

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

4 October 2012 (am)

Subject CT8 – Financial Economics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

<p><i>In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.</i></p>
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- 1** The effective risk free interest rate is 4% p.a. Company AA has issued a one year zero coupon bond with a yield of 6% p.a. and Company BB has issued a one year zero coupon bond with a yield of 8% p.a. All rates are annually compounded.

Recovery rates on the bonds in the event of default are zero and there are no frictional costs.

- (i) Calculate the risk neutral implied default probability of each bond. [2]
 - (ii) Calculate the 95% VaR and 95% TailVaR at the end of the year for the following portfolios, assuming defaults by AA and BB are independent:
 - (a) £100 invested in AA bonds.
 - (b) £100 invested in BB bonds.
 - (c) £50 invested in AA bonds and £50 invested in BB bonds.[6]
 - (iii) Comment on your answers to (ii). [4]
- [Total 12]

- 2** A non-dividend-paying stock has a current price of $S_0 = 400p$. Over each of the next three years its price could increase by 20% (so $S_{t+1} = 1.2S_t$), or decrease by 20% (so $S_{t+1} = S_t / 1.2$). The continuously compounded risk-free rate is 6% p.a. The stock price move in each year is independent of the move in other years.

A non-standard derivative pays off $\sqrt{S_3}$ after three years, provided that at some point over three years the stock price has moved up in one year and then immediately down in the following year. Otherwise, the derivative pays zero.

Calculate the current price of this non-standard derivative. [8]

- 3** (i) State the three main assumptions of Modern Portfolio Theory. [3]

Assume Modern Portfolio Theory holds true.

- (ii) Write down equations for the expected return, E and variance, V of a portfolio of N securities, defining any notation used. [3]
 - (iii) Describe how an efficient portfolio can be found. [4]
- [Total 10]

- 4** A non-dividend paying stock is currently priced at $S_0 = £80$. Over each of the next two three-month periods it is expected to go up by 6% or down by 5% on each period. The continuously compounded risk free interest rate is 5% p.a.
- (i) Calculate the value of a six-month European call option with a strike price of £82. [5]
 - (ii) Calculate the value of a six-month European put option with a strike price of £82.
 - (a) Directly. [2]
 - (b) Using put-call parity. [3]
 - (iii) Explain whether, if the put option were American, it would ever be optimal to exercise early. [4]
- [Total 14]
- 5** State eight desirable characteristics of a term-structure model. [8]
- 6** (i) State the Stochastic Differential Equations for the short rate $r(t)$ in the Vasicek model and the Cox-Ingersoll-Ross model. [2]
- (ii) Explain the impact of a movement in the short rate on the volatility term in both models. [2]
- [Total 4]
- 7** A three-month European call option on a non-dividend paying stock in Universal Widget Inc with a strike price of \$1.30 has current price of \$0.8557.
- The continuously compounded risk free rate is 0.5% p.a. The current stock price is \$1.20. Assume all the Black-Scholes assumptions hold.
- (i) Calculate the implied volatility for the underlying stock to within 1% p.a. [2]
- It is known that in three months Universal Widget Inc will embark on a major restructuring. It is anticipated that this will double the volatility of the stock price thereafter.
- (ii) Write down a formula in terms of the underlying Brownian motion, Z , for the stock price in three months' and in six months' time. [3]
 - (iii) Derive the corresponding price of a six month European put on the Universal Widget Inc stock with strike price \$1.20 [6]
- [Total 11]

- 8** (i) Describe the Merton model for pricing a defaultable bond. [4]

A very highly geared company, Risky plc, has issued zero coupon bonds payable in three year time for a nominal amount of £3,200m.

A Black-Scholes model for the value of the company is adopted.

- (ii) Derive an expression for the value of the debt. [3]

The current gross value of the company is £6,979m. The continuously compounded risk-free interest rate is 2% p.a. and the price of £100 nominal of the bond is £92.603.

An insurance company is offering default insurance on Risky plc. They will charge a premium of £55,000 for a contract which pays £1m at the end of three years if Risky plc defaults.

- (iii) Discuss whether there is an arbitrage opportunity. [4]
[Total 11]

- 9** Consider a market where there are two risky assets A and B and a risk free asset. Both risky assets have the same market capitalisation.

Assume that all the assumptions of the CAPM hold.

- (i) State the composition of the market portfolio. [1]
(ii) Derive the expressions for the variance of the market portfolio and for the beta of each asset, in terms of the variance of each asset and of their covariance. [4]

Assume now that the risk-free rate is $r_f = 10\%$, the expected return of the market portfolio is $r_M = 18\%$, the variance of asset A is 4%, the variance of asset B is 2% and their covariance is 1%.

- (iii) Derive the value for the expected return on asset A and asset B. [4]

An investor wants an expected return of 20%.

- (iv) Calculate the composition of the corresponding portfolio. [2]
(v) Derive the corresponding standard deviation using the Capital Market Line. [2]
[Total 13]

- 10** Let A and B be two investment portfolios taking values in $[a,b]$ with cumulative probability distribution functions of returns F_A and F_B respectively, and let the investor's smooth utility function be U .
- (i) Write down the equation that the function U satisfies if the investor prefers more to less. [1]
 - (ii) Explain what it would mean for portfolio A to first order stochastically dominate portfolio B . [2]
 - (iii) Prove, by considering the expected utility of investments in either A or B , that if portfolio A first order stochastically dominates portfolio B , then the investor prefers A to B . [6]
- [Total 9]

END OF PAPER