EXAMINATION

28 April 2010 (am)

Subject CT8 — Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all nine questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Let $\left(X_t; t \geq 0\right)$ be a stochastic process satisfying:

$$X_t = X_0 + \int_0^t \mu_s ds + \int_0^t \sigma_s dW_s$$

where W is a standard Brownian motion.

Let $f : \Re \times \Re \to \Re$ be a function, twice partially differentiable with respect to *x*, once with respect to *t*.

(i) State the stochastic differential equation for $f\left(t, X_t\right)$. [2]

Let $dX_t = -\gamma X_t dt + \sigma dW_t$.

(ii) Prove that the solution of this stochastic differential equation is given by:

$$X_t = X_0 \exp\left(-\gamma t\right) + \sigma \int_0^t \exp\left(\gamma\left(s - t\right)\right) dW_s$$ [6]

[Total 8]

2 Consider a stock paying a dividend at a rate δ and denote its price at any time *t* by S_t. The dividend earned between *t* and *T*, $T \geq t$, is $S_t(e^{\delta(T-t)} - 1)$.

Let C_t and P_t be the price at time *t* of a European call option and European put option respectively, written on the stock *S*, with strike price *K* and maturity $T \geq t$. The instantaneous risk-free rate is denoted by *r*.

Prove put-call parity in this context by adapting the proof of standard put-call parity that applies to put and call options on a non-dividend paying stock. [8]

3 Consider a two-period binomial model for a non-dividend paying stock whose current price is $S_0 = 100$. Assume that:

- over each six-month period, the stock price can either move up by a factor u = 1.2 or down by a factor d = 0.8
- the continuously compounded risk-free rate is r = 5% per six-month period

(i) (a) Prove that there is no arbitrage in the market.
(b) Construct the binomial tree.
[2]

(ii) Calculate the price of a standard European call option written on the stock S with strike price $K = 100$ and maturity one year. [5]

Consider a special type of call option with strike price $K = 100$ and maturity one year. The underlying asset for this special option is the average price of the stock over one year, calculated as the average of the prices at times 0, 0.5 and 1 measured in years.

(iii) Calculate the initial price of this call option assuming it can be exercised only at time 1. [5]
[Total 12]

4 Consider the following stochastic differential equation for the instantaneous risk free rate (also referred to as the short-rate):

$$dr(t) = a\left(b - r(t)\right)dt + \sigma dW_t$$

Its solution is given by:

$$r(t) = r_0 \exp(-at) + b\left(1 - \exp(-at)\right) + \sigma \exp(-at)\int_0^t \exp(as)\, dW_s$$

You may also use the fact that for $T > t$:

$$\int_t^T r(u)du = b(T-t) + \left(r(t) - b\right)\frac{1 - \exp\left(-a(T-t)\right)}{a} + \frac{\sigma}{a}\int_t^T \left(1 - \exp\left(-a(T-s)\right)\right)dW_s$$

(i) Derive the price at time t of a zero-coupon bond with maturity T. [10]

(ii) (a) State the main drawback of such a model for the short-rate.

(b) State the name and stochastic differential equation of an alternative model for the short-rate that is not subject to the drawback.
[2]
[Total 12]

5 A European call option on a stock has an exercise date one year away and a strike price of 320p. The underlying stock has a current price of 350p. The option is priced at 52.73p. The continuously compounded risk-free interest rate is 4% p.a.

(i) Estimate the stock price volatility to within 0.5% p.a. assuming the Black-Scholes model applies. [5]

A new derivative security has just been written on the underlying stock. This will pay a random amount D in one year's time, where D is 100 times the terminal value of the call option capped at 1p (i.e. 100 times the lesser of the terminal value and 1p).

(ii) (a) State the payoff for this derivative security in terms of two European call options.

(b) Calculate the fair price for this derivative security.

[5]

(iii) Calculate the risk neutral probability that the stock price is greater than 320p.

[4]

[Total 14]

6 (i) Describe the-two state model for credit ratings under the real world measure.

[9]

(ii) Explain how the two state model is generalised in the Jarrow-Lando-Turnbull model. [3]

[Total 12]

7 (i) State the Cameron-Martin-Girsanov Theorem. [3]

(ii) Derive the value of a which makes $\exp(\sigma B_t - at)$ a martingale when B is a standard Brownian Motion. [3]

In a Black-Scholes market, the stock price is given by:

$S_t = S_0 \exp(0.2B_t + 0.2t)$, where B is a standard Brownian Motion under the real-world measure.

A derivative security written on this stock in the same market has price:

$D_t = 2\exp(0.6B_t + 0.39t)$ at time t.

(iii) (a) Calculate the value of c such that $B_t + ct$ is a standard Brownian Motion under the Equivalent Martingale Measure.

(b) Calculate the risk-free rate of interest.

[8]

[Total 14]

8 Outline the main points you would make in a discussion of the statement:

The efficient markets hypothesis states that the market price is always correct and therefore it is not possible for investors to make money from investing in shares.

[10]

9 An asset is worth 100 at the start of the year and is funded by a senior loan and a junior loan of 50 each. The loans are due to be repaid at the end of the year; the senior one with interest at 6% p.a. and the junior one with interest of at 8% p.a. Interest is paid on the loans only if the asset sustains no losses.

Any losses of up to 50 sustained by the asset reduce the amount returned to the investor in the junior loan by the amount of the loss. Any losses of more than 50 mean that the investor in the junior loan gets 0 and the amount returned to the investor in the senior loan is reduced by the excess of the loss over 50.

The probability that the asset sustains a loss is 0.25. The size of a loss, L, if there is one, follows a uniform distribution between 0 and 100.

(i) Calculate the variances of return for the investors in the junior and senior loans. [8]

(ii) Calculate the shortfall probabilities for the investors in the junior and senior loans, using the full return of the amounts of the loans as the respective benchmarks. [2]

[Total 10]

END OF PAPER