INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

7 October 2015 (am)

Subject CT8 – Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a new page.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Describe the definitions, assumptions and key results of consumer choice theory. [8]

2 An investor makes decisions using a quadratic utility function, $U(w) = a + bw + cw^2$.

(i) Write down the absolute and relative risk aversion for this utility function. [2]

The investor currently has wealth of £100, and using her utility function $U(100) = 610$.

The investor is offered a gamble with a profit of £20 with probability p, and a loss of £20 with probability $(1 - p)$. She will accept this gamble only if $p \geq 0.55$.

(ii) Explain what this implies about the investor's risk aversion. [1]

The investor accepts the gamble and wins. She now has wealth of £120.

The investor is offered the same gamble again, with a profit of £20 with probability p, and a loss of £20 with probability $(1 - p)$. Based on her new wealth, she will now accept this gamble only if $p \geq 0.5625$.

(iii) Determine a, b and c. [4]

(iv) Determine the maximum wealth for which the function $U(w)$ satisfies the requirement of non-satiation. [2]

[Total 9]

3 (i) Define an "efficient portfolio" in the context of mean-variance portfolio theory. [1]

(ii) State the assumptions required for the existence of efficient portfolios. [2]

Suppose an investor invests his wealth in N securities, $i = 1, ..., N$, with x_i denoting the proportion of wealth invested in security i.

(iii) Write down a formula for the expected return on this portfolio. [1]

(iv) Write down a formula for the variance of the return on this portfolio. [1]

Now suppose the investor invests in only two securities, A and B.

(v) Derive the proportion x_A that should be invested in security A to minimise the portfolio variance. [4]

[Total 9]

4 (i) Describe the Arbitrage Pricing Theory (APT) using a two-index model for illustration. [5]

A two-index APT model has been built, using a market (traded) index, I_M, and a currency index, I_C . The model for expected returns is:

$$E_i = \lambda_0 + b_{i,M}\ \lambda_M + b_{i,C}\,\lambda_C,$$

where:

- E_i is the expected return on traded security i.
- λ_M and λ_C are the expected returns on the market and currency indices respectively.
- $b_{i,M}$ and $b_{i,C}$ are the sensitivities of the returns of security i with respect to the market and currency indices respectively.

(ii) Justify why λ_0 must be 0. [3]

Suppose that a traded security has return R and the market has the following characteristics:

- $\mathrm{Cov}(R,I_M) = 0.02$.
- $\mathrm{Var}(I_M) = 0.04$.
- $\mathrm{Var}(I_C) = 0.01$.
- The correlation between I_M and I_C is $\mathrm{Corr}(I_M,I_C) = -0.4$.
- The expected return on the security is $E_R = 0.09$.
- $\lambda_M = 0.07$ and $\lambda_C = 0.02$.

(iii) Calculate $b_{i,M}$ and $b_{i,C}$. [4]

(iv) Determine $\mathrm{Cov}(R,I_C)$. [3]

[Total 15]

5 An actuary plans to retire in five years' time, and hopes to celebrate retirement with a round-the-world cruise. The cruise will cost €20,000. The actuary chooses to save for the cruise by buying non-dividend paying shares with price S_t governed by the Stochastic Differential Equation:

$$dS_t = S_t(\mu dt + \sigma dZ_t)$$

where:

- Z_t is a standard Brownian motion.
- $\mu = 10\%$.
- $\sigma = 20\%$.
- t is the time from now measured in years; and
- $S_0 = 1$.

The instantaneous, constant, continuously compounded risk-free rate of interest is 4% p.a.

(i) Derive the distribution of S_t. [2]

(ii) Calculate the amount, A, that the actuary will need to invest in the shares to give a 40% probability of having savings of at least €20,000 in five years' time. [3]

(iii) Calculate the following risk measures at $t = 5$ applied to the difference between the value of the share holding and €20,000, if the actuary invests €10,000 at $t = 0$:

(a) standard deviation
(b) 95% Value at Risk relative to €20,000

[4]
[Total 9]

6 Consider a non-dividend paying share with price S_t at time t in a market with continuously compounded risk-free rate of interest r.

(i) Show that the fair price of a forward contract on the share maturing at time T is $K = S_0e^{rT}$. [4]

A share is currently worth $S_0 = £5$. The continuously compounded risk-free rate of interest is 3% p.a. for $0 \leq t < 1$, 5% p.a. for $1 \leq t < 2$ and 2% p.a. for $2 \leq t \leq 4$.

(ii) Calculate the fair price at $t = 0$ of a forward contract written on the share with delivery at $t = 4$. [1]

An investor enters into the above forward contract at time $t = 0$. At time $t = 1$ the share price has increased to £6.

(iii) Calculate the value to the investor of the forward contract at $t = 1$. [2]
[Total 7]

7 A non-dividend paying share currently trades at $S_0 = \$10$. An investor is considering buying a European call option on the share with a strike price of \$12 and expiry in five years. The continuously compounded risk-free rate of interest is 4% p.a.

(i) Determine lower and upper bounds for the price of the call option at time 0. [2]

The call option is currently priced at \$1.50. The assumptions of the Black-Scholes model apply.

(ii) Calculate the implied volatility of the share. [4]

(iii) Determine the corresponding hedging portfolio in shares and cash for 100 call options. [2]

[Total 8]

8 Let X be a continuous random variable with distribution function F. Define $Y = F(X)$.

(i) Prove that Y is a $U(0,1)$ random variable i.e. $P(Y < y) = y$ for $0 \le y \le 1$. [2]

Suppose that S_t is the price at time t (in years) of a share in a Black-Scholes market with $S_0 = £1.11$, the continuously compounded risk-free rate of interest $r = 2\%$ p.a. and the volatility $\sigma = 22\%$ p.a.

(ii) Determine the values of a and b such that $\Phi((\ln S_2 - a)/b)$ is a $U(0,1)$ random variable under the risk-neutral measure, where Φ is the cumulative distribution function of the standard normal distribution. [3]

Suppose that a derivative pays $D_2 = £100[\Phi((\ln S_2 - a)/b)]^2$ at maturity $t = 2$, where a and b take the values calculated in part (ii).

(iii) Determine the fair price of this derivative. [3]

Suppose that another derivative security pays the amount D_2 at time $t = 2$, but only if D_2 is at least £25.

(iv) Determine the fair price of this derivative. [3]

[Total 11]

9 (i) Describe the Merton model for credit risk. [5]

A company is about to issue a zero-coupon bond which will redeem in $T = 5$ years at £12.3 billion. The value of current issued share capital is £12.5092 billion and the company has no other debt. The continuously compounded risk-free rate of interest is 5% p.a. and the volatility of the company's gross asset value is assumed to be 30% p.a.

Assume that the share price will not change on issue and that the assumptions of the Black-Scholes model apply.

(ii) Write down the relationship between the current equity value of the company and F_T, the final gross value of the company's assets. [1]

(iii) Estimate F_{0+}, the gross value of the company's assets just after the bond issue, using the Black-Scholes formula and interpolation. [7]

(iv) Determine the corresponding credit spread on the loan. [2]
[Total 15]

10 Consider a market with the following properties:

t	$F(t-1,t)$	$B(0,t)$	$R(0,t)$	$C(t)$
0	-	-	-	£100.00
1	2%	(b)	2.0%	£102.02
2	4%	£94.18	(c)	£106.18
3	3%	£91.39	3.0%	(d)
4	(a)	£86.94	3.5%	£115.03

where:

- t is time.
- $F(s,t)$ is the forward rate at time 0 from time s to time t.
- $B(s,t)$ is the price of a zero coupon bond at time s maturing at time t with a nominal value of £100.
- $R(s,t)$ is the spot rate of interest at time s for the period s to t.
- $C(t)$ is the value of a cash account at time t.

(i) Calculate the values of (a), (b), (c) and (d) in the table above. [4]

At time 0 an investor buys £1,000 nominal of zero coupon bonds maturing at time 2, and £2,000 nominal of zero coupon bonds maturing at time 4. At time 1 interest rate expectations have changed as set out below.

t	$F(t-1,t)$
1	-
2	5%
3	4%
4	6%

(ii) Calculate the loss the investor will make if she sells the bonds at time 1. [3]

The investor decides to keep the bonds rather than selling them at time 1.

(iii) Comment on whether the investor can restructure her portfolio to recover her loss if interest rates remain unchanged. [2]
[Total 9]

END OF PAPER