INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2012 examinations

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and who are using past papers as a revision aid, and also those who have previously failed the subject. The Examiners are charged by Council with examining the published syllabus. Although Examiners have access to the Core Reading, which is designed to interpret the syllabus, the Examiners are not required to examine the content of Core Reading. Notwithstanding that, the questions set, and the following comments, will generally be based on Core Reading.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report. Other valid approaches are always given appropriate credit; where there is a commonly used alternative approach, this is also noted in the report. For essay-style questions, and particularly the open-ended questions in the later subjects, this report contains all the points for which the Examiners awarded marks. This is much more than a model solution – it would be impossible to write down all the points in the report in the time allowed for the question.

T J Birse
Chairman of the Board of Examiners

July 2012

General comments on Subject CT5

CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions actuarial work.

Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given where appropriate to different valid points made which do not appear in the solutions below.

In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

Comments on the April 2012 paper

The general performance was better this session than in recent diets and many students scored well with a very pleasing increase in the number passing. Questions that were done less well were 2, 10, 12, 13 and 15(i) and (iii) and here more commentary is given to students to assist with further revision.

Most of the short questions were very straightforward where an answer could be produced quickly and this is where many successful candidates scored particularly well. Students should note that for long questions a reasonable level of credit is given if they can describe the right procedures although to score well reasonable accurate numerical calculation is necessary.

1 (a) $_{4|5}q_{[60]+1}$ is the probability that a life now aged 61 exact who entered the selection period 1 year ago will die between the ages of 65 and 70 both exact

(b) $_{4|5}q_{[60]+1} = \left(l_{65} - l_{70}\right) / l_{[60]+1} = (8821.2612 - 8054.0544) / 9209.6568$
$= 0.0833$

A gentle starter generally done well

2 The death benefit in year 12 is £36,000

Profit emerging per policy in force at the start of the year is:

$$({}_{11}V + P - e) \times (1+i) - (q_{x+t} \times S) - p_{x+t} \times {}_{12}V$$

$$= (25{,}130 + 3{,}000 - 90) \times 1.04 - 36{,}000 \times 0.03 - (1 - 0.03) \times 28{,}950 = £0.10$$

This question overall caused problems and students sometimes had only a vague recall of the iterative formula in line 3. The most common error was to forget expenses and the survival factor before the closing reserve. It was also not on many occasions appreciated that the accumulation minus the benefit costs gave the profit.

3 (a)

$$a_{50:\overline{15|}} = a_{50} - v^{15}\left(\frac{l_{65}}{l_{50}}\right)(a_{65})$$

$$= (14.044 - 1) - 0.41727 \times \frac{8{,}821.2612}{9{,}712.0728} \times (10.569 - 1)$$

$$= 9.417$$

(b)

$$(IA)^{1}_{50:\overline{15|}} = (IA)_{50} - v^{15}\left(\frac{l_{65}}{l_{50}}\right)((IA)_{65} + 15A_{65})$$

$$= 4.84555 - 0.41727 \times \frac{8{,}821.2612}{9{,}712.0728}(5.50985 + 15 \times 0.40177)$$

$$= 0.47329$$

In (a) a surprising number of students thought that the required function could be derived from the a due function for the same term minus 1 which is, of course, wholly incorrect.

Otherwise the question was generally well done.

4 The value is $100000\bar{A}_{\overline{60:55}}$ where 60 relates to the male life and 55 the female life.

$$\begin{aligned}100000\bar{A}_{\overline{60:55}} &= 100000\times(\bar{A}_{60}+\bar{A}_{55}-\bar{A}_{60:55})\\ &= 100000\times(1-\ln(1.04)(\bar{a}_{60}+\bar{a}_{55}-\bar{a}_{60:55}))\\ &= 100000\times(1-\ln(1.04)([\ddot{a}_{60}-1/2]+[\ddot{a}_{55}-1/2]-[\ddot{a}_{60:55}-1/2]))\\ &= 100000\times(1-0.039221\times(15.632+18.210-14.756-0.5))\\ &= £27104\end{aligned}$$

Generally well done. Other methods such as multiplying non continuous functions by $(1.04)^{1/2}$ to obtain the continuous one were quite acceptable.

5 The reserves required at the beginning of policy years 6, 4, 3 and 2 are:

$$\begin{aligned}{}_5V &= \frac{4}{1.025} = 3.902\\ {}_3V &= \frac{1}{1.025} = 0.976\\ {}_2V &= \frac{1}{1.025}(6+.995\times {}_3V) = 6.801\\ {}_1V &= \frac{1}{1.025}(12+.995\times {}_2V) = 18.309\end{aligned}$$

Revised cash flow in policy year 5 $= 5-0.995\times {}_5V = 1.118$

Revised cash flow in policy year 1 $= -40-0.995\times {}_1V = -58.218$

=> revised profit vector: (–58.22, 0, 0, 0, 1.12, 0, 8, 20, 25, 30)

Generally well done by well prepared students who were able to recall the techniques involved.

6 (a) $p_{67} = \exp(-\int_{67}^{68} \mu \; dx)$ where μ is the constant force.

$$\Rightarrow \mu = -\ln(p_{67}) = -\ln(1-q_{67}) = -\ln(0.982176)$$
$$\Rightarrow \mu = 0.017985$$

(b) Using the constant force assumption:

$$_{0.5}q_{\,67.25} = 1 - {}_{0.5}p_{\,67.25} = 1 - \exp(-\int_{67.25}^{67.75} \mu \; dx)$$
$$= 1 - \exp(-0.5 \times \mu) = 1 - \exp(-0.5 \times 0.017985)$$
$$= 0.008952$$

Generally well done. However some students ignored the instruction to use (a) to get (b) choosing the more direct route. The examiners penalised this approach, although some credit was given.

7 Pension schemes usually have a fixed Normal Pension Age (NPA).

Age retirement benefits may be provided on early or late retirement.

Pension usually depends on pensionable service at retirement, as defined in the scheme rules, e.g. complete years of membership.

Pension for each year of service is usually related to pensionable salary, for example 1/80ths of pensionable salary for each year of service. 1/80 is described as the accrual rate.

Pensionable salary can be defined as:

1. Salary at retirement (final salary)
2. Annual salary averaged over a few years before retirement (final average salary)
3. Annual salary averaged the whole of service (career average salary)

Pensions are commonly increased in payment to offset the effect of inflation.

Some benefit may be in the form of cash, sometimes by converting pension to cash.

There can be a spouse's pension for married pensioners which is often a percentage of the main pension on the member's prior death.

Pensions may also be paid for an initial guarantee period like 5 years.

Other relevant comments were credited. No credit was given for any discussion on ill-health retirement as this was not required from the question.

Many students scored reasonable marks.

8 Occupation can have several direct and indirect effects on mortality and morbidity.

Occupation determines a person's environment for 40 or more hours each week. The environment may be rural or urban, the occupation may involve exposure to harmful substances e.g. chemicals, or to potentially dangerous situations e.g. working at heights. Much of this is moderated by health and safety at work regulations.

Some occupations are more healthy by their very nature e.g. bus drivers have a sedentary and stressful occupation while bus conductors are more active and less stressed. Some work environments e.g. publicans, give exposure to a less healthy lifestyle.

Some occupations by their very nature attract more healthy or unhealthy workers. This may be accentuated by health checks made on appointment or by the need to pass regular health checks e.g. airline pilots. However, this effect can be produced without formal checks, e.g. former miners who have left the mining industry as a result of ill-health and then chosen to sell newspapers. This will inflate the mortality rates of newspaper sellers.
A person's occupation largely determines their income, and this permits them to adopt a particular lifestyle e.g. content and pattern of diet, quality of housing. This effect can be positive and negative e.g. over indulgence.

Generally well done and credit was given for any other relevant points.

9 (i) Initial Expense
Renewal Expense
Claim Expense
Overhead Expense

(ii) <u>Initial Expense</u>

Underwriting (allowed for on a per policy basis although medical expenses might be sum assured related) **<u>or</u>**;
Processing proposal and issuing policy (allowed for on a per policy basis) **<u>or</u>**;
Commission (allowed for directly and usually premium related) **<u>or</u>**;
Marketing (allowed for on a per policy basis on estimated volumes)

<u>Renewal Expense</u>

Administration (allowed for on a per policy per annum basis with allowance for inflation) **<u>or</u>**;
Commission (allowed for directly and usually premium related) **<u>or</u>**;
Investment Expense (charged as a deduction from investment funds).

<u>Claim Expense</u>

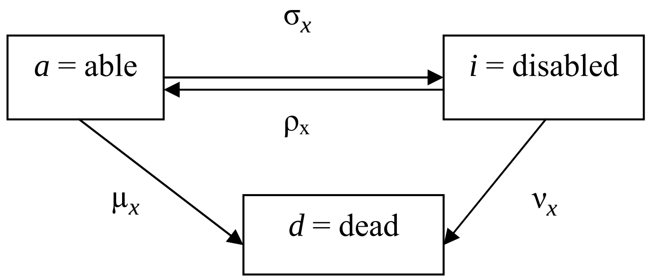
Calculation and payment of benefit (allowed for on a per policy per annum basis with allowance for inflation)

Overhead Expense

Central services e.g. premises, IT, legal (allowed for on a per policy per annum basis with allowance for inflation)]

Many students did not give a full answer referring only to direct and indirect expenses for which only partial credit was given, Also many did not give the full number of distinctly different examples. Other relevant examples were credited however.

10 The multiple state transition model is:



Define the force of interest δ

$$\text{Value of death benefit } = 10,000\int_0^\infty e^{-\delta t}({}_t p_x^{aa}.\mu_{x+t} + {}_t p_x^{ai}.\nu_{x+t})dt$$

$$\text{Value of disablement benefit } = 1,000\int_0^\infty e^{-\delta t}({}_t p_x^{aa}.\sigma_{x+t})dt$$

Generally the diagram was completed satisfactorily. Many students took the view that returning to the able state from the disabled one was impossible and thus omitted the return arrow. This was accepted so long as the assumptions were stated.

The resultant formulae were, however, on the whole poorly done.

11 (i) *Crude Mortality Rate*

Advantage – do not need population and deaths split by age
Disadvantage – differences in age structure between populations will be confounded

Directly Standardised Mortality Rate

Advantage – only reflects differences in mortality rates

Disadvantage – requires age specific mortality rates for the observed population

(ii) SMR = actual deaths / expected deaths

Expected deaths = 100000*0.00464+95000*0.00797+80000*0.01392 = 2335

SMR = 1250 / 2335 = 0.535

A straightforward question generally done well by well prepared students. Some students struggled to find the distinctive advantages and disadvantage given above.

12 The expected value is $100000\bar{A}^{1}_{x:\overline{n|}} + 50000A_{x:\overline{n|}}^{\ \ \ 1}$

$$\bar{A}^{1}_{x:\overline{n|}} = \int_0^{10} \exp((-\ln(1.05) - .03)t) \times .03dt$$

$$= 0.03\int_0^{10} \exp(-0.07879t)dt = 0.03\left[-\frac{\exp(-0.07879t)}{0.07879}\right]_0^{10}$$

$$= \frac{0.03}{0.07879}\left[1 - \exp(-0.7879)\right] \quad =0.20759$$

$$A_{x:\overline{n|}}^{\ \ \ 1} = \exp(-0.7879) = 0.45480$$

The expected value is thus $100000 \times 0.20759 + 50000 \times 0.45480 = £43499$

For the variance the rate used is $(1.05)^2 - 1 = 10.25\%$ and $\ln(1.1025) = 0.09758$.

Hence:

$$^2\bar{A}^{1}_{x:\overline{n|}} = 0.03\int_0^{10} \exp(-0.12758t)dt = 0.03\left[-\frac{\exp(-0.12758t)}{0.12758}\right]_0^{10}$$

$$= \frac{0.03}{0.12758}\left[1 - \exp(-1.2758)\right] \quad =0.16949$$

$$^2A_{x:\overline{n|}}^{\ \ \ 1} = \exp(-1.2758) = 0.27921$$

The variance is thus $\left(100000\right)^2 \times 0.16949 + \left(50000\right)^2 \times 0.27921 - (43499)^2 = (22378)^2$

The part relating to the expected value was generally done well. However by contrast the part relating to the variance was done poorly. Many students failed to realise that the integration process was the same as for the expected value with the exception of building in the 10.25% interest rate.

13 Let P be the monthly premium. Then:

EPV of premiums:

$$12P\ddot{a}^{(12)}_{[20]:\overline{40|}} @ 6\% = 184.6092P$$

where

$$\ddot{a}^{(12)}_{[20]:\overline{40|}} = \ddot{a}_{[20]:\overline{40|}} - \frac{11}{24}\left(1 - v^{40}\ {}_{40}p_{[20]}\right)$$

$$= 15.801 - \frac{11}{24}\left(1 - 0.09722 \times \frac{9287.2164}{9980.2432}\right) = 15.801 - 0.4169 = 15.3841$$

EPV of benefits:

$$85,000\left(q_{[20]}v^{0.5} + {}_{1|}q_{[20]}(1+b)v^{1.5} + \ldots + {}_{39|}q_{[20]}(1+b)^{39}v^{39.5} + (1+b)^{40}\ {}_{40}p_{[20]}v^{40}\right)$$

where $b = 0.0192308$

$$= 85,000 \times \frac{(1.06)^{0.5}}{(1+b)}\left(q_{[20]}(1+b)v + {}_{1|}q_{[20]}(1+b)^2v^2 + \ldots + {}_{39|}q_{[20]}(1+b)^{40}v^{40}\right)$$
$$+85,000(1+b)^{40}v^{40}\ {}_{40}p_{[20]}$$

$$= \frac{85,000}{(1+b)} \times (1.06)^{0.5} \times A^{1}_{[20]:\overline{40|}} @ i' + 85,000v^{40}\ {}_{40}p_{[20]} @ i'$$

where

$$i' = \frac{1.06}{1+b} - 1 = 0.04$$

and $\quad A^{1}_{[20]:\overline{40|}} @ i' = A_{[20]:\overline{40|}} - v^{40}\ {}_{40}p_{[20]}$

$$= 0.21746 - 0.20829 \times \frac{9287.2164}{9980.2432} = 0.21746 - 0.19383 = 0.02363$$

EPV of benefits

$$= \frac{85,000 \times (1.06)^{0.5}}{(1+b)} \times 0.02363 + 85,000 \times 0.19383$$

$$= 2,028.911 + 16,475.550 = 18,504.461$$

EPV of expenses

$$= 4.8P + 325 + 0.025 \times 12 \times P\ddot{a}^{(12)}_{[20]:\overline{40}|}$$

$$- 0.025P + 65 \times \left[\ddot{a}_{[20]:\overline{40}|} - 1 \right] + 5 \times \left[(I\ddot{a})_{[20]:\overline{40}|} - 1 \right]$$

$$= 9.3902P + 325 + 65 \times 14.801 + 5 \times 208.366 = 9.3902P + 2,328.895$$

where

$$(I\ddot{a})_{[20]:\overline{40}|} = (I\ddot{a})_{[20]} - v^{40}\,{}_{40}p_{[20]} \left[40\ddot{a}_{60} + (I\ddot{a})_{60} \right]$$

$$= 262.666 - 0.09722 \times \frac{9287.2164}{9980.2432} \times \left[40 \times 11.891 + 113.516 \right]$$

$$= 262.666 - 53.300 = 209.366$$

Equation of value gives

$$184.6092P = 18,504.461 + 9.3902P + 2,328.895$$

$$\Rightarrow P = \frac{20,833.356}{175.219} = £118.90$$

The difficult part of this question was related to the EPV of Expenses and most students failed to complete this complex part. The rest of the question was however generally reasonably done by well prepared students.

14 (i) Annual net premium for the decreasing term assurance is given by:

$$P = \frac{210,000A^{1}_{40:\overline{20}|} - 10,000(IA)^{1}_{40:\overline{20}|}}{\ddot{a}_{40:\overline{20}|}}$$

where $A^{1}_{40:\overline{20}|} = A_{40:\overline{20}|} - v^{20}\,{}_{20}p_{40}$

$$= 0.46433 - 0.45639 \times \frac{9287.2164}{9856.2863} = 0.46433 - 0.43004 = 0.03429$$

and $(IA)^{1}_{40:\overline{20}|} = (IA)_{40} - v^{20}\,{}_{20}p_{40} \left[20A_{60} + (IA)_{60} \right]$

$$= 7.95699 - 0.43004 \times \left[20 \times 0.45640 + 8.36234 \right] = 0.43544$$

$$P \;= \frac{210,000 \times 0.03429 - 10,000 \times 0.43544}{13.927} = 204.39$$

(ii) Reserve at the <u>end</u> of the 10th policy year given by:

$${}_{10}V \;= 110,000 A^{1}_{50:\overline{10}|} - 10,000 (IA)^{1}_{50:\overline{10}|} - P\ \ddot{a}_{50:\overline{10}|}$$

where

$$A^{1}_{50:\overline{10}|} = A_{50:\overline{10}|} - v^{10}\,{}_{10}p_{50}$$

$$= 0.68024 - 0.67556 \times \frac{9287.2164}{9712.0728} = 0.68024 - 0.64601 = 0.03423$$

and

$$(IA)^{1}_{50:\overline{10}|} = (IA)_{50} - v^{10}\,{}_{10}p_{50}\left[10A_{60} + (IA)_{60}\right]$$

$$= 8.55929 - 0.64601 \times \left[10 \times 0.45640 + 8.36234\right] = 0.20875$$

$${}_{10}V \;= 110,000 \times 0.03423 - 10,000 \times 0.20875 - 204.39 \times 8.314$$

$$= 3,765.30 - 2,087.50 - 1,699.30 = -21.50$$

Therefore, sum at risk per policy in the 10th policy year is:

DSAR = 110,000 – (–21.50) = 110,021.50

Mortality profit = EDS – ADS

$$EDS = 625 \times q_{49} \times 110,021.50 = 625 \times 0.002241 \times 110,021.50 = 154,098.86$$

$$ADS = 3 \times 110,021.50 = 330,064.50$$

i.e. mortality profit = – 175,965.36 (i.e. a loss)

(iii) The death strain at risk per policy in the 10th policy year for this decreasing term assurance is very large (approximately equal to the sum assured payable in the event of death).

The actual number of deaths during the 10th policy year (at 3) is approximately double that expected (at 1.4) which accounts for the mortality loss.

However, a mortality experience investigation would need to consider a longer time period and ideally, a larger number of policies to determine whether actual mortality experience is heavier than expected.

Question generally done well by well prepared student. This was a straightforward question of its type.

15 (i) Gross future loss random variable =

$$150{,}000v^{K_{[57]}+1} + 350 + 50a_{\overline{K_{[57]}}|} - P(0.975\ddot{a}_{\overline{K_{[57]}+1)}|} - 0.125) \quad \text{if } K_{[57]} < 3$$

$$350 + 50a_{\overline{2}|} - P(0.975\ddot{a}_{\overline{3)}|} - 0.125) \quad \text{if } K_{[57]} \geq 3$$

(ii) E (Gross future loss random variable) = 0

$$\Rightarrow 150{,}000A^{1}_{[57]:\overline{3}|} + 350 + 50\left[\ddot{a}_{[57]:\overline{3}|} - 1\right] = P\left[0.975\ddot{a}_{[57]:\overline{3}|} - 0.125\right]$$

where $A^{1}_{[57]:\overline{3}|} = A_{[57]:\overline{3}|} - v^3\,{}_3p_{[57]} = 0.84036 - 0.83962 \times \dfrac{9287.2164}{9451.5938} = 0.84036 - 0.82502 = 0.01534$

and $\ddot{a}_{[57]:\overline{3}|} = 2.820$

$$\Rightarrow 150{,}000 \times 0.01534 + 350 + 50 \times 1.820 = 2.6245P$$

$$\Rightarrow P = \frac{2{,}301.0 + 350 + 91.0}{2.6245} = 1{,}044.77$$

(iii) Mortality table:

x	t	$q_{[x]+t-1}$	$p_{[x]+t-1}$	${}_{t-1}p_{[x]}$
57	1	0.004171	0.995829	1.000000
58	2	0.006180	0.993820	0.995829
59	3	0.007140	0.992860	0.989675

Cash flows (per policy at start of year) assuming annual premium is denoted by P:

Year	*1*	*2*	*3*
Premium	P	P	P
Expenses	$0.15P + 350$	$0.025P + 50$	$0.025P + 50$
Interest	$0.051P - 21$	$0.0585P - 3$	$0.0585P - 3$
Claim	625.65	927.00	1071.00
Profit vector	$0.901P - 996.65$	$1.0335P–980.00$	$1.0335P–1124.00$
Cumulative probability of survival	1.000000	0.995829	0.989675
Profit signature	$0.9010P - 996.650$	$1.0292P - 975.912$	$1.0228P - 1112.395$
Discount factor	0.94340	0.890000	0.83962
NPV of profit	$0.85P - 940.240$	$0.9160P - 868.562$	$0.8588P - 933.989$

Therefore:

$$\sum_{1}^{3} NPV = 0 = 2.6248P - 2742.791 \Rightarrow P = \frac{2742.791}{2.6248} = 1{,}044.95$$

which is consistent with the premium calculated in (ii) above (allowing for rounding)

(iv) (a) profit is deferred but as the earned interest rate is equal to the risk discount rate, there is no change to the *NPV* or premium

(b) profit is deferred and because the risk discount rate is greater than the earned interest rate, *NPV* falls. Therefore, the premium would need to be increased to satisfy the same profit criterion.

Most students struggled with part (i) but well prepared ones completed parts (ii) and (iv) satisfactorily. Part (iii) caused students great difficulties as often occurs with this approach and many even failed to realise that the answers to (ii) and (iii) should numerically be the same within rounding.

END OF EXAMINERS' REPORT