INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

24 April 2014 (am)

Subject CT8 – Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a new page.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Outline the key findings in behavioural finance. [10]

2 (i) State the expression for the return on a security, i, in the single-index model, defining all terms used. [2]

(ii) Explain the difference between the single-index model and the Capital Asset Pricing Model. [1]

Suppose the market has expected return 6% and standard deviation 10%. Two securities have expected returns 8% and 10%, and standard deviations 15% and 20%. The correlation between these two securities and the market is 0.25 and 0.4 respectively. Assume the single-index model described in (i) holds.

(iii) Calculate the constant parameters in the expression for the return of these two securities. [5]

(iv) Explain how a multi-index model would be expected to perform relative to the single-index model in terms of fitting data and predicting future security price moves. [2]

[Total 10]

3 Let W be a standard Brownian motion.

(i) State the continuous-time log-normal model of a security price S, defining all the terms used. [2]

Let f be a function of t and W_t^2.

(ii) (a) Find a function f such that $f(t, W_t^2)$ is a F_t-martingale, with F the Brownian filtration.

Hint: $E(W_t^2 | \mathcal{F}_s) = W_s^2 + t - s$ for all $t \geq s$.

(b) Use Ito's lemma to show that $f(t, W_t^2)$ is a process with zero drift.

[4]

Let X be the process defined as $X_t = t^{\alpha} W_{t^{\beta}}$.

(iii) Derive the values of α and β for which X_t defines a standard Brownian motion. [6]

[Total 12]

4 Consider the following long position in European and American call options written on a stock, with strikes and times to expiry as set out in the table below.

Option	*European/American*	*Strike price*	*Time to expiry*
A	American	400	3 years
B	American	400	2 years
C	American	420	3 years
D	European	400	3 years
E	European	400	2 years

Rank these options in order of value to the extent that this is possible. [5]

5 Consider the following model for the short-rate *r*:

$$dr_t = \mu r_t dt + \sigma dZ_t$$

where μ and σ are fixed parameters and Z is a standard Brownian motion.

(i) Comment on the suitability of this model for the short-rate. [4]

An alternative model for the short-rate is the Vasicek model:

$$dr_t = a(\mu - r_t)dt + \sigma dZ_t.$$

(ii) Derive an expression for $\int_t^T r(u)du$. [6]

(iii) State the distribution of $\int_t^T r(u)du$. [1]

[Total 11]

6 (i) State the equation for the capital market line in the Capital Asset Pricing Model (CAPM), defining all the terms used. [3]

In a market where the CAPM is assumed to hold, the expected annual return on the market portfolio is 12%, the variance is 4%% and the effective risk-free annual rate is 4%. An Agent wants an expected annual return of 18% on a portfolio worth £1,200,000.

(ii) Calculate the standard deviation of the return on the corresponding efficient portfolio. [2]

(iii) Calculate the amount of money invested in each component of the Agent's portfolio. [3]

[Total 8]

7 In a Black-Scholes market, let S be the price of a stock and D be the price of a derivative written on S, with maturity T, where $D_t = g(t, S_t)$ for any $t < T$ and $g(T, x) = f(x)$.

(i) Write down the partial differential equation (PDE) that g must satisfy, including the boundary condition for time T. [3]

Suppose that the derivative pays S_T^n / S_0^{n-1} at time T, where n is an integer greater than 1.

(ii) Show, using (i), that the price of the derivative at time t is given by $D_t = (S_t^n / S_0^{n-1})e^{\mu(T-t)}$ for some μ which you should determine. [6]

[Total 9]

8 (i) State and prove the put-call parity for a stock paying no dividends. [5]

In a Black-Scholes market, a European call option on the dividend-free stock, with strike price \$120 and expiry $T = 1$ year is priced at \$10.09. The continuously compounded risk-free rate is 2% p.a. and the stock is currently priced at \$110.

(ii) Estimate the implied volatility of the stock to the nearest 1%. [4]

A European put option on the same stock has strike price \$121 and the same maturity. An investor holds a portfolio which is long one call and short one put.

(iii) Sketch a graph of the payoff at maturity of the portfolio against the stock price [2]

(iv) (a) Determine an upper and a lower bound on the value of the portfolio at maturity.

(b) Deduce bounds for the current put price. [3]

(v) Determine the fair price of the put. [2]

[Total 16]

9 Outline the evidence against normality assumptions in models of market returns. [8]

10 A company has two zero-coupon bonds in issue. Bond A redeems in 1 year and the current price of £100 nominal is £92.50. Bond C redeems in 2 years and the current price of £100 nominal is £74.72.

The continuously compounded risk-free rate is 2.5% p.a. for the next two years.

(i) Write down the formula for the general zero-coupon bond price in the two-state model for credit ratings, defining all the terms used. [2]

(ii) Determine the implied risk-neutral probability of default for bond A, assuming this model holds, and a recovery rate of 50% for bond A. [3]

If bond A defaults then bond C automatically defaults with a recovery rate of zero, whereas if bond A does not default then bond C may still default in the second year, but with a recovery rate of 50%.

(iii) Modify your answer to (i) to give a formula for the current price of bond C. [3]

(iv) Calculate the risk-neutral probability of default for bond C. [3]

[Total 11]

END OF PAPER