INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

24 April 2015 (pm)

Subject CT8 – Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 (i) State in words the four axioms of the Expected Utility Theorem. [4]

(ii) State the conditions for an investor to be non-satiated and risk neutral in terms of their utility function, $U(w)$. [2]

An investor makes investment decisions using utility function $U(w) = (w^{\gamma} - 1) / \gamma$.

(iii) Derive the relative risk aversion function for $U(w)$. [2]

(iv) Describe how the relative risk aversion of $U(w)$ changes with w. [1]
[Total 9]

2 Consider an asset the annual return, X, on which has probability density function $f(x)$.

(i) Define the 5% Value at Risk for this asset. [1]

(ii) Define the expected shortfall of the return on this asset below 2%. [2]

Assume X has a Normal distribution with mean μ = 5% and variance σ^2 = 100%%.

(iii) Calculate the 5% Value at Risk. [2]

(iv) Discuss the limitations of using Value at Risk to measure the downside risk in an investment portfolio. [2]
[Total 7]

3 A share is currently priced at 640p. A writer of 100,000 units of a one year European put option with an exercise price of 630p has delta-hedged the option with a portfolio which holds cash and is short 24,830 shares. The continuously compounded risk-free rate of interest is 3% p.a. and no dividends are payable during the life of the option.

The assumptions of the Black-Scholes model apply.

(i) (a) Write down an expression for the delta of the option.

(b) Calculate its value in this case. [4]

(ii) Prove that the volatility of the share implied by the delta is 7.1% p.a. (assuming it is less than 100%). [5]

(iii) (a) Calculate the price of the option.

(b) Determine the value of the cash holding in the hedging portfolio. [4]
[Total 13]

4 Suppose that X is an AR(1) process given by the equation:

$$X_{t+1} = 0.75X_t + 0.25e_{t+1}$$

where the e_n are independent, identically distributed $N(0,1)$ random variables independent of X_0.

(i) (a) State the distribution of X_{t+1} conditional on the value of X_t .

(b) Show that the distribution of X_{t+s} conditional on the distribution of X_s is $N(0.75^t X_s, 0.25^2 + 0.25^4 + \ldots + 0.25^{2t})$. [3]

(ii) (a) Show that the distribution of X_t converges to a stationary distribution.

(b) State this distribution. [2]

The process $\ln(Y_t)$, with Y_t the multiplicative increase in a retail price index in year t, is believed to be an AR(1) process with the same parameters as X above.

(iii) Determine $E[Y_3|Y_0 = 1.21]$. [3]

(iv) Determine the long-run mean annual increase in the retail price index. [3]

[Total 11]

5 Let $(X_t; t \geq 0)$ be a stochastic process satisfying $dX_t = \mu_t dt + \sigma_t\, dW_t$ where W_t is a standard Brownian motion.

Let $f(t,x)$ be a function, twice partially differentiable with respect to x, once with respect to t.

(i) State the stochastic differential equation for $f(t, X_t)$. [2]

Let $dX_t = \lambda X_t dt + \sigma dW_t$.

(ii) Solve this differential equation, by considering $X_t = U_t e^{\lambda t}$ or otherwise. [6]

[Total 8]

6 Consider a non-dividend paying share with price S_t at time t.

(i) State and prove the put-call parity relationship for this share. [5]

Two options written on this share have the following characteristics:

1. a European call option maturing in two years, strike price $10.15, option price $3.87

2. a European put option maturing in two years, strike price $10.15, option price $0.44

The continuously compounded risk-free rate of interest is 4% p.a.

(ii) Calculate the share price implied by the option prices. [2]

(iii) Determine the implied volatility of the share to the nearest 1%. [5]
[Total 12]

7 (i) Define delta, gamma and vega for an individual derivative. [3]

A bank is considering selling a European call option on a share, and wants to hedge some of its risk. The share is non-dividend paying and has the following properties:

Strike price = $50
Option price = $17.91
Underlying share price = $60
Volatility = 25% p.a.
Time to expiry = 3 years

The continuously compounded risk-free rate of interest is 3% p.a. and the vega for this option is $29.00.

(ii) Calculate delta for this option. [1]

(iii) Identify a delta-hedged replicating portfolio using the share and the risk-free asset. [2]

Assume that the volatility has instantaneously increased to 27% p.a., with everything else except the option price remaining the same.

(iv) Estimate the new option price. [2]
[Total 8]

8 (i) State the main assumptions of mean-variance portfolio theory. [3]

There are only three assets available on a stock exchange:

Asset 1, expected return 2%, standard deviation 4%
Asset 2, expected return 4%, standard deviation 12%
Asset 3, expected return 3%, standard deviation 8%

The correlation between the returns on assets 1 and 3 is 0.75. The return on asset 2 is uncorrelated with the returns on the other two assets.

An investor in this market wants to minimise the variance of his portfolio.

(ii) Determine the Lagrangian function that can be used to find the minimum variance portfolio for a given expected return. [3]

Let x_i denote the weight of asset i (i = 1, 2, 3) in the minimum variance portfolio with an expected return of 4%.

(iii) Show, by taking partial derivatives of the Lagrangian function in part (ii), that:

$$x_1 = -0.45, x_2 = 0.55, x_3 = 0.9. \quad [4]$$

(iv) Comment on how the portfolio would change if short-selling was not allowed. [1]

[Total 11]

9 (i) Outline the three types of credit risk model. [3]

(ii) Describe how the Merton model can be used to estimate the risk-neutral probability of default. [2]

Let r be the constant continuously compounded risk-free rate and δ be the constant recovery rate for a defaultable zero-coupon bond in a two state model for credit rating with a deterministic transition intensity.

(iii) State the formula for the bond price. [1]

(iv) Determine the risk-neutral default intensity if the zero-coupon bond price is given by:

$$B(t,T) = e^{-r(T-t)}\,[1 - (1 - \delta)(1 - \exp\{-(T^3 - t^3)/6\})]. \quad [2]$$

(v) Calculate the fair price of an insurance contract which pays £1,200,000 after *two years* if the bond defaults in the *first year* and the continuously compounded risk-free rate is 2% p.a. [3]

[Total 11]

10 There are two risk-free zero coupon bonds trading in a market, Bond X and Bond Y.

The short-rate of interest, r_t, follows a Vasicek model:

$$dr_t = \alpha(\mu - r_t)dt + \sigma dW_t$$

where W_t is a standard Brownian motion.

(i) Write down the formula for the price of a risk-free zero coupon bond at time t, with bond maturity at time T, under the Vasicek model. [3]

In this market the parameters for the Vasicek model are $\alpha = 0.5$, $\mu = 4\%$ and $\sigma = 10\%$. The short-rate at time 0, $r(0)$, is 2% p.a. Bond X matures at time 1, and Bond Y matures at time 3. Both bonds are for a nominal value of \$100.

(ii) Calculate the fair price of Bond X. [3]

Bond Y has a fair price at time 0 of \$90.

(iii) Derive the market-implied risk-free spot rate of interest with maturity 3 years. [2]

(iv) Derive the market-implied risk-free forward rate of interest from time 1 to time 3. [2]

[Total 10]

END OF PAPER