INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2016 (with mark allocations)

Subject CT8 – Financial Economics
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

F Layton
Chairman of the Board of Examiners
June 2016

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Financial Economics subject is to develop the necessary skills to construct asset liability models and to value financial derivatives. These skills are also required to communicate with other financial professionals and to critically evaluate modern financial theories.

2. The marking approach for CT8 is flexible in the sense that different answers to those shown in the solution can earn marks if they are relevant and appropriate. Marks for the methodology are also awarded.

B. General comments on *student performance in this diet of the examination*

1. Students performed relatively well on bookwork questions, although many missed the opportunity to be awarded full marks for these due to relatively superficial knowledge.

2. The majority of the students though seemed to struggle on the applications part of the questions, through not being able to put together the pieces of information given and use them. In a few instances this resulted in students re-calculating given data from basic principles and therefore running out of time. Further, there is often a lack of knowledge of how to use the distribution tables to compute probabilities (in the specific case of this exam paper, the normal distribution), and relative sloppiness in getting the details right.

C. Pass Mark

The Pass Mark for this exam was 60%.

Solutions

Q1 (i) $U'(w) = 1/w$ [½]

$U''(w) = -1/w^2$ [½]

Absolute risk aversion = $A(w) = -U''(w)/U'(w)$ [½]

$= 1/w$ [½]

$A'(w) = -1/w^2$ [½]

Relative risk aversion = $R(w) = -wU''(w)/U'(w)$ [½]

$= 1$ [½]

$R'(w) = 0$ [½]

[Total 4]

(ii) $R'(w)$ =0 thus the log utility function exhibits constant relative risk aversion. [1]

This is consistent with an investor who keeps a constant proportion of wealth invested in risky assets as she gets richer. [2]

[Max 2]

(iii) Wealth after the uncertain event will be either:

$$100 \times (1.3a + (1 - a)) = 100 + 30a \text{ with probability } 0.75$$ [½]

or:

$$100 \times (0.4a + (1 - a)) = 100 - 60a \text{ with probability } 0.25.$$ [½]

Thus expected utility of wealth is:

$$0.75 \times \ln(100 + 30a) + 0.25 \times \ln(100 - 60a).$$ [2]

[Max 2]

(iv) Differentiate with respect to a:

$$30 \times 0.75/(100 + 30a) - 60 \times 0.25/(100 - 60a).$$ [2]

Set equal to zero:

$30 \times 0.75 / (100 + 30a) - 60 \times 0.25 / (100 - 60a) = 0$
$30 \times 0.75 / (100 + 30a) = 60 \times 0.25 / (100 - 60a)$
$30 \times 0.75 \times (100 - 60a) = 60 \times 0.25 \times (100 + 30a)$
$22.5 \times (100 - 60a) = 15 \times (100 + 30a)$
$2250 - 1350a = 1500 + 450a$
$750 = 1800a$
$a = 0.4167$ [2]

Check for maximum:

Differentiate with respect to a again:

$$-30^2 \times 0.75/(100 + 30a)^2 - 60^2 \times 0.25/(100 - 60a)^2.$$

This must be negative because of the square terms, hence this is a local maximum. [1]

So invest £41.67 in Asset Aand £58.33 in Asset B. [2]

[Max 5]

[TOTAL 13]

Early parts of this question were largely completed well, though some students used the incorrect formulae despite them appearing in the tables (sign problems mainly). The majority of the students were able to correctly identify the nature of the utility function in terms of index of relative risk aversion but failed to comment about the proportion of the assets that the investor will invest in risky assets. The majority of students also failed to express the expected utility of wealth, and calculated the utility of expected wealth instead.

Q2 (i) Variance of return is defined as:

$$\int_{-\infty}^{\infty} (\mu - x)^2 f(x)dx,$$

where μ is the mean return at the end of the chosen period. [1]

(ii) Shortfall probability = $\int_{-\infty}^{L} f(x)dx$. [1]

(iii) The shortfall probability required is the probability that the return is lower than 480/500 – 1 = –4% i.e. $P(N(6\%, 23\%) \le 4\%)$ [1]

$= P(Z \le (-4\% - 6\%)/\sqrt{(23\%)})$ [½]

$= P(Z \le -0.20851)$ [½]

$= 0.417$ [1]

[Max 2]

(iv) (a) This may imply that the investor has a quadratic utility function. [1]

(b) This corresponds to a utility function which has a discontinuity at the minimum required return. [1]

[Total 2]

[TOTAL 6]

Well prepared students scored well on the bookwork parts of this question, although some students failed to define in full the notation used in part (i). Many students had problems in calculating the shortfall probability using the distribution of the normal random variable, and in recognising that the corresponding utility function has a discontinuity.

Q3 (i) $V = \Sigma_i \; x_i^2 \; V_i + \Sigma_i \, \Sigma_{j, \, j \neq i} \, x_i \, .x_j \, C_{ij}$. [Total 1]

(ii) The aim is to choose x_i to minimise V … [1]

… subject to the constraints

$$\Sigma_i \, x_i = 1 \qquad [1]$$

and expectation of return $E = E_P$, say, in order to plot the minimum variance curve. [1]

One way of solving such a minimisation problem is the method of Lagrangian multipliers. [1]

The Lagrangian function is:

$$W = V - \lambda(E - E_P) - \mu(\Sigma_i \, x_i - 1). \qquad [1]$$

To find the minimum we set the partial derivatives of W with respect to all the x_i and λ and μ equal to zero. [1]

The result is a set of linear equations that can be solved. [1]

The usual way of representing the results of the above calculations is by plotting the minimum standard deviation for each value of E_P as a curve in expected return – standard deviation ($E - \sigma$) space. [1]

In this space, with expected return on the vertical axis, the efficient frontier is the part of the curve lying above the point of the global minimum of standard deviation. [1]

Any portfolio on this efficient frontier is an efficient portfolio. [1]
[Max 5]

(iii) Where all assets are independent, the covariance between them is zero and the formula for variance becomes:

$$V = \Sigma_i \; x_i^2 \; V_i \, . \qquad [1]$$

If we assume that equal amounts are invested in each asset, then with N assets the proportion invested in each is $1/N$. Thus:

$$V = \Sigma_i \; (1/N)^2 \; V_i \qquad [1]$$
$$= 1/N[\Sigma_i \; V_i/N] = 1/N \, \bar{V} \qquad [1]$$

where $\bar{V}$ represents the average variance of the stocks in the portfolio.

As N gets larger and larger, the variance of the portfolio approaches zero. [1]
[Max 3]

(iv) With equal investment, the proportion invested in any one asset x_i is $1/N$ and the formula for the variance of the portfolio becomes:

$$V = \Sigma_i \ (1/N)^2 \ V_i + \Sigma_i \Sigma_j \ (1/N)(1/N).C_{ij}\,. \qquad [1]$$

Factoring out $1/N$ from the first summation and $(N - 1)/N$ from the second yields:

$$V = 1/N \, \Sigma_i \ V_i \, /N + (N - 1)/N \, \Sigma_i \Sigma_j \ C_{ij}/N(N - 1). \qquad [1]$$

Replacing the summation by averages we have:

$$V = 1/N \ V_i + (N - 1)/N.\bar{C}. \qquad [1]$$

The contribution to the portfolio variance of the variances of the individual securities goes to zero as N gets very large. [1]

This shows that the individual risk of securities can be diversified away. [1]

The contribution of the covariance terms approaches the average covariance as N gets large. However, this does not represent specific risk i.e. risk relating to individual securities. [1]
[Max 3]
[TOTAL 12]

Early parts of this question were largely completed well. The majority of the students proceeded without problems although a few provided answers only for the general case of dependent assets. Some students answered parts (iii) and (iv) using the single index model despite the question being clear that mean-variance portfolio theory was being examined.

Q4 (i) The composition of the market portfolio is as follows:

Market capitalisation	30,000	70,000	
w_i	0.3	0.7	[Total 1]

(ii) Mean returns: Asset 2: 21.8% Asset 3: 14.9% [1]

Consequently:

$$Er_M = \sum_{i=2}^{3} w_i Er_i = 16.97\% \qquad [1]$$

$$\text{std. dev}\left(r_M\right) = \sigma_M = \sqrt{E\left[\left(\sum_{i=2}^{3} w_i r_i\right)^2\right] - \left(Er_M\right)^2} = 3.57\%\,. \qquad [2]$$

The market price of risk is given by $\left(Er_M - r_f\right)/\sigma_M$ [1]

And since the risk-free rate is 5.0%, this equates to:

(0.1697 – 0.05)/0.0357 = 3.35 [1]

[Max 5]

(iii) From the Security Market Line it follows that $\beta_i = \left(Er_i - r_f\right)/\left(Er_M - r_f\right)$. [1]

Hence $\beta_2 = 1.40$ and $\beta_3 = 0.83$. [1 mark each]

[Max 2]

(iv) The assumptions made are unrealistic. [1]
Empirical studies do not provide strong support for the model. [1]
It does not account for taxes. [1]
Or inflation. [1]
Or situations in which there is no riskless asset. [1]
It does not consider multiple time periods. [1]
Or optimisation of consumption over time. [1]
Investors don't always use the same "currency" [1]
Markets are not always perfect [1]
Investors don't always have the same expectations [1]
Cannot lend/borrow unlimited amounts at the same risk-free rate [1]
Difficult to check as need to think about investment markets as well as capital markets [1]
Unrealistic to invest in the market portfolio in practice as so many stocks [1]

[Max 3]

[TOTAL 11]

Many students answered all parts of this question correctly. A few either made calculation mistakes, or did not cover a wide enough range of limitations of the CAPM. Some students confused calculating the market price of risk with the risk premium.

Q5 (i) The three forms are: Strong – market prices reflect all current information relevant to the stock, including information which is not public. [1]

Semi-strong – market prices reflect all current, publicly available information relevant to the stock. [1]

Weak – market prices reflect all information available in the past history of the stock price. [1]
[Total 3]

(ii) Tests need to make assumptions (which may be invalid) such as normality of returns or stationarity. [1]

Transaction costs may prevent the exploitation of anomalies, so that the EMH might hold net of transaction costs. [1]

Allowance for risk: the EMH does not preclude higher returns as a reward for risk; however the EMH does not tell us how to price such risks. [1]

Testing the strong form EMH is problematic as it requires access to information that is not in the public domain. [1]

It can be difficult to define "public information" or to determine exactly when information becomes public. [1]
It is impossible to test all of the possible trading rules that might be used by technical analysts. [1]

The assumptions made about how security prices should react to new information may be invalid. [1]
[Max 2]
[TOTAL 5]

Standard bookwork question which was largely well answered. Some students referred to the investor knowing the information rather than the security price reflecting the information or that the security price reflected "only" the relevant information rather than "all" relevant information.

Q6 (i) This portfolio is described as *self-financing* if $dV(t)$ is equal to $\phi_t \, dS_t + \psi_t \, dB_i$. That is, at $t + dt$, there is no inflow or outflow of money necessary to make the value of the portfolio back up to $V(t + dt)$. [Total 2]

(ii) Consider two self-financing portfolios:

- Portfolio A: holding the call (long position) and a sold put (short position) at time t. [1]

  Its value at time t is $C_t - P_t$ [1]

and at time T, it is $S_T - K$. [1]

- Portfolio B: holding a fraction $e^{-\delta(T-t)}$ of the underlying asset for $S_t\, e^{-\delta(T-t)}$ and shorting (borrowing) cash of $Ke^{-r(T-t)}$ at time t. [1]

  Its value at time t is then $S_t\, e^{-\delta(T-t)} - Ke^{-r(T-t)}$. [1]

  Its value at maturity is then $S_T - K$ by taking into account the dividends which are paid continuously at rate δ. [1]

By the principle of no arbitrage… [1]
… both portfolios must have the same value at all time t, since they have the same value at time T. [1]

Hence: $C_t - P_t = S_t\, e^{-\delta(T-t)} - Ke^{-r(T-t)}$ [1]

[Max 6]

[TOTAL 8]

> Standard bookwork question. The majority of the students answered correctly although quite a few did not justify their argument on the basis of the no arbitrage principle. Alternative valid approaches (including different portfolio combinations) were of course acceptable.

Q7 (i) Let K be the forward price. Now compare the setting up of the following portfolios at time 0:

A: one long forward contract. [1]
B: borrow Ke^{-rT} cash and buy one share at S_0. [1]

If we hold both of these portfolios up to time T then both have a value of $S_T - K$ at T. [1]

By the principle of no arbitrage… [1]
… these portfolios must have the same value at all times before T. [1]

In particular, at time 0 both portfolios must have value zero (since the value of a forward contract at $t = 0$ is zero). [1]

Since portfolio B has value $S_0 - Ke^{-rT}$ at $t = 0$, this can only be zero if $K = S_0 e^{rT}$. [1]

[Max 5]

(ii) $K = €20 \times e^{2\times 0.01}$ [1]
$= €20.40$ [1]

[Max 1]

(iii) Value = $(S_t e^{r(2-t)} - 20.40)e^{-r(2-t)} = S_t - 20.40e^{r(t-2)}$. [Total 2]

(iv) Using (iii), we get value = €20 – €20.40$e^{-0.04}$ [1]
= €0.40 at time 1 [1]
[Max 1]

(v) Using (iii):

delta = $d/dS_t(S_t - 20.40e^{r(t-2)})$ [1]
[½ mark for the definition of the greek, ½ mark for the actual formula]
= 1 [1]

theta = $d/dt(S_t - 20.40e^{r(t-2)})$ [1]
[½ mark for the definition of the greek, ½ mark for the actual formula]

= $-20.40\, r\, t\, e^{r(t-2)} = -0.784$ at $t = 1$ [1]
vega = 0 [1]
as the value does not depend directly on the volatility of the share [1]
[Max 3]
[TOTAL 12]

Early parts of this question were answered well. The majority of the students though confused the forward price (i.e. the delivery price) with the value to the investor of the forward contract in part (iii), and consequently struggled with the remaining parts, even if the large majority knew the definition of each Greek.

Q8 (i) (a) The market is arbitrage free if and only if there exists a probability measure under which discounted asset prices are martingales. [1]

In this case, the probability exists if and only if $d < e^{r\Delta t} < u$. [1]

(b) $d = 0.95 < e^{0.04} < 1.1 = u$ hence the condition is verified. [1]
[Max 2]

(ii)

Stock tree

Time	*0*	*1*	*2*	*3*
	100.00	110.00	121.00	133.10
		95.00	104.50	114.95
			90.25	99.28
				85.74

[½ mark for each of the prices of the stock at time 3]

The price C_0 of the option is computed via risk-neutral valuation. [½]

Let $\hat{p}$ denote the risk-neutral probability of an up movement, then:

$$\hat{p} = \{\exp(0.04) - 0.95\} \,/\, \{1.1 - 0.95\} = 0.6054 \qquad [1]$$

and

$$C_0 = e^{-rT} \sum_{k=0}^{3} \binom{3}{k} \hat{p}^k (1-\hat{p})^{3-k} \left(S_0 u^k d^{3-k} - K\right)^+$$

$$= e^{-rT} \left(\hat{p}^3 \times 30.10 + 3\hat{p}^2 (1-\hat{p}) \times 11.95\right) = 10.52. \qquad [4]$$

[Max 4]

For information, the detailed tree-based workings are provided below:

CALL

Time	*0*	*1*	*2*	*3*
	10.52	15.45	22.04	30.10
		4.04	6.95	11.95
			0.00	0.00
				0.00

(iii) As the premium is set so that the option price is zero, by risk-neutral valuation it follows that:

$$c = e^{rT} \frac{C_0}{\hat{P}(S_T > K)} \qquad [2]$$

$$= e^{rT} \frac{C_0}{\hat{p}^3 + 3\hat{p}^2(1-\hat{p})} \qquad [2]$$

$$= 18.09 \qquad [1]$$

[Max 3]

Alternatively for the above:

From the figures in part (ii), must have:

$[(30.1 - c) \times \hat{p}^3 + (11.95 - c) \times 3\,\hat{p}^2\,(1 - \hat{p})] \times \exp\text{-}(3 \times 0.04) = 0$ [2]

$c = [30.1 \times \hat{p}^3 + 11.95 \times 3\,\hat{p}^2\,(1 - \hat{p})]\,/\,[\,\hat{p}^3 + 3\,\hat{p}^2\,(1 - \hat{p})]$ [2]

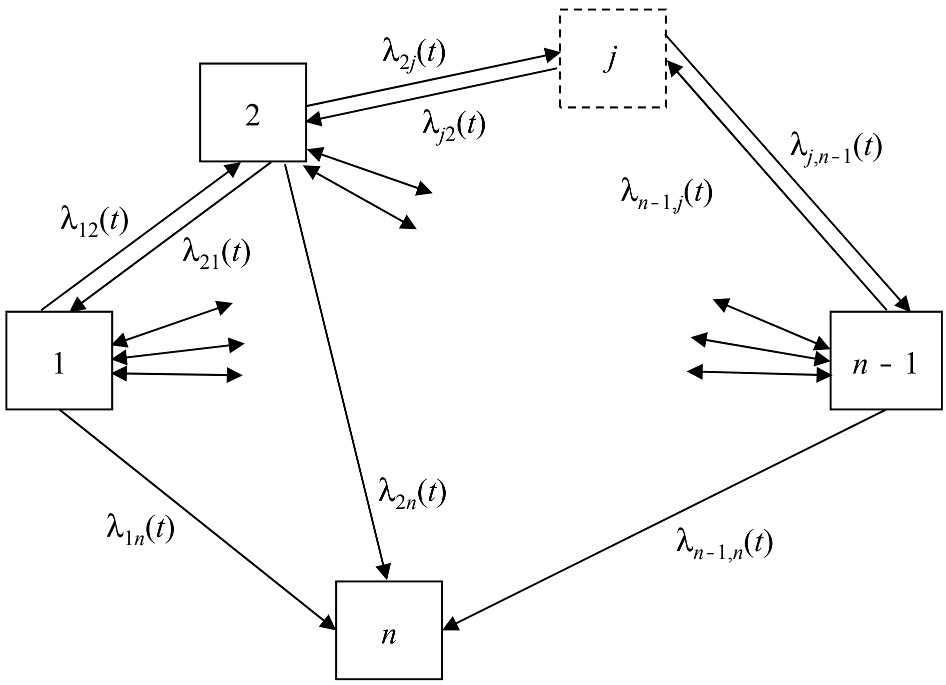
= 18.09 [1]

[Max 3]

[TOTAL 9]

Generally well answered, although some made calculation mistakes in obtaining the prices of the given option contracts. In particular, too many students simply used 1.04 for exp(0.04), which is of course not correct – a simple check with the calculator would have shown this. The majority of students adopted the correct approach to solve part (iii).

Q9 (i)



[3 marks for diagram]

The n states represent $n - 1$ credit ratings plus default. [1]

$\lambda_{ij}(t)$ are the deterministic transition intensities from state i to state j at time t under the real world measure P. [1]

[Max 4]

[1½ marks for diagram applied to specific example – 3 states model]

(ii) $p_D(2) = p_{HD} + p_{HH} \times p_{HD} + p_{HU} \times p_{UD}$ [1]
$= 0.02 + (1 – 0.1 – 0.02) \times 0.02 + 0.1 \times 0.3$ [1]
$= 0.0676$ [1]
[Max 2]

(iii) £87.63 $= e^{-2\times 0.04}((1 - p_D(2)) \times 100 + p_D(2) \times 100x)$ [1]
$= e^{-2\times 0.04}((1 – 0.0676) \times 100 + 0.0676 \times 100x)$ [1]
so $x = 25\%$ [1]
[Max 2]

(iv) $87.63 = 100e^{-2\times(r+c)}$ where c is the credit spread. [1]
So $c = 2.6\%$. [1]
[Max 1]

(v) The impact on cashflows would be that the bond might return the x% of its nominal value earlier than time 2, so the value of the bond would increase.
[Total 1]
[TOTAL 10]

Generally, students answered this question correctly, although in the first part quite a few considered only the particular case of the three states given in the rest of the question (but which had not yet been introduced for part (i)). A few students used the Merton formula for default to solve part (iii) of the question, which was not appropriate for this model.

Q10 (i) By substitution and direct integration between 0 and t:

$$R_t = \theta t\,. \quad [1]$$

$$+\left(r_0 - \theta\right)\frac{\left(1-e^{-kt}\right)}{k} \quad [2]$$

$$+\frac{\sigma}{k}\int_0^t \left(1-e^{-k(t-s)}\right)\,dW_s \quad [3]$$

In virtue of the properties of the stochastic integral, R_t follows a Normal distribution [1]
with the given mean (as the integral has zero mean) [1]
and the given variance – see Result 3.2 in Core Reading (Ito isometry). [1]
[Max 6]

(ii) From risk-neutral valuation, the price of a bond is given by:

$$P(0,t) = E\left(e^{-\int_0^t r_s ds} \right) \quad [2]$$

$$= E\left(e^{-R_t}\right) \quad [1]$$

which is the first moment of $\exp(-R_t)$. [1]

As $-R_t$ is normally distributed, the moment generating function gives the first moment of $-R_t$ as required:

$$= e^{-E(R_t)+\frac{\mathrm{Var}(R_t)}{2}}. \quad [2]$$

Equivalently, use the results regarding the mean of the lognormal random variable as per the Formulae & Tables.

[Max 3]

(iii) Notice that the variance can be written as:

$$\mathrm{Var}(R_t) = \frac{\sigma^2}{k^2}\left(t - B(t) - \frac{1-e^{-kt}}{k} + \frac{1-e^{-2kt}}{2k}\right) \quad [2]$$

$$= -\frac{\sigma^2}{k^2}\left(B(t)-t\right) + \frac{\sigma^2}{2k}\left(-2\frac{1-e^{-kt}}{k^2} + \frac{1-e^{-2kt}}{k^2}\right)$$

$$= -\frac{\sigma^2}{k^2}\left(B(t)-t\right) - \frac{\sigma^2}{2k}B(t)^2.$$

The result follows by substitution. [2]

[Max 4]

(iv) The main drawback of the Vasicek model is that the short rate can take negative values with positive probability. [1]

[TOTAL 14]

There was a wide range of quality of answers for this question. Generally, students answered correctly the first and last parts of this question. Many students managed to get through a few steps for part (ii), though often with algebra issues; whilst in part (iii) there were relatively few comprehensive attempts.

END OF EXAMINERS' REPORT