INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

30 September 2014 (am)

Subject CT8 – Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

CT8 exam paper September 2014

Please note, regarding Question 9: The five marks for part (v) of Question 9 should have been split as two marks for part (iv) and three marks for part (v).

1 Outline consumer choice theory. [9]

2 An investor wishes to allocate her capital between a service company S and a manufacturing company M. The investor believes that returns on shares in S have mean 10% and variance 16%% while returns on shares in M have mean 8% and variance 25%%. The correlation between the two companies is 0.3.

Assume the investor chooses their investments according to mean-variance portfolio theory.

(i) Explain which company's share she would prefer. [2]

Assume the investor's preferences are described by a standard quadratic utility function.

(ii) State which assumption of the mean-variance portfolio theory can be relaxed. [1]

(iii) Calculate the expected return and the standard deviation of a portfolio which is invested three quarters in S and one quarter in M. [2]

(iv) Calculate the minimum variance portfolio. [2]

A new study suggests that in the future, S will make more sales to M, when M is delivering strong profits.

(v) Describe the effect this will have on portfolios composed of M and S, including the minimum-variance portfolio. [3]
[Total 10]

3 Let $(Z_t\,; t \geq 0)$ be a standard Brownian motion.

(i) Calculate the probability of the event that $Z_1 > 0$ and $Z_2 < 0$. [5]

Hint: Write $Z_1 = W$, $Z_2 = W + X$, where W and X are both independent, identically distributed $N(0,1)$ random variables.

(ii) State the model for geometric Brownian motion. [1]

(iii) Explain why the standard Brownian motion is less suitable than the geometric Brownian motion as a model of stock prices. [2]
[Total 8]

4 A non-dividend paying stock currently trades at \$65. Every two years the stock price either increases by a multiplicative factor 1.3, or decreases by a multiplicative factor 0.8. The effective risk-free rate is 4% p.a.

Calculate the price of an American put option written on the stock with strike price \$70 and maturity four years, using a two period binomial model. [9]

5 Let S be the price of a non-dividend paying share, and let r be the continuously compounded risk-free rate.

(i) Derive the forward price at time zero for the forward contract on S with maturity T. [4]

Assume that, at time zero, the share price is 500, and that the forward contract has maturity two years. The share pays a dividend of 5% of the share price every six months with the next dividend due in two months, and the continuously compounded risk-free rate is 3% p.a.

(ii) Determine the forward price for this contract. [4]

(iii) Comment on whether the high dividend yield relative to the risk-free rate offers an arbitrage opportunity. [2]
[Total 10]

6 Consider a one-factor model for the short- rate r.

(i) Explain why a tradeable asset has to be introduced in order to build an arbitrage-free model. [2]

Consider a specific bond with maturity T_1, suppose its price satisfies the following Stochastic Differential Equation (SDE) under the real-world probability measure P:

$$dB(t,T_1) = B(t,T_1)[m(t,T_1)dt + S(t,T_1)dW(t)]$$

where W is a standard Brownian Motion, $m(t, T_1)$ is the drift and $S(t, T_1)$ is the volatility.

(ii) (a) State the market price of risk.

(b) Explain what it represents.

(c) Show how it can be used in transforming the SDE above from the real-world probability measure P to a risk-neutral probability measure Q.
[4]

(iii) Show how the above results would be used in calculating zero-coupon bond prices. [3]

(iv) Explain how this is typically done in practice. [2]
[Total 11]

7 Let B be a standard Brownian motion.

(i) Derive the probability density function of $\max_{0 \le s \le t}(B_s + \mu s)$, where μ is a constant, using the formula in section 7.2 of the Actuarial Formulae and Tables. [3]

In a Black-Scholes market, let S be the stock price.

(ii) Give the expression for the fair price at time t of a derivative written on S paying an amount D_T at time T, defining any terms you use. [3]

Suppose that S has an initial price of S_0 = £1.20 and a volatility σ = 30% p.a. and that the continuously compounded risk-free rate is r = 3% p.a.

(iii) Calculate the fair-price at time zero of the derivative paying £10 at time $T = 2$ if and only if $\max_{0 \le s \le T}(B_s + \mu s) >$ £1.44. [4]

[Total 10]

8 In a Black-Scholes model, the delta of a call option is $\Delta = \Phi(d_1)$.

(i) Define delta. [1]

Suppose that the stock price at time zero is S_0 = \$100, the continuously compounded risk-free rate is 3% and that a European call option written on S with strike price \$109.42 and maturity $t = 1$ year has a delta of $\Delta = 0.42074$.

(ii) Find the implied volatility of the stock to the nearest 1%. [3]

An exotic option written on S with strike prices K_1 and K_2 and exercise times τ and T is defined as follows:

- The option *may be exercised at time* τ in which case the holder receives \$100 if and only if the price of the underlying, S_τ is at least K_1.

- *If the option is not exercised at time* τ, then the holder will receive an amount c *if and only if* the price at expiry T, S_T, satisfies $S_T / S_\tau \ge K_2$.

(iii) Explain why, if $c \le$ \$100, the option will always be exercised at time τ when S_τ is at least K_1. [2]

(iv) Give a formula for the value of the option just after the first exercise time τ (i.e. just after the first exercise option has expired). [2]

(v) Explain why this value does not depend on the stock price at time τ. [2]

Suppose that K_1 = \$10, $K_2 = e^{-0.09}$, τ = 1 year, T = 2 years and c = \$200.

(vi) Determine the fair price of the exotic option just after time one and hence at time one and at time zero. [3]

[Total 13]

9 A company has issued a loan in the form of a zero-coupon bond which redeems in one year from now. The bond is priced at £92.78 per £100 nominal and the recovery rate in the event of a default is assumed to be 50%. The continuously compounded risk-free rate for one year is 3% p.a.

(i) Write down the formula for the bond price under the two-state model, defining all the terms used. [2]

(ii) Calculate the risk-neutral probability that the bond defaults. [3]

Assume that the Merton model holds and that the annual volatility of the company's total assets is 13%.

(iii) Give an expression for the risk-neutral probability that the company defaults, defining any other terms you use. [3]

(iv) Calculate the ratio of nominal loan to total asset value, assuming that the risk-neutral default probability is the same as calculated in (ii).

(v) Calculate the ratio of loan value to total asset value and hence determine the percentage of total assets represented by equity value at time zero. [5]
[Total 13]

10 (i) (a) Describe the lognormal model for securities prices including the definition of the parameters used.

(b) State the corresponding mean and variance for the security price. [4]

A security price S is assumed to follow a lognormal model. The price now is S_0 = €200. The expected price at time 1 (in years) is $E(S_1) = 200e^{0.4}$ and the variance is $\text{Var}(S_1) = 40000e^{-0.4}$.

(ii) Determine the parameter values for the corresponding lognormal model. [3]
[Total 7]

END OF PAPER