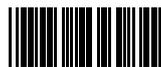# INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

8 October 2015 (pm)

Subject CT5 – Contingencies
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 14 questions, beginning your answer to each question on a new page.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

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1   Calculate:

   (a) $_{25}p_{40}$

   (b) $_{10|}q_{[53]}$

   (c) $\bar{a}_{55:\overline{10|}}$

   Basis:

   Mortality   AM92
   Interest   4% per annum   [3]

2   Derive (to the nearest integer) the median of the complete future lifetime of a person aged 30 exact who is subject to the force of mortality shown below:

$$\mu_{30+t} = \begin{cases} .01 & 0 \le t < 10 \\ .02 & 10 \le t < 20 \\ .03 & 20 \le t \end{cases}$$

                                                                                           [3]

3   (i)   Describe the difference between an overhead expense and a direct expense in the context of calculating premiums for a life assurance policy.   [1]

   (ii)   (a)   State an example of an overhead expense and a direct expense.

          (b)   Describe the manner in which each example in (a) is usually allowed for in the calculation of premiums.   [3]
                                                                                    [Total 4]

4   (i)   Describe how education may affect mortality.   [1]

   (ii)   List three examples of the influence of education on mortality.   [3]
                                                                                    [Total 4]

5   A special annuity pays 5,000 per annum for five years increasing to 6,000 per annum for the next five years and increasing further to 7,000 thereafter. The payments for the first five years are guaranteed and thereafter are contingent on survival. The annuity is payable monthly in advance.

Calculate the expected present value of this annuity for a life aged 60 exact. Show all your workings.

Basis:

Mortality   PMA92C20
Interest   4% per annum

[5]

6   The employees of a manufacturing company are subject to two modes of decrement, mortality and withdrawal from employment.

The independent forces of mortality and withdrawal for employees aged 50 and 51 are given in the following table:

Age	μ_x^d	μ_x^w
50	0.0010	0.15
51	0.0015	0.10

Calculate, showing all your workings, the probability that a new employee aged 50 exact will die as an employee at age 51 last birthday. State any assumptions that you make. [5]

7   A critical illness scheme provides a benefit of 100,000 on death or earlier diagnosis of a critical illness.

(i)   Draw and label the appropriate transition diagram. [3]

(ii)   Set out an expression for the expected present value of this benefit. [3]
[Total 6]

8   Calculate, showing all your workings, $a_{73.25}^{(4)}$.

Basis:

Mortality   PFA92C20 (assume that the force of mortality is constant between ages 73 and 74 only)
Interest   4% per annum [7]

9 A pension scheme provides a pension of one-sixtieth of final pensionable salary on retirement, due to age or ill-health, for each year of service (part years included). Final pensionable salary is average salary over the three years before retirement. Normal retirement age is 65. Members contribute 5% of pensionable salary each year.

(i) Calculate the expected present value of the combined past and future benefits for a member aged 45 exact with 10 years of past service and salary in the previous year of 25,000. [5]

(ii) Calculate the present value of the member's future contributions. [2]

Basis:

Pension Scheme Table in the Formulae and Tables for Examinations

[Total 7]

10 (i) Define in words the Area Comparability Factor. [2]

The table below shows an extract from a study of mortality for Country A and Area N:

| | Country A | | Area N | |
Age	Population	Number of deaths	Population	Number of deaths
60	100,235	566	25,366	125
61	95,666	621	22,159	121
62	92,386	635	21,864	135

(ii) Calculate, showing all your workings, the Area Comparability Factor for Area N using Country A as the standard population. [3]

(iii) Calculate, showing all your workings, the directly standardised mortality rate for Area N. [2]

[Total 7]

11 An assurance policy provides a benefit of 10,000 payable immediately on the death of the last survivor of a male life aged 55 exact and a female life aged 50 exact.

(i) Calculate, showing all your workings, the expected present value for this policy. [5]

(ii) Derive an expression for the variance of the value of this policy. [3]

Basis:

Mortality   PFA92C20
Interest   4% per annum

[Total 8]

12 A life insurance company issues a two year unit-linked endowment assurance policy to a male life aged 45 exact. Level premiums of 6,000 per annum are payable yearly in advance throughout the term of the policy or until earlier death with 98% of each premium being allocated to units. A policy fee of 50 is deducted from the bid value of units at the start of each policy year. The units are subject to a bid-offer spread of 6%. An annual management charge of 1.25% of the bid value of units is deducted at the end of each policy year.

If the policyholder dies during the term of the policy, the death benefit of 200% of the bid value of the units is payable at the end of the policy year of death.

The policyholder may only surrender the policy at the end of the first policy year. On surrender, the bid value of units less a surrender value penalty of 500 is payable.

On maturity, 100% of the bid value of the units is payable.

Management charges are deducted from the unit fund before death, surrender and maturity benefits are paid.

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund	5.0% per annum in year 1 4.5% per annum in year 2
Rate of interest on non-unit fund cash flows	3.0% per annum in both years 1 & 2
Mortality	AM92 Select
Surrenders	2.5% of all policies in force at the end of policy year 1
Initial expense	225
Renewal expense	80 on the second premium date
Initial commission	7.5% of first premium
Renewal commission	2.5% of the second premium
Death claim expense	90
Maturity claim expense	55
Risk discount rate	6% per annum

(i) Calculate, showing all your workings, the non-unit fund cash flows in the first and second years of the policy if the policyholder:

(a) dies in the first year of the policy.
(b) surrenders in the first year of the policy.
(c) dies in the second year of the policy.
(d) survives to the end of the policy.

[7]

(ii) Derive the expected present value of profit for the policy in the event that the policyholder:

(a) dies in the first year of the policy.
(b) surrenders in the first year of the policy.
(c) dies in the second year of the policy.
(d) survives to the end of the policy.

[5]

(iii) Calculate, showing all your workings, the expected present value of the profit for the policy. [1]

[Total 13]

13 A life assurance company issues a policy to a male life aged 40 exact which provides the following benefits:

- An annuity of 30,000 per annum, payable annually in advance starting on the policyholder's 65th birthday and continuing for life thereafter. The annuity increases by 1,500 each year, with the first increase given on the policyholder's 66th birthday.

- A decreasing term assurance with a death benefit, payable immediately on death, which is given by the formula:

$$10,000 \times (25-t) \qquad t = 0, 1, 2, \ldots, 24$$

where t denotes the curtate duration in years since inception of the policy. Death benefit cover ceases at age 65.

The policy is paid for by level monthly premiums payable in advance from the date of issue for 25 years, but ceasing on earlier death.

The company uses the following premium basis for the policy:

Mortality	AM92 Select
Interest	4% per annum
Initial commission	35% of the total premiums payable in the first policy year
Initial expenses	225
Renewal commission	5% of the second and subsequent monthly premiums
Renewal expense	55 per annum at the start of the second and subsequent policy years
Death benefit claim expense	275
Annuity payment expense	2.5% of each annuity payment

The renewal expense and the death benefit claim expense are both assumed to increase continuously at 4% compound per annum from inception of the policy and to cease at age 65, or earlier death.

Calculate, showing all your workings, the monthly premium for the policy.   [13]

14 (i) Write down in the form of symbols, and also describe, the expression "death strain at risk". [2]

On 1 January 2011, a life insurance company issued the following three types of policies to male lives aged 55 exact:

A 5-year pure endowment assurances with a sum assured of 75,000.

B 5-year term assurances with a sum assured of 75,000, where the death benefit is payable at the end of the year of death.

C 5-year single premium temporary immediate annuities with an annual benefit payable in arrear of 15,000.

For policies A and B, premiums are payable annually in advance throughout the policy term or until earlier death.

(ii) Calculate, showing all your workings, the death strain at risk for each type of policy during 2014. [8]

Basis:

Mortality   AM92 Ultimate for policies A and B
             PMA92C20 for policy C
Expenses   Ignore
Interest   4% per annum

At the beginning of 2014, the numbers of policies in force were:

Pure endowment assurances   984
Term assurances   3,950
Temporary immediate annuities   495

During 2014, the actual deaths were 5 from policy A, 22 from policy B and 2 from policy C.

(iii) Calculate, showing all your workings, the total mortality profit or loss to the company for 2014 using the same basis as in (ii). [5]

[Total 15]

END OF PAPER