# EXAMINATION

April 2006

## Subject CT8 — Financial Economics
## Core Technical

EXAMINERS' REPORT

**Introduction**

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

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Chairman of the Board of Examiners

June 2006

**Comments**

Please see individual comments on questions 4 and 6. No other comments given.

**1** (i) Let $V = V_A\alpha_A^2 + (1-\alpha_A)^2 V_B + 2\alpha_A(1-\alpha_A)C_{AB}$

$$\frac{\partial V}{\partial \alpha_A} = V_A 2\alpha_A - 2(1-\alpha_A)V_B + (2(1-\alpha_A) - 2\alpha_A)C_{AB}$$

Set equal to zero gives

$$0 = V_A 2\alpha_A - (2-2\alpha_A)V_B + 2(1-2\alpha_A)C_{AB}$$

$$0 = \alpha_A(V_A + V_B - 2C_{AB}) - V_B + C_{AB}$$

$$\Rightarrow \alpha_A = \frac{V_B - C_{AB}}{V_A + V_B - 2C_{AB}}$$

In this case

$$\alpha_A = \frac{12\%\% - 0.25\times(24\%\%\times 12\%\%)^{½}}{24\%\% + 12\%\% - 2\times 0.25\times(24\%\%\times 12\%\%)^{½}}$$

= 28.2%

$\Rightarrow$ 71.8% of asset B

(ii) As a portfolio is diversified, the return on the portfolio is less exposed to the specific risk of any one component.

This means that as portfolios are diversified the correlation components become less important, therefore variance of return is minimised.

**2** (i) The market price of risk is $(E_m - r)/\sigma_m$.

Asset $C$ is the risk free asset therefore $r$ = 5%.

$E_m$ = (10,000 × (5% × 0.3 + 4% × 0.2 + 7% × 0.5) +

20,000 × (5% × 0.3 + 7% × 0.2 + 3% × 0.5) +

10,000 × (2% × 0.3 + 6% × 0.2 + 9% × 0.5)) ÷ 40,000

= 5.225%

$$\sigma_m^2 = \left(\frac{10,000\times 5\% + 20,000\times 5\% + 10,000\times 2\%}{40,000} - 5.225\%\right)^2 \times 0.3 +$$

$$\left(\frac{10,000\times 4\% + 20,000\times 7\% + 10,000\times 6\%}{40,000} - 5.225\%\right)^2 \times 0.2 +$$

$$\left(\frac{10,000\times 7\% + 20,000\times 3\% + 10,000\times 9\%}{40,000} - 5.225\%\right)^2 \times 0.5$$

$= 4.4312 \times 10^{-5} = 0.66567\%^2$

$\Rightarrow$ The market price of risk is (5.225% –5%) / 0.66567%

= 33.8%.

(ii) Empirical studies do not provide strong support for the model.

It does not account for taxes, inflation or where there is no riskless asset.

It does not consider multiple time periods or optimisation of consumption over time.

**3** (i) Credit should be awarded for any five from the following:

(a) $B_t$ has <u>independent increments</u>, i.e. $B_t - B_s$ is independent of $\{B_r, r \le s\}$ whenever $s < t$.

(b) $B_t$ has <u>stationary increments</u>, i.e. the distribution of $B_t - B_s$ depends only on $t - s$.

(c) $B_t$ has <u>Gaussian increments</u>, i.e. the distribution of $B_t - B_s$ is $N(0, t - s)$.

(d) $B_t$ has <u>continuous sample paths</u> $t \to B_t$.

(e) $B_0 = 0$.

(f) $\mathrm{Cov}(B_s, B_t) = \min(s, t)$ since, for $s > t$, $\mathrm{Cov}(B_t, B_t) = t$ and $\mathrm{Cov}(B_s - B_t, B_t) = 0$.

(g) $\{B_t, t \ge 0\}$ is a Markov process: this follows directly from the independent increment property.

(h) $\{B_t, t \ge 0\}$ is a martingale: 0.1 demonstrates that $E(B_s \mid F_t) = B_t$.

(i) $\{B_t, t \geq 0\}$ returns infinitely often to 0, or indeed to any other level.

(j) If $\{B_1(t), t \geq 0\}$ is defined by

$$B_1(t) = \frac{1}{\sqrt{c}} B_{ct}$$

then $\{B_1(t), t \geq 0\}$ is also a standard Brownian motion. (This is the *scaling property* of Brownian motion.)

(k) If $\{B_2(t), t \geq 0\}$ is defined by

$$B_2(t) = tB_{1/t}$$

then $\{B_2(t), t \geq 0\}$ is also a standard Brownian motion. (This is the *time inversion property* of Brownian motion.)

(ii) **Advantages**

- The mean and variance of return are proportional to the length of the time interval considered.
- Returns over non-overlapping time intervals are independent of each other.
- Cannot use past history to identify whether prices are cheap or dear implying weak form market efficiency consistent with empirical observations.
- Does not permit negative share prices.

**Disadvantages**

- Estimates of volatility vary widely over time periods. This is supported by implied volatility from option prices.
- The model is not mean reverting, which is contradicted by some evidence of momentum effects and reversion after market crashes.
- Does not reflect jumps and discontinuities observed in the market.

**4** (i)

- The model should be arbitrage free.
- Interest rates should be positive.
- Interest rates should exhibit some element of mean reversion.
- The model should be computationally tractable.
- Gives a reasonable range of possible yield curves.
- Fits historical data.
- Can be calibrated to current market data.
- Flexible to cope with a range of derivatives.

***Comments on question 4(ii):***

*Please note that there was a typographical error in this question where the first "T" in the expression should have been a "minus" sign.*

*Candidates who identified this and proceeded on the assumption that there had been a typographical error were given appropriate credit, as were candidates who noted that the expression led to inconsistencies. The "solution" below shows the correct technical approach applied to the question as it stood. All candidates' scripts were assessed to take into account any additional impact of this error.*

(ii) Using Ito's lemma:

$$dB(t, T) = B(t, T)\,(-Tr_t dt - ½\sigma^2(T-t)^2 dt + T(T-t)\,dr_t + ½T^2(T-t)^2\sigma^2 dt)$$

(a) Therefore as the market price of risk is

$$\gamma(t, T) = (m(t, T) - r_t)\,/\,s(t, T)$$

where $dB(t, T) = B(t, T)\,(m(t, T)\,dt + s(t, T)\,dz_t)$

$$m(t, T) = T((T-t)\,\mu - 1)\,r_t + ½(T-t)^2\,\sigma^2(T^2-1)$$

$$s(t, T) = \sigma(T-t)\,.\,T$$

Therefore $\gamma(t) = \dfrac{(r_t(T(T-t)\mu - 2) + ½(T-t)^2\sigma^2(T^2-1))}{T(T-t)\sigma}$

(b) SDE for $r_t$ is $dr_t = \sigma d\tilde{z}_t$ under the risk neutral measure $Q$

$$dr_t = \frac{r_t(\mu - (T(T-t)\mu - 2) + ½(T-t)^2\sigma^2(T^2-1))dt}{T(T-t)} + \sigma d\tilde{z}_t$$

**5** (i) If a market is strong form efficient then senior managers could not make abnormal profits. The existence of a ban suggests abnormal profits could be made, which suggests markets are not strong form efficient.

(ii) Different stock exchanges have different disclosure levels therefore different markets have different levels of efficiency.

Even if information is publicly available, there is a cost and possibly a time delay to obtain it. This erodes the advantages of obtaining the information.

Individuals do not have access to the management of companies that fund managers do. They spend time and money interviewing senior management.

(iii) In aggregate, active managers hold the market, which is identical to the passive manager. Therefore the average active manager should perform in line with the passive manager.

If markets are inefficient, we would expect active managers with above average skill to perform better than passive managers.

The existence of active managers suggests a belief that markets are inefficient.

**6** (i) Let $Q(t)$ be the inflation index at time $t$

$$\frac{Q(t-1)}{Q(t-2)} = 1.0275 = e^{I(t-1)}$$

therefore $I(t-1) = \ln(1.0275)$

The 95% confidence interval for $I(t)$ is therefore at the upper level

$$I(t) = 0.03 + 0.55(\ln(1.0275) - 0.03) + 0.45 \times 1.96$$

$$= 0.910$$

at the lower level

$$I(t) = 0.03 + 0.55(\ln(1.0275) - 0.03) - 0.45 \times 1.96$$

$$= -0.854$$

***Comments on question 6 (i)***

*Candidates who used 0.045 as in the original Wilkie calibration (instead of 0.45) were given appropriate credit.*

(ii) Inflation in many countries tends to be mean-reverting because central banks and governments attempt to manage it close to target ranges

(iii) The model used for inflation is not suitable for share prices for the following reasons:

(1) Strong mean reversion implies prices are 'predictable' so high returns are possible with little risk through active market timing. This runs counter to much empirical evidence.

(2) Lots of evidence for share price 'jumps' in market prices that are not reflected in the model.

(3) Share prices tend to increase rather than mean revert therefore a stationary process is not suitable.

(4) The model permits negative share prices, which is highly unrealistic.

**7** (i) The martingale approach gives much more clarity in the valuation process. By providing an explicit expectation to evaluate.

It gives the replicating strategy for the derivative.

It can be applied to exotic options where the PDE approach cannot.

(ii)

- Risk neutral pricing approach is the same as the martingale approach, i.e. values are derived from the risk neutral world.

- Deflators values the derivative in a real world probability measure with a stochastic adjustment factor.

  The approach is the same as risk-neutral pricing, the only difference is that calculations are presented using the real world measure and a stochastic adjustment factor versus a risk neutral measure. Intuitively the deflator approach can also give information about real world expected outcomes.

**8** (i) The value of the promise can be thought as part of a call option contract. A call option consists of a contract to deliver a share in return for the payment of an exercise, where the share price exceeds the exercise price.

The promise made to the employees is the first part of the call option, the promise to deliver a share provided the price exceeds a certain level. Therefore, the first component of the Black Scholes formula gives the value

Value = $S.N(d_1)$ where $d_1 = \left(\dfrac{\ln s/k + r + ½\sigma^2}{\sigma\sqrt{T}}\right)T$

$$= 1{,}000 \times N\left(\frac{\ln\left(\frac{1}{1.5}\right) + 0.04 + ½0.3^2}{0.3}\right)$$

$$= N(-1.0682) \times 1{,}000$$

$$= £142.70$$

(ii) Limiting the gain under the contract can be represented by a portfolio of the above promise less a call option with exercise price of £2.

The value of the call option is therefore

$1{,}000(N(d_1) - e^{-0.04}N(d_2) * 2)$ $d_1$ as above

$d_2 = d_1 - \sigma\sqrt{T}$

where

$d_1 = -2.027$
$d_2 = -2.327$

The value of the call option is therefore

$$1{,}000 \times 0.00215 = £2.15$$

The value of the revised promise is therefore

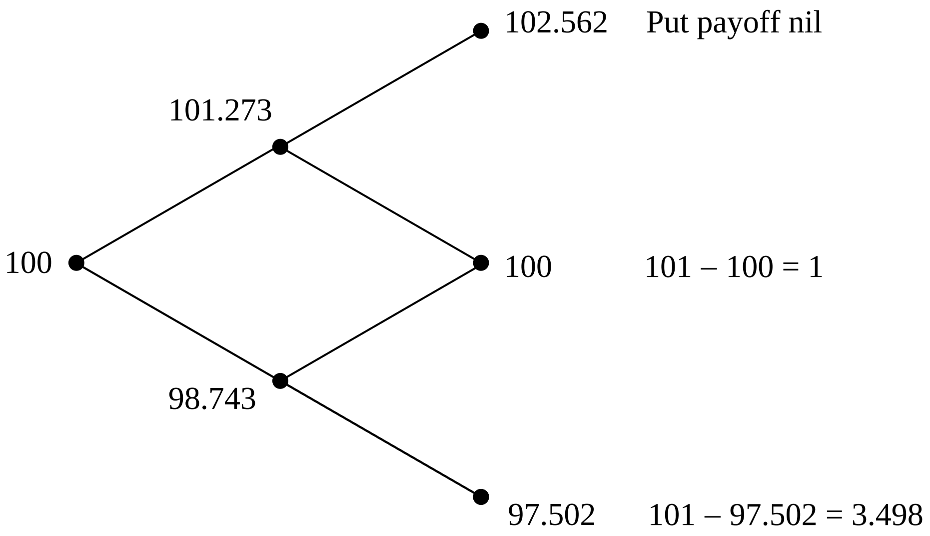
$$£142.70 - £2.15 = £140.53$$

(iii) (a) The employee has taken a view about the expected growth in share price. The result is a value that is not consistent with risk neutral pricing.

This means that, if there were a market in these contracts the price the employee has derived is not equal to the price the market would place on it.

(b) If the employer were willing to buy such promises at their suggested price, an arbitrageur would sell of £300 and hedge their position at £142.70. Resulting in substantial risk free profits.

**9** (i)




$u = \exp(\sigma \,.\, 250^{-\frac{1}{2}}) = 1.01273$

The risk neutral probability of an upstep is

$$q = \frac{e^{0.016\%} - 0.98743}{1.01273 - 0.98743}$$

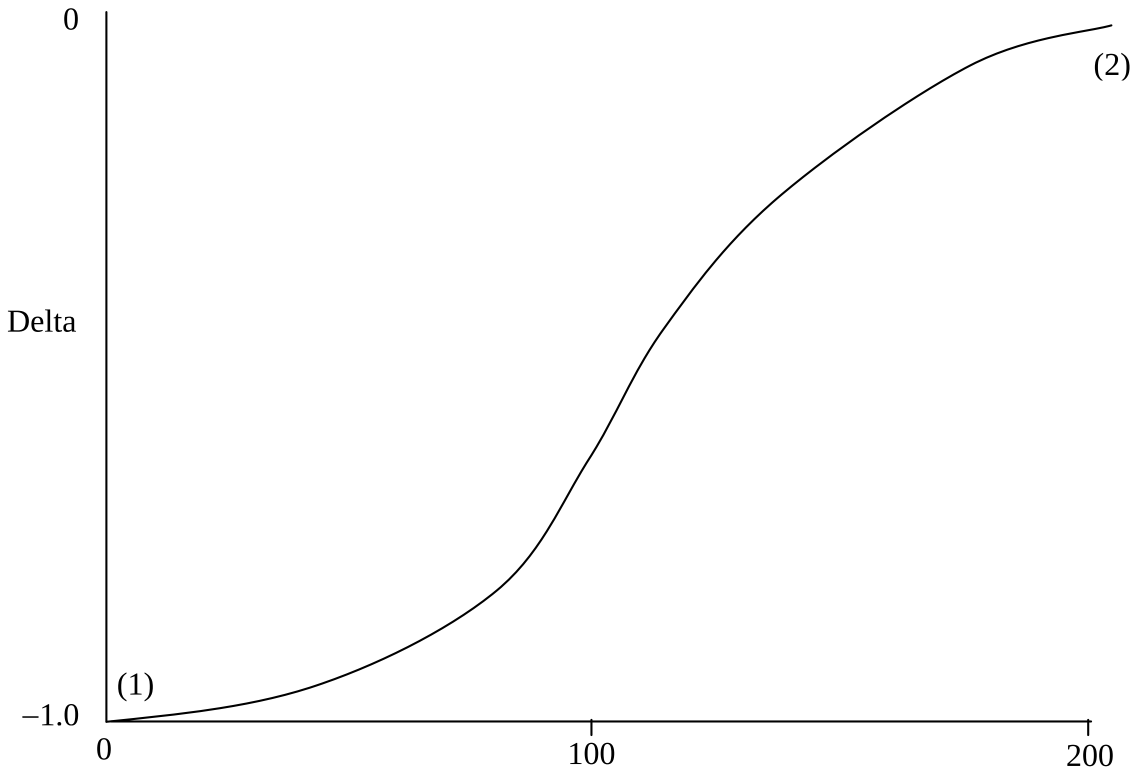
$$= 0.50316$$

$\Rightarrow$ The value of the put option is therefore

$$e^{-2\times 0.016\%}(0.50316(1 - 0.50316) \times 2 \times 1 + (1 - 0.50316)^2 \times 3.498)$$

$$= 1.3635$$

(ii) (a)




(b) Key feature:

Delta is negative as the value of a put option falls when share price rises.

(1) When the share price is very low the value is almost 100 therefore delta is almost − 1.

(2) When share price is high the value is almost nil therefore delta is almost nil.

(iii) Price of call option

$$= e^{-2\times 0.016\%}(0.50316^2 \times 1.562) = 0.3953$$

(iv) Put call parity means for all time $t < T$, then

$$c_t + k\mathrm{e}^{-r(T-t)} = p_t + s_t$$

$c_t$ = call option with exercise price $k$ and expiry $T$
$p_t$ = put option with exercise price $k$ and expiry $T$

Consider

A one call plus cash of $ke^{-\sigma(T-t)}$
B one put plus share

At expiry if $S_T > K$ then portfolio A is worth $S_T$ as is portfolio B.

If $S_T \leq K$ then portfolio A is worth $k$ as is portfolio B.

By the principle of no arbitrage since the payoffs are identical at time $T$ the value of the portfolios must be identical at time $t < T$.

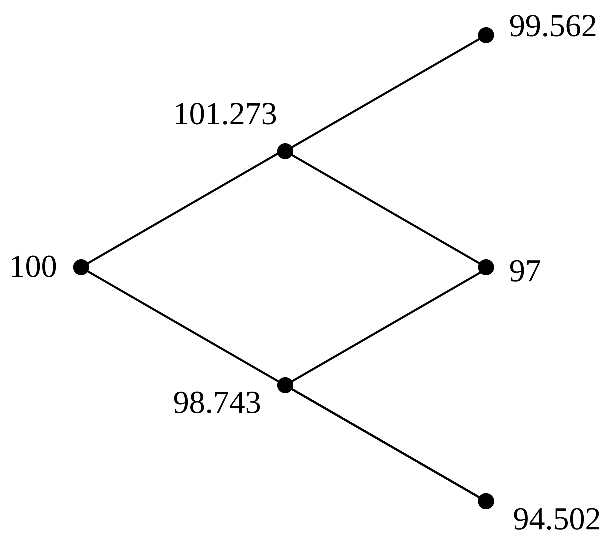
(v) Put call parity gives

$$c_t = 1.374 + 100 - 101e^{-0.016\%\times 2}$$

$$= 0.4063$$

This is not equal to the value above because the binomial model is a discrete time approximation to a continuous model. Therefore, the difference in value is due to discretisation error.

(vi) Share price




**Call option**

If the call option is held to expiry it will be worthless as it will expire underwater.

If the option holder exercise at day 1 when the share price has risen they will have a positive gain. Therefore, exercising early, if the share price has risen will be advantageous.

**Put option**

It is clear that it is not advantageous for the option holder to exercise early as follows:

If share price is 101.273 then gain = nil.
If the option is held to expiry there is a positive gain.
If the share price is 98.743 the gain is 2.257.
If held to expiry the gain must be at least 101 – 97 = 4.

## END OF EXAMINERS' REPORT