EXAMINATION

7 April 2005 (am)

Subject CT8 — Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

1 An investor holds a portfolio consisting of N assets in equal proportions.

Derive an expression for the variance of the portfolio as N gets very large.

[You may assume that all assets have variance less than a certain level V_{max}. You may also assume that the average covariance is $\bar{c}$.]

[6]

2 An investor wishes to measure the investment risk presented by an asset which has the following distribution:

State	*Return*	*Probability*
1	10%	0.5
2	20%	0.3
3	50%	0.2

(i) Evaluate three different measures of investment risk for this asset. Where necessary, you may assume a benchmark return of 25%. [4]

(ii) (a) State two key properties of Value at Risk (VaR).

(b) VaR is frequently calculated assuming a normal distribution of returns. State an advantage and a disadvantage of this approach.

[2]

[Total 6]

3 An investor has the choice of the following assets that earn rates of return as follows in each of the four possible states of the world:

State	*Probability*	*Asset 1*	*Asset 2*	*Asset 3*
1	0.2	5%	5%	6%
2	0.3	5%	12%	5%
3	0.1	5%	3%	4%
4	0.4	5%	1%	7%
Market capitalisation		10,000	17,546	82,454

Determine the market price of risk assuming CAPM holds.

Define all terms used. [6]

4 Let R_i denote the return on security i given by the following multifactor model

$$R_i = a_i + b_{i,1} I_1 + b_{i,2} I_2 + \ldots + b_{i,L} I_L + c_i$$

a_i and c_i are the constant and random parts respectively of the component of the return unique to security i.

$I_1, \ldots I_L$ are the changes in a set of L indices.

$b_{i,k}$ is the sensitivity of security i to factor k.

(i) State the category of the above model where:

(a) index 1 is a price index
index 2 is the yield on government bonds
index 3 is the annual rate of economic growth

(b) index 1 is the level of Research and Development expenditure
index 2 is the price earnings ratio

[2]

(ii) Determine the number of parameters to be estimated in a single index model and in a multifactor model. [4]

[Total 6]

5 The following unusual model has been proposed for the (real-world) stochastic behaviour of the short term interest rate:

$$dr_t = \mu r_t \, dt + \sigma dZ_t,$$

where $\mu > 0$ and σ are fixed parameters and Z is a standard Brownian motion under the proposed real-world measure P.

Under the same measure P, a (zero coupon) bond with maturity T has price at time t $B(t, T) = \exp(-(T - t)\, r_t + \sigma^2 (T - t)^3/6)$.

(a) Derive the SDE satisfied by $B(t, T)$.

(b) Determine the market price of risk and deduce the corresponding SDE for r, under the risk neutral measure Q.

[7]

6 One particular company over which an investment bank writes European call options has experienced a severe fall in its share price. However, analysts have not revised their expectation that the share price will grow to £4 in six months. The table below shows the share price together with the price of the options.

Date	*Share price*	*Option price*
1 November	£3.00	£0.90
2 November	£2.00	£0.60

You may assume that the basic Black-Scholes framework is used to price the options.

(i) Explain why the option price has fallen even though the expected return has increased according to the analysts. [3]

(ii) State any requirements for the option price to have fallen to its level on 2 November. [3]

[Total 6]

7 (i) Outline the approach adopted by Shiller to test for "excessive volatility" and state the criticisms of his work. [7]

(ii) State one difficulty of testing the strong form of the efficient market hypothesis and state the general conclusion of studies carried out on it. [2]

[Total 9]

8 (i) State the martingale representation theorem, including conditions for its application, defining all terms used. [3]

Let S_t denote the price of an underlying security at time t; r denotes the risk free rate of return expressed in continuously compounded form, B_t represents an accumulated "bank account" at time t that earns the risk free rate of return.

Let X be any derivative payment contingent on F_T, payable at some fixed future time T, where F_T is the sigma algebra generated by S_u for $0 \le u \le T$.

You may assume that, under the equivalent measure Q, the process

$D_t = e^{-rt} S_t$ is a martingale

and that

$$dS_t = B_t(rD_t dt + dD_t)$$

(ii) Show that the value of this derivative at time $t < T$ is

$$V_t = e^{-r(T-t)} E_Q[X \mid F_t]$$ [11]

[Total 14]

9 (i) Describe the role that the inflation model plays within the Wilkie model. [3]

The Wilkie model proposes an AR(1) process for the continuously compounded rate of inflation $I(t)$ that can be written as:

$$I(t) = a + bI(t-1) + \varepsilon(t)$$

Where $\varepsilon(t) \sim N(0, t^2)$ and a and b are constants with $-1 < b < 1$.

(ii) Derive an expression for the long term average rate of inflation in terms of a and b. [1]

(iii) Explain why a model of the form above would not be suitable for share prices. [3]

(iv) Explain why a lognormal model may be used for share prices and state its weaknesses. [8]
[Total 15]

10 An investment bank has issued a derivative on a share (with share price, S, of 100) that provides for the following payoff after two months:

$$F(S) = \ln(S - 90) \quad \text{if } S > 90$$
$$= 0 \quad \text{otherwise}$$

You may assume that:

- There exists a risk free asset that earns 5% per month, continuously compounded.
- The expected effective rate of return on the share is 2% per month.
- The monthly standard deviation of the log share price is 10%.

(i) By using a two period recombining model of future share prices, derive the state price deflators at time 2. The parameters determining the share price after an up-jump and down-jump should be determined by considering the standard deviation of the log share price. [9]

(ii) Using the state price deflators from (i) derive the value at time zero of the option. [3]

The delta of this derivative at time zero is 7% and the gamma is 10%. The bank which issued the derivative wishes to delta hedge its position in the most efficient manner. Assume that the share price can also be modelled in continuous time with a geometric Brownian motion with volatility (diffusion parameter) of 0.1 consistent with a Black-Scholes framework.

(iii) Determine the delta hedging portfolio, as a combination of the risk free asset, the underlying share, and a European Call option on the share with term of 3 months and exercise price of 100. [13]
[Total 25]

END OF PAPER