INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2013 examinations

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

D C Bowie
Chairman of the Board of Examiners

July 2013

General comments on Subject CT5

CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions actuarial work.

Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given where appropriate to different valid points made which do not appear in the solutions below.

In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

Comments on the April 2013 paper

The general performance was similar this session to previous ones although it was felt that this paper was a little easier than some previous ones. Questions that were done less well were 9, 10 (variance), 11, 12(b) and 14(ii). The examiners hope that the detailed solutions given below will assist students with further revision.

However most of the short questions were very straightforward where an answer could be produced quickly and this is where many successful candidates scored particularly well. Students should note that for long questions some credit is given if they can describe the right procedures although to score well reasonably accurate numerical calculation is necessary.

1 (a) $_{10|5}q_{40} = (l_{50} - l_{55}) / l_{40} = (9712.0728 - 9557.8179) / 9856.2863$
$= 0.01565$

(b) $\bar{a}_{65} = \ddot{a}_{65} - 1/2 = 11.776$

(c) $_{15}p_{[46]} = l_{61} / l_{[46]} = 9212.7143 / 9783.3371 = 0.94167$

Generally question done well.

2 The constant force of decrement is consistent with the Kolmogorov equations where the transition intensities are constant.

Thus:

$$(aq)_x^{\alpha} = \frac{\mu_\alpha}{(\mu_\alpha + \mu_\beta)} \times (1 - e^{-(\mu_\alpha + \mu_\beta)})$$

$$= \frac{0.1}{0.3} \times (1 - e^{-0.3}) = 0.086394$$

Generally question done well. Other approaches given credit.

3 Climate and geographical location are closely linked. Levels and patterns of rainfall and temperature lead to an environment which is amicable to certain kinds of diseases e.g. those associated with tropical regions.

Effects can also be observed within these broad categories e.g. the differences between rural and urban areas in a geographical region. Some effects may be accentuated or mitigated depending upon the development of an area e.g. industry leading to better roads and communications.

Natural disasters (such as tidal waves and famines) will also affect mortality and morbidity rates, and may be correlated to particular climates and geographical locations.

Generally question done well. Other valid points given credit.

4

- Terminal bonuses are allocated when a policy matures or becomes a claim as a result of the death of the life assured.

- Terminal bonuses are usually allocated as a percentage of the basic sum assured and the bonuses allocated prior to a claim.

- The terminal bonus percentage rate will vary with the term of the policy at the date of payment.

- Because the policy is being terminated, the terminal bonus rate is usually chosen so as to distribute all the surplus available to the policy based on asset share.

- Distributing available surplus as a terminal bonus delays the distribution of surplus and may allow the insurer to choose investments that are more volatile in the short term but are expected to be more profitable in the long term.

Generally question done well. Other valid points given credit. In particular comments about effects on lapse rates were an important extra point.

5 Past Service:

Value is $\dfrac{10}{60}*40000*\dfrac{({}^{z}M^{ra}_{30}+{}^{z}M^{ia}_{30})}{s_{29}D_{30}}=\dfrac{1}{6}*40000*\dfrac{128026+64061}{4.991*7874}=32585.5$

Future Service:

Value is $\dfrac{1}{60}*40000*\dfrac{({}^{z}\bar{R}^{ra}_{30}+{}^{z}\bar{R}^{ia}_{30})}{s_{29}D_{30}}=\dfrac{1}{60}*40000*\dfrac{4164521+1502811}{4.991*7874}=96140.1$

Total Value is 32585.5+96140.1 = £128,726 rounded

Generally question done well. It was not necessary to give the total in the last line for full credit.

6 The death benefit in policy year 10 is £65,000 which increases by £1,500 each year and the maturity value is £80,000. Therefore:

(a) Net premium P for the policy is given by

$$P=\frac{\left(50,000A^{1}_{30:\overline{20}|}+1,500(IA)^{1}_{30:\overline{20}|}+80,000\times v^{20}\times\dfrac{l_{50}}{l_{30}}\right)}{\ddot{a}_{30:\overline{20}|}}$$

(b) Net premium prospective policy reserve at duration t = 9 is given by:

$${}_{9}V^{\text{Pro}}=63,500A^{1}_{39:\overline{11}|}+1,500(IA)^{1}_{39:\overline{11}|}+80,000\times v^{11}\times\frac{l_{50}}{l_{39}}-P\ddot{a}_{39:\overline{11}|}$$

Well prepared students scored good marks but many made elementary mistakes the most common of which was 48500 as the 1st factor in the numerator of the first formula above.

The alternative solution for the numerator in (a) is:

$50000A_{30:\overline{20}|}+1500(IA)_{30:\overline{20}|}$

And for (b) overall:

$${}_9V = 63500A_{39:\overline{11}|} + 1500(IA)_{39:\overline{11}|} - P\ddot{a}_{39:\overline{11}|}$$

7 When a life table is constructed it is assumed to reflect the mortality experience of a homogeneous group of lives i.e. all the lives to whom the table applies follow the same stochastic model of mortality represented by the rates in the table. This means that the table can be used to model the mortality experience of a homogeneous group of lives which is suspected to have a similar experience.

If a life table is constructed for a heterogeneous group then the mortality experience will depend on the exact mixture of lives with different experiences that has been used to construct the table. Such a table could only be used to model mortality in a group with the same mixture. It would have very restricted uses.

For this reason separate mortality tables are usually constructed for groups which are expected to be heterogeneous. This can manifest itself as class selection e.g. separate tables for males and females, whole life and term assurance policyholders, annuitants and pensioners, or as time selection e.g. separate tables for males in England and Wales in 1980–82 (ELT14) and 1990–92 (ELT15).

Sometimes only parts of the mortality experience are heterogeneous e.g. the experience during the initial select period for life assurance policyholders, and the remainder are homogeneous e.g. the experience after the end of the select period for life assurance policyholders. In such cases the tables are separate (different) during the select period, but combined after the end of the select period. In fact there are separate (homogeneous) mortality tables for each age at selection, but they are tabulated in an efficient (space saving) way.

Generally question done well. Other valid points given credit.

8 (i) The crude mortality rate is defined as

$$\frac{\sum_x E^c_{x,t}\, m_{x,t}}{\sum_x E^c_{x,t}} = \frac{\text{actual deaths}}{\text{total exposed to risk}}$$

It is a weighted average of $m_{x,t}$ using $E^c_{x,t}$ as weights where:

$E^c_{x,t}$ Central exposed to risk in population being studied between ages x and $x + t$.

$m_{x,t}$ Central rate of mortality either observed or from a life table in population being studied for ages x to $x + t$.

The directly standardised mortality rate is defined as

$$\frac{\sum_x {}^sE^c_{x,t}\, m_{x,t}}{\sum_x {}^sE^c_{x,t}}$$

It is a weighted average of $m_{x,t}$ using ${}^sE^c_{x,t}$ as weights where

${}^sE^c_{x,t}$ Central exposed to risk for a standard population between x and $x + t$.

(ii)

Age	*Lives*	*Deaths*	*ELT15 M rate*	*Expected Deaths*
65	125000	2937	0.02447	3058.8
66	130000	3301	0.02711	3524.3
67	140000	3756	0.02997	4195.8
		9994		10778.9

Standardised mortality ratio is 9994/10778.9 = 0.927

Generally question done well. It was not necessary to make the weighted average remarks in line 3 and 7 above to obtain full marks.

9 $\text{PV}=10000\ddot{a}^{(12)}_{\overline{10}|}+5000*(\ddot{a}^{(12)}_{\overline{65:62}}+\ddot{a}^{(12)}_{65})+10000*(\bar{A}_{65}+\bar{A}_{62})$

where 65 relates to the male life and 62 the female.

$$10000\ddot{a}^{(12)}_{\overline{10}|}=10000*(i/d^{(12)})*a_{\overline{10}|}=1021537*8.1109=82855.85$$

$$\ddot{a}^{(12)}_{\overline{65:62}}=\left(\ddot{a}_{65}-\frac{11}{24}\right)+\left(\ddot{a}_{62}-\frac{11}{24}\right)-\left(\ddot{a}_{65:62}-\frac{11}{24}\right)=13.666+15.963-12.427-\frac{11}{24}$$

$$=16.744$$

$$\ddot{a}^{(12)}_{65}=\left(\ddot{a}_{65}-\frac{11}{24}\right)=13.666-\frac{11}{24}=13.208$$

$$10000\bar{A}_{65}=10000*(1-\ln(1.04)*(13.666-.5))=4836.20$$

$$10000\bar{A}_{62}=10000*(1-\ln(1.04)*(15.963-.5))=3935.30$$

$$\text{Total Value} = 82855.85 + 5000*(16.744 + 13.208) + 4836.20 + 3935.30$$

$$= £241387 \text{ rounded}$$

Well prepared students completed this question satisfactory but others had problems with the joint life portion. A very few students concluded that the question wording could be taken to mean that for the joint part the annuity ceases altogether on the female life death and examiners agreed that this was a potential ambiguity and the alternative approach was allowable. This alternative approach gave an answer of £228,994.

10 The expected value is:

$$1000(A_{40} + v^{20}\frac{l_{60}}{l_{40}}A_{60} + v^{40}\frac{l_{80}}{l_{40}}A_{80})$$

$$= 1000*(0.23056 + (0.45639*\frac{9287.2164}{9856.2863}*0.45640) + (0.20829*\frac{5266.4604}{9856.2863}*0.73775))$$

$$= 1000*(0.23056 + 0.19627 + 0.08211)$$

$$= £509 \text{ to nearer } £$$

To get the variance we calculate the second moment by defining the benefit as three temporary assurances, two of which are deferred, thus:
Benefit from age 40–60

$$(1000)^2*[{}^2A_{40} - v^{20}\frac{l_{60}}{l_{40}}\,{}^2A_{60}] \ (v \text{ at } 8.16\%)$$

$$= (1000)^2*(0.06792 - 0.20829*\frac{9287.2164}{9856.2863}*0.23723)$$

$$= 1,000,000*.021361 = 21,361$$

Benefit from age 60–80

$$(2000)^2*v^{20}\frac{l_{60}}{l_{40}}*[{}^2A_{60} - v^{20}\frac{l_{80}}{l_{60}}\,{}^2A_{80}] \ (v \text{ at } 8.16\%)$$

$$= (2000)^2*0.20829*\frac{9287.2164}{9856.2863}(0.23723 - 0.20829*\frac{5266.4604}{9287.2164}*0.56432)$$

$$= 4,000,000*.033478 = 133,911$$

Benefit from age 80

$$(3000)^2 * v^{40} \frac{l_{80}}{l_{40}} * {}^2A_{80} \text{ (}v\text{ at 8.16\%)}$$

$$= (3000)^2 * 0.04338 * \frac{5266.4604}{9856.2863} * 0.56432$$

$$= 9{,}000{,}000 * .013082 = 117{,}735$$

Second moment:

= 21,361 + 133,911 + 117,735

= 273,007

Variance = 273,007 – $(509)^2$

= 13,926

= $(£118)^2$

The calculation for the mean was generally well done but the calculation for the variance was poorly done overall.

11 (i) The probability is $(e^{(-20*.05)})^2 = e^{-2} = 0.13534$

(ii) The value is

$$1000*(1+(1.03/1.04)(e^{-.05})^2 + (1.03/1.04)^2(e^{-.1})^2 + (1.03/1.04)^3(e^{-.15})^2 + ...)$$
$$= 1000*(1/(1-(1.03/1.04)e^{-.1})) = 1000/0.10386 = £9628$$

(iii) The value is

$$\int_0^{20} (100000*(1.04)^{-t} * (2e^{-.05t}(1-e^{-.05t})*.05)dt$$

$$= 10000\int_0^{20} (e^{-t(\ln(1.04)+.05)} - e^{-t(\ln(1.04)+.1)})dt$$

$$= 10000\int_0^{20} (e^{-.089221t} - e^{-.139221t})dt = 10000\left[-\frac{e^{-.089221t}}{.089221} + \frac{e^{-.139221t}}{.139221}\right]_0^{20}$$

$$= 10000*\left(-\frac{e^{-1.78442}}{.089221} + \frac{e^{-2.78442}}{.139221} + \frac{1}{.089221} - \frac{1}{.139221}\right)$$

$$= 10000 * (-1.88180 + 0.44365 + 11.20812 - 7.18282)$$

$$= £25872$$

Parts (ii) and (iii) were poorly done. In (ii) many students failed to realise that the expression needed was a geometric series rather than an integral.

12 (a) Let P be the annual premium for the policy. Then (functions at 6%):

EPV of premiums:

$$P\ddot{a}_{[50]} = 14.051P$$

EPV of benefits:

$$75,000A_{[50]}$$

EPV of expenses:

$$P + 325 + \left(75 + 0.025P\right)a_{[50]}$$

Equation of value gives:

$$P\ddot{a}_{[50]} = 75,000A_{[50]} + 325 + P + \left(0.025P + 75\right)a_{[50]}$$

$$P \times 14.051 = 75,000 \times 0.20463 + 325 + P + \left(0.025P + 75\right) \times 13.051$$

$$\Rightarrow P = \frac{16,651.075}{12.724725} = 1,308.56$$

(b) The insurer's loss random variable for this policy is given by (where K and T denote the curtate and complete future lifetime of a policyholder):

$$L = 75,000v^{K_{[50]}+1} + 325 + P' + \left(0.025P' + 75\right)a_{\overline{K_{[50]}|}} - P'\ddot{a}_{\overline{K_{[50]}+1|}}$$

We need to find a value of t such that

$$P\left(L > 0\right) = P\left(T < t\right) = 0.1 \Rightarrow P\left(T \geq t\right) = 0.9$$

Using AM92 Select, we require:

$$\frac{l_{[50]+t}}{l_{[50]}} \geq 0.9 \Rightarrow l_{[50]+t} \geq 0.9l_{[50]} = 0.9 \times 9706.0977 = 8735.488$$

As $l_{65} = 8821.2612$ *and* $l_{66} = 8695.6199$ then t lies between 15 and 16 so $K_{[50]}$ = 15.

We therefore need the minimum premium such that

$$L = 0 = 75{,}000v^{16} + 325 + P' + \left(0.025P' + 75\right)a_{\overline{15|}} - P'\ddot{a}_{\overline{16|}}$$

$$\Rightarrow 0 = 75{,}000 \times 0.39365 + 325 + P' + \left(0.025P' + 75\right) \times 9.712254 - 10.712254P'$$

$$\Rightarrow P' = \frac{30{,}577.169}{9.46944765} = 3{,}229.03$$

Part (a) was done well. However very few students completed part (b).

13 (i) If P is the initial premium payable, then

EPV of premiums

$$= P\left[1 \times \frac{l_{56}}{l_{56}} + 0.75 \times \frac{l_{57}}{l_{56}} \times v + 0.5 \times \frac{l_{58}}{l_{56}} \times v^2 + 0.25 \times \frac{l_{59}}{l_{56}} \times v^3\right]$$

$$= \frac{P}{9515.104}\left[9515.104 + 0.75 \times 9467.2906 \times 0.9434 + 0.5 \times 9413.8004 \times .89 + 0.25 \times 9354.004 \times .83962\right]$$

$$= 2.350603P$$

EPV of benefits

$$= 100{,}000\left[q_{56} \times v + 0.75 \times p_{56} \times q_{57} \times v^2 + 0.5 \times {}_2p_{56} \times q_{58} \times v^3 + 0.25 \times {}_3p_{56} \times q_{59} \times v^4\right]$$

$$= 100{,}000\left[\begin{array}{l} 0.005025 \times 0.9434 + 0.75 \times 0.994975 \times 0.00565 \times 0.89 \\ +0.5 \times 0.989353 \times 0.006352 \times 0.83962 + 0.25 \times 0.983069 \times 0.00714 \times 0.79209 \end{array}\right]$$

$$= 1252.116$$

EPV of renewal expenses =

$$= 35\left[\ddot{a}^{@i'}_{56:\overline{4|}} - 1\right] = 35 \times 2.745 = 96.075$$

where $i' = \dfrac{1.06}{1.0192308} - 1 = 0.04$

EPV of other expenses =
$125+0.25P+0.03\times(EPV\ of\ \ premiums-1)$
$=125+0.25P+0.040518P$

Equation of value gives:

$2.350603P$ = 1252.116 + 96.075 + 125 + 0.25P + 0.040518P

P = 715.11

(ii) Prospective gross premium policy reserve at the end of the 1st policy year given by:

$${}_1V = EPV(\,future\,benefits + \exp enses - premiums) \text{ where:}$$

EPV of premiums

$$= P\left[0.75\times\frac{l_{57}}{l_{57}}+0.5\times\frac{l_{58}}{l_{57}}\times v\ \ +0.25\times\frac{l_{59}}{l_{57}}\times v^2\right]$$
$$=\frac{715.11}{9467.2906}\left[0.75\times 9467.2906+0.5\times 9413.8004\times.9434+0.25\times 9354.004\times.89\right]=1028.952$$

EPV of benefits

$$=100,000\left[0.75\times q_{57}\times v\ \ +0.5\times\ \ p_{57}\times q_{58}\times v^2+0.25\times {}_2p_{57}\times q_{59}\times v^3\right]$$

$$=100,000\left[\begin{array}{l}0.75\times 0.00565\times 0.9434+0.5\times 0.99435\times 0.006352\times 0.89\\ +0.25\times 0.9880339\times 0.00714\times 0.83962\end{array}\right]$$

$$=828.911$$

EPV of renewal expenses

$$=35\times 1.0192308\times \ddot{a}^{@i'}_{57:\overline{3}|}=35.673\times 2.870=102.382$$

EPV of renewal commission
= 0.03 × EPV of premiums = 30.867

Therefore ${}_1V=828.911+102.382+30.867-1028.952=-66.79$

(iii) Therefore, sum at risk per policy in the 1st policy year is:

DSAR = 100,000 – (– 66.79) = 100,066.79
Mortality profit = EDS – ADS

$$EDS = 5000 \times q_{56} \times 100,066.79 = 5000 \times 0.005025 \times 100,066.79 = 2,514,178.1$$

$$ADS = 27 \times 100,066.79 = 2,701,803.3$$

i.e. mortality profit = –187,625.2 (i.e. a loss)

Mortality profit =

$$= 5000 \times ({}_0V + P - E) \times (1+i) - S \times actual\ deaths - {}_1V \times number\ of\ policies\ \text{in}\ force$$
$$= 5000 \times (0 + 715.11 - 0.25 \times 715.11 - 125) \times 1.06 - 100,000 \times 27 - (-66.79) \times 4973$$
$$= -187,791.1$$

i.e. approximately the same figure as derived in (c) above

Reasonably well done by well prepared students. Partial credit was given in (b) for showing understanding of the processes involved.

14 (i) Let P be the annual premium required to meet the company's profit criteria.

Multiple decrement table – although deaths can be assumed to be uniformly distributed over the year, surrenders occur only at the year end. Therefore:

$$(aq)_x^d = q_x^d \text{ and } (aq)_x^w = q_x^w(1 - q_x^d)$$

x	q_x^d	q_x^w	$(aq)_x^d$	$(aq)_x^w$	$(ap)_x$	${}_{t-1}(ap)_x$
67	0.016042	0.08	0.016042	0.07872	0.905242	1
68	0.017922	0.04	0.017922	0.03928	0.942795	0.905242
69	0.020003	0.00	0.020003	0.0	0.979997	0.853458

Unit fund cashflows (per policy at start of year)

	Year 1	*Year 2*	*Year 3*
Value of units at start of year	0	0.490295P	1.584731P
Allocation	0.5P	1.1P	1.1P
Bid/offer	0.025P	0.055P	0.055P
Interest	0.019P	0.061412P	0.105189P
Management charge	0.003705P	0.011975P	0.020512P
Value of units at end of year	0.490295P	1.584731P	2.714408P

Non-unit fund cashflows

	Year 1	*Year 2*	*Year 3*
Unallocated premium	0.5*P*	–0.1P	–0.1*P*
Bid/offer	0.025*P*	0.055*P*	0.055*P*
Expenses	0.125*P*+235	0.025*P*+45	0.025*P*+45
Interest	0.012*P*–7.05	–0.0021*P*–1.35	–0.0021*P*–1.35
Management charge	0.003705*P*	0.011975*P*	0.020512*P*
Claim expense	7.10715	4.29015	1.500225
End of year cashflows	0.415705*P*–249.15715	–0.060125*P*–50.64015	–0.051588*P*–47.850225
Probability in force	1	0.905242	0.853458
Discount factor	0.943396	0.889996	0.839619
Expected present value of profit	0.392174*P*–235.0539	–0.048440*P*–40.7987	–0.036967*P*–34.2886

NPV of profit = .10P = 0.306767P – 310.1412 => *P* = £1500.0
(i.e. NPV of profit = £150.0)

(ii) The profit vector for the policy is (374.401, –140.827, –125.232)

In order to set up reserves in order to zeroise future expected negative cash flows, we require:

$$ {}_2V = \frac{125.232}{1.03} = 121.584 $$
$$ {}_1V \times 1.03 - (ap)_{68} \times {}_2V = 140.827 \Rightarrow {}_1V = 248.016 $$

revised cash flow in year 1 $= 374.401 - (ap)_{67} \times {}_1V = 149.887$

and NPV of profit = 149.887/1.06 = 141.402

Again reasonably well done by well prepared students for part (i). Part (ii) caused more difficulties however. As before partial credit was given for showing understanding of the processes involved.

END OF EXAMINERS' REPORT