.26	0.816298	658.15

Total PVFNP = 700.58

(b) Without reserves

Year	*Opening reserve*	*Premium*	*Initial expense*	*Interest*	*Annuity claim*	*Annuity expense*	*Closing reserve*	*Profit vector*
1	0.00	42000.00	770.00	2061.50	14909.52	56.31	0.00	28325.67
2	0.00	0.00	0.00	0.00	14892.80	57.93	0.00	–14950.73
3	0.00	0.00	0.00	0.00	14873.42	59.59	0.00	–14933.01

Year	*Profit vector*	*Profit signature*	*Discount factor*	*PVFNP*
1	28325.67	28325.67	0.934579	26472.59
2	–14950.73	–14860.54	0.873439	–12979.78
3	–14933.01	–14736.85	0.816298	–12029.66

Total PVFNP = 1463.15

(ii) The net present value is smaller when reserves are set up because we are tying up money in the reserves which are subject to a lower rate of interest (5%) than the risk discount rate (7%) c.f. 700.58 compared to 1463.15.

(iii) With reserves, the net present value of the expected profit will increase if the risk discount rate is reduced from 7% per annum to 4% per annum because the positive profit signature in year 2 and 3 become more significant (note: NPV increases from 700.58 to 803.10).

Without reserves, the net present value of the expected profit will fall if the risk discount rate is reduced from 7% per annum to 4% per annum because the negative profit signature in year 2 and 3 become more significant (note: NPV decreases from 1463.16 to 395.81).

The net present value of expected profits without reserves would now be less than the net present value of expected profits with reserves. This is because the reserves are now subject to a higher rate of interest (5%) than the risk discount rate (4%) (note: 803.10 compared to 395.81).

Many well prepared students answered this question well. Other than accuracy of the numbers the main omission was the detail expected in part (iii). Note that in part (i) a general understanding of the methods needed to solve the problem earned proportionate credit even if the numerical accuracy was not always apparent.

13 (i) Let P be the annual premium for the contract. Then:

EPV of premiums is:

$$P\ddot{a}^{6\%}_{30:\overline{35|}} = 15.150P$$

EPV of benefits:

$$60,000\left[\frac{1}{1.0192308}\times(1.06)^{0.5}\times A^{1}_{30:\overline{35|}} + v^{35}\,{}_{35}p_{30}\right]@\,4\%$$

$$= 60,000[0.04176 + 0.22523] = 16,019.40$$

where

$$A_{30:\overline{35|}} = 0.26657$$

$$v^{35}\,{}_{35}p_{30} = 0.25342\times\frac{8821.2612}{9925.2094} = 0.22523$$

$$A^{1}_{30:\overline{35|}} = A_{30:\overline{35|}} - v^{35}\,{}_{35}p_{30} = 0.04134$$

EPV of expenses:

$$250 + 0.575P + 0.025P\ddot{a}^{6\%}_{30:\overline{35|}} = 250 + 0.95375P$$

Equation of value gives

$$15.15P = 16,019.40 + 250 + 0.95375P$$
$$\Rightarrow P = £1146.04$$

(ii) Gross future loss random variable

= *PV* future benefit payment + *PV* future expenses – *PV* of future premiums

$$= G(K_{30+t}) + 0.025 \times 1146.04 \ddot{a}_{\overline{\min[K_{30+t}+1, 35-t]}|} - 1146.04 \ddot{a}_{\overline{\min[K_{30+t}+1, 35-t]}|}$$

where

$$G(K_{30+t}) = 60,000 \times (1.0192308)^{t+K_{30+t}} \times v_{.06}^{T_{30+t}} \quad \text{if} \quad K_{30+t} < 35-t$$

or

$$G(K_{30+t}) = 60,000 \times (1.0192308)^{35} \times v_{.06}^{35-t} \quad \text{if} \quad K_{30+t} \geq 35-t$$

(iii) Sum assured and attaching bonuses at 31 December 2012

$$= 60,000(1.0192308)^{10} = 72,589.97$$

gross prospective reserve at the end of the 10th policy year is given by:

$${}_{10}V = 72,589.97\left[\frac{1}{1.0192308} \times (1.06)^{0.5} \times A^{1}_{40:\overline{25}|} + v^{25}\,{}_{25}p_{40}\right] @ 4\% - 0.975 \times 1146.04 \ddot{a}^{6\%}_{40:\overline{25}|}$$

where

$$A_{40:\overline{25}|} = 0.38907$$

$$v^{25}\,{}_{25}p_{40} = 0.37512 \times \frac{8821.2612}{9856.2863} = 0.33573$$

$$A^{1}_{40:\overline{25}|} = A_{40:\overline{25}|} - v^{25}\,{}_{25}p_{40} = 0.05334$$

$$\ddot{a}^{6\%}_{40:\overline{25}|} = 13.288$$

$$\Rightarrow {}_{10}V = 72,589.97\left[0.05388 + 0.33573\right] - 0.975 \times 1146.04 \times 13.288$$
$$= 28,281.78 - 14,847.87 = £13,433.91$$

Part (i) was generally well done. Part (ii) was poorly done which is often the case for these types of question. Part (iii) gave more difficulties but was generally completed successfully by well prepared students.

END OF EXAMINERS' REPORT