

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

19 April 2013 (pm)

Subject CT5 – Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

<p><i>In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.</i></p>
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1 Calculate:

(a) ${}_{10|5}q_{40}$

(b) \bar{a}_{65}

(c) ${}_{15}P_{[46]}$

Basis:

Mortality AM92

Interest 4% per annum [3]

2 Calculate $(aq)_x^\alpha$.

Basis:

Mortality: $\mu_x^\alpha = 0.1$ and $\mu_x^\beta = 0.2$ for all x
 α, β are independent decrements [3]

3 Describe how climate and geography affect mortality and morbidity. [3]

4 Describe the use of terminal bonus within the reversionary bonus system. [3]

5 A pension scheme provides a pension on retirement of one-sixtieth of final pensionable salary for each year of service. Final pensionable salary is average salary received in the three years before retirement. Normal retirement age is 65 exact. The same level of pension is payable on retirement on the grounds of ill-health or otherwise prior to age 65.

Calculate the expected present value of past and future benefits for a life currently aged 30 exact with 10 years of past service and salary in the previous year of £40,000.

Basis:

PEN Tables in Formulae and Tables for Actuarial Examination.

[4]

- 6** A life insurance company issues a 20-year increasing endowment assurance policy which provides a sum assured given by the formula:

$$£ [50,000 + 1,500t] \quad t = 1, 2, \dots, 20$$

where t denotes the policy year.

The sum assured is payable on maturity at age 50 exact or at the end of year of death if earlier. Premiums on the policy are payable annually in advance.

Write down an expression for:

- (a) the net premium for the policy.
- (b) the net premium prospective policy reserve for the policy immediately before the tenth premium is paid.

[4]

- 7** Explain why it is necessary to have different mortality tables for different classes of lives. [6]

- 8** (i) Define the measures of crude mortality rate and directly standardised mortality rate. You should include a definition of all symbols used. [5]

The data in the table below is for a sub-population for the year 2012.

<i>Age</i>	<i>Number of lives</i>	<i>Number of deaths</i>
65	125,000	2,937
66	130,000	3,301
67	140,000	3,756

- (ii) Calculate the standardised mortality ratio for this sub-population using ELT15 (Males) as the standard population. [2]

[Total 7]

- 9** A male life currently aged 65 exact purchases a special joint life annuity of £10,000 per annum payable monthly in advance together with additional benefits detailed below.

On the death of the male life, the annuity reduces to £5,000 per annum payable monthly in advance to a female life until her death, assuming she survives him. The female life is currently aged 62 exact.

The policy additionally provides benefits of:

- An annuity certain (extra to the above and not dependent on the survival status of each life) of £10,000 per annum payable monthly in advance and paid only for ten years, and
- £10,000 payable immediately on the death of each life.

Calculate the expected present value of the total benefits.

Basis:

Mortality	Male life	PMA92C20
	Female life	PFA92C20

Interest 4% per annum

Expenses Nil

[7]

- 10** A special whole life assurance policy issued to a life aged 40 exact provides a benefit of £1,000 on death within 20 years of inception, £2,000 on death between 20 and 40 years from inception and £3,000 on death thereafter. Benefits are payable at the end of the year of death.

Calculate the expected present value and variance of the present value of this policy.

Basis:

Mortality	AM92 Ultimate
Interest	4% per annum

[8]

11 Two lives are both aged 45 exact.

Calculate:

- (i) The probability of both lives surviving to age 65 exact. [1]
- (ii) The present value of an annuity of £1,000 per annum increasing by 3% each year payable annually in advance so long as both lives survive. [3]
- (iii) The present value of a 20-year term assurance with a benefit of £100,000 payable immediately on the second death. [5]

Basis:

Mortality $\mu_x = 0.05$ for all x for both lives

Interest 4% per annum

[Total 9]

12 A life insurance company issues whole life assurance policies to lives aged 50 exact for a sum assured of £75,000 payable at the end of the year of death. Premiums are payable annually in advance.

- (i) Calculate the annual gross premium for each policy using the basis below. [4]
- (ii) Calculate the minimum annual gross premium that the company should charge in order that the probability of making a loss on any one policy would be 10% or less. [6]

Basis:

Mortality AM92 Select

Interest 6% per annum

Initial commission 100% of the annual gross premium

Initial expenses £325

Renewal commission 2.5% of each annual gross premium excluding the first

Renewal expenses £75 per annum at the start of the second and subsequent policy years

[Total 10]

- 13** A life insurance company issues 5,000 four-year decreasing term assurance policies on 1 January 2012 to a group of male lives aged 56 exact at that date.

Premiums are payable annually in advance on each policy. The initial annual gross premium P reduces to $.75P$, $.5P$ and $.25P$ at the beginning of the second, third and fourth policy year respectively.

The sum assured on each policy is payable at the end of year of death and is given by the formula:

$$£100,000 \times [1 - 0.25t] \quad t = 0, 1, 2, 3$$

where t denotes the curtate duration in years since the inception of the policy.

- (i) Calculate the initial annual gross premium P for each policy using the basis below. [7]
- (ii) Determine the prospective gross premium reserve for each policy in force at the end of the first policy year using the same basis. [5]
- (iii) Calculate the mortality profit or loss for this portfolio of business for the calendar year 2012 given that 27 policyholders died during that year. [2]

Actual expenses incurred and interest earned by the company on this portfolio of business during 2012 was the same as that assumed in the premium basis.

- (iv) Derive the mortality profit or loss for the calendar year 2012 using the recursive relationship between the opening and closing prospective reserves in the first policy year. [2]

Basis:

Mortality	AM92 Ultimate
Interest	6% per annum
Initial commission	25% of the first annual premium
Initial expenses	£125
Renewal commission	3% of each annual premium excluding the first
Renewal expenses	£35 per annum at the start of the second and subsequent policy years. The renewal expense is assumed to increase by 1.92308% compound per annum from inception of the policy.
[Total 16]	

- 14** A life insurance company issues a three-year unit-linked endowment assurance policy to a life aged 67 exact. Level premiums are payable yearly in advance throughout the term of the policy or until earlier death. In the first year, 50% of the premium is allocated to units and 110% in the second and third years. The units are subject to a bid-offer spread of 5% and an annual management charge of 0.75% of the bid value of units is deducted at the end of each policy year.

Management charges are deducted from the unit fund before death and surrender benefits are paid.

If the policyholder dies during the term of the policy, the death benefit of the bid value of the units is payable at the end of the year of death. The policyholder may surrender the policy only at the end of each year immediately before a premium is paid. On surrender or on survival to the end of the term, the bid value of the units is payable at the end of the year of exit.

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund	4% per annum
Rate of interest on non-unit fund cash flows	3% per annum
Mortality	90% AM92 Ultimate
Surrenders	8% at end of first year, 4% at end of second year based on policies in force at that time.
Initial expenses	£235
Renewal expenses	45 per annum on the second and third premium dates
Initial commission	12.5% of first premium
Renewal commission	2.5% of the second and third years' premiums
Claim expense	£75 on deaths and surrenders only

The company sets premiums so that the net present value of the profit for the policy is 10% of the annual premium, using a risk discount rate of 6% per annum.

- (i) Calculate the premium for the policy on the assumption that the company **does not** zeroise future expected negative cash flows. [12]
- (ii) Calculate the net present value of the profit on the policy on the assumption that the company **does** set up reserves in order to zeroise future expected negative cash flows. [5]

[Total 17]

END OF PAPER