INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

20 April 2017 (pm)

Subject CT8 – Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 11 questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 (i) State the expected utility theorem. [2]

A risk averse investor makes decisions using a quadratic utility function:

$$U(w) = w + dw^2.$$

(ii) Derive an upper bound for d for this investor. [2]

(iii) Explain why the investor can only use this utility function to make decisions over a limited range of wealth, w. Your answer should include a statement of this range. [2]

The investor states that the upper limit of wealth where she can use this utility function is w = $1,000.

(iv) Determine the value of d in the investor's utility function. [1]

The investor wins a prize of $250 in a gameshow. She is then offered the opportunity to exchange this prize for a larger prize of $600 if she can answer one more question correctly. However, she will receive no prize at all if she gets the question wrong. She estimates her chances of answering the question correctly to be 50%.

(v) Determine whether the investor should take this opportunity to exchange. [3]

[Total 10]

2 Describe the empirical evidence relating to the continuous-time lognormal model for security prices. [8]

3 Consider a non-dividend-paying security with price S_t at time t. The security price follows the stochastic differential equation:

$$dS_t = S_t(\mu dt + \sigma dZ_t)$$

where:

- Z_t is a standard Brownian motion
- $\mu = 16\%$ per annum
- $\sigma = 25\%$ per annum
- t is the time from now measured in years
- $S_0 = 1$

(i) Derive the distribution of S_t. [4]

An investor has taken out a house loan, with a repayment of £100,000 due in six years' time.

(ii) Determine the amount that the investor would need to invest in the security to give a 75% probability of having an investment value of at least £100,000 in six years' time. [4]

The investor only has £50,000 available, which he invests in this security at time $t = 0$.

(iii) Calculate the following risk measures applied to the difference between the value of the security and £100,000 at time $t = 6$:

(a) 90% Value at Risk relative to £100,000
(b) expected shortfall or surplus relative to £100,000

[5]

(iv) Comment on the implications for the investor of your answers to part (iii). [2]

(v) Suggest two changes that the investor might therefore make to his portfolio. [2]

[Total 17]

4 (i) Define the following Greeks algebraically:

(a) delta
(b) vega
(c) theta
(d) gamma

[4]

Consider a call option with price c_t at time t (in years) written on an underlying non-dividend-paying asset with price S_t at time t and volatility σ.

Using Taylor's expansion, it can be shown that the change in value of the option is approximately given by:

$$dc_t = \text{delta} \times dS_t + 0.5 \times \text{gamma} \times (dS_t)^2 + \text{theta} \times dt + \text{vega} \times d\sigma$$

At time $t = 0$, the underlying asset price is €23 and the volatility is 20% per annum. The option is priced at €6.17 and has the following properties:

- delta = 0.822
- vega = 0.104
- theta = –0.855
- gamma = 0.033

At time $t = 1$, the security price has fallen to €20 and its volatility is now 15% per annum.

(ii) Estimate the value of the call option at time $t = 1$. [2]

The delta of a call option is always positive, whilst the delta of a put option is always negative.

(iii) Justify this result. [2]

The vega of both call and put options is always positive.

(iv) Justify this result. [1]

[Total 9]

5 Consider a three-period binomial tree model for a stock price process S_t, under which the stock price either rises by 18% or falls by 15% each month. No dividends are payable.

The continuously compounded risk-free rate is 0.25% per month.

Let S_0 = \$85.

Consider a European put option on this stock, with maturity in three months (i.e. at time $t = 3$) and strike price \$90.

(i) Calculate the price of this put option at time $t = 0$. [4]

(ii) Calculate the risk-neutral probability that the put option expires out-of-the-money. [2]

(iii) Assess whether the probability calculated in part (ii) would be higher or lower under the real-world probability measure. [No further calculation is required.] [3]

[Total 9]

6 The market price S_t of a traded security satisfies the following stochastic differential equation:

$$dS_t = (\mu - \lambda\sigma)S_t dt + \sigma S_t dW_t,$$

where W_t is a standard Brownian motion under the probability measure P^*.

(i) Determine the value of λ such that the discounted asset price process $\tilde{S}_t = e^{-rt} S_t$ is a martingale under the given probability measure. [3]

(ii) Explain whether the probability measure P^* is the real-world or risk-neutral measure, for the value of λ obtained in part (i). [3]

[Total 6]

7 The current price of a non-dividend-paying share is £7 and its volatility is thought to be 40% per annum. The continuously compounded risk-free interest rate is 5% per annum.

A European call option on this share has a strike price of £6.50 and term to maturity of one year.

(i) Calculate the price of this call option, assuming that the Black-Scholes model applies. [4]

The market price for the option is actually £2.

(ii) Show that the volatility of the share implied by the true market price of the option is 60% per annum, to the nearest 1% per annum. [6]
[Total 10]

8 (i) List the desirable characteristics of a term structure model. [3]

Let $B(t,T)$ be the price at time $t > 0$ of a zero-coupon bond which pays a value of 1 when it matures at time T.

Let $F(t,S,T)$ be the forward rate at time t for a deposit starting at time $S > t$ and expiring at time $T > S$.

Consider the following two investment strategies implemented at time t:

A	At time t: Purchase one zero-coupon bond maturing at time T. Continue to hold the bond to time T.
B	At time t: Purchase $\alpha = e^{-F(t,S,T)(T-S)}$ zero-coupon bonds maturing at time $S < T$. At time S: Invest the redemption amount from the bond at the forward rate $F(t,S,T)$ and continue to hold this deposit to time T.

(ii) Show that:

$$B(t,T) = e^{-F(t,S,T)(T-S)} B(t,S).$$ [4]

(iii) Derive an expression for $B(t,T)$ in terms of the instantaneous forward rate, using the result from part (ii). [3]
[Total 10]

9 Let R_i denote the return on security i in a two-factor model.

(i) Write down the return equation for this two-factor model, defining all additional notation that you use. [2]

(ii) Describe the three main categories of multifactor models. [3]
[Total 5]

10 In a market in which the Capital Asset Pricing Model (CAPM) holds, there are two securities with the following attributes (expressed per annum):

	security	*A*	*B*
$E(r_i)$		0.196	0.164
$\mathrm{Cov}(r_i, r_j)$	A	0.05	0.01
	B	0.01	0.03

(i) Determine the composition of the market portfolio with expected return 18% per annum. [2]

(ii) Calculate the beta of each security, under the assumption that the risk-free rate of interest is 10% per annum. [2]

(iii) State the limitations of the CAPM. [3]
[Total 7]

11 (i) Describe the three main approaches to modelling credit risk. [5]

Consider the Merton model for credit risk. Let $F(t)$ be the total value at time t of a corporate entity which has issued zero-coupon debt with promised repayment amount L due at time T.

(ii) State the condition under which the corporate entity is assumed to default in this model. [1]

(iii) State the type of process that $F(t)$ can be assumed to follow. [1]

(iv) Give an expression for the risk-neutral probability of default of the corporate entity at time 0, defining any additional notation used. [2]
[Total 9]

END OF PAPER