INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

25 April 2012 (am)

Subject CT8 – Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 (i) State the assumptions underlying the Black-Scholes market. [3]

(ii) State the defining characteristics of Brownian Motion. [2]

(iii) Explain what an examination of past option prices tell us about the assumptions in (i) and (ii). [4]

[Total 9]

2 In a market where the CAPM holds there are five risky assets with the following attributes per year.

Asset number	*1*	*2*	*3*	*4*	*5*
Expected return	6%	5%	8%	13%	11%
Market capitalisation (in $)	2.6m	3.9m	5.2m		1.3m
Beta				1.5	

The risk-free rate is r = 1% p.a.

(i) Calculate the expected return on the market portfolio. [1]

(ii) Deduce the market capitalisation of asset 4 and the betas of all the other assets. [3]

(iii) Calculate the beta of a portfolio P which is equally weighted in the five assets and the risk-free asset. [1]

(iv) Explain whether or not this portfolio P lies on the Capital Market Line. [2]

[Total 7]

3 A non-dividend paying stock has a current price of $S_0 = 150\text{p}$ and trades in a market which is arbitrage free and has a constant effective risk-free rate of interest r. After one year the price of the stock could increase to 280p, or decrease to 120p. Over the following year the price could increase from 280p either to 420p or to 322p. If the stock price had decreased to 120p, then over the following year it could increase to 168p or decrease to 112p.

(i) Determine the range of values that the annual risk-free rate of interest could take. [3]

Assume that r takes the value 20% p.a.

(ii) Calculate the price at time 0 of a non-standard derivative which pays off $(S_2 - 100)^2$ at the end of two years. [6]

[Total 9]

4 Let c be the price of a four- month European call option on a dividend paying share. Assume the strike price is \$30, the underlying is currently valued at \$28 and a dividend of \$0.50 is expected in 2 months. The continuously compounded risk-free rate is constant and equal to 5% p.a.

(i) Derive upper and lower bounds on the price c of this call option, taking into account the dividend. [5]

The price of a put option with the same underlying, the same strike price and the same maturity is \$3.

(ii) Calculate the price c of the call option exactly. [5]
[Total 10]

5 Let X be a random variable denoting the rate of return on the fund ABC. The distribution of X is $N\left(\mu,\sigma^2\right)$.

(i) Define $VaR_\alpha(X)$ with $\alpha \in [0,1]$. [1]

(ii) Show that:

$$VaR_\alpha = -\left(\mu + \sigma\Phi^{-1}(\alpha)\right)$$

where Φ denotes the cumulative Normal distribution function.

(Hint: Consider the probability that X is less than VaR_α). [4]

(iii) Derive an expression for $TailVaR_\alpha\left(X\right)$ given that:

$$TailVaR_\alpha = \frac{1}{\alpha}\mathbb{E}\left(X|X < VaR_\alpha\right).$$ [4]

An investor holds £350m invested in ABC, the expected return on the fund is 10% and the standard deviation of that return is 25%.

(iv) Calculate the VaR and TailVaR of this investment when $\alpha = 0.01$. [2]
[Total 11]

6 (i) Write down a stochastic differential equation for the short rate $r(t)$ for the Vasicek model. [1]

(ii) State the type of process of which the Vasicek model is a particular example. [1]

(iii) Solve the stochastic differential equation in (i). [5]

(iv) State the distribution of $r(t)$ for t given. [1]

(v) Derive the expected value and the second moment of $r(t)$ for t given. [3]

(vi) Outline the main drawback of the Vasicek model. [1]
[Total 12]

7 The remuneration package for the CEO of a quoted company in the tax year 2012/13 includes a bonus proportional to the excess of the share price over 100p at 5 April 2013 at a rate of £50,000 per penny.

The company's Finance Director wants to hedge the cost of this bonus as at 6 April 2012. The share price at that date is $S_0 = 90$p.

The continuously compounded interest rate is 1% p.a. and the share price volatility is 18% p.a.

(i) Explain the bonus in terms of an option on the share price. [2]

(ii) Calculate the hedging portfolio of shares and cash the Finance Director should hold to hedge the liability for the CEO's bonus. [3]

The CEO will be liable to tax at 80% on the excess over £1m of this bonus and at 40% up to £1m. The Finance Director realises that if she purchases for the CEO a portfolio of a call options with a strike of 100p and $-b$ call options with a strike of 120p and gives this portfolio to the CEO on 6 April 2012 then the proceeds will be liable for tax at only 40%.

(iii) (a) Calculate the values of a and b which ensure that the CEO would receive the same net bonus. [5]

(Hint: Consider the different situations depending on whether one or both options are exercised).

(b) Calculate the amount this transaction will save the company. [2]
[Total 12]

8 In a Black-Scholes market, a special option with strike price a and maturity T on an underlying (non-dividend bearing) stock with price process S, pays 100p at time T if and only if the stock price at time T, S_T, is more than a. Let $I_a(x)$ denote the function which takes the value 1 if x>a and 0 otherwise.

(i) Write down a formula, in terms of expectation, I_a, and the underlying stock price, for the price $D_0(a)$ at time 0 of this security, specifying any other notation that you use. [2]

(ii) Write down an equation connecting the price, $C_0(K)$ of the call option on S with maturity T and strike price K, to the price of the special option on S, using the fact that $\max(x-k,0) = \int_k^\infty I_a(x)\,da$. [2]

(iii) Find a formula for the price of the special option on S, by differentiating the Black-Scholes formula with respect to K. [3]

Suppose $S_0 = 110$p, the continuously compounded risk-free rate is 1% p.a., and the volatility of S is 20% p.a.

(iv) Calculate the price for a derivative security which pays $S_1 - 20$p if $S_1 > 120$p and 0 otherwise. [3]
[Total 10]

9 (i) Describe the two state model for credit ratings and its generalisation to the Jarrow-Lando-Turnbull model. [4]

Companies A and B are joint investors in a high risk project to build a new space plane. Each of the two companies' zero-coupon bonds are modelled according to a two-state model. Company A's bonds have a recovery rate of $\delta_A = 60\%$, while Company B's have a recovery rate of $\delta_B = 50\%$. All bonds mature in nine months. Company A's bonds have a current price of \$82 per \$100 nominal, Company B's bonds have a current price of \$79 per \$100 nominal. The continuously compounded risk-free rate is 1.5% p.a.

(ii) Calculate the implied risk-neutral default intensities λ_A and λ_B, assuming that they are constant. [4]

A competitor to the space plane project now starts to sell a derivative security which pays \$100,000 at the end of nine months if and only if both companies default within the nine months (a double-default). The current price for the derivative is \$7900.

(iii) Calculate the implied risk neutral probability of a double-default and the corresponding constant rate. [2]

(iv) Calculate the maximum price for this derivative, by considering the maximum possible double-default rate.
[4]
[Total 14]

10 (i) Describe Arbitrage Pricing Theory (APT) in the context of factor models. [4]

(ii) State the two major weaknesses of APT. [2]
[Total 6]

END OF PAPER