EXAMINATION

26 April 2010 (am)

Subject CT5 — Contingencies
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Explain what the following represent:

(a) $l_{[x]+r}$

(b) ${}_{n|m}q_x$

(c) d_x

[3]

2 Define spurious selection, giving two distinct examples. [3]

3 Calculate the standardised mortality ratio for the population of Urbania using the following data:

	Standard Population		*Urbania*	
Age	*Population*	*Deaths*	*Population*	*Deaths*
60	2,500,000	26,170	10,000	130
61	2,400,000	29,531	12,000	145
62	2,200,000	32,542	11,000	173

[3]

4 A life insurance company offers an increasing term assurance that provides a benefit payable at the end of the year of death of 10,000 in the first year, increasing by 100 on each policy anniversary.

Calculate the single premium for a five year policy issued to a life aged 50 exact.

Basis:

Rate of interest	4% per annum
Mortality	AM92 Select
Expenses	Nil

[4]

5 A population is subject to the force of mortality $\mu_x = e^{0.0002x} - 1$.

Calculate the probability that a life now aged 20 exact:

(i) survives to age 70 exact [2]

(ii) dies between ages 60 exact and 70 exact [3]

[Total 5]

6 You are provided with the following extract from a life table:

x	l_x
50	99,813
51	97,702
52	95,046

Calculate ${}_{0.75}p_{50.5}$ using two different methods. [5]

7 A company is about to establish a pension scheme that will provide an age retirement benefit of n/60ths of final pensionable salary where n is total number of years of service. Final pensionable salary is the average salary in the three years before retirement.

An employee who will become a member of the pension scheme is currently aged 55 exact has and will be granted exactly 20 years of past service. The employee's salary in the year before the valuation date was £40,000.

(i) Calculate the present value of benefits for this member (including future service). [3]

(ii) Calculate the contribution required to fund this benefit as a percentage of future salaries. [3]

Basis:

Pension Scheme from the Formulae and Tables for Actuarial Examinations

[Total 6]

8 100 graduates aged 21 exact decide to place the sum of £1 per week into a fund to be shared on their retirement at age 66 exact.

(i) Show that each surviving member can expect to receive on retirement a fund of approximately £7,240. [4]

Basis:

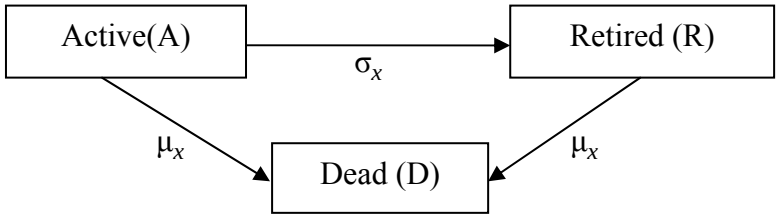
Rate of interest 4% per annum
Mortality AM92 Ultimate

One of the survivors uses the accumulated fund to buy a weekly annuity payable for 10 years certain. After 10 years the annuity is payable at two-thirds of the initial level for the rest of life.

(ii) Calculate the weekly amount of the annuity on the basis used in part (i). [2]

[Total 6]

9 A life insurance company models the experience of its pension scheme contracts using the following three-state model:



(i) Derive the dependent probability of a life currently Active and aged x retiring in the year of age x to $(x + 1)$ in terms of the transition intensities. [2]

(ii) Derive a formula for the independent probability of a life currently Active and aged x retiring in the year of age x to $(x + 1)$ using the dependent probabilities. [4]

[Total 6]

10 The decrement table extract below is based on the historical experience of a very large multinational company's workforce.

Age (x)	*Number of employees* $(al)_x$	*Deaths* $(ad)^d_x$	*Withdrawals* $(ad)^w_x$
40	10,000	25	120
41	9,855	27	144
42	9,684		

Recent changes in working conditions have resulted in an estimate that the annual independent rate of withdrawal is now 75% of that previously used.

Calculate a revised table assuming no changes to the independent death rates, stating your results to one decimal place. [7]

11 Thiele's differential equation for the policy value at duration t ($t > 0$), ${}_t\overline{V}_x$, of an immediate life annuity payable continuously at a rate of £1 per annum from age x is:

$$\frac{\partial}{\partial t}{}_t\overline{V}_x = \mu_{x+t} \times {}_t\overline{V}_x - 1 + \delta \times {}_t\overline{V}_x$$

(i) Derive this result algebraically showing all the steps in your working. [5]

(ii) Explain this result by general reasoning. [3]

[Total 8]

12 On 1 January 2005, a life insurance company issued 1,000 10-year term assurance policies to lives aged 55 exact. For each policy, the sum assured is £50,000 for the first five years and £25,000 thereafter. The sum assured is payable immediately on death and level annual premiums are payable in advance throughout the term of this policy or until earlier death.

The company uses the following basis for calculating premiums and reserves:

Mortality	AM92 Select
Interest	4% per annum
Expenses	Nil

(i) Calculate the net premium retrospective reserve per policy as at 31 December 2009. [6]

(ii) (a) Give an explanation of your numerical answer to (i) above.

(b) Describe the main disadvantage to the insurance company of issuing this policy.

(c) Give examples of how the terms of the policy could be altered so as to remove this disadvantage.

[3]

There were, in total, 20 deaths during the years 2005 to 2008 inclusive and a further 8 deaths in 2009.

(iii) Calculate the total mortality profit or loss to the company during 2009. [3]

[Total 12]

13 A life insurance company issues a 3-year unit-linked endowment assurance policy to a male life aged 45 exact.

Level premiums of £4,000 per annum are payable yearly in advance throughout the term of the policy or until earlier death. 95% of the premium is allocated to units in the first policy year, 100% in the second and 105% in the third. A policy fee of £50 is deducted from the bid value of units at the start of each year. The units are subject to a bid-offer spread of 5% on purchase. An annual management charge of 1.75% of the bid value of units is deducted at the end of each policy year.

Management charges are deducted from the unit fund before death, surrender and maturity benefits are paid.

If the policyholder dies during the term of the policy, the death benefit of 125% of the bid value of the units is payable at the end of the policy year of death. On maturity, 100% of the bid value of the units is payable.

The policyholder may surrender the policy only at the end of the first and second policy years. On surrender, the bid value of the units less a surrender penalty is payable at the end of the policy year of exit. The surrender penalty is £1,000 at the end of the first policy year and £500 at the end of the second policy year.

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund	5.5% per annum in year 1 5.25% per annum in year 2 5.0% per annum in year 3
Rate of interest on non-unit fund cash flows	4.0% per annum
Mortality	AM92 Select
Initial expenses	£200
Renewal expenses	£50 per annum on the second and third premium dates
Initial commission	15% of first premium
Renewal commission	2.0% of the second and third years' premiums
Rate of expense inflation	2.0% per annum
Risk discount rate	7.0% per annum

For renewal expenses, the amount quoted is at outset and the increases due to inflation start immediately. In addition, you should assume that at the end of the first and second policy years, 12% and 6% respectively of all policies still in force then surrender immediately.

(i) Calculate the profit margin for the policy. [13]

(ii) Calculate the expected present value of profit for the policy if the company assumed that there were no surrenders at the end of each of the first and second policy years. [3]

[Total 16]

14 A life insurance company issues a 30-year with profits endowment assurance policy to a life aged 35 exact. The sum assured of £100,000 plus declared reversionary bonuses are payable on survival to the end of the term or immediately on death if earlier.

(i) Show that the quarterly premium payable in advance throughout the term of the policy or until earlier death is approximately £616.

Pricing basis:

Mortality:	AM92 Select
Interest:	6% per annum
Initial commission:	100% of the first quarterly premium
Initial expenses:	£250 paid at policy commencement date
Renewal commission:	2.5% of each quarterly premium from the start of the second policy year
Renewal expenses:	£45 at the start of the second and subsequent policy years
Claim expense:	£500 on death; £250 on maturity
Future reversionary bonus:	1.92308% of the sum assured, compounded and vesting at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year of death)

[10]

At the end of the 25th policy year, the actual past bonus additions to the policy have been £145,000.

(ii) Calculate the gross prospective policy reserve at the end of that policy year immediately before the premium then due.

Policy reserving basis:

Mortality:	AM92 Ultimate
Interest:	4% per annum
Bonus loading:	4% of the sum assured and attaching bonuses, compounded and vesting at the end of each policy year
Renewal commission:	2.5% of each quarterly premium
Renewal expenses:	£90 at the start of each policy year
Claim expense:	£1,000 on death; £500 on maturity

[6]
[Total 16]

END OF PAPER