INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2017

Subject CT8 – Financial Economics
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Luke Hatter
Chair of the Board of Examiners
December 2017

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Financial Economics subject is to develop the necessary skills to construct asset liability models and to value financial derivatives. These skills are also required to communicate with other financial professionals and to critically evaluate modern financial theories.

2. The marking approach for CT8 is flexible in the sense that different answers to those shown in the solution can earn marks if they are relevant and appropriate. Marks for the methodology are also awarded.

B. General comments on *student performance in this diet of the examination*

1. Students performed relatively well on bookwork questions, although many missed the opportunity to be awarded full marks due to relatively superficial knowledge.

2. Some students seemed to struggle on the application parts of the questions, because they were not able to combine and use the information given to them in the question.

C. Pass Mark

The Pass Mark for this exam was 60.

Solutions

Q1 (i) (a) Absolute dominance exists when one investment portfolio provides a higher return than another in all possible circumstances. [1]

(b) The first order stochastic dominance theorem states that, assuming an investor prefers more to less, A will dominate B (i.e. the investor will prefer portfolio A to portfolio B) if: [½]

$F_A(x) \le F_B(x)$, for all x, and [½]

$F_A(x) < F_B(x)$ for some value of x. [½]

(c) The second order stochastic dominance theorem applies when the investor is risk averse, as well as preferring more to less. [½]

In this case, the condition for A to dominate B is that

$\int_a^x F_A(y)dy \le \int_a^x F_B(y)dy$, for all x, [½] with the strict inequality holding for some value of *x*, [½]

and where a is the lowest return that the portfolios can possibly provide. [½]

[Max 4]

(ii)

PDF		–5%	–3%	0%	3%	5%
	1	0.2	0.2	0.2	0.2	0.2
	2	0.3	0.2	0.1	0.2	0.2
	3	0.1	0.3	0.2	0.3	0.1

CDF		–5%	–3%	0%	3%	5%
	1	0.2	0.4	0.6	0.8	1
	2	0.3	0.5	0.6	0.8	1
	3	0.1	0.4	0.6	0.9	1

[1 mark for CDF table]

∫CDF		–5%	–3%	0%	3%	5%
	1	0	0.004	0.016	0.034	0.05
	2	0	0.006	0.021	0.039	0.055
	3	0	0.002	0.014	0.032	0.05

[2 marks for ∫CDF table]

(a) None [1]

(b) Second order [1]

(c) First order [1]

Most students knew the definitions of the different types of dominance and stated them clearly, either in words or formulae. Fewer students were able to determine the types of dominance exhibited by the assets in part (ii). In particular, very few students integrated the CDF correctly to check for second order dominance. There was one anomaly in the question whereby students were asked to "consider four assets" when only three were given in the table.

Q2 (i) (a) A portfolio is inefficient if the investor can find another portfolio with the same expected return and lower variance, [½]

or the same variance and higher expected return. [½]

(b) A portfolio is efficient if the investor cannot find a better one in the sense that it has both the same or higher expected return [½]

and the same or lower variance. [½]

(ii) The assumptions are:

(a) Investors are never satiated. [At a given level of risk, they will always prefer a portfolio with a higher expected return to one with a lower return.] [½]

(b) Investors dislike risk. [For a given level of return, they will always prefer a portfolio with lower expected variance to one with higher variance.] [½]

(iii) $V = a^2 V_A + (1-a)^2 V_B + 2a(1-a)(V_A V_B)^{0.5} C_{AB}$ [1]

$= 0.16a^2 + 0.25\,(1-a)^2 - 2a(1-a)\,(0.16 \times 0.25)^{0.5} \times 0.2$ [1]

$= 0.49a^2 - 0.58a + 0.25$ [1]

(iv) $R = aR_A + (1-a)R_B$ [1]
$= -0.02a + 0.07$

So $R^2 = 0.0004a^2 - 0.0028a + 0.0049$ [1]

So $V = 1225R^2 - 142.5R + 4.2225$ [1]

(v) $1225R^2 - 142.5R + 4.2225 = 16R - 200R^2$ [1]

So $R = 0.0670$ or 0.0442 [2]

Hence $a = 0.1497$ or 1.2889 [1]

(vi) The second solution implies a proportion of –0.2889 invested in asset B [1]

so would not be allowed, hence only the first solution would remain. [1]

This question was largely well-answered. Many students made mistakes in the algebra but these were penalised only for the mistake itself if the remaining workings were correct. Some students found the point where the efficient frontier and the indifference curve were tangential but did not check that they touched.

Q3 (i) (a) reduce [½]

(b) reduce [½]

(c) increase [½]

(d) reduce [½]

(ii) (a) This is because there is a lower intrinsic value (or, where the intrinsic value is currently zero, a smaller chance that the option is in-the-money at maturity). [1]

(c) This is because there is again a lower intrinsic value, or a smaller chance that the option is in-the-money at maturity. [1]

(d) This is because the higher the volatility of the underlying share, the greater the chance that the share price can move significantly in favour of the holder of the option before expiry. [1]

(e) This is because the money saved by purchasing the option rather than the underlying share has to be invested at this lower rate of interest, thus decreasing the value of the option. [1]

(iii) $c_t + Ke^{-r(T-t)} = p_t + S_t$ [1]

(iv) $0.5 + 6e^{-3r} = 1 + 5$ [1]

$\Rightarrow r = 2.9\%$ p.a. [1]

(v) As no dividend is paid, American call options will never be exercised before maturity. Therefore, their price should be the same as for European call options with the same characteristics. [1]

It is sometimes optimal to exercise an American put option early, so the value of an American put option can be higher than a European put option. [1]

and the formula becomes an inequality:

$$c_t + Ke^{-r(T-t)} \le p_t + S_t$$ [1]

> *Almost all students stated the correct option price changes in part (i), and most gave good reasons for the changes in part (ii). The question did not state that the assumptions underlying the Black-Scholes formula applied so explanations based on this were not valid. Fewer students were able to explain clearly what the impact was of changing the options to be American , though well-prepared students built an answer around the impact of exercising each option early.*

Q4 **(i)** The market is arbitrage free if and only if there exists a probability measure under which discounted asset prices are martingales [1]

In this case, the probability exists iff $d < e^{r\delta t} < u$ [1]

The given market does not satisfy this property as $e^{r\delta t} = 1.0126 < d = 1.05 < u = 1.25$ [1]

Alternatively, it can be seen that investment in the stock will gain more than the risk-free rate… [1]

… under any possible outcome / with no downside risk. [1]
[Max 2]

(ii) (a) The investor could buy the stock at 100 by borrowing money at the risk-free rate of interest. [1]

In three months, the investor then could sell the stock and repay the loan + interest [1]

(b) This would result in a profit of either 23.74 – in the case in which the stock is worth 125 [1]

Or 3.74 – in the case in which the stock is worth 105 [1]

(iii) The price C_0 of the option is computed via risk-neutral valuation; let $\hat{p}$ denote the risk-neutral probability of an up movement, then

$$\hat{p} = \frac{e^{0.2\times 0.25} - 1.05}{1.25 - 1.05} = 0.0064 \quad [1]$$

$$C_0 = e^{-0.2\times 0.25}(15\times(1-\hat{p})) = 14.18. \quad [2]$$

This question was broadly well-answered. Part (ii) caused the most difficulty, with some students struggling to construct valid portfolios or trying to explain in general terms without any specific portfolios. There were also some solutions to part (iii) involving probabilities either less than zero or greater than one, which were clearly not valid.

Q5 (i) Suppose that Z_t is a standard Brownian motion under *P*. [1]

Furthermore, suppose that γ_t is a previsible process. [½]

Then there exists a measure *Q* equivalent to *P* [½]

and where $\tilde{Z}_t = Z_t + \int_0^t \gamma_s ds$ is a standard Brownian motion under *Q*. [1]

Conversely, if Z_t is a standard Brownian motion under *P* and if *Q* is equivalent to *P* then there exists a previsible process γ_t such that $\tilde{Z}_t = Z_t + \int_0^t \gamma_s ds$ is a Brownian motion under *Q*. [1]

[Max 3]

(ii) Under the risk-neutral probability measure, the discounted value of asset prices are martingales. [1]

(iii) First determine the SDE of $\tilde{S}_t = e^{-rt} S_t$:

$$d\tilde{S}_t = (\mu - r)\tilde{S}_t dt + \sigma \tilde{S}_t dW_t,$$ [1]

Then change the Brownian motion and the probability measure (using the CMG theorem) so that the above reads

$$d\tilde{S}_t = (\mu - r - \lambda\sigma)\tilde{S}_t dt + \sigma \tilde{S}_t d\hat{W}_t,$$ [1]

If $\lambda = {(\mu - r)}/{\sigma}$ then the drift term is zero, as required (for a martingale). [1]

(iv) $$\lambda = \frac{0.04 + r - r}{0.4} = 0.1$$ [1]

(v) The risk driver is the same, therefore the market price of risk is identical. [1]

Hence $\gamma = \frac{0.06}{\lambda} = \frac{0.06}{0.1} = 0.6$. [1]

> *This question was answered poorly on the whole. Few students knew the Cameron-Martin-Girsanov theorem well enough to score full marks. The later parts were answered better, with many students picking up marks in parts (iv) and (v).*

Q6 **(i)** $C_t = E(e^{-r(T-t)} C_T \mid F_t)$ [1]

where F_t denotes the filtration at time $t > 0$, [½]

C_T is the payoff under the derivative [½]

at maturity time T, [½]

C_t is the derivative value at time t, [½]

and the expectation is taken under the risk-neutral martingale measure. [½]
[Max 3]

Data: $S = 50; K = 49; r = 5\%; \sigma = 25\%; T = 0.5$

(ii) The Black-Scholes formula returns:

$d1 = 0.3441$ [½]

$d2 = 0.1673$ [½]

$N(d1) = 0.6346$ [½]

$N(d2) = 0.5664$ [½]

So $\text{Call} = 50 \times 0.6346 - 49e^{-0.05\times0.50} \times 0.5664 = 4.66$ [2]

(iii) Same as European call (as the stock is non-dividend-paying), i.e. 4.66 [1]

(iv) Using put-call parity (or otherwise):

$p_t = c_t + Ke^{-r(T-t)} - S_t$ [1]

Hence $p_t = 2.45$. [1]

(v) If the stock is dividend-paying, the payment of the dividends would cause the value of the underlying asset to fall – which follows from the no arbitrage principle [1]

Alternatively: in valuing the option we must take account of the fact that dividends are payable on the underlying asset which do not feed through to the holder of the option. [1]

Therefore the price of the European call would decrease… [½]

… since by buying the option instead of the underlying share the investor forgoes the income [½]

Similarly, the price of the European put would increase [½]

The American call would now be more expensive than the European call due to potential early exercise opportunity [1]

[Max 3]

This question was answered well by most students. There were a number of numerical mistakes in the Black-Scholes calculations in part (ii) despite this being a very common skill examined in CT8. Knowledge of how dividends affect the option pricing was weak. There were also many cases of students rounding d1 and d2 too aggressively in the Black-Scholes calculations resulting in a materially incorrect answer.

Q7 (i) It allows negative interest rates. [1]

(ii) The extent of the problem depends on the probability of negative interest rates… [½]

… within the timescale of the problem in hand (or, for example, less of an issue if the time horizon is short)… [1]

… and their likely magnitude if they can go negative. [1]

It also depends on the economy being modelled, as negative interest rates have been seen in some countries. [1]

[Max 3]

(iii) The CIR model does not allow interest rates to go negative. [1]

This is because the volatility under the CIR increases in line with the square root of $r(t)$. [1]

Since this reduces to zero as $r(t)$ approaches zero… [½]

… and provided the volatility parameter is not too large… [½]

… $r(t)$ will never actually reach zero. [½]

… provided $\sigma^2 \leq 2\alpha\mu$

[Max 3]

(iv) Letting $t \to \infty$, we note that the mean converges to b. [1]

Hence interest rates under the model are mean reverting [½]

To the long-run mean b [½]

(v) Letting $t \to \infty$, we note that the variance converges to $\sigma^2 / 2a$ [1]

Hence, the variance of the short rate is inversely proportional to a [½]

This implies that the convergence of the rate to the long run mean b is faster the bigger a [1]

So a controls the speed of the mean convergence [1]

[Max 2]

Most students here knew that the Vasicek model allows negative interest rates, and were able to explain why the Cox-Ingersoll-Ross model does not. Part (ii) required students to think about why negative interest rates might (or might not) be a problem in the real world –many students just repeated bookwork about the Vasicek model and scored no marks. Parts (iv) and (v) were answered fairly well, though not all students worked through the algebra correctly.

Q8 **(i)** $E[R_i]$ is the expected return on security *i* [½]

$b_{i,k}$ is the response of (or sensitivity of) the rates of return on security *i* to factor *k* [1]

λ_k is the risk premium per unit of exposure corresponding to factor *k* [1]
[Max 2]

(ii) The risk-free portfolio has zero exposure to all risk factors [1]

i.e. $b_{i,k} = 0$ for all i, k [½]

And the expected return on the risk-free portfolio is r_f [½]

From which the result follows: $\lambda_0 = r_f$. [½]
[Max 2]

(iii) We need to solve the linear system: [1]

$$0.18 = 0.075 + 1.5\lambda_1 + 0.5\lambda_2$$
$$0.15 = 0.075 + 0.5\lambda_1 + 1.5\lambda_2$$

which returns $\lambda_1 = 0.06, \lambda_2 = 0.03$ [1 mark each]

(iv) The given portfolio represents an arbitrage opportunity [1]

as the given expected return does not satisfy the given APT equation [1]

Indeed $E[R_3] = 0.075 + 0.06 \times 0.75 + 0.03 \times 0.7 = 0.141$ [1]

It is therefore not a feasible portfolio… [1]

… under an assumption of no arbitrage [½]
[Max 3]

> *This question was answered well by most students. Parts (ii) and (iii) caused some difficulty, though most students spotted what they needed to do.*

Q9 The Merton model for credit risk is based on the Black-Scholes formula. Hence.

(i) The current value of the debt, say D_0, of the firm is the value of a risk-free zero coupon bond with the same face value and maturity of the firm debt corrected by the cost of default. That is, where F_0 is the value of the firm's assets (=110) and L is the face value of the debt (=100):

$$D_0 = Le^{-rT} - (Le^{-rT}N(-d_2) - F_0N(-d_1))$$ [1½]

Alternative approach: Or equivalently it is the current value of the firm's assets less the value of equity, where the latter is the value of a call option on the assets of the company with strike price equal to L, i.e.:

$$D_0 = F_0 - (F_0N(d_1) - Le^{-rT}N(d_2))$$ *[Alternative 1½]*

d_1	0.6289
d_2	0.0699

[½ for each]

$N(d_1)$	0.7353
$N(d_2)$	0.5279
$N(-d_1)$	0.2647
$N(-d_2)$	0.4721

[½ for each, but only give for either + or –, i.e. Max 1]

Giving overall value of either (in €m):

$100e^{-0.1} - (100e^{-0.1} \times 0.4721 - 110 \times 0.2647)$ or

$110 - (110 \times 0.7353 - 100e^{-0.1} \times 0.5279)$ [1]

= €76.88m as required [½]

(ii) The yield to maturity solves $D_0 = Le^{-yT}$ [1]

i.e. 76.88 = $100e^{-5y}$ [1]

Consequently $y = 5.26\%$ per annum [1]

Note: may quote 5.25% if didn't round to 76.88.

Note to markers: give full marks for correct answer, even if no working shown.

(iii) Credit spread: $y - r$ [1]

= 3.26% per annum. [1]

Most students applied the Merton model correctly, though some just explained the model. Parts (ii) and (iii) were also answered well by most students.

END OF EXAMINERS' REPORT