EXAMINATION

September 2005

Subject CT8 — Financial Economics
Core Technical

EXAMINERS' REPORT

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

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Chairman of the Board of Examiners

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1 (i) $\text{Var}(R) = 500{,}000^2 \, \text{Var}(U) = 2.5 \times 10^{11} \times 1/12 = 2.08333 \times 10^{10}$

(ii) Downside semi-variance of $R = 2.5 \times 10^{11} \times$ upside semi-variance of U; the upside semi-variance of U is by symmetry 1/24 so downside semi-variance of R is 1.04166×10^{10}.

(iii) $P(R < 100{,}000) = P(U > 0.4) = 0.6$

(iv) If $\text{VaR}_{5\%}(R) = t$ then $P(R \leq -t) = 0.05$, so

$$P(300{,}000 - 500{,}000U \leq -t) = P(U > 0.6 + (t / 500{,}000)) = 5\%,$$

hence (since $P(U > x) = 1 - x$), $0.4 - (t / 500{,}000)) = 0.05$, so

$$t = 500{,}000\,(0.35) = 175{,}000.$$

2 (i) The market portfolio is (2/7, 3/7, 2/7), so

$$R_M = (2R_A + 3R_B + 2R_C) / 7.$$

Thus

$$\text{Cov}(R_i, R_M) = [2\,\text{Cov}(R_i, R_A) + 3\text{Cov}(R_i, R_B) + 2\,\text{Cov}(R_i, R_C)] / 7.$$

So,

$$\text{Cov}(R_A, R_M) = [.32 + .12 + .04] / 7 = .06857$$

$$\text{Cov}(R_B, R_M) = 0.22/7 = .03143,$$

and

$$\text{Cov}(R_C, R_M) = .09/7 = .01286,$$

and

$$\sigma_M^2 = [2\,\text{Cov}(R_M, R_A) + 3\,\text{Cov}(R_M, R_B) + 2\,\text{Cov}(R_M, R_C)] / 7 = .03674.$$

We conclude that $\beta_A = 1.8664$, $\beta_B = 0.8555$ and $\beta_C = 0.3500$.

Finally, solving

$$r_i - r_0 = \beta_i(r_M - r_0), \text{ we get } r_A = 0.4,\ r_B = 0.2 \text{ and } r_C = 0.1.$$

(ii) The corresponding single index model is

$$R_i = (1 - \beta_i)\, r_0 + \beta_i\, R_M + \varepsilon_i$$

where the ε_i s are uncorrelated with each other and with R_M, and ε_i has variance equal to

$$\mathrm{Var}(R_i) - \beta_i^2 \sigma_M^2 \,,$$

so that, setting

$$\mathrm{Var}(\varepsilon_i) = \sigma_i^2,\ \ \sigma_A^2 = 0.0320,\ \sigma_B^2 = 0.0131 \text{ and } \sigma_C^2 = 0.0055.$$

(iii) The single index model is not the same as the CAPM model because the covariances of asset returns are different in the two models: in the single index model

$$\mathrm{Cov}(R_i, R_j) = \beta_i\, \beta_j\, \sigma_M^2 \,,$$

so for example we would obtain

$$\mathrm{Cov}(R_A, R_B) = 0.0587,$$

whereas in the CAPM model

$$\mathrm{Cov}(R_A, R_B) = 0.04.$$

3 (i) Delta: the rate of change in derivative price with respect to change in the price of underlying asset.

Gamma: the rate of change of delta with respect to change in the price of underlying asset.

Theta: the rate of change in the value of the derivative with respect to change in time to expiration.

lambda: the rate of change in the value of the derivative with respect to change in the assumed continuous dividend yield on the underlying asset.

rho: the rate of change in the value of the derivative with respect to change in the risk-free rate of interest.

vega: the rate of change in the value of the derivative with respect to the (assumed) volatility of the underlying asset.

(ii) Assuming that the portfolio under management is delta hedged at discrete times, the two most important Greeks are gamma and vega. Between rebalancing at the trading times, delta will drift away from zero as the underlying asset prices move. If the portfolio is gamma-hedged at the discrete trading times then the amount of such drift will be small (comparable to the square of the change in underlying price).

The underlying volatilities used in hedging calculations are all estimates. If these are incorrect then delta hedging may be incorrect, consequently it is appropriate to attempt to immunise a portfolio against (small) errors in volatility estimates. Just as in delta hedging, achieving a portfolio vega of zero achieves this. Consequently, good risk managers will seek to achieve a (close to) zero vega for the bank's portfolio.

4 (i) No frictions; short-selling permitted; small investor (i.e. does not "move the market"); market is arbitrage-free; stock price is given by

$$dS_t = \mu_t S_t \, dt + \sigma S_t \, dZ_t$$

where Z is a standard Brownian motion.

All are, in some sense, implausible. Friction (spreads and commission) is present; short-selling is available but on very different terms; "small investor" not true for an investment bank; stock-market returns are not compatible with normality (fat tails, jumps); arbitrages occur (for short periods).

(ii) Let N be the number of options written, then $N\Phi(d_1) = 250{,}000$. Now the value of the bank's portfolio is

$$1.8\, N\Phi(d_1) - 2e^{-.015}\, N\Phi(d_2) = 1.8 \times 250{,}000 - 413{,}057 = 36{,}943$$

(iii) So $\Phi(d_2) / \Phi(d_1) = 413{,}057 / (250{,}000 \times 2e^{-.015}) = 0.8386$.

With $\sigma = 10\%$: $d_1 = -1.2425$, $d_2 = -1.3132$, $\Phi(d_1) = .1070$, $\Phi(d_2) = .0946$

so $\Phi(d_2) / \Phi(d_1) = 0.8841$

With $\sigma = 30\%$: $d_1 = -.3199$, $d_2 = -.5320$, $\Phi(d_1) = .3745$, $\Phi(d_2) = .2974$

so $\Phi(d_2) / \Phi(d_1) = 0.7941$

Linear interpolation gives an estimate of

$$10 + 20(.8841 - .8386) / (.8841 - .7941) = 20.1\%$$

for the implied volatility.

(iv) Consequently,

$$N = 250{,}000 / \Phi(d_1) = 874126 \text{ contracts.}$$

5 (i) Consider an investment of x in the stock and y in cash at time t: the value of the holding at time $t + 1$ is

$$x\beta + y(1 + r),$$

if there is an up-jump and is

$$x\alpha + y(1 + r),$$

if there is an down-jump. The value (with $t = 0$) is supposed to be b in the first case and a in the second. So, we need to solve:

$$x\beta + y(1 + r) = b, (1)$$

$$x\alpha + y(1 + r) = a, (2).$$

Subtracting (2) from (1) we get:

$$x = (b - a) / (\beta - \alpha) \text{ and}$$

$$y = (a\beta - b\alpha) / (\beta - \alpha)(1 + r)$$

(ii) $x + y = X_t = [qb + (1 - q)a](1 + r)^{-1}$

$\Rightarrow q(b - a) + a = (x + y)(1 + r)$
$\Rightarrow q = [(x+ y)(1+ r) - a] / (b - a)$ (3)

(iii) Using (3), we see from the first derivative that $q = (10/a) - 1$, while, from the second we see that $q = (c_2/2a) - 1/2$, so we deduce that $c_2 = 20 - a$.

6 (i) Consider a portfolio which is, initially, short one forward contract, holds 1 share and is short c in cash. At the delivery date for the forward contract, the portfolio contains 1 share and is short ce^{rt}, where r is the risk free rate and t is the duration of the contract.

So, immediately after delivery the portfolio contains zero shares and is short $ce^{rt} - p$ in cash, where p is the forward price.

Setting $c = pe^{-rt}$, the portfolio contains nothing. It follows that the portfolio should have a zero set-up cost so, so $p = S_0e^{rt}$.

(ii) Consider a portfolio which is, initially, short one forward contract, holds s shares and is short c in cash. At each dividend date the dividend is used to buy more shares. At the delivery date for the forward contract, the portfolio contains 1.03^4s shares and is short ce^{rt}, where r is the risk free rate and t is the duration of the contract.

So, immediately after delivery the portfolio contains $1.03^4s - 1$ shares and is short $ce^{rt} - p$ in cash, where p is the forward price.

Setting $s = 1 / 1.03^4$, and $c = pe^{-rt}$, the portfolio contains nothing. It follows that the portfolio should have a zero set-up cost so $0 = c - 10s$, so $p = 10se^{rt} = £9.98$.

7 (i) We expect a strong negative correlation.

Dividend yield = dividend/price so if there is a strong price rise it's likely to be accompanied by a decrease in yield.

(ii) Mean-reverting: this is in line with historical evidence in most markets.
Non-negative: dividend yield cannot be negative.

(iii) Not mean-reverting: consistent with weak form EMH. Empirical evidence is mixed.

Constant volatility: this is inconsistent with empirical evidence

Normal distribution: markets jump and returns have fat tails, so inconsistent with empirical evidence.

8 (i) $dr_t = \alpha(\mu - r_t)\, dt + \sigma dZ_t$

$f(X_t, t) = e^{a(T-t)-X_t}$

Now $X_t = b(T-t)r_t + \int_0^t r_s ds$

$$\Rightarrow dX_t = b'(T-t)r_t dt + b(T-t)dr_t + r_t dt$$

$$= -e^{-\alpha(T-t)}r_t dt + \left[1 - e^{-\alpha(T-t)}\right](\mu - r_t)dt$$

$$+ r_t dt + \left[1 - e^{-\alpha(r-t)}\right]\frac{\sigma}{\alpha}dZ_t$$

$$= \left\{\mu\left(1 - e^{-\alpha(T-t)}\right)\right\}dt + \left\{\frac{\sigma}{\alpha}\left(1 - e^{-\alpha(T-t)}\right)\right\}dZ_t$$

$$= A_t dt + B_t dZ_t$$

Using Ito

$$df(X_t, t) = \frac{\partial f}{\partial x}B_t dZ_t + \left[\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}A_t + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}B_t^2\right]dt$$

$$= -f(X_t, t)B_t dZ_t - f(X_t, t)\, a'(T-t)dt$$

$$- f(X_t, t)A_t dt + \frac{1}{2}f(X_t, t)B_t^2 dt$$

$$= f(X_t, t)\left\{\left[a'(T-t) - A_t + \frac{1}{2}B_t^2\right]dt - B_t dZ_t\right\}$$

(ii) (a) To be a martingale the […] term in (i) must be zero:

$$a'(T-t) = A_t - \frac{1}{2}B_t^2$$

$$= \alpha\mu b(T-t) - \frac{1}{2}\sigma^2 b^2(T-t)$$

(b) $B(T,T) = 1 \Rightarrow e^{a(0)-r_T b(0)} = 1$

$$\Rightarrow a(0) = 0$$

9 (i) This means short term interest rates would in the long term increase if $\mu > 0$ geometrically. This is not desirable as it does not reflect reality.
The model also has the following properties:

The change in rate is dependent on the current rate. This is undesirable as typical rates mean revert.

The model requires constant volatility over time. This is not desirable as volatility of short term interest rates changes over time.

(ii) (a) Incorporates mean reversion.
Arbitrage free.
Allows negative interest rates.

(b) Incorporates mean reversion.
Arbitrage free.
Volatility high/low when rates high/low.
Does not allow negative interest rates.
More difficult to implement than Vasicek model

10 (i) **Over-reaction tests**

- past winners tend to be future losers (or vice versa)

- certain accounting ratios appear to have predictive power (e.g. BV/MV or E/P)

- IPOs and other new offerings have poor subsequent performance

Under-reaction

- stock prices react slowly to earnings announcements
- abnormal excess returns for parent/subsidiary following a demerger
- abnormal negative returns following mergers

(ii) Over-reaction or under-reaction to the arrival of public information would appear to contradict
the semi-strong form of the EMH since excess returns could be earned.

However, some of the tests (such as accounting ratios) may not allow properly for risk and the results are therefore not incompatible with the EMH.

Many of these tests appear to be time-period specific.

(iii) Assume that investors are non satiated (always prefer more expected return to less) and risk averse (in the sense of wanting to avoid volatility of returns).

An efficient portfolio is one with the highest expected return for a given level of volatility **and** the lowest volatility for the expected return.

END OF EXAMINERS' REPORT