EXAMINATION

22 April 2009 (am)

Subject CT8 — Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Describe what is meant by an arbitrage opportunity. [3]

2 One of your colleagues says that the stock market is not efficient because some accounting ratios have been shown to have predictive powers.

(i) Explain which of the main forms of efficiency is most relevant to this situation. [2]

(ii) Comment on whether you agree with your colleague. [2]

(iii) Explain the difference between active and passive fund management in terms of the concept of market efficiency. [2]

[Total 6]

3 (i) Outline the relevant empirical evidence and theoretical arguments regarding the behaviour of stock prices for each of the properties below.

(a) The volatility of returns over time.
(b) The expected value of returns over time.
(c) Whether stock prices are mean reverting.
(d) The statistical distribution of returns.

[8]

(ii) (a) Discuss the extent to which a continuous time lognormal model of security prices can capture the statistical properties empirically observed or expected in the stock market.

(b) Outline other possible processes which may be used.

[6]

[Total 14]

4 (i) State how investors are assumed to make decisions in modern portfolio theory (MPT). [1]

(ii) Define an efficient portfolio in the context of MPT. [1]

An investor can invest in only three assets which are uncorrelated with one another. The assets have the following characteristics:

Asset A	*Expected Rate of Return*	*Standard Deviation*
A	9%	18%
B	5%	8%
C	4%	0%

(iii) Calculate the efficient frontier for the investor taking into account the numbers provided in the table above. [8]

(iv) Explain how an investor with a quadratic utility function would select a portfolio from those making up the efficient frontier. [1]

[Total 11]

5 (i) Define the market price of risk in the CAPM. [1]

The table below gives the annual returns conditional on the state of the economy for all the assets in an investment market.

Economic State	*Asset*			*Probability*
	Stock	*Property*	*Bonds*	
Recession	0%	1%	2%	0.1
Normal	5%	3%	3%	0.7
Boom	10%	7%	3%	0.2
Value of asset (bn)	100	50	100	

(ii) Calculate the market price of risk given that the risk free annual rate of return is 2.5%. [4]

(iii) Discuss the particular issue a young investor might face in using the CAPM. [2]

[Total 7]

6 Describe three different approaches to modelling credit risk. [6]

7 (i) Set out a formula for the stock-price, S_t, in the Black-Scholes model under the equivalent martingale measure. [2]

A European call option on a stock has an exercise date one year away and a strike price of £2. The underlying stock has a current price of £1.80. The continuously compounded risk free rate of interest is 5% p.a. The option is priced at 20p.

(ii) Estimate the volatility of the stock price process to within 1% p.a., assuming the Black-Scholes model applies. [5]

A new derivative security has just been written on the underlying stock. This will pay a random amount D in one year's time, where D is £1 if $S_{0.5} > £2$ and $S_1 > 2S_{0.5}$, is 50p if $S_{0.5} < £2$ and $S_1 > 2\,S_{0.5}$ and is zero otherwise.

(iii) Derive an expression in terms of the distribution function and/or density function of the standard normal distribution for the fair price for this derivative security. [6]

[Total 13]

8 (i) Explain what is meant by self-financing in the context of continuous-time derivative pricing, defining all notation used. [4]

(ii) Define the delta of a derivative, defining all notation and terms used other than those already defined in your answer to (i). [2]

(iii) Explain how delta and self-financing are used in the martingale approach to valuing derivatives. [4]

[Total 10]

9 The zero-coupon bond market is assumed to be arbitrage-free and complete. Consider the following model for the instantaneous forward rate process:

$$df(t,T) = a(t,T)dt + \sigma(t,T)dW_t$$

where $(W_t; t \geq 0)$ is a standard Brownian motion with respect to the risk-neutral probability measure **Q**.

(i) State how the price of a zero-coupon bond is related to the instantaneous forward rate. [2]

Using Itô calculus, it is possible to prove that the dynamics for the zero-coupon bond price are given under **Q** as follows:

$$\frac{dB(t,T)}{B(t,T)} = m(t,T)dt + S(t,T)dW_t,$$

where

$$m(t,T) = r(t) - \int_t^T a(t,s)ds + \left(\int_t^T \sigma(t,s)ds\right)^2$$

$$S(t,T) = -\int_t^T \sigma(t,s)ds.$$

(ii) Explain the relationship between a and σ under the condition that the bond market is complete. Give reasons for your answer. [6]

[Total 8]

10 Consider a three-period binomial model for a stock with the following parameters: $u = 1.2$, $d = 0.9$ and $S_0 = 60$. Assume that the discretely compounded risk-free rate of interest is $r = 11\%$ per period.

(i) (a) Verify that there is no arbitrage in the market.
(b) Construct the binomial tree.

[3]

(ii) Calculate the price of a standard European call option with maturity date in three periods and strike price $K = 60$. [7]

A new "knock-in" option is introduced which has the following characteristics:

> If the value of the stock crosses the level 80 during the whole life of the option, the contract holder has the right to obtain the difference between the value of the stock at maturity (in three periods) and 60.

(iii) Calculate the price of this new option. [4]

[Total 14]

11 Consider a forward contract on gold. Suppose that there is a fixed storage cost of £c per ounce, paid at the end of the period and c is the same for any time period less than one year. Let S_t be the spot price of one ounce of gold at time t and r be the continuously compounded risk-free rate of interest which is assumed to be constant.

Derive the price at time t of a forward contract written on one ounce of gold at the start of the year, with maturity T years ($T < 1$).

[8]

END OF PAPER