INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2014 examinations

Subject CT8 – Financial Economics
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

D C Bowie
Chairman of the Board of Examiners

June 2014

General comments on Subject CT8

Subject CT8 introduces the main concepts and principles of financial economics. These are developed in later subjects in the ST series of exams. This subject combines various types of skills. In particular, along with CT7, it is one of the first where candidates are expected to write lengthy passages of reasoned thought, rather than just complete calculations. This is a skill that will be new to many, and candidates are advised to pay particular attention to the answers to this type of question by studying many past papers.

Comments on the April 2014 paper

The general performance was good. Candidates generally found this paper challenging, but well-prepared candidates scored well across the whole paper and the best candidates scored close to full marks. As in previous diets, questions that required an element of application of the core reading to situations that were not immediately familiar proved more challenging to most candidates.

1 **Anchoring and adjustment**

Anchoring is a term used to explain how people will produce estimates. They then adjust away from this initial anchor to arrive at their final judgement.

Prospect theory

A theory of how people make decisions when faced with risk and uncertainty. It replaces the conventional risk averse / risk seeking decreasing marginal utility theory.

Prospect theory is associated with the concept of:

Framing (and question wording)

The way a choice is presented ("framed") and, particularly, the wording of a question in terms of gains and losses, can have an enormous impact on the answer given or the decision made.

Myopic loss aversion

This is similar to prospect theory, but considers repeated choices rather than a single "gamble".

Estimating probabilities

Issues (other than anchoring) which might affect probability estimates include:

- *Dislike of "negative" events* – the "valence" of an outcome (the degree to which it is considered as negative or positive) has an enormous influence on the probability estimates of its likely occurrence.

- *Representative Heuristics* – people find more probable that which they find easier to imagine. As the amount of detail increases, its *apparent* likelihood may increase (although the *true* probability can only decrease steadily).

- *Availability* – people are influenced by the ease with which something can be brought to mind. This can lead to biased judgements when examples of one event are inherently more difficult to imagine than examples of another.

Overconfidence

People tend to overestimate their own abilities, knowledge and skills. This may be a result of:

- *Hindsight bias* – events that happen will be thought of as having been predictable prior to the event, events that do not happen will be thought of as having been unlikely prior to the event.

- *Confirmation bias* – people will tend to look for evidence that confirms their point of view (and will tend to dismiss evidence that does not justify it).

Mental accounting

People show a tendency to *separate* related events and decisions and find it difficult to *aggregate* events.

Effect of options

Other issues include:

- *Primary effect* – people are more likely to choose the *first* option presented, but

- *Recency effect* – in some instances, the *final* option that is discussed may be preferred! (The gap in time between the presentation of the options and the decision may influence this dichotomy.)

- Other research suggests that people are more likely to choose an *intermediate* option than one at either end!

- A greater range of options tends to discourage decision-making. On the other hand, a higher probability is attributed to options explicitly stated than when included in a broader category.

- *Status Quo bias* – people have a marked preference for keeping things as they are.

- *Regret aversion* – by retaining the existing arrangements, people minimise the possibility of *regret* (the pain associated with feeling responsible for a loss).

- *Ambiguity aversion* – people are prepared to pay a premium for rules.

This question was generally well answered by candidates who knew the 8 key findings of behavioural finance. A surprising number of candidates confused this question with the Efficient Market Hypothesis. Some candidates discussed the shortcomings of assuming that consumers are rational.

2 (i) The single-index model expresses the return on a security as

$$R_i = \alpha_i + \beta_i R_M + \varepsilon_i$$

where R_i is the return on security i,

α_i and β_i are constants,

R_M is the return on the market,

ε_i is a random variable representing the component of R_i not related to the market.

(ii) The single-index model is purely empirical and is not based on any theoretical relationships between β_i and the other variables, which are assumed in CAPM.

(iii) The β_i are the ratio of the covariances of the securities with the market divided by the variance of the market.

So, $\beta_1 = \dfrac{15\times 0.25\times 10}{100} = 0.375$ and $\beta_2 = \dfrac{20\times 0.4\times 10}{100} = 0.8$.

Hence, $\alpha_1 = 8 - 0.375\times 6 = 5.75$ and $\alpha_2 = 10 - 0.8\times 6 = 5.2$.

(iv) As you are fitting more parameters, in-sample results should give a better fit (although not necessary a higher information criterion).

In terms of prediction, adding additional indices are unlikely to improve predictions, assuming the market is reasonably efficient.

Part (i) was well answered by most candidates. Part (ii) was poorly answered by most candidates. Part (iii) was well answered by most candidates. Some candidates failed to derive both alpha and beta, but instead only provided a single set of parameters. Part (iv) was well answered by most candidates. Several candidates failed to answer both parts of the question (i.e. fitting the data and predicting future security prices). A number of candidates also concluded that the multi-factor model would be better at predicting future security prices.

3 $S_t = S_0 \exp(\mu_t + \sigma W_t)$

Where μ is the drift
and σ is the volatility

Using the hint, we see that $E\left(W_t^2 - t \mid \mathcal{F}_s\right) = W_s^2 - s$ for all $s < t$.

(and $E(|(W_t^2 - t)|) < \infty$) .

Then using Ito's lemma we can see that

$$d(W_t^2 - t) = 2W_t dW_t,$$

so indeed the process has zero drift and is a martingale.

For X to be a Brownian Motion we need

$$t = \mathrm{Var}(X_t) = t^{2\alpha}\mathrm{Var}\left(W_{t^\beta}\right) = t^{2\alpha+\beta} \Rightarrow 2\alpha + \beta = 1$$

We can then consider the increment in the process from t = 1(when the value of X_t is simplified). So suppose that $t > 1$:

$$E\left(\left(t^{\alpha}W_{t^{\beta}} - W_1\right)^2\right) = t^{2\alpha+\beta} + 1 - 2t^{\alpha}\min\left(t^{\beta},1\right).$$

But for X_t to be a standard Brownian motion we must also have:

$$E\left(\left(t^{\alpha}W_{t^{\beta}} - W_1\right)^2\right) = t - 1$$

Since $t^{2\alpha+\beta} = t$ we must have $t^{\alpha}\min\left(t^{\beta},1\right)$ = 1, so $\alpha = 0$ whence $\beta = 1$ if $\beta > 0$.

This is not the only solution.

If $\beta < 0$ then $t^{\alpha}t^{\beta} = 1$ and so $\alpha + \beta = 0$ and thus $\alpha = 1$ and $\beta = -1$ also works.

You may recognise the second solution to this problem as the time inversion property of standard Brownian motion.

Alternatively:

The law of any Gaussian stochastic process is completely determined by its expectation and its covariance function.

For X_t to be standard Brownian motion, we require:
$E\ X_t = 0$ and cov (X_s, X_t) min(s, t).

Now Cov(X_s , X_t) = Cov($s^{\alpha}X_s^{\beta}$, $t^{\alpha}X_t^{\beta}$) = $s^{\alpha}\ t^{\alpha}\min(s^{\beta}, t^{\beta})$

Setting this equal to min(s, t) gives the two pairs of solutions required: $\alpha = 0$, $\beta = 1$ and $\alpha = 1$, $\beta = -1$.

Part (i) was well answered by most candidates. Part (ii) was well answered by most candidates. Part (iii) was very poorly answered in general. Few if any considered the finite moment condition. Only a few students managed to score more than a couple of marks in part (iii) because they didn't try to check the relevant Gaussian parameters.

4 Call options with lower strike prices are more valuable, so A > C.

American call options are more valuable than European call options so A > D and B > E.

American call options with longer time to expiry are more valuable so A > B.

So, A is the most valuable, B is more valuable than E, and we cannot pass comment on the relative value of other pairs with the information available.

Most managed this quite well, though a lot of students who found the correct inequalities also derived some spurious ones as well. The most common mistake seemed to be not understanding American options and not noticing that dividends might be payable.

5 (i) Interest rates may not be positive.

Interest rates do not display mean reversion.

This model is computationally tractable.

This model won't give a realistic range of yield curves.

It won't fit historical data well.

It cannot be calibrated to current market data.

It is not very flexible (single factor model).

It is arbitrage-free.

(ii) Since the Vasicek model is an Ornstein-Uhlenbeck process we can solve the SDE for the short rate to get:

$$r(u) = r(t)e^{-a(u-t)} + \mu\left(1 - e^{-a(u-t)}\right) + \sigma e^{-au}\int_t^u e^{as}\,dZ_s\,.$$

Hence

$$\int_t^T r(u)\,du = r(t)\int_t^T e^{-a(u-t)}du + \mu\int_t^T \left[1 - e^{-a(u-t)}\right]du + \sigma\int_t^T e^{-au}\int_t^u e^{as}\,dZ_s\,du$$

and so, carrying out the deterministic integrals, we find:

$$\int_t^T r(u)\,du = \mu(t-t) + \left[r(t)-\mu\right]\frac{1-e^{-a(T-t)}}{a} + \sigma\int_t^T \frac{1-e^{-a(T-s)}}{a}\,dZ_s$$

So, $\int_t^T r(u)\,du$ is a Gaussian random variable.

Students scored well on part (i) which was standard bookwork. In part (ii) many students only solved the Vasicek SDE rather than deriving an expression for the integral as asked.

6 (i) The capital market line is given by

$$r_P - r_0 = \sigma_P / \sigma_M(r_M - r_0),$$

where

r_P is the expected return on an efficient portfolio, P;
r_M is the expected return on the market portfolio;
r_0 is the risk-free rate;
σ_P is the standard deviation of the return on the portfolio, P;
σ_M is the standard deviation of the return on the market portfolio.

(ii) r_P is 18% and so

$$14 = 8\sigma_P / 2, \text{ thus } \sigma_P = 0.035 = 3.5\%.$$

(iii) The efficient portfolio is a mix of the market portfolio and the risk-free asset. If the weights (which sum to 1) are w_M and w_0 then the expected return is $12w_M + 4\,w_0$ so $8\,w_M = 14$ and $w_M = 1.75$, $w_M = -0.75$.

Thus the efficient portfolio has £2,100,000 in the market portfolio and is short £900,000 in cash.

The majority of students scored full marks on a straight-forward question.

Some weaker students confused the variance with the standard deviation when applying the formula for CAPM. Others lost marks for minor calculation errors.

7 (i) The PDE is the Black-Scholes PDE:

$$\tfrac{1}{2}\sigma^2 x^2 g_{xx} + (r - q)x g_x - rg + g_t = 0$$

with boundary condition as above: $g(T, x) = f(x)$.

(ii) The proposed solution implies that for this derivative the function g is given by $g(t, x) = (x^n / S_0^{n-1})e^{\mu(T-t)}$, where n is an integer great than 1.

This gives $xg_x = ng$, $x^2 g_{xx} = n(n-1)g$ and $g_t = -\mu g$.

Thus, to solve the PDE we need $\mu = \tfrac{1}{2}\sigma^2 n(n-1) + (n-1)r - nq$.

A quick check shows that g satisfies the boundary condition:
$g(T, x) = x^n/S_0^{n-1}$.

Not generally well-answered. Most students managed part (i) but few got any marks for part (ii). A surprising number of students answered part (i) correctly but failed to try the obvious route of substituting the equation from part (ii) into the formula from part (i).

Students were not penalised if they took the dividend rate q to be 0.

8 (i) Consider the portfolio which is long one call plus cash of $Ke^{-r(T-t)}$ and short one put.

The portfolio has a payoff at the time of expiry of S_T.

Since this is the value of the stock at time T, the stock price should be the value at any time $t < T$: that is

$$C_t + Ke^{-r(T-t)} - P_t = S_t.$$

(ii) This relationship is known as *put-call parity*.

The Black-Scholes formula gives us that $S_0\,\Phi(d_1)\,Ke^{-rT}\,\Phi(d_2)$, with

$$S_0 = 110,\ K = 120,\ r = .02,\ T = 1$$

so that

$$d_1 = (\log(S_0 / K) + r + \tfrac{1}{2}\sigma^2 T) / \sigma\sqrt{T} = (\log(11/12) + .02 + \tfrac{1}{2}\sigma^2) / \sigma,$$
$$d_2 = d_1 - \sigma.$$

Guessing and repeated interpolation gives $\sigma = 30\%$.

(iii)

(iv) (a) The payoff from the portfolio, D, satisfies

$$S_1 - 121 \le D \le S_1 - 120.$$

It follows that the initial price, V, of the portfolio should satisfy

$$S_0 - 121e^{-r} \le V \le S_0 - 120\, e^{-r},$$

i.e. $-8.604 \le V \le -7.624$.

(b) And this implies that $17.714 \le P_0 \le 18.694$.

(v) The Black-Scholes price (using the formula in the tables) is \$18.35.

Most students scored highly with the proof of the put-call parity.

Part (ii): a lot of students checked two trial values, and then interpolated to get something at or near 30%. They didn't always check that their answer gave the right answer.

Part (iii): few students managed to sketch the graph correctly. There was often confusion over the payoff profile between 120\$ and 121\$.

Part (iv): few students understood the question, mainly because they hadn't sketched the graph in part (iii).

Part (v): a common mistake was using put-call parity to work out the value of the put and not spotting that it had a different strike.

9 A strand of empirical research questions the use of the normality assumptions in market returns. In particular,

- market crashes appear more often than one would expect from a normal distribution. While the random walk produces continuous price paths, jumps or discontinuities seem to be an important feature of real markets.

- Furthermore, days with no change, or very small change, also happen more often than the normal distribution suggests. This would seem to justify the consideration of Levy processes.

- Q-Q plots of the observed changes in the FTSE All Share index against those which would be expected if the returns were lognormally distributed show substantial differences. This demonstrates that the actual returns have many more extreme events, both on the upside and downside, than is consistent with the lognormal model.

- a quintic polynomial distribution whose parameters have been chosen to give the best fit to the data, clearly provides an improved description of the returns observed, in particular more extreme events are observed than is the case with the lognormal model. The rolling volatilities of a simulation from the non-normal distribution show significant differences over different periods. This volatility process has the same characteristics as the observed volatility from the equity market.

This question was very poorly answered by most candidates, with very few candidates scoring more than 4/8. Several candidates scored zero marks for providing a discussion on the normal distribution itself, as opposed to the assumption of normality in market returns. Candidates were generally able to generate the first two points (market crashes occur more often than expected, jumps, etc. and that there are a larger number of days with little or no movement). Almost no one discussed the use of Q-Q plots or a quintic polynomial. Again, some candidates noted the points highlighted by Anna Bishop around the Hausdorff fractal dimension.

10 (i) $B(t,T) = e^{-r(T-t)}[1-(1-\delta)(1-\exp(-\int_t^T \lambda_s ds))]$, where B is the bond price, λ is the risk-neutral default rate, δ is the recovery rate, and r is the risk-free rate.

(ii) Using the formula, $0.925 = e^{-0.025}[1-0.5(1-\exp(-\int_0^1 \lambda_s ds))]$

so that

$$Q(\text{bond } A \text{ defaults}) = (1-\exp(-\int_0^1 \lambda_s ds)) = 0.10317$$

(iii) Now bond C pays:

- nothing with probability $(1-\exp(-\int_0^1 \lambda_s ds))$

- 50% with probability $(1-\exp(-\int_1^2 \lambda_s ds))\ \exp(-\int_0^1 \lambda_s ds)$
- 100% with probability $\exp(-\int_0^2 \lambda_s ds)$

Thus

$$C_0 = 0.7472 = e^{-0.05}[\exp(-\int_0^2 \lambda_s ds) + 0.5(1-\exp(-\int_1^2 \lambda_s ds))\ \exp(-\int_0^1 \lambda_s ds)]$$
$$= e^{-0.05}[0.5\exp(-\int_0^2 \lambda_s ds) + 0.5\exp(-\int_0^1 \lambda_s ds)]$$

(iv) From the second expression in (iii) and the answer to (ii) we obtain

$$Q(\text{bond } C \text{ defaults}) = 1-\exp(-\int_0^2 \lambda_s ds)$$
$$= (1 - 2e^{0.05}.7472 + (1 - .10317)) = 0.32581.$$

Most students scored highly on part (i), part (ii).

Most struggled on part (iii).

A small number of students managed to work out the non-conditional probability of C defaulting with only a handful coming to the full answer.

END OF EXAMINERS' REPORT