EXAMINATION

13 September 2006 (am)

Subject CT8 — Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 8 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

1 An investor is contemplating an investment with a return of £R, where:

$$R = 250{,}000 - 100{,}000N,$$

and N is a Normal [1, 1] random variable.

Calculate each of the following four measures of risk:

(a) variance of return
(b) downside semi-variance of return
(c) shortfall probability, where the shortfall level is £50,000
(d) Value at Risk at the 5% level

[8]

2 A non-dividend-paying stock has a current price of 800p. In any unit of time $(t, t+1)$ the price of the stock either increases by 25% or decreases by 20%. £1 held in cash between times t and $t+1$ receives interest to become £1.04 at time $t+1$. The stock price after t time units is denoted by S_t.

(i) Calculate the risk-neutral probability measure for the model. [4]

(ii) Calculate the price (at $t = 0$) of a derivative contract written on the stock with expiry date $t = 2$ which pays 1,000p if and only if S_2 is not 800p (and otherwise pays 0). [4]

[Total 8]

3 (i) Explain what is meant by "self-financing" in the context of continuous-time derivative pricing, defining all notation used. [4]

(ii) Define the delta of a derivative, defining all notation and terms used other than those already defined in your answer to (i). [2]

(iii) Explain, in general terms, how delta and self-financing are used in the martingale approach to valuing derivatives. [5]

[Total 11]

4 The Wilkie model has been used to produce stochastic simulations of inflation rates.

The following runs were made:

- 1,000 simulations of one-year
- one simulation of 1,000 years

and the standard deviations were calculated.

(i) Explain why you would expect the standard deviations calculated in each run to be different. [4]

(ii) State the conditions under which the standard deviations in the two runs would be expected to be the same. [2]

(iii) Discuss the advantages and disadvantages of using economic theory rather than statistical models to construct and calibrate a stochastic model. [5]

[Total 11]

5 (i) List five desirable characteristics of a model for the term structure of interest rates. [5]

(ii) State the Stochastic Differential Equation satisfied by the short rate in the Vasicek model for the term structure of interest rates. [1]

(iii) Comment on the appropriateness of the Vasicek model in the light of your answer to part (i). [5]

[Total 11]

6 An investor can invest in only two assets with the following characteristics (annualised):

Asset	*Expected rate of return*	*Standard deviation*
A	10%	20%
B	5%	0%

(i) Show that the efficient frontier for the investor is a straight line passing through the points (0, 0.05) and (0.1, 0.075) in (standard deviation, expected return) space. [5]

A third security C becomes available to the investor. It has an annualised expected return of 6% and an annualised standard deviation of 10%. It is uncorrelated with A and B.

(ii) Determine the portfolio using only A and C that maximises:

$$\frac{\text{expected return} - 5\%}{\text{standard deviation}}.$$ [6]

(iii) Using (ii), or otherwise, show that the new efficient frontier using A, B and C passes through the point (0.1, 0.0769). [6]

[Total 17]

7 Consider the following two-factor model of security returns:

$$R_i = \alpha_i + \beta_{i1}I_1 + \beta_{i2}I_2 + \varepsilon_i$$

where:

- R_i = return on security i
- α_i, β_{i1}, β_{i2} are security-specific parameters
- I_1 and I_2 are the changes in the 2 factors on which the model is based
- ε_i is an independent random normal variate with variance σ_i^2.

(i) Describe briefly three categories of model that could help in choosing the factors, I_1 and I_2. [6]

Suppose the factors I_1 and I_2 are chosen to be total return indices with I_1 based on the whole market and I_2 based on the 50 stocks with the highest dividend yield.

(ii) Explain in detail how the two factors can be transformed into two orthogonal factors, one of which is the same as the index on which I_1 is based. [3]

(iii) Derive an expression for the variance of the returns on the security in terms of the variances of the changes of the orthogonal factors and σ_i^2. [3]

(iv) Explain in words the expression in (iii). [2]

[Total 14]

8 (i) State the SDE of a non-dividend paying stock price in the Black-Scholes model, under the EMM defining all symbols used. [2]

(ii) Give the general formula for the price of a derivative security which has a terminal value of C at time T. [2]

(iii) A special option on a share pays £1 at time T if (and only if) the share price at time T lies in the interval $[a, b]$.

Prove that the price of such an option is given by:

$$e^{-rT}[\Phi(d(b)) - \Phi(d(a)] \text{ where } d(x) = \frac{\ln\left(\dfrac{x}{S_0}\right) - \left(r - \dfrac{\sigma^2}{2}\right)T}{\sqrt{T}\sigma}$$

where S_0 = price of underlying stock, r = continuously compounded rate of return on the risk free asset and σ = volatility parameter of stock price process. [5]

A fund manager currently charges an annual management fee of 0.5% of the value of the funds under management at the end of a one-year contract.

The value of the funds under management are governed by the following SDE:

$$dS_t = S_t(\mu dt + \sigma dZ_t)$$

where S_t = value of funds under management
Z_t = standard Brownian motion
$\mu = 0.08$
$\sigma = 0.25$

The funds generate no income during the year.

The continuously compounded risk-free rate is 5% per annum.

The owner of the funds wishes to change the management fee to be performance-related.

Specifically the fee, KS_1 is set so that:

$$K = \begin{cases} 0.1\% & \text{if } S_1 < S_0 \\ 1\% & \text{if } S_1 > U \\ 0.5\% & \text{otherwise} \end{cases}$$

(iv) Calculate the value at time 0 of the management fee under the original fee structure if $S_0 = 100$. [1]

(v) Calculate U so that the management fee under the performance-related fee structure has the same value at time 0 as the fixed fee in (iv). [10]

Hint: the fee can be written as a basic fee plus two call options plus two options of the form in (iii). [Total 20]

END OF PAPER