

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2018

Subject CT8 – Financial Economics Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Luke Hatter
Chair of the Board of Examiners
June 2018

A. General comments on the aims of this subject and how it is marked

1. The aim of the Financial Economics subject is to develop the necessary skills to construct asset liability models and to value financial derivatives. These skills are also required to communicate with other financial professionals and to critically evaluate modern financial theories.
2. The marking approach for CT8 is flexible in the sense that different answers to those shown in the solution can earn marks if they are relevant and appropriate. Marks for the methodology are also awarded.

B. General comments on student performance in this diet of the examination

1. Performance by candidates on this paper was, on the whole, considerably worse than in recent sittings.
2. In general, the real differentiators in those who scored well were attention to detail in their algebraic steps, and the breadth of knowledge in being able to score the bookwork marks and even attempt most questions. The majority of candidates seemed unable to gather from the text of the question the relevant information, and translate it in the appropriate equivalent statistical concepts. For example, candidates struggled with formulating the probability that an event occurs in appropriate mathematical terms, and determining from the information in the question the direct way to recover required variances and covariances. This showed a lack of sufficient confidence with the fundamental statistical concepts which Financial Economics so heavily relies on.
3. Students performed relatively well on bookwork questions, although many missed the opportunity to be awarded full marks due to relatively superficial knowledge.
4. The majority of candidates seemed to struggle on the application parts of the questions, because they were not able to use and combine the information given to them in the question.

C. Pass Mark

The Pass Mark for this exam was 60.

Solutions

Q1

- i)
- a. $U'(w) = w^{-0.5} > 0$ for $w > 0$ [1]
 b. $U''(w) = -0.5w^{-1.5} < 0$ for $w > 0$ [1]
- ii) $R(w) = w*(-U''(w)/U'(w)) = 0.5$ [2]
- iii) $E[U] = 0.6U(1.69a) + 0.1U(6.25b) + 0.3U(0)$ [½]
 $= 0.6*2*((1.69a)^{0.5}-1) + 0.1*2*((6.25b)^{0.5}-1) - 0.6$ [½]
 $= 1.2*(1.3a^{0.5}-1) + 0.2*(2.5b^{0.5}-1) - 0.6$
 $= 1.56a^{0.5} + 0.5b^{0.5} - 1.4 - 0.6$ [½]
 $= 1.56a^{0.5} + 0.5(1000-a)^{0.5} - 2$ [½]
 $dE[U]/da = 0.78a^{-0.5} - 0.25(1000-a)^{-0.5}$ [1]
 Setting $dE[U]/da = 0$ gives
 $0.78a^{-0.5} = 0.25(1000-a)^{-0.5}$ [½]
 $\Rightarrow 0.78 = 0.25a^{0.5}(1000-a)^{-0.5}$
 $\Rightarrow 3.12 = (a/(1000-a))^{0.5}$ [½]
 Squaring both sides
 $\Rightarrow 9.7344 = a/(1000-a)$ [½]
 $\Rightarrow 9,734.4 = 10.7344a$ or $8.7344a$
 $\Rightarrow a = £906.8$ or $£1,114.50$ [½]
 Rejecting the figure $>£1,000$ gives
 $a = £906.80$ and $b = £93.20$ [1]
 Checking the second derivative
 $d^2E[U]/da^2 = -0.39a^{-1.5} - 0.25(1000-a)^{-1.5} < 0$ [½]
 hence this is a maximum [½]

[Note to markers: rounding accepted]

- iv) $E[U] = 49.801$ [1]
[Note to markers: assign [1] mark to answers containing the expected wealth]
- v) $U(1000) = 61.2456$ [1]
 So the maximum expected utility of wealth is less than the current utility of wealth. [½]
 This is because the odds offered pay out less than would be required based on the investor's estimated probabilities of each horse winning. [½]
 Based on expected utility, the investor would be better off not betting at all. [½]
 There may be other horses in the race where this position is reversed. [½]

[Note to markers: assign [1] mark to any valid comment]

[Max 2]

Part i) and ii) were very well answered with most students scoring full marks.

Many students found iii) challenging with only a few able to formulate the expected utility correctly allowing for the possibility of neither horse winning. Some students understood the method required of setting the first derivative to zero and so were able to score method marks even if they were unable to obtain the correct formula for the expected utility. Very few candidates checked for conditions on the second order derivative.

Quite a few candidates were able to derive the expected wealth correctly for iv) based on their answer in iii) and so were able to score this mark. Some candidates were able to score a mark for calculating the utility of the initial wealth although few were able to make sensible comments to score additional marks.

Q2

The estimation of parameters is one of the most time-consuming aspects of stochastic asset modelling. [½]

The simplest case is the purely statistical model, where parameters are calibrated entirely to past time series. Provided the data is available, and reasonably accurate, the calibration can be a straightforward and mechanical process. [½]

Of course, there may not always be as much data as we would like, and the statistical error in estimating parameters may be substantial. [½]

Furthermore, there is a difficulty in interpreting data which appears to invalidate the model being fitted. [½]

For example, what should be done when fitting a Gaussian model in the presence of large outliers in the data? [½]

Perhaps the obvious course of action is to reject the hypothesis of normality, and to continue building the model under some alternative hypothesis. After all, in many applications, the major financial risks lie in the outliers, so it seems foolish to ignore them. [½]

In practice, a more common approach to outliers is to exclude them from the statistical analysis, and focus attention instead on the remaining residuals which appear more normal. [½]

The model standard deviation may be subjectively nudged upwards after the fitting process, in order to give some recognition to the outliers which have been excluded. [½]

It has often been the practice in actuarial modelling to use the same data set to specify the model structure, to fit the parameters, and to validate the model choice. [½]

A large number of possible model structures are tested, and testing stops when a model which is found which passes a suitable array of tests. [½]

Unfortunately, in this framework, we may not be justified in accepting a model simply because it passes the tests. [½]

Many of these tests (for example, tests of stationarity) have notoriously low power, and therefore may not reject incorrect models. [½]

Indeed, even if the “true” model was not in the class of models being fitted, we would still end up with an apparently acceptable fit, because the rules say we keep generalising until we find one. [½]

This process of generalisation tends to lead to models which wrap themselves around the data, resulting in an understatement of future risk, and optimism regarding the accuracy of out-of-sample forecasts. [½]

In the context of economic models, the calibration becomes more [½]
complex. The objective of such models is to simplify reality by imposing certain stylised facts about how markets would behave in an ideal world.

This theory may impose constraints, for example on the relative volatilities of bonds and currencies. Observed data may not fit these constraints perfectly. [½]

In these cases, it is important to prioritise the features of the economy that are most important to calibrate accurately for a particular application. [½]

[Note to markers: please award ½ mark for any valid idea presented by the candidates. We only need the concept to earn a half mark, not all the detail above.]

[Max 5]

This was generally poorly answered with most candidates picking up one or two marks at most. Many candidates focused on why the CAPM and APT models were unrealistic rather than focusing specifically on the challenges of estimating parameters and hence were unable to score well.

Q3

- i) EITHER B_t has independent increments, OR $B_t - B_s$ is independent of $\{B_r, r \leq s\}$ whenever $s < t$, OR BOTH
 EITHER B_t has stationary increments, OR the distribution of $B_t - B_s$ depends only on $t - s$, OR BOTH
 EITHER B_t has Gaussian increments, OR the distribution of $B_t - B_s$ is $N(0, t - s)$, OR BOTH
 B_t has continuous sample paths $t \rightarrow B_t$.
 $B_0 = 0$.

[1 Mark each]

[Total of 5]

- ii) $A(0) = \exp(0) = 1$, so the student buys 1,000 units of the asset. [½]
 $E[A(5)] = \exp(0.5 \cdot 1^2 \cdot 5) = 12.182$ [1]
 So the expected value of the investment is \$12,182. [½]
- iii) $P(\text{Investment} < \$10,000) = P(A(5) < 10)$ [½]
 $= P(Z < (\ln(10))/\sqrt{5})$ [½]
 $= P(Z < 1.03)$ [½]
 $= 0.8484$ (0.85) [½]

Part i) was bookwork and was well answered with the majority of candidates scoring full or close to full marks.

Parts ii) and iii) proved challenging for many candidates although the techniques required were quite standard. Students in general struggled with identifying the correct parameters and which expectation and probability to calculate.

Q4

- i) $F(4) = 10\exp(4 \cdot 0.05) = £12.21$ [1]
- ii) We can obtain d_1 and d_2 from the Black Scholes formula as:
 $d_1 = 0.1214$ [1]
 $d_2 = -0.1186$ [1]
 Call price = £0.96 [1]
- iii) Paid £12.21 for a stock worth £12 = loss of £0.21 [1]
- iv) Paid £0.96 for call option = $0.96 \cdot \exp(4 \cdot 0.05) = £1.17$ at time 4 [1]
 Call expires worthless, hence loss of £1.17 [1]

[Note to marker: please award ½ mark for using £0.96 as the loss on the position]

- v) Mr Jones must pay for the optionality. [1]
 He makes a loss if the option expires worthless, but that loss is never [1]
 larger than £1.17.
 This capped loss has a cost. [1]

[Note to Markers: please accept valid answers carrying forward an erroneous £0.96 as loss]

[Max 2]

- vi) If the stock is worth more than £12.21 at time 4 then Mrs Jones will make [½]
 a profit

and this profit will always be larger than Mr Jones' profit because he had [1]
 to buy the option.

If the stock is worth less than £12.21 at time 4 then Mrs Jones will make [½]
 a loss
 of £12.21 minus stock price [1]

Mr Jones will always make a loss of £1.17 [½]
 So the crossover is at a stock price of $12.21 - 1.17 = £11.04$ [1]

[Max 3]

ALTERNATIVE ANSWER: Profit from Call > Profit from forward [1]

$$\text{Max}\{S(T) - 12.21, 0\} - 1.17 > S(T) - 12.21 \quad [½]$$

$$\Rightarrow \text{Max}\{-12.21, -S(T)\} > -11.04 \quad [½]$$

$$\Rightarrow S(T) < 11.04 \quad [1]$$

[Note to Markers: please accept valid answers carrying forward an erroneous £0.96 as loss]

In general, parts i), ii) and iii) were well answered although there were some calculation errors. Some candidates though did not reflect on the magnitude of their answer to determine if it was realistic.

For iv) many candidates did not accumulate the premium paid to expiry as required by the question and so were not able to score full marks.

In part v) most candidates did not explain properly why a premium is required to enter the option and hence lost a mark.

Part vi) was quite poorly answered and quite a few candidates struggled as they were unable to consider all the different possible outcomes.

Q5

- i) We need $E[e^{-rt}B_t | F_s] = e^{-rs}B_s$ [½]

If the bond has already defaulted by time s then $e^{-rs}B_s = e^{-rs}30e^{-r(3-s)} = e^{-rt}B_t$ [1]

Otherwise $e^{-rt}B_t = 30e^{-3r}$ if the bond defaults before time t , or [½]

$e^{-rt}B_t = e^{-3r}(30(1 - e^{-\lambda(3-t)}) + 100e^{-\lambda(3-t)})$ otherwise [½]

Then $E[e^{-rt}B_t | F_s] = 30e^{-3r}(1 - e^{-\lambda(t-s)}) + e^{-3r}(30(1 - e^{-\lambda(3-t)}) + 100e^{-\lambda(3-t)}) e^{-\lambda(t-s)}$ [½]

$= e^{-3r}(30(1 - e^{-\lambda(3-s)}) + 100e^{-\lambda(3-s)})$ [½]

$= e^{-rs}B_s$ hence this is a martingale [½]

- ii) (a) Let the portfolio contain x cash and y bonds.

We need the value at time 3 to be \$35 if the bond has defaulted,
so $xe^{3r} + 30y = 35$ [1]

We also need the value at time 3 to be zero if the bond has not defaulted,
so $xe^{3r} + 100y = 0$ [1]

Hence $x = 50 e^{-3r}$ and $y = -0.5$. [1]

(b) The price at time zero must be the cost of buying this portfolio [½]

$= x + yB_0 = 50e^{-3r} - 0.5e^{-3r}(30(1 - e^{-3\lambda}) + 100e^{-3\lambda})$ [1]

$= 35e^{-3r}(1 - e^{-3\lambda})$ [½]

- iii) Let the portfolio value be V . [1]

We need the value of the portfolio at time 0 to be $E[e^{-3r}V_3]$ under probability measure P .

$E[e^{-3r}V_3] = 35e^{-3r} * [\text{Probability that bond defaults}]$ [1]

$= 35e^{-3r}(1 - e^{-3\lambda})$ hence the requirement holds. [1]

This fits with the fact that being able to hedge a derivative price without
arbitrage means we can price it under the Equivalent Martingale Measure. [1]

[Max 3]

This was a difficult question that was very poorly answered with candidates either not attempting or only superficially attempting this question. Overall the lowest scoring question from the paper.

For i), the majority of candidates appeared unfamiliar with the term “equivalent martingale measure”, and did not realise they needed to focus on the discounted price process.

There were more attempts for ii) although the majority were unable to formulate the required equations for the replicating portfolio.

Part iii) was very poorly attempted and any reasonable comment was given credit.

Q6

- i) EITHER: The market is arbitrage free if and only if there exists a probability measure under which discounted asset prices are martingales. [1]

OR

The probability exists if $1 - d < e^{r\delta t} < 1 + u$. [1]

OR BOTH [max 1]

[Note to markers: please accept solutions with $d < \exp(r\delta t) < u$, and $\delta t = 1$.]

(Note to markers: both answers are acceptable)

- ii)

| | Stock tree | | | |
|------|------------|--------|--------|--------|
| time | 0 | 1 | 2 | 3 |
| | 95.00 | 114.00 | 136.80 | 164.16 |
| | | 76.00 | 91.20 | 109.44 |
| | | | 60.80 | 72.96 |
| | | | | 48.64 |

The price P_0 of the option is computed via Risk Neutral Valuation; let \hat{p} denote the risk neutral probability of an up movement, then

$$\hat{p} = \frac{e^{0.05} - 0.80}{1.20 - 0.80} = 0.6282 \quad [1]$$

and

$$P_0 = e^{-rT} \sum_{k=0}^3 \binom{3}{k} \hat{p}^k (1 - \hat{p})^{3-k} (K - S_0 u^k d^{3-k})^+ \quad [2]$$

$$= e^{-0.05 \times 3} (0.56 \times 3\hat{p}^2(1 - \hat{p}) + 37.04 \times 3\hat{p}(1 - \hat{p})^2 + 61.36 \times (1 - \hat{p})^3) \\ = 11.23$$

[1]

The detailed working are provided below – in case attempts to answer this question go through the whole tree.

| | PUT | | | |
|------|-------|-------|-------|-------|
| time | 0 | 1 | 2 | 3 |
| | 11.23 | 4.87 | 0.20 | 0.00 |
| | | 23.53 | 13.44 | 0.56 |
| | | | 43.84 | 37.04 |
| | | | | 61.36 |

- iii) The given market conditions imply that $1 - d = 0.95, 1 + u = 1.05$; however the discount factor is $e^{0.05} = 1.0513$ [1]

Hence the condition of no arbitrage is violated and no pricing is possible [1]

[Alternatively, candidates can recalculate the risk neutral probability \hat{p} , which in this case would give 1.0127, hence there is arbitrage in the market
Alternatively, candidates can obtain a negative option price].

Parts i) and ii) were well answered although there were a few calculation errors in ii). Lost marks were mostly for getting an incorrect probability value, forgetting the combination factor in the final calculation or slipping up with the numbers. A few students also got confused and tried to price a call option.
There were quite a few good answers to iii) although about half did not spot that the no arbitrage condition would not hold under the new scenario.

Q7

- i) The market is complete if for any contingent claim X there is a replicating strategy (Φ_t, Ψ_t) [1]

i.e. is a self-financing strategy, defined for $0 \leq t < U$, capable of reproducing the derivative terminal payment at U without risk, for an initial investment of $V(0)$ at time 0. [1]

[Max 1]

- ii) The SDE of $\tilde{S}_t = e^{-rt} S_t$:

$$d\tilde{S}_t = (\mu - r)\tilde{S}_t dt + \sigma\tilde{S}_t dW_t, \quad [1]$$

For this process to be a martingale, the drift should be zero [1]

but $r \neq \mu$ in general. Hence it is not a martingale. [1]

[Alternative solutions based on solving the SDE and checking the martingale condition are equally acceptable. This is equivalent to check that the identity $E[\tilde{S}_T|F_t] = \tilde{S}_t$ for $\tilde{S}_t = S_0 \exp((\mu - r - 0.5\sigma^2)t + \sigma W_t)$ does not hold.]

- iii) We need to change the Brownian motion by means of the Girsanov Theorem. Let $\hat{W}_t = W_t + \lambda t$ be a Brownian motion under a new probability measure Q [1]

then the above SDE becomes: [1]

$$d\tilde{S}_t = (\mu - r - \lambda\sigma)\tilde{S}_t dt + \sigma\tilde{S}_t d\hat{W}_t,$$

For the martingale property to hold set the drift to zero, which implies

$$\lambda = (\mu - r)/\sigma. \quad [1]$$

[Alternative solution carrying equal marks: change the Brownian motion as above and take the conditional expectation under the new measure; this returns $E^Q[\tilde{S}_T|F_t] = \tilde{S}_t \exp(\mu - r - \lambda\sigma)(T - t)$. The martingale condition requires $\lambda = (\mu - r)/\sigma$.]

In general, candidates who scored marks seemed to have a good understanding of the underlying theory.

For i) many did not know the required bookwork definition although this did not affect the rest of the question.

Part ii) was reasonably answered although many calculated an expression for $S(t)$ and struggled to show that the expectation condition was satisfied rather than directly use the SDE.

Part iii) was not well answered and generally only the stronger candidates scored well on this part.

Q8

- i) Data: $S_0 = 65, K = 55, \sigma = 25\% \text{ p.a.}, T = 0.5 \text{ year}, r = 2\%$
Let C_t be the price of the European call.

The Black-Scholes formula returns [½ Mark each]

$$d_1 = 1.09$$

$$d_2 = 0.9132$$

$$N(d_1) = 0.8621$$

$$N(d_2) = 0.8194$$

$$\text{Therefore } C_0 = 65 \times 0.8621 - 55e^{-0.02 \times 0.5} \times 0.8194 \quad [1]$$

$$= 11.42 \quad [1]$$

- ii) $\text{delta} = \frac{\partial C}{\partial S}$ [1]
[Note to markers: please award ½ mark for stating $N(d_1)$]
- iii) In the Black-Scholes model $\text{delta} = N(d_1)$ [1]
 Using the results from above $\text{delta} = 0.8621$ [1]
- iv) $\text{delta}_{\text{put}} = \text{delta}_{\text{call}} - 1$ [1]
 Therefore, $\text{delta}_{\text{put}} = -0.1379$ [1]
[Note to markers: if signs are incorrect in the formula and the actual value, award ½ mark for $\text{delta}_{\text{put}} = 0.1379$ only]

This question was generally well answered with quite a few scoring full marks or close to full marks. Many got (iv) correct, but a significant number of candidates got the sign wrong.

Q9

- i) Using continuous compounding.
[Note to markers: please accept any correct attempt using different compounding convention]
- a. $-\frac{1}{0.25} \ln \frac{97.5}{100} = 10.13\%$ [1]
 b. $-\frac{1}{0.5} \ln \frac{94.9}{100} = 10.47\%$ [1]
 c. $0.04 \times (94.9 + 90) + 104 \times e^{-0.1068 \times 1.5} = 96$ [1]
 d. $\frac{(0.1054 \times 1 - 0.1047 \times 0.5)}{1 - 0.5} = 10.60\%$ [1]
 e. $\frac{(0.1068 \times 1.5 - 0.1054 \times 1)}{1.5 - 1} = 10.97\%$ [1]
- ii) Two standard models for the short rate of interest are the Vasicek model and the CIR model.
 The corresponding SDEs are respectively
- $$dr_t = k(\theta - r_t)dt + \sigma dW_t$$
- $$dr_t = k(\theta - r_t)dt + \sigma \sqrt{r_t} dW_t$$
- [1 mark each]

Alternatively: another standard model is the Hull and White model which extends the Vasicek model to allow for time-inhomogeneity, therefore the parameters in the SDE are time dependent.

Parts i) a) and b) were very well answered and d) was also well answered by many. Parts c) and e) proved difficult with only a few getting the marks here.

Part ii) was very well answered.

Q10

i) Data:

$$S_0 = 15, K = 12, \sigma = 20\% \text{ p. a. }, T = 0.25 \text{ year}, r = 2\%.$$

Let C_t be the price of the European call.

The Black-Scholes formula returns

[½ mark each]

$$d_1 = 2.3314$$

$$d_2 = 2.2314$$

$$N(d_1) = 0.9901$$

$$N(d_2) = 0.9872$$

$$\begin{aligned} \text{Therefore } C_0 &= 15 \times 0.9901 - 12e^{-0.02 \times 0.25} \times 0.9872 \\ &= 3.0650 \end{aligned}$$

[1]

[1]

ii) Probability of expiring in the money: $P(S_T > K) = N(d_2)$

[½]

hence from above $P(S_T > K) = 0.9872$.

[½]

iii) Risk neutral valuation applied to the given digital option returns

$$E(e^{-rT} 1_{S_T > K}) = e^{-rT} P(S_T > K) \quad [1]$$

From above it follows that the price is $e^{-0.02 \times 0.25} \times 0.9872 = 0.98225$ [1]

iv) Limitations

[½ mark each]

- a. Share prices can jump. This invalidates assumption that the stock price evolves as geometric Brownian motion, as this process has continuous sample paths. However, hedging strategies can still be constructed which substantially reduce the level of risk.
- b. The risk-free rate of interest does vary and in an unpredictable way. However, over the short term of a typical derivative, the assumption of a constant risk-free rate of interest is not far from reality. (More specifically the model can be adapted in a simple way to allow for a stochastic risk-free rate, provided this is a predictable process.)
- c. Unlimited short selling may not be allowed, except perhaps at penal rates of interest. These problems can be mitigated by holding mixtures of derivatives which reduce the need for short selling. This is part of a suitable risk management strategy.
- d. Shares can normally only be dealt in integer multiples of one unit, not continuously, and dealings attract transaction costs. Again we are still able to construct suitable hedging strategies which substantially reduce risk.

- e. Distributions of share returns tend to have fatter tails than suggested by the lognormal model.

[Note to Markers: please award ½ mark for any valid idea/comment. Just the concept is enough for a half mark, no need for all the detail above.]

[Max 2]

Part i) was well answered although there were some calculation errors. Parts ii) and iii) proved challenging for some although quite a few candidates were able to score full marks. Part iv) was generally well answered although quite a number of candidates struggled to generate enough points to score full marks for a standard bookwork question.

Q11

- i) CAPM assumptions [½ mark each]

- a. All investors have the same one-period horizon.
- b. All investors can borrow or lend unlimited amounts at the same risk-free rate.
- c. The markets for risky assets are perfect. Information is freely and instantly available to all investors and no investor believes that they can affect the price of a security by their own actions.
- d. Investors have the same estimates of the expected returns, standard deviations and covariances of securities over the one-period horizon.
- e. All investors measure in the same “currency” e.g. pounds or dollars or in “real” or “money” terms.

[Max 2]

- ii) By definition the beta of each security is $\beta_i = Cov(R_i, R_M)/Var(R_M)$ [½]
 where R_i is the rate of return on security i , $R_M, Var(R_M)$ are respectively [½]
 the rate of return on the market portfolio and its variance
 Hence

$$\beta_M = \frac{Cov(R_M, R_M)}{Var(R_M)} = 1$$

as required [1]

(Note to markers: the same conclusion can be reached from the Security Market Line, and is equally acceptable)

- iii) As the market portfolio is the weighted portfolio of the risky securities in the market, and the given weights are 0.4 and 0.6, then

$$Cov(R_i, R_M) = Cov(R_i, 0.4R_1 + 0.6R_2)$$

[1]

from which it follows that [½ mark each]

$$\text{Cov}(R_1, R_M) = 0.4\text{Var}(R_1) + 0.6\text{Cov}(R_1, R_2) = 0.00089$$

$$\text{Cov}(R_2, R_M) = 0.4\text{Cov}(R_1, R_2) + 0.6\text{Var}(R_2) = 0.00150$$

Also:

$$\text{Var}(R_M) = 0.4^2 * \text{Var}(R_1) + 0.6^2 * \text{Var}(R_2) + 2 * 0.4 * 0.6 * \text{Cov}(R_1, R_2) = 0.00125$$

Consequently $\beta_1 = 0.70915, \beta_2 = 1.1939$ [½ each]

[Note to Markers: please accept any correct attempt with rounded figures. For

$$\text{Cov}(R_1, R_M) = 0.4\text{Var}(R_1) + 0.6\text{Cov}(R_1, R_2) = 0.0009$$

we obtain $\beta_1 = 0.72, \beta_2 = 1.2$]

iv) From the Security Market Line it follows that $R_f = (ER_i - \beta_i ER_M) / (1 - \beta_i)$

From the data $ER_1 = 6.40\%, ER_2 = 9.90\%$. [½ marks each]

Consequently

$$ER_M = \sum_{i=1}^2 w_i ER_i = 8.5\% \quad [½]$$

and $R_f = 0.012797$ [½]

[Note to Markers: if the rounding above and the corresponding betas are used, then from the equation for asset 1 we obtain $R_f = 0.01$, whilst from the equation for asset 2 we obtain $R_f = 0.015$. Please accept any valid attempt.]

Part i) was answered reasonably although many candidates were unable to identify the additional assumption of CAPM compared to modern portfolio theory.

Part ii) was very well answered.

Part iii) was poorly answered with most candidates unable to calculate the required covariances correctly. Many candidates indeed attempted this task by only looking at the information regarding the rate of return in each state (and corresponding probability), rather than actually using the provided variance/covariance matrix

Part iv) was answered well and credit was given for calculating the expected returns even if this was done in iii).

END OF EXAMINERS' REPORT