INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

28 September 2017 (pm)

Subject CT8 – Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all nine questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 (i) Define the following terms:

(a) absolute dominance
(b) first order stochastic dominance
(c) second order stochastic dominance

[4]

Consider four assets which will deliver a one-year return r_i on asset i with probabilities as set out below:

	$P(r_i = -5\%)$	$P(r_i = -3\%)$	$P(r_i = 0\%)$	$P(r_i = +3\%)$	$P(r_i = +5\%)$
Asset 1	0.2	0.2	0.2	0.2	0.2
Asset 2	0.3	0.2	0.1	0.2	0.2
Asset 3	0.1	0.3	0.2	0.3	0.1

(ii) Determine which type of dominance, if any, is exerted by:

(a) asset 2 over asset 3.
(b) asset 3 over asset 1.
(c) asset 1 over asset 2. [6]

[Total 10]

2 (i) Define in the context of mean-variance portfolio theory:

(a) an inefficient portfolio
(b) an efficient portfolio [2]

(ii) State the two assumptions about investor behaviour that are needed for the existence of efficient portfolios. [1]

An investment universe includes two assets, A and B, with expected return on asset i of r_i and variance v_i as set out below:

Asset i	*Expected return* r_i	*Variance of return* v_i
A	$r_A = 0.05$	$v_A = 0.16$
B	$r_B = 0.07$	$v_B = 0.25$

The correlation of returns is $c_{AB} = -0.2$.

In an efficient portfolio, let a be the proportion which is held in asset A.

(iii) Express the portfolio variance V in terms of a quadratic function in a, showing your workings. [3]

Let R be the expected return on the portfolio.

(iv) Express the portfolio variance V in terms of a quadratic function in R, using your result from part (iii) and showing your workings. [Your expression should not include a.] [3]

The expression in part (iv) represents the efficient frontier.

An investor uses a utility function that gives rise to an indifference curve $V = 16R - 200R^2$.

(v) Determine the two portfolios on the efficient frontier that also lie on the investor's indifference curve. [4]

(vi) Comment on the implications for part (v) if short selling is not allowed in the market. [2]

[Total 15]

3 Consider a European call option with price c_t written on an underlying non-dividend-paying security with price S_t at current time t.

(i) State whether each of the following changes in underlying factors would increase or reduce the price of this option:

(a) a fall in the price of the underlying security
(b) an increase in the strike price of the option
(c) an increase in the volatility of the underlying security price
(d) a fall in the risk-free rate of interest

[You should assume that each change occurs on a standalone basis, i.e. all other factors are unchanged.] [2]

(ii) Explain each of your statements in part (i). [4]

Consider a European put option with price p_t written on the same underlying security, with the same strike price K and the same maturity T as the call option described above.

The continuously compounded risk-free rate of interest is r.

(iii) Write down a formula that relates the values of c_t and p_t. [1]

The call option has value £0.50 at time $t = 0$, and the put option has value £1.00. Both options are written on a security with current value S_0 = £5, and both options have strike price £6.00 and maturity $T = 3$ years.

(iv) Determine the continuously compounded risk-free rate r. [2]

(v) Suggest, with justification, how the formula in part (iii) can be rewritten as an inequality if both options are American options. [3]

[Total 12]

4 Consider a one-period binomial tree model for the stock price process S_t.

Let $S_0 = \$100$ and assume that in three months' time the stock price is either \$125 or \$105. No dividends are payable on this stock.

Assume also that the continuously compounded risk-free rate is 5% per annum.

(i) Verify that this market is not arbitrage-free by considering the relationship between the risk-free rate and the stock price movements. [2]

(ii) (a) Identify a portfolio which would generate an arbitrage profit.
(b) Calculate this profit.
[4]

Now assume that the continuously compounded risk-free rate is 20% per annum. Consider a European put option on this stock, expiring in three months' time and with strike price $K = \$120$.

(iii) Calculate the current price of this put option. [3]
[Total 9]

5 (i) State the Cameron-Martin-Girsanov theorem. [3]

(ii) State an important property of the discounted value of a security price process under the risk-neutral measure. [1]

The price process S_t of a traded security satisfies the following stochastic differential equation:

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

where W_t is a standard Brownian motion under the real-world probability measure, and μ and σ are constants, with $\sigma > 0$.

Let $r > 0$ be the continuously compounded risk-free rate of interest.

(iii) Show, using parts (i) and (ii), that $W_t + \lambda t$ is a Brownian motion under the risk-neutral probability measure, if $\lambda = {}^{(\mu - r)}\!/_{\sigma}$. [3]

(iv) Calculate the value of λ in the case in which $\mu = 0.04 + r$ and $\sigma = 0.4$. [1]

Another traded asset has a price process satisfying the stochastic differential equation

$$dA_t = (0.06 + r)A_t dt + \gamma A_t dW_t.$$

(v) Determine the value of the volatility coefficient γ, using your result from part (iv). [2]
[Total 10]

6 (i) Write down an expression for the price of a derivative in a Black-Scholes market in terms of an expectation under the risk-neutral measure, defining any additional notation that you use. [3]

Consider an option on a non-dividend-paying stock when the stock price is £50, the exercise price is £49, the continuously compounded risk-free rate of interest is 5% per annum, the volatility is 25% per annum, and the time to maturity is six months.

(ii) Calculate the price of the option using the Black-Scholes formula, if the option is a European call. [4]

(iii) Determine the price of the option if it is an American call. [1]

(iv) Calculate the price of the option if it is a European put. [2]

(v) Determine how the prices of the contracts in parts (ii) to (iv) would change in the case of a dividend-paying underlying stock. [Note that you do not have to perform any further calculations.] [3]
[Total 13]

7 (i) State the main potential drawback of the Vasicek model. [1]

(ii) Discuss the extent to which this drawback may be a problem. [3]

(iii) Explain how the Cox-Ingersoll-Ross model avoids this drawback. [3]

The Vasicek term structure model is described by the following stochastic differential equation:

$$dr_t = a(b - r_t)dt + \sigma dW_t,$$

and a, b, $\sigma > 0$.

Under this model, the short rate r_t follows a Normal distribution with mean

$$E(r_t) = r_0 e^{-at} + b\,(1 - e^{-at})$$

and variance $\text{Var}\,(r_t) = \dfrac{\sigma^2}{2a}(1 - e^{-2at})$.

(iv) Assess, using the information provided above, whether the model generates interest rates that are mean reverting and, if so, the value to which they revert. [2]

(v) Assess, using the information provided above, the relevance of the parameter a to any mean reversion. [2]
[Total 11]

8 In a market in which the Arbitrage Pricing Theory (APT) model holds, the expected return is given by

$$E[R_i] = \lambda_0 + \lambda_1 b_{i,1} + \lambda_2 b_{i,2} + \ldots + \lambda_n b_{i,n}$$

(i) Define all the terms in this equation. [2]

Let r_f denote the risk-free rate of interest.

(ii) Construct a risk-free portfolio to prove that $\lambda_0 = r_f$. [2]

Assume that $r_f = 0.075$. Consider a two-factor model (i.e. $n = 2$) and two well-diversified portfolios (P1 and P2) with the following features:

	P1	*P2*
$E[R_i]$	0.18	0.15
$b_{i,1}$	1.5	0.5
$b_{i,2}$	0.5	1.5

(iii) Determine the values of λ_1 and λ_2. [3]

Suppose that in the market there is another portfolio with the following features:

$$E[R_3] = 0.16,\ b_{3,1} = 0.75,\ b_{3,2} = 0.7.$$

(iv) Comment on the feasibility of such a portfolio under the APT model assumptions. [3]

[Total 10]

9 Consider the Merton model for credit risk.

Assume that a firm has issued a zero-coupon bond maturing in five years' time with maturity value €100m, and that the current value of the firm's assets is €110m.

Further assume that the estimated volatility of the firm's assets is 25% per annum and the risk-free rate of interest is 2% per annum continuously compounded.

(i) Show that the current value of the debt of the firm is €76.88m. [5]

(ii) Calculate the yield to maturity of the debt. [3]

(iii) Calculate the credit spread on the debt. [2]

[Total 10]

END OF PAPER