INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2017

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Luke Hatter
Chair of the Board of Examiners
December 2017

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Contingencies subject is to provide a grounding in the mathematical techniques which can be used to model and value cashflows dependent on death, survival, or other uncertain risks.

2. CT5 introduces the fundamental building blocks of all life insurance and pensions actuarial work.

B. General comments on *student performance in this diet of the examination*

Well prepared students did very well in this relatively straightforward exam where the main questions of challenge were Q7, Q9(ii), Q12 and Q13(iii). Q1 was in addition done very poorly.

There was evidence that may people attempted this examination without robust preparation and in these cases many of the simpler questions were not answered satisfactorily.

C. Pass Mark

The Pass Mark for this exam was 60.

Solutions

Q1

- Salaries and salary related expenses [½]
- Buildings and other property costs [½]
- Computing and associated costs [½]
- Costs involved with the investment of funds [½]

[Total 2]

> This question was generally poorly answered. Many students simply ignored the reference to inflation and gave a generalised answer on costs which was not what was required.

Q2

When homogeneous groups are formed we usually tacitly infer that the factors used to define each group are the cause of the differences in mortality observed between the groups. However, there may be other differences in composition between the groups, and it is these differences rather than the differences in the factors used to form the groups that are the true causes of the observed mortality differences.

Ascribing mortality differences to groups formed by factors which are not the true causes of these differences is termed spurious selection. [1½]

For example, when the population of England and Wales is divided by region of residence, some striking mortality differences are observed. However, a large part of these differences can be explained by the different mix of occupations in each region. The class selection ascribed to regions is spurious and is in part the effect of compositional differences in occupation between the regions.

In statistical terminology the occupational differences in mortality are confounded (mixed up) with the regional differences. [1½]

[Total 3]

> Generally well done. Other valid examples were given credit, particularly with reference to changing underwriting standards as explained in the Core Reading

Q3

$$ {}_{2.25}p_{85.5} = {}_{0.5}p_{85.5}\,p_{86}\,{}_{0.75}p_{87} $$

$$ = \frac{l_{86}}{l_{85.5}} \times \frac{l_{87}}{l_{86}} \times \frac{l_{87.75}}{l_{87}} $$

$$ = \frac{l_{87.75}}{l_{85.5}} $$

$$ = \frac{0.75 l_{88} + 0.25 l_{87}}{0.5 l_{86} + 0.5 l_{85}} $$

$$ = \frac{0.75 \times 11{,}874 + 0.25 \times 14{,}280}{0.5 \times 16{,}917 + 0.5 \times 19{,}756} $$

$$ = \frac{12{,}475.5}{18{,}336.5} $$

$$ = 0.68036 $$

$$ {}_{2.25}q_{85.5} = 1 - {}_{2.25}p_{85.5} $$

$$ = 0.31964 $$

[2 marks for first 4 lines plus 2 for calculations]
[Total 4]

Generally done well.

Q4

The expected cost of paying benefits usually increases as the life ages and the probability of a claim by death increases. [1]

Level premiums received in the early years of a contract are more than enough to pay the benefits that fall due in those early years, but in the later years the premiums are too small to pay for the benefits. It is therefore prudent for the premiums that are not required in the early years of the contract to be set aside, or reserved, to fund the shortfall in the later years of the contract. [2]

If premiums received that were not required to pay benefits were spent by the company, perhaps by distributing to shareholders, then later in the contract the company may not be able to find the money to pay for the excess of the cost of benefits over the premiums received. [1]
[Total 4]

Many students did not clearly describe the fundamental timing issue given in the second paragraph above. A small credit was given if students mentioned legislative requirements.

Q5

(i)

$$\bar{A}_{47:\overline{11|}} = (1.04)^{1/2} \times A^{1}_{47:\overline{11|}} + (1.04)^{-11} \times \frac{l_{58}}{l_{47}}$$

$$= 1.0198 \times (A_{47} - (1.04)^{-11} \times \frac{l_{58}}{l_{47}} \times A_{58}) + (1.04)^{-11} \times \frac{l_{58}}{l_{47}}$$

$$= 1.0198 \times (0.29635 - 0.64958 \times \frac{9413.8004}{9771.0789} \times 0.42896) + 0.64958 \times \frac{9413.8004}{9771.0789}$$

$$= 0.02845 + 0.62583$$

$$= 0.65428$$

[2 marks lines 1 and 2; 1 mark for calculation]

(ii)

$$\ddot{a}^{[4]}_{[53]:\overline{13}|} = \ddot{a}^{[4]}_{[53]} - (1.04)^{-13} \times \frac{l_{66}}{l_{[53]}} \times \ddot{a}^{[4]}_{66}$$

$$= (\ddot{a}_{[53]} - 0.375) - (1.04)^{-13} \times \frac{l_{66}}{l_{[53]}} \times (\ddot{a}_{66} - 0.375)$$

$$= 16.163 - 0.60057 \times \frac{8695.6199}{9621.1006} \times 11.521$$

$$= 9.909$$

[1 mark lines 1 and 2; 1 mark for calculation]
[Total 5]

Generally done well. Some students lost marks in (i) by mistakenly calculating the temporary assurance function only. Others forgot the $(1.04)^{1/2}$ adjustment for immediate claim payment.

Q6

(i)

$$\ddot{a}_{40:\overline{4}|} = 1 + \frac{0.992}{1.05} + \frac{0.981}{(1.05)^2} + \frac{0.967}{(1.05)^3}$$

$$= 1 + 0.94476 + 0.88980 + 0.83533$$

$$= 3.6699$$

[2]

(ii)

$$A_{40:\overline{4}|} = 1 - d(5\%) \times \ddot{a}_{40:\overline{4}|} = 1 - \frac{0.05}{1.05} \times 3.6699$$

$$= 0.82524$$

$$A^{1}_{40:\overline{4}|} = A_{40:\overline{4}|} - (1.05)^{-4} \times \frac{l_{44}}{l_{40}} = 0.82524 - 0.82270 \times 0.947 = 0.04614$$

[2 marks 1st 2 lines in and 2 marks for 3rd line]
[Total 6]

Generally well done. Despite the question wording requirement in (ii), some students attempted to calculate $A^{1}_{40:\overline{4|}}$ in the long method way rather than the simple premium conversion route. This received full credit if the answer was correct.

Q7

$$(aq)^{\beta}_{40} = \int_0^1 ({}_t p^{\alpha}_{40} \times {}_t p^{\beta}_{40} \times \mu^{\beta}_{40+t})dt$$

$${}_t p^{\alpha}_{40} = \exp(-\int_0^t \mu^{\alpha}_{40+s} ds) = \exp(-\int_0^t (\frac{1}{110-(40+s)})ds) = \exp(-\int_0^t (\frac{1}{70-s})ds$$

$$= \exp(\ln\left[70-s\right]_0^t) = \frac{70-t}{70}$$

$${}_t p^{\beta}_{40} = \exp(-\int_0^t \mu^{\beta}_{40+s} ds) = \exp(-0.03t)$$

$$\mu^{\beta}_{40+t} = 0.03$$

$$\Rightarrow (aq)^{\beta}_{40} = \int_0^1 ((\frac{70-t}{70}) \times e^{-0.03t} \times .03)dt = 0.03\int_0^1 (e^{-0.03t} - \frac{t}{70} \times e^{-0.03t})dt$$

$$= 0.03 \times \left[-\frac{e^{-0.03t}}{0.03}\right]_0^1 - \frac{0.03}{70}\int_0^1 t \times e^{-0.03t} dt$$

$$= (1-e^{-.03}) - \frac{0.03}{70} \times 0.490112$$

$$= 0.029554 - 0.000210$$

$$= 0.029344$$

[Total 7]

This was a very challenging question which many students did not attempt fully. Many that did assumed μ^{α}_{40} was a constant 1/70 but this was not correct although the numerical answer was nearly the same (reasonable credit was given for this approach however), The question did not give the assumption that μ^{α}_{x} was constant through each year of age x and to score high marks this non constant factor needed to be taken into account.

Q8

(i)

Past benefits:

$$= 35000 \times \frac{15}{80} \times \left(\frac{{}^zM_{45}^{ra} + {}^zM_{45}^{ia}}{s_{44}D_{45}} \right) = 6562.5 \times \left(\frac{128026 + 52554}{8.375 \times 2329} \right) = 60755.4$$

Future benefits:

$$= 35000 \times \frac{1}{80} \times \left(\frac{{}^z\bar{R}_{45}^{ra} + {}^z\bar{R}_{45}^{ia}}{s_{44}D_{45}} \right) = 437.5 \times \left(\frac{2244130 + 609826}{8.375 \times 2329} \right) = 64013.4$$

Total Value $= 60755.4 + 64013.4 = 124769$ rounded

[2 marks for each formula plus 1 for result]

(ii)

Let the contribution rate be k

Then $\frac{k}{100} \times 35000 \times \frac{{}^s\bar{N}_{45}}{s_{44}D_{45}} = 124769$

$$\Rightarrow k = \frac{124769 \times 100 \times 8.375 \times 2329}{253080 \times 35000} = 27.47$$

i.e. 27.47% of salary

[2 marks line 2 plus 1 for result]
[Total 8]

Generally done well. The most common error was the definition of s and forgetting to calculate both forms of retirement benefit in (i).

Q9

(i)

$$\text{EPV} = A^{1}_{50:\overline{10}|} + 0.75 \times v^{10} \times \frac{l_{60}}{l_{50}} \times A_{60}$$

$$= (A_{50} - v^{10} \times \frac{l_{60}}{l_{50}} \times A_{60}) + 0.75 \times v^{10} \times \frac{l_{60}}{l_{50}} \times A_{60}$$

$$= A_{50} - 0.25 \times v^{10} \times \frac{l_{60}}{l_{50}} \times A_{60}$$

$$= 0.32907 - 0.25 \times 0.67556 \times \frac{9287.2164}{9712.0728} \times 0.45640$$

$$= 0.25536$$

[2 marks first 3 lines; 1 mark for result]

(ii)

First calculate 2nd moment

$$\text{Value} \quad = ({}^{2}A_{50} - v^{20} \times \frac{l_{60}}{l_{50}} \times {}^{2}A_{60}) + (0.75)^{2} \times v^{20} \times \frac{l_{60}}{l_{50}} \times {}^{2}A_{60}$$

$$= {}^{2}A_{50} - 0.4375 \times v^{20} \times \frac{l_{60}}{l_{50}} \times {}^{2}A_{60}$$

$$= 0.13065 - 0.4375 \times 0.45639 \times \frac{9287.2164}{9712.0728} \times 0.23723$$

$$= 0.08535$$

$$\text{Variance} \quad = 0.08535 - (0.25536)^{2} = 0.02014 = (0.14192)^{2}$$

[3 marks for the first 3 lines; 2 marks for remainder]
[Total 8]

Part (i) was generally answered well'

Part (ii) was poorly answered where the main issue was developing correctly the 2nd moment value above especially the treatment of the 0.75 multiplier.

Q10

(i) (a) Crude mortality rate is the ratio of the total number of deaths in a category to the total exposed to risk in the same category. [1]

(b) Directly standardised mortality rate is the mortality rate of a category weighted according to a standard population. [1]

(c) Indirectly standardised mortality rate is an approximation to the directly standardised mortality rate being the crude rate for the standard population multiplied by the ratio of actual to expected deaths for the region. [1]

(d) Standardised Mortality Ratio is the ratio of the actual deaths in the category to the expected deaths in the same category using the mortality rates from the standard population. [1]

(ii) The crude death rate is $\dfrac{287}{150000} = 0.001913$ [½]

The Directly Standardised Mortality Rate is:

$$\left(\frac{(1000000\times\frac{42}{40000})+(1600000\times\frac{135}{75000})+(900000\times\frac{110}{35000})}{3500000}\right)$$

$$=\frac{1050+2880+2828.6}{3500000}=0.001931$$ [2]

The Indirectly Standardised Mortality Rate can be calculated as follows:

Expected deaths for regional group:

$$\left(\frac{40000\times1300}{1000000}+\frac{75000\times3200}{1600000}+\frac{35000\times2500}{900000}\right)$$

$$=52+150+97.22=299.22$$

So the Indirectly Standardised Mortality rate is:

$$\frac{0.002\times287}{299.22}=0.001918$$ [2]

The Standardised Mortality Ratio is $\dfrac{287}{299.22} = 0.9592$ [½]

[Total 9]

Generally straightforward and done well.

Q11

(i)

t	*Profit vector*	${}_{t-1}p_{65}$	*Profit signature*
1	185.21	1	185.21
2	–121.52	0.985757	–119.79
3	–5.28	0.970044	–5.12
4	12.95	0.952754	12.34

[1] [½]

Let X be the reserve required at t =1 in order to zeroise negative cash flows at t = 2 and t = 3.

Then:

$X = 119.79v + 5.12v^2$ at 5% $= 118.73$ [1]

Revised cash flow at t = 1 is 185.21 – 118.73 = 66.48

Hence profit signature is: (66.48, 0, 0, 12.34) [½]

(ii) Multiple decrement table – although deaths can be assumed to be uniformly distributed over the year, surrenders occur only at the year end. Therefore:

$(aq)^d_x = q^d_x$ and $(aq)^s_x = q^s_x(1 - q^d_x)$

x	q^d_x	q^s_x	$(aq)^d_x$	$(aq)^s_x$	$(ap)_x$	${}_{t-1}(ap)_x$
65	0.014243	0.03	0.014243	0.029573	0.956184	1
66	0.015940	0.03	0.015940	0.029522	0.954538	0.956184
67	0.017824	0.00	0.017824	0.00	0.982176	0.912714
68	0.019913	0.00	0.019913	0.00	0.980087	0.896446

[2½]

t	*Revised profit vector*	${}_{t-1}(ap)_{65}$	*Revised profit signature*
1	$185.21+50(aq)^s_{65}$ = 186.69	1	186.69

2	$-121.52+50\left(aq\right)^{s}_{66}=-120.04$	0.956184	–114.78
3	–5.28	0.912714	–4.82
4	12.95	0.896446	11.61

[2]

Let Y be the reserve required at t =1 in order to zeroise negative cash flows at t = 2 and t = 3.

Then:

$Y = 114.78v + 4.82v^2$ at $5\% = 113.69$ [1]

Revised cash flow at t = 1 is 186.69 – 113.69 = 73.00

Hence revised profit signature is: (73.00, 0, 0, 11.61) [½]

(iii) NPV of revised profit signature $= 73.00v + 11.61v^4$ at $8\% = 76.13$ [1]

[Total 10]

Straightforward and done well by students who had prepared thoroughly. Reasonable credit was given for method even if the values were not always accurate.

Q12

(i)

The sum assured and attaching bonus at the beginning of each policy year is:

Policy year	*Sum assured + Bonus*
1	78,000
2	81,000
3	84,000

Let NP be the net premium for the policy. Then

$$NP = \frac{75000A_{62:\overline{3}|} + 3000(IA)_{62:\overline{3}|}}{\ddot{a}_{62:\overline{3}|}}$$

$$= \frac{75000 \times 0.89013 + 3000 \times 2.64053}{2.857} = 26139.78$$

where

$$(IA)_{62:\overline{3}|} = (IA)_{62} - v^3 \frac{l_{65}}{l_{62}}\left(3A_{65} + (IA)_{65}\right) + 3v^3 \frac{l_{65}}{l_{62}}$$

$$= 8.20491 - 0.889 \times \frac{8821.2612}{9129.7170}\left(3 \times 0.52786 + 7.89442\right) + 3 \times 0.889 \times \frac{8821.2612}{9129.7170}$$

$$= 8.20491 - 8.14126 + 2.57688 = 2.64053$$

[4]

Alternatively

$$NP = \frac{78000 q_{62} v + 81000\, p_{62} q_{63} v^2 + 84000\, p_{62}\, p_{63} v^3}{\ddot{a}_{62:\overline{3}|}}$$

$$= (758.40 + 840.953 + 73082.462)/2.857 = 26139.94$$

The net premium reserve ${}_tV$ at duration t for the policy is given by:

$${}_1V = 81,000 q_{63} v + 84,000\, p_{63} v^2 - P\ddot{a}_{63:\overline{2}|}$$

$$= 81,000 \times 0.011344 \times 0.96154 + 84,000 \times 0.988656 \times 0.92456 - 26,139.78 \times 1.951$$

$$= 883.52 + 76781.72 - 50998.71 = 26,666.53$$

$${}_2V = 84,000 v \ - P\ddot{a}_{64:\overline{1}|}$$

$$= 84,000 \times 0.96154 - 26,139.78 \times 1.0$$

$$= 80,769.23 - 26,139.78 = 54,629.45$$

[4]

(ii)

Let P be the annual office premium for the policy. Then the expected cash flows for the policy are:

Yr	*Opening reserve*	*Premium*	*Expense*	*Interest*	*Death claim*	*Maturity claim*	*Closing reserve*
1	0.00	P	0.15P	0.0425P	788.74	0.00	26396.88
2	26666.53	P	0.05P	1333.33+0.0475P	918.86	0.00	54009.73
3	54629.45	P	0.05P	2731.47+0.0475P	1068.14	82931.86	0.00
	[½]	[½]	[½]	[1]	[1]	[½]	[1]

Yr	*Profit vector*	${}_{t-1}(ap)_x$	*Profit signature*	*Discount factor*	*Present value of profit*
1	0.8925*P*–27185.62	1.000000	0.89250*P*–27185.62	0.94340	0.84198*P*–25646.91
2	0.9975*P*–26928.73	0.989888	0.98741*P*–26656.43	0.89000	0.87880*P*–23724.22
3	0.9975*P*–26639.08	0.978659	0.97621*P*–26070.58	0.83962	0.81965*P*–21889.38

[½] [½] [½] [½] [½]

Total present value of profit = 2.54043*P* – 71260.51 [½]

However we require:

Total present value of profit = 0 to achieve an internal rate of return of 6% p.a. [½]

Therefore *P* = 71260.51/2.54043 = 28050.57 [½]

[Total 17]

Many students in part (i) overlooked the fact that the bonus was guaranteed throughout from outset i.e. it was effectively a product with an increasing sum assured. Therefore the net premium calculation needed to reflect this guarantee rather than use the traditional net premium non-profit basis.

Part (ii) was quite challenging and credit was given for method employed even when not all the arithmetic calculations were correct.

Q13

(i)

Let P be the annual net premium for the increasing term assurance policy. Then the equation of value is given by:

$$P = \frac{\dfrac{50,000}{1.0192308} A^{1}_{[40]:\overline{25|}} @ 4\% + 25PA_{[40]:\overline{25|}}^{\ \ 1} @ 6\%}{\ddot{a}^{@6\%}_{[40]:\overline{25|}}}$$ [2½]

where at 4%

$$A^{1}_{[40]:\overline{25|}} = A_{[40]:\overline{25|}} - v^{25}\,{}_{25}p_{[40]}$$

$$= 0.38896 - 0.37512 \times \frac{8821.2612}{9854.3036} = 0.38896 - 0.33580 = 0.05316$$

[½]

and at 6%

$$A_{[40]:\overline{25}|}^{\;\;\;\;\;\;1} = v^{25}\,{}_{25}p_{[40]}$$

$$= 0.233 \times \frac{8821.2612}{9854.3036} = 0.20857$$

[½]

$$\Rightarrow P \;= \frac{\frac{50,000}{1.0192308} \times 0.05316 + 25P \times 0.20857}{13.29} \Rightarrow P = \frac{2607.849}{8.07575} = 322.95$$ [½]

(ii)

Reserve at the <u>end</u> of the 17th policy year given by:

$${}_{17}V \;= 50,000\left(1.0192308\right)^{16} A_{57:\overline{8}|}^{1} @\,4\% + 25PA_{57:\overline{8}|}^{\;\;\;\;1} @\,6\% - P\;\ddot{a}_{57:\overline{8}|} @\,6\%$$ [2½]

where at 4%

$$A_{57:\overline{8}|}^{1} = A_{57:\overline{8}|} - v^8\,{}_8p_{57}$$

$$= 0.73701 - 0.73069 \times \frac{8821.2612}{9467.2906} = 0.73701 - 0.68083 = 0.05618$$

[½]

and at 6%

$$A_{57:\overline{8}|}^{\;\;\;\;1} = v^8\,{}_8p_{57}$$

$$= 0.62741 \times \frac{8821.2612}{9467.2906} = 0.58460$$

[½]

$$\Rightarrow {}_{17}V \;= 50,000\left(1.0192308\right)^{16} \times 0.05618 + 25 \times 322.95 \times 0.58460 - 322.95 \times 6.433$$

$$= 3,809.957 + 4,719.914 - 2,077.537 = 6,452.334$$

[½]

Therefore, death strain at risk (*DSAR*) per policy in the 17th policy year is:

$$DSAR = 50,000\left(1.0192308\right)^{16} - 6,452.334 = 61,363.425$$ [1]

Mortality profit = expected death strain (*EDS*) – actual death strain (*ADS*) [½]

$$EDS \;= 1425 \times q_{56} \times 61,363.425 = 1425 \times 0.005025 \times 61,363.425 = 439,400.475$$

$ADS = 10 \times 61{,}363.425 = 613{,}634.25$ [1]

i.e. mortality profit = –174,233.77 (i.e. a loss) [½]

(iii)

Reserve at the <u>end</u> of the 16th policy year given by:

$${}_{16}V = 50{,}000(1.0192308)^{15} A^{1}_{56:\overline{9}|} @\,4\% + 25PA_{56:\overline{9}|}^{\;\;1} @\,6\% - P\ \ddot{a}_{56:\overline{9}|} @\,6\%$$ [2½]

where at 4%

$$A^{1}_{56:\overline{9}|} = A_{56:\overline{9}|} - v^9\ {}_9p_{56}$$

$$= 0.70993 - 0.70259 \times \frac{8821.2612}{9515.1040} = 0.70993 - 0.65136 = 0.05857$$

[½]

and at 6%

$$A_{56:\overline{9}|}^{\;\;1} = v^9\ {}_9p_{56}$$

$$= 0.5919 \times \frac{8821.2612}{9515.1040} = 0.54874$$

[½]

$$\Rightarrow {}_{16}V = 50{,}000(1.0192308)^{15} \times 0.05857 + 25 \times 322.95 \times 0.54874 - 322.95 \times 7.038$$

$$= 3{,}897.026 + 4{,}430.390 - 2{,}272.922 = 6{,}054.494$$

[½]

Let *i'* be the actual interest rate earned. Then the interest profit is given by:

$$[i' - 0.06] \times 1425 \times ({}_{16}V + P) = 174{,}233.77$$

$$[i' - 0.06] \times 9{,}087{,}857.7 = 174{,}233.77$$

$$[i' - 0.06] = 0.019172 \Rightarrow i' = 7.92\%$$ [2]

[Total 17]

Parts (i) and (ii) generally well done by students who had thoroughly prepared. The main error in (ii) was having the wrong duration for the reserve.

Part (iii) was more challenging and done less well.

Again credit was given for understanding the method even if calculations were not always accurate.

END OF EXAMINERS' REPORT