INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2016

Subject CT5 – Contingencies Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Luke Hatter
Chair of the Board of Examiners
December 2016

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Contingencies subject is to provide a grounding in the mathematical techniques which can be used to model and value cashflows dependent on death, survival, or other uncertain risks.

2. CT5 introduces the fundamental building blocks of all life insurance and pensions actuarial work.

3. Credit is given to students who produce alternative correct numerical solutions. In the case of descriptive answers credit is also given where appropriate valid points are made which do not appear in the solutions below.

4. In questions where definitions of symbols and then formulae are requested, a different notation system produced by a student to that used by the Examiners is acceptable provided it is used consistently, is relevant and is properly defined and used in the answer.

5. Students should note that for long questions reasonable credit is given if they can describe the right procedures although to score high marks reasonably accurate numerical calculation is necessary.

B. General comments on *student performance in this diet of the examination*

1. In general this paper was very well done by students who had prepared fully for the exam. Most questions were straightforward and capable of being answered in the allotted time. The questions that gave most difficulty were 3, 5, 11 and 12.

2. Detailed solutions are given below together with commentary from the examiners which we hope will be of assistance.

C. Pass Mark

The Pass Mark for this exam was 60.

Solutions

Q1 (a) $EPV = 100000 \times (1.04)^{1/2} \times A_{45} = 101980.4 \times 0.27605 = 28151.7$ [1]

(b) $$\text{Variance} = (100000)^2 \times {}^2A_{45} \times (1.04) - (28151.7)^2$$
$$= 10^{10} \times 0.09458 \times 1.04 - 7.925 \times 10^8$$
$$= 10^8 \times 1.9113$$
$$= (13825)^2$$

[3]
[Total 4]

Generally well done. The most common error was in (b) where students did not square the 100000 figure.

Q2 The reserves required at the beginning of policy years 7, 3 and 2 are:

$${}_6V = \frac{3}{1.015} = 2.956$$ [½]

Revised cash flow in policy year 6 $= 5 - 0.9975 \times {}_6V = 2.052$ [1]

$${}_2V = \frac{10}{1.015} = 9.852$$ [½]

$${}_1V = \frac{1}{1.015}(10 + .9975 \times {}_2V) = 19.535$$ [½]

Revised cash flow in policy year 1 $= -50 - 0.9975 \times {}_1V = -69.486$ [1]

=> revised profit vector: (–69.49, 0, 0, 5, 5, 2.05, 0, 15, 40, 60) [½]
[Total 4]

Again well done in general.

Q3 (i) Value is given by:

$$100{,}000 \times \frac{2}{3} \times \frac{{}^zM_{42}^{ia}}{s_{41}D_{42}} = 100{,}000 \times \frac{2}{3} \times \frac{56093}{7.980 \times 2799} = 167{,}422$$ [2]

(ii) Value of this benefit given by:

$$100{,}000 \times 4 \times \frac{{}^{s}M_{42}^{d}}{s_{41} D_{42}}$$ [2]

[Total 4]

> This question was poorly done overall. The most common errors were using 42 instead of 41 for the salary index, using *R* functions instead of *M* and valuing age retirement instead of ill health. In (ii) all that was required for the limited marks was the final formula shown whilst students often spent a long time preparing detailed formulae

Q4 (a)

$$_{10|5}q_{65} = \frac{l_{75} - l_{80}}{l_{65}} = \frac{6879.1673 - 5266.4604}{8821.2612} = 0.18282$$ [1½]

(b)

$$\ddot{a}^{(12)}_{[30]:\overline{15}|} = \ddot{a}^{(12)}_{[30]} - v^{15} \times \frac{l_{45}}{l_{[30]}} \times \ddot{a}^{(12)}_{45}$$

$$= \left(\ddot{a}_{[30]} - \frac{11}{24}\right) - v^{15} \times \frac{l_{45}}{l_{[30]}} \times \left(\ddot{a}_{45} - \frac{11}{24}\right)$$

$$= \left(21.837 - \frac{11}{24}\right) - 0.555265 \times \frac{9801.3123}{9923.7497} \times \left(18.823 - \frac{11}{24}\right)$$

$$= 21.378 - 10.071$$

$$= 11.307$$

[2½]

> A straightforward typical CT5 short question which was generally well done.

Q5

- Life insurance companies provide a service of pooling independent homogeneous risks. If a company is able to do this then as a result of the Central Limit Theorem the profit per policy will be a random variable that follows the normal distribution with a known mean and standard deviation. [1]

- The company can use this result to set premium rates which ensure that the probability of a loss of a portfolio of policies is at an acceptable level. [1]

- The independence of risks usually follows naturally from the way in which life insurance policies are sold. Rarely does the death of one policyholder influence the mortality of another policyholder. [1]

- Careful underwriting is the mechanism by which the company ensures that its risk groups are homogeneous. The risk groups are defined by the use of rating factors, e.g. age, sex, medical history, height, weight, lifestyle. [1]

- In theory, a company should continue to add rating factors to its underwriting system until the differences in mortality between the different categories of the next rating factor are indistinguishable from the random variation between lives that remains after using the current list of rating factors. [1]

- In reality the ability of prospective policyholders to provide accurate responses to questions and the cost of collecting information also limit the extent to which rating factors can be used. [1]

- For example, a proposal form should not ask for information which requires a specialist knowledge. Height is acceptable, but blood pressure is not. For example, the cost of undertaking extensive blood tests has to be weighed against the expected cost of claims that will be "saved" as a result of having this information. [1]

- From a marketing point of view proposers are anxious that the process of underwriting should be straightforward and speedy. [1]

- In practice, rating factors will be included if they avoid any possibility of selection against the company, and satisfy the time and cost constraints of marketing. This decision is often driven by competitive pressures. If several companies introduce a new rating factor, which in fact influences mortality levels significantly, then other companies will need to follow this lead or risk adverse selection against them. [1]

[Max 6]

Students did not score well on this question often only covering a limited number of points. The above solution is a full one. Any other valid points not shown were credited.

Q6 (i) $${}_{2.5}q_{80.75} = 1 - {}_{2.5}p_{80.75}$$

$${}_{2.5}p_{80.75} = {}_{0.25}p_{80.75} \times p_{81} \times p_{82} \times {}_{0.25}p_{83}$$

$$= \frac{l_{81}}{l_{80.75}} \times \frac{l_{82}}{l_{81}} \times \frac{l_{83}}{l_{82}} \times \frac{l_{83.25}}{l_{83}}$$

$$= \frac{l_{83.25}}{l_{80.75}} = \frac{110-83.25}{110-80.75} = \frac{26.75}{29.25} = \frac{107}{117} \text{ using UDD}$$

$${}_{2.5}q_{80.75} = 1 - \frac{107}{117} = \frac{10}{117} \text{ as required}$$

[3]

(ii) $$\ddot{a}_{80:\overline{4}|} = 1 + \frac{{}_1p_{80}}{1.05} + \frac{{}_2p_{80}}{1.05^2} + \frac{{}_3p_{80}}{1.05^3}$$

$$= 1 + \frac{29}{30} \times \frac{1}{1.05} + \frac{28}{30} \times \frac{1}{1.05^2} + \frac{27}{30} \times \frac{1}{1.05^3}$$

$$= 1 + 0.9206 + 0.8466 + 0.7775$$

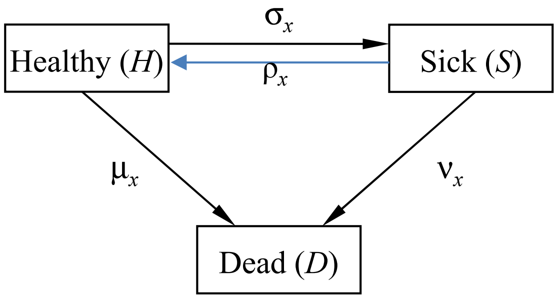
$$= 3.545 \text{ to 3 decimal places}$$

[3]
[Total 6]

Generally well done. Students who obtained decimal answers in (i) were given full credit if equivalent although this made the work more time consuming.

Students answered part (ii) very well.

Q7 (i) Transition model:



[5 marks for complete diagram]

(ii) Temporary Initial Selection – assuming that there is initial underwriting to exclude unhealthy lives we would expect to see lower levels of disability in the initial period.

Class Selection – we would expect to see different rates of sickness, recovery and mortality for different classes of policyholder, e.g. Male/Female, age.

[1 each example, max 2]
[Total 7]

A very straightforward question, generally well done. Candidates lost marks in part (i) if labelling was unclear or incomplete. Note it was not necessary to define symbols in this case.

In (ii) other valid examples were credited. Marks were lost if the reason for the selection chosen was not explained.

Q8 (i) The main advantage of the use of single figure indices is their simplicity for summary and comparison compared to the use of a set of age specific rates. Some indices are particularly designed for comparison with the mortality in a standard population, e.g. mortality rates used for premium calculation. This makes their use particularly relevant in an actuarial context.

The main disadvantage of the use of single figure indices is the loss of information as a result of summarising the set of age specific rates, and any distortions that may be introduced by the choice of weights for the averaging process. [2]

(ii)

	Country			*Sub-population*			
Age	*Population*	*Number of deaths*	*Mortality rate*	*Population*		*Actual deaths*	*Expected deaths*
40–44	834,561	3,510	0.0042	123,978	0.0029	360	521
45–49	779,862	3,153	0.0040	116,853	0.0033	386	467
50–54	750,234	3,620	0.0048	102,800	0.0051	524	493
Total						1,270	1,481

[3 for complete table]

The standardised mortality ratio = 1,270 / 1,481 = 0.858 [1]

(iii) The value less than 1 indicates that the mortality of the sub-population is lighter than for the country as a whole, after allowing for changes in the structure of the population [2]

[Total 8]

A straightforward question generally well done. Despite asking for words only some students still gave formulae in (i).

In (ii) the final answer is very sensitive to rounding. Retaining decimals for Expected Deaths refines the answer to 0.852.

Q9 Let b be the simple bonus rate (expressed as a percentage of the sum assured). Then the equation of value at 4% p.a. interest is (where $P = 3{,}090$):

$$P(.975\ddot{a}_{[30]:\overline{35|}} + 0.025) = (125{,}000 \times (1-b) + 375)A_{[30]:\overline{35|}} + 125{,}000bv^{35}\,{}_{35}p_{[30]}$$

$$+125{,}000b(IA)_{[30]:\overline{35|}} + 325 + 0.75P$$

[4½]

where

$$(IA)_{[30]:\overline{35|}} = (IA)_{[30]} - v^{35}\,{}_{35}p_{[30]}(35A_{65} + (IA)_{65}) + 35v^{35}\,{}_{35}p_{[30]}$$

$$= 6.91644 - 0.25342 \times \frac{8821.2612}{9923.7497}(35 \times 0.52786 + 7.89442) + 35 \times 0.25342 \times \frac{8821.2612}{9923.7497}$$

$$= 6.91644 - 0.22527 \times 26.36952 + 7.88431 = 8.86049$$

[3]

Equation of value becomes:

$$P(.975\times 19.072+0.025)=(125{,}375-125{,}000b)\times 0.26647+125{,}000b\left(0.25342\times\frac{8821.2612}{9923.7497}+8.86049\right)$$
$$+325+0.75P$$

$$\Rightarrow 57{,}536.42=33{,}408.68+1{,}102{,}411b+2{,}642.5$$

$$\Rightarrow b=\frac{21{,}485.24}{1{,}102{,}411}=0.0195$$

[1½]
[Total 9]

> Generally well done. The main difficulty was the correct valuation of the *IA* factor in the above equation

Q10 (i) Let *P* be the net premium for the policy payable annually in advance. Then, equation of value becomes:

$$P\ddot{a}_{[40]:\overline{25}|}=25{,}000(A_{[40]:\overline{25}|}+v^{25}\,{}_{25}p_{[40]})$$

$$15.887P=25{,}000\left(0.38896+0.37512\times\frac{8821.2612}{9854.3036}\right)=25{,}000(0.38896+0.33580)$$

$$\Rightarrow P=£1{,}140.49$$ [2]

Net premium reserve at the end of the 17^{th} policy year is

$${}_{17}V=25{,}000(A_{57:\overline{8}|}+v^{8}\,{}_{8}p_{57})-P\ddot{a}_{57:\overline{8}|}$$

$$=25{,}000\left(0.73701+0.73069\times\frac{8821.2612}{9467.2906}\right)-1{,}140.49\times 6.838$$

$$=35{,}445.98-7{,}798.67=27{,}647.31$$ [2]

Death strain at risk per policy = 25,000 – 27,647.31 = −2,647.31 [1]

$$EDS=5{,}374q_{56}\times -2{,}647.31=5{,}374\times 0.005025\times -2{,}647.31=-71{,}488.85$$
$$ADS=24\times -2{,}647.31=-63{,}535.44$$ [1½]

mortality profit = −71,488.85 + 63,535.44 = -£7,953.41 i.e. a loss [½]

(ii) The death strain at risk is negative. Hence, the life insurance company makes money on deaths. [1]

Less people die than expected during the year considered so the company makes a mortality loss. [1]
[Total 9]

> Generally done well by fully prepared students. The main error was in trying to use premium conversion to fix the premium in (i) which is not correct. Students also used ${}_{18}V$ instead of ${}_{17}V$ in the net premium reserve.

Q11

$$\text{EPV} = 50000 \times \left(a_{\overline{10|}}^{(12)} + v^{10} \times \frac{l_{70}}{l_{60}} \times a_{70}^{(12)} \right)$$

$$+20000 \times \left[v^{10} \times \left(1 - \frac{l_{70}}{l_{60}} \right) \times \frac{l_{72}}{l_{62}} \times a_{72}^{(12)} + v^{10} \times \frac{l_{70}}{l_{60}} \times \frac{l_{72}}{l_{62}} \times a_{70|72}^{(12)} \right] \quad [4]$$

$$a_{\overline{10|}}^{(12)} = \frac{i}{i^{(12)}} \times a_{\overline{10|}} = 1.018204 \times 8.1109 = 8.259 \quad [½]$$

$$v^{10} = 0.67556 \qquad \frac{l_{70}}{l_{60}} = \frac{9238.134}{9826.131} = 0.94016 \qquad \frac{l_{72}}{l_{62}} = \frac{9193.860}{9804.173} = 0.93775 \quad [1½]$$

$$a_{70}^{(12)} = \ddot{a}_{70} - \frac{13}{24} = 11.020 \quad [½]$$

$$a_{72}^{(12)} = \ddot{a}_{72} - \frac{13}{24} = 11.593 \quad [½]$$

$$a_{70|72}^{(12)} = a_{72}^{(12)} - a_{70:72}^{(12)} = 11.593 - (9.404 - \frac{13}{24}) = 2.731 \quad [1½]$$

$$EPV = 50000 \times (8.259 + 0.67556 \times 0.94016 \times 11.020)$$
$$+20000 \times 0.67556 \times 0.93775 \times (0.05984 \times 11.593 + 0.94016 \times 2.731)$$

$$= 762909.1 + 41321.1$$

$$= 804230 \text{ rounded} \quad [1½]$$

[Total 10]

> This question was often poorly done because students did not distinguish the four payment situations properly.

Q12 (i)

$$\text{GFLRV} = 275 + 0.3P + 0.05P \times a^{4\%}_{\overline{\min(K_{[50]},14)}|} + 68 \times a^{0\%}_{\overline{\min(K_{[50]},14)}|} - P \times \ddot{a}^{4\%}_{\overline{\min(K_{[50]}+1,15)}|}$$

$$+ v^{T_{[50]}}(450{,}000 - 30{,}000 \times K_{[50]}) + 315 \qquad (\text{if } K_{[50]} < 15) \qquad [4]$$

(ii) $$P\ddot{a}^{4\%}_{[50]:\overline{15}|} = 207 + 0.25P + 0.05P\ddot{a}^{4\%}_{[50]:\overline{15}|} + 68\ddot{a}^{0\%}_{[50]:\overline{15}|}$$

$$+480{,}000\bar{A}^{1}_{[50]:\overline{15}|} - 30{,}000(I\bar{A})^{1}_{[50]:\overline{15}|} + 315 \times {}_{15}q_{[50]} \qquad [4]$$

$$\ddot{a}^{4\%}_{[50]:\overline{15}|} = 11.259$$

$$\ddot{a}^{0\%}_{[50]:\overline{15}|} = (1 + e_{[50]}) - {}_{15}p_{[50]}(1 + e_{65})$$

$$= 30.583 - \left(\frac{8{,}821.2612}{9{,}706.0977}\right)17.645$$

$$= 30.583 - (0.90884)17.645$$

$$= 14.547 \qquad [1]$$

$$\bar{A}^{1}_{[50]:\overline{15}|} = (1.04)^{0.5}(A_{[50]\overline{15}|} - v^{15}\,{}_{15}p_{[50]})$$

$$= (1.04)^{0.5}(0.56695 - 0.55526 \times 0.90884)$$

$$= 0.06354 \qquad [1]$$

$$(I\bar{A})^{1}_{[50]:\overline{15}|} = (1.04)^{0.5}[(IA)_{[50]} - v^{15}\,{}_{15}p_{[50]}(15A_{65} + (IA)_{65}]$$

$$= (1.04)^{0.5}[8.56390 - 0.55526 \times 0.90884(15 \times 0.52786 + 7.89442)]$$

$$= 0.59590 \qquad [1½]$$

$$11.259P = 207 + 0.25P + 0.05P \times 11.259 + 68 \times 14.547 + 480{,}000 \times 0.06354$$

$$-30{,}000 \times 0.59590 + 315 \times (1 - 0.90884)$$

$$\Rightarrow 10.4461P = 207 + 989.196 + 30{,}499.2 - 17{,}877.0 + 28.715$$

$$\Rightarrow P = 13{,}847.11 / 10.4461 = 1{,}325.58 \qquad [½]$$

(iii) $$_{14}V = q_{64}v^{0.5}[30{,}000 + 315\times(1.04)^{14.5}] + 68\times(1.04)^{14} - 0.95P$$

$$= (0.012716)(0.980581)[30{,}000 + 556.28] + 117.75 - 0.95\times 1{,}325.58$$

$$= 381.01 + 117.75 - 1{,}259.30 = -760.54$$ [2]

[Total 14]

> This proved to be the hardest question on the paper. Many struggled to get the Random Variable solution in part (i) (although correct alternatives were credited).
>
> In part (ii) the main issue was the Increasing Assurance functions.
>
> In part (iii) it is very common for students to have difficulties with the calculation of retrospective reserves.

Q13 (i) The dependent rates of decrement are calculated for policy years 1 and 2 using:

$$(aq)_x^j = \frac{\mu^j}{\mu^d + \mu^m + \mu^s}\left[1 - e^{-(\mu^d + \mu^m + \mu^s)}\right]$$

where d denotes mortality, m marriage and s surrenders

$\Rightarrow$

$$(aq)_x^d = \frac{\mu^d}{\mu^d + \mu^m + \mu^s}\left[1 - e^{-(\mu^d + \mu^m + \mu^s)}\right] = \frac{0.01}{0.235}\left[1 - e^{-(0.235)}\right] = 0.008912$$

$$(aq)_x^m = \frac{\mu^m}{\mu^d + \mu^m + \mu^s}\left[1 - e^{-(\mu^d + \mu^m + \mu^s)}\right] = \frac{0.15}{0.235}\left[1 - e^{-(0.235)}\right] = 0.133678$$

$$(aq)_x^s = \frac{\mu^s}{\mu^d + \mu^m + \mu^s}\left[1 - e^{-(\mu^d + \mu^m + \mu^s)}\right] = \frac{0.075}{0.235}\left[1 - e^{-(0.235)}\right] = 0.066839$$

The dependent rates of decrement are calculated for policy year 3 using:

$$(aq)_x^j = \frac{\mu^j}{\mu^d + \mu^m}\left[1 - e^{-(\mu^d + \mu^m)}\right]$$

where d denotes mortality and m marriage

$\Rightarrow$

$$(aq)_x^d = \frac{\mu^d}{\mu^d + \mu^m}\left[1 - e^{-(\mu^d + \mu^m)}\right] = \frac{0.01}{0.16}\left[1 - e^{-(0.16)}\right] = 0.009241$$

$$(aq)_x^m = \frac{\mu^m}{\mu^d + \mu^m}\left[1 - e^{-(\mu^d + \mu^m)}\right] = \frac{0.15}{0.16}\left[1 - e^{-(0.16)}\right] = 0.138615$$ [4]

Multiple decrement table:

t	$(aq)_{x+t}^d$	$(aq)_{x+t}^m$	$(aq)_{x+t}^s$	$(ap)_{x+t}$	${}_{t-1}(ap)_x$
1	0.008912	0.133678	0.066839	0.790571	1.000000
2	0.008912	0.133678	0.066839	0.790571	0.790571
3	0.009241	0.138615	0.00	0.852144	0.625002

(ii) The expected cash flows for the policy are:

Yr	*Opening reserve*	*Premium*	*Expense*	*Interest*	*Death claim*	*Surrender claim*	*Marriage claim*	*Maturity claim*	*Closing reserve*
1	0.00	9516.00	142.74	328.06	89.12	318.02	1367.49	0.00	0.00
2	0.00	9516.00	142.74	328.06	178.24	636.04	2734.98	0.00	0.00
3	0.00	9516.00	142.74	328.06	277.23	0.00	4253.98	25564.31	0.00
		[½]	[½]	[½]	[1½]	[1½]	[1½]	[1]	

(iii)

Yr	*Profit vector*	${}_{t-1}(ap)_x$	*Profit signature*	*Discount factor*	*Present value of profit*
1	7926.70	1.000000	7926.70	0.96154	7621.84
2	6152.07	0.790571	4863.65	0.92456	4496.74
3	–20394.20	0.625002	–12746.42	0.88900	–11331.57
	[½]		[½]		

Total present value of profit = 787.01 [1]

(iv) The cash flows show that for this policy, the expected profit vector is positive for policy years 1 and 2 but negative (significantly) for the last policy year (which is expected due to the maturity value being paid at the end of the term of the policy). The company may not have sufficient funds available to pay claims in policy year 3 and therefore, it would be prudent for the company to hold reserves at the beginning and end of each policy year. [2]

[Total 15]

Generally the question was answered well by fully prepared students. Credit was given to students who could outline the processes even if the final calculations were not always correct.

END OF EXAMINERS' REPORT