INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

20 April 2011 (am)

Subject CT8 — Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 (i) State the six assumptions underlying the Black-Scholes market.

(ii) Give the four defining characteristics of a Brownian Motion Z, such that $Z_0 = 0$.

[5]

2 (i) State the main assumptions of modern portfolio theory. [2]

Three assets have the following characteristics:

Asset i	*Expected return E_i*	*Volatility σ_i*
1	4%	6%
2	6%	12%
3	8%	18%

The correlation between assets 1 and 2 is 0.75; while the correlation between asset 3 and both of the other two assets is zero.

(ii) (a) State the Lagrangian function that can be minimised to find the minimum variance portfolio associated with a given expected return, defining any notation used.

(b) By taking five partial derivatives of this function, calculate the minimum variance portfolio which yields an expected return of 5%.

[7]
[Total 9]

3 A securities market has only three risky securities, A, B and C with the following annual return attributes:

	Asset A	*Asset B*	*Asset C*
Market capitalisation	£100bn	£150bn	£250bn
Annual expected return	4%	r_B	6%

Assume that:

- the assumptions of the Capital Asset Pricing Model hold
- the market price of risk is 10% per annum
- the risk free rate is 3.3% per annum
- the expected annual return on the market portfolio is 5.3% per annum.

(i) Calculate σ_M, the standard deviation of the annual return on the market portfolio. Quote any results that you use. [1]

(ii) Calculate r_B, the expected annual return on asset *B*. [2]

(iii) Calculate the covariance of the annual returns on each asset with the annual return on the market portfolio. State any further results that you use. [4]
[Total 7]

4 (i) Outline the three forms of the efficient market hypothesis. [6]

XYZ has just announced that its profits are up by 52% on last year. On the announcement XYZ shares fell in price by 20%. Analysts had been predicting a rise in profits of 65%. A friend says that this shows that the efficient markets hypothesis is false.

(ii) Comment on this statement. [3]
[Total 9]

5 Assume that a non-dividend-paying security with price S_t at time t can move to either $S_t u$ or $S_t d$ at time $t + 1$. The continuously compounded rate of interest is r, and $u > e^r > d$. A financial derivative pays α if $S_{t+1} = S_t u$ and β if $S_{t+1} = S_t d$.

A portfolio of cash (amount x) and the underlying security (value y) at time t exactly replicates the payoff of the derivative at time $t + 1$.

(i) Derive expressions for x and y in terms of r, u, d, α and β. [4]

(ii) Derive an expression for the risk-neutral probability of the security having value $S_t u$ at time $t + 1$ in terms of $(x + y)$, r, α and β. [2]

Assume $S_t = 100$, $u = 1.25$, $d = 0.8$ and $r = 0$.

(iii) (a) Calculate the prices of at-the-money call and put options.
(b) Check that the put-call parity holds for this model.
[4]
[Total 10]

6 (i) Describe the lognormal model for security prices. [2]

A security price, S_t, is assumed to follow a lognormal model with drift $\mu = 4.28\%$ per annum and volatility 12% per annum. The price now is $S_0 = €1.83$. The continuously compounded risk-free rate of interest is 2% per annum.

(ii) Calculate, as at this date, the probability, p, that ($S_1 > €2.20$). [2]

Someone now offers you an option which will pay €1000 if and only if the stock price $S_1 > €2.20$. They propose to charge $€1000e^{-0.02}p$.

(iii) Explain whether or not you would buy this option. [4]

Assume now that the value of 4.28% for μ has been estimated from observations of the security price over 10 years using the estimator $\mu' = \{\log(S_0) - \log(S_{-10})\}/10$.

(iv) (a) Specify the distribution of $\mu' - \mu$.

(b) Deduce the probability that $\mu' - \mu > 3\%$. [4]

(v) (a) Explain how your answer to (iii) would change if you knew that $\mu < 1.28\%$ rather than 4.28%.

(b) Comment on this in the light of your answer to part (iv)(b). [3]

[Total 15]

7 (a) List five factors that effect the price of a European put option on a non-dividend paying share.

(b) State how the premium for a European put option would change if each of these factors increased.

[5]

8 Assume the Black-Scholes model applies.

(i) State an expression for the price of a derivative security with payoff D at maturity date T in terms of the risk-neutral measure. [2]

An at the money European call option on a stock has an exercise date one year away and a strike price of £118.57. The option is priced at £10. The continuously compounded risk-free rate is 1% per annum.

(ii) (a) Estimate the implied volatility to within 1% per annum.

(b) Calculate the corresponding hedging portfolio in shares and cash for 1000 options on the share, quoting any results that you use.

(c) Calculate the option's Vega.

[10]

(iii) Price a put on the same stock with the same expiry date and a strike price of £110. [2]

The hedging portfolio of the call option has the same value, the same Delta and the same Vega as the option.

The Delta of the put option is −0.29975 and its Vega is 39.435.

(iv) Determine the hedging portfolio of the call option in terms of shares, cash and the put option. [4]

[Total 18]

9 In an extension of the Merton model, a very highly geared company has two tiers of debt, a senior debt and a junior debt. Both consist of zero coupon bonds payable in three years time. The senior debt is paid before the junior debt.

Let F_t be the value of the company at time t, L_1 the nominal of the senior debt and L_2 the nominal of the junior debt.

(i) (a) State the value of the senior debt at maturity.
(b) Deduce the value of the junior debt at maturity.

[4]

The current gross value of the company is £3.2m. The nominal of the senior debt is £1.2m and that of the junior debt is £2m. The continuously compounded risk-free rate is 4% per annum, the volatility of the value of the company is 30% per annum and the price of £100 nominal of the senior bond is £88.26.

(ii) Calculate the theoretical price of £100 nominal of the junior debt. [6]

[Total 10]

10 Let $B(t,T)$ be the price at time t of a zero-coupon bond paying £1 at time T, r_t be the short-rate of interest, $\mathbb{P}$ be the real world probability measure and $\mathbb{Q}$ the risk neutral probability measure.

(i) Write down two equations for the price of a zero-coupon bond, one of which uses the risk-neutral approach to pricing and the other of which uses the state-price-deflator approach to pricing. [2]

(ii) State the Stochastic Differential Equation (SDE) of the short rate r_t under Q for the Vasicek model and the general type of process this SDE represents. [3]

(iii) Solve the SDE for the short rate r_t from (ii). [5]

(iv) Deduce the form of the distribution of the zero-coupon bond price under this model. [2]

[Total 12]

END OF PAPER