

# **EXAMINERS' REPORT**

April 2010 Examinations

## **Subject CT5 — Contingencies Core Technical**

### **Introduction**

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

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Chairman of the Board of Examiners

July 2010

### **Comments**

These are given in italics at the end of each question.

- 1**
- (i) The number of lives still alive at age  $x + r$  out of  $l_x$  lives alive at age  $x$  subject to select mortality.
  - (ii) The probability that a life age  $x$  will die between age  $x + n$  and  $x + n + m$ .
  - (iii) The number of lives that die between  $x$  and  $(x + 1)$  out of  $l_x$  lives alive at  $x$ .

*Question generally answered well.*

- 2** Spurious selection occurs when mortality differences ascribed to groups are formed by factors which are not the true causes of these differences.

For example mortality differences by region may be put down to the actual class structure of the region itself whereas a differing varying mix of occupations region by region could be having a major effect. So Region is spurious and being confounded with occupation.

Another example might be in a company pension scheme which might be showing a significant change in mortality experience which could be viewed as change over time. However withdrawers from the scheme may be having an effect as their mortality could be different. To that degree Time Selection may be spurious.

*Question generally answered well. Credit was given for a wide range of valid examples.*

- 3** The Standardised mortality ratio is the ratio of actual deaths in the population divided by the expected number of deaths in the population if the population experienced standard mortality.

Actual number of deaths for Urbania =  $130+145+173 = 448$

Mortality rates in standard population are:

Age 60:  $26,170 / 2,500,000 = 0.0104680$

Age 61:  $29,531 / 2,400,000 = 0.0123046$

Age 62:  $32,542 / 2,200,000 = 0.0147918$

Expected number of deaths for Urbania

$= 0.010468 \times 10,000 + 0.0123046 \times 12,000 + 0.0147918 \times 11,000 = 415$

SMR =  $448/415 = 107.95\%$

*Question generally answered well.*

$$\begin{aligned}
 \mathbf{4} \quad EPV &= (10,000 - 100)A_{[50]:5}^1 + 100(IA)_{[50]:5}^1 \\
 &= 9,900(A_{[50]} - v^5 {}_5P_{[50]}A_{55}) + 100((IA)_{[50]} - v^5 {}_5P_{[50]}(5A_{55} + (IA)_{55})) \\
 &= 9,900(0.32868 - v^5 \frac{9557.8179}{9706.0977} * 0.38950) \\
 &\quad + 100 * \left( 8.5639 - v^5 \frac{9557.8179}{9706.0977} (5 * 0.38950 + 8.57976) \right) \\
 &= 132.96 + 4.34 \\
 &= 137.30
 \end{aligned}$$

Many students answered the question well. The most common error was the use of 10,000 as the multiplier before the temporary assurance function rather than 9,900.

$$\begin{aligned}
 \mathbf{5} \quad {}_tP_x &= \exp\left(-\int_x^{x+t} \mu_s ds\right) \\
 &= \exp\left(-\int_x^{x+t} (e^{0.0002s} - 1) ds\right) \\
 &= \exp\left(-\int_x^{x+t} e^{0.0002s} ds + \int_x^{x+t} ds\right) \\
 &= \exp\left(-\frac{\left[e^{0.0002(x+t)} - e^{0.0002x}\right]}{0.0002} + t\right)
 \end{aligned}$$

(i) Probability =

$$\begin{aligned}
 &= \exp\left(-\frac{\left[e^{0.0002x70} - e^{0.0002x20}\right]}{0.0002} + 50\right) \\
 &= 0.6362
 \end{aligned}$$

- (ii) This is the probability that the life survives to 60 and then dies between 60 and 70

$$\begin{aligned} \text{Probability} &= {}_{40}P_{20}(1 - {}_{10}P_{60}) \\ &= \exp\left(-\frac{[e^{0.0002 \times 60} - e^{0.0002 \times 20}]}{0.0002} + (60 - 20)\right) \cdot \left(1 - \exp\left(-\frac{[e^{0.0002 \times 70} - e^{0.0002 \times 60}]}{0.0002} + (70 - 60)\right)\right) \\ &= 0.725x(1 - 0.8773) \\ &= 0.0889 \end{aligned}$$

*This question was answered poorly overall. It was an unusual representation of the  $\mu_x$  function but other than that was a straight forward probability and integration question.*

**6**  $p_{50} = 97,702 / 99,813 = 0.978850$   
 $p_{51} = 95,046 / 97,702 = 0.972815$

*Uniform distribution of deaths*

$$\frac{{}_{P_{50:0.25}P_{51}}}{0.5P_{50}} = \frac{p_{50}(1 - 0.25(1 - p_{51}))}{(1 - 0.5(1 - p_{50}))} = \frac{0.978850 * (1 - 0.25 * (1 - 0.972815))}{(1 - 0.5 * (1 - 0.978850))} = 0.982588$$

*Constant force of mortality*

$$\begin{aligned} \mu_t &= -\ln(p_t) \\ \mu_{50} &= -\ln(0.978850) = 0.021377 \\ \mu_{51} &= -\ln(0.972815) = 0.027561 \\ {}_{0.5}P_{50} * {}_{0.25}P_{51} &= e^{-0.5 * 0.021377} * e^{-0.25 * 0.027561} = 0.989368 * 0.993133 = 0.982574 \end{aligned}$$

*Generally answered well. A limited number of students used the Balducci Assumption as one of their answers. This is not in the CT5 Course whilst the above 2 methods clearly are. This method was however credited – solution not published as not in CT5*

- 7** (i) *Age retirement benefit*

$$\frac{1}{60} 40,000 \frac{(20 {}^z M_{55}^{ra} + {}^z \bar{R}_{55}^{ra})}{s_{54} D_{55}}$$

$$= \frac{1}{60} 40,000 \frac{(20 * 128,026 + 963,869)}{9.745 * 1,389}$$

$$= 173,584$$

(ii) Contributions

$$K 40,000. \frac{{}^s\bar{N}_{55}}{s_{54}D_{55}}$$

$$= K.40,000x \frac{88,615}{9.745 * 1,389}$$

$$= 261,868K$$

Therefore  $K = 173,584 / 261,868$  i.e. 66.3%

*Most students answered reasonably well. Most common error was the wrong  $s_x$  function. Also some students included early retirement calculations which were not asked for.*

*Also students often did not include the past service benefits in the final contribution rate believing the final result would have been too high (the question however was quite specific on providing past benefits).*

$$8 \quad (i) \quad \text{Fund} = 52 * \frac{1.04^{(66-21)} \bar{a}_{21:\overline{45}|}}{45 P_{21}}$$

$$\bar{a}_{21:\overline{45}|} = \ddot{a}_{21:\overline{45}|} - \frac{1}{2} * (1 - v^{45} * l_{66} / l_{21}) = \ddot{a}_{21:\overline{45}|} - \frac{1}{2} * \left( 1 - 0.17120 * \frac{8695.6199}{9976.3909} \right)$$

$$= \ddot{a}_{21:\overline{45}|} - 0.42539$$

$$\ddot{a}_{21:\overline{45}|} = \ddot{a}_{21:\overline{44}|} + v^{44} * l_{65} / l_{21} = 21.045 + .17805 * \frac{8821.2612}{9976.3909} = 21.202$$

$$\Rightarrow \bar{a}_{21:\overline{45}|} = 20.777$$

$$\text{therefore fund} = \frac{52 * 1.04^{45} (20.777)}{\left( \frac{8695.6199}{9976.3909} \right)} = 7,240$$

- (ii) Let annuity be £P per week. Then EPV of annuity at 66 is

$$\begin{aligned}
 & 52P(\bar{a}_{10|} + \frac{2}{3} * v^{10} {}_{10}P_{66} \cdot \bar{a}_{76}) \\
 &= 52P \left[ \frac{(1-v^{10})}{\ln(1.04)} + \frac{2}{3} * 0.675564 * \frac{6589.9258}{8695.6199} (8.169 - 0.5) \right] \\
 &= 52P[8.272 + 2.618] \\
 &= 566.26P
 \end{aligned}$$

Therefore pension is given by

$$7,240 = 566.26P$$

$$P = 12.79$$

Many students struggled with this question and indeed a large number did not attempt it. As will be seen from the solution above the actuarial mathematics involved are relatively straightforward.

Note that 52.18 (i.e. 365.25/7) would have been an acceptable alternative to 52 as the multiplier which will of course have adjusted the answer slightly.

- 9** (i) We are looking to derive  $(aq)_x^r$  in terms of  $\sigma_x$  and  $\mu_x$

Use the Kolmogorov equations (assuming the transition intensities are constant across a year age):

$$\begin{aligned}
 \frac{\partial}{\partial t} {}_t(aq)_x^r &= \sigma e^{-(\sigma+\mu)t} \\
 (aq)_x^r &= \frac{\sigma}{(\sigma+\mu)} (1 - e^{-(\sigma+\mu)})
 \end{aligned}$$

- (ii) Similarly

$$(aq)_x^d = \frac{\mu}{(\sigma+\mu)} (1 - e^{-(\sigma+\mu)})$$

Note that:

$$\begin{aligned}
 1 - ((aq)_x^r + (aq)_x^d) &= e^{-(\sigma+\mu)} \\
 \Rightarrow \sigma + \mu &= -\log(1 - ((aq)_x^r + (aq)_x^d))
 \end{aligned}$$

So

$$(aq)_x^r = \frac{\sigma}{(-\log(1 - ((aq)_x^r + (aq)_x^d)))} ((aq)_x^r + (aq)_x^d)$$

this can be rearranged to show

$$-\sigma = \frac{(aq)_x^r}{(aq)_x^r + (aq)_x^d} \log(1 - ((aq)_x^r + (aq)_x^d))$$

Given that:

$$q_x^r = 1 - e^{-\sigma},$$

then

$$q_x^r = 1 - \left[ 1 - ((aq)_x^r + (aq)_x^d) \right]^{(aq)_x^r / ((aq)_x^r + (aq)_x^d)}$$

*In general this was poorly answered with most students making a limited inroad to the question.*

*However, the question did not specify that constant forces must be assumed. So, a valid alternative to part (i) is:*

$$(aq)_x^r = \int_0^1 {}_t(ap)_x \sigma_{x+t} dt = \int_0^1 \exp \left[ -\int_0^t (\mu_{x+r} + \sigma_{x+r}) dr \right] \sigma_{x+t} dt$$

*This makes no assumptions and provides an answer in the form asked for in the question, and so would merit full marks. If constant forces are assumed, the above expression will turn into the answer in the above solution.*

*For part (ii) a solution is only possible if some assumption is made. The following alternatives could be valid:*

(1) *Assume dependent decrements are uniformly distributed over the year of age*

*With this assumption, deaths occur on average at age  $x + \frac{1}{2}$ , so:*

$$q_x^r = \frac{(ad)_x^r + \frac{1}{2}(ad)_x^d \times q_x^r}{(al)_x} = (aq)_x^r + \frac{1}{2}(aq)_x^d \times q_x^r \Rightarrow q_x^r = \frac{(aq)_x^r}{1 - \frac{1}{2}(aq)_x^d}$$

*(This is covered by the Core Reading in Unit 8 Section 10.1.3.)*

(2) *Assume independent decrements are uniformly distributed over the year of age*

This leads to two simultaneous equations:

$$q_x^d = \frac{(aq)_x^d}{1 - \frac{1}{2}q_x^r} \quad \text{and} \quad q_x^r = \frac{(aq)_x^r}{1 - \frac{1}{2}q_x^d}$$

which results in a quadratic equation in  $q_x^r$ . (This is covered by the Core Reading Unit 8 Section 10.1.6.)

Whilst a full description has been given above to assist students, in reality those who successfully attempted this question did assume constant forces.

**10** First calculate  $(aq)_x^d$  and  $(aq)_x^w$

Age ( $x$ )	Number of employees ( $al$ ) $_x$	$(aq)_x^d$	$(aq)_x^w$
40	10,000	.00250	.01200
41	9,855	.00274	.01461
42	9,684		

From this table and relationship

$$q_x^d = (aq)_x^d / (1 - \frac{1}{2} * (aq)_x^w) \quad \text{and} \quad q_x^w = (aq)_x^w / (1 - \frac{1}{2} * (aq)_x^d)$$

Calculate  $q_x^d$  and  $q_x^w$

$$q_{40}^d = .00250 / (1 - .006) = .00252 \quad \text{and} \quad q_{41}^d = .00274 / (1 - .00731) = .00276$$

$$q_{40}^w = .01200 / (1 - .00125) = .01201 \quad \text{and} \quad q_{41}^w = .01461 / (1 - .00137) = .01463$$

Adjusting for the 75% multiplier of independent withdrawal decrements:

$$(aq)_{40}^d = .00252 * \left( 1 - \frac{1}{2} * \frac{3}{4} * .01201 \right) = .00251$$

$$(aq)_{41}^d = .00276 * \left( 1 - \frac{1}{2} * \frac{3}{4} * .01463 \right) = .00274$$

$$(aq)_{40}^w = .01201 * \frac{3}{4} * \left( 1 - \frac{1}{2} * .00252 \right) = .00900$$

$$(aq)_{41}^w = .01463 * \frac{3}{4} * \left( 1 - \frac{1}{2} * .00276 \right) = .01096$$

Using the above data the Table can now be reconstructed

Age (x)	Number of employees (al) <sub>x</sub>	Deaths (ad) <sub>x</sub> <sup>d</sup>	Withdrawals (ad) <sub>x</sub> <sup>w</sup>
40	10,000	(10000*.00251)=25.1	10000*.00900=90.0
41	9,884.9	(9,884.9*.00274)=27.1	9,884.9*.01096=108.3
42	9749.5		

It should be noted that if more decimal places are used in the aq factors then the deaths at 40 become 25.0 so full credit was given for this answer also.

Because of the limited effect on the answer from the original table students were asked to show the result to 1 decimal place. Many failed to do so and were penalised accordingly.

- 11** (i) Policy value at duration  $t$  of an immediate annuity payable continuously at a rate of £1 per annum and secured by a single premium at age  $x$  is given by:

$$\begin{aligned}
 {}_t\bar{V}_x &= \bar{a}_{x+t} = \int_0^{\infty} e^{-\delta s} {}_s p_{x+t} ds \\
 \Rightarrow \frac{\partial}{\partial t} {}_t\bar{V}_x &= \frac{\partial}{\partial t} \bar{a}_{x+t} = \frac{\partial}{\partial t} \int_0^{\infty} e^{-\delta s} {}_s p_{x+t} ds = \int_0^{\infty} e^{-\delta s} \frac{\partial}{\partial t} {}_s p_{x+t} ds \\
 \frac{1}{{}_s p_{x+t}} \times \frac{\partial}{\partial t} {}_s p_{x+t} &= \frac{\partial}{\partial t} \ln({}_s p_{x+t}) = \frac{\partial}{\partial t} (\ln l_{x+t+s} - \ln l_{x+t}) = -\mu_{x+t+s} + \mu_{x+t} \\
 \Rightarrow \frac{\partial}{\partial t} {}_s p_{x+t} &= {}_s p_{x+t} (-\mu_{x+t+s} + \mu_{x+t}) \\
 \Rightarrow \frac{\partial}{\partial t} {}_t\bar{V}_x &= \int_0^{\infty} e^{-\delta s} {}_s p_{x+t} (\mu_{x+t} - \mu_{x+t+s}) ds \\
 &= \mu_{x+t} \times \bar{a}_{x+t} - \int_0^{\infty} e^{-\delta s} {}_s p_{x+t} \times \mu_{x+t+s} ds \\
 &= \mu_{x+t} \times \bar{a}_{x+t} - \left\{ \left[ -e^{-\delta s} {}_s p_{x+t} \right]_0^{\infty} - \delta \int_0^{\infty} e^{-\delta s} {}_s p_{x+t} ds \right\} \\
 &= \mu_{x+t} \times \bar{a}_{x+t} - 1 + \delta \times \bar{a}_{x+t}
 \end{aligned}$$

$$= \mu_{x+t} \times {}_t\bar{V}_x - 1 + \delta \times {}_t\bar{V}_x$$

- (ii) Consider a short time interval  $(t, t + dt)$  then equation implies:

$${}_{t+dt}\bar{V} - {}_t\bar{V} = \mu_{x+t} \times {}_t\bar{V}_x \times dt - 1 \times dt + \delta \times {}_t\bar{V}_x \times dt + o(dt)$$

where

$\mu_{x+t} \times {}_t\bar{V}_x \times dt$  = reserve released as a result of deaths in time interval  $(t, t + dt)$

$-1 \times dt$  = annuity payments made in time interval  $(t, t + dt)$

$\delta \times {}_t\bar{V}_x \times dt$  = interest earned on reserve over time interval  $(t, t + dt)$

*In general very poorly answered on what was a standard bookwork question.*

- 12** (i) Annual premium  $P$  for the term assurance policy is given by:

$$P = \frac{25,000\bar{A}_{[55]:10}^1 + 25,000\bar{A}_{[55]:5}^1}{\ddot{a}_{[55]:10}}$$

where

$$\begin{aligned} & 25,000\bar{A}_{[55]:10}^1 + 25,000\bar{A}_{[55]:5}^1 \\ &= 25,000 \times (1+i)^{1/2} \times \left( (A_{[55]} - v^{10} {}_{10}P_{[55]}A_{65}) + (A_{[55]} - v^5 {}_5P_{[55]}A_{60}) \right) \\ &= 25,000 \times 1.019804 \times \left( \begin{aligned} & (0.38879 - 0.67556 \times \frac{8821.2612}{9545.9929} \times 0.52786) \\ & + (0.38879 - 0.82193 \times \frac{9287.2164}{9545.9929} \times 0.4564) \end{aligned} \right) \\ &= 25,495.10 \times ((0.38879 - 0.32953) + (0.38879 - 0.36496)) = 2118.39 \end{aligned}$$

Therefore

$$P = \frac{2118.39}{8.228} = 257.46$$

Net Premium Retrospective Reserves at the end of the fifth policy year is given by:

$$(1+i)^5 \times \frac{l_{[55]}}{l_{60}} \times \left[ P\ddot{a}_{[55]:5} - 50,000\bar{A}_{[55]:5}^1 \right]$$

$$= 1.21665 \times \frac{9545.9929}{9287.2164} \times [257.46 \times 4.59 - 50,000 \times 1.019804 \times (0.38879 - 0.36496)]$$

$$= -41.71$$

- (ii) **Explanation** – more cover provided in the first 5 years than is paid for by the premiums in those years. Hence policyholder “in debt” at time 5, with size of debt equal to negative reserve.

**Disadvantage** – if policy lapsed during the first 5 years (and possibly longer), the company will suffer a loss which is not possible to recover from the policyholder.

#### Possible alterations to policy structure

Collect premiums more quickly by shortening premium payment term or make premiums larger in earlier years, smaller in later years

Change the pattern of benefits to reduce benefits in first 5 years and increase them in last 5 years.

- (iii) Mortality Profit = EDS – ADS

$$\text{Death strain at risk} = 50,000 - (-42) = 50,042$$

$$\begin{aligned} EDS &= (1000 - 20) \times q_{59} \times 50,042 \\ &= 980 \times 0.00714 \times 50,042 = 350,154 \end{aligned}$$

$$ADS = 8 \times 50,042 = 400,336$$

$$\text{Total Mortality Profit} = 350,154 - 400,336 = -£50,182 \text{ (i.e. a mortality loss)}$$

*Quite reasonably answered by the well prepared student.*

*In (i) it should be noted that in this case the retrospective and prospective reserves are equal. If the student recognised this, explicitly stated so and then did the easier prospective calculation full marks were given. No credit was given for a prospective calculation without explanation.*

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Annual premium	£4000.00	Allocation % (1st yr)	95.0%
Risk discount rate	7.0%	Allocation % (2nd yr)	100.0%
Interest on investments (1st yr)	5.5%	Allocation % (3rd yr)	105.0%
Interest on investments (2nd yr)	5.25%	B/O spread	5.0%
Interest on investments (3rd yr)	5.0%	Management charge	1.75%
Interest on non-unit funds	4.0%	Surrender penalty (1st yr)	£1000
Death benefit (% of bid value of units)	125%	Surrender penalty (2nd yr)	£500
		Policy Fee	£50

	£	% prem
Initial expense	200	15.0%
Renewal expense	50	2.0%
Expense inflation	2.0%	

(i) Multiple decrement table:

$x$	$q_x^d$	$q_x^s$
45	0.001201	0.12
46	0.001557	0.06
47	0.001802	0.00

$x$	$(aq)_x^d$	$(aq)_x^s$	$(ap)$	${}_{t-1}(ap)$
45	0.001201	0.11986	0.878943	1.000000
46	0.001557	0.05991	0.938536	0.878943
47	0.001802	0.00000	0.998198	0.824920

Unit fund (per policy at start of year)

	<i>yr 1</i>	<i>yr 2</i>	<i>yr 3</i>
value of units at start of year	0.000	3690.074	7693.641
Alloc	3800.000	4000.000	4200.000
B/O	190.000	200.000	210.000
policy fee	50.000	50.000	50.000
Interest	195.800	390.604	581.682
management charge	65.727	137.037	213.768
value of units at year end	3690.074	7693.641	12001.554

Cash flows (per policy at start of year)

	yr 1	yr 2	yr 3
unallocated premium + pol fee	250.000	50.000	-150.000
B/O spread	190.000	200.000	210.000
expenses	800.000	131.000	132.020
Interest	-14.400	4.760	-2.881
man charge	65.727	137.037	213.768
extra death benefit	1.108	2.995	5.407
surrender penalty	119.856	29.953	0.000
end of year cashflow	-189.926	287.755	133.461
probability in force	1	0.878943	0.824920
discount factor	0.934579	0.873439	0.816298
expected p.v. of profit	133.280		
premium signature	4000.000	3285.769	2882.069
expected p.v. of premiums	10167.837		
profit Margin	1.31%		

- (ii) Revised profit vector (-309.781, 257.802, 133.461)  
 Revised profit signature (-309.781, 257.492, 133.093)

$$\text{Revised PVFNP} = -289.515 + 224.904 + 108.643 = 44.032$$

Again most well prepared students made a good attempt at this question. The most common error was to ignore dependent decrements.

Substantial credit was given to students who showed how they would tackle this question even if they did not complete all the arithmetical calculations involved.

## 14

- (i) Let  $P$  be the quarterly premium. Then:

EPV of premiums:

$$4P\ddot{a}_{[35]:\overline{30}|}^{(4)} @ 6\% = 56.1408P$$

where

$$\ddot{a}_{[35]:\overline{30}|}^{(4)} = \ddot{a}_{[35]:\overline{30}|} - \frac{3}{8}(1 - {}_{30}P_{[35]}v^{30})$$

$$= 14.352 - \frac{3}{8} \left( 1 - \frac{8821.2612}{9892.9151} \times 0.17411 \right)$$

$$= 14.0352$$

EPV of benefits:

$$100,000(q_{[35]}v^{0.5} + {}_1|q_{[35]}(1+b)v^{1.5} + \dots + {}_{29}|q_{[35]}(1+b)^{29}v^{29.5})$$

$$+ 100,000 \times (1+b)^{30} v^{30} {}_{30}P_{[35]}$$

where  $b = 0.0192308$

$$= \frac{100,000}{(1+b)^{0.5}} \times \frac{(1.06)^{0.5}}{(1+b)^{0.5}} \left( q_{[35]}(1+b)v + {}_1|q_{[35]}(1+b)^2v^2 + \dots + {}_{29}|q_{[35]}(1+b)^{30}v^{30} \right)$$

$$+ 100,000(1+b)^{30} v^{30} {}_{30}P_{[35]}$$

$$= \frac{100,000}{(1+b)} \times (1.06)^{0.5} \times A_{[35]:30}^1 @ i' + 100,000 v^{30} {}_{30}P_{[35]} @ i'$$

$$= \frac{100,000 \times (1.06)^{0.5}}{(1+b)} \times \left( 0.32187 - 0.30832 \times \frac{8821.2612}{9892.9151} \right)$$

$$+ 100,000 \times 0.30832 \times \frac{8821.2612}{9892.9151}$$

$$= 4,742.594 + 27,492.112 = 32,234.706$$

where

$$i' = \frac{1.06}{1+b} - 1 = 0.04$$

EPV of expenses (at 6%)

$$= P + 250 + 0.025 \times 4P \ddot{a}_{[35]:30}^{(4)} - 0.025 \times 4P \ddot{a}_{[35]:1}^{(4)} + 45 \left[ \ddot{a}_{[35]:30} - 1 \right]$$

$$+ 500 \bar{A}_{[35]:30}^1 + 250 v^{30} {}_{30}P_{[35]}$$

$$= P + 250 + 0.025 \times 56.1408P - 0.025 \times 4P \times 0.97857 + 45 \times 13.352$$

$$+500 \times 1.06^{0.5} \left( 0.18763 - 0.17411 \times \frac{8821.2612}{9892.9151} \right) + 250 \times 0.17411 \times \frac{8821.2612}{9892.9151}$$

$$= 2.30566P + 906.322$$

where

$$\ddot{a}_{[35]|\bar{1}}^{(4)} = \ddot{a}_{[35]|\bar{1}} - \frac{3}{8} \left( 1 - P_{[35]} v \right)$$

$$= 1 - \frac{3}{8} \left( 1 - \frac{9887.2069}{9892.9151} \times 0.9434 \right) = 0.97857$$

Equation of value gives:

$$56.1408P = 32,234.706 + 2.30566P + 906.322$$

$$\Rightarrow P = \frac{33,141.028}{53.8351} = \text{£}615.60$$

(ii) Gross prospective policy value (calculated at 4%) is given by:

$$V^{\text{prospective}} = \frac{245,000}{(1+b)} (1+i)^{1/2} A_{60:\bar{5}}^1 @ i'' + 245,000 \times v^5 {}_5P_{60} @ i'' + 0.025 \times 4P\ddot{a}_{60:\bar{5}}^{(4)} + 90\ddot{a}_{60:\bar{5}} - 4P\ddot{a}_{60:\bar{5}}^{(4)}$$

$$+ 1000\bar{A}_{60:\bar{5}}^1 + 500v^5 {}_5P_{60}$$

$$= \frac{245,000}{(1.04)^{0.5}} \times A_{60:\bar{5}}^1 @ i'' + 245,000 \times v^5 \frac{l_{65}}{l_{60}} @ i'' + 90\ddot{a}_{60:\bar{5}} - 0.975 \times 4P\ddot{a}_{60:\bar{5}}^{(4)}$$

$$+ 1000 \times 1.04^{0.5} \left( A_{60:\bar{5}} - v^5 \frac{l_{65}}{l_{60}} \right) + 500v^5 \frac{l_{65}}{l_{60}}$$

$$\text{where } \ddot{a}_{60:\bar{5}}^{(4)} = \ddot{a}_{60:\bar{5}} - \frac{3}{8} \left( 1 - v^5 \times \frac{l_{65}}{l_{60}} \right) = 4.55 - \frac{3}{8} \left( 1 - 0.82193 \times \frac{8821.2612}{9287.2164} \right) = 4.4678$$

$$\text{and } i'' = \frac{1.04}{1.04} - 1 = 0 \Rightarrow \bar{A}_{60:\bar{5}}^1 @ i'' = \frac{\sum_0^4 d_{60+t}}{l_{60}} = \frac{465.9551}{9287.2164} = 0.05017$$

$$= \frac{245,000}{(1.04)^{0.5}} \times 0.05017 + 245,000 \times 0.94983 + 90 \times 4.55 - 0.975 \times 4 \times 615.60 \times 4.4678$$

$$+ 1000 \times 1.04^{0.5} (0.82499 - 0.78069) + 500 \times 0.78069$$

$$= 12,052.954 + 232,708.35 + 409.5 - 10,726.473 + 45.177 + 390.345 = 234,880$$

*Part (i) answered reasonably well. Students had more problems with (ii)*

**END OF EXAMINERS' REPORT**