INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2017

Subject CT8 – Financial Economics
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Luke Hatter
Chair of the Board of Examiners
July 2017

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Financial Economics subject is to develop the necessary skills to construct asset liability models and to value financial derivatives. These skills are also required to communicate with other financial professionals and to critically evaluate modern financial theories.

2. The marking approach for CT8 is flexible in the sense that different answers to those shown in the solution can earn marks if they are relevant and appropriate. Marks for the methodology are also awarded.

B. General comments on *student performance in this diet of the examination*

1. Students performed relatively well on bookwork questions, although many missed the opportunity to be awarded full marks due to relatively superficial knowledge.

2. The majority of the students seemed to struggle on the application parts of the questions, because they were not able to use and combine the information given to them in the question. In a few instances, students did not know how to go from the lognormal distribution to the Normal and then to the standard Normal. Further, there is often a lack of knowledge of how to use the distribution tables to compute probabilities (in the specific case of this exam paper, the normal distribution), and relative inaccuracy in getting the details right.

C. Pass Mark

The Pass Mark for this exam was 60.

Solutions

Q1 (i) The expected utility theorem states that a function, $U(w)$, can be constructed representing an investor's utility of wealth, w, at some future date. [1]

Decisions are made on the basis of maximising the expected value of utility under the investor's particular beliefs about the probability of different outcomes. [1]

(ii) $U'(w) = 1 + 2dw$, and [½]

$U''(w) = 2d$. [½]

Because the investor is risk averse, we must have $U''(w) < 0$ *(alternatively to satisfy the condition of diminishing marginal utility of wealth (risk aversion))* [½]

So we must have $d < 0$. [½]

(iii) The condition of non-satiation requires $U'(w) > 0$. [½]

Hence $1 + 2dw > 0$ and $w < -1/(2d)$ [½]

So the quadratic utility function can only satisfy the condition of non-satiation over a limited range of w:

Specifically $-\infty < w < -1/(2d)$ [1]

(iv) $1{,}000 = -1/2d$ [½]

$\Rightarrow d = -1/2000 = -0.0005$ [½]

(v) $U(250) = 250 - 0.0005 \times 250^2 = 218.75$ [1]

$E[U(\text{exchange})] = 0.5 \times U(600) + 0.5 \times U(0)$ [½]

$= 0.5 \times (600 - 0.0005 \times 600^2) = 210$ [½]

So the investor should not accept the opportunity to exchange… [½]

… because the expected utility of the exchange opportunity is lower than that of the prize. [½]

[Total 10]

Generally well answered. In part (i) many students covered the axioms on which the theory is based rather than the theorem, as asked. In part (ii) some candidates confused non satiation with risk aversion. A significant number of candidates did not know how to reply to part (iv). In part (v) students appeared to have difficulties distinguishing between utility of expected wealth and expected utility of wealth.

Q2 Share prices are always positive, which is consistent with this model. [½]

The increments of share prices are proportional to the share price itself. [½]

However, estimates of σ vary widely according to what time period is considered. [1]

Examination of historic option prices suggests that volatility expectations fluctuate markedly over time. [1]

One way of modelling this behaviour is to take volatility as a process in its own right. This can explain why we have periods of high volatility and periods of low volatility. [1]

One class of models with this feature is known as ARCH: autoregressive conditional heteroscedasticity. [½]

A more contentious area relates to whether the drift parameter μ is constant over time. [½]

There are good theoretical reasons to suppose that μ should vary over time. [½]

For example, if interest rates are high, we might expect the equity drift, μ, to be high as well. [½]

One unsettled empirical question is whether markets are mean reverting, or not. [½]

There appears to be some evidence for this… [½]

… but the evidence rests heavily on the aftermath of a small number of dramatic crashes. [½]

Furthermore, there also appears to be some evidence of momentum effects. [½]

A further strand of empirical research questions the use of the normality assumptions in market returns. [½]

Actual returns tend to have many more extreme events, both on the upside and downside, than is consistent with such a model. [1]

While the random walk produces continuous price paths, jumps or discontinuities seem to be an important feature of real markets. [1]

Furthermore, days with no change, or very small change, also happen more often than the normal distribution suggests. [1]

However, whilst a non-normal distribution can provide an improved description of the actual returns observed, the improved fit to empirical data comes at the cost of losing the tractability of working with normal (and lognormal) distributions. [1]

Market jumps are consistent with the arrival of information in packets rather than continuously. [½]

After a crash, many investors may have lost a significant proportion of their total wealth; it is not irrational for them to be more averse to the risk of losing what remains. [½]

Many orthodox statistical tests are based around assumptions of normal distributions. If we reject normality, we will also have to re-test various hypotheses. In particular, the evidence for time-varying mean and volatility is greatly weakened. [1]

[Max 8]

Standard bookwork question. Overall most of the candidates described some key points worth some marks, but not everyone covered all the necessary points to get full marks. Most students focussed on the appropriateness of the normality assumptions. A few students instead discussed Brownian motion rather than the lognormal model.

Q3 (i) Using Ito's Lemma:

$d\log S_t = 1/S_t\, dS_t - 1/(2S_t^2)(dS_t)^2$ [½]

$= (\mu - \sigma^2/2)dt + \sigma dZ_t$ [1]

Integrating both sides gives

$\log S_t = \log S_0 + (\mu - \sigma^2/2)t + \sigma Z_t$ [½]

$\Rightarrow S_t = S_0 \exp((\mu - \sigma^2/2)t + \sigma Z_t)$ [½]

As Z_t is normal, [1]

then S_t is lognormal [½]

with parameters

$(\mu - \sigma^2/2)t = 0.12875t$ [½]

and $\sigma^2 t = 0.0625t$ [½]

[Max 4]

(ii) To find the initial investment we need the 25th percentile of $\log S_t$ over 6 years, i.e. with parameters $0.12875 \times 6 = 0.7725$ [½]

and $0.0625 \times 6 = 0.375$. [½]

25th percentile is calculated from:

$P((\log S_6 - 0.7725) / \sqrt{(0.375)} < X) = 0.25$ under a normal distribution [1]

$\Leftrightarrow X = -0.6745$ [½]

$\Leftrightarrow \log S_6 = -0.6745\times\sqrt{(0.375)} + 0.7725 = 0.35945$ [½]

$\Leftrightarrow S_6 = 1.4325$ [½]

So:

Initial investment required = £100,000 / 1.4325 = £69,806 [½]

(iii) (a) We need the 10th percentile of $\log S_6$, which is

$P((\log S_6 - 0.7725) / \sqrt{(0.375)} < X) = 0.1$ [1]

$\Leftrightarrow X = -1.2816$ [½]

$\Leftrightarrow \log S_6 = -1.2816\times\sqrt{(0.375)} + 0.7725 = -0.01232$ [½]

$\Leftrightarrow S_6 = 0.98776$ [½]

So the VaR is £100,000 – (£50,000×0.98776) = £50,612 [½]

(b) $E[S_6] = \exp(\mu + \sigma^2/2) = \exp(0.7725 + 0.1875) = 2.61170$ [1]

So expected value = 50,000 × 2.61170 = £130,585 [½]

Hence expected surplus of £30,585.

(iv) The investor has an expected surplus, and therefore expects to repay the loan… [1]

… but there is also a chance of a very large shortfall. [1]

He therefore may wish to change the components of his portfolio, to reduce the risk of not being able to pay off the loan. [1]
[Max 2]

[Note to markers: please award marks for any reasonable point which is consistent with answers in 3(ii) and 3(iii) – even if those results were wrong.]

(v) The investor might move his investments to an asset with a lower expected return but also lower variance. [1]

The investor might decide to diversify his portfolio between a large number of different securities. [1]

The investor might decide to pay off some of the loan early. [1]

The investor might decide to buy an insurance product rather than using securities. [1]

[Max 2]

[Total 17]

> Large variety of answers on this question where well prepared candidates scored full marks and less prepared candidates struggled. The main issues seem to be the parts of questions requiring calculations. In part (iii) most students struggled to calculate the VaR or calculated it at the 10% level rather than 90%, as asked. For both part (iv) and (v) most of the candidates did not manage to give valid points.

Q4 (i) Let f be the value of the derivative, S the price of the underlying, σ its volatility, T maturity of the derivative and t current time.

(a) $\Delta = \frac{\partial f}{\partial s} \equiv \frac{\partial f}{\partial s}(t, S_t)$ [1]

(b) $\nu = \frac{\partial f}{\partial \sigma}$ [1]

(c) Either $\Theta = \frac{\partial f}{\partial t}$ or $= \frac{\partial f}{\partial (T-t)}$ [1]

(d) $\Gamma = \frac{\partial^2 f}{\partial s^2}$ [1]

(ii) Change in value of option = 0.822×(–3) + 0.5 × 0.033 × $(–3)^2$ – 0.855× 1 + 0.104 × (–0.05) = –3.178 [1]

So new value of option = 6.17 – 3.178 = €2.992 [1]

(iii) The delta for a call option is always positive because an increase in the share price makes an option to buy the share for a set price more valuable. So as the share price increases, the call option price also increases, hence the relative change (the delta) is positive. [1]

Similarly, the delta for a put option is always negative because an increase in the share price makes the option to sell the share for a set price less valuable. So as the share price increases, the put option price reduces, hence the relative change (the delta) is negative. [1]

(iv) The more volatile an asset is, the more valuable the choice offered by an option. [1]

[Total 9]

Generally well answered by most candidates (especially part (i)). However some candidates failed to calculate the correct figures in part (ii). In part (iii) the majority of candidates assumed that Black Scholes applied which is why full marks were not awarded.

Q5 (i)

Stock tree

time	*0*	*1*	*2*	*3*
	85.00	100.30	118.35	139.66
		72.25	85.26	100.60
			61.41	72.47
				52.20

The price C_0 of the option is computed via risk-neutral valuation; let $\hat{p}$ denote the risk-neutral probability of an up movement, then:

$$\hat{p} = \frac{e^{0.0025} - 0.85}{1.18 - 0.85} = 0.4621$$ [1]

And

$$C_0 = e^{-rT} \sum_{k=0}^{3} \binom{3}{k} \hat{p}^k \left(1-\hat{p}\right)^{3-k} \left(K - S_0 u^k d^{3-k}\right)^+$$ [2]

$$= e^{-0.0075}\left(17.53 \times 3\hat{p}\left(1-\hat{p}\right)^2 + 37.80 \times \left(1-\hat{p}\right)^3\right) = 12.82$$ [1]

[Max 4]

The detailed working are provided below – in case attempts to answer this question go through the whole tree [and it carries same marks as above].

PUT

time	*0*	*1*	*2*	*3*
	12.82	5.05	0.00	0.00
		19.55	9.41	0.00
			28.36	17.53
				37.80

(ii) Risk-neutral probability that the put option expires out-of-the-money = $P(S_3 > K)$ [1]

Hence $P(S_3 > K) = 3\hat{p}^2(1-\hat{p}) + \hat{p}^3 = 0.4433$ [1]

(iii) Under the risk-neutral probability the expected rate of return on the stock is the risk-free rate of return, i.e. 0.25% per month. [1]

Under the real-world probability measure instead, the stock is expected to earn a much higher rate of return… [1]

… in order to justify the higher risk it carries compared to the risk-free bond [1]

Consequently we would expect the probability of the put option to expire out-of-the-money to be higher than 0.4433. [1]

[Max 3]

[Total 9]

Generally well answered, although not as well answered as binomial tree questions in past exams. A common mistake was failing to appreciate the number of possible combinations comprising the probability in part (ii). There were a lot of numerical slips; candidates struggled with the reasoning required for part (iii).

Q6 (i) The SDE of $\tilde{S}_t = e^{-rt}S_t$ is:

$$d\tilde{S}_t = (\mu - r - \lambda\sigma)\tilde{S}_t dt + \sigma\tilde{S}_t d\hat{W}_t,$$ [1]

For the martingale property to hold, set the drift to zero [1]

which implies $\lambda = {(\mu - r)}\big/{\sigma}$. [1]

(ii) By substitution of the value of λ in the given SDE we obtain

$dS_t = rS_t dt + \sigma S_t dW_t,$ [1]

In other words the expected rate of return on the stock is given by the risk-free rate of interest [1]

Hence the probability measure P^* is risk-neutral [1]

[Total 6]

> Large variety of answers on this question: only well prepared candidates scored well – some candidates did not answer. In particular, the main difficulty was recognising that for the martingale property to hold, the drift needed to be zero.

Q7 (i) Data: $S_0 = 7, K = 6.50, \sigma = 40\%$ p.a., $T = 1$ year, $r = 5\%$.

The Black-Scholes formula returns:

$d1 = 0.5103$ [½]

$d2 = 0.1103$ [½]

$N(d1) = 0.6951$ [½]

$N(d2) = 0.5439$ [½]

So $C_0 = 7 \times 0.6951 - 6.50e^{-0.05} \times 0.5439$ [1]

$= 1.50$ [1]

(ii) 60% returns a call price of 1.995, following the same calculations as in part (i)
$d1 = 0.5068$, [½]
$d2 = -0.0932$, [½]
$N(d1) = 0.6939$, [1]
$N(d2) = 0.4629$ [1]

We need to check that the volatility is closer to 60% than 61%, so we calculate the call price with a volatility of 60.5%. We need this price to be larger than 2. [1]

60.5% returns a call price of 2.0073
$d1 = 0.5076$, [½]
$d2 = -0.0974$, [½]
$N(d1) = 0.6941$, [1]
$N(d2) = 0.4612$ [1]

Therefore, since the 60% result is smaller than 2 and the 60.5% result is larger than 2, the implied volatility is 60% to the nearest 1%. [1]

[Max 6]

[Total 10]

Some struggled to get the d1 and d2 terms of the Black-Scholes formula correctly. Many candidates just verified 60% as the volatility instead of proving that this was the closest volatility to within 1%. Any meaningful values used by candidates for verification purposed has been awarded accordingly

Q8 (i) The model should be arbitrage-free. [½]

Interest rates should ideally be positive. [½]

Interest rates should exhibit some element of mean reversion. [½]

The model should be computationally tractable / produce simple formulae for bond and option prices. [½]

It should produce realistic dynamics. [½]

It should give a full range of possible yield curves. [½]

It should fit historical data. [½]

Can be calibrated easily to current market data. [½]

Flexible to cope with a range of derivatives. [½]

[Max 3]

(ii) Both strategies pay a value of 1 at time T [1]

By the no arbitrage principle… [1]

… if they have the same value at time T then they must have the same value at time t [1]

Hence $B(t,T) = \alpha\ B(t,S)$ [1]

and so the requested relationship follows, with $\alpha = e^{-F(t,S,T)(T-S)}$. [½]

[Max 4]

(iii) From the expression in part (ii), it follows that:

$$F(t,S,T) = -\frac{\log B(t,T) - \log B(t,S)}{T-S}. \quad [1]$$

The instantaneous forward rate is defined as $f(t,T) = \lim_{S \to T} F(t,S,T)$, [½]

i.e.

$$f(t,T) = -\lim_{S \to T} \frac{\log B(t,T) - \log B(t,S)}{T-S} = -\frac{\partial \log B(t,T)}{\partial T}. \quad [1]$$

Solving the last equality with respect to the ZCB price, we obtain:

$$B(t,T) = e^{-\int_t^T f(t,s)ds} \quad [½]$$

[Total 10]

> Large variety of answers and only well prepared candidates scored well. In part (i) many candidates failed to list all the key points; in part (ii) a number of candidates failed to use the given portfolio to obtain the required result and, instead, used the result they had to show as given. In part (iii) candidates struggled to recognise that they needed to define the instantaneous forward rate.

Q9 (i) $R_i = a_i + b_{i,1} I_1 + b_{i,2} I_2 + c_i$ [½]

where a_i and c_i are the constant and random parts respectively of the component of the return unique to security *i* [½]

I_1, I_2 are the changes in a set of the two indices [½]

$b_{i,k}$ is the sensitivity (factor beta) of security *i* to factor *k* [½]

(ii) **Macroeconomic factor models**

These use observable economic time series as the factors, such as the annual rates of inflation and economic growth, short term interest rates, the yields on long term government bonds, and the yield margin on corporate bonds over government bonds. [1]

Fundamental factor models

These use company specific variables as the factors, e.g. the level of gearing, the price earnings ratio, the level of research and development spending, the industry group to which the company belongs. [1]

Statistical factor models

These do not rely on specifying the factors independently of the historical returns data. Instead a technique called principal components analysis can be used to determine a set of indices which explain as much as possible of the observed variance. [1]

[Total 5]

Well answered by most candidates. Generally candidates answered part (i) correctly with some candidates failing to define correctly c_i and some failing to write down the equation correctly. Most of the candidates obtained marks in part (ii), with some failing to give a proper definition of statistical models and some failing to name the three main categories of multifactor models.

Q10 (i) The market portfolio is the weighted portfolio of the risky securities in the market, consequently

$$Er_M = 18\% = w_A Er_A + w_B Er_B$$ [1]

As $w_A + w_B = 0.5$, then $w_A = w_B = 0.5$. [1]

(ii) From the security market line

$$\beta_i = \frac{Er_i - r_f}{Er_M - r_f}$$

Therefore $\beta_A = 1.2$ and $\beta_B = 0.8$. [1 each]

(iii) Empirical studies do not provide strong support for the model. [½]

The underlying assumptions are not realistic. [½]

Investors cannot necessarily borrow or lend unlimited amounts at the same risk-free rate. [½]

The markets for risk assets may not be perfect. [½]

Investors may not have the same estimates of expected returns, standard deviations and covariances of securities. There are basic problems in testing the model since, in theory, account has to be taken of the entire investment universe open to investors, not just capital markets. [½]

It does not account for taxes. [½]

It does not account for inflation. [Or: some investors may measure in real terms and some in money terms.] [½]

It does not account for situations in which there is no riskless asset. [½]

The basic model does not allow for currency risk. [Or: investors may not measure in the same currency.] [½]

It does not consider multiple time periods. [Or: investors do not all have the same one-period time horizon.] [½]

It does not consider optimisation of consumption over time. [½]
[Max 3]
[Total 7]

Well answered by most candidates. However, in part (ii) there was some evidence of students not being able to calculate the market portfolio correctly instead calculating the minimum-variance portfolio under Mean-Variance Portfolio Theory.

Q11 (i) **Structural models** [½]

Structural models aim to link default events explicitly to the fortunes of the issuing corporate entity. [1]

An example of a structural model is the Merton model. [½]

Reduced form models [½]

Reduced form models use observed market statistics rather than specific data relating to the issuing corporate entity. [1]

The market statistics most commonly used are the credit ratings… [½]

… issued by credit rating agencies such as Standard and Poor's and Moody's. [½]

The output of such models is a distribution of the time to default. [½]

Intensity-based models [½]

An intensity-based model is a particular type of continuous-time reduced form model. [1]

It typically models the "jumps" between different states,… [½]
… which are usually credit ratings,… [½]
… using transition intensities. [½]
[Max 5]

(ii) Default occurs if the value of the assets is not enough to cover the face value of the debt at maturity

Or alternatively: F(T) < L. [1]

(iii) $F(t)$ follows a geometric Brownian motion (or continuous time lognormal model). [1]

(iv) Hence, by risk-neutral valuation, the Merton model-based probability of default is:

$$P\left(F\left(T\right)<L\right)=1-N\left(\frac{\ln\frac{F\left(0\right)}{L}+\left(r-q-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right)$$

where $P(.)$ is the probability under the risk-neutral measure, $F(0)$ is the current value of the firm, σ is its volatility, r is the risk-free rate and q denotes any potential payout cashflow. [2]
[Total 9]

Candidates familiar with the study material scored well. Part (i) was often answered correctly although some candidates failed to describe properly the main approaches and some mixed the names of the approaches. In part (iii) many candidates missed the "*q*".

END OF EXAMINERS' REPORT