INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

19 April 2016 (am)

Subject CT8 – Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a new page.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 An investor measures the utility of her wealth using the utility function $U(w) = \ln(w)$ for $w > 0$.

(i) Derive the absolute and relative risk aversions for this investor's utility function, and the first derivative of each. [4]

(ii) Comment on what this tells us about the proportion of her assets that this investor will invest in risky assets. [2]

The investor has £100 available to invest in two possible assets, Asset A and Asset B. The future value of Asset A depends on an uncertain future event.

- Every £1 invested in Asset A will be worth £1.30 with probability 0.75 and £0.40 with probability 0.25.

- Asset B is risk-free, so every £1 invested in Asset B will always be worth £1.

The investor does not discount future asset values when making investment decisions. She decides to invest a proportion a of her wealth in Asset A and the remaining proportion $1 - a$ in Asset B.

(iii) Express her expected utility of wealth in terms of a. [2]

(iv) Determine the amount that she should invest in each of Asset A and B to maximise her expected utility, using your result from part (iii). [5]

[Total 13]

2 Consider an asset whose return follows the probability density function $f(x)$.

(i) Write down a formula for the variance of the return on the asset, defining any additional notation you use. [1]

(ii) Write down a formula for the shortfall probability for the return on the asset below a level L. [1]

The returns on an asset follow a Normal distribution with mean $\mu = 6\%$ per annum and variance $\sigma^2 = 23\%$ per annum. An investor buys €500 of the asset.

(iii) Determine the shortfall probability for the value of the asset in one year's time below a value of €480. [2]

(iv) Explain what can be deduced about an investor's utility function if the investor makes decisions based on:

(a) the variance of returns.
(b) the shortfall probability of returns.

[2]

[Total 6]

3 Consider a market with N securities. Let x_i denote the weight of security i in a portfolio, V_i the variance of the return on security i and C_{ij} the covariance between the returns on security i and security j.

(i) Write down an expression for V, the variance of the return on the portfolio. [1]

(ii) Describe how an efficient portfolio can be found under mean-variance portfolio theory. [You do not have to include details of the partial derivatives and their solutions.] [5]

(iii) Show that investors can diversify away specific risk by investing equal amounts in an increasing number of independent securities. [3]

(iv) Show that the result in part (iii) still holds true when the securities are correlated. [3]

[Total 12]

4 In a market where the assumptions of the Capital Asset Pricing Model (CAPM) hold, there are a risk-free asset and two risky assets with the following attributes:

		Rate of return (per annum)		
State	*Probability*	*Asset 1*	*Asset 2*	*Asset 3*
1	0.2	5.0%	15.0%	26.0%
2	0.3	5.0%	22.0%	15.0%
3	0.1	5.0%	10.0%	24.0%
4	0.4	5.0%	28.0%	7.0%
Market capitalisation			30,000	70,000

(i) Determine the composition of the market portfolio. [1]

(ii) Determine the market price of risk. [5]

(iii) Calculate the beta of each risky asset. [2]

(iv) State the limitations of the CAPM. [3]

[Total 11]

5 (i) Define the three forms of the Efficient Markets Hypothesis. [3]

(ii) State two reasons why it is hard to test whether any of the three forms hold in practice. [2]

[Total 5]

6 Suppose that at time t a portfolio (ϕ_t, ψ_t) is held, where ϕ_t represents the number of units of a stock, with price S_t, held at time t and ψ_t is the number of units of a cash bond, with price B_t, held at time t. The processes ϕ and ψ are previsible.

Let $V(t) = \phi_t S_t + \psi_t B_t$ be the value of the portfolio at time t.

(i) Explain what it means for this portfolio to be self-financing. [2]

Consider a stock paying a continuous dividend at a rate δ and denote its price at any time t by S_t.

Let C_t and P_t be the price at time t of a European call option and European put option respectively, written on the stock S, each with strike price K and maturity $T \geq t$.

The instantaneous risk-free rate is denoted by r.

(ii) Prove put-call parity in this context by constructing two self-financing portfolios whose value must be equal by the principle of no arbitrage. [6]
[Total 8]

7 Consider a non-dividend-paying share with price S_t at time t (in years) in a market with continuously compounded risk-free rate of interest r.

(i) Show that the fair price at $t = 0$ of a forward contract on the share maturing at time T is $K = S_0 e^{rT}$. [5]

A share is currently worth $S_0 = €20$. The continuously compounded risk-free rate of interest is 1% per annum.

(ii) Calculate the fair price at $t = 0$ of a forward contract written on the share with delivery at $t = 2$. [1]

(iii) Give an expression for the value to the investor of the forward contract in part (ii) at time $t \leq 2$, in terms of S_t, t and r. [2]

An investor enters into the above forward contract at time $t = 0$. At time $t = 1$ the risk-free rate of interest has increased to 4% per annum. The share price has not changed.

(iv) Calculate the value to the investor of the forward contract at $t = 1$. [1]

(v) Determine each of the following Greeks for the contract value at time $t = 1$:

- delta
- theta
- vega

[3]
[Total 12]

8 Consider a three-period binomial tree model for the stock price process S_t.

Let $S_0 = 100$ and let the price rise by 10% or fall by 5% at each time step.

Assume also that the risk-free rate is 4% per time period, continuously compounded.

(i) (a) State the conditions under which the market is arbitrage free.

(b) Verify that there is no arbitrage in the given market. [2]

(ii) Calculate the price of a European call option on this stock, with maturity at the end of the third period and a strike price of 103. [4]

A special option, called a European "Paylater" call option, has the following payoff at maturity T:

$$(S_T - K - c) \text{ if } S_T > K$$

and zero otherwise. K is the strike price and c is the premium paid for the option.

The premium is paid at maturity, and is only paid if the option expires in-the-money.

Further, the option premium is set such that the value of the option at time $t = 0$ is zero.

Assume that $K = 103$ and the maturity of the contract is at time $t = 3$.

(iii) Determine the premium c of this contract. [3]
[Total 9]

9 (i) Draw a diagram to illustrate the Jarrow-Lando-Turnbull model for credit default, defining any notation used. [4]

Consider a three-state credit model for a company in discrete time. The states are Healthy (H), Unhealthy (U) or Defaulted (D). Transition probabilities from state i to state j, p_{ij}, are constant:

$p_{HU} = 0.1$
$p_{UH} = 0.05$
$p_{HD} = 0.02$
$p_{UD} = 0.3$
$p_{Dj} = 0$ for all $j \neq D$

Denote the probability that the company is in state i at time t (years) as $p_{\mathrm{i}}(t)$.

A company is in the Healthy state at time 0.

(ii) Calculate $p_D(2)$, i.e. the probability that the company is in the Default state at time 2. [2]

The company issues a zero-coupon bond at time 0, with maturity at time 2 and nominal value £100. The continuously compounded risk-free rate of interest is 4% per annum.

Assume that the bond returns its nominal value at time 2 if the company is not in default, or x% of its nominal value at time 2 if the company is in default.

The fair price of the bond at time 0 is £87.63.

(iii) Calculate the value of x, the assumed percentage recovery on default. [2]

(iv) Calculate the credit spread on the bond. [1]

(v) Comment on the impact on the current price of the bond if it returned x% of its nominal value at the time of default rather than at time 2. [1]
[Total 10]

10 In the Vasicek model, the short rate of interest under the risk-neutral probability measure is given by:

$$r_t = \theta + e^{-kt}(r_0 - \theta) + \sigma \int_0^t e^{-k(t-u)} dW_u$$

where k, θ, $\sigma > 0$ and W is a standard Brownian motion.

Consider the related process:

$$R_t = \int_0^t r_s ds$$

where r_t is the short rate defined above.

(i) Show that R_t has a Normal distribution with mean and variance given by:

$$E(R_t) = \theta t + (r_0 - \theta)\,\frac{1-e^{-kt}}{k} \quad \text{and}$$

$$\mathrm{Var}(R_t) = \frac{\sigma^2}{k^2}\left(t - \frac{2(1-e^{-kt})}{k} + \frac{1-e^{-2kt}}{2k}\right).$$ [6]

Let $P(0,t)$ be the price at time 0 of a zero-coupon bond with redemption date $t > 0$.

(ii) Show that, under the Vasicek model:

$$P(0,t) = e^{-E(R_t) + \frac{\mathrm{Var}(R_t)}{2}}.$$ [3]

(iii) Show, by using the results from parts (i) and (ii), that:

$$P(0,t) = A(t)e^{-B(t)r_0}$$

where $B(t) = \dfrac{1-e^{-kt}}{k}$

and $A(t) = \exp\left[(B(t)-t)\left(\theta - \dfrac{\sigma^2}{2k^2}\right) - \dfrac{\sigma^2}{4k}B(t)^2\right].$ [4]

(iv) State the main drawback of the above model for the term structure of interest rates. [1]

[Total 14]

END OF PAPER