INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

24 April 2018 (am)

Subject CT8 – Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 11 questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

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1 A horse racing fan assesses her utility of wealth using the utility function

$$U(w) = 2(w^{0.5}-1).$$

(i) Prove algebraically that the horse racing fan is:
(a) non-satiated
(b) risk averse. [2]

(ii) Prove that the horse racing fan exhibits constant relative risk aversion. [2]

The horse racing fan is attending a race and she intends to place bets on two horses. The table below shows the pay-out per £1 bet on each of these horses if it wins the race, and the investor's estimated probabilities of each horse winning the race. The pay-out is the total paid and is not in addition to the bet being returned.

Horse	*Winning pay-out per £1 bet*	*Probability of winning*
A	£1.69	60%
B	£6.25	10%

The horse racing fan has total wealth of £1,000 and she will bet all of her wealth on this race. Negative bets are not allowed.

(iii) Calculate the amount she should bet on each horse to maximise her expected utility of wealth. [7]

(iv) Calculate the expected wealth resulting from the bets in part (iii). [1]

(v) Explain how and why this differs from the utility of the horse racing fan's initial wealth. [2]
[Total 14]

2 Describe the difficulties in estimating parameters for asset pricing models. [5]

3 The value of an investment asset follows the equation $A(t) = \exp(B_t)$, where B_t follows standard Brownian motion.

(i) State the five defining properties that apply to B_t as a standard Brownian motion. [5]

An actuarial student invests \$1,000 in the asset at time 0.

(ii) Calculate the expected value of this investment at time 5. [2]

(iii) Calculate the probability that the value of the student's investment is less than \$10,000 at time 5. [2]
[Total 9]

4 Mr and Mrs Jones both wish to buy stocks in Widgets Inc. They don't have enough money right now, so they are considering buying either forwards or options on the stocks, both with a term of 4 years.

The stock price at time 0 is £10 with standard deviation of 12% per annum. The stock does not pay any dividend. The continuously compounded risk-free rate of interest is 5% per annum.

(i) Calculate the 4 year forward price on one stock. [1]

(ii) Calculate the price at time 0 of a 4 year call option on one stock with a strike price of £12.21. [3]

Mrs Jones enters into one forward contract, while Mr Jones buys one call option. At time 4 the stock is worth £12.

(iii) Calculate the accumulated profit or loss at time 4 for Mrs Jones. [1]

(iv) Calculate the accumulated profit or loss at time 4 for Mr Jones. [2]

(v) Explain why Mr Jones makes a loss despite having an option that does not force him to buy the stock. [2]

(vi) Calculate the range of stock prices at time 4 which would leave Mr Jones better off than Mrs Jones. [3]

[Total 12]

5 Consider a zero-coupon bond B_t with three years to maturity. The bond pays \$100 at maturity if it has not defaulted, or \$30 if it has defaulted. The continuously compounded risk-free rate is r. In a two-state model the default intensity is λ under the probability measure P, and the bond price is:

$B_t = 30e^{-r(3-t)}$ if the bond has already defaulted by time t, or
$B_t = e^{-r(3-t)}(30(1 - e^{-\lambda(3-t)}) + 100e^{-\lambda(3-t)})$ if the bond has not yet defaulted by time t

(i) Show that P is an equivalent martingale measure. [4]

A derivative pays \$35 at time 3 if the bond has defaulted and \$0 otherwise.

(ii) (a) Determine a constant portfolio containing the bond and cash which replicates this derivative.
(b) Derive an expression for the arbitrage-free price for the derivative at time 0 in terms of r and λ. [5]

(iii) Explain how your answers to parts (i) and (ii) are related through the value of the portfolio in part (ii) also being a martingale. [3]

[Total 12]

6 Consider a three-period binomial tree model for the non-dividend paying stock price process S_t, in which the stock price either rises by u% or falls by d% each period till maturity. Let r denote the continuously compounded risk-free rate of interest.

(i) State the conditions under which this market is arbitrage free. [1]

Let S_0 = £95 and assume this price either rises or falls by 20% each year for the next three years. Assume also that the risk-free rate is 5% per annum continuously compounded.

(ii) Calculate the price of a vanilla European put option with maturity in three years and strike price 110. [4]

Assume a change in market conditions such that the same share price now either rises or falls by 5% each year for the next three years.

(iii) Determine how this change would impact on the option price. [2]
[Total 7]

7 (i) Define a complete market. [1]

The price process of a traded security satisfies the following stochastic differential equation

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

where W_t is a Brownian motion under the real-world probability measure P. Let $r > 0$ be the continuously compounded risk-free rate of interest, with $r \neq \mu$.

(ii) Show that the discounted stock price $e^{-rt}S_t$ is not a martingale under the real-world probability measure P. [3]

(iii) Demonstrate how the discounted asset price $e^{-rt}S_t$ can be a martingale under an equivalent martingale measure Q. [3]
[Total 7]

8 The current price of a non-dividend paying stock is £65 and its volatility is 25% per annum. The continuously compounded risk-free interest rate is 2% per annum.

Consider a European call option on this share with strike price £55 and expiry date in six months' time. Assume that the Black-Scholes model applies.

(i) Calculate the price of the call option. [4]

(ii) Define algebraically the delta of the call option. [1]

(iii) Calculate the value of the delta of the call option. [2]

(iv) Calculate the value of the delta of a European put option written on the same underlying, with the same strike and maturity as above. [2]

[Total 9]

9 Consider a market with the following bonds in issue.

Principal value	*Expire (years) T*	*Coupon (annual*)*	*Price*	*Zero rate R(0, T)*	*Forward rate F(0, S, T)*
100	0.25	0	97.5	(a)	
100	0.5	0	94.9	(b)	F(0, 0.25, 0.5) = 10.81%
100	1	0	90.0	10.54%	F(0, 0.5, 1) = (d)
100	1.5	8%	(c)	10.68%	F(0, 1, 1.5) = (e)
(* half the stated coupon is paid every 6 months)					

(i) Calculate the values of (a), (b), (c), (d), (e) in the table above. [5]

(ii) Write down the stochastic differential equations of two standard models for the short rate of interest. [2]

[Total 7]

10 Consider a call option on a non-dividend paying stock S when the stock price is £15, the exercise price, K, is £12, the continuously compounded risk-free rate of interest is 2% per annum, the volatility is 20% per annum and the time to maturity is three months.

(i) Calculate the price of the option using the Black-Scholes model. [4]

(ii) Determine the (risk neutral) probability of the option expiring in the money. [1]

A special option called a "digital cash-or-nothing" option has a payoff in three months' time of:

$$\begin{array}{ll} 1 & \text{if } S_T > K \\ 0 & \text{otherwise} \end{array}$$

(iii) Calculate the price of the digital option. [2]

(iv) Describe the limitations of the Black-Scholes model. [2]

[Total 9]

11 Consider a market in which the Capital Asset Pricing Model (CAPM) holds.

(i) List the assumptions, additional to those used in modern portfolio theory, of the CAPM. [2]

(ii) Prove that the market portfolio has unit beta. [2]

In the same market as above, there are two assets with the following attributes.

Rate of return (per annum)					*Variance/Covariance Matrix*		
State	*Probability*	*Asset 1*	*Asset 2*			*Asset 1*	*Asset 2*
1	0.2	5.00%	11.00%		Asset 1	0.00068	0.00102
2	0.3	10.00%	15.00%		Asset 2	0.00102	0.00181
3	0.1	8.00%	12.00%				
4	0.4	4.00%	5.00%				
Market capitalisation		40,000	60,000				

(iii) Calculate the beta of each security. [3]

(iv) Determine the value of the risk-free rate of interest which is consistent with the results obtained in part (iii), under the assumption that the CAPM holds. [2]
[Total 9]

END OF PAPER