EXAMINATION

8 October 2010 (am)

Subject CT8 — Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all nine questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 An investor holds an asset that produces a random rate of return, R, over the course of a year. The distribution of this rate of return is a mixture of normal distributions, i.e. R has a normal distribution with a mean of 0% and standard deviation of 10% with probability 0.8 and a normal distribution with a mean of 30% and a standard deviation of 10% with a probability of 0.2.

S is the normally distributed random rate of return on another asset that has the same mean and variance as R.

(i) Calculate the mean and variance of R. [3]

(ii) Calculate the shortfall probabilities for R and for S using:

(a) a benchmark rate of return of 0%
(b) a benchmark rate of return of –10% [4]

(iii) Comment on what the variance and shortfall probabilities at both benchmark levels illustrate about the asset returns, by referring to the calculations in (i) and (ii). [3]
[Total 10]

2 (i) Explain what is meant by "excessive volatility" of share prices. [2]

(ii) State two examples of empirical evidence of the "under-reaction" of share prices to events. [4]
[Total 6]

3 Discuss whether one-factor models are good models for the short-rate of interest (instantaneous risk free rate). Include discussion of extensions that may be considered to improve the model. Illustrate your discussion by defining and referring to particular models. [10]

4 (i) In the context of credit risk for defaultable bonds:

(a) give three examples of a credit event
(b) give three examples of an outcome of a default
(c) define the recovery rate [7]

(ii) Describe the two-state model for credit ratings. [4]

Two companies have zero coupon defaultable bonds in issue. Bond A has £2m nominal in issue. Bond B has £3m nominal in issue. Both bonds redeem in exactly 2 years time.

Under a risk neutral measure, each bond defaults (not necessarily independently) at a constant rate. Both bonds have a 60% recovery rate.

Assume:

- a continuously compounded risk free rate of interest of 3% p.a.
- the issue of bond A is priced at £1.6m
- the issue of bond B is priced at £2.2m

(iii) Evaluate the two default rates (under a risk-neutral measure). [4]

There is also a traded derivative security, D, priced at £52 which pays £100 after 2 years if (and only if) at least one of the bonds defaults.

(iv) (a) Determine a hedging portfolio for the security which pays £100 after 2 years if and only if both bonds default by considering fixed portfolios consisting of bond A, bond B and security D and a risk-free zero-coupon bond paying £100 at redemption in exactly 2 years.

(b) Calculate the fair price for the security that pays £100 if and only if both bonds default.

[8]
[Total 23]

5 (i) State an expression for the price of a derivative security in a Black-Scholes market in terms of the risk-neutral measure. [2]

A European call option on a stock has an exercise date one year away and a strike of £6. The underlying stock has a current price of £5.50. The option is priced at 60p. The stock price volatility has been estimated from other option prices as 20%.

(ii) Estimate the risk free rate of interest to within 0.5% p.a. assuming the Black-Scholes model applies. [5]

A new derivative security has just been written on the underlying stock. This will pay a random amount D in one year's time, where $D = S_1^2$.

(iii) Calculate the fair price for this new derivative security, quoting any further results that you use. [5]

(iv) Determine the initial hedging portfolio (in units of the underlying stock and cash) for this new derivative security. [4]

[Total 16]

6 Under the real-world measure P, W is a standard Brownian motion and the price of a stock, S, is given by $S_t = S_0\exp(\sigma W_t + (\mu - 1/2\,\sigma^2)t)$. The continuously compounded risk-free rate of interest is r and a zero coupon bond with maturity T has price $B_t = e^{-r(T-t)}$. Suppose that in the market any contract which pays $f(S_T)$ at time T is valued at:

$$p_t = E[e^{-r(T-t)} f(S_T)\,\Lambda_T | F_t],$$

where:

$$\Lambda_t = \exp(mW_t - 1/2m^2 t) \text{ for } t \leq T \text{ for some real number } m.$$

(i) (a) Prove, using Ito's formula, that Λ_t is a martingale.

(b) Show that $E[\exp(mW_t)] = \exp(1/2m^2 t)$.

[5]

(ii) (a) Derive an expression for p_0 when $f(x) = x$.

(b) Show that there is an arbitrage in the market unless $m = (r - \mu)/\sigma$.

[5]

[Total 10]

7 (i) Define delta, gamma and vega for an individual derivative. [3]

(ii) Explain how gamma and vega can be used in the risk management of a portfolio that is delta-hedged. [4]

[Total 7]

8 Consider a particular stock and denote its price at any time t by S_t. This stock pays a dividend D at time T'.

Let C_t and P_t be the price at time t of a European call option and European put option respectively, written on S, with strike price K and maturity $T \geq T' \geq t$. The instantaneous risk-free rate is denoted by r.

Prove the put-call parity in this context by adapting the proof of standard put-call parity.

[Hint: assume that when the dividend is paid it is used to pay off any borrowed positions required as part of the proof.] [7]

9 Consider a two-period binomial model for a non-dividend paying stock whose current price is $S_0 = 100$. Assume that:

- over each of the next six-month periods, the stock price can either move up by a factor $u = 1.2$ or down by a factor $d = 0.8$
- the continuously compounded risk-free rate is $r = 6\%$ per period

(i) (a) Prove that there is no arbitrage in the market.

(b) Construct the binomial tree for the model. [2]

(ii) Calculate the price of a standard European call option written on the stock S with strike price $K = 100$ and maturity one year. [5]

Consider a special European call option with strike price $K = 100$ and maturity one year. The owner of such an option has the right to exercise her option at the end of the year only if the stock price goes above the level $L = 130$ during or at the end of the year.

(iii) (a) Calculate the initial price of this call option.

(b) Comment on the relationship between the price of the special call option and the option in (ii). [4]

[Total 11]

END OF PAPER