EXAMINATION

8 September 2005 (am)

Subject CT8 — Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

1 An investor is contemplating an investment with a return of £R, where:

$$R = 300{,}000 - 500{,}000U$$

where U is a uniform [0, 1] random variable.

Calculate each of the following four measures of risk:

(a) variance of return

(b) downside semi-variance of return

(c) shortfall probability, where the shortfall level is £100,000

(d) Value at Risk at the 5% level [8]

2 A market consists of three securities A, B and C with capitalisations of £22bn, £33bn and £22bn respectively. Annual returns on the three shares (R_A, R_B and R_C) have the following characteristics:

Asset	*Standard deviation*
A	40%
B	20%
C	10%

The expected rate of return on the market portfolio is 22.86% p.a.

The correlation between the returns on each pair of distinct securities is 0.5.

The risk-free rate of return is 3.077% p.a. No adjustments to an investor's portfolio are possible within the year.

(i) Prove that the expected returns on A, B and C are 40%, 20% and 10% respectively if the CAPM is assumed to hold. [9]

(ii) Derive a single index model (with the index equal to R_M, the random return on the market portfolio) with the same expected returns and variances as in the CAPM. You are required to calculate the values of all parameters in the model. [4]

(iii) Prove that this single index model is not completely consistent with the CAPM model. [3]

[Total 16]

3 (i) Define in words Δ, Γ, θ, λ, ρ, and $\mathcal{V}$ for an individual derivative. [6]

(ii) Explain how Γ and $\mathcal{V}$ can be used in the risk management of a portfolio that is delta-hedged. [4]

[Total 10]

4 (i) State the assumptions underlying the Black-Scholes option pricing formula and discuss how realistic they are. [6]

An investment bank has written a number, N, of European call options on a non-dividend paying stock with strike price 200p, current stock price 180p, time to expiry of six months and an assumed continuously-compounded interest rate of 3% p.a. The bank is delta-hedging the option position assuming the Black-Scholes framework holds and currently holds 250,000 shares of the stock and is short £413,057 in cash.

(ii) By using the hedging position and the Black-Scholes formula for the value of the option, derive two equations satisfied by N and σ, the bank's assumed volatility. [3]

(iii) Estimate σ by interpolation. [5]

(iv) Deduce the value of N. [1]

[Total 15]

5 A binomial model for a non-dividend-paying security with price S_t at time t is as follows: the price at time $(t + 1)$ is either αS_t (down-jump) or βS_t (up-jump). £1 held in cash between times t and $t + 1$ receives interest to become £$(1 + r)$ at time $t + 1$. The parameters satisfy $\beta > 1 + r > \alpha$.

A derivative security with price X has the following payoff at time $t + 1$:

$$X_{t+1} = b \text{ if } S_{t+1} = \beta S_t$$
$$\quad\;\; = a \text{ if } S_{t+1} = \alpha S_t$$

A portfolio of cash (amount y) and stock (value x) at time t exactly replicates the payoff of the derivative at time $t + 1$.

(i) Derive expressions for x and y in terms of b, β, a, α and r. [3]

(ii) Derive an expression for q in terms of $(x + y)$, a, b and r, where q is the risk-neutral probability of an up-jump. [2]

(iii) Suppose that $r = 0\%$. Two derivatives each have a payoff of a if $S_{t+1} = \alpha S_t$, but the first derivative pays $2a$ and the second $3a$ if $S_{t+1} = \beta S_t$. The price at time t of the first derivative is 10. Derive an expression, in terms of a, for the price at time t of the second derivative. [3]

[Total 8]

6 (i) Explain why, in the absence of arbitrage, the forward price for a forward contract on one share (over a period where no dividends are payable) is $S_0 e^{rt}$, where S_0 is the initial price of the share, r is the continuously-compounded risk-free rate of interest and t is the time to delivery of the contract. [4]

(ii) Determine a fair (forward) price for a forward contract on a share (currently priced at £10) with delivery in 20 months when the share pays a dividend of 3% of the share price every six months, the continuous risk-free rate is 7% p.a. and the next dividend is due in one month's time.
[You may assume that dividends are immediately re-invested.] [5]
[Total 9]

7 Consider the following discrete time models for (log) share prices and (log) dividend yields:

$$\ln S_{t+1} = \ln S_t + \mu + \sigma Z_{t+1}$$
$$\ln D_{t+1} = \ln\delta + \alpha(\ln D_t - \ln \delta) + \eta W_{t+1},$$

where

S_t = share price at time t
D_t = dividend yield at time t,

W_t and Z_t are both serially uncorrelated standard normal random variables but are correlated with each other and μ, σ, δ and η are positive parameters and $0 < \alpha < 1$.

The unit of time in this model is a month.

(i) Explain the magnitude and sign of the correlation coefficient you would expect between Z_t and W_t. You do not have to calculate the correlation coefficient or derive an expression for it. [2]

(ii) State two properties of the dividend yield model and comment on their realism. [2]

(iii) State three properties of the share price model and comment on them relative to empirical evidence and, if relevant, the efficient markets hypothesis. [3]
[Total 7]

8 Suppose that under the unique Equivalent Martingale Measure, Q, for a term structure model, the SDE satisfied by the instantaneous interest rate r is

$$dr_t = \alpha(\mu - r_t)\, dt + \sigma dZ_t,$$

where $\alpha > 0$, μ and σ are fixed parameters and, under Q, Z is a standard Brownian Motion.

The process X is defined by

$$X_t = r_t\, b(T - t) + \int_0^t r_s\, ds,$$

where the function b is given by $b(s) = (1 - e^{-\alpha s}) / \alpha$.

The function f is given by $f(x, t) = \exp(a(T - t) - x)$, where a is a differentiable function.

(i) Apply Ito's formula to $f(X_t, t)$. [6]

[Hint: First show that $dX_t = A_t dt + B_t dZ_t$ where $A_t = \alpha\mu b(T - t)$ and $B_t = \sigma b(T - t)$]

(ii) (a) Find a differential equation which the function a must satisfy for $f(X_t, t)$ to be a martingale.

(b) Determine an additional condition on a which is necessary for a bond with unit payoff at time T to have a price given by the formula

$$B(t, T) = f(X_t, t)\ \exp\left(\int_0^t r_s ds\right).$$ [4]

[Total 10]

9 The following model has been suggested for the short term interest rate at time t, r_t:

$$dr_t = \mu r_t\, dt + \sigma r_t\, dZ_t$$

where σ and μ are fixed parameters and Z_t is a standard Brownian motion.

(i) Outline three properties of this model and comment on their desirability. [3]

(ii) Outline the properties of the following two models for interest rates:

(a) the one-factor Vasicek model
(b) the Cox-Ingersoll-Ross model

[3]
[Total 6]

 PLEASE TURN OVER

10 (i) Describe four examples of tests that have been done to assess informational efficiency in stock markets. [4]

(ii) Explain to what extent the results of such tests should affect the assessment of the validity or otherwise of the efficient markets hypothesis. [3]

(iii) Explain what an efficient portfolio is in the context of modern portfolio theory, being careful to include a description of what is assumed about investors. [4]

[Total 11]

END OF PAPER