INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

25 September 2013 (pm)

Subject CT8 – Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 (i) (a) State the expected utility theorem.
(b) State the four axioms from which it can be derived.
[5]

(ii) Explain of the concepts of non-satiation and risk aversion, showing how they can be expressed in terms of a utility function. [2]

A quadratic utility function is given by the equation $U(w) = w + bw^2$. The value of absolute risk aversion at a value of wealth of one unit is 0.25

(iii) Calculate the value of b and the range over which $U(.)$ satisfies the condition of non-satiation. [3]
[Total 10]

2 (i) Describe the single-index model of security returns, defining any terms used.
[2]

The single-index model is to be used in a particular market.

(ii) Determine the following results:

(a) the expected return on a security
(b) the variance of returns on a security; and
(c) the covariance of returns between two securities

in the market, using the parameters described in part (i). [3]

(iii) Show that investors can diversify away specific risk in this model by holding equal weights in an increasing number of securities. [4]

(iv) State the potential impact of adding additional indices to the model:

(a) in terms of explaining historic data.
(b) in terms of forecasting security returns.
[2]
[Total 11]

3 (i) Outline the three forms of the Efficient Markets Hypothesis (EMH). [3]

(ii) Discuss the following two scenarios in the light of the EMH:

Scenario 1: Company A’s share price falls suddenly, immediately after news of an earthquake in the capital city of one of its major markets.

Scenario 2: Company B’s share price falls suddenly, when a long-awaited and publicly negotiated merger is completed.
[3]
[Total 6]

4 In the Wilkie model, the force of inflation in year t, $I(t)$, is modelled as an AR(1) process as follows:

$$I(t) = m + a(I(t-1) - m) + Z(t),$$

where, for any t, all $Z(t)$ are independent, identically distributed $N(0, \sigma^2)$ random variables.

Let $m = 0.03$; $a = 0.6$ and $\sigma^2 = 2.5 \times 10^{-5}$. The force of inflation was 3.3% in 2012.

(i) Calculate a 95% confidence interval for the force of inflation in 2013. [3]

A final salary pension fund is assuming inflation of 1% p.a. for the period 1 January 2012 to 31 December 2013.

(ii) Comment on the appropriateness of this assumption assuming that the Wilkie model is correct. [2]

[Total 5]

5 The share price in Santa Insurance Co, S_t, is currently 97p and can be modelled by the stochastic differential equation:

$$dS_t = 0.4 S_t dt + 0.5 S_t dB_t$$

where B_t is a standard Brownian motion.

(i) (a) Determine $d\log S_t$, using Ito's Lemma.

(b) Calculate the expectation and variance of the Santa Insurance Co share price in two years' time. [6]

The share price in Rudolf Financial Services plc, R_t, is also currently at 97p and can be modelled by the stochastic differential equation:

$$dR_t = -0.4 R_t dt + 0.5 dB_t$$

Let $U_t = e^{0.4t} R_t$

(ii) (a) Calculate dU_t.

(b) Calculate the expectation and variance of the Rudolf Financial Services plc share price in two years' time. [6]

[Total 12]

6 A non-dividend-paying stock has a current price of 300p. Over each of the next two three-month periods its price will either go up by 30p or down by 30p. Price movements for each period are independent of each other. An investment in a cash account returns 2% per quarter. A European call option on the stock pays out in six months based on a strike price of 290p. The price of the stock is to be modelled using a binomial tree approach with three-month time steps.

(i) Calculate the value of the call option today using a risk-neutral pricing approach. [3]

Assume that the real world probability of the stock price moving up in each of the next three month periods is 0.7

(ii) (a) Calculate the values of the state price deflator after six months

(b) Calculate and the value of the call option today using your answers to part (ii)(a).

(c) Compare this to your answer to part (i). [5]

Assume that the real world probability has now dropped from 0.7 to 0.6.

(iii) (a) Explain, without performing any further calculations, how the state price deflator would change in value.

(b) Comment on the impact that this would have on the option price. [2]

[Total 10]

7 The continuously compounded risk-free rate of interest is r, and a stock, with maturity T, pays dividends continuously at rate q.

(i) Determine the forward price at time 0 for a forward contract on the stock. [3]

(ii) Show that there exists a portfolio that earns the risk free rate r, containing:

- the stock
- a European call option on the stock
- and a European put option on the stock [4]

[Total 7]

8 (i) Write down a stochastic differential equation for the short rate r in the Vasicek model defining any notation used. [1]

(ii) List the desirable and undesirable features of this model for the term structure of interest rates. [4]

(iii) (a) Solve the stochastic differential equation from your answer to part (i).

(b) Comment on the statistical properties of r_T, $T > t$.

[7]

[Total 12]

9 A one-year European call option on a non-dividend paying stock in Company ABC has a strike of \$150.

The continuously compounded risk-free rate is 2% p.a. The current stock price is \$117.98. Assume that the market follows the assumptions of a Black-Scholes model.

An institutional investor holds a delta-hedged portfolio with 100,000 call options, no cash and short 18,673 shares of Company ABC.

(i) Calculate the delta of the call option. [2]

(ii) Calculate the implied volatility for the underlying. [4]

(iii) Calculate the price of a one-year put on the same stock with a strike of \$150. [2]

The investor retains their holding of call options and trades in the put and the stock to achieve a delta and gamma-hedged portfolio.

(iv) Calculate the investor's new holdings of the put and the stock. [4]
[Total 12]

10 (i) Describe the Merton model for pricing a bond subject to default risk. [4]

A very highly geared company – XYZ plc – has issued zero-coupon bonds payable in four years' time. The debt is a nominal \$120m.

(ii) Give expressions for the value of the debt in four years' time and today, adopting a Black-Scholes model for the value of XYZ plc. [4]

The current gross value of XYZ plc is \$180m. The continuously compounded risk-free interest rate is 2% p.a. and the continuously compounded credit spread on the bond is 4.5% p.a.

(iii) Calculate the price of the bond today. [1]

(iv) Estimate to the nearest 1% the implied volatility of the value of XYZ. [3]

(v) Determine the implied risk-neutral probability of default. [3]
[Total 15]

END OF PAPER