EXAMINATION

April 2005

Subject CT8 — Financial Economics
Core Technical

EXAMINERS' REPORT

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

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Chairman of the Board of Examiners

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1 Let V_i denote the variance of the ith asset.
C_{ij} denote the covariance of the ith and jth assets ($i \neq j$).

The variance of the investor's portfolio is therefore

$$V = \sum_i \frac{1}{N^2} V_i + \sum_i \sum_{j \neq i} \frac{1}{N^2} C_{ij}$$

Let $\bar{V} = \frac{1}{N} \sum V_i$, and $\bar{c} = \frac{1}{N(N-1)} \sum_i \sum_{j \neq i} C_{ij}$

$$V = \frac{1}{N} \bar{V} + \frac{(N-1)}{N} \bar{c}$$

As $N \to \infty$

$$\frac{1}{N} \bar{V} \to 0 \text{ because } \frac{1}{N} \bar{V} < \frac{1}{N} V_{\max}$$

and $\frac{1}{N} V_{\max} \to 0$ as $N \to \infty$

therefore $V \to \bar{c}$ as $N \to \infty$

2 (i) The mean return μ is $0.1 \times 0.5 + 0.2 \times 0.3 + 0.5 \times 0.2 = 21\%$

Variance of return

$(0.1 - 0.21)^2 \times 0.5 + (0.2 - 0.21)^2 \times 0.3 + (0.5 - 0.21)^2 \times 0.2 = 2.29\%\%$

Semi variance of return

$= (0.1 - 0.21)^2 \times 0.5 + (0.2 - 0.21)^2 \times 0.3 = 0.608\%\%$

Shortfall probability

$50\% + 30\% = 80\%$

(ii) (a)

- It is a statistical measure of downside risk.
- It assesses the potential minimum loss over given time with given degree of confidence.

(b) Advantage: normal distribution is easy to manipulate to calculate VaRs based on only two parameters.
Disadvantage: results may be misleading with skewed or "fat tailed" distribution.

3 The market price of risk is $(E_m - r)/\sigma_m$ where asset 1 is the risk free asset so $r = 5\%$.

$$E_m = (17{,}546/100{,}000) \times (0.2 \times 5\% + 0.3 \times 12\% + 0.1 \times 3\% + 0.4 \times 1\%)$$

$$+ (82{,}454/100{,}000) \times (0.2 \times 6\% + 0.3 \times 5\% + 0.1 \times 4\% + 0.4 \times 7\%)$$

$$= 5.79472\%$$

$$\sigma_m^2 = 0.2 \times (0.17546 \times 5\% + 0.82454 \times 6\% - 5.79472\%)^2$$

$$+ 0.3 \times (0.17546 \times 12\% + 0.82454 \times 5\% - 5.79472\%)^2$$

$$+ 0.1 \times (0.17546 \times 3\% + 0.82454 \times 4\% - 5.79472\%)^2$$

$$+ 0.4 \times (0.17546 \times 1\% + 0.82454 \times 7\% - 5.79472\%)^2$$

$$= 0.000045402$$

$$= (0.674\%)^2$$

$\Rightarrow$ market price of risk is $(5.79472\% - 5\%) / 0.674\%$

$$= 1.179 = 118\%$$

4 (i) (a) Macroeconomic.
(b) Fundamental.

(ii) The single index model requires the return on the market, plus for each security: α_i, β_i and σ_i .

Therefore $3N + 1$ data items are required

The multifactor model requires:

L means of indices (Note: some candidates may assume that they have 0 mean, which is acceptable.)

$\frac{L(L+1)}{2}$ covariances

N a_i's

NL sensitivities

N standard deviations of c_i

Therefore $\frac{L(L+3)}{2} + N(L+2)$ data items are required

5 (a) $B(t, T) = f(r_t, t)$, where

$$f(x, t) = \exp(-(T-t)x + \sigma^2(T-t)^3 / 6),$$

so, by Ito's lemma,

$$dB(t, T) = B(t, T)((\sigma^2(T-t)^2 / 2 - (T-t)\mu r_t + r_t - \sigma^2(T-t)^2 / 2)dt - \sigma(T-t)dZ_t)$$

$$= B(t, T)((-(T-t)\mu r_t + r_t)dt - \sigma(T-t)dZ_t)$$

(b) The market price of risk is

$$\gamma(t, T) = (m(t, T) - r_t) / S(t, T),$$

where

$$dB(t, T) = B(t, T)(m(t, T)dt + S(t, T)\, dZ_t), \text{ so } \gamma(t, T) = \mu/\sigma r_t$$

and so under the risk-neutral measure, Q, $dr_t = \sigma dW_t$, where W is a standard BM under Q.

6 (i) Options are priced by "relative valuation" techniques (i.e. risk neutral valuation).

This approach is equivalent to building a hedging strategy for the option and does not take account of the expected return on the share. Since the hedging strategy involves holding some shares, the drop in price will result in a drop of the value of the option even though the expected future share price has remained the same.

(ii) Unless the option is deep in the money, the drop in price of the option will be less than proportional to the share price and hence some combination of the following must also have occurred:

- dividends increased
- share price volatility decreased
- risk free interest rate decreased

7 (i) Shiller used a discounted cashflow model of equities going back to 1870.

A perfect foresight price was determined using actual dividends paid and a terminal value for the stock.

If markets are rational there would be no systematic forecast errors (i.e. error between the perfect foresight price and the actual price).

If markets are efficient, the perfect foresight price matches with share price.

Strong evidence was found that contradicted the EMH.

- Criticisms of terminal stock price.
- Choice of constant discount rate.
- Bias in estimates of variance because of autocorrelation.
- Non-stationarity of the series.

(ii)

- Researchers require access to information that is not in the public domain.
- Studies suggest that it is difficult to out perform with inside information.

8 (i) Suppose X_t is a martingale with respect to a measure P, that is for any $t < s$

$$E_P[X_s|F_t] = X_t$$

and that the volatility of X_t is always non-zero.

Suppose Y_t is another martingale with respect to P.

Then, there exists a unique previsible process ϕ_t such that

$$Y_t = Y_0 + \int_0^t \phi_s \, dX_s$$

Or equivalently $dY_t = \phi_t \, dX_t$

Full credit for either integral or differential form above.

(ii) Let $E_t = e^{-rT}E_Q[X|F_t] = e^{-rt}V_t$, which is a martingale with respect to Q.

Using the martingale representation theorem, there exists a unique previsible process ϕ_t such that

$$dE_t = \phi_t dD_t$$

Let $\psi_t = E_t - \phi_t D_t$

Suppose that at time t we hold

ϕ_t units of asset S_t
ψ_t of cash B_t

The value of the portfolio at time t is

$$\phi_t S_t + \psi_t B_t = V_t$$

over the period t to $t + dt$ the change in the value of the portfolio is

$$\phi_t dS_t + \psi_t dB_t$$

$$\begin{aligned}
\phi_t dS_t + \psi_t dB_t &= \phi_t B_t(rD_t dt + dD_t) + \psi_t r B_t dt \\
&= B_t[\phi_t dD_t + r(\phi_t D_t + \psi_t)dt] \\
&= B_t[\phi_t dD_t + rE_t dt] \\
&= B_t[dE_t + rE_t dt] \\
&= B_t dE_t + E_t dB_t + dB_t\, dE_t \\
&= dV_t
\end{aligned}$$

Therefore (ϕ_t, ψ_t) is self financing

$V_T = E_Q[X|F_T] = X$, therefore (ϕ_t, ψ_t) is replicating so V_t is the value of the claim.

9 (i) The Wilkie model can be described as a cascade or hierarchical model, with inflation being the key component. Variations of dividend yields, growth and interest rates are affected by shocks in the inflation model and moving averages of past inflation.

(ii) $I_\infty = a + bI_\infty$

$$\Rightarrow I_\infty = \frac{a}{1-b}$$

(iii)

- AR process is stationary, share prices have tended to increase over time.

- AR(1) implies a systematic element to the changes in prices which is inconsistent with high risk and return.

- Non-normality, jumps in share prices.

- Prices can be negative.

(iv) Log-normal distribution makes the maths for option pricing simple (i.e. tractable solutions).

Returns in non-overlappng periods are independent, which is consistent, with the EMH, for example.

It does not allow negative share prices.

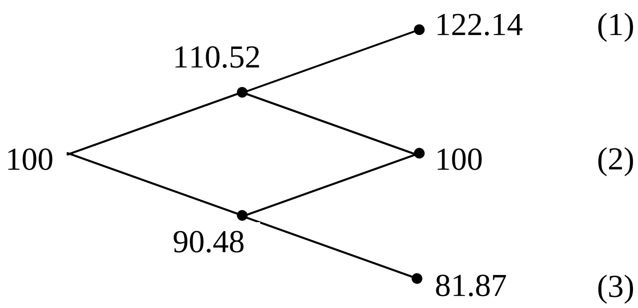
Mean and variance are proportional to time period.

Weaknesses:

- The variance may not be stable over time.
- The mean (drift) may not be constant over time.
- Share prices may be considered to be mean-reverting.
- Share prices exhibit jumps.

10 (i) $u = \exp(0.1) = 1.10517$

$d = 1/1.10517 = 0.904837$



Since the real world expected return is 2% per month, we can derive the real world probability of an up-jump

$$102 = p \times 110.52 + (1 - p) \times 90.48$$

$$\Rightarrow p = \frac{(102 - 90.48)}{(110.52 - 90.48)} = 57.5\%$$

The risk neutral probability of an up-jump is

$$q = \frac{e^{5\%} - 0.904837}{1.10517 - 0.904837} = 73.1\%$$

The state price deflator A_2 at time 2 is defined as follows:

Node

(1) $e^{-0.1}\left(\frac{q}{p}\right)^2 = 1.4624$

(2) $e^{-0.1}\left(\frac{q}{p}\right)\left(\frac{1-q}{1-p}\right) = 0.72809$

(3) $e^{-0.1}\left(\frac{1-q}{1-p}\right)^2 = 0.36249$

(ii) The value is $E_P[A_2 f(S_2)]$ where S_2 is the share price at time 2.

$= (57.5\%)^2 \times 1.4624 \times \log(32.14) + 2 \times 57.5\% \times (1 - 57.5\%) \times \log(10) \times 0.72809$

$= 2.4972$

(iii) The value of the European call option is

$$V = 100\Phi(d_1) - 100e^{-0.05\times3}\Phi(d_2)$$

Where $d_1 = \dfrac{\log\frac{100}{100} + (0.05 + \frac{1}{2}0.1^2)\times 3}{0.1\sqrt{3}} = 0.95263$

$$d_2 = \frac{\log\frac{100}{100} + (0.05 - \frac{1}{2}0.1^2)\times 3}{0.1\sqrt{3}} = 0.7794256$$

Therefore value is

$$100(0.829611) - 100e^{-0.05 \text{ x } 3}\, 0.7821347$$

$$= 15.642$$

The delta of the European Call option is given by

$$\Delta = \Phi(\text{d1}) \text{ where } d_1 = \frac{\log\frac{S}{K} + (r + \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$$

$$= 0.95263$$

$$\Delta = 0.82961$$

The gamma of the European call option is given by

$$\Gamma = \frac{\phi(d_1)}{s\sigma\sqrt{t}} = \frac{\phi(0.95263)}{100\times 0.1\sqrt{3}} = \frac{1}{\sqrt{2\pi}}.\frac{e^{-\frac{1}{2}\times 0.95263^2}}{100\times 0.1\sqrt{3}}$$

$$= 1.46\%$$

Clearly the risk free rate has nil delta and gamma.

The underlying share has $\Delta = 1$ and $\Gamma = 0$. Therefore, equating

delta	0.07	=	$1.x_2 + 0.82961x_3$
gamma	0.10	=	$0.0146x_3$
and value	2.4972	=	$1.x_1 + 100x_2 + 15.642x_3$

Solving these three equations in three unknowns gives

$$x_3 = \frac{0.1}{0.0146} = 6.8493$$

$$\Rightarrow \quad 0.07 = x_2 + 0.82961 \times 6.8493$$

$$x_2 = -5.6123$$

$$\Rightarrow \quad 2.4972 = x_1 + 100 \times -5.6123 + 15.642 \times 6.8493$$

$$\Rightarrow \quad x_1 = 456.59$$

Therefore

hold 456.59 in risk free asset
sell 5.6123 of underlying share
hold 6.8493 European Call option

END OF EXAMINERS' REPORT