INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

17 April 2013 (pm)

Subject CT8 – Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 List the key advantages and disadvantages of the following measures of investment risk in the context of a portfolio of bonds subject to credit risk:

- Variance of return
- Downside semi-variance of return
- Shortfall probability
- Value at Risk
- Tail Value at Risk

[10]

2 Consider a mean-variance portfolio model with two securities, S_A and S_B, where the expected return and the variance of return for S_B are twice the corresponding values for S_A. Suppose the correlation between the returns on the two securities is ρ.

(i) (a) Determine the values of ρ which allow the possibility of constructing a zero-risk portfolio, by calculating the variance of the return on a portfolio with weights x_A and x_B invested in the two assets.

(b) Calculate the portfolio weights that lead to the most efficient zero-risk portfolio.

(c) Calculate the expected return on the portfolio in part (i)(b) in terms of the expected return on S_A.

[5]

(ii) Calculate the maximum expected return for an investor:

(a) if portfolio weights are unlimited.

(b) if the investor can short sell at most one unit of either security and the total he has to invest is one unit.

[2]

(iii) Calculate the expected return on the minimum variance portfolio if the covariance between the two securities is 60% of the variance of S_A. [2]

[Total 9]

3 An analyst states that "It is common practice in actuarial modelling to use the same data set to specify the model structure, to fit the parameters, and to validate the model choice. A large number of possible model structures are tested, and testing stops when a model is found which passes a suitable array of tests."

Indicate, giving evidence and examples, why this procedure may be inappropriate.

[5]

4 In a market where the CAPM holds there are five assets with the following attributes.

Asset	*A*	*B*	*C*	*D*	*E*	*Probability of being in state*
Annual return in						
State 1	3%	3%	3%	3%	3%	0.25
State 2	5%	7%	2%	8%	3%	0.5
State 3	7%	5%	8%	1%	3%	0.25
Market Capitalisation	10m	20m	40m	30m		

(i) Calculate the expected annual return on the market portfolio and σ_M, the standard deviation of the annual return on the market portfolio. [4]

(ii) Calculate the market price of risk under CAPM. [2]

(iii) Calculate the beta of each asset. [6]

(iv) Outline the limitations of the CAPM. [3]
[Total 15]

5 (i) State the five key features of a standard Brownian motion B_t. [5]

Consider a stochastic differential equation

$$dX_t = Y_t dB_t + A_t dt,$$

where A_t is a deterministic process and Y_t is a process adapted to the natural filtration of B_t.

(ii) Write down Ito's lemma for $f(t, X_t)$, where f is a suitable function. [2]

(iii) Determine $df(t, X_t)$ where $f(t, X_t) = e^{2tX_t}$. [2]
[Total 9]

6 Suppose that at time t we hold the portfolio (a_t, b_t, c_t) where a_t, b_t and c_t represent the number of units held at time t of securities with respective price processes A_t, B_t and C_t. Assume (a_t, b_t, c_t) are previsible. Let V_t be the value of this portfolio at time t.

(i) Explain what it means for (a_t, b_t, c_t) to be previsible. [1]

(ii) Write down an equation for the instantaneous change in the value of the portfolio, including cash inflows and outflows, at time t. [2]

(iii) Give the condition for this portfolio to be self-financing. [2]

(iv) Define a replicating strategy for a derivative with payoff X at a future time U, contingent on the path taken by a_t, b_t and c_t. [2]

(v) Describe how the no-arbitrage condition and a self-financing strategy can be used to value the derivative in (iv) at time 0. [2]

(vi) Give a condition for the market to be complete. [1]
[Total 10]

7 A non-dividend-paying stock in an arbitrage-free market has a current price of 150p. Over each of the next two years its price will either be multiplied by a factor of 1.2 or divided by 1.2. The continuously compounded risk-free rate is 1% p.a. The value of an option on the stock is 50p.

Denote by P_{uu} the value of the payoff if both stock price moves are up, P_{ud} for the value of the payoff if one move is up and one is down (this is the same whichever order the price moves occur), and P_{dd} for the value of the payoff if both stock price moves are down. The price of the stock is to be modelled using a binomial tree approach with annual time steps.

(i) Derive, and simplify an equation for P_{uu} in terms of P_{ud} and P_{dd}. [4]

(ii) Calculate, using your answer to part (i), or otherwise, the range of values that P_{uu} could take. [2]

(iii) Determine the value of the option in each of the two cases below, assuming that P_{uu} takes its maximum possible value:

(a) If the first stock price move is up.
(b) If the first stock price move is down.

[3]
[Total 9]

8 (i) Describe three limitations of one-factor term structure models. [5]

(ii) Write down, defining all terms and notation used, the two-factor Vasicek model. [3]
[Total 8]

9 In a Black-Scholes market, we consider a special option with strike K and expiry in 2 years on an underlying (non-dividend bearing) stock with price process S_t. Its payoff at maturity is $100\text{Max}(S_2/S_1 - 1;0)$ if and only if the stock price has not exceeded \$2 by time 1. The volatility of the stock is 25% p.a. and the continuously compounded risk-free rate is 3% p.a. The initial stock price is \$1.

(i) Calculate $Q(\text{Max}_{t<1}S_t < 2)$, where Q is the EMM, using the formula in the actuarial tables and the representation of a geometric Brownian Motion. [3]

(ii) (a) Write down an expression for the price of this option at time 1. You should consider separately the two cases $(\text{Max}_{t<1}S_t) < 2$ and $(\text{Max}_{t<1}S_t) \geq 2$.

(b) Show that the value of this option at time 1 is \$11.348 in the case $(\text{Max}_{t<1} S_t) < 2$.

Hint: S_2/S_1 is independent of the values of S_t up to time 1 under the EMM.

(c) Determine, using the result in (i), the fair price at time 0 for the option. [9]
[Total 12]

10 (i) Describe the two-state model for credit defaults. [4]

Company A's bonds are modelled according to a two-state model. Company A has two zero-coupon bonds in issue, both with a recovery rate of $\delta = 60\%$. Bond 1 matures in one year, bond 2 in two years' time. Bond 1 has a continuously compounded credit spread of 4%, bond 2 has a continuously compounded credit spread of 5%. The continuously compounded risk-free rate is 1.5% p.a.

(ii) (a) Calculate the price per \$100 nominal of each bond in one year and in two years' time.

(b) Deduce the implied risk-neutral probabilities of no default in one year and in two years' time. [6]

(iii) Determine the implied values of the default intensities, assuming that they are constant for each of the two years. [3]
[Total 13]

END OF PAPER