INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

26 September 2018 (pm)

Subject CT8 – Financial Economics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 11 questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Describe the key findings in behavioural finance. [10]

2 An investor has taken out a \$100,000 loan at a 10% per annum rate of interest, annually compounded. The investor uses the loan to buy a portfolio of stocks whose value follows a lognormal distribution, with parameters $\mu = 12\%$ per annum and $\sigma^2 = 25\%$ per annum.

The investor plans to sell the stocks and repay the loan after five years.

(i) Calculate the mean and the variance of the lognormal distribution. [1]

(ii) Calculate the probability that the investor will have enough money to repay the loan plus interest. [2]

After five years the stocks are only worth \$120,000 so the investor cannot afford to repay the loan plus interest.

(iii) Calculate the one-year 95% value at risk of the investor's stock portfolio from time $t = 5$ to time $t = 6$. [2]

The annual standard deviation of the investor's stock portfolio at time $t = 5$ is \$81,708.

The bank agrees to continue the loan for another five years, as long as the investor can prove that the annual standard deviation of his portfolio is no higher than \$40,000 at time $t = 5$.

(iv) Calculate the proportion of the investor's stock portfolio he would have to sell in order to bring the value at risk down to a level acceptable to the bank. [1]

The bank also offers a cash deposit account returning a 6% per annum rate of interest, annually compounded.

The investor sells the proportion of the stock portfolio in part (iv) and invests the funds in the cash deposit account. The bank therefore continues the loan for another five years.

(v) Calculate the probability that the investor's stocks and cash deposit combined are sufficient to repay the loan plus interest at time 10. [3]

(vi) (a) Comment on this result.
(b) Propose an alternative course of action for the investor. [2]
[Total 11]

3 In a market in which the Arbitrage Pricing Theory (APT) model holds, the expected return is given by:

$$E[R_i] = \lambda_0 + \lambda_1 b_{i,1} + \lambda_2 b_{i,2} + \ldots$$

(i) Define all the terms in the equation. [2]

Assume the risk-free rate $r_f = 0.04$. Consider two well diversified portfolios P_i with the following features in a two factor model:

	P1	*P2*
$E\ R_i$	15.50%	11.95%
$b_{i,1}$	(a)	(b)
$b_{i,2}$	1.5	0.7

(ii) Determine the values (a) and (b) for $\lambda_1 = 0.05$ and $\lambda_2 = 0.06$. [3]
[Total 5]

4 An investor has £100 and is considering investing in two different stocks. The prices of both stocks are assumed to follow the lognormal model with the parameters below.

Stock	*Current price*	*Drift μ*	*Volatility σ*
A	£5	5%	20%
B	£5	8%	30%

(i) Calculate the expected value at time 3 of £100 invested in:
(a) stock A
(b) stock B. [2]

(ii) Calculate the standard deviation at time 3 of £100 invested in:
(a) stock A
(b) stock B. [4]

The investor decides to invest £50 in each stock.

(iii) Calculate the expected value of the investor's portfolio at time 3. [1]

The correlation of the two stocks is 0.3.

(iv) Calculate the standard deviation of the value of the investor's portfolio at time 3. [3]

(v) Comment on the expected return and standard deviation of the portfolio compared to investing the whole £100 in one stock. [4]
[Total 14]

5 The Ornstein-Uhlenbeck process is the solution to the equation $dX_t = -\gamma X_t dt + \sigma dB_t$ where γ and σ are positive parameters.

Derive the solution for X_t. [8]

6 Consider a call option c_t and a put option p_t written on a non-dividend paying stock S_t.

(i) Prove the put-call parity relationship by constructing two portfolios that produce the same value at maturity. [4]

A stock market includes four options set out below. All the options are for a term of 10 years and relate to a single non-dividend paying stock, currently priced at \$5. The continuously compounded risk-free rate is 3% per annum.

	Type	*Strike price*	*Option price*
Option A	European Call	\$8	\$0.32
Option B	European Put	\$8	?
Option C	European Put	\$10	?
Option D	American Put	\$10	?

(ii) Calculate the price of Option B. [2]

(iii) Determine lower and upper bounds for the price of option C. [2]

(iv) Determine lower and upper bounds for the price of option D. [2]
[Total 10]

7 A company is currently financed entirely by equity with 100,000 shares in issue and no debt. The current share price is \$1. The company has total assets of \$100,000 with volatility of 15% per annum.

The company is considering raising \$250,000 by issuing zero-coupon debt with a five-year maturity date. The continuously compounded risk-free rate of interest is 3% per annum.

The company intends to set the redemption value of the debt such that the share price will remain unchanged under the Merton model.

(i) Give the value of the company's assets immediately after issuing the debt. [1]

(ii) Calculate the redemption value of the debt using the Merton model. [5]

(iii) Calculate the credit spread on the debt. [2]

One year later, the company is struggling. The share price has fallen to \$0.50 and the current value of the debt has fallen to \$50 per \$100 of redemption value.

(iv) Calculate the proportionate fall in the value of:
  (a) the equity.
  (b) the debt. [2]

(v) Suggest why the value of the equity has fallen by proportionately more than the fall in the value of the debt. [3]
[Total 13]

8 Consider a binomial tree model for the non-dividend paying stock with price S_t. Assume this price either rises by 30% or falls by 20% each quarter (3 months) for the next three quarters. Assume also that the risk-free rate is 2% per annum continuously compounded. Let S_0 = £60.

(i) Calculate the price of a vanilla European call option with maturity in nine months' time and a strike price of £55. [3]

(ii) Calculate the price of a vanilla European put option with the same maturity and strike price as the contract in part (i). [1]

Assume the investor has a portfolio formed by a short position in the call option given in part (i) and a long position in the put option given in part (ii).

(iii) Determine how the value of the portfolio would differ if the possible change in the stock price was a fall of 30% instead of 20%. [3]
[Total 7]

9 The price process of a non-dividend paying stock S_t satisfies the following stochastic differential equation

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

where W_t is a Brownian motion under the real-world probability measure P. Let $V(t)$ be the value at t of a self-financing portfolio, consisting of Φ_t stocks and Ψ_t cash bond.

(i) Show that $d(e^{-rt}V(t)) = \Phi_t d(e^{-rt}S_t)$. [3]

(ii) Determine the conditions under which the discounted value $e^{-rt}V(t)$ is a martingale. [3]

[Total 6]

10 (i) State the main assumptions underpinning the Black-Scholes model. [3]

Consider a put option on a non-dividend paying stock when the stock price is £8, the exercise price is £9, the continuously compounded risk-free rate of interest is 2% per annum, the volatility is 20% per annum. and the time to maturity is three months.

(ii) Calculate the price of the option using the Black-Scholes model. [4]

(iii) Discuss how the price of the contract in part (ii) would change if the rate of interest increases. (There is no need to carry out further calculations.) [2]

[Total 9]

11 Consider a market in which the Capital Asset Pricing Model (CAPM) holds.

(i) Write down the equation of the Security Market Line, defining all the notation you use. [2]

In this market, the risk-free rate of interest is 9.44% per annum. There are only two risky assets in the market with the following attributes.

Rate of return (per annum)					*Variance/Covariance Matrix*		
State	*Probability*	*Asset 1*	*Asset 2*			*Asset 1*	*Asset 2*
1	0.2	10.00%	11.00%		Asset 1	0.00142	0.00379
2	0.3	15.00%	30.00%		Asset 2	0.00379	0.01146
3	0.1	18.00%	25.00%				
4	0.4	20.00%	40.00%				

(ii) Determine the weight of each asset in the market portfolio to be consistent with $\beta_1 = 0.46$, $\beta_2 = 1.36$. [3]

(iii) Calculate the Market Price of Risk. [2]
[Total 7]

END OF PAPER