INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS’ REPORT

September 2018

Subject CT8 – Financial Economics
Core Technical

Introduction

The Examiners’ Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners’ preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Mike Hammer
Chair of the Board of Examiners
December 2018

A. General comments on *the aims of this subject and how it is marked*

1. The aim of the Financial Economics subject is to develop the necessary skills to construct asset liability models and to value financial derivatives. These skills are also required to communicate with other financial professionals and to critically evaluate modern financial theories.

2. The marking approach for CT8 is flexible in the sense that different answers to those shown in the solution can earn marks if they are relevant and appropriate. Marks for the methodology are also awarded.

B. General comments on student performance in this diet of the examination

1. In general, the real differentiators in those who scored well were attention to detail in their algebraic steps, and the breadth of knowledge in being able to score the knowledge marks and even attempt most questions. A number of candidates did not gather relevant information from the text of the question, and translate it in the appropriate equivalent statistical concepts. For example, candidates struggled with formulating the probability that an event occurs in appropriate mathematical terms, and determining from the information in the question the direct way to recover required variances and covariances.

2. Students performed relatively well on knowledge based questions, although many missed the opportunity to be awarded full marks.

C. Pass Mark

The Pass Mark for this exam was 60.

Solutions

Q1

Anchoring and adjustment Anchoring is a term used to explain how people will produce estimates. They then adjust away from this initial anchor to arrive at their final judgement. [1]
Prospect theory A theory of how people make decisions when faced with risk and uncertainty. It replaces the conventional risk averse / risk seeking decreasing marginal utility theory. [1]
Framing (and question wording) The way a choice is presented ("framed") and, particularly, the wording of a question in terms of gains and losses, can have an enormous impact on the answer given or the decision made. [1]
Myopic loss aversion This is similar to prospect theory, but considers repeated choices rather than a single "gamble". [1]
Estimating probabilities Issues (other than anchoring) which might affect probability estimates include: [0.5]
• *Dislike of "negative" events* – the "valence" of an outcome (the degree to which it is considered as negative or positive) has an enormous influence on the probability estimates of its likely occurrence. [0.5]
• *Representative Heuristics* – people find more probable that which they find easier to imagine. As the amount of detail increases, its *apparent* likelihood may increase (although the *true* probability can only decrease steadily). [0.5]
• *Availability* – people are influenced by the ease with which something can be brought to mind. This can lead to biased judgements when examples of one event are inherently more difficult to imagine than examples of another. [0.5]
Overconfidence People tend to overestimate their own abilities, knowledge and skills. This may be a result of: [0.5]
• *Hindsight bias* – events that happen will be thought of as having been predictable prior to the event, events that do not happen will be thought of as having been unlikely prior to the event. [0.5]
• *Confirmation bias* – people will tend to look for evidence that confirms their point of view (and will tend to dismiss evidence that does not justify it). [0.5]
Mental accounting People show a tendency to *separate* related events and decisions and find it difficult to *aggregate* events. [1]
Effect of options Other issues include:
• *Primary effect* – people are more likely to choose the *first* option presented, but [0.5]
• *Recency effect* – in some instances, the *final* option that is discussed may be preferred! (The gap in time between the presentation of the options and the decision may influence this dichotomy.) [0.5]
• Other research suggests that people are more likely to choose an *intermediate* option than one at either end! [0.5]
• A greater range of options tends to discourage decision-making. On the other hand, a higher probability is attributed to options explicitly stated than when included in a broader category. [0.5]
• *Status Quo bias* – people have a marked preference for keeping things as they are. [0.5]
• *Regret aversion* – by retaining the existing arrangements, people minimise the possibility of *regret* (the pain associated with feeling responsible for a loss). [0.5]
• *Ambiguity aversion* – people are prepared to pay a premium for rules. [0.5]

[Max 10]

> *The majority of students scored either full marks or nearly full marks on this knowledge based question.*
>
> *Some students confused Behavioural Finance with Expected Utility Theory or the Efficient Market Hypothesis, for which there were no marks on offer.*

Q2

i) Mean = $\exp(\mu+0.5*\sigma^2)$ = exp(0.12+0.5*0.25) = 1.2776 [0.5]
Variance = $\exp(2\mu+\sigma^2)*(\exp(\sigma^2)-1)$ = exp(2*0.12+0.25)*(exp(0.25)-1) = 0.4636 [0.5]

ii) L(5) = 100,000 * 1.1^5 = 161,051 [0.5]
P(S(5) > 161,051) = P(S(5)/S(0) > 1.61051) = P(ln(S(5)/S(0)) > ln(1.61051))
= P(Z > ((ln(1.61051) – 5*0.12) / (0.25*5)^0.5) [0.5]
= P(Z > -0.110416) [0.5]
= 54.4% [0.5]
[If students have used continuous compounding ($e^{0.1*5}$) deduct one mark]

iii) P(S(6)/S(5) < t) = 0.05 => P(Z < (ln(t)-0.12) / 0.25^0.5) = 0.05 [0.5]
=> (ln(t)-0.12) / 0.25^0.5 = -1.645 [0.5]
=> t = exp(-1.645 * 0.25^0.5 + 0.12) = 0.4954 [0.5]
=> VaR = £120,000 * 0.4954 = $59,446 [0.5]
*[Or for £120,000 * (1 – 0.4954) = £60,552 lose one mark]*

iv) The investor can retain 40,000 / 81,708 = 48.95% of his stocks, so he would need to sell $61,254 of stocks. [1]

v) The loan at time 10 will be 161,051 * 1.1^5 = $259,374. [0.5]
The cash deposit account holds $61,254 at time 5, hence 61254 * 1.06^5 = $81,972 at time 10. [0.5]
So we need the stocks to be worth at least 259,374-81,972 = $177,402 at time 10. [0.5]
This needs a return of 177,402 / 58,746 = 3.0198 [0.5]
P(S(10)/S(5) > 3.0198) = P(Z > (ln(3.0198) – 5*0.12) / (0.25*5)^0.5) = P(Z > 0.4519) [0.5]
= 32.6% [0.5]

vi) There is a slightly less than 50:50 chance that the investor would be able to repay the loan at time t=5. [1]
The cash account pays a lower rate of interest than the loan charges, so the investor would be better off repaying $61,254 of the loan at time 5 if this is possible. [1]
The investor could also seek other assets that deliver a higher potential return. [1]

Or the investor could try to find other funds to cut his losses and repay the loan at time 5. [1]

[Max 2]

Overall, students did not score well on the application of the lognormal model.
A proportion of students confused the lognormal model and the solution to Geometric Brownian Motion so had the incorrect drift term. Another common mistake was not including the time factor in the drift and volatility components.
Most students made suggestions in (vi) for how the investor could reduce the risk in their portfolio.

Q3

i. $E[R_i]$ is the expected return of security *i*; [0.5]
$b_{i,k}$ is the responses of the rates of return on security *i* to factor *k* (alternatively the sensitivity of security *i* to index *k*). [1]
λ_k is the risk premium corresponding to factors *k*. [0.5]

ii. The risk free portfolio has zero exposure to any risk factor, i.e. $b_{i,k} = 0$ for all k, which implies $\lambda_0 = r_f$. [1]

Then, we look for the solution to

$$0.155 = 0.04 + 0.05b_{1,1} + 0.06 \times 1.5$$
$$0.1195 = 0.04 + 0.05b_{2,1} + 0.06 \times 0.7$$
[0.5 each]

which returns $b_{1,1} = 0.5, b_{2,1} = 0.75$ [0.5 each]

The majority of students scored either full marks or nearly full marks on this knowledge question.

For part (i), common mistakes were confusing the different parameters such as describing lambas as sensitivities and vice versa.

For part (ii), common mistakes were not including the risk-free rate in the equations or minor calculation errors.

Q4

i)

a. $E[A_3] = A_0\exp(\mu t+0.5\sigma^2 t) = 100\exp(0.05*3+0.5*0.2^2*3) = £123.37$ [1]

b. $E[B_3] = B_0\exp(\mu t+0.5\sigma^2 t) = 100\exp(0.08*3+0.5*0.3^2*3) = £145.50$ [1]

ii)

a. $SD[A_3] = \sqrt{(A_0^2\exp(2\mu t+\sigma^2 t)(\exp(\sigma^2 t)-1))}$ [0.5]
$= \sqrt{(100^2\exp(2*0.05*3+0.2^2*3)(\exp(0.2^2*3)-1))}$ [1]
$= £44.05$ [0.5]

b. $SD[B_3] = \sqrt{(B_0^2\exp(2\mu t+\sigma^2 t)(\exp(\sigma^2 t)-1))}$ [0.5]
$= \sqrt{(100^2\exp(2*0.08*3+0.3^2*3)(\exp(0.3^2*3)-1))}$ [1]
$= £81.01$ [0.5]

iii) $E[P_3] = 0.5E[A_3] + 0.5E[B_3] = £134.44$ [1]

iv) $V[P_3] = 0.5^2V[A_3] + 0.5^2V[B_3] + 2*\text{Correlation}*0.5*0.5*SD[A_3]*SD[B_3]$ [1]
$= 0.25*44.05^2 + 0.25*81.01^2 + 2*0.3*0.5*0.5*44.05*81.01$
$= 2{,}661.03$ [1]
$\Rightarrow SD[P_3] = £51.59$ [1]

v) The expected return of the portfolio falls halfway between the expected return on each of the one-stock investment strategies. [1]
But the standard deviation is well below halfway between the two one-stock strategies.
[1]
The price of risk for stock A is 23.37/44.05 = 0.53 [1]
The price of risk for stock B is 45.5/81.01 = 0.56 [1]
But the price of risk for the portfolio is 34.44/51.59 = 0.67 [1]
So the portfolio delivers a better expected return per unit of risk [1]
This is because the assets are not fully correlated… [1]
Which shows the benefit of diversification. [1]
[Max 4]

In general, students struggled with this question. The most common difficulty was making the link between the price and the number of shares held.

For part (iv), some students did not calculate a standard deviation for the portfolio that was consistent with their answers in part (ii). Their portfolio standard deviation was either much higher or much lower.

Some students calculated the proportion invested in the minimum variance portfolio under Mean Variance Portfolio Theory despite this not being asked for in the question.

Q5

Since $ce^{-\gamma t}$ is the general solution of $dX_t = -\gamma X_t\,dt$, we look for a solution in the form

$$X_t = U_t\, e^{-\gamma t}.$$ [1]

Hence, we have:

$$dU_t = d(e^{\gamma t} X_t).$$ [1]

We use Itô's Lemma to determine $d(e^{\gamma t} X_t)$. [1]

Applying Itô's Lemma to the function $f(t, X_t) = e^{\gamma t} X_t$ gives: [1]

$$d(e^{\gamma t} X_t) = e^{\gamma t}\,\sigma dB_t + \left[\gamma e^{\gamma t} X_t + e^{\gamma t}(-\gamma X_t) + \frac{1}{2}\times 0\times\sigma^2\right]dt$$ [1]

$$= \sigma e^{\gamma t} dB_t.$$ [1]

Written as an integral equation, this reads:

$$e^{\gamma t} X_t = e^{\gamma 0} X_0 + \int_0^t \sigma e^{\gamma s} dB_s$$ [1]

i.e. $X_t = X_0 e^{-\gamma t} + \sigma\int_0^t e^{\gamma(s-t)}\, dB_s.$ [1]

[Or using a Taylor expansion:

$df(X_t,t) = \frac{df(X_t,t)}{dX_t} dX_t + \frac{1}{2}\frac{d^2 f(X_t,t)}{dX_t^{\,2}} dX_t^{\,2} + \frac{df(X_t,t)}{dt} dt$ [1]

Where $\frac{df(X_t,t)}{dX_t} = e^{\gamma t}$, $\frac{d^2 f(X_t,t)}{dX_t^{\,2}} = 0$ and $\frac{df(X_t,t)}{dt} = \gamma X_t e^{\gamma t}$ [1]

So $df(X_t,t) = e^{\gamma t} dX_t + \gamma X_t e^{\gamma t} dt$ [1]

But $dX_t = -\gamma X_t dt + \sigma dB_t$ so $df(X_t,t) = e^{\gamma t}\sigma dB_t$ [1]

Integrating between 0 and t gives $\int_0^t df(X_s, s) = \sigma \int_0^t e^{\gamma s} dB_s$ [1]

So $f(X_t, t) - f(X_0, 0) = \sigma \int_0^t e^{\gamma s} dB_s$ [1]

So $X_t e^{\gamma t} - X_0 = \sigma \int_0^t e^{\gamma s} dB_s$ [1]

So $X_t = X_0 e^{-\gamma t} + \sigma \int_0^t e^{\gamma(s-t)} dB_s$] [1]

[Or using an integrating factor:

Use the integrating factor $e^{\gamma s}$. [1]

Then $e^{\gamma s} dX_s = -\gamma X_s e^{\gamma s} ds + e^{\gamma s} \sigma dB_s$ [1]

So $\gamma X_s e^{\gamma s} ds + e^{\gamma s} dX_s = e^{\gamma s} \sigma dB_s$ [1]

So $\frac{d}{ds}(e^{\gamma s} dX_s) = e^{\gamma s} \sigma dB_s$ [1]

Then $\int_0^t \frac{d}{ds}(e^{\gamma s} dX_s) = \int_0^t e^{\gamma s} \sigma dB_s$ [1]

So $e^{\gamma t} X_t - e^{\gamma 0} X_0 = \sigma \int_0^t e^{\gamma s} \sigma dB_s$ [1]

So $e^{\gamma t} X_t = X_0 + \sigma \int_0^t e^{\gamma s} \sigma dB_s$ [1]

So $X_t = X_0 e^{-\gamma t} + \sigma \int_0^t e^{\gamma(s-t)} dB_s$] [1]

> *The majority of students scored more than half marks on this knowledge based question.*
>
> *Common approaches to solve the SDE included using an integrating factor or Ito's Lemma.*
>
> *Some students went on to calculate the distribution of the solution and its long-term mean and variance despite not being asked for this in the question.*

6 Q6

i) Portfolio A = one call plus cash of Kexp(-r(T-t)) [1]
Portfolio B = one put plus one share [1]
Both portfolios have value max{K,S_T} at expiry, hence by the principle of no arbitrage they must have the same value at all earlier times. [1]

Hence $c_t + Ke^{-r(T-t)} = p_t + S_t$ [1]
[Note to markers: any alternative valid solution is acceptable]

ii) By put-call parity [0.5]
B = 0.32 + 8exp(-0.03*10) – 5 [1]
= $1.25 [0.5]

iii) C >= max{ 0 , 10exp(-0.03*10) – 5 } [0.5]
= $2.41 [0.5]
C<= 10exp(-0.03*10) [0.5]
= $7.41 [0.5]

Alternatively, option C is the same as B but with a strike price $2 higher. It can never be in the money by more than $2 more than B, so it can never be worth more than $2 more than B. [1]
Hence it can't be worth more than $3.25. [1]
[Max 2]

iv) D >= 10 – 5 = $5 [1]

D <= $10 [1]
(The American option can be exercised early, but its value will never be more than $10.)

[Or the American option is worth at least as much as the European option for 0.5 marks.]

Part (i) was a knowledge based question with well-prepared students having little difficulty answering. Common mistakes included using incorrect portfolios or not explicitly applying the assumption of no arbitrage to prove the portfolios had the same value at the beginning.

In parts (iii) and (iv) most students identified the correct bounds but some struggled with the American option.

Q7

i) 100,000 + 250,000 = $350,000 [1]

ii) Under the Merton model, we consider that the shareholders have a call option on the company's assets, with a strike price equal to the nominal value of the debt. [1]
We want the share price to remain unchanged, so the value of the 'call option' after the debt has been issued must be $1 per share = $100,000 in total. [1]

We need to find the nominal value of the debt at maturity, which will be the strike price of this option. [1]

[Max 2 so far]

Trying a strike price of $250,000 gives a call value of $137,811 [1]
Trying a strike price of $350,000 gives a call value of $71,960 [1]

Sample answers to help with marking:

Strike price	*d1*	*d2*	*Call option value*
$200,000	*2.2834*	*1.948*	*$178,363*
$250,000	*1.6181*	*1.2827*	*$137,811*
$300,000	*1.0745*	*0.7391*	*$101,703*
$350,000	*0.6149*	*0.2795*	*$71,960*
$400,000	*0.2168*	*-0.1186*	*$49,148*

Interpolating gives a strike price (i.e. nominal debt value at maturity) of $307,419 (actual value is $302,582) [1]

iii) The required yield on the debt is i where $250{,}000e^{5i} = 302{,}582 \Rightarrow i = 3.82\%$ [1]
=> credit spread = 3.82% - 3% = 0.82% [1]

iv) The equities fell to 50% of their original value [1]
The debt fell to (0.5 * 302,582) / 250,000 = 60.5% of its original value [1]
[Alternatively the debt fell by 39.5%.]

v) The debt ranks above the equities on company default. [1]
Hence the debt holders have a more secure investment. [1]
They will almost always receive something at maturity, and may receive the whole value. [1]
The equity holders will receive nothing on default. [1]
And might receive nearly nothing even if the company does not default. [1]

[Max 3]

[Max 3]

Most students struggled with this question with most failing to score more than a few marks.

In general, students did not understand they had been given the value of equity and debt and had to solve for the redemption value that was consistent with the values given.

Common mistakes included calculating the value of the equity using the current value of the debt as a redemption value and then proceeding through the question. This led to students using a

> *present value of debt that was higher than the redemption value, leading to a negative credit spread.*

Q8

i.

	Stock tree			
time	0	3 month	6 month	9 month
	60.00	78.00	101.40	131.82
		48.00	62.40	81.12
			38.40	49.92
				30.72

The price C_0 of the option is computed via Risk Neutral Valuation; let $\hat{p}$ denote the risk neutral probability of an up movement, then

$\hat{p} = \frac{e^{0.02*0.25}-0.80}{1.30-0.80} = 0.41$ [1]

and

$C_0 = e^{-rT}\sum_{k=0}^{3}\binom{3}{k}\hat{p}^k(1-\hat{p})^{3-k}max\left(0, S_0u^kd^{3-k}-K\right)$ [1]

$= e^{-0.02*0.75}(76.82\times\hat{p}^3 + 26.12\times 3\hat{p}^2(1-\hat{p})) = 12.87$ [1]

The detailed workings are provided below – in case attempts to answer this question go through the whole tree.

	CALL			
time	0	3 month	6 month	9 month
	12.87	25.30	46.67	76.82
		4.35	10.66	26.12
			0.00	0.00
				0.00

ii. Either from the put-call parity or by repeating calculation: $P_0 = 7.05$ [1]

iii. The value of the position is $C_0 - P_0 = S_0 - Ke^{-rT}$ [1].
This value would not change as it is independent of the expected movements of the stock (i.e. its volatility) [2]
[Or students can recalculate the value for full marks.]

> *Parts (i) and (ii) were well-answered by the majority of candidates.*

For part (ii), some students calculated the put price from first principles rather than simply using the put-call parity relationship. This was a valid approach but took up more time in the exam.

For part (iii), the majority of students re-calculated the prices of the put and call directly, though many made mistakes and hence failed to conclude that the portfolio value remains unchanged.

Q9

i. From the definition, $V\ (t)\ = \Phi_t S_t\ + \Psi_t B_t$ [1]
therefore $e^{-rt}V\ (t)\ = \Phi_t e^{-rt}S_t\ + \Psi_t$ [1]
because the portfolio is self-financing it follows that $d(e^{-rt}V\ (t))\ = \Phi_t d(e^{-rt}S_t)$ [1]
as required.

ii. Using the result from the previous part, the martingale property for the discounted value of the portfolio is the same as for the discounted stock price. [1]
This requires a change of measure to adjust for the market price of risk $\lambda = {(\mu - r)}/{\sigma}$ [2]
[Or we can apply Taylor's theorem to d($S_t\ e^{-rT}$) and check that the drift is zero.]
[Or we could use Ito's Lemma.]
[Or just $\lambda = r$ as a possible solution for one mark.]

Several approaches were used to prove that this is a martingale. A common approach included Ito's Lemma, while other students used the five-step method and applied the Martingale Representation Theorem.

In part (ii), some students simply repeated the definition or properties of a martingale rather than considering the conditions for the discounted share price process that would make it a martingale.

Q10

i. The assumptions underlying the Black-Scholes model are as follows:
1. The price of the underlying share follows a geometric Brownian motion. [1/2]
2. There are no risk-free arbitrage opportunities. [1/2]

3. The risk-free rate of interest is constant, the same for all maturities and the same for borrowing or lending. [1/2]
4. Unlimited short selling (that is, negative holdings) is allowed. [1/2]
5. There are no taxes or transaction costs. [1/2]
6. The underlying asset can be traded continuously and in infinitesimally small numbers of units. [1/2]

ii.

Data: $S = 8; K = 9; r = 2\%; \sigma = 20\%;\ T = 0.25$

By the Black-Scholes formula:

$-d_1 = 1.0778$ [0.5]

$-d_2 = 1.1778$ [0.5]

$N(-d_1) = 0.8594$ [0.5]

$N(-d_2) = 0.8806$ [0.5]

Therefore $P_0 = 9e^{-0.02\times 0.25} \times 0.8806 - 8 \times 0.8594$ [1]

$= 1.01$ [1]

iii. As interest rates increase in the market, the expected return required by investors in stock tends to increase [0.5]

However, the present value of any future cash flow generated by option contracts decreases [0.5]

The combined impact of these two effects is to decrease the value of the put option [1]

Rho is negative for a put option [0.5]

put options become less valuable in times of increasing interest rates because they effectively defer the selling of a share and so delay access to the cash required to obtain the risk-free rate

[0.5]

[Or students could explain how the terms in the formula change.]

[Max 2]

This was well-answered overall by the majority of students.

For part (i), some students included assumptions from Expected Utility Theory, the Efficient Market Hypothesis or CAPM which scored no marks.

For part (ii), simple calculation errors were the most common mistake.

Q11

i. SML: $ER_i = R_f + \beta_i(ER_M - R_f)$ [1]

for

- ER_i: expected return on Asset *i*. [1/4]
- R_f: risk-free rate. [1/4]
- β_i: beta factor of security *i* defined as $Cov(R_i, R_M)/Var(R_M)$. [1/4]

- ER_M: expected return on the market portfolio. [1/4]
  [Round up to nearest half mark.]

ii. Note that $ER_M = x_1 ER_1 + x_2 ER_2$ [1/4], and $x_1 + x_2 = 1$ [1/4]
Substitute into the SML and solve for x_1, so that
$x_1 = \frac{ER_i - \beta_i ER_2 - R_f(1-\beta_i)}{(ER_1 - ER_2)\beta_i}$. [1]
From the data: $ER_1 = 16.30\%$ [1/4]
$ER_2 = 29.70\%$ [1/4]
Substituting either for Asset 1 or Asset 2, $x_1 = 0.4$ [1/2]
and therefore $x_2 = 0.6$ [1/2]
*[Alternatively, the beta of the market portfolio is 1, so x_1*0.46 + x_2*1.36 = x_1*0.46 + (1-x_1)*1.36 = 1 => x_1=0.4]*
[Round up to nearest half mark.]

iii. $Var(R_M) = 0.4^2 * Var(R_1) + 0.6^2 * Var(R_2) + 2 * 0.4 * 0.6 * Cov(R_1, R_2) = 0.00617$.
[1]
Consequently $\left(ER_M - R_f\right)/\sigma_M = 1.897$. [1]

The majority of students scored well in this question.

For part (i), some students confused the Security Market Line and the Capital Market Line despite these being given in the Actuarial Tables.

END OF EXAMINERS' REPORT