EXAMINATION

13 April 2007 (am)

Subject CT8 — Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 9 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

1 (i) Derive an expression for the theta of an option under the Black-Scholes model involving delta and gamma. [4]

(ii) Explain why a deep out of the money call option in the Black-Scholes world will experience a rate of return close to the risk-free rate of return. [2]
[Total 6]

2 Consider a set of risky assets in a mean-variance framework where:

V_i = variance of the return for asset i
C_{ij} = covariance between the returns of assets i and j where $i \neq j$

(i) Derive an expression for the variance of a portfolio of N such assets where x_i is the relative weight of asset i in the portfolio. Assume that the weights sum to unity and that short selling is prohibited. [3]

(ii) Show that the variance of the returns of a very large portfolio of equally weighted allocations to the assets depends mainly on the average covariance between the asset returns. [5]
[Total 8]

3 Consider a two period recombining binomial model for S_t, the price of a non-dividend paying security at times $t = 0$, 1 and 2, with real world dynamics:

$$\begin{aligned} S_{t+1} &= S_t\, u \quad \text{with probability } p \\ &= S_t\, d \quad \text{with probability } 1-p \end{aligned}$$

and $u > d > 0$.

There also exists a risk-free instrument that offers a continuously compounded rate of return of 5% per period.

The state price deflator in this model after one period is:

$$\begin{aligned} A_1 &= 0.7610 \quad \text{when } S_1 = S_0 u \\ &= 1.5220 \quad \text{when } S_1 = S_0 d \end{aligned}$$

The price of a derivative at time 0 that pays 1 at time 2 if $S_2 < S_0$ is 0.1448.

(i) Calculate the value of p. [5]

(ii) Calculate the risk-neutral probability measure. [2]

(iii) Calculate the price at time 0 of a derivative that pays 1 at time 2 if $S_2 > S_0$ using the risk-neutral probability measure derived in (ii). [2]
[Total 9]

4 An investor is contemplating an investment with a payoff of R, where R has the probability density function f, given by:

$$f(t) = 0 : t < 0.5$$
$$f(t) = c/t^4 : t \geq 0.5,$$

for $c = 0.375$. All amounts are in units of £ million.

Calculate the following two measures of risk for the net return when the cost of the investment is 0.7:

(a) downside semi-variance of return
(b) Value at Risk at the 5% level

[9]

5 Consider a single period multifactor model of security returns where:

$$R_i = \alpha_i + \sum_{j=1}^{K} \beta_{ij} I_j + \varepsilon_i$$

where:

R_i = return on security i

α_i, β_{ij} are security specific parameters

ε_i = cross-sectionally independent random component of return that is also independent of all I_j

I_j = cross-sectionally independent rate of change in factor j

(i) Derive an expression for the covariance between the returns of two securities in terms of the statistical properties of the factors using the model above. [4]

(ii) Explain the implications of your expression in (i) for constructing a diversified portfolio. [3]

(iii) Explain how the multifactor model above can be used to form an asset pricing theory when combined with the principle of no-arbitrage. [5]
[Total 12]

6 (i) State the SDEs under the risk-neutral measure for $r(t)$, the default-free instantaneous rate of interest at time t, under the following two models, defining all notation used:

(a) Hull & White
(b) 2-factor Vasicek

[4]

(ii) State the advantages of the Hull & White model over the single factor Vasicek model. [4]

(iii) Explain the limitations of using a model with only one factor, taking into account both theoretical and empirical considerations. [4]
[Total 12]

7 Consider a special type of scheme that has an obligation to pay £10,000 in exactly one year. There is currently £9,000 in the fund created to meet this obligation and it is invested in the shares of an infinitely divisible non-dividend paying security with price S_t governed by the SDE:

$$dS_t = S_t\left(\mu\, dt + \sigma\, dZ_t\right)$$

where:

Z_t is a standard Brownian motion
$\mu = 0.10$
$\sigma = 0.20$
t is the time since the start of the year
$S_0 = 1$
a risk free rate of return of 5% p.a. compounded continuously is available.

(i) Derive the distribution of S_1. [4]

(ii) Calculate the following risk measures applied to the surplus of the scheme where the surplus of the scheme is defined as the difference between the value of the fund and the obligation at the end of the year:

(a) variance
(b) shortfall probability relative to a benchmark surplus of £0

[6]

(iii) Calculate the cost of a put option to protect against the surplus being negative at the end of the year. [3]
[Total 13]

8 (i) Explain the difference between an efficient market and an arbitrage-free market. [4]

Empirical investigations of stock market returns have revealed a fractal dimension of 1.4.

(ii) Explain what this means about the distribution of returns. [2]

(iii) Explain how mean-reversion in the stock market can be consistent with an efficient market. [2]

(iv) Outline the claim and test of excessive volatility in stock markets made by Shiller, along with four criticisms made of the test. [7]

[Total 15]

9 (i) Describe three types of credit risk model. [6]

Assume that a company has fixed debt of £40m with term 10 years, the value of the equity in the company is £20m and the Merton model for credit risk holds true. The risk free rate of return is 5% p.a. and there are no other dividends or interest payments.

(ii) Explain how to calculate the (risk neutral) probability of default. You do not have to calculate the probability, but should state how each value would be calculated. [6]

In a particular two state model for credit rating with deterministic transition intensity, the risk free rate is a constant, r, the recovery rate is δ and the zero coupon bond price is given by:

$$B(t,T) = e^{-r(T-t)}\left[1-(1-\delta)\left(1-e^{-\frac{(T^2-t^2)}{4}}\right)\right].$$

(iii) (a) State the general formula for the zero coupon bond prices in a two state model for credit ratings.

(b) Deduce the risk-neutral default intensity for the particular two state model above.

[4]

[Total 16]

END OF PAPER