EXAMINATION

1 October 2009 (am)

Subject CT8 — Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 (i) State the general form of the equation used in multifactor models of security returns, defining any terms you use. [3]

(ii) Describe the different categories of factors that are used in these models and illustrate your answer with suitable examples. [6]
[Total 9]

2 (i) In the context of time series models of financial markets explain the difference between cross-sectional and longitudinal properties of statistical distributions. [2]

(ii) Discuss the difference between cross-sectional and longitudinal estimates of stock volatility assuming:

(a) stock prices follow a multiplicative random walk.
(b) the Wilkie model is being used to model financial variables.
[4]
[Total 6]

3 A small bank wishes to improve the performance of its investments by investing £1m in high returning assets. An investment bank has offered the bank two possible investments:

Investment A: A diversified portfolio of shares and derivatives which can be assumed to produce a return of £R_1 million where $R_1 = 0.1 + N$, where N is a normal $N(1,1)$ random variable.

Investment B: An over-the-counter derivative which will produce a return of £R_2 million where the investment bank estimates:

$$R_2 = \begin{cases} 1.5 & \text{with probability } 0.99 \\ -5.0 & \text{with probability } 0.01. \end{cases}$$

The chief executive of the bank says that if one investment has a better expected return and a lower variance than the other then it is the best choice.

(i) (a) Calculate the expected return and variance of each investment A and B.

(b) Discuss the chief executive's comments in the light of your calculations.

[6]

(ii) Calculate the following risk measures for each of the two investments A and B:

(a) semi-variance of return
(b) shortfall probability of the returns falling below 0
(c) shortfall probability of the returns falling below –2

[3]

(iii) (a) Define other suitable risk measures that could be calculated.
(b) Discuss what the risk measures in (iii) (a) would show. [4]

(iv) Compare the merits of the two investments A and B. [2]
[Total 15]

4 An investor invests a proportion x_i of the assets in his portfolio in the ith of N securities.

(i) State the expected return and variance of his portfolio. Define any notation you use. [2]

Securities with the properties in the table below are available to an investor. The statistics in the table refer to the next year.

	A	B
Expected return	4%	3%
Variance of return	16%%	4%%
Correlation coefficient between assets	$\rho_{AB} = 1$	

The investor combines the securities to form a portfolio.

(ii) Calculate the relative amount which should be invested in each security to give a portfolio with the minimum possible variance. (Note: you may assume that short selling securities is allowable.) [4]

(iii) Show that if it is possible to borrow at the rate of 1% p.a. over the next year, it is possible for the investor to make a risk free profit over the year without using any of his own capital. [4]
[Total 10]

5 A derivative security entitles the holder to a payment, at time T, of $\max_{0 \le t \le T} S_t$, where S_t is the price at time t of a security.

Assume that S satisfies $S_t = S_0 \exp(\sigma B_t + (r - 1/2\, \sigma^2)t)$ under the risk neutral measure, where B is a standard Brownian motion and r is the risk-free rate of interest.

(i) Derive the probability density of $\max_{0 \le s \le t} B_s + \mu s$. (Hint: use the formula in section 7.2 of the Formulae and Tables for Actuarial Examinations). [4]

(ii) Determine an expression for p_t, the fair price of the derivative security at time t. You need not evaluate the resulting integral. [4]
[Total 8]

6 (i) Describe the two-state model for credit-ratings. [4]

In a two state model a zero-coupon defaultable bond is due to redeem at par in two years' time. If default occurs the recovery rate is δ. The continuously compounded risk free rate of return is r, Under the probability measure P_λ, the default intensity is constant and equal to λ and the defaultable bond price is D_t, given by:

$$D_t = \delta e^{-r(2-t)}, \text{ if default has occurred prior to time } t$$

$$= e^{-r(2-t)}(\delta(1 - e^{-\lambda(2-t)}) + e^{-\lambda(2-t)}) \text{ otherwise.}$$

(ii) Show that P_λ is an equivalent martingale measure for this model. [3]

A derivative contract pays \$1,000 after two years if and only if the bond has defaulted.

(iii) (a) Determine a constant portfolio in the defaultable bond and cash which replicates the derivative.

(b) Calculate the fair price for the derivative. [5]

(iv) Explain how your answer to (iii) relates to the fact stated in part (ii). [3]
[Total 15]

7 (i) State the main assumptions underlying the Black-Scholes model for a security price. [4]

(ii) Comment on how realistic these assumptions are in practice. [4]
[Total 8]

8 (i) Define a state-price deflator in the context of continuous time models for security prices. [3]

(ii) Give a formula for the state-price deflator in the Black-Scholes model when the risk-free rate of interest is r and the stock price satisfies:

$$dS_t = S_t(\mu dt + \sigma dZ_t)$$

under the real-world measure P, where Z is a standard Brownian motion. [3]

A derivative contract pays $\exp(\gamma Z_1)$ if $Z_1 > 1$, and zero otherwise, where $Z_0 = 0$ and $\gamma = (\mu - r) / \sigma$.

(iii) Calculate the price p_t, at each time t, of this derivative contract, using your answer to part (ii), or otherwise. [5]

[Total 11]

9 Comment on the difference between real-world and risk-neutral measures in the context of the valuation of derivative securities using a binomial tree. [4]

10 Prove that it is never optimal to exercise an American call written on a non-dividend paying stock before maturity. [7]

11 (i) Define the market price of risk in the context of pricing zero coupon bonds using diffusion models for the short-rate of interest. Define any notation you use. [2]

(ii) Prove that the market price of risk at a given time t is constant for all zero-coupon bonds with maturities $T > t$ in the case where the diffusion model for the short-rate of interest has only one factor. Define any notation you use. [5]

Hint: construct a self-financing strategy involving zero coupon bonds of maturities T_1 and T_2 and a cash account.

[Total 7]

END OF PAPER