INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

6 October 2016 (pm)

Subject CT8 – Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 10 questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Consider an asset whose return follows the probability density function $f(x)$.

(i) Write down a formula for the Value at Risk for the asset, at confidence level p. [1]

(ii) Write down a formula for the downside semi-variance of the return on the asset, defining any additional notation you use. [1]

(iii) State the arguments for and against using semi-variance as a risk measure. [2]

A farmer has a small apple tree which produces one harvest of apples per year. The number of apples the tree produces follows a Poisson distribution with a mean and variance of 8.

(iv) Determine the 10% Value at Risk level for the number of apples produced. [3]

(v) Determine the expected shortfall below a harvest of 5 apples. [3]
[Total 10]

2 (i) State the main assumptions of mean-variance portfolio theory. [4]

Consider a mean-variance portfolio model with two securities, with respective returns S_A and S_B, where the expected return $E[S_B] = 0.25E[S_A]$ and the variance of return $V[S_B] = 0.25V[S_A]$.

Let the correlation between the returns on the two securities be ρ.

(ii) Determine, in terms of $E[S_A]$, the expected return on the minimum variance portfolio if:

(a) $\rho = 0$
(b) $\rho = 1$

[4]

(iii) (a) Calculate the variance of the return on the minimum variance portfolio for part (ii)(b).

(b) Comment on the risk in this portfolio.

[2]
[Total 10]

3 In a market where the assumptions of the Capital Asset Pricing Model hold, there are two risky assets with the following attributes:

Security	*A*	*B*
Expected return (p.a.)	20%	16%

(i) Determine the composition of the market portfolio with expected return 18% per annum. [2]

(ii) Calculate the beta of each security under the assumption that the risk-free rate of interest is 10% per annum. [2]
[Total 4]

4 Let R_i denote the return on security i given by the following multifactor model:

$$R_i = a_i + b_{i,1} I_1 + b_{i,2} I_2 + \ldots + b_{i,L} I_L + c_i$$

where a_i and c_i are the constant and random parts respectively of the component of the return unique to security i, $I_1, \ldots, I_L$ are the changes in a set of the L indices and $b_{i,k}$ is the sensitivity (factor beta) of security i to factor k.

(i) State the category of the above model where:

(a) index 1 is a price index, index 2 is the yield on government bonds and index 3 is the annual rate of economic growth.

(b) index 1 is the level of Research and Development expenditure, index 2 is the price earnings ratio, index 3 is the level of gearing.
[2]

Consider the following two-factor model for the returns on three assets A, B and C:

Asset	*A*	*B*	*C*
a_i	0.03	0.05	0.1
$b_{i,1}$	1	3	1.5
$b_{i,2}$	−4	2	1.5

(ii) Determine the equation for the expected return on a portfolio which:

(a) equally weights the three securities.
(b) has weights $x_A = -0.5, x_B = 1.5, x_C = 0$.
[4]

(iii) Construct a portfolio of securities *A, B, C* that has a factor beta of 2 on the first factor and 1 on the second factor, i.e. the expected return on the portfolio is:

$$R_P = a_P + 2 I_1 + I_2 + c_P.$$ [3]
[Total 9]

5 (i) State the key arguments against modelling market returns using a Gaussian random walk. [3]

(ii) Describe the difference in time series modelling between a cross-sectional property and a longitudinal property, including their dependence on the initial conditions imposed on the model. [4]

[Total 7]

6 Let p_t denote the value at time t (measured in years) of a European put option on a non-dividend-paying stock with price S_t. The option matures at time T and has a strike price K. The continuously compounded risk-free rate of interest is r.

(i) Derive a lower bound for p_t in terms of S_t and K. [4]

Consider a market with the following two non-dividend-paying stocks:

Stock	*Volatility*	*Current price*
S_1	10%	£10
S_2	20%	£10

The following options are available on those stocks:

	Derivative	*Underlying asset*	*Strike price*	*Time to expiry*	*Current price*
(a)	European call option	S_1	£8	1	£2.50
(b)	European call option	S_2	£8	1	£2.26
(c)	European put option	S_1	£8	1	£1.55
(d)	European put option	S_1	£12	1	£1.20
(e)	American put option	S_1	£12	1	£1.13

The continuously compounded risk-free rate is 3% per annum.

(ii) Identify, with reasons, five discrepancies in these option prices. [5]

[Total 9]

7 Consider a binomial tree model for the stock price S_t. Let $S_0 = 50$ and let the price rise by 10% or fall by 5% each month for the next three months. Assume also that the risk-free rate is 5% per annum continuously compounded.

(i) State the conditions under which the market is arbitrage free. [2]

(ii) Calculate the price at time $t = 0$ of a European call option on this stock, which expires in three months and is struck at-the-money (i.e. strike price $K = 50$). [4]

A special option, called a knock-out barrier option, goes out of existence (i.e. expires without any payoff or value) if the underlying asset reaches a pre-specified barrier $b > 0$ either from above (down-and-out) or from below (up-and-out).

The down-and-out call has the following payoff at time T:

$$\max(S_T - K, 0) \text{ if } \min_{0 \le t \le T} S_t \ge b,$$

$$0 \text{ otherwise.}$$

Assume this special option is written on the given stock, has the same strike price and maturity as the European call option described in part (ii) and the barrier b is fixed at 48.

(iii) Calculate the price of this contract using the binomial tree model and risk-neutral valuation. [3]

(iv) Determine the price of the down-and-out contract when $b = 40$, without performing any further calculations. [2]
[Total 11]

8 Consider a non-dividend-paying stock, with price S_t , and a European call option on that stock, whose value can be modelled using the Black-Scholes model.

(i) Write down the formula for the delta of this option under this model. [1]

Suppose that the stock price at time 0 is S_0 = \$40 and the continuously compounded risk-free rate is 2% per annum. The call option has strike price \$45.91, term to maturity 5 years and a delta of $\Delta = 0.6179$.

(ii) Determine the implied volatility of the stock to the nearest 1%. [4]

A second stock with price R_t is currently priced at R_0 = \$30 and has volatility $\sigma_R = \sqrt{15\%}$ per annum.

An exotic option pays an amount c at time T if $S_1/S_0 < k_S$ and $R_1/R_0 < k_R$.

(iii) Give a formula for the value of the option at time 0 if the two stocks are independent, defining any additional notation used. [2]

(iv) Explain how the structure of the option could be simplified if the assets were perfectly correlated. [3]

Assume now that the stock prices are independent. The option has term $T = 1$ year, payoff c = \$50 and strike prices $k_S = 0.8$ and $k_R = 0.6$.

(v) Determine the value of the option at time 0. [5]
[Total 15]

9 (i) Write down the properties of the following two models for interest rates:

(a) the one-factor Vasicek model
(b) the Cox-Ingersoll-Ross model

[You are not required to give any formulae for the models.] [4]

The Vasicek term structure model is described by the following stochastic differential equation:

$$dr_t = a(b - r_t)dt + \sigma dW_t,$$

with initial value r_0 and a, b, $\sigma > 0$.

(ii) Show, by solving the Vasicek stochastic differential equation, that:

$$r_t = r_0 e^{-at} + b\left(1 - e^{-at}\right) + \sigma \int_0^t e^{-a(t-s)} dW_s \ .$$ [4]

(iii) Determine the expectation, the variance and the distribution of the short rate r_t. [3]

[Total 11]

10 A company has issued zero-coupon bonds payable in five years' time for a nominal amount of £100m. The company has also issued 1 million non-dividend-paying shares. A Black-Scholes model for the value of the company is adopted.

(i) Derive an expression for the value of the debt at time 0 using the Merton model, in terms of the total value of the company and the value of a call option. [4]

The current total value of the company is £200m. The continuously compounded risk-free interest rate is 1% per annum.

The current arbitrage-free prices of options on the company's shares, with maturity in five years' time and a strike price of £100, are as follows:

- put option = £17.30
- call option = £27.55

(ii) Calculate, using put-call parity, the value of the zero-coupon bonds per £100 nominal. [3]

The volatility of the total value of the company is 17% per annum.

(iii) Determine the approximate change in the share price and the bond price that would arise from a £1m increase in the total value of the company.
[Hint: consider the delta of an appropriate option.] [4]

(iv) Comment on the relative change in the share and bond prices in part (iii). [2]

(v) Comment, without carrying out any calculations, on how the relative change in part (iii) would differ if the total value of the company was lower. [1]
[Total 14]

END OF PAPER