INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS’ REPORT

April 2015 examinations

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners’ Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners’ preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context at the date the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

F Layton
Chairman of the Board of Examiners

June 2015

General comments on Subject CT5

CT5 introduces the fundamental building blocks that stand behind all life insurance and pensions actuarial work.

Credit is given to students who produce alternative viable numerical solutions. In the case of descriptive answers credit is also given where appropriate to different valid points made which do not appear in the solutions below.

In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

Comments on the April 2015 paper

The general performance was higher than usual this session compared to previous ones although it was felt that this paper was roughly of the same standard as previous ones. Questions that were done less well were Q1, Q10 part (ii), Q11, Q13 part (ii) and Q13 part (iv) and Q14 part (ii). The examiners hope that the detailed solutions given below will assist students with further revision.

However most of the short questions were very straightforward where an answer could be produced quickly and this is where many successful candidates scored particularly well. Students should note that for long questions reasonable credit is given if they can describe the right procedures although to score high marks reasonable accurate numerical calculation is necessary.

1

$$\ddot{a}_{50:\overline{4}|} = 1+\frac{(1-.05)}{1.06}+\frac{(1-.05)(1-.06)}{(1.06)^2}+\frac{(1-.05)(1-.06)(1-.06(1.1))}{(1.06)^3}$$

$$= 1+0.89623+0.79477+0.70029=3.39129$$

$$A_{50:\overline{4}|} = 1-d(6\%)\ddot{a}_{50:\overline{4}|} = 1-\frac{.06}{1.06}(3.39129)=0.80804$$

This question gave many students difficulties. The answer was most easily obtained quickly using premium conversion formulae as above. The alternative method of direct computation is, of course, possible but is more involved.

2 The standard of housing encompasses not only all aspects of the physical quality of housing (e.g. state of repair, type of construction, heating, sanitation) but also the way in which the housing is used e.g. overcrowding and shared cooking.

These factors have an important influence on morbidity, particularly that related to infectious diseases (e.g. from tuberculosis and cholera to colds and coughs) and thus on mortality in the longer term.

The effect of poor housing is often confounded with the general effects of poverty.

A straightforward bookwork question generally well answered. The main omission by students was the comment in the 3rd paragraph.

3

$$(aq)_x^{\alpha} = \frac{\mu_x^{\alpha}}{\mu_x^{\alpha}+\mu_x^{\beta}}\left(1-e^{-(\mu_x^{\alpha}+\mu_x^{\beta})}\right) \text{ and}$$

$$(aq)_x^{\beta} = \frac{\mu_x^{\beta}}{\mu_x^{\alpha}+\mu_x^{\beta}}\left(1-e^{-(\mu_x^{\alpha}+\mu_x^{\beta})}\right)$$

Thus $(aq)_x = (aq)_x^{\alpha}+(aq)_x^{\beta} = \left(1-e^{-5\mu_x^{\beta}}\right)$

Question was generally well done. Students who left the final answer in integral form also received full credit.

4 (a) $${}_{10|15}q_{60} = \frac{l_{70} - l_{85}}{l_{60}} = \frac{8054.0544 - 3385.2479}{9287.2164} = 0.50271$$

(b) $${}_{12}p_{[50]+1} = \frac{l_{63}}{l_{[50]+1}} = \frac{9037.3973}{9686.9669} = 0.93294$$

(c) $$a^{(4)}_{40:\overline{10}|} = a^{(4)}_{40} - \frac{v^{10} l_{50}}{l_{40}} a^{(4)}_{50} \text{ at } 6\%$$

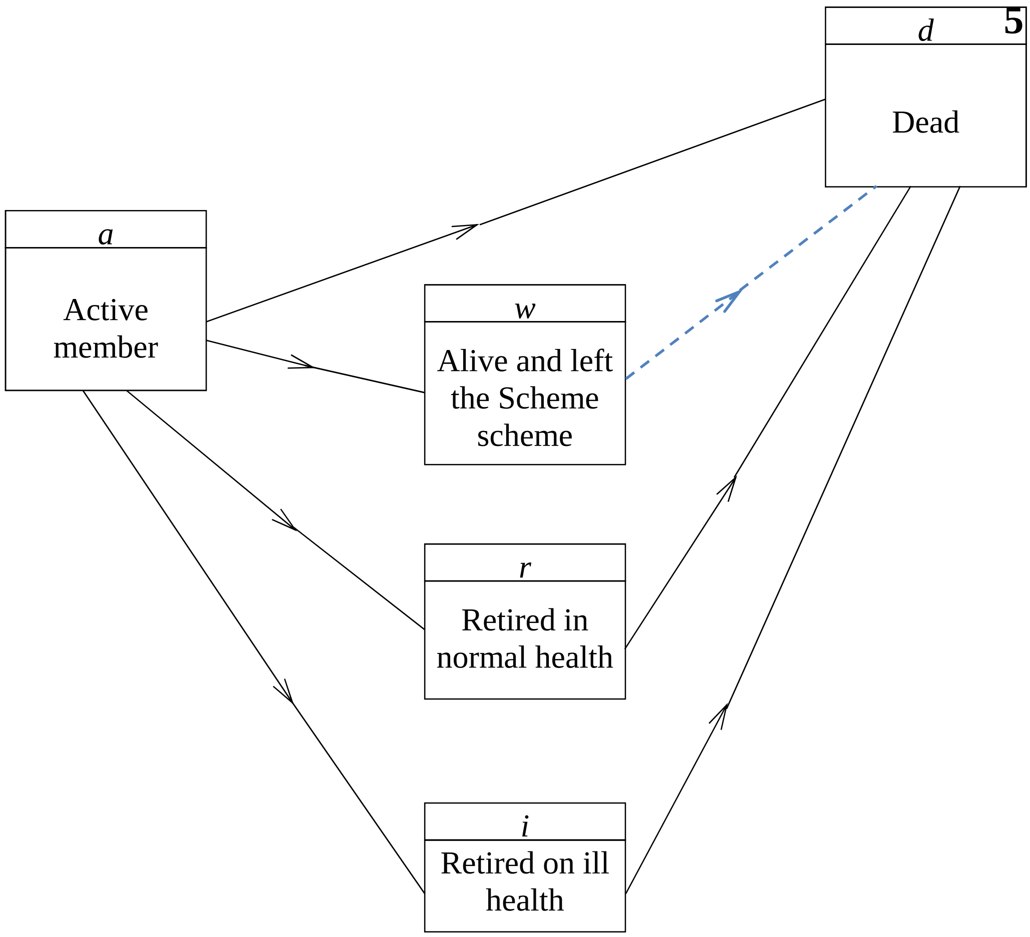
$$= \left(\ddot{a}_{40} - \frac{5}{8}\right) - \frac{v^{10} l_{50}}{l_{40}} \left(\ddot{a}_{50} - \frac{5}{8}\right)$$

$$= (15.491 - 0.625) - \frac{0.55839 \times 9712.0728}{9856.2863}(14.044 - 0.625)$$

$$= 14.866 - 7.383 = 7.483$$

Parts (a) and (b) were straightforward and well done. Many students in (c) did not obtain the correct relationship for the adue function in line 2 of the formulae above.

5



Straightforward question generally well done. Note there is in reality no connection from Withdrawn to Dead as this is not a feature of the PEN tables and lives have left the scheme experience altogether. Also there are no probabilities shown for the PEN tables for states w, r and i to d so students who did not include these in the diagram were given full credit.

6

	$q_{[x]}$	$q_{[x-1]+1}$	$q_{[x-2]+2}$
55	**0.003358**		
56		**0.004903**	
57			**0.005650**

EPV premiums $= 900\{1 + v*p_{[55]} + v^2* p_{[55]}*p_{[55]+1}\}$
$= 900\{1 + v*(1 – 0.003358) + v^2*(1 – 0.003358)*(1 – 0.004903)\}$
$= 900(1 + 0.96761 + 0.93482) = 2612.19$

EPV benefits $= 150{,}000\{v*q_{[55]} + v^2*p_{[55]}*q_{[55]+1} + v^3*p_{[55]}*p_{[55]+1}*q_{57}\}$
$= 150{,}000 \{0.0032602 + 0.0046060 + 0.0051279\} = 1949.12$

EPV expenses $= 260 + 70\{v*p_{[55]} + v^2* p_{[55]}*p_{[55]+1}\}$
$= 260 + 70\{v*(1 – 0.003358) + v^2*(1 – 0.003358)*(1 – 0.004903)\}$
$= 260 + 70(0.96761 + 0.93482) = 393.17$

EPV profit $= 2612.19 – 1949.12 – 393.17 = 269.90$

Alternatively, using cash flow approach:

Yr	*premium*	*expense*	*interest*	*claim*	*profit vector*	*cumulative probability of survival*	*discount factor*	*net present value*
1	900.00	260.00	19.20	503.70	155.50	1.000000	.97087	150.97
2	900.00	70.00	24.90	735.45	119.45	0.996642	.94260	112.22
3	900.00	70.00	24.90	847.50	7.40	0.991755	.91514	6.72

Total net present value of profit = 269.91

This question was generally well done by well prepared students.

7 (a)

$$ {}_{1.75}p_{82.75} = {}_{0.25}p_{82.75} \times p_{83} \times {}_{0.5}p_{84} $$

$$ = (1 - {}_{0.25}q_{82.75})(1 - q_{83})(1 - {}_{0.5}q_{84}) $$

$$ = \left(1 - \frac{0.25q_{82}}{(1 - 0.75q_{82})}\right)(1 - q_{83})(1 - 0.5q_{84}) $$

$$ = \left(1 - \frac{0.25 \times 0.11279}{(1 - 0.75 \times 0.11279)}\right) \times (1 - 0.12235) \times (1 - 0.5 \times 0.13270) $$

$$ = 0.79418 $$

(b)

$$ {}_{1.75}p_{82.75} = {}_{0.25}p_{82.75} \times p_{83} \times {}_{0.5}p_{84} $$

$$ = (p_{82})^{0.25} \times p_{83} \times (p_{84})^{0.5} $$

$$ = (1 - 0.11279)^{0.25} \times (1 - 0.12235) \times (1 - 0.13270)^{0.5} $$

$$ = 0.79325 $$

This question was generally well done.

8 (a) When a life table is constructed it is assumed to reflect the mortality experience of a homogeneous group of lives. This table can then be used to model the experience of a homogeneous group of lives which is suspected to have a similar experience.

If a table is constructed for heterogeneous group then the mortality experience will depend on the exact mixture of lives with different experiences used to construct the table. Such a table could only be used to model mortality in a group with the same mixture.

For this reason separate mortality tables are usually constructed for groups which are expected to be heterogeneous.

(b) Choose from:

- Full choice available here from
- Temporary Initial Selection
- Class Selection
- Adverse Selection
- Time Selection
- Spurious Selection

A straight bookwork question generally well done.

9 (i) Let P be the annual premium for the contract. Then:

EPV of premiums is:

$$P\ddot{a}_{[45]:\overline{20}|} = 11.888P$$

EPV of benefits and claim expense:

$$125,325A_{[45]} = 125,325 \times 0.15918 = 19,949.23$$

EPV of other expenses:

$$0.75P + 0.05P\left[\ddot{a}_{[45]:\overline{20}|} - 1\right] = 1.2944P$$

Equation of value gives

$$11.888P = 19,949.23 + 1.2944P$$

$$\Rightarrow P = \frac{19,949.23}{10.5936} = £1,883.14$$

(ii) gross prospective reserve

$$= 125,000A_{60} - 1883.14\ddot{a}_{60:\overline{5}|} = 125,000 \times 0.32692 - 1883.14 \times 4.39$$
$$= 40,865.0 - 8,266.98 = 32,598.02$$

Generally well done. The main omission that some students counted the claim expense within gross prospective reserve.

10 (i) $$\bar{A}^{1}_{40:50} = \int_0^\infty v^t\,{}_tp_{40:50}\mu_{40+t}dt = .04\int_0^\infty e^{-(.04+.06+\ln 1.05)t}dt = .04\int_0^\infty e^{-0.14879t}dt$$

$$= .04\left[-\frac{e^{-.14879t}}{.14879}\right]_0^\infty = \frac{.04}{.14879} = 0.26884$$

(ii) $$\bar{a}_{40:50:\overline{20}|} = \int_0^{20} v^t\,{}_tp_{40:50}dt = \int_0^{20} e^{-.14879t}dt$$

$$= \left[-\frac{e^{-0.14879t}}{0.14879}\right]_0^{20} = \frac{1}{0.14879}(1 - e^{-2.976}) = 6.378$$

$$\bar{a}_{40:50:\overline{30}|} = \int_0^{30} v^t \, {}_t p_{40:50} dt = \int_0^{30} e^{-.14879t} dt$$

$$= \left[-\frac{e^{-0.14879t}}{0.14879} \right]_0^{30} = \frac{1}{0.14879}(1-e^{-4.464}) = 6.643$$

Let Premium = P, then

$$P(0.75 \times 6.643 + .25 \times 6.378) = 75,000 \times 0.26884$$

$$P = \frac{20163}{6.577} \Rightarrow P = 3065.7$$

Generally part (i) was done well but part (ii) was poorly done. A large proportion of students did not appreciate how to derived the premium relationship described in the question. Another common error was to take the force of interest as 5% rather than ln(1.05)%.

11 The annuity can be written as (with 65 denoting the male life and 62 the female):

$$50000a^{(12)}_{\overline{65:62}} + 25000a^{(12)}_{65:62} + 25000a^{(12)}_{65} + 20000(v^{10}\,{}_{10}p_{\overline{65:62}} + v^{20}\,{}_{20}p_{\overline{65:62}})$$

$$a^{(12)}_{65} = \ddot{a}_{65} - \frac{13}{24} = 13.666 - \frac{13}{24} = 13.124$$

$$a^{(12)}_{65:62} = \ddot{a}_{65:62} - \frac{13}{24} = 12.427 - \frac{13}{24} = 11.885$$

$$a^{(12)}_{\overline{65:62}} = \ddot{a}_{65} + \ddot{a}_{62} - \ddot{a}_{65:62} - \frac{13}{24} = 13.666 + 15.963 - 12.427 - \frac{13}{24} = 16.660$$

$$v^{10}\,{}_{10}p_{\overline{65:62}} = \frac{1-(1-l_{75}/l_{65})(1-l_{72}/l_{62})}{(1.04)^{10}}$$

$$= \frac{1-(1-8405.16/9647.797)(1-9193.86/9804.173)}{1.48024}$$

$$= 0.67015$$

$$v^{20}\,{}_{20}p_{\overline{65:62}} = \frac{1-(1-l_{85}/l_{65})(1-l_{82}/l_{62})}{(1.04)^{20}}$$

$$= \frac{1-(1-4892.878/9647.797)(1-7147.965/9804.173)}{2.19112}$$

$$= 0.39545$$

So value is:

(50000*16.660)+(25000*11.885)+(25000*13.124)+20000*(.67015+.39545)

= 1479537

Other formulae approaches credited. Also the final answer is very sensitive to rounding and full credit was given to +/–00 to the answer.

Many students found difficulty in reproducing the correct annuities to make up the total value.

12 Let P be the monthly premium. Then:

EPV of premiums:

$$12P\ddot{a}^{(12)}_{[40]:\overline{25|}}\ @\,6\% = 155.1272P$$

where

$$\ddot{a}^{(12)}_{[40]:\overline{25|}} = \ddot{a}_{[40]:\overline{25|}} - \frac{11}{24}\left(1 - {}_{25}p_{[40]}v^{25}\right)$$

$$= 13.290 - \frac{11}{24}\left(1 - \frac{8821.2612}{9854.3036}\times 0.233\right) = 12.9273$$

EPV of benefits:

$$72,750\bar{A}^{\,1}_{[40]:\overline{25|}} + 2250\left(I\bar{A}\right)^{1}_{[40]:\overline{25|}} + 131,250A_{[40]:\overline{25|}}^{\ \ \ \ \ 1}\quad @\,6\%$$

$$= 72,750\times 0.04032 + 2250\times 0.62876 + 131,250\times 0.208574$$

$$= 2,933.561 + 1,414.71 + 27,375.3375 = 31,723.609$$

where

$$\bar{A}^{\;1}_{[40]:\overline{25}|} = 1.06^{0.5} A^{\;1}_{[40]:\overline{25}|} = 1.06^{0.5}\left[A_{[40]:\overline{25}|} - v^{25}\,{}_{25}p_{[40]}\right]$$

$$= 1.06^{0.5}\left[0.24774 - 0.233 \times \frac{8821.2612}{9854.3036}\right] = 0.04032$$

$$(I\bar{A})^{\;1}_{[40]:\overline{25}|} = 1.06^{0.5}(IA)^{\;1}_{[40]:\overline{25}|} = 1.06^{0.5}\left[(IA)_{[40]} - v^{25}\,{}_{25}p_{[40]}\left(25A_{65} + (IA)_{65}\right)\right]$$

$$= 1.06^{0.5}\left[3.85489 - 0.208574 \times (25 \times 0.40177 + 5.50985)\right] = 0.62876$$

EPV of expenses:

$$= 1.15P + 210 + 0.025 \times 12 \times P\ddot{a}^{(12)}_{[40]:\overline{25}|} - 0.025P + 85\left[\ddot{a}^{@i'}_{[40]:\overline{25}|} - 1\right]$$
$$= 1.15P + 210 + 0.025 \times 12 \times P \times 12.9273 - 0.025P + 85 \times [15.887 - 1]$$
$$= 5.00319P + 1,475.395$$

where

$$i' = \frac{1.06}{1+b} - 1 = 0.04$$

Equation of value gives:

$$155.1272P = 31,723.609 + 5.00319P + 1475.395$$

$$\Rightarrow P = \frac{33,199.00}{150.1240} = £221.14$$

Well prepared students completed this question satisfactorily. Others found difficulty in deriving in particular the expense values. Credit was given in part to the correct approach even if the final arithmetic proved to be inaccurate.

13 (i) Let P be the net annual premium. Then:

EPV of premiums:

$$P\ddot{a}_{45:\overline{15}|} = 11.386P$$

EPV of benefits:

$$60000\bar{A}^{1}_{45:\overline{10}|} + 40000\bar{A}^{1}_{45:\overline{15}|}$$

$$= 60000\frac{i}{\delta}\left(A_{45} - v^{10}\,{}_{10}p_{45}A_{55}\right) + 40000\frac{i}{\delta}\left(A_{45} - v^{15}\,{}_{15}p_{45}A_{60}\right)$$

$$= 60000\frac{0.04}{0.039221}\left(0.27605 - 1.04^{-10}\frac{9557.8179}{9801.3123}0.38950\right)$$

$$+\ 40000\frac{0.04}{0.039221}\left(0.27605 - 1.04^{-15}\frac{9287.2164}{9801.3123}0.45640\right)$$

$$= 1190.567 + 1465.406 = £2655.973$$

Equation of value gives:

$$P = \frac{2655.973}{11.386} = £233.27$$

(ii) The net premium reserve at 31.12.13 is given by:

$${}_{10}V_{45:\overline{15}|} = 40000\bar{A}^{1}_{55:\overline{5}|} - 233.27\ddot{a}_{55:\overline{5}|} = 40000\frac{i}{\delta}\left(A_{55} - v^{5}\,{}_{5}p_{55}A_{60}\right) - 233.27 \times 4.585$$

$$= 40000\frac{0.04}{0.039221}\left(0.38950 - 1.04^{-5}\frac{9287.2164}{9557.8179}0.4564\right) - 233.27 \times 4.585$$

$$= 1019.53 - 1069.54 = -£50.01$$

(iii) Explanation:

Policyholder "in debt" at time 10 (with size of debt equal to the negative reserve) as more life cover provided in the first 10 years than is paid for by the level premiums in those years.

Disadvantages:

If policy is lapsed during first ten years (possibly longer) the company will suffer a loss.

Not possible to recover this loss from policyholder.

Possible alterations:

Collect the premiums more quickly e.g. shorten premium paying term, make premiums larger in earlier years, smaller in later years.

Change the pattern of benefits to reduce benefits in first ten years and increase them in last five years.

(iv) During 2013, we have:

Death strain at risk = $100{,}000\,(1.04)^{1/2} + 50.01 = 102{,}030.40$

EDS = $2878 q_{54} \times 102{,}030.40 = 2878 \times 0.003976 \times 102{,}030.40 = 1{,}167{,}526.52$

ADS = $12 \times 102{,}030.40 = 1{,}224{,}364.80$

Mortality profit = $1{,}167{,}526.52 - 1{,}224{,}364.80$ = –£56,838.28 (i.e. a loss)

Question done well for students who had prepared. Common errors were in (ii) where immediate payment on death not computed and not getting the profit correct in (iv). Students were given reasonable credit if they showed understanding of the problem even if all arithmetical calculations not correct.

14 (i) Multiple decrement table:

x	q_x^d	q_x^s
58	0.004649	0.1
59	0.006929	0.1
60	0.008022	0.1

x	$(aq)_x^d$	$(aq)_x^s$	(ap)	$_{t-1}(ap)$
58	0.004649	0.09954	0.895816	1.000000
59	0.006929	0.09931	0.893764	0.895816
60	0.008022	0.09920	0.892780	0.800648

Unit fund (per policy at start of year)

	Yr 1	*Yr 2*	*Yr 3*
value of units at start of year	0.00	2206.33	5072.05
allocation	2250.00	2850.00	3450.00
B/O spread	112.50	142.50	172.50
interest	85.50	196.55	333.98
management charge	16.67	38.33	65.12
value of units at end of year	2206.33	5072.05	8618.41

Cash flows (per policy at start of year)

	Yr 1	*Yr 2*	*Yr 3*
unallocated premium	750.00	150.00	–450.00
B/O spread	112.50	142.50	172.50
expenses	425.00	130.00	130.00
interest	8.75	3.25	–8.15
management charge	16.67	38.33	65.12
extra death benefit	31.58	27.22	3.06
profit vector	431.34	176.86	–353.59

profit vector	431.34	176.86	–353.59
probability in force	1.0	0.895816	0.800648
profit signature	431.34	158.43	–283.10
discount factor	0.943396	0.889996	0.839619
expected p.v. of profit	406.92	141.01	–237.69

Total NPV of expected profit = 310.24

	Yr 1	*Yr 2*	*Yr 3*
premium signature	3000.00	2687.45	2401.94
discount factor	1.0	0.943396	0.889996
expected p.v. of premiums	3000.00	2535.33	2137.72

Total PV of premiums = 7673.05

Profit margin = $\dfrac{310.24}{7673.05} = 4.04\%$

(ii) To calculate the expected provisions at the end of each year we have (utilising the end of year cash flow figures and decrement tables in (i) above):

$${}_2V = \frac{353.59}{1.02} = 346.66$$

$${}_1V \times 1.02 - (ap)_{59} \times {}_2V = -176.86 \Rightarrow {}_1V = 130.36$$

The revised cash flow for year 1 will become:

$$431.34 - (ap)_{58} \times {}_1V = 314.56$$

Hence the table below can now be completed for the revised net present value of expected profit.

	Yr 1	*Yr 2*	*Yr 3*
revised end of year cash flow	314.56	0	0
probability in force	1	0.895816	0.800648
discount factor	0.943396	0.889996	0.839619
expected p.v. of profit	296.76		

Profit margin = $\dfrac{296.76}{7673.05} = 3.87\%$

Question again done well by students properly prepared. Part (ii) gave more issues as many students could not seem to remember the zeroisation procedure.

Again reasonable credit given for understanding the process where computational errors had occurred.

END OF EXAMINERS' REPORT