EXAMINATION

16 April 2008 (am)

Subject CT8 — Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Define Gamma and Vega for a derivative written on a portfolio of assets in a market where the assumptions underpinning the Black-Scholes model hold. [3]

2 State the stochastic differential equation for geometric Brownian motion and its solution. (No proof is required.) [4]

3 Consider a two-period Binomial model of a stock whose current price $S_0 = 100$. Suppose that:

- over each of the next two periods, the stock price can either move up by 10% or move down by 10%
- the continuously compounded risk-free rate is $r = 8\%$ per period

(i) Show that there is no arbitrage in the market. [1]

(ii) Calculate the price of a one-year European call option with a strike price $K = 100$. [4]
[Total 5]

4 (i) Outline the two-state model for credit ratings assuming constant transition intensity. [5]

(ii) State the formula for the zero-coupon bond price in terms of the risk-neutral default rate λ, when this rate is deterministic. [3]
[Total 8]

5 An investor is considering investing in one of two assets. The distribution of returns from each asset is shown below:

Asset 1		*Asset 2*	
Return (%)	*Probability (%)*	*Return (%)*	*Probability (%)*
-1	$8^1/_3$	0	50
11	$91^2/_3$	20	50

(i) Calculate for each asset:

(a) the variance
(b) semi-variance
(c) and shortfall probability

Where necessary assume a benchmark return of 0%. [4]

(ii) Explain which asset an investor with a quadratic utility function would choose. [2]

(iii) State the reasons why variance of return is frequently used as a measure of risk. [3]
[Total 9]

6 (i) Outline the assumptions used in modern portfolio theory regarding investor behaviour that are necessary to specify efficient portfolios. [3]

(ii) An investor can construct a portfolio using only two assets X and Y. The statistical properties of the two assets are shown below:

	X	*Y*
Expected return	12%	8%
Variance of return	30%	15%
Correlation coefficient between assets *X* and *Y*	0.5	

Assuming that the investor cannot borrow to invest:

(a) Determine the composition of the portfolio which will give the investor the highest expected return.

(b) Calculate the composition of the portfolio which will give the investor the minimum variance.

[3]

(iii) Explain and sketch how the investor would choose a utility maximising portfolio. [3]

[Total 9]

7 (i) State the assumptions, additional to those used in modern portfolio theory, that allow the capital asset pricing model (CAPM) to be consistent with an equilibrium model of prices in the whole market. [5]

(ii) Explain why in the CAPM all investors should hold all risky assets in proportion to the market capitalisation of those assets. [2]

In an investment market there are three risky assets available. The table below shows the returns each of the assets will earn in the three possible states of the world and the current market capitalisation of the assets. Assume a risk free rate of return of 4% is available.

States	*Probability*	*Asset 1*	*Asset 2*	*Asset 3*
1	0.4	5%	6%	7%
2	0.1	8%	2%	1%
3	0.5	3%	5%	4%
Market Capitalisation		30,000	50,000	30,000

(iii) Calculate the market price of risk under the CAPM. [4]

[Total 11]

8 Consider a continuous time log-normal model for a security price, S, with parameters μ and σ.

(i) Write down formulae for:

(a) the log-return of the process
(b) the expected value of an investment at a specified future time
(c) the variance of the value of an investment at a specified future time

[3]

(ii) Explain what the model implies about market efficiency. [2]

(iii) Outline the empirical evidence for and against the model. [5]

[Total 10]

9 Consider two call options, which are identical (same maturity, same underlying asset) except for the strike price. Denote by $C(K)$ the price at time 0 of the call option with strike price K. Stating the key arguments required, prove that, if there are no arbitrage opportunities, the following relation holds true, for $K_1 \le K_2$.

$$\forall \lambda \in [0,1] \quad \lambda C(K_1) + (1-\lambda)C(K_2) \ge C(\lambda K_1 + (1-\lambda)K_2)$$ [10]

10 In a situation where the zero-coupon bond market is arbitrage-free and complete, consider the following Vasicek model for the short-rate process:

$$dr(t) = a(b - r(t))dt + \sigma dW_t$$

where $(W_t; t \ge 0)$ is a standard Brownian motion with respect to the risk-neutral probability measure **Q**.

(i) State the general expression $r(t)$ of the solution of this stochastic differential equation. [2]

(ii) Derive an expression for $\int_t^T r(u)du$, where t and T are given.

Hint: consider the stochastic differential equation of $r(u)$, for $u \ge t$.

[6]

(iii) State the distribution of $\int_t^T r(u)du$. [1]

(iv) Derive the price of a zero-coupon bond at time t with maturity $T \ge t$ related to the distribution of $\int_t^T r(u)du$. [6]

[Total 15]

11 A stock is currently priced at €8.20. A writer of 100,000 units of a one year European call option on this stock with an exercise price of €8 has hedged the option with a portfolio of 75,000 shares and a loan. The annual risk-free interest rate (continuously compounded) is 7% and no dividends are payable during the life of the option.

Assume the Black-Scholes pricing formula applies.

(i) (a) Derive an expression for the Delta of the option.
(b) State the value of the Delta in this case.
[4]

(ii) Calculate the implied volatility of the stock to within 0.1% p.a., assuming that it is below 100%. [5]

(iii) Calculate:

(a) the value of the loan
(b) the price of the option [4]

(iv) (a) Calculate the current price of a one year European put option with the same exercise price.

(b) State any assumptions you make in your calculation in (iv)(a).
[3]
[Total 16]

END OF PAPER