

# **INSTITUTE AND FACULTY OF ACTUARIES**

## **EXAMINERS' REPORT**

September 2016

### **Subject CT8 – Financial Economics Core Technical**

#### **Introduction**

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Luke Hatter  
Chair of the Board of Examiners  
December 2016

**A. General comments on the *aims of this subject and how it is marked***

1. The aim of the Financial Economics subject is to develop the necessary skills to construct asset liability models and to value financial derivatives. These skills are also required to communicate with other financial professionals and to critically evaluate modern financial theories.
2. The marking approach for CT8 is flexible in the sense that different answers to those shown in the solution can earn marks if they are relevant and appropriate. Marks for the methodology are also awarded.

**B. General comments on *student performance in this diet of the examination***

1. Students performed relatively well on bookwork questions, although many missed the opportunity to be awarded full marks due to relatively superficial knowledge.
2. The majority of the students seemed to struggle on the applications part of the questions, because they were not able to use and combine the information given to them in the question. In a few instances this resulted in students re-calculating given data from basic principles and therefore running out of time. Further, there is often a lack of knowledge of how to use the distribution tables to compute probabilities (in the specific case of this exam paper, the normal distribution), and relative inaccuracy in getting the details right.

**C. Pass Mark**

The Pass Mark for this exam was 60.

## Solutions

**Q1** (i)  $\text{VaR}(X) = -t$  where  $P(X < t) = p$ . [1]

(ii)  $\int_{-\infty}^{\mu} (\mu - x)^2 f(x) dx$  where  $\mu$  is the mean return at the end of the chosen period.  
[Or equivalently  $\int_{-\infty}^{\mu} (x - \mu)^2 f(x) dx$ ] [1]

(iii) For:

Most investors do not dislike uncertainty of returns as such; rather they dislike the possibility of low returns. One measure that seeks to quantify this view is downside semi-variance. [1]

Against:

Semi-variance is not easy to handle mathematically. [½]

Semi-variance takes no account of variability above the mean. [½]

Furthermore if returns on assets are symmetrically distributed semi-variance is proportional to variance, so it gives no extra information. [1]

Semi-variance measures downside relative to the mean rather than another benchmark that might be more relevant to the investor. [½]  
[Max 2]

(iv)  $P(X = 0) = (8^0 e^{-8})/0! = 0.00034$   
 $P(X = 1) = (8^1 e^{-8})/1! = 0.00268$   
 $P(X = 2) = (8^2 e^{-8})/2! = 0.01073$   
 $P(X = 3) = (8^3 e^{-8})/3! = 0.02863$   
 $P(X = 4) = (8^4 e^{-8})/4! = 0.05725$   
 So  $P(X \leq 4) = 0.00034 + 0.00268 + 0.01073 + 0.02863 + 0.05725 = 0.09963$

*Alternatively, directly from the Formulae & Tables:  $P(X \leq 4) = 0.09963$*  [1]

$P(X = 5) = (8^5 e^{-8})/5! = 0.09160$   
 So  $P(X \leq 5) = 0.191236$  (or directly from the Formulae & Tables) [1]

So the 10% VaR level is 5 (or -5) apples. [1]

(v)  $P(X = 0) \times (5 - 0) = 0.002$   
 $P(X = 1) \times (5 - 1) = 0.011$   
 $P(X = 2) \times (5 - 2) = 0.032$   
 $P(X = 3) \times (5 - 3) = 0.057$   
 $P(X = 4) \times (5 - 4) = 0.057$  [2]

Summing the above, we get 0.159

So the expected shortfall below 5 apples is 0.159 apples

[1]  
[Total 10]

Well-prepared students scored well here, though some confused semi-variance with expected shortfall. Many students calculated probabilities though they are listed in the Formulae & Tables.

A common mistake was to calculate the VaR and Expected Shortfall using the distribution function rather than the probability mass functions, i.e.  $P(X \leq x)$  rather than  $P(X = x)$ .

- Q2** (i) Investors select their portfolios on the basis of the expected return and the variance of that return over a single time horizon. [1]

The expected returns, variance of returns and covariance of returns are known for all assets and pairs of assets. [1]

Investors are never satiated. At a given level of risk, they will always prefer a portfolio with a higher return to one with a lower return. [1]

Investors dislike risk. For a given level of return they will always prefer a portfolio with lower variance to one with higher variance. [1]

- (ii) We use the following notation for  $i=A,B$ :

$$E(S_i) = E_i$$
$$V(S_i) = V_i$$

and  $C_{AB}$  is the covariance between the returns of Asset  $A$  and Asset  $B$ .

- (a) From the Core Reading

$$x_A = \frac{V_B - C_{AB}}{V_A - 2C_{AB} + V_B}.$$

[1]

$$\text{So } x_A = (0.25V_A - 0) / (V_A - 0 + 0.25V_A) = 0.2$$

$$\text{and } x_B = 0.8. \quad [1/2]$$

$$\text{Hence expected return} = 0.2 \times E_A + 0.8 \times 0.25E_A \quad [1/2]$$

$$= 0.4E_A. \quad [1/2]$$

(b) Now  $C_{AB} = \sqrt{(V_A \times 0.25 \times V_A)} = 0.5V_A$ . [1]

So  $x_A = (0.25V_A - 0.5V_A) / (V_A - 2 \times 0.5V_A + 0.25V_A) = -1$

and  $x_B = 2$ . [½]

Hence expected return  $= -1 \times E_A + 2 \times 0.25E_A$  [½]

$= -0.5E_A$ . [½]

[Max 4]

(iii) (a) The variance of the return on the portfolio in (b) is:

$(-1)^2 \times V_A + 2^2 \times V_B + 2 \times (-1) \times 2 \times 0.5V_A$  [½]

$= 0$ . [½]

(b) So we have created a risk-free portfolio. [1]

[Total 10]

In part (i) the majority of students stated all the assumptions of MVPT but marks were only available for the main assumptions (as asked for in the question).

Part (ii) was a straightforward calculation based on bookwork and most students scored well. A proportion of students attempted to derive the formula for the minimum variance portfolio from scratch rather than using the formula given in the Core Reading – this was time-consuming and the number of marks on offer should have been a good guide that this was not required.

- Q3** (i) The market portfolio is the weighted portfolio of the risky securities in the market, [½]  
consequently  $Er_M = 18\% = w_1Er_1 + w_2Er_2$ . [½]

As  $w_1 + w_2 = 1$ , then  $w_1 = w_2 = 0.5$ . [1]

- (ii) From the Security Market Line:

$$\beta_i = \frac{Er_i - r_f}{Er_M - r_f} \quad [1]$$

therefore  $\beta_A = 1.25$  and  $\beta_B = 0.75$ . [1]

[Total 4]

A straightforward question and many students scored full marks. Part (i) was well answered by all students but most students didn't define the market portfolio when deriving its composition.

- Q4** (i) (a) Macroeconomic. [1]  
(b) Fundamental. [1]
- (ii) The results follow from the fact that the factor beta of a portfolio on a given factor is the portfolio-weighted average of the individual securities' betas on that factor. This also applies to the constant and the random part. [1]

Working as follows:

<i>Asset</i>	<i>A</i>	<i>B</i>	<i>C</i>
$a_i$	0.03	0.05	0.1
$b_{i,1}$	1	3	1.5
$b_{i,2}$	-4	2	1.5
Weights			
<i>P1</i>	0.33	0.33	0.33
<i>P2</i>	-0.5	1.5	0
	<i>P1</i>	<i>P2</i>	
$a_P$	0.06	0.06	
$b_{P,1}$	1.83	4	
$b_{P,2}$	-0.17	5	

Hence:

$$(a) \quad E_P = 0.06 + 1.83E[I_1] - 0.17E[I_2] + E[c_P] \quad [1]$$

$$\text{for } c_P = (c_A + c_B + c_C)/3. \quad [1]$$

$$(b) \quad E_P = 0.06 + 4E[I_1] + 5E[I_2] + E[c_P] \quad [1]$$

$$\text{for } c_P = -0.5c_A + 1.5c_B. \quad [1]$$

[Max 4]

- (iii) The required portfolio has weights such that the portfolio-weighted averages of the betas equal to the target beta. Hence, we need to solve the linear system:

$$\begin{bmatrix} 1 & 3 & 1.5 \\ -4 & 2 & 1.5 \end{bmatrix} \begin{pmatrix} x_A \\ x_B \\ x_C \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad [1\frac{1}{2}]$$

which returns solution  $x_A = 1/8$ ,  $x_B = 3/8$  and  $x_C = 1/2$  (recall that the weights sum up to 1). [1/2 each = 1 1/2 total]  
[Total 9]

Part (i) was well answered by almost all students. Part (ii) was also answered well, but some students wrote the expected return on the portfolio in terms of the return on the indices rather than the expected values of the indices. Most students correctly derived the equations to solve in part (iii) with the majority also solving the equations correctly.

## Q5 (i)

- Market crashes appear more often than one would expect from a normal distribution. (The real world distribution has “fat tails”.) [1/2]
- While the random walk produces continuous price paths, jumps or discontinuities seem to be an important feature of real markets. [1/2]
- Days with no change, or very small change, also happen more often than the normal distribution suggests. (The real world distribution is “more peaked”.) [1/2]
- The assumption of independent increments is contradicted by empirical evidence of mean reversion and momentum effects. [1/2]
- The assumption of a constant volatility is contradicted by empirical evidence. [1/2]

- It can be argued that expected returns on shares are likely to vary with bond yields, which contradicts the assumption of a constant mean. [½]
- Random walks have a fractal dimension of  $1\frac{1}{2}$ , (whereas) empirical investigations of market returns often reveal a fractal dimension around 1.4. [½]
- Market returns are often (negatively) skewed [½]  
[Max 3]

(ii)

- A *cross-sectional property* fixes a time horizon and looks at the distribution over all the simulations. [1]
- For example, we might consider the distribution of inflation next year. [½]
- Implicitly, this is a distribution conditional on the past information which is built into the initial conditions, and is, of course, common to all simulations. [½]
- If those initial conditions change, then the implied cross-sectional distribution will also change. [½]
- As a result, cross-sectional properties are difficult to validate from past data, since each year of past history typically started from a different set of conditions. [½]
- However, the prices of derivatives today should reflect market views of a cross-sectional distribution. [½]
- Cross-sectional information can therefore sometimes be deduced from the market prices of options and other derivatives. [½]
- A *longitudinal property* picks one simulation and looks at a statistic sampled repeatedly from that simulation over a long period of time. [1]
- For example, we might consider one simulation and fit a distribution to the sampled rates of inflation projected for the next 1,000 years. [½]
- For some models, this longitudinal distribution will converge to some limiting distribution (ergodic distribution) as the time horizon lengthens. [½]
- Furthermore this limiting distribution is common to all simulations. [½]



- Unlike cross-sectional properties, longitudinal properties do not reflect market conditions at a particular date but, rather, an average over all likely future economic conditions. [½]

[Max 4]

[Total 7]

Part (i) was standard bookwork which was answered well.

In part (ii), most students defined a cross-sectional and longitudinal property correctly but failed to note their dependence on the initial condition and their differences, as asked in the question. There were easy marks to be had by giving an example of each, which few students did.

- Q6** (i) Consider a portfolio, A, consisting of a European put on a non-dividend-paying share and a share. [½]

At time  $T$ , portfolio A has a value of at least  $K$ , which is equal to that of the cash alternative at time  $t$  of  $Ke^{-r(T-t)}$ . [½]

Thus by the principle of no arbitrage... [½]

$$p_t + S_t \geq Ke^{-r(T-t)}. \quad [1]$$

$$\text{So } p_t \geq Ke^{-r(T-t)} - S_t. \quad [1]$$

Moreover  $p_t \geq 0$  as the payoff is always  $\geq 0$  so  $p_t \geq \max(Ke^{-r(T-t)} - S_t, 0)$  [1]  
[Max 4]

- (ii)
- (b) should be greater than (a) because the underlying asset is more volatile. [1]
  - (c) should be £0.26 by put/call parity (assuming that (a) is correct). [1]  
[Alternatively, (a) should be £3.79 if (c) is correct, or both could be incorrect.]
  - (d) should be higher than (c) because the strike price is higher. [1]
  - (d) is below the lower bound of £1.65. [1]

- (e) should be higher than (d) because an American option is always worth at least as much as a European option. [1]  
[Total 9]

Part (i) was largely well answered, though common mistakes included using incorrect portfolios or trivial arguments that did not really constitute a proof.

Part (ii) was well answered by the majority of students, though few identified all five differences. Some students calculated theoretical prices using the Black-Scholes formula, but this was time-consuming and not necessary to identify the discrepancies. Many students failed to check the lower bound for the put option despite having proved it in part (i).

- Q7** (i) The market is arbitrage free if and only if there exists a probability measure under which discounted asset prices are martingales. [1]

In this case, the probability exists if and only if  $d < e^{r\Delta t} < u$ . [1]

Using the figures in the question,  $0.95 < e^{0.25 \times 0.05} < 1.1$  [1]

Alternatively we need  $0 < q < 1$  where  $q = \frac{e^r - d}{u - d}$  [1]

[Max 2]

(ii)

<i>Stock price tree</i>				
<i>Time</i>	<i>0</i>	<i>1</i>	<i>2</i>	<i>3</i>
	50.00	55.00	60.50	66.55
		47.50	52.25	57.48
			45.13	49.64
				42.87

[1 mark for the final prices; the bottom one is not necessary]

The price  $C_0$  of the option is computed via risk-neutral valuation; let  $\hat{p}$  denote the risk-neutral probability of an up movement, then:

$$\hat{p} = \{ \exp(0.05/12) - 0.95 \} / \{ 1.1 - 0.95 \} = 0.3612 \quad [1]$$

and

$$\begin{aligned} C_0 &= \exp(-rT) \sum_{k=0}^3 \binom{3}{k} \hat{p}^k (1-\hat{p})^{3-k} (S_0 u^k d^{3-k} - K)^+ \\ &= \exp(-rT) (\hat{p}^3 \times 16.55 + 3 \times \hat{p}^2 (1-\hat{p}) \times 7.48) = 2.62. \end{aligned} \quad [2]$$

Detailed workings:

Time	CALL			
	0	1	2	3
	<b>2.62</b>	5.56	10.71	16.55
		0.97	2.69	7.48
			0.00	0.00
				0.00

- (iii) The only relevant trajectories, given the barrier set at 48, are the up-up-up, up-up-down and up-down-up (i.e. the ones leading to a state in which the call option is in the money). [1]

The price by risk-neutral valuation, therefore, is 2.0.

[1 for computation]

[1 for values of probabilities of the relevant trajectories – see table below]

Workings as follows:

Split the stock tree				DOC barrier		$K$	50.0000	Exp. Value
0	1	2	3	PATH	min	$b$	48.0000	
						Payoff	Probs	
50.0000	55.0000	60.5000	66.5500	uuu	55.0000	16.5500	0.0471	0.7797
50.0000	55.0000	60.5000	57.4750	uud	55.0000	7.4750	0.0833	0.6229
50.0000	55.0000	52.2500	57.4750	udu	52.2500	7.4750	0.0833	0.6229
50.0000	55.0000	52.2500	49.6375	udd	49.6375	0.0000	0.1474	0.0000
50.0000	47.5000	52.2500	57.4750	duu	47.5000	0.0000	0.0833	0.0000
50.0000	47.5000	52.2500	49.6375	dud	47.5000	0.0000	0.1474	0.0000
50.0000	47.5000	45.1250	49.6375	ddu	45.1250	0.0000	0.1474	0.0000
50.0000	47.5000	45.1250	42.8688	ddd	42.8688	0.0000	0.2607	0.0000

$D_0$  2.0003

Alternative approach for the second and third marks:

Therefore option value = value of standard call option from part (ii) – the value of the payoff under the duu path [1]

$$= 2.62 - 7.475 \times .0833 \times \exp(-.05/4) = 2.00 \quad [1]$$

- (iv) If the barrier is at 40, the barrier option is equivalent to the vanilla call option in part (ii), so the price is 2.62 [1]  
as it is never knocked-out. [1]  
[Total 11]

This was well-answered with the majority of students scoring full marks. Some students simply defined arbitrage in part (i) rather than stating the conditions for the market to be arbitrage free.

Common mistakes included using an incorrect formula for the risk-neutral probability or failing to correctly apply the annual risk free rate to the required monthly time steps. Some students tried to price the option using the Black-Scholes formula without appreciating that the underlying Black-Scholes assumptions may not hold.

- Q8** (i) Delta =  $\Delta = \Phi(d_1)$  [1]  
using standard Black-Scholes notation. [1]
- (ii)  $\Delta = \Phi(d_1) = 0.6179$  means that  $d_1 = 0.3$  [1]  
So  $0.3 = (\log(40/45.91) + (0.02 + 0.5\sigma^2) \times 5) / \sigma\sqrt{5}$  [1]  
So  $-0.0378 - 0.6708\sigma + 2.5\sigma^2 = 0$  [½]  
Solving the quadratic gives  $\sigma = 0.3161$  or  $\sigma = -0.0478$  [1]  
Rejecting the negative root gives  $\sigma = 32\%$  (or may quote variance = 10%) [½]
- (iii) Under the risk-neutral probability measure  $Q$ , the fair price of the option is  $ce^{-rT} Q(S_I/S_0 < k_S) Q(R_I/R_0 < k_R)$  [2]
- (iv) Under the Black-Scholes model, if the stocks are perfectly correlated then  $S_I/S_0 = R_I/R_0$ . [1]  
So if  $k_S < k_R$  then the option only depends on stock  $S$  and has value  $ce^{-rT} Q(S_I/S_0 < k_S)$  [1]  
Similarly if  $k_S > k_R$  then the option only depends on stock  $R$  and has value  $ce^{-rT} Q(R_I/R_0 < k_R)$  [½]  
If  $k_S = k_R$  then the option can be defined in terms of the price of either stock as  $ce^{-rT} Q(S_I/S_0 < k_S) = ce^{-rT} Q(R_I/R_0 < k_S)$  [½]

So overall the option can be defined in terms of the lower of  $k_S$  and  $k_R$ , and either of the stock increases, i.e. has value

$$ce^{-rT} Q(R_I/R_0 < \min(k_S, k_R)) = ce^{-rT} Q(S_I/S_0 < \min(k_S, k_R)) \quad [1]$$

[Max 3]

$$\begin{aligned} \text{(v)} \quad & ce^{-rT} Q(S_T/S_0 < k_S) Q(R_T/R_0 < k_R) \\ &= 50e^{-0.02} Q(S_T/S_0 < 0.8) Q(R_T/R_0 < 0.6) \\ &= 50e^{-0.02} Q(S_1 < 0.8 \times 40) Q(R_1 < 0.6 \times 30) \quad [1] \\ &= 50e^{-0.02} (1 - \Phi((\log(S_1/0.8S_1) + (r - 0.5\sigma_S^2))/\sigma_S)) (1 - \Phi((\log(R_1/0.6R_1) \\ &\quad + (r - 0.5\sigma_R^2))/\sigma_R)) \quad [1] \\ &= 50e^{-0.02} (1 - \Phi((\log(1/0.8) + 0.02 - 0.5 \times 0.32^2)/0.32)) (1 - \Phi((\log(1/0.6) \\ &\quad + 0.02 - 0.5 \times 0.15^2)/\sqrt{0.15})) \quad [1/2] \\ &= 50e^{-0.02} (1 - (0.59982)) (1 - \Phi(1.1769)) \quad [1/2] \\ &= 50e^{-0.02} (1 - 0.7257) (1 - 0.88039) \quad [1] \\ &= \$1.61 \text{ (using } \sigma = 0.32, \text{ or } \$1.59 \text{ using an exact } \sigma = 0.3161) \quad [1] \end{aligned}$$

[Total 15]

Most students scored full marks in parts (i) and (ii). A number of students used trial and error to find the volatility instead of simply solving the quadratic equation.

Students struggled with parts (iv) and (v) with many only scoring low marks. Most students calculated part (v) using the distribution of the share price rather than  $\Phi(d_2)$  as an alternative solution. The best students showed their workings so that some marks could be awarded even if the final answer was not correct.

**Q9** (i) (a) It incorporates mean reversion [1/2]

It is time homogenous, i.e. the future dynamics of  $r(t)$  only depend upon the current value of  $r(t)$  rather than what the present time  $t$  actually is. [1]

It is arbitrage free. [1/2]

It allows negative interest rates. [1/2]

It is easy to implement since the characteristic functions of all related quantities are available. [1]  
 It has constant volatility [½]  
 [Max 2]

- (b) It incorporates mean reversion.... [½]  
 ... is arbitrage free... [½]  
 ... and time homogenous. [½]

Volatility depends on the level of the rates: it is high/low when rates are high/low. [1]

It does not allow negative interest rates. [½]

However it is more involving to implement than Vasicek model [½]  
 as it is linked to the chi-squared distribution. [½]

It is a one factor model [½]  
 [Max 2]

- (ii) Use Itô's lemma on the auxiliary process  $X_t = e^{at} r_t$  : [1]

$$\frac{dX}{dr} = e^{at}, \frac{d^2 X}{dr^2} = 0, \frac{dX}{dt} = ae^{at} r_t. \quad [1]$$

And so Itô gives:

$$dX_t = [e^{at} a(b - r_t)dt + ae^{at} r_t]dt + e^{at} \sigma dW_t. \quad [1]$$

And hence:

$$dX_t = de^{at} r_t = abe^{at} dt + \sigma e^{at} dW_t. \quad [½]$$

By direct integration from 0 to  $t$ , it follows that:

$$e^{at} r_t = r_0 + b(e^{at} - 1) + \sigma \int_0^t e^{as} dW_s \quad [1]$$

and hence, as required,  $r_t = r_0 e^{-at} + b(1 - e^{-at}) + \sigma \int_0^t e^{-a(t-s)} dW_s. \quad [½]$

[Max 4]

[Alternatively using an integrating factor of  $e^{at}$  gives the same result.]

- (iii) From Result 3.2 of the Core Reading,  $r_t$  follows a Normal distribution with mean: [1]

$$Er_t = r_0 e^{-at} + b(1 - e^{-at}) \quad [1]$$

and variance

$$\text{Var}(r_t) = E(r_t - Er_t)^2 = E\left[\left(\sigma \int_0^t e^{-a(t-s)} dW_s\right)^2\right] \quad [1]$$

$$= \sigma^2 \int_0^t e^{-2a(t-s)} ds \quad [1]$$

$$= \frac{\sigma^2}{2a} (1 - e^{-2at}). \quad [\frac{1}{2}]$$

[Max 3]  
[Total 11]

This was standard bookwork that was well-answered by the majority of students, though some students gave formulae for the models despite being told not to in the question.

Common mistakes in calculating the variance were to forget to square the integrand or change the differential from  $dW_s$  to  $ds$ .

- Q10** (i) Under the Merton model the value at redemption is  $\min(F(T), £100\text{m})$ , where  $F(t)$  is the gross value of the company at time  $t$ . [1]

Thus the value at time 0 is:

$$e^{-5r} E[\min(F(5), 100)] \quad [1]$$

$$= e^{-5r} E[F(5) - \max(F(5) - 100, 0)] \quad [1]$$

(where the expectation is under the risk-neutral measure).

Thus the value at time 0 equals  $F(0) - C$ . [1]

where  $C$  is a call option on the total value of the company with strike £100m and time to maturity five years. [1]  
[Max 4]

Alternatively:

In the Merton model, assuming no other claims on the company's assets:

$$F(0) = E(0) + B(0) \quad [1]$$

So:  $B(0) = F(0) - E(0)$

At time  $T$ , if:

- $F(T) \geq L$ , the shareholders will repay the debt and  $E(T) = F(T) - L$  [½]
- $F(T) < L$ , the shareholders will default and  $E(T) = 0$  [½]

i.e. the shares are equivalent to a European call option on the assets of the company with maturity  $T$  and a strike price equal to the par value of the debt,  $L$ , and a current price of  $C$ . [1]

So:  $B(0) = F(0) - C$  [1]

where  $C$  is a call option with a term of five years and a strike price of £100m. [1]  
[Max 4]

- (ii) Put-call parity gives a share price of £105.37 ( $= 27.55 + 100e^{-0.05} - 17.3$ ) [1]  
hence the total value of all shares in issue is £105.37m. [½]

The total bond value is therefore £200m – £105.37m = £94.63m. [1]

This is £94.63 per £100 nominal. [½]

- (iii) The sensitivity of the share price to a change in the company's gross value is  $dS_t/dV_t$ . [1]

If we regard  $S_t$  as a call option on the asset value  $V_t$  (current value £200m, strike £100m) then this is the Greek delta. [1]

From the Black-Scholes formula and the volatility above we find that  
 $d_1 = 2.145$  [1]  
so delta =  $\Phi(d_1) = 0.984$ , [1]  
so a £1m increase in asset value will give a £0.984m increase in the total share value [½]  
and a £0.016m increase in total bond value. [1]

This is an increase of £0.984 in the share price and an increase of £0.016 in the bond price per £100 nominal. [½]  
[Max 4]

- (iv) The current value of the company is well in excess of the nominal value of the bonds... [½]



... so bondholders are highly likely to receive the full nominal amount on maturity. [½]

The bond price is therefore not very sensitive to small changes in the value of the company... [½]

... and the share price moves almost in line with the value of the company. [½]

- (v) If the company value was lower then the value received by the bondholders at maturity would be more likely to fall short of the nominal amount. [½]

So any change in company value would impact the bond price more (and hence impact the share price less). [½]

[Total 14]

Part (i) was standard bookwork with most students scoring well.

Students struggled with parts (ii) and (iii), but many picked up some marks by showing all their working.

## **END OF EXAMINERS' REPORT**