EXAMINATION

26 September 2007 (am)

Subject CT8 — Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 8 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

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1 (i) Define the Δ, Γ, θ, λ, ρ and ν for an individual derivative security. [6]

(ii) Explain put-call parity and use it to calculate the delta and gamma of a European call option on a non-dividend paying stock with the same strike and maturity as the put option. [4]

A portfolio with a delta of zero consists of cash, European put options, P, on a stock and 1 million shares of the underlying (non-dividend paying) stock . The delta of a single put option is –0.212, while the gamma is 0.377.

Two further derivatives on the stock are also traded: a European call option, C, with the same strike and maturity as P and another derivative security, D, with a delta of 0.222 and a gamma of 0.111.

(iii) Calculate n, the number of put options in the original portfolio of cash, stock and P. [2]

(iv) Calculate the numbers of derivatives D and C that have to be purchased and added to the portfolio so that both the delta and gamma of the expanded portfolio are zero. [4]
[Total 16]

2 A binomial model for a non-dividend-paying security with price S_t at time t is as follows: the price at time $(t + 1)$ is either $1.25S_t$ (up-jump) or $0.8S_t$ (down-jump). Cash receives interest of 10% per time unit.

(i) Calculate the risk neutral probability measure for this model. [3]

The value of S_0 is 100. A derivative security with price D_t at time t pays the following returns at time 2:

$D_2 = 1$: if $S_2 = 156.25$
$D_2 = 2$: if $S_2 = 100$
$D_2 = 0$: if $S_2 = 64$.

(ii) Determine D_1 when $S_1 = 125$ and when $S_1 = 80$ and hence calculate the value of D_0. [5]

(iii) Derive the corresponding hedging strategy, i.e. the combination of the underlying security and the risk free asset required to hedge an investment in the derivative security. [4]

(iv) Comment on your answer to (iii) in the light of your answer to part (ii). [1]
[Total 13]

3 (i) State the assumptions underlying the Black-Scholes option pricing formula and discuss how realistic they are. [6]

A discounted stock price can be written as:

$$\tilde{S}_t = \cosh(\sigma Z_t)\exp(-\sigma^2 t),$$

where Z_t is a standard Brownian motion under the real world measure $\mathbb{P}$.

Hint: $\cosh(x) = (e^x + e^{-x})/2$.

(ii) Apply Ito's formula to derive an SDE satisfied by $\tilde{S}_t$. [5]

(iii) Explain why the discounted stock price (under $\mathbb{P}$) is not a martingale. [1]

(iv) State the SDE satisfied by $\tilde{S}_t$ under the equivalent martingale measure. [2]
[Total 14]

4 (i) State the general zero-coupon bond pricing formula in terms of random short rates. Define all notation used. [2]

(ii) State the SDEs defining the dynamics of the short rates under the risk-neutral measure for each of the Vasicek, Cox-Ingersoll-Ross and Hull & White models. Define all notation used. [6]

(iii) State the zero-coupon bond price formula for the Vasicek model. [3]

(iv) Comment on the limitations of one-factor interest rate models. [3]
[Total 14]

5 Consider a special company that has just issued a zero-coupon bond of nominal value £10m with maturity 10 years. The value of the assets of the company is £20m and this value is expected to grow at an average of 10% per annum compound with an annual volatility of 40%. The company is expected to be wound up after 10 years when the assets will be used to pay off the bond holders with the remainder being distributed to the equity holders.

A constant risk-free rate of return of 5% p.a. compounded continuously is available in the market.

Calculate the credit spread on the debt for the zero-coupon bond. State any additional assumptions that you make.

Hint: use the Merton model. [8]

6 A market consists of three assets A, B and C. Annual returns on the three assets (R_A, R_B and R_C) have the following characteristics:

Asset	*Expected return %*	*Standard deviation %*
A	9	20
B	6	20
C	3	10

The correlation between the returns are as follows: $Corr(R_A, R_B) = -¼$, $Corr(R_B, R_C) = -½$ and $Corr(R_A, R_C) = -½$.

(i) Calculate the variance of the returns of each asset and the covariances between the returns of each pair of assets. [5]

(ii) Define an efficient portfolio for the corresponding mean-variance portfolio model. [2]

Efficient portfolios for this model are of the form $(\frac{2}{9}, \frac{2}{9}, \frac{5}{9})^T + c(4, 1, -5)^T$, for a suitable choice of c, where the vector represents proportions of the investor's capital invested in assets A, B and C respectively.

(iii) (a) Determine an expression in terms of c for the variance of an efficient portfolio.

(b) Deduce the global minimum variance and the portfolio that attains it. [4]

Assume that the risk-free rate of interest is 4% p.a.

(iv) (a) Determine the tangent that passes through (0%, 4%) to the original efficient frontier in (standard deviation, mean return) space.

(b) Deduce the market capitalisations of the three assets consistent with the Capital Asset Pricing Model if the total market capitalisation is £180 bn. [6]

[Total 17]

7 Consider a corporate bond that will return £1 per bond to an investor at the end of a year provided the borrower does not default during the year. The constant annual probability of default is 4%.

Investor 1 holds one thousand such bonds that depend on the same borrower.

Investor 2 holds one thousand such bonds, each of which depends on a different borrower. Each borrower defaults (or not) independently of the other borrowers, but with the same probability of 4%.

All bonds were purchased at par.

(i) For each investor calculate (using suitable approximations if necessary):

(a) the variance of the investment
(b) the 95% value at risk
(c) the 90% value at risk
(d) the probability of shortfall relative to a target level of shortfall of 0 [10]

(ii) Comment on your answers to (i). [3]
[Total 13]

8 (i) Outline the three forms of the Efficient Markets Hypothesis. [3]

(ii) State two reasons why it is hard to test whether any of the three forms hold in practice. [2]
[Total 5]

END OF PAPER